# For Board Exams 2022-23
# Math Question Sample Paper 1

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

## Section A

1. The value of the integral $\int_{-2}^{2} \left|1-x^2\right| dx$ is

a) 4 b) 0

c) 2 d) -2

2. If O is the origin, OP = 3 with direction ratios proportional to - 1, 2, - 2 then the coordinates of P are

a) (3, 6 , - 9 ) b) ( 1 , 2 , 2 )

c) ( - 1 , 2 , - 2 ) d) $\left(\frac{-1}{9}, \frac{2}{9}, \frac{-2}{9}\right)$

3. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|$ = 13, then the value(s) of n can be

a) $\pm 12$ b) $\pm 8$

c) Only 12 d) Only 8

4. A five-digit number is written down at random. The probability that the number is divisible by 5 and no two consecutive digits are identical, is

a) None of these b) $\frac{1}{5}\left(\frac{9}{10}\right)^3$

c) $\frac{1}{5}$ d) $\left(\frac{3}{5}\right)^4$

5. $\int_0^{\frac{\pi}{2}} \sqrt{1+\cos 2x}\, dx = ?$

a) $\sqrt{3}$ b) 2

c) $\sqrt{2}$ d) none of these

6. If E and F are independent, then which one of the following is right?

a) P (E|F) = P (E'), P (F) ≠ 0E' is complement of E

b) P (E|F) = P (E), P (F) ≠ 0

c) P (E|F) = P (F), P (F) ≠ 0

d) P (E|F) =P (E' ∪ F)E' is complement of E

7. If the area above the x-axis, bounded by the curves y = $2^{kx}$ and x = 0, and x = 2 is $\frac{3}{\log_e 2}$ then the value of k is

a) 1

b) -1

c) 2

d) $\frac{1}{2}$

8. A line makes equal angles with co-ordinate axis. Direction cosines of this line are

a) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

b) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

c) $\pm(1,1,1)$

d) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

9. Consider the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and b = $4\hat{i} - 4\hat{j} + 7\hat{k}$.

What is the scalar projection of $\vec{a}$ on $\vec{b}$?

a) $\frac{23}{9}$

b) $\frac{17}{9}$

c) 1

d) $\frac{19}{9}$

10. Solution of $\frac{dy}{dx} = 1 + x + y + xy$ is

a) $\log|1+y| = 2x + \frac{x^2}{2} + C$

b) $\log|1+y| = x + \frac{x^2}{2} + C$

c) None of these

d) $\log|1+y| = x + \frac{x^2}{2} + Cy$

11. The area bounded by the curves y = |x - 1| and y = 1 is given by

a) 1

b) $\frac{1}{2}$

c) 2

d) none of these

12. $\int \frac{dx}{\sqrt{x-x^2}} =$?

a) None of these

b) $\sin^{-1}(x - 1) + C$

c) $\sin^{-1}(2x - 1) + C$

d) $\sin^{-1}(x + 1) + C$

13. Let f(x) = $(x - a)^2 + (x - b)^2 + (x - c)^2$. Then f(x) has a minimum at x =

a) $\frac{a+b+c}{3}$

b) $3\sqrt{abc}$

c) none of these

d) $\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

14. If A, B are symmetric matrices of same order, then AB – BA is a

a) Symmetric matrix

b) Zero matrix

c) Identity matrix

d) Skew symmetric matrix

15. Find the area of the triangle with vertices (0,0), (4,2), and (1,1).

a) 1 sq.unit

b) 2 sq.unit

c) 0 sq.unit

d) 5 sq.unit

16. If A, B are two n $\times$ n non - singular matrices, then what can you infer about AB?

a) AB is singular
b) $(AB)^{-1}$ does not exist
c) AB is non-singular
d) $(AB)^{-1} = A^{-1}B^{-1}$

17. One branch of $\cos^{-1}$ other than the principal value branch corresponds to

a) $[2\pi, 3\pi]$
b) $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$
c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
d) $(0, \pi)$

18. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is

a) $y = \frac{x^4+c}{x^2}$
b) $y = \frac{x^4+c}{4x^2}$
c) $y = \frac{x^2+c}{4x^2}$
d) $y = \frac{x^2}{4} + c$

19. **Assertion (A):** The absolute maximum value of the function $2x^3 - 24x$ in the interval [1, 3] is 89.

**Reason (R):** The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order n.

**Reason (R):** If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $\frac{1}{|A|}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

## Section B

21. For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$

22. Find the general solution of $\frac{dy}{dx} + y = 1 \, (y \neq 1)$

23. Using matrix method, solve the system of equations

$x + 2y = 1;$

$3x + y = 4.$

OR

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + 1 = 0$. Thus, find $A^{-1}$.

24. if $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are distinct non-zero vectors represented by directed line segments from the origin to the points A, B, C and D respectively, and if $\vec{b} - \vec{a} = \vec{c} - \vec{d}$, then prove that ABCD is a parallelogram.

25. If P (A) = 0.3, P (B) = 0.6, P $(\frac{B}{A})$ = 0.5, find P $(A \cup B)$.

## Section C

26. Evaluate the integral: $\int \frac{\sin x}{\cos 2x} dx$

27. Find one-parameter families of solution curves of the differential equation: $x\frac{dy}{dx}$ + y = $x^4$

OR

Find the equation of the curve which passes through the origin and has the slope x + 3y - 1 at any point (x, y) on it.

28. Write down a unit vector in XY-plane making an angle of $30^o$ in anti-clockwise direction with the positive direction of x - axis.

OR

If $\vec{a} = 2\hat{i} + 5\hat{j} - 7\hat{k}, \vec{b} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} - 3\hat{k}$, compute $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$ and verify that these are not equal.

29. Evaluate: $\int$ sin x sin 2x sin 3x

OR

Prove that: $\int\limits_{a/4}^{3a/4} \frac{\sqrt{x}}{(\sqrt{a-x}+\sqrt{x})} dx = \frac{a}{4}$.

30. Find the values of a and b so that the function f given by $f(x) = \begin{cases} 1, \text{if } x \leq 3 \\ ax + b, \text{if } 3 < x < 5 \\ 7, \text{if } x \geq 5 \end{cases}$ is continuous at x = 3 and x = 5

31. Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$.

## Section D

32. Minimize and Maximize Z = x + 2y subject to $x + 2y \geqslant 100, 2x - y \leqslant 0, 2x + y \leqslant 200, x, y \geqslant 0$ .

33. Let R be a relation on $N \times N$, defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ $N \times N$. Show that R is an equivalence relation.

OR

Each of the following defines a relation on N:

i. x is greater than y, x, y ∈ N

ii. x + y = 10, x, y ∈ N

iii. xy is square of an integer x, y ∈ N

iv. x + 4y = 10x, y ∈ N.

Determine which of the above relations are reflexive, symmetric and transitive.

34. Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

OR

Find the shortest distance $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$ .

35. $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a \sin t$ find $\frac{dy}{dx}$

## Section E

36. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(iii)Verify that volume of the box is maximum at $x = 3$ cm by second derivative test?

**OR**

Find the maximum volume of the box.

37. **Read the text carefully and answer the questions:**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a $1 \times 2$ matrix and B be a $2 \times 1$ matrix, representing the investment and interest rate on each bond respectively.


(i) Represent the given information in matrix algebra.

(ii) If ₹15000 is invested in bond X, then find total amount of interest received on both bonds?

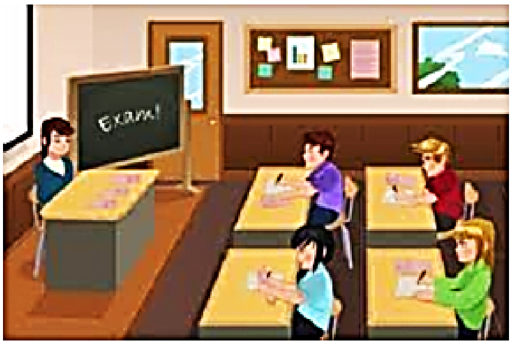
(iii) If the trust fund obtains an annual total interest of ₹ 3200, then find the investment in two bonds.

OR

If the amount of interest given to old age home is ₹500, then find the amount of investment in bond Y.

38. **Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.




(i) Find the probability that Ajay gets Grade A in all subjects.

(ii) Find the probability that he gets Grade A in no subjects.