Assignment-9

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Outline

Question

Papoulis book exercise 6

Q-39 The random variable x and y are independent, x is $N(0, \sigma^2)$. and y is uniform in the interval $(0.\pi)$. Show that if $z = x + acosy$, then

$$f_z(z) = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_0^\pi e^{-(z-acosy)^2/2\sigma^2}\, dy$$

Solution

$$\underline{z} = \underline{x} + \underline{s} \qquad \underline{s} = a\cos\underline{y}$$

$$f_z(z) = f_x(z) * f_s(z)$$

$$f_s(s) = \begin{cases} \dfrac{1}{\pi\sqrt{a^2 - s^2}} & |a| < a \\ \\ 0 & |a| < a \end{cases}$$

$$f_z(z) = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-a}^{a} \frac{e^{-(z-s)^2/2\sigma^2}}{\sqrt{a^2 - s^2}}\, ds$$

$$f_z(z) = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-(z-a\cos y)^2/2\sigma^2}\, dy$$