Exemplar - 10.13.6.16

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Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is 6 , 7 , 12

Solution:

Product: Let the random variables be defined as:

Variable	Values	Description
X	$1 \leq X \leq 6$	First Dice Roll
Y	$1 \leq Y \leq 6$	Second Dice Roll

Assuming both dice rolls and equally likely,:

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The probability mass function is:

$$p_{XY}(k) = \Pr(XY = k) \quad (3)$$

$$= \Pr\left(X = \frac{k}{Y}\right) \quad (4)$$

$$= E\left(p_X\left(\frac{k}{Y}\right)\right) \quad (5)$$

$$= \sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) p_Y(i) \quad (6)$$

$$= \frac{1}{6} \sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) \quad (7)$$

$$= \frac{1}{6} \sum_{i=1}^{6} \frac{[k \bmod i = 0]}{6} \left[\frac{k}{i} \leq 6\right] \quad (8)$$

$$= \frac{1}{36} \sum_{i=1}^{6} [k \bmod i = 0] \left[\frac{k}{i} \leq 6\right] \quad (9)$$

Thus, the probability of getting product 6 is:

$$\Pr(XY = 6) = \left(\frac{1}{36} \sum_{i=1}^{6} [6 \bmod i = 0] \left[\frac{6}{i} \leq 6\right]\right) \quad (10)$$

$$= \frac{1}{36} (1 + 1 + 1 + 1 + 0 + 0) \quad (11)$$

$$= \frac{4}{36} \quad (12)$$

$$= \frac{1}{9} \quad (13)$$

Probability of getting product 7 is:

$$\Pr(XY = 7) = \left(\frac{1}{36} \sum_{i=1}^{6} [7 \bmod i = 0] \left[\frac{7}{i} \leq 6\right]\right) \quad (14)$$

$$= \frac{1}{36} (0 + 0 + 0 + 0 + 0 + 0) \quad (15)$$

$$= \frac{0}{36} \quad (16)$$

$$= 0 \quad (17)$$

Probability of getting product 12 is:

$$\Pr(XY = 12) = \left(\frac{1}{36} \sum_{i=1}^{6} [12 \bmod i = 0] \left[\frac{12}{i} \leq 6\right]\right) \quad (18)$$

$$= \frac{1}{36} (0 + 1 + 1 + 1 + 0 + 1) \quad (19)$$

$$= \frac{4}{36} \quad (20)$$

$$= \frac{1}{9} \quad (21)$$