SA Parameters Model

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1 Introduction

Notation:

1. Set of females: $F = \{f_1, ..., f_p\}$
2. Set of males: $M = \{m_1, ..., m_q\}$
3. Set of actions: A
4. Set of males acted on by female f_i: $M_i = \{m \in M : a_{f_i,m} \in A\}$
5. Set of females acted on by male m_j: $F_j = \{f \in F : a_{m_j,f} \in A\}$

In this Model each user, u, is represented by two parameters, selectiveness (s_u) and attractiveness (a_u). And the probability of like from user u_1 to user u_2 is now given as: $P(a_{u_1,u_2} = 1) = \sigma(s_{u_1}, a_{u_2})$. Where σ is a like probability function discussed in the previous Notation doc. Let θ be the set of all parameters.

As the model is a classifier, I'll call a like action, positive, and a hide action, negative.

2 SA Parameters Model v1

This was the version of the model discussed in meeting on 28th March. In this version we consider both Female-to-Male and Male-to-Female actions together. We can construct a Markov Network for this model as in Figure 1. The parameters represented by circular nodes are hidden continuous variables and actions represented by square nodes are observed binary variables. The problem can then be viewed as that of finding the most likely hidden state of the Markov Network.

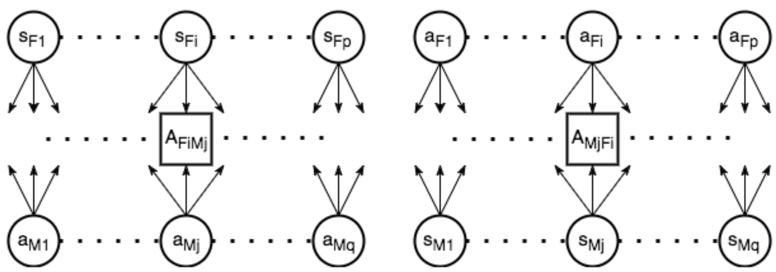

Figure 1: Markov Network for the SA Parameters Model v1.

Now, for any state of the Markov network:

$$Likelihood, L(\theta) = \prod_{i=1}^{p} \prod_{m \in M_i} P(a_{f_i,m}) * \prod_{j=1}^{q} \prod_{f \in F_j} P(a_{m_j,f})$$

Where $P(a_{u_1,u_2}) = 1[a_{u_1,u_2} = 1]\{\sigma(s_{u_1}, a_{u_2})\} + 1[a_{u_1,u_2} = 0]\{1 - \sigma(s_{u_1}, a_{u_2})\}$. Let $\sigma(s, a) = g(s*a)$ where g is the sigmoid function. Using the property of sigmoid,

$$g'(x) = g(x) * (1 - g(x)) = g(x) * g(-x)$$

we have the gradient of log-likelihood w.r.t. s_{f_i}:

$$\frac{\partial LL(\theta)}{\partial s_{f_i}} = \sum_{m \in M_i} 1[a_{u_1,u_2} = 1](1 - g(s_{f_i} * a_m)) * a_m + 1[a_{u_1,u_2} = 0](-g(s_{f_i} * a_m)) * a_m$$

Similarly, the expressions for gradient w.r.t. a_{f_i}, s_{m_j}, a_{m_j} can also be found.

Some more details:

1. A regularizer on the parameters can also be multiplied to the likelihood.
2. The during prediction, an action is predicted to be like if $sigma(s, a) > \gamma$ where γ is the threshold. Till the 28th March meeting it was fixed at 0.5.

3 SA Parameters Model v2

This is a 2nd version of the same model. Here I realized that the female-male and male-female actions would more naturally be modeled separately as shown in 2 and 3. Finding the most likely state of former gives the female **s** and male **a** parameters and the latter gives female **a** and male **s** parameters. This allows me to apply different regularization and set different thresholds for male-female actions and female-male actions classification.

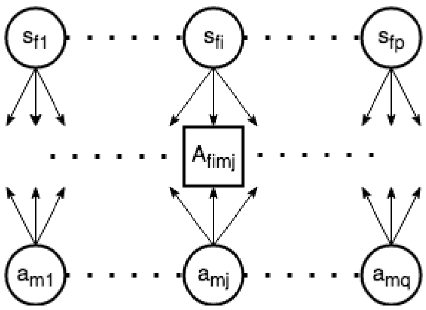

Figure 2: Markov Network for the Female to Male actions.

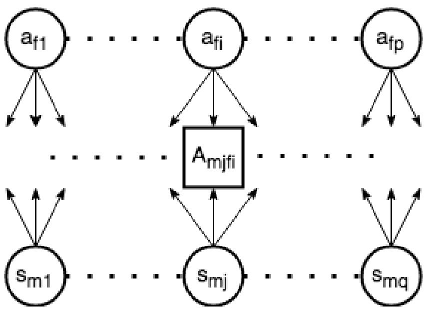

Figure 3: Markov Network for the Male to Female actions.

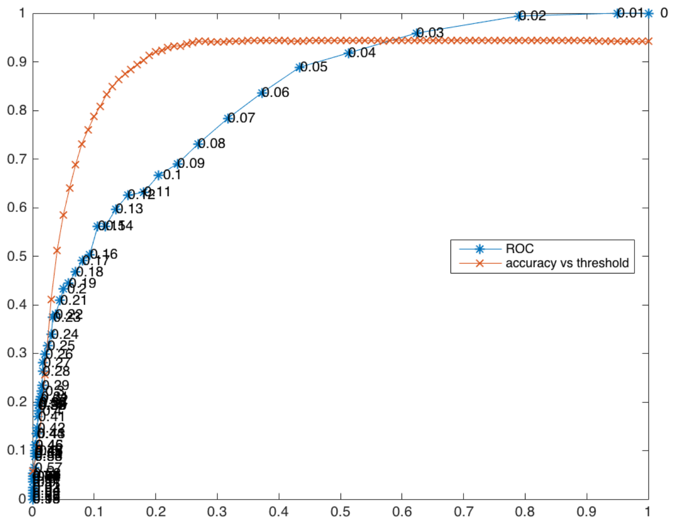

Figure 4: Accuracy and ROC curve for female-male actions. (C = 1)

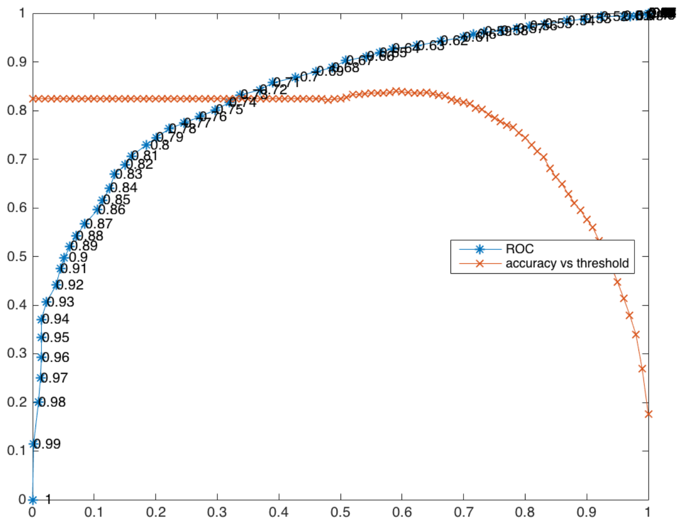

Figure 5: Accuracy and ROC curve for male-female actions. (C = 1)

Observations from the ROC and accuracy curves 4 and 5:

1. For Female to Male Actions, the model has a low true positive rate i.e. it tends to predict even likes as hides.

2. For Male to Female Actions, the model has a high false positive rate i.e. it tends to predict even hides as likes.

3. In both cases this leads to a low ROC. The condition can be improved by varying the threshold. For male-female actions, a increasing the threshold, $\gamma = 0.7$, gives much better performance. Conversely, for female-male actions, decreasing the threshold, $\gamma = 0.2$ gives better performance.