Assignment-8

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Question

Papoulis book exercise 5

Q-47 We are given an even convex function g(x) and a random variable x whose density f(x) is symmetrical as in fig. P5-47 with a single maximum at $x = \eta$. Show that the mean E(g(x - a)) of the random variable g(x - a) is minimum if $a = \eta$.

Solution

From the assumption it follows that

$g'(-x) = -g'(x) \qquad g''(x) \geq 0 \qquad f(x-\eta) = f(\eta - x)$

Hence, if $I(a) = Eg(x-a)$, then

$I'(a) = -\int_{-\infty}^{\infty} g'(x-a)f(x)dx \qquad I'(\eta) = 0$

$I''(a) = \int_{-\infty}^{\infty} g''(x-a)f(x)dx \geq 0 \qquad$ all a

Hence, I(a) is minimum for $a = \eta$.