Math Image T-1

Total Time: 1 Hr | Total Marks: 20

Science

1) The plane x + 2y – z = 4 cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius- $\sqrt{g^2 + f^2 + w^2 - c}$

Correct Answer: 1

Solution:

R = Radius of sphere is $\sqrt{g^2 + f^2 + w^2 - c}$

$= \sqrt{\frac{1}{4} + \frac{1}{4} + 2} \therefore R = \sqrt{\frac{5}{2}}$

d = distance from centre to the plane is equal to

$d = \left| \frac{\frac{1}{2} + 0 + \frac{1}{2} - 4}{\sqrt{1^2 + 2^2 + 1^2}} \right|$

$d = \frac{3}{\sqrt{6}}$

Radius of the circle

Radius of sphere – perpendicular distance from centre of sphere to plane

$= \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - \left(\frac{9}{6}\right)} = \sqrt{\frac{15}{6} - \frac{9}{6}} = 1$

2) If the plane 2ax – 3ay + 4az + 6 = 0 passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals-

Correct Answer: -2

Solution:

Equation of given spheres are

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13 \ldots .$ (i)

and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8 \ldots .$ (ii)

whose centres are (-3, 4, 1) and (5, -2, 1)

Midpoint of (-3, 4, 1) and (5, -2, 1) is (1, 1, 1)

Since, the given plane 2ax – 3ay +4 az + 6 = 0 passes through (1, 1, 1)

2a – 3a + 4a + 6 = 0

3a = -6

a = -2

3) The angle between the lines 2x = 3y = –z and 6x = –y = –4z is-

Correct Answer: 90

Solution:

$\frac{x}{2} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{x}{-3}$

$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\cos\theta = \frac{6 - 24 + 18}{\sqrt{3^2 + 2^2 + (-6^2)}\sqrt{2^2 + (-12)^2 + (-3)^2}}$

$\therefore \theta = 90^o$

4) The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane x + 2y + 2z + 7 = 0 is-

Correct Answer: 3

Solution:

The radius and centre of sphere

$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is

$\sqrt{1^2+1^2+4+19} = 5$ and center (-1, 1, 2)

PB from centre to the plane

$\frac{|-1+2+4+7|}{\sqrt{1+2^2+2^2}} = 4$

Now $(AB)^2 = AP^2 - PB^2$

$= 25 - 16$

$= 9$

$AB = 3$

5) A person standing on the bank of a river, observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 m away from the tree the angle of elevation becomes 30°. The breadth of the river is

Correct Answer: 20

Solution:

Let CD h ( ) = be the height of the tree and BC x ( ) = be the width of the river.

In ΔCBD, $\tan 60^\circ = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ . . . (i)

and in ΔCAD , $\tan 30^\circ = \frac{CD}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40+x}$

$\Rightarrow h\sqrt{3} = 40 + x$

$\Rightarrow 3x = 40 + x$

$\Rightarrow 2x = 40$

$\therefore$ x = 20 m