For Board Exams 2022-23
Math Question Sample Paper 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int$ x $\cos^2$ x dx = ?

a) $\frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$ + C b) $\frac{x^2}{4} - \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$ + C

c) None of these d) $\frac{x^2}{4} + \frac{x\sin 2x}{4} - \frac{\cos 2x}{8}$ + C

2. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

a) $cos^{-1}(\frac{2}{3})$ b) $tan^{-1}(-\frac{2}{3})$

c) none of these d) $cos^{-1}(\frac{3}{2})$

3. If the position vectors of P and Q are $\hat{i} + 3\hat{j} - 7\hat{k}$ and $5\hat{i} - 2\hat{j} + 4\hat{k}$ respectively , then the cosine of the angle between $\overrightarrow{PQ}$ and y-axis is

a) $\frac{4}{\sqrt{162}}$ b) $\frac{11}{\sqrt{162}}$

c) $\frac{5}{\sqrt{162}}$ d) $-\frac{5}{\sqrt{162}}$

4. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find P(A ∩ B)

a) 0.15 b) 0.10

c) 0.14 d) 0.12

5. $\int cos3x.\,dx$

a) -3 sin 3x + C b) $-\frac{1}{3}\sin 3x + C$

c) 3 sin 3x + C d) $\frac{1}{3}\sin 3x + C$

6. If A ⊆ B, then which one of the following is not correct?

a) P(B/A) = $\frac{P(B)}{P(A)}$

b) P(A/B) = $\frac{P(A)}{P(B)}$

c) P(A $\cap \bar{B}$) = 0

d) P(A/(A $\cup$ B)) = $\frac{P(A)}{P(B)}$

7. The area enclosed by the curve $xy^2 = a^2(a - x)$ and the y – axis is

a) π

b) 2π

c) πa^2

d) none of these

8. Find the angle between the following pairs of lines: $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k}$ +$\lambda \left(3\hat{i} + 2\hat{j} + 6\hat{k}.\right)$ and $\vec{r} = 7\hat{i} - 6\hat{k}$ + $\mu \left(\hat{i} + 2\hat{j} + 2\hat{k}.\right)$,$\lambda, \mu \in R$

a) $\theta = \cos^{-1}(\frac{19}{21})$

b) $\theta = \sin^{-1}\left(\frac{19}{21}\right)$

c) $\theta = \cot^{-1}\left(\frac{19}{21}\right)$

d) $\theta = \tan^{-1}\left(\frac{19}{21}\right)$

9. The 2 vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a $\triangle$ABC. The length of the median through A is

a) $\frac{\sqrt{48}}{2}$

b) $\sqrt{18}$

c) $\frac{\sqrt{34}}{2}$

d) None of these

10. The solution of the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$ is

a) $(x + C)e^{x+y} = 0$

b) $(x + y)e^{x+y} = 0$

c) $(x - C)e^{x+y} + 1 = 0$

d) $(x - C)e^{x+y} = 1$

11. The area bounded by the curve y = f (x), x-axis, and the ordinates x = l and x - b is (b - 1) sin (3b + 4). Then, f (x) is

a) sin (3x + 4)

b) sin (3x + 4) + 3 (x -1) cos (3x + 4)

c) None of these

d) (x -1) cos(3x + 4)

12. $\int e^x \left\{\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right\}$ dx = ?

a) None of these

b) $e^x \sin^{-1}$ + C

c) $\frac{-e^x}{\sin^{-1} x}$ + C

d) $e^x \cdot \frac{1}{\sqrt{1-x^2}}$ + C

13. The maximum value of $\left(\frac{1}{x}\right)^x$ is:

a) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

b) e^e

c) e

d) $e^{1/e}$

14. If A is 3 $\times$ 4 matrix and B is a matrix such that A^T B and BA^T are both defined. Then, B is of the type

a) 4 $\times$ 4

b) 4 $\times$ 3

c) 3 $\times$ 3

d) 3 $\times$ 4

15. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then A^{-1} = ?

a) -adj A

b) adj A

c) -A

d) A

16. If $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj A is

a) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

b) $\begin{bmatrix} -d & -b \\ -c & a \end{bmatrix}$

c) $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$

d) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

17. Range of $\sin^{-1}x$ is

a) None of these

b) $[0, \pi]$

c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

d) $\left[0, \frac{\pi}{2}\right]$

18. The solution of the DE $x\frac{dy}{dx} = \cot y$ is

a) x sec y = C

b) x cos y = C

c) x tan y = C

d) none of these

19. **Assertion (A):** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8.

Reason (R): If f be a function defined on an interval I and c $\in$ l and let f be twice differentiable at c, then x = c is a point of local minima if f'(c) = 0 and f''(c) > 0 and f(c) is local minimum value of f.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If A is a 3×3 non-singular matrix, then $|A^{-1} \text{ adj } A| = |A|$.

Reason (R): If A and B both are invertible matrices such that B is inverse of A, then AB = BA = I.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = ?$

22. Solve the differential equation: $x\frac{dy}{dx} - y = (x - 1)e^x$

23. Solve the system of linear equation, using matrix method 2x + 3y + 3z = 5; x - 2y + z = - 4; 3x - y - 2z = 3

OR

If the points (x, - 2), (5, 2) and (8, 8) are collinear, find x using determinants.

24. Represent graphically a displacement of 50 km, 60^0 west of north.

25. A coin is tossed three times determine P(E|F),

where E: at most two tails, F: at least one tail.

Section C

26. Evaluate: $\int_0^{\pi/4} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$

27. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

OR

In the differential equation show that it is homogeneous and solve it: $(x^2 - y^2)dx + 2xy\, dy = 0$

28. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a} \times \vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

Prove that the points $\hat{i}$ - $\hat{j}$, 4$\hat{i}$ + 3$\hat{j}$ + $\hat{k}$ and 2$\hat{i}$ - 4$\hat{j}$ + 5$\hat{k}$ are the vertices of a right-angled triangle.

29. Evaluate the integral $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ using substitution.

OR

Evaluate: $\int \frac{2x}{x^3-1}dx$

30. Find $\frac{dy}{dx}$, when x = $e^\theta(\sin\theta + \cos\theta)$, y = $e^\theta(\sin\theta - \cos\theta)$.

31. Find the area of the region bounded by the curve y = x^2, the x-axis, and the lines x = 1 and x = 3.

Section D

32. Solve the Linear Programming Problem graphically:

Maximize Z = 3x + 4y subject to the constraints:$x + y \leq 4, x \geq 0, y \geq 0$

33. Show that the relation R on the set A = {x ∈ Z ; 0 ≤ x ≤12}, given by R = {(a, b) : a = b}, is an equivalence relation. Find the set of all elements related to 1.

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A ⇒B defined by$f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. Find the coordinates of the point P where the line through A (3, - 4, -5) and B (2, -3, 1) crosses the plane passing through three points L (2, 2, 1), M(3, 0, 1) and N(4, -1, 0). Aso, find the ratio in which P divides the line segment AB.

OR

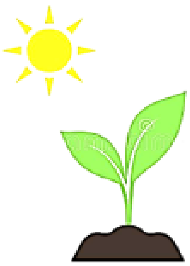
Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z - 3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

35. Find all the points of discontinuity of f defined by f(x) = |x| – |x + 1|.

Section E

36. **Read the text carefully and answer the questions:**

The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = 4x - $\frac{1}{2}x^2$ where x is the number of days exposed to sunlight.



(i) Find the rate of growth of the plant with respect to sunlight.

(ii) What is the number of days it will take for the plant to grow to the maximum height?

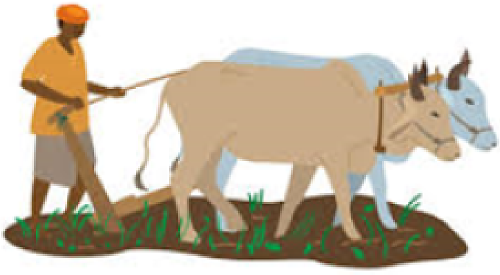
(iii)Verify that height of the plant is maximum after four days by second derivative test and find the maximum height of plant.

OR

What will be the height of the plant after 2 days?

37. **Read the text carefully and answer the questions:**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{array}{cc} \begin{array}{ccc} \mathit{Urad} & \mathit{Masoor} & \mathit{Mung} \end{array} & \\ \begin{pmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} & \begin{array}{c} \mathit{Ankit} \\ \mathit{Girish} \end{array} \end{array}$$

October sales (in ₹):

$$A = \begin{array}{cc} \begin{array}{ccc} \mathit{Urad} & \mathit{Masoor} & \mathit{Mung} \end{array} & \\ \begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} & \begin{array}{c} \mathit{Ankit} \\ \mathit{Girish} \end{array} \end{array}$$

(i) Find the combined sales of Masoor in September and October, for farmer Girish.

(ii) Find the combined sales of Urad in September and October, for farmer Ankit.

(iii)Find a decrease in sales from September to October.

OR

If both the farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October.

38. **Read the text carefully and answer the questions:**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.

(i) Find the probability that India won the second match, if India has already loose the first match.

(ii) Find the probability that India losing the third match, if India has already lost the first two matches.