Assignment-11

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Outline

Question

Papoulis book exercise 11

Q-10 Show that if

$$E\{x_n, x_k\} = \sigma^2\delta[n-k] \quad X(\omega) = \sum_{n=-\infty}^{\infty} x_n e^{-jn\omega T}$$

and $E\{x_n\} = 0$, then $E\{X(\omega)\} = 0$ and
$E\{X(u)X^*(v)\} = \sum_{n=-\infty}^{\infty} \sigma_n^2 e^{-jn(u-v)T}$

Solution

$$X(u) = \sum_{n=-\infty}^{\infty} x_n e^{-jnuT} \tag{1}$$

$$X^*(u) = \sum_{k=-\infty}^{\infty} x_k e^{-jkuT} \tag{2}$$

$$X(u)X^*(u) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (x_n e^{-jnuT} x_k e^{-jkuT}) \tag{3}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (x_n x_k)(e^{-j(nu-kv)T}) \tag{4}$$

$$E(X(u)X^*(u)) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (e^{-j(nu-kv)T})E(x_n x_k) \tag{5}$$

Solution

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sigma_n^2 \delta(n-k)(e^{-j(nu-kv)T}) \tag{6}$$

$$= \sum_{n=-\infty}^{\infty} \sigma_n^2 e^{-jn(u-v)T} \tag{7}$$