

Fourier Series

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Abstract—This manual provides a simple introduction to Fourier Series

1 PERIODIC FUNCTION

Let

$$x(t) = A_0 |\sin(2\pi f_0 t)| \quad (1.1)$$

1.1 Plot $x(t)$.

Solution: This the required plot.

The required plot can also be downloaded

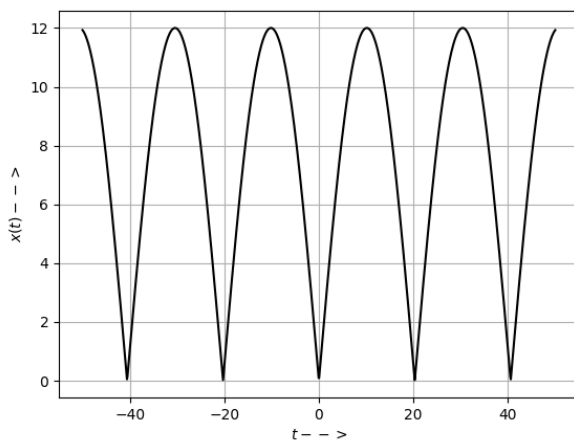


Fig. 1.1: Plot of $x(t)$

using the given link

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wget link_1

1.2 Show that $x(t)$ is periodic and find its period.

Solution:

$$x\left(t + \frac{1}{2f_0}\right) = A_0 \left| \sin\left(2\pi f_0 \left(t + \frac{1}{2f_0}\right)\right) \right| \quad (1.2)$$

$$= A_0 |\sin(2\pi f_0 t + \pi)| \quad (1.3)$$

$$= A_0 |\sin(2\pi f_0 t)| \quad (1.4)$$

So the period of the function $x(t)$ is $\frac{1}{2f_0}$

2 FOURIER SERIES

Consider $A_0 = 12$ and $f_0 = 50$ for all numerical calculations.

2.1 If

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t} \quad (2.1)$$

show that

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-j2\pi k f_0 t} dt \quad (2.2)$$

2.2 Find c_k for (1.1)

2.3 Verify (1.1) using python.

2.4 Show that

$$x(t) = \sum_{k=0}^{\infty} (a_k \cos j2\pi k f_0 t + b_k \sin j2\pi k f_0 t) \quad (2.3)$$

and obtain the formulae for a_k and b_k .

2.5 Find a_k and b_k for (1.1)

2.6 Verify (2.3) using python.

3 FOURIER TRANSFORM

3.1

$$\delta(t) = 0, \quad t \neq 0 \quad (3.1)$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (3.2)$$

3.2 The Fourier Transform of $g(t)$ is

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft} dt \quad (3.3)$$

3.3 Show that

$$g(t - t_0) \xleftrightarrow{\mathcal{F}} G(f)e^{-j2\pi ft_0} \quad (3.4)$$

$$(3.5)$$

3.4 Show that

$$G(t) \xleftrightarrow{\mathcal{F}} g(-f) \quad (3.6)$$

$$3.5 \quad \delta(t) \xleftrightarrow{\mathcal{F}} ?$$

$$3.6 \quad e^{-j2\pi f_0 t} \xleftrightarrow{\mathcal{F}} ?$$

$$3.7 \quad \cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} ?$$

3.8 Find the Fourier Transform of $x(t)$ and plot it.
Verify using python.

3.9 Show that

$$\text{rect}(t) \xleftrightarrow{\mathcal{F}} \text{sinc}(t) \quad (3.7)$$

Verify using python.

3.10 $\text{sinc}(t) \xleftrightarrow{\mathcal{F}} ?$. Verify using python.

4 FILTER

4.1 Find $H(f)$ which transforms $x(t)$ to DC 5V.

4.2 Find $h(t)$.

4.3 Verify your result using through convolution.

5 FILTER DESIGN

5.1 Design a Butterworth filter for $H(f)$.

5.2 Design a Chebyshev filter for $H(f)$.

5.3 Design a circuit for your Butterworth filter.

5.4 Design a circuit for your Chebyshev filter.