Design & Analysis of Algorithm

(Prof. Parminder Kaur Wadhwa)

Introduction :-

Algorithm and its importance, mathematical foundations :- Growth functions, Complexity Analysis of Algorithms

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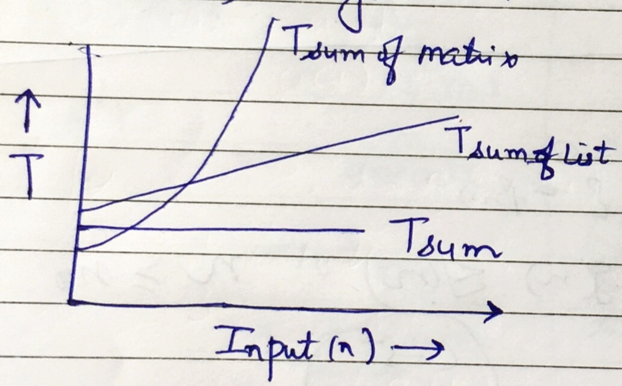
Algorithm

Problem P $\rightarrow$ A1 ? choose
$\rightarrow$ A2
$\rightarrow$:

$n \rightarrow$ input size

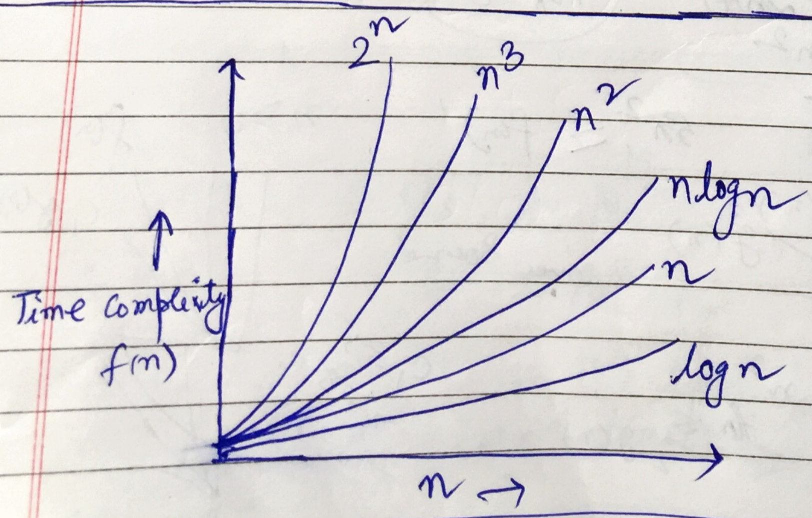
$n \uparrow$, algorithm ? behave ?



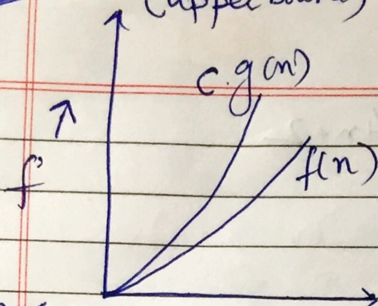
T sum of matrix for.... for... $O(n^2)$

T sum of list $\Rightarrow$ $O(n)$
for loop
sum = sum + i

T sum = a+b k
$O(1)$



Worst case — Big oh (O) (upper bound)



$f(n) \le c.g(n)$

Tight bound (Θ) Theta

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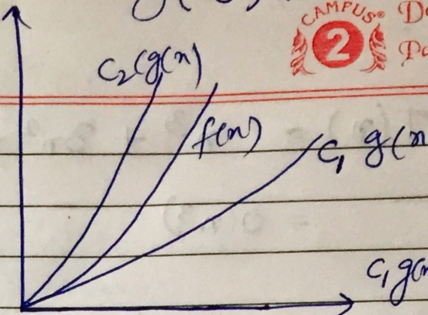
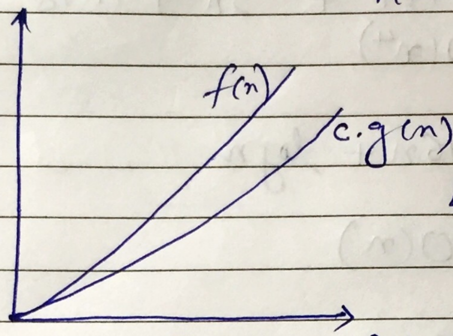


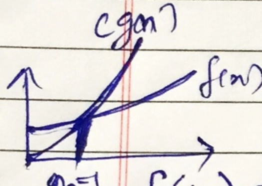
$c_1 g(n) \le f(n) \le c_2 g(n)$



Lower Bound (Ω) Omega

$f(n) \ge c.g(n)$

Best case



$f(n) = 5n^2 + 2n + 1$

$g(n) = n^2$

$c = 8 \quad = (5+2+1)$

$f(n) \le 8n^2$ for $(n \ge 1)$

$O(n^2)$

$2n \ngtr 2n^2$

$1 \ngtr n^2$

$f(n) = 5n^2 + 2n + 1 = \Omega(n^2)$

$g(n) = n^2$

$c = 5$

$n_0 = 0$

$n \ge 0$

$2n + 1 \ge 1$

$5n^2 \le f(n)$, $n \ge 0$

$5n^2 \le f(n) \le 8n^2$

$\boxed{c_1 = 5,\ c_2 = 8,\ n_0 = 1}$

$f(n) = \Theta(n^2)$

Examples:—

1. $T(n) = n^3 + 3n^2 + 4n + 2$

$= O(n^3)$

2. $T(n) = 17n^4 + 3n^3 + 4n + 8$

$= O(n^4)$

3. $T(n) = 16n + \log n$

$= O(n)$

4. $T(n) = 3 + 4$

$\Rightarrow O(1)$

5. $T(n) = a + b$

$= O(1)$

	times	cost
6. for (i=0; i≤n; i++)	(n+1) times	c_1
{		
// stmts	n times	c_2
}		

$$T(n) = c_1(n+1) + c_2 n$$

$$= c_1 n + c_1 + c_2 n$$

$$= (c_1 + c_2)n + c_1$$

⇒ It is in the form:-

$$T(n) = an+b$$

$$\doteq O(n)$$

i.e. linear relationship

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(7) for ($\overset{i=0 \text{ to } n}{\rule{2em}{0.4pt}}$) ———— $(n+1)$

{

for ($\overset{\text{for } i=0 \text{ to } n}{\rule{2em}{0.4pt}}$) $n(n+1)$

{

// stmt $n(n+1) - 1$

}

}

$c_1(n+1) + c_2 * n(n+1) + c_3((n)(n+1) - 1)$

$= c_1 n + c_1 + c_2 n^2 + c_2 n + c_3(n^2 + n - 1)$

$= c_1 n + c_1 + \underline{c_2 n^2} + c_2 n + \underline{c_3 n^2} + c_3 n - c_3$

$= (c_2 + c_3) n^2 + (c_2 + c_3 + c_1) n + (c_1 - c_3)$

$= an^2 + bn + c$

$\Rightarrow O(n^2)$

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8. for (0 → n) —— (n+1)

{

for (0 → n) $n(n+1)$

{

for (0 — n) $[(n)(n+1)-1](n+1)$

$= (n^2+n-1)(n+1)$

{ $= n^3+n^2 \cancel{-n} + n^2 \cancel{+n} -1$

~~stmt~~ $= n^3+2n^2-1$

} → n^3+2n^2

}

}

$$T(n) = c_1(n+1) + c_2(n)(n+1) + c_3(n^3+2n^2-1) + c_4(n^3+2n^2)$$

$$= \underline{c_1 n} + c_1 + c_2 n^2 + c_2 + c_3 n^3 + 2c_3 n^2 - c_3 + c_4 n^3 + 2c_4 n^2$$

$$= (c_3+c_4)n^3 + (2c_3+c_2+2c_4)n^2 + c_1 n + (c_1+c_2-c_3)$$

Of the form:-

$\Rightarrow an^3 + bn^2 + cn + d$

$\Rightarrow O(n^3)$

9. fragments:-

for ($\underline{0 \to n}$)
{

} → $(n+1)$ times

$O(n)$

for ($0 \to n$)
{
for ($\underline{0 \to n}$)
{

}
}

$O(n^2)$

$T(n) = O(n) + O(n^2)$

$T(n) \Rightarrow O(n^2)$

Analysis of :-

p. **Algorithm to compute sum**

	Cost	Times
$S \leftarrow 0$	c_1	1 time
for $i \Rightarrow 1$ to n do	c_2	$(n+1)$ times
$S \leftarrow S + A[i]$	c_3	n times
return S	c_4	1 time

$$T(n) = c_1 * 1 + c_2 * (n+1) + c_3 * n + c_4 * 1$$

$$= c_1 + c_2 n + c_2 + c_3 n + c_4$$

$$= (c_2 + c_3) n + (c_1 + c_2 + c_4)$$

$\Rightarrow$ In the form

$$an + b$$

$$= O(n)$$

11. Analysis of Insertion Sort

```
for j <- 2 to length [A]                      c1   n
{   do key <- A[j]                            c2   n-1
       i <- j-1                               c3   n-1
    while (i > 0 and A[i] > key)              c4   Σ_{j=2}^{n} K
    {
       do A[i+1] <- A[i]                      c5   Σ_{j=2}^{n} K-1
          i <- i-1                            c6   Σ_{j=2}^{n} K-1
    }
    A[i+1] <- key }                           c7   n-1
}
```

$$\Rightarrow T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} K$$
$$+ c_5\left(\sum_{j=2}^{n} K-1\right) + c_6\left(\sum_{j=2}^{n} K-1\right) + c_7(n-1)$$

As $\Rightarrow \sum_{j=1}^{n} = \frac{n(n+1)}{2}$ $\quad 1+2+3 \ldots$

$\Rightarrow \sum_{j=2}^{n} \Rightarrow \frac{n(n+1)}{2} - 1$ $\quad 2+3+\ldots$

$\therefore$

$$T(n) = c_1 n + c_2(n-1) + c_3(n-1) + c_4\left(\frac{n(n+1)}{2} - 1\right)$$
$$+ c_5\left(\left(\frac{n(n+1)}{2} - 1\right) - 1\right) + c_6\left(\left(\frac{n(n+1)}{2} - 1\right) - 1\right) + c_7(n-1)$$

$\Rightarrow an^2 + bn + c \Rightarrow O(n^2)$

Practice Questions :-

1. Do the analysis of selection sort algorithm.

(Hint :- $O(n^2)$)

2. Do the analysis of bubble sort algorithm.

(Hint: → $O(n^2)$)