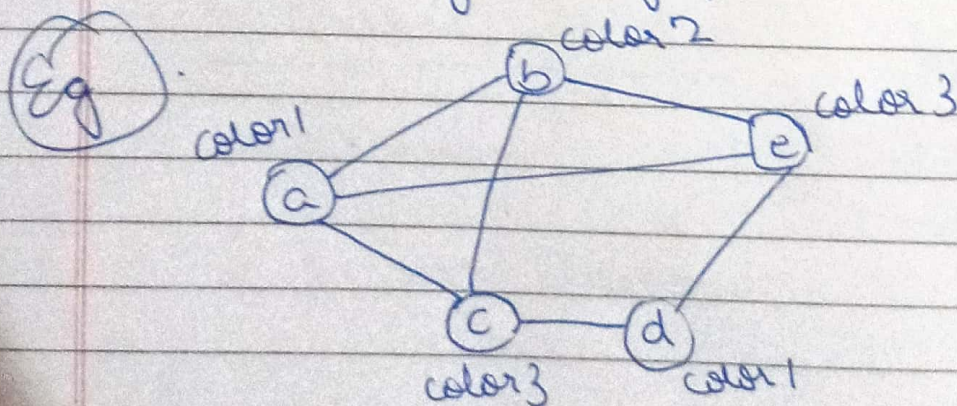


Q5 What do you know about graph coloring problem?  
Explain how it can be solved using backtracking.

Ans Let  $G$  be a graph and  $m$  be a given positive integer.

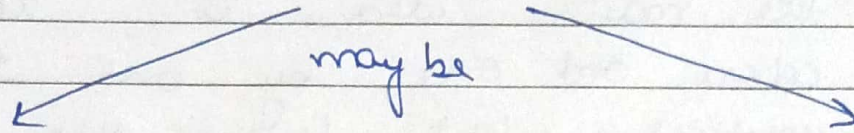
We want to discover whether the nodes of  $G$  can be coloured in such a way that no two adjacent nodes have the same color yet only  $m$  colors are used. This is called as  $m$ -colorability decision problem.

The  $m$ -colorability optimization problem asks for the smallest integer  $m$  for which the graph  $G$  can be colored. This integer is referred to as the chromatic number of the graph.





## Graph Coloring Problem



a decision problem

(for a given number  $m$ , whether it is possible to color the graph such that no two adjacent nodes have same color)

an optimisation problem

(find the minimum value of  $m$  required to color the graph such that no two adjacent nodes have same color)

This has practical application in map coloring.

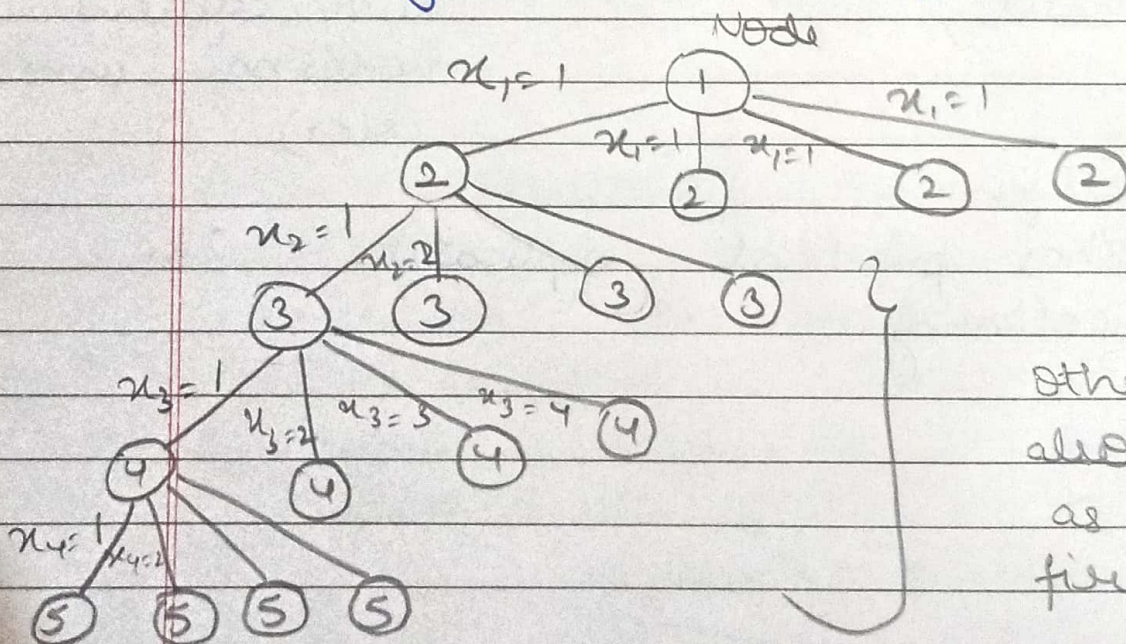


## Graph Coloring using backtracking

By using the backtracking method, the main idea is to assign colors ~~and~~ one by one to different vertices right from the first vertex.

- Before color assignment, check if the adjacent vertices have same or different color by considering already assigned colors to the adjacent vertices.

### Using depth first generation



Other nodes are also expanded as per depth first generation.



# Algorithm

```
void mcoloring (int k)
{
  // k is the index of the next vertex
  // to color
  do
  {
    Next Value (k);           // Assign to
                              // x[k] a legal
                              // color
    if (!x[k]) break;         // No new color
                              // possible
                              // for decision problem
    if (k == n)               // colored all nodes
                              // then display result
    {
      for (int i = 1; i <= n; i++)
        cout << x[i],
        cout << endl;
    }
    else mcoloring (k+1);
  } while (1);
}
```

Worst case time complexity  $O(nm^n)$