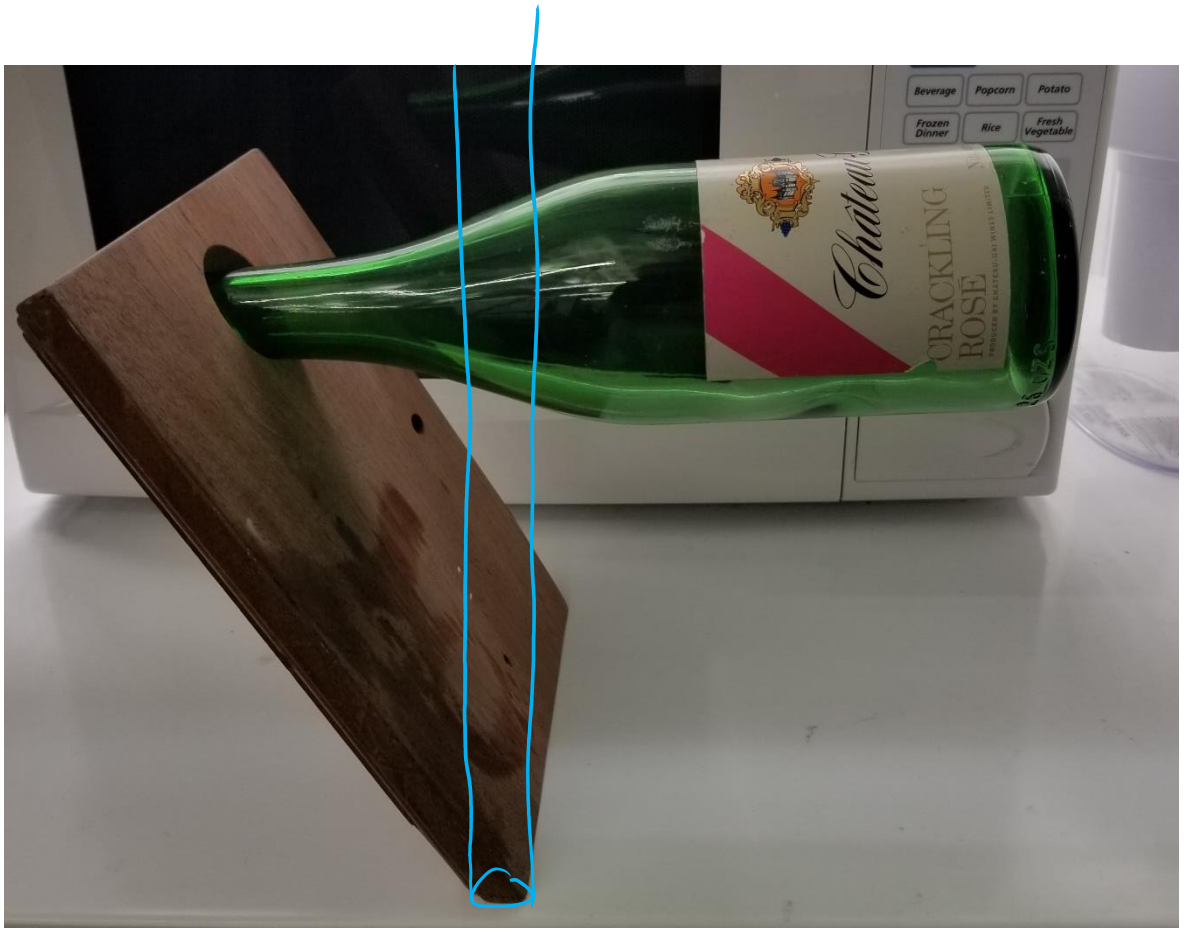
How does this balance?



Chapter 12 – Rotation of a Rigid Body

- Centre of mass and moment of inertia today
- Torque and cross product Friday
- Rolling motion and rotational energy } Monday
- Angular momentum

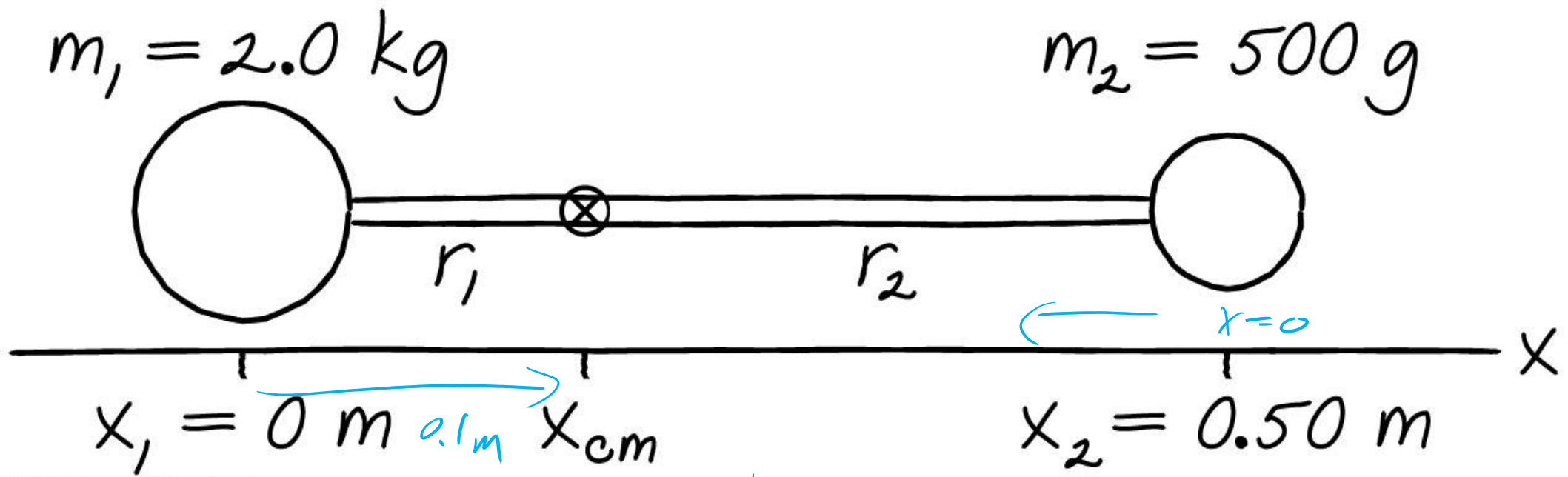
Center of Mass

$$x_{\text{cm}} = \frac{1}{M}\sum_i m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

centre mass

$$y_{\text{cm}} = \frac{1}{M}\sum_i m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \qquad (12.4)$$

$$x_{\text{cm}} = \frac{1}{M}\int x\, dm \qquad \text{and} \qquad y_{\text{cm}} = \frac{1}{M}\int y\, dm \qquad (12.5)$$



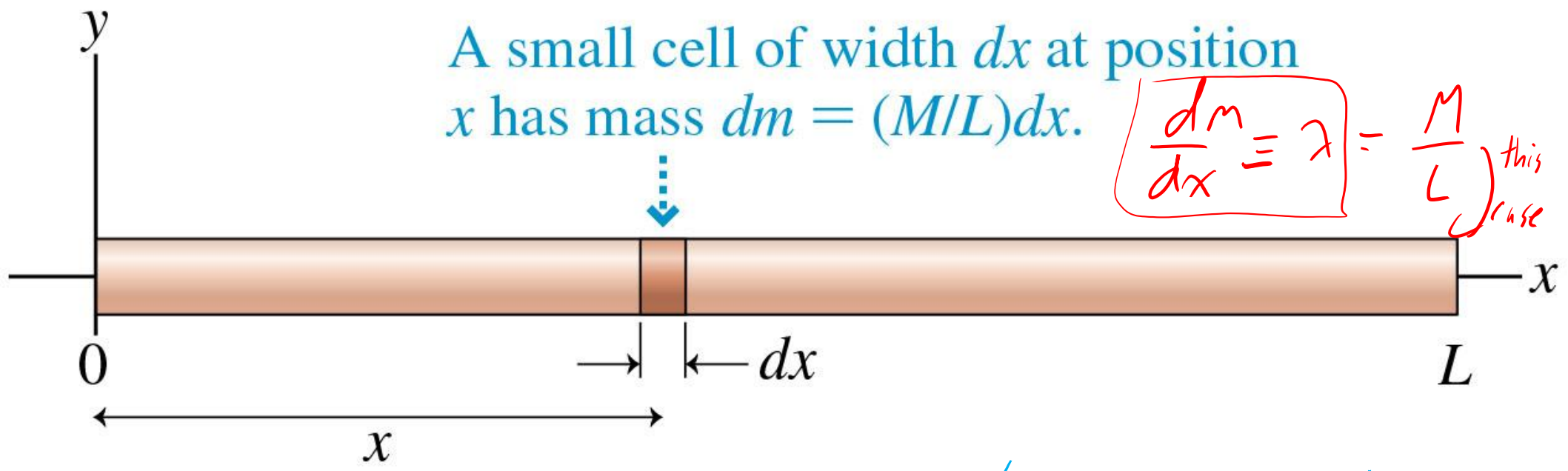
$$x_{cm} = \frac{1}{M}\sum m_i x_i$$

$$= \frac{2000 \times 0 + 500 \times 0.5}{2500} = 0.1\ m$$

$$x_{cm} = \frac{1}{M}\sum m_i x_i$$

$$= \frac{500 \times 0 + 2000 \times 0.5}{2500}$$

$$= 0.4\ m$$



$\frac{dm}{dx} = \lambda = \frac{M}{L}$ this case

$$x_{cm} = \frac{1}{M}\int_0^M x\,dm = \frac{1}{M}\int_0^L x\,\frac{M}{L}\,dx = \frac{1}{M}\,\frac{M}{L}\int_0^L x\,dx = \frac{1}{L}\left[\frac{1}{2}x^2\right]_0^L$$

$$= \frac{1}{L}\left(\frac{1}{2}L^2 - 0\right)$$

$$= \frac{1}{2}L$$

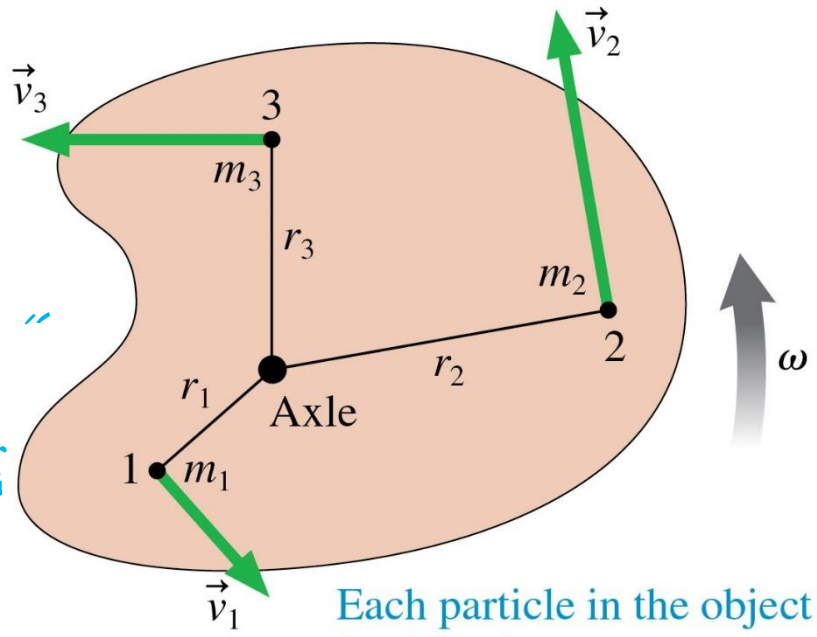
$K = \sum \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i \omega_i^2 r_i^2$

$= \frac{1}{2} \omega^2 \boxed{\sum m_i r_i^2}$

rotational "mass"

rotational inertia

$v = \omega r$



Each particle in the object has kinetic energy as the object rotates.

The **moment of inertia**

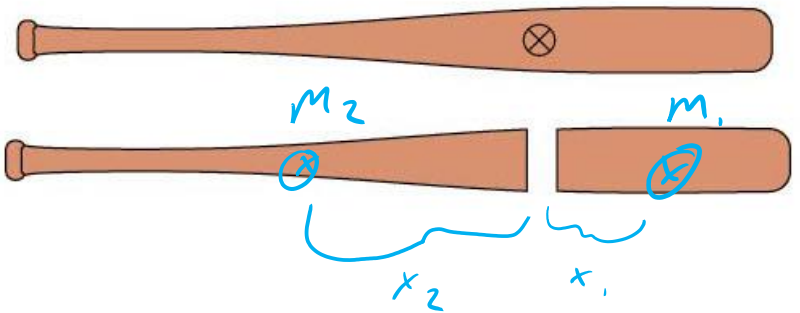
$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, I about a parallel axis distance d away is given by the **parallel-axis theorem:** $I = I_{cm} + Md^2$.

Team Up Questions

STOP TO THINK 12.1 A baseball bat is cut into two pieces at its center of mass. Which end is heavier?

a. The handle end (left end).
b. The hitting end (right end).
c. The two ends weigh the same.



$$x_{cm} = 0 = \frac{1}{M}\sum m_i x_i$$

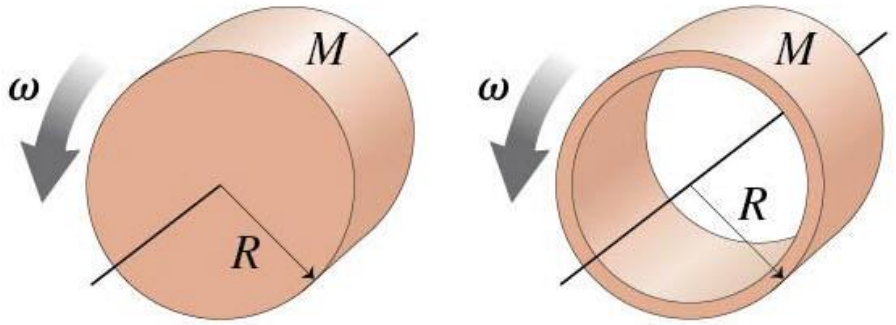
$$= \frac{m_1 x_1 - m_2 x_2}{m_1 + m_2} \rightarrow m_1 x_1 = m_2 x_2$$

$x_2 > x_1 \rightarrow m_1 > m_2$

Team Up Questions

STOP TO THINK 12.2 A solid cylinder and a cylindrical shell, each with radius R and mass M, rotate about their axes with the same angular velocity ω. Which has more kinetic energy?

a. The solid cylinder.
b. The cylindrical shell.
c. They have the same kinetic energy.
d. Neither has kinetic energy because they are only rotating, not moving.



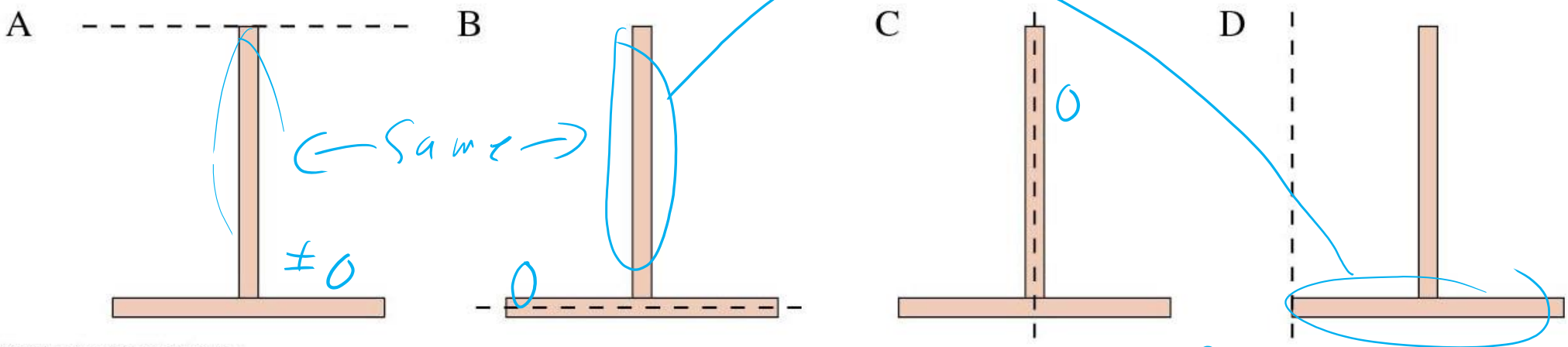
$K = \frac{1}{2} I \omega^2$

$\omega \rightarrow$ Same

$I \sim mr^2$

Team Up Questions

STOP TO THINK 12.3 Four Ts are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moments of inertia I_A to I_D for rotation about the dashed line.



Same

A > D

← Same →

≠ 0

0

0

A > B

B = C

D > C

A > D > B < C

D > B

How does this balance?