test1-paper-171-10

4. [5 marks] **Proofs and property-based testing.**

Recall the function from the previous question:

```
def sum_of_squares(numbers: set[int]) -> int:
    """Return the sum of the squares of each number in numbers.

    If numbers = {n_1, n_2, ..., n_k}, this function returns

        (n_1 ** 2) + (n_2 ** 2) + ... + (n_k ** 2)

    >>> sum_of_squares({1, 2, 3})  # 1 + 4 + 9
    14
    """
    # Assume we have a correct implementation of this function
```

(a) [1 mark] Find a **non-empty** set of integers numbers such that sum_of_squares(numbers) == 0. Briefly justify your response (but a formal proof is not required).

Such a set would be numbers = {0}. Since the square of zero is zero and the sum function on [0] returns 0, the function returns 0 Thus the condition, sum-of-squares({0}) == 0 is true.

(b) [2 marks] Prove that for all sets of integers numbers, if len(numbers) ≥ 2 then sum_of_squares(numbers) > 0. We have provided a start of the proof for you.

Proof. Let numbers be an arbitrary set of integers, and assume len(numbers) ≥ 2, i.e., that numbers contains at least two elements.

Complete the proof in the space below, and on the next page if you need more space.

We know that the square of zero is zero and the square of every other integer is greater than zero. Since numbers is a set it cannot have duplicate values. Furthermore, numbers has atleast two elements so atleast one of them must be a non-zero integer. Therefore the collection inside the sum function has one positive element and no negative elements. The sum of one positive and

Continue your proof from Part (b) here if you need more space.

other non-negative elements must be positive. Thus if len(numbers) ≥ 2 then sum_of_squares(numbers) > 0. Hence Proved.

(c) **[2 marks]** Complete the following property-based test for sum_of_squares that tests whether the following property holds:

$$\forall n \in \mathbb{Z},\ \texttt{sum\_of\_squares}(\{n\}) = n^2.$$

```
from hypothesis import given
from hypothesis.strategies import integers


@given(n=integers())
def test_single(n: int) -> None:
    """Test that sum_of_squares satisfies the property stated above.
    """
    # TODO: Complete this test
    assert sum_of_squares({n}) == n ** 2
```