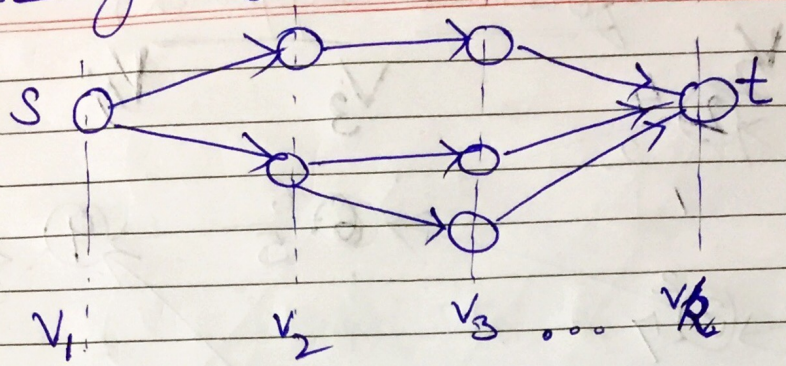
MULTISTAGE GRAPH PROBLEM
(using Dynamic Programming)



k stage graph

(To find minimum-cost path from source (S) to sink (t))

⇒ Involves sequence of decisions from one stage to another.

Two approaches :— (1) Forward Approach
(2) Backward Approach

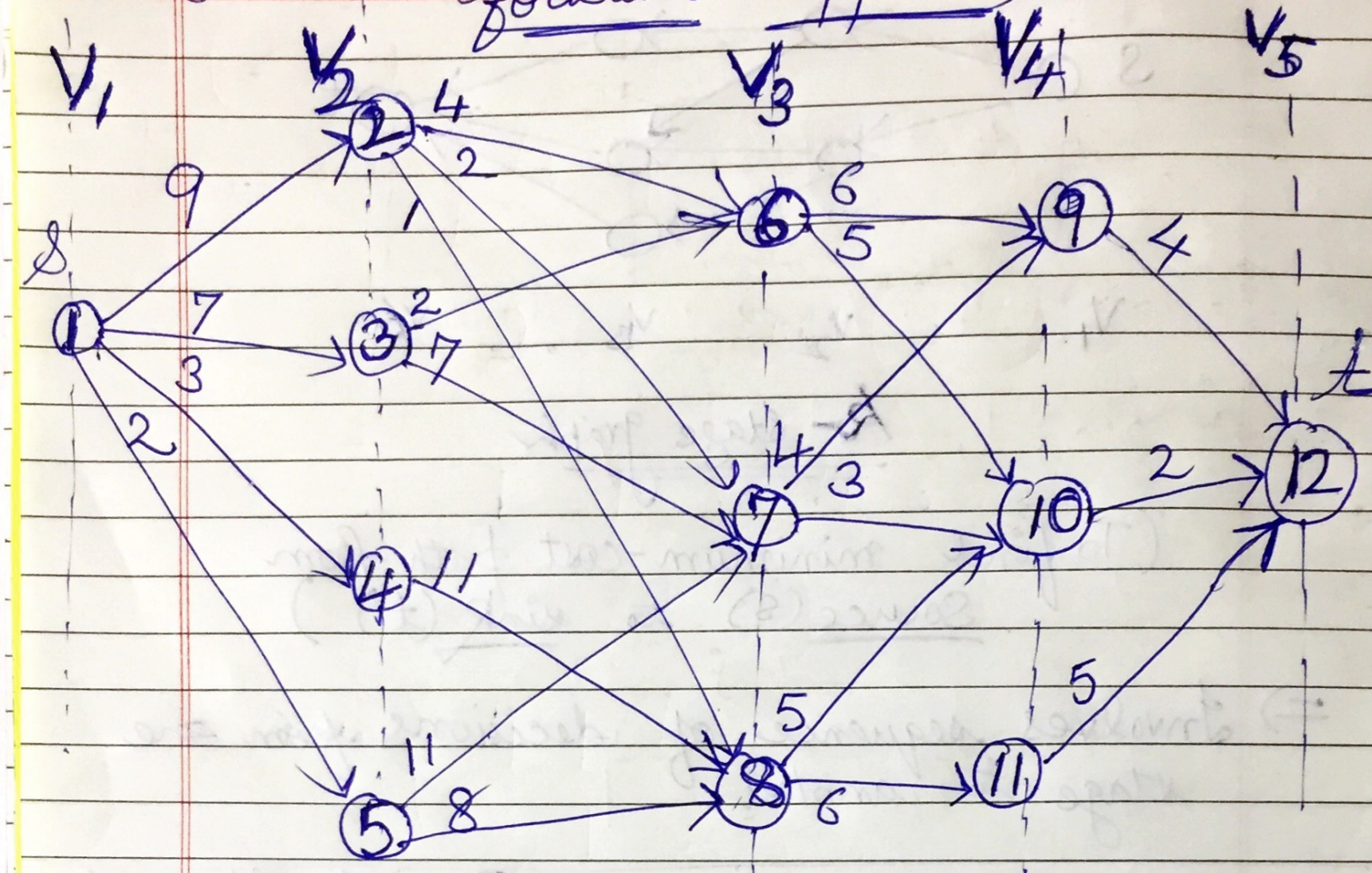
A dynamic programming technique for a k-stage graph problem says that every s to t path is the result of a sequence of (k-2) decisions.
(principle of optimality holds)

(1)

k=4	(k-2) = 4-2 = 2 decisions	As only one edge is from source & one from sink

Multistage graph problem

(using dynamic programming & forward approach)



A - five stage graph

Forward approach :-

next stage

$$cost(i, j) = \min\{c(j, l) + cost(i+1, l)\}$$

$$cost(i,j) = \min\{c(j,l) + cost(i+1,l)\}$$

stage, vertex, max value

$cost(4,9)$ = $\min\{4 + cost(5,12)\}$

$cost(4,9) = \{4+0\} = 4$

$cost(4,10) = \{2 + cost(5,12)\} = \{2+0\} = 2$

$cost(4,11) = \{5 + cost(5,12)\} = \{5+0\} = 5$

$cost(3,6) = \min\{6 + cost(4,9),\ 5 + cost(4,10)\}$

stage, vertex

$= \min\{6+4, 5+2\} = \min\{10, 7\} = 7$

$cost(3,7) = \min\{4 + cost(4,9),\ 3 + cost(4,10)\}$

stage, vertex

$= \min\{4+4, 3+2\} = \min\{8, 5\}$

$= 5$

$cost(3,8) = \min\{5 + cost(4,10),\ 6 + cost(4,11)\}$

stage, vertex

$= \min\{5+2, 6+5\} = \min\{7, 11\} = 7$

stage → vertex

cost(2,2)

$= \min\{ 4 + cost(3,6)$

$2 + cost(3,7)$

$1 + cost(3,8)\}$

$= \min\{ 4+7,$

$2+5,$

$1+7\}$

$= \min\{11, 7, 8\}$

$= 7$

cost(2, 3)

stage — vertex

$= \min\{2 + \text{cost}(3,6),\ 7 + \text{cost}(3,7)\}$

$= \min\{2+7,\ 7+5\} = \min\{9, 11\}$

$= 9$

$\text{cost}(2,4) = \min\{11 + \text{cost}(3,8)\}$

stage — vertex $= \{11+7\} = 18$

$\text{cost}(2,5) = \min\{11 + \text{cost}(3,7),\ 8 + \text{cost}(3,8)\}$

stage — vertex

$= \min\{11+5,\ 8+7\}$

$= \min\{16, 15\}$

$= 15$

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stage

vertex

cost (1, 1)

$= \min\{ 9 + cost(2,2),$ ~~cost~~

$7 + cost(2,3),$

$3 + cost(2,4),$

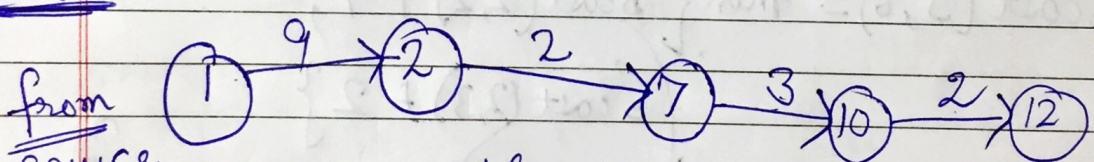
$2 + cost(2,5) \}$

$= \min\{ 9+7, 7+9, 3+18, 2+15 \}$

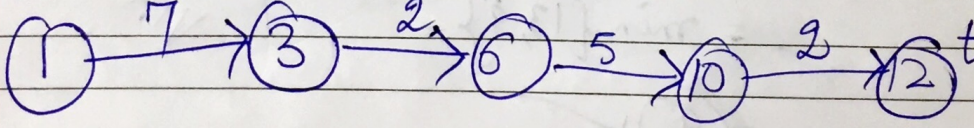
$= \min\{ 16, 16, 21, 17 \}$

$= 16 \Rightarrow$ minimum cost path from source to sink

Solution:-



Total cost = 16



Total cost = 16

Multistage graph problem

(using dynamic programming & backward approach)

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$$bcost(i, j) = \min\{ bcost(i-1, l) + c(l, j) \}$$

(l → next vertex)

bcost(1,1) ⇒ source

bcost(2,2) = min { bcost(1,1) + 9 } = 0+9 = 9
bcost(2,3) = min { bcost(1,1) + 7 } = 0+7 = 7
bcost(2,4) = min { bcost(1,1) + 3 } = 0+3 = 3
bcost(2,5) = min { bcost(1,1) + 2 } = 0+2 = 2

bcost(3,6) = min { bcost(2,2) + 4, bcost(2,3) + 2 }

= min { 9+4, 7+2 }

= min { 13, 9 }

= 9

via vertex 3

bcost(3,7)

stage, vertex

$= \min\{ bcost(2,2) + 2,$

$bcost(2,3) + 7,$

$bcost(2,5) + 8 \}$

$= \min\{ 9+2, 7+7, 2+11 \}$

$= \min\{ 11, 14, 13 \}$

$= 11$

via vertex 2

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bcost(3,8)

$= \min\{ bcost(2,2) + 1,$

$bcost(2,4) + 11,$

$bcost(2,5) + 8 \}$

$= \min\{ 9+1, 3+11, 2+8 \}$

$= \min\{ 10, 14, 10 \}$

$= 10$

via vertex 2 or vertex 5

bcost(4, 9)

$= \min\{ bcost(3,6) + 6,$
$\quad bcost(3,7) + 4 \}$

$= \min\{ 9+6, 11+4 \}$

$= \min\{ 15, 15 \}$

$= \min \Rightarrow (15)$ ✓

via vertex 6
or vertex 7

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bcost(4, 10)

$= \min\{ bcost(3,6) + 5,$
$\quad bcost(3,7) + 3,$
$\quad bcost(3,8) + 5 \}$

$= \min\{ 9+5, 11+3, 10+5 \}$

$= \min\{ 14, 14, 13 \}$

$= (14)$

via vertex 6
or vertex 7

bcast (4, 11)

= min { bcast (3, 8) + 6 }

= 10 + 6

= (16) ✓

Via vertex 8

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bcast (5, 12)

= min { bcast (4, 9) + 4,
bcast (4, 10) + 2,
bcast (4, 11) + 5 }

= min { 15+4, 14+2, 16+5 }

= min { 19, 16, 21 }

= (16)

Via vertex 10

Solutions:— O(V+E)