Assignment-2

AI1110: Probability And Random Variables
IIT Hyderabad

Shivanshu Ai21btech11027
April 11,2022

Q7(b) Evaluate the given finite integral.

$$\int_{-6}^{3} |x+3]\, dx$$

Solution:-

$$|x+3| = \begin{cases} (x+3) \text{ if } x+3 \geq 0 \\ -(x+3) \text{ if } x+3 < 0 \end{cases} \quad (1)$$

$$|x+3| = \begin{cases} (x+3) \text{ if } x \geq -3 \\ -(x+3) \text{ if } x < -3 \end{cases} \quad (2)$$

using the property
$\int_a^b x\,dx = \int_a^c x\,dx + \int_c^b x\,dx$

$$I = \int_{-6}^{3} |x+3|\, dx \quad (3)$$

$$= \int_{-6}^{-3} |x+3|\, dx + \int_{-3}^{3} |x+3|\, dx \quad (4)$$

$$= \int_{-6}^{-3} -(x+3)\, dx + \int_{-3}^{3} (x+3)\, dx \quad (5)$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{3} \quad (6)$$

$$= -\left[\frac{(-3)^2 - (-6)^2}{2} + 3(-3-(-6))\right] \quad (7)$$

$$+\left[\frac{(3)^2-(-3)^2}{2} + 3(3-(-6))\right]$$

$$= -\left[\frac{(9)-(36)}{2} + 3(3))\right] \quad (8)$$

$$+\left[\frac{(9)-(9)}{2} + 3(3))\right]$$

$$= \frac{27}{2} - 9 + 0 + 18 \quad (9)$$

$$= \frac{27}{2} + 9 \quad (10)$$

$$= \frac{45}{2} \quad (11)$$

$$= 22\frac{1}{2} \quad (12)$$

(13)

$\implies \int_{-6}^{3} |x+3]\, dx = 22\frac{1}{2}$ unit.