Assignment-2

AI1110: Probability And Random Variables
IIT Hyderabad

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April 11,2022

Q7(b) Evaluate the given finite integral.

$$\int_{-6}^{3} |x+3|\,dx$$

Solution:-

$$|x+3| = \begin{cases} (x+3) \text{ if } x+3 \geq 0 \\ -(x+3) \text{ if } x+3 < 0 \end{cases} \quad (1)$$

$$|x+3| = \begin{cases} (x+3) \text{ if } x \geq -3 \\ -(x+3) \text{ if } x < -3 \end{cases} \quad (2)$$

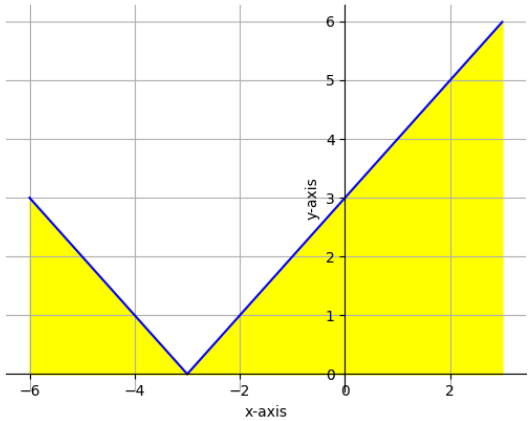

Fig. 1. Graph of $f(x) = |x+3|$

using the property
$\int_a^b x\,dx = \int_a^c x\,dx + \int_c^b x\,dx$

$$I = \int_{-6}^{3} |x+3|\,dx \quad (3)$$

$$= \int_{-6}^{-3} |x+3|\,dx + \int_{-3}^{3} |x+3|\,dx \quad (4)$$

$$= \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{3} (x+3)\,dx \quad (5)$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{3} \quad (6)$$

$$= -\left[\frac{(-3)^2 - (-6)^2}{2} + 3(-3-(-6))\right] \quad (7)$$

$$+\left[\tfrac{(3)^2-(-3)^2}{2} + 3(3-(-6))\right]$$

$$= -\left[\frac{(9)-(36)}{2} + 3(3))\right] \quad (8)$$

$$+\left[\tfrac{(9)-(9)}{2} + 3(3))\right]$$

$$= \frac{27}{2} - 9 + 0 + 18 \quad (9)$$

$$= \frac{27}{2} + 9 \quad (10)$$

$$= \frac{45}{2} \quad (11)$$

$$= 22\frac{1}{2} \quad (12)$$

$\implies \int_{-6}^{3} |x+3|\,dx = 22\frac{1}{2}$ unit.