

Using Backtracking Technique to solveSum of Subsets

Suppose we are given n distinct positive numbers (usually called weights) and we desire to find all combinations of these numbers whose sums are m .

This is called as the sum of subsets problem.

For example, if $n=4$

Required sum $= m = 31$

Given weights are:-

$$(w_1, w_2, w_3, w_4) \\ = (11, 13, 24, 7)$$

Then, the desired subsets whose sum is $m=31$
are:-

$$(11, 13, 7) \text{ and } (24, 7)$$

The number of nodes in solution space by using backtracking for sum of subsets is 2^n .

The worst-case time for the backtracking algorithm solving sum of subsets problem is $O(p(n) 2^n)$, (where $p(n)$ is polynomial in n).

Considering a backtracking solution involving the fixed-tuple-size strategy.

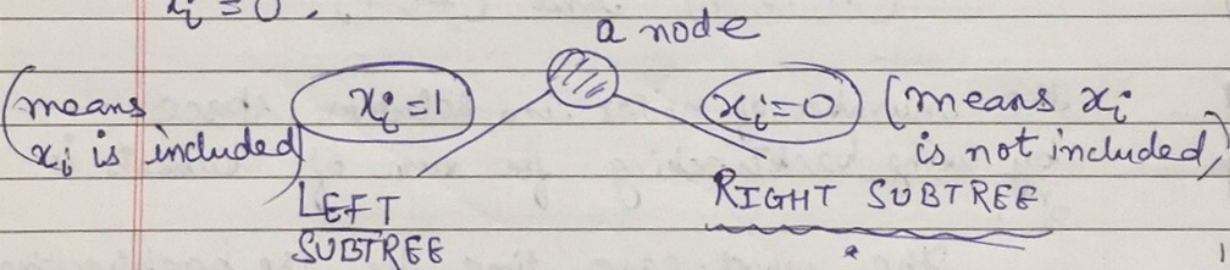
In this case the element x_i of the solution vector is either one or zero depending on whether the weight w_i is included or not.

The solution for: - $n=4$
 $m=31$

(w_1, w_2, w_3, w_4)
 $(11, 13, 24, 7)$

are: - $(1, 1, 0, 1)$
 $(0, 0, 1, 1)$ } fixed-tuple size of the solution

For a node at level i the left child corresponds to $x_i = 1$ and the right to $x_i = 0$.



The bounding function for sum of subsets problem is :-

$$B_k(x_1, \dots, x_k) = \text{true}$$

iff

$$\sum_{i=1}^k w_i x_i + \sum_{i=k+1}^n w_i \geq m$$

and

$$\sum_{i=1}^k w_i x_i + w_{k+1} \leq m$$

The first condition is :- that the sum computed till k i.e. $\sum_{i=1}^k w_i x_i$ when is added to the sum of remaining weights i.e. from $(k+1)$ & onwards i.e. $\sum_{i=k+1}^n w_i$ should be greater than or equal to m .

The second condition is :- that when new next weight i.e. w_{k+1} is added to the already computed weights i.e. $\sum_{i=1}^k w_i x_i$ should be less than or equal to m . It means it should not exceed m because m is the required sum.

When both first & second conditions are TRUE, then that {solution} is considered otherwise not.
 [state]

Sum of subsets - for example :- $m = 31$ (required sum)
 Elements :- x_1, x_2, x_3, x_4
 $11, 13, 24, 7$

11
 +13
 24
 +07
 55

no element is selected
 0, 55
 sum of remaining weights

