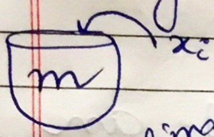
0/1 Knapsack Problem

using Dynamic Programming



optimal sol. → maximize $\sum_{1 \le i \le n} p_i x_i$ (objective function)

feasible solution — subject to $\sum_{1 \le i \le n} w_i x_i \le m$ (constraint)

0/1 Knapsack :— x_i is either 0 or 1

Using dynamic programming (such that the principle of optimality holds) :—

$$f_n(m) = \max \{ f_{n-1}(m),\ f_{n-1}(m - w_n) + p_n \}$$

not including n^{th} element

including n^{th} element

reduce the remaining weight

gain the profit

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Generalizing the equation :-

$$f_i(y) = \max\{ f_{i-1}(y) , f_{i-1}(y - w_i) + p_i \}$$

To represent $f_i(y)$ an ordered set S^i is used such that :-

$$S^i = \{ f(y_j) , y_j \}$$

P   W

$P = f(y_j)$ $W = y_j$

Each member of S^i is a pair (P, W).

Compute S^i first ⇐ do not include i^{th} element

then S^i_1 , ⇐ include i^{th} element

then S^{i+1} by merging S^i & S^i_1

including & not including cases of i^{th} but not including $(i+1)^{th}$

$S^i \rightarrow$ P&W when i^{th} element is NOT included

$S_1^i \rightarrow$ P&W when i^{th} element is INCLUDED

$S^{i+1} \rightarrow S^i \quad S_1^i$

(merging)

$\left.\begin{array}{l} i^{th} \times \\ i^{th} \checkmark \end{array}\right\}$ but $(i+1)^{th}$ not included

$S_1^{i+1} \rightarrow$ include $(i+1)^{th}$ element

(do it for all elements).

$S^i \quad S_1^i$

(merging)

$S^{i+1} \rightarrow$ if S^{i+1} contains two pairs (P_j, W_j) and (P_k, W_k)

with the property that:-

$P_j \leq P_k$ and $W_j \geq W_k$

Profit is increased but Weight is decreased

then discard (P_j, W_j) pair

called as:- Discarding or purging rule or dominance rule.

also purge if $w > m$

0/1 Knapsack Problem

(solving using dynamic programming)

$n = 3$ $\quad (p_1, p_2, p_3) = (1, 2, 5)$

$m = 6$ $\quad (w_1, w_2, w_3) = (2, 3, 4)$

$S^0 = \{(0,0)\}$

$S^0_1 = \{(1,2)\}$ $\quad \| \overset{P\ W}{(1,2)}$

$S^1 = \{(0,0), (1,2)\}$

$S^1_1 = \{(2,3), (3,5)\}$ $\quad \| \overset{P\ W}{(2,3)}$

$S^2 = \{(0,0)\ (1,2)\ (2,3), (3,5)\}$

$S^2_1 = \{(5,4), (6,6), (7,7), (8,9)\}$ $\quad \| \overset{P\ W}{(5,4)}$

$S^3 = \{(0,0), (1,2), (2,3), \overset{p_j\ w_j}{\cancel{(3,5)}}, \overset{p_k\ w_k}{(5,4)},$

$(6,6), (7,7), (8,9)\}$ purge d_s

(p_j, w_j) (p_k, w_k)

$p_j \leq p_k$ | $w_j \geq w_k$

purged as $w > m$

$S^3 = \{(0,0), (1,2), (2,3), (5,4), (6,6)\}$

Trace Back

$P_1 = 1 \mid P_2 = 2 \mid P_3 = 5$

$W_1 = 2 \mid W_2 = 3 \mid W_3 = 4$

$i=0 \quad i=1 \quad i=2$

max → (6, 6)

✓ first appeared for S^2_1 third element included

$x_1 \quad x_2 \quad x_3$

$x_3 = 1 \;\; (5,4)$

∴

$$\begin{array}{r} (6,6) \\ -\,(5,4) \\ \hline = \boxed{(1,2)} \end{array}$$

✓ (1,2) is first appeared at S^0_1 first element included

x_1	x_2	x_3
1	0	1
(1,2)		(5,4)

$$\begin{array}{r} \boxed{1,2} \\ -\,(1,2) \\ \hline 0 \\ \hline \times \end{array}$$

→ rest are not included

(x_1, x_2, x_3)

Solution $(1, 0, 1)$ for 0/1 Knapsack problem.