Digital Signal Processing

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Contents

***Abstract*—This manual provides a simple introduction to digital signal processing.**

1 Software Installation

Run the following commands

```
sudo apt-get update
sudo apt-get install libffi-dev libsndfile1 python3
    -scipy python3-numpy python3-matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://github.com/Shivanshu8211/
    EE3900/blob/master/codes/Sound_Noise.
    wav
```

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band noise and execute the code.
Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav'
    )

#sampling frequency of Input signal
sampl_freq=fs
#Hello
#order of the filter
order=4

#cutoff frquency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and denominator
    polynomials respectively
b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b, a,
    input_signal)
#output_signal = signal.lfilter(b, a,
    input_signal)

#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav',
    output_signal, fs)
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above

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5.1 kHz.

3 Difference Equation

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{3.1}$$

Sketch $x(n)$. **Solution:** Graph of x(n) has been plotted in part 1 of Fig. 3.2.

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$
$$y(n) = 0, n < 0 \tag{3.2}$$

Sketch $y(n)$.
Solution: The following code yields Fig. 3.2.

```
wget https://github.com/Shivanshu8211/
    EE3900/blob/master/codes/xnyn.py
```

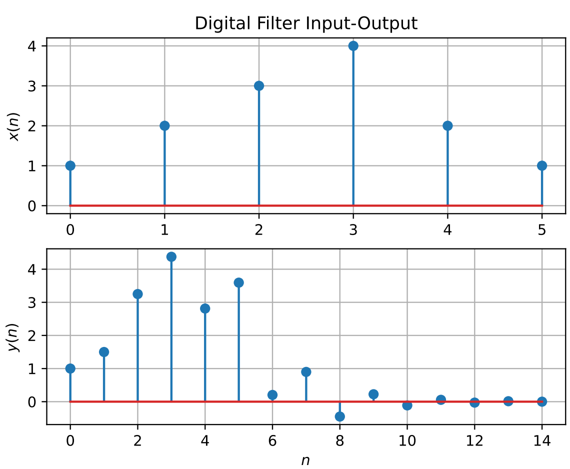

Fig. 3.2

3.3 Repeat the above exercise using a C code. **Solution:** The following code is in c and doing the same function as the above one is doing.

```
c −code
```

4 Z-transform

4.1 The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.3}$$

Solution: Given that,

$$X(z) = \mathcal{Z}\{x(n)\} \tag{4.4}$$
$$= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4.5}$$

So,

$$\mathcal{Z}\{x(n-1)\} = \sum_{n=-\infty}^{\infty} x(n-1) z^{-n} \tag{4.6}$$

let $n - 1 = k$,

$$= \sum_{k=-\infty}^{\infty} x(k) z^{-(k+1)} \tag{4.7}$$
$$= z^{-1} \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{4.8}$$
$$= z^{-1} \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{4.9}$$
$$= z^{-1} X(z) \tag{4.10}$$

From (4.1),

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.11}$$
$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1} \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.12}$$
$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{4.13}$$

Hence proved.

4.2 Obtain X(z) for x(n) defined in problem 3.1. **Solution:** Given

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{4.14}$$

and the Z-transform of $x(n)$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.15}$$

$$\sum_{n=0}^{5} x(n)z^{-n} = z^0 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5} \tag{4.16}$$

4.3 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.17}$$

from (3.2) assuming that the Z-transform is a linear operation.

Solution: since Z-transform is a linear operator therefore

y(z) = Y(z) and x(z) = X(z)

So,

on applying (4.13) in (3.2), we get

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.18}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{4.19}$$

4.4 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.20}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.21}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.22}$$

Solution: The Z-transform of δn is,

$$\mathcal{Z}\{\delta n\} = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} \tag{4.23}$$

$$= \delta(0) z^0 \tag{4.24}$$

$$= 1 \tag{4.25}$$

and the Z-transform of unit-step function $u(n)$ is,

$$U(n) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} \tag{4.26}$$

$$= 0 + \sum_{n=0}^{\infty} 1.z^{-n} \tag{4.27}$$

$$= 1 + z^{-1} + z^{-2} + \ldots \tag{4.28}$$

and from (4.21),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.29}$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{4.30}$$

using the fomula for the sum of an infinite geometric progression.

4.5 Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.31}$$

Solution: The Z- transform will be

$$\mathcal{Z}\{a^n u(n)\} = \sum_{n=0}^{\infty} a^n z^{-n} \tag{4.32}$$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \ldots. \tag{4.33}$$

Above is a infinite geometric series with first term 1 and common ratio as $\frac{a}{z}$ and it can be written as,

$$\mathcal{Z}\{a^n u(n)\} = \frac{1}{1 - \frac{a}{z}} \because |a| < |z| \tag{4.34}$$

Therefore,

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{4.35}$$

4.6 Let

$$H(e^{\jmath\omega}) = H(z = e^{\jmath\omega}). \tag{4.36}$$

Plot $|H(e^{\jmath\omega})|$. Comment. $H(e^{\jmath\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$.

Solution: The following code plots Fig. 4.6.

```
wget https://github.com/Shivanshu8211/
    EE3900/blob/master/codes/dtft.py
```

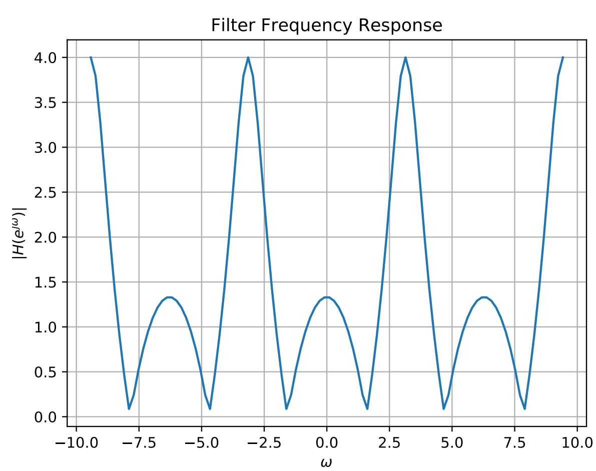

Fig. 4.6: $|H(e^{\jmath\omega})|$

4.7 Express x(n) in terms of $H(e^{j\omega})$

Solution: Using (4.19) and (4.17)

$$H\left(e^{j\omega}\right) = \frac{1 + e^{-2j\omega}}{1 + \frac{e^{-j\omega}}{2}} \tag{4.37}$$

$$\implies \left|H\left(e^{j\omega}\right)\right| = \frac{\left|1 + e^{-2j\omega}\right|}{\left|1 + \frac{e^{-j\omega}}{2}\right|} \tag{4.38}$$

$$= \frac{\left|1 + e^{2j\omega}\right|}{\left|e^{2j\omega} + \frac{e^{j\omega}}{2}\right|} \tag{4.39}$$

$$= \frac{\left|1 + e^{2j\omega}\right|}{\left|e^{j\omega}\right| \left|e^{j\omega} + \frac{1}{2}\right|} \tag{4.40}$$

And we know that $\left|e^{j\omega}\right| = 1$

$$= \frac{\left|1 + \cos 2\omega + j \sin 2\omega\right|}{\left|e^{j\omega} + \frac{1}{2}\right|} \tag{4.41}$$

$$= \frac{\left|4\cos^2(\omega) + 4j\sin(\omega)\cos(\omega)\right|}{\left|2e^{j\omega} + 1\right|} \tag{4.42}$$

$$= \frac{\left|4\cos(\omega)\right| \left|\cos(\omega) + j\sin(\omega)\right|}{\left|2\cos(\omega) + 1 + 2j\sin(\omega)\right|} \tag{4.43}$$

$$\therefore \left|H\left(e^{j\omega}\right)\right| = \frac{\left|4\cos(\omega)\right|}{\sqrt{5 + 4\cos(\omega)}} \tag{4.44}$$