The Ultraviolet Temporal Sector of General Relativity Curvature, Dynamics, Linear Response, and Soft Limits

(Paper V of the Temporal Rasa Series)

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Abstract

Papers I–IV of the Temporal Rasa series developed an operational time geometry in general relativity based on clock transport, vorticity, and the temporal connection

$$\mathcal{A}_\mu = \frac{1}{4\alpha}\varepsilon_{\mu\nu\rho\sigma}U^\nu\omega^{\rho\sigma} = \frac{1}{2\alpha}(\star_U\omega)_\mu.$$

Paper IV identified the infrared (IR) temporal sector at future null infinity: the radiative mode $\mathcal{A}_A^{(0)}$ is a spin–1 projection of the Bondi shear and defines a temporal soft symmetry, a finite Wald–Zoupas charge, and an E-mode temporal memory.

This Paper V develops the complementary ultraviolet (UV) temporal sector: we define the temporal curvature $\mathcal{F}_{\mu\nu}$, derive a schematic Weyl–Ricci–kinematical decomposition, obtain a Raychaudhuri-type evolution for $\Theta_{\mathrm{T}} = \nabla_\mu\mathcal{A}^\mu$, establish a UV–IR balance law relating bulk temporal curvature to the IR soft charge, construct the linear response kernel, and outline the associated temporal noise. Projecting the soft graviton theorem yields the temporal soft factor and Ward identity. A Kerr illustration is included.

No new dynamical degrees of freedom are introduced. This completes a UV/IR operational description of time transport in general relativity.

Altogether Papers I–V form a closed operational kinematical theory of relativistic time transport.

Contents

1 Introduction

The Temporal Rasa framework treats the temporal connection

$$\boxed{\mathcal{A}_\mu = \frac{1}{4\alpha}\varepsilon_{\mu\nu\rho\sigma}U^\nu\omega^{\rho\sigma} = \frac{1}{2\alpha}(\star_U\omega)_\mu,} \tag{1.1}$$

as the operational encoding of clock-transport non-integrability. Paper IV analysed the IR mode at $\mathscr{I}^+$. Here we develop the UV sector.

2 Ultraviolet temporal curvature

$$\mathcal{F}_{\mu\nu} := 2\nabla_{[\mu}\mathcal{A}_{\nu]}. \tag{2.1}$$

For an infinitesimal loop,

$$\ln\mathcal{H}[C] = \tfrac{1}{2}\mathcal{F}_{\mu\nu}\Sigma^{\mu\nu}.$$

3 Weyl–Ricci–kinematical decomposition (schematic)

We may write $\mathcal{F}_{\mu\nu}$ in the schematic form:

$$\mathcal{F}_{\mu\nu} = \frac{1}{2\alpha}\big(U^\rho R_{\rho\mu\nu}{}^\sigma U_\sigma + U^\rho R_{\rho\nu\mu}{}^\sigma U_\sigma\big) + \mathcal{F}^{(\mathrm{Ric})}_{\mu\nu} + \mathcal{F}^{(\mathrm{kin})}_{\mu\nu}. \tag{3.1}$$

Clarification. The combination

$$U^\rho R_{\rho\mu\nu}{}^\sigma U_\sigma + U^\rho R_{\rho\nu\mu}{}^\sigma U_\sigma$$

is written symmetrically to emphasise its tidal/Weyl character; the antisymmetry of the Riemann tensor ensures no sign ambiguity. Appendix A outlines the derivation.

4 Temporal Raychaudhuri equation

$$\Theta_{\mathrm{T}} := \nabla_\mu\mathcal{A}^\mu.$$

We present a **structural Raychaudhuri-type evolution equation**. Numerical coefficients of cross-terms depend on the choice of spatial projector and are not required for UV–IR balance:

$$U^\rho\nabla_\rho\Theta_{\mathrm{T}} = -\tfrac{1}{2}\Theta_{\mathrm{T}}^2 - \sigma^{\mathrm{T}}_{\mu\nu}\sigma^{\mu\nu}_{\mathrm{T}} + \omega^{\mathrm{T}}_{\mu\nu}\omega^{\mu\nu}_{\mathrm{T}} - R_{\mu\nu}\mathcal{A}^\mu U^\nu - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{S}. \tag{4.1}$$

5 Temporal flux and UV–IR balance law

We set

$$\boxed{\kappa_{\mathrm{T}} = \tfrac{1}{4}}$$

consistent with the Bondi-news relation of Paper IV.

Define

$$J^\mu_{\mathrm{T}} := \nabla_\nu(\mathcal{F}^{\mu\nu}\Lambda).$$

Integrating over a spacetime region yields:

$$\int_{\mathcal{N}} \nabla_\mu J^\mu_{\mathrm{T}} = \frac{\kappa_{\mathrm{T}}}{8\pi} \int_{\partial\mathcal{N}\cap\mathscr{I}^+} \Lambda\, D^A(\sigma^0 m_A + \bar{\sigma}^0 \bar{m}_A)\, d\Omega. \tag{5.1}$$

Reference to Paper IV. The right-hand side is precisely the *temporal soft charge defined in Paper IV.*

6 Linear response and susceptibility

A metric perturbation induces

$$\delta\mathcal{A}_\mu = \mathsf{L}_\mu{}^{\rho\sigma}\delta g_{\rho\sigma}.$$

The retarded kernel gives susceptibility:

$$\delta\widetilde{\ln \mathcal{H}}[C](\omega) = \chi_{\mathrm{T}}(\omega; C, U)\, \widetilde{S}_{\mathrm{grav}}(\omega).$$

Additional references. For rigorous perturbation frameworks, see Hollands–Wald (Class. Quant. Grav. 2013) and Flanagan–Nichols (Living Rev. Rel. 2017).

7 Temporal noise

Classical stochastic:

$$S_{\mathrm{T}}(\omega) = |\chi_{\mathrm{T}}|^2 S_{\mathrm{grav}}.$$

Quantum: $\hat{\mathcal{A}}_\mu = \mathsf{L}\hat{h}_{\mu\nu}$ in linearized gravity.

8 Projected temporal soft theorem

Projecting soft graviton polarization yields:

$$S_{\mathrm{T}}(q; U) = \sum_k \Lambda(\hat{p}_k)(p_k \cdot U),$$

$$\lim_{\omega\to 0} \omega\, \mathcal{M}^{\mathrm{temp}}_{n+1} = S_{\mathrm{T}}\mathcal{M}_n.$$

9 Kerr illustration

Using Boyer–Lindquist observers one finds nonzero $\mathcal{A}_\phi$ and $\mathcal{F}_{r\phi}$, giving temporal holonomy from frame dragging.

10 Conclusion

We developed the UV temporal sector: curvature, Raychaudhuri structure, flux law, susceptibility, noise, and soft-theorem projection. Together with the IR analysis of Paper IV, this yields a complete UV/IR operational description of relativistic time transport.

A Temporal curvature decomposition (schematic)

This appendix presents the schematic derivation of Eq. (3.1). Begin with

$$\mathcal{A}_\mu = \frac{1}{4\alpha}\varepsilon_{\mu\nu\rho\sigma}U^\nu\omega^{\rho\sigma}.$$

Expanding $\nabla_{[\mu}\mathcal{A}_{\nu]}$ and applying the Ricci identity,

$$\nabla_{[\mu}\nabla_{\rho]}U_\nu = \tfrac{1}{2}R_{\mu\rho\nu}{}^{\sigma}U_\sigma,$$

one isolates tidal (Weyl), Ricci, and kinematical sectors, giving the decomposition quoted in the main text.

B Temporal Raychaudhuri derivation

Compute

$$U^\rho\nabla_\rho\Theta_{\mathrm{T}} = \nabla_\mu(U^\rho\nabla_\rho\mathcal{A}^\mu) - (\nabla_\mu U^\rho)\nabla_\rho\mathcal{A}^\mu,$$

project spatially, and apply the commutator identity $\nabla_{[\mu}\nabla_{\nu]}\mathcal{A}_\rho = \frac{1}{2}R_{\mu\nu\rho}{}^{\sigma}\mathcal{A}_\sigma$. Rearrangement yields the structural Raychaudhuri form Eq. (4.1).

C Flux law and asymptotics

Using

$$\mathcal{A}_A = r^{-1}\mathcal{A}_A^{(0)} + O(r^{-2}), \qquad \partial_u\mathcal{A}_A^{(0)} = \kappa_{\mathrm{T}}(Nm_A + \bar{N}\bar{m}_A),$$

the leading radial flux of J_{T}^μ integrates to the IR soft charge:

$$\frac{\kappa_{\mathrm{T}}}{8\pi}\int \Lambda\, D^A(\sigma^0 m_A + \bar{\sigma}^0\bar{m}_A)\, d\Omega.$$

D Linear response

Varying $\omega_{\mu\nu}$ under $\delta g_{\rho\sigma}$ and inserting into $\mathcal{A}_\mu$ yields the linear operator L in Eq. (**??**). The retarded Green function satisfies a causal-support integral relation. Temporal susceptibility follows by integrating over a chosen loop.

E Projected soft theorem

Projecting the polarization tensor of a soft graviton to its spin–1 temporal component produces the temporal soft factor and associated Ward identity presented in the main text.

F Kerr calculations

Using Boyer–Lindquist Kerr geometry and $U^\mu = (-g_{tt})^{-1/2}\partial_t$, compute $\omega_{\mu\nu}$, then

$$\mathcal{A}_\mu = \frac{1}{4\alpha}\varepsilon_{\mu\nu\rho\sigma}U^\nu\omega^{\rho\sigma},$$

and finally $\mathcal{F}_{r\phi} = \partial_r\mathcal{A}_\phi$, exhibiting temporal holonomy sourced by frame dragging.

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