Assignment for Research and Development

Key trick (rotation–translation to a canonical form)

From the parametrization

$xy = t\cos\theta - e^{M|t|}\sin(0.3t)\sin\theta + X, = 42 + t\sin\theta + e^{M|t|}\sin(0.3t)\cos\theta,$

translate by $(X,42)$ and rotate by angle $-\theta$:

$x' = (x-X)\cos\theta + (y-42)\sin\theta = t$

$y' = -(x-X)\sin\theta + (y-42)\cos\theta = e^{M|t|}\sin(0.3t).$

So **after the correct shift (X,42) and rotation by θ**, the data must satisfy the 1-D curve

$y' = e^{M|x'|}\sin(0.3x').$

This means we can estimate θ and X by finding the rotation/shift that makes the transformed points (x_i', y_i') fit the above relation best; then M is determined by the vertical growth of y' versus x'.

Practical estimator (single robust fit)

Given observed points (x_i, y_i), define for any candidate (θ, M, X):

$x_i'(\theta, X) = (x_i - X)\cos\theta + (y_i - 42)\sin\theta,$

$y_i'(\theta, X) = -(x_i - X)\sin\theta + (y_i - 42)\cos\theta,$

$\hat{y}_i(\theta, M, X) = e^{M|x_i'|}\sin(0.3x_i').$

Minimize the robust L1-like residuals $r_i = y_i' - \hat{y}_i$ over the bounds

$0^\circ < \theta < 50^\circ,\ \ -0.05 < M < 0.05,\ \ 0 < X < 100.$