Chapter 1

Systematic Uncertainties

There is always a possibility that the final result(s) produced from any data analysis will be shifted from the true or ideally expected value because the final result(s) are derived from the measured or estimated values of one or more other variables, whose values themselves usually have some systematic measurement or estimation uncertainties.

The systematic effects due to a particular variable are studied by making a small change in the variable while holding the others constant, and measuring by how much the end result(s) changed.

In this analysis, ten sources of systematic errors are studied as listed below:

1. Possible Error in the overall scaling factor
2. Effect due to the contaminations of polarized H in the target and π^-in the scattered electrons.
3. Possible error in the beam energy measurement
4. Possible error in the CC-inefficiency estimation
5. Effect due to the e^+e^-pair symmetric contamination
6. Possible error in the estimation of radiation lengths (especially RADA)
7. Model variation using preliminary version (v1) of A_1 model by Guler/Kuhn (2008-9)
8. Model variation using old version of A_2 resonance model

9. Model variation of F_2 (and proportionally of F_1)
10. Model variation of R (F_2 changed)

For the ease of description later on, these ten components will be referred to by the index "k" with its value indicating the position in the list. So, the error due to scaling factor will be identified with k=1 and so on.

Possible Error due to the overall scaling factor This error is due to the uncertainties in the overall scaling factor (SF), which is a convolution of various unmeasured constants such as $P_b P_t$, packing fraction etc (see Sec. **??**). This contribution is estimated by assuming that the uncertainties in SF is not more than 10%. Thus considering the worst case scenario of 10% error in SF, we estimate the corresponding error in g_1 as follows:

$$\Delta g_1^{SF}(W,Q^2) = g_1^{std}(W,Q^2) + \frac{\Delta n^{data}(W,Q^2) - 1.1 \cdot \Delta n^{std}(W,Q^2)}{1.1 \cdot B(W,Q^2)} - g_1^{data}(W,Q^2) \quad (1.1)$$

with "std" shorthand used for "standard" model or the corresponding simulation i.e. the ones provided by RCSLACPOL when the asymmetry A_1 was not artificially increased to $A_1 + 0.1$. Here, Δn^{data} and Δn^{std} represent the polarized count differences for the experimental and simulated (without artificially changing A_1) data respectively.

Error from Polarized H in target and π^-contaminations This contribution from polarized H in target and π^-contamination is evaluated as follows,

$$\Delta g_1^{cont}(W,Q^2) = g_1^{std}(W,Q^2) + \frac{\Delta n^{data}(W,Q^2) \cdot 1.025 - \Delta n^{std}(W,Q^2)}{B(W,Q^2)} - g_1^{data}(W,Q^2) \quad (1.2)$$

where we assume that the contamination is not more than 2.5%, which is consistent with what was found from our own study of the contamination.

Possible error in the beam energy measurement This contribution is evaluated assuming the error in beam energy measurement is not more than 10 MeV, so either the experimental data or the standard-simulation data can be analyzed assuming the beam energy was different by 10 MeV. In this analysis, the simulated data was analyzed assuming that the beam energy was 10 MeV more than that used during the event generation or simulation which was the same as that used for analyzing the experimental data. For example, for the lower beam energy data set, 1339 MeV was used to analyze the experimental data as well as to simulate the corresponding data, but during the analysis of the simulated data, the energy value used was 1349 MeV. Here, the change in energy results in changes in both Q^2 and W.

$$\Delta g_1^{Eb}(W,Q^2) = g_1^{std}(W,Q^2) + \frac{\Delta n^{data}(W,Q^2) - \Delta n^{std}_{Eb+}(W,Q^2)}{B(W,Q^2)} - g_1^{data}(W,Q^2) \tag{1.3}$$

where Δn^{std}_{Eb+} is now the simulated Δn^{std} obtained by analyzing the data from the standard simulation as usual but with a beam energy that was 10 MeV more than the standard value.

Possible error in the CC-inefficiency estimation This contribution is estimated by assuming a maximum of 50% error in the estimated inefficiency as follows:

$$\Delta g_1^{Eb}(W,Q^2) = g_1^{std}(W,Q^2) + \frac{\Delta n^{data}(W,Q^2) - \Delta n^{std}_{0.5CCi}(W,Q^2)}{B(W,Q^2)} - g_1^{data}(W,Q^2) \tag{1.4}$$

where $\Delta n^{std}_{0.5CCi}$ is now the simulated Δn^{std} obtained after applying 50% more inefficiency instead of the actually estimated value.

Possible error due to e^+e^-pair symmetric contamination The contribution due to e^+e^-pair symmetric contamination is calculated as follows:

$$\Delta g_1^{Eb}(W,Q^2) = g_1^{std} + \frac{\Delta n^{data} \cdot \frac{1}{1+f(e^+e^-)} - \Delta n^{std}}{B(W,Q^2)} - g_1^{data}(W,Q^2) \tag{1.5}$$

where f(e^+e^-) is the e^+e^-fraction from the EG1b fit by N. Guler [57] (used the closest available energies).

Radiative correction uncertainty Here, we need to change the parameter that most influences radiative corrections, the number of radiation lengths before (RADB) and after (RADA) the scattering. By increasing both numbers by 10%, we should have a safe upper limit on practically all uncertainties coming from the radiative procedure itself. But, to simplify the situation, we increased the RADA parameter in RCSLACPOL by 20% and repeated the full-statistic simulation. As a result the simulated count difference in each kinematic bin changed from $\Delta n^{standard}$ to a new value Δn^{rad}. This change can be converted to the corresponding inferred change in g_1 by using the same proportionality factors $B(W, Q^2)$ as used earlier in the g_1 (or A_1F_1) extraction/calculation. In other words, for a given kinematic bin this particular contribution to the systematic error is calculated as:

$$\Delta g_1^{rad}(W, Q^2) = g_1^{std} + \frac{\Delta n^{data}(W, Q^2) - \Delta n^{standard}(W, Q^2)}{B(W, Q^2)} - g_1^{data}(W, Q^2) \tag{1.6}$$

where the proportionality factor $B(W, Q^2)$ for the bin is exactly the same as that used to calculate g_1 earlier.

1.0.1 Model uncertainties

The remaining four components in the total systematic uncertainty (the last four in the list 1) account for the model uncertainty contributions. For this purpose, we changed the values of two of the model parameters "AsymChoice" and "SFchoice" (each takes value of 11, in the standard case)

We repeated the full statistics simulation four more times by changing the values of two RCSLACPOL parameters "AsymChoice" and "SFchoice" (which controls the values of model asymmetries and the structure functions, with each taking a value of 11 in the standard case) one by one corresponding to the following four model variations:

1. Variation-1: AsymChoice=12, SFchoic=11
2. Variation-2: AsymChoice=15, SFchoic=11
3. Variation-3: AsymChoice=11, SFchoic=12
4. Variation-4: AsymChoice=11, SFchoic=13

where, the different values of the two RCSLACPOL parameters correspond to the following model choices:

1. **AsymChoice** values are used to determine specific A_1/A_2 models used in the RCSLACPOL program

(a) 11: Standard Resonance Model 2008-9 Guler/Kuhn (**Used for standard simulation**)

(b) 12: Variation of A_1 model

(c) 15: Variation of A_2 resonance model: Vary the virtual photon asymmetry A_2 in the resonance region within its fit errors.

2. **SFchoice** values are used to determine specific F_1/F_2 models.

   (a) 11: 2009 version of $F_1^n/F_1^p/F_1^d$ by Peter Bosted/Eric Christie 2009, HERMES (**Used for standard simulation**) (with d in F_1^d denoting a deuteron).

   (b) 12: Same version as 11, but with fit errors added to F_2 (and proportionally F_1)

   (c) 13: Same version as 11, but with fit errors subtracted from R (F_2 unchanged)

After the simulation data for the above four cases were available, four more data tables (TM1,TM2,TM3 and TM4) were produced for the corresponding model values of g_1, A_1, F_1 etc. Then, the contributions to the systematic error from each of these four cases of model variation were given as follows:

$$\Delta g_1^i(W,Q^2) = g_1^{standard}(W,Q^2) - g_1^i(W,Q^2) + \frac{\Delta n^i(W,Q^2) - \Delta n^{standard}(W,Q^2)}{B(W,Q^2)} \quad (1.7)$$

with "i" indicating any of the four cases of model variation, g_1^i being the model prediction for the i^{th} case as obtained from the corresponding data table "TMi" and the proportionality factor $B(W,Q^2)$ again being exactly the same as used to calculate g_1 as earlier.

1.1 Combining uncertainties

Contributions from the 10 individual components are estimated and then a total contribution is estimated by first combining the corresponding individual compenents from the two beam energies and finally combining them all by calculating the RMS of the ten combined contributions.

1.1.1 Combining errors from the two beam energies

In principle, each of the individual contributions to the systematic error can also be combined using the same equations as for combining g_1 and A_1F_1 (see above). However, we must be careful to distinguish between correlated and uncorrelated errors. If for a given (W, Q^2) bin, data is available only from one beam energy, then combined error for the k^{th}component is simply the error from that beam energy. If there are measurements from both beam energies, we combine them with statistical weights as follows:

1. The variations due to scale factor (k=1), beam energy (k=3) and CC-efficiency (k=4) are all un-correlated and, therefore, added in quadrature as follows:

$$\delta g_1(\text{k=8,10,11, combined}) = \sqrt{\left(\sum_i \frac{(\delta g_1)^2(i)}{(\Delta g_1)^2(i)}\right) / Sum2} \tag{1.8}$$

   where, δ represents the k^{th}component of the systematic error, whereas, 'Sum2', 'i' and Δ have the same meanings as before.

2. while all other variations are correlated between the two beam energies and should be averaged linearly (WITH sign):

$$\delta g_1(\text{other k, combined}) = \left(\sum_i \frac{(\delta g_1)(i)}{(\Delta g_1)^2(i)}\right) / Sum2 \tag{1.9}$$

   with

$$Sum2 = \sum_i \frac{1}{(\Delta g_1)^2(i)} \tag{1.10}$$

providing the statistical weight, where the index 'i' represents two beam energy (1.3 and 2.0 GeV) data sets, and Δg_1 indicates the statistical error in $g_1$in the corresponding kinematic bin.

1.1.2 Combining errors from the ten sources

Once each of the k^{th} component of the systematic errors are combined between the two beam energies, we then proceed to combine them all to get a grand total. This is done by simply adding the ten E_b-combined systematic errors in quadrature and taking the square-root of the sum as follows:

$$TotalSystematicError = \sum_{k=1}^{10} (\Delta g_1)_k^2 \tag{1.11}$$