Assignment-10

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Outline

Question

Papoulis book exercise 6

Q-39 The process x(t) is real with autocorrelation R(τ).

(a) Show that

$$P\{|x(t+\tau) - x(t)| \geq a\} \leq 2[R(0) - R(\tau)]/a^2$$

(B) Express P{$|x(t+\tau) - x(t)| \geq a$} in terms of the second-order density of x(t).

Solution

(a) As we know for any $\epsilon > 0$,

$$P\{|x-\eta| \geq \epsilon\} \leq \frac{\sigma^2}{\epsilon^2} \tag{1}$$

with $x = x(t+r) - x(t)$ and also from (8 - 101):

$$P\{|x(t+\tau) - x(t)| \geq a\} \leq \frac{2[R(0) - R(\tau)]}{a^2} \tag{2}$$

$$= 2[R(0) - R(\tau)]/a^2 \tag{3}$$

(b) The above probability equals the mass in the region (shaded) $x_2 - x_1 > a$ and $x_2 - x_1 < -a$.
Hence,

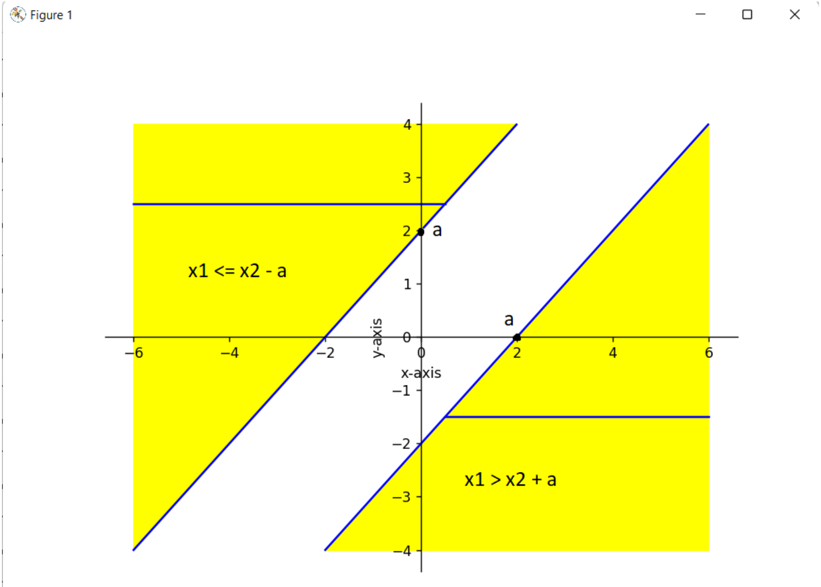
Figure 1
x1 <= x2 - a
x1 > x2 + a
a
a
y-axis
x-axis
4
3
2
1
0
−1
−2
−3
−4
−6
−4
−2
0
2
4
6

Solution

In second order density of x(t)
P$\{|x(t+\tau)-x(t)| \geq a\}$
$= \int_{-\infty}^{\infty}\int_{-\infty}^{x_2-a} f(x_1,x_2;\tau)\,dx_1\,dx_2 + \int_{-\infty}^{\infty}\int_{x_2+a}^{\infty} f(x_1,x_2;\tau)\,dx_1\,dx_2$
(This the region that we have plotted in previous slide)