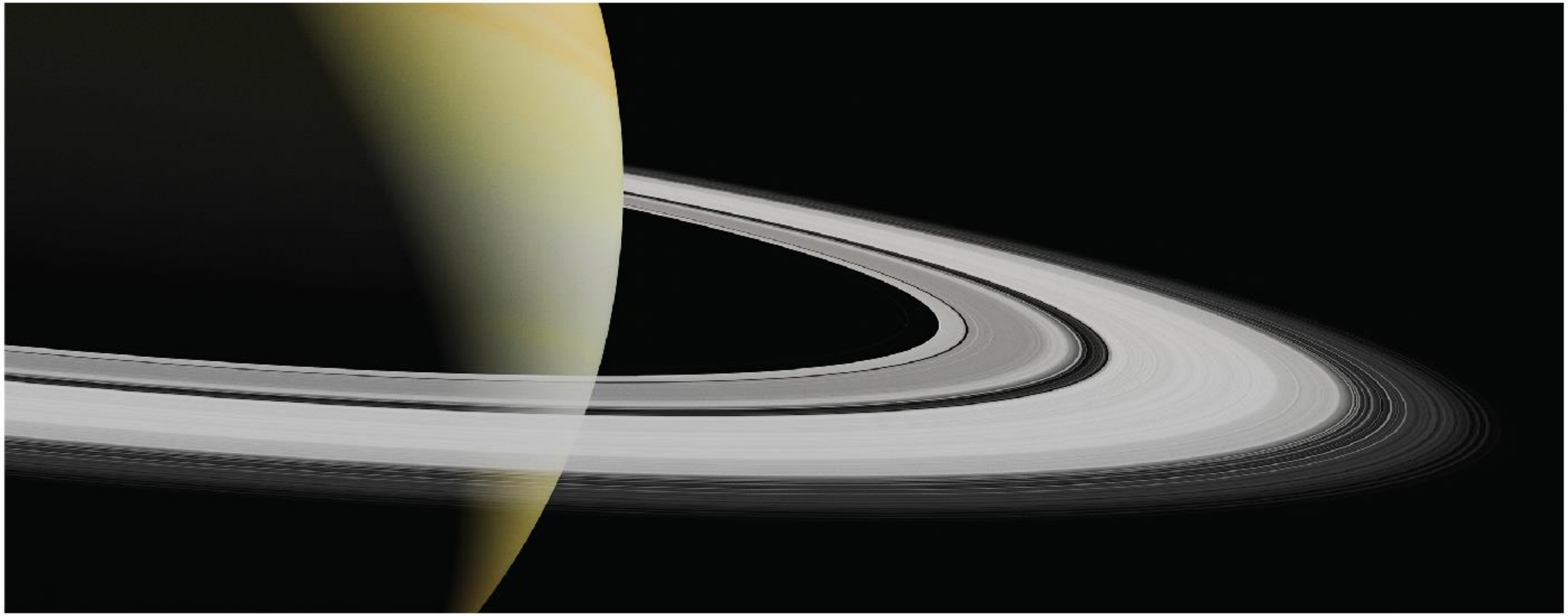
Chapter 13 – Newton’s theory of gravity

- Newton’s theory of gravity
- Kepler’s Laws
- Orbits



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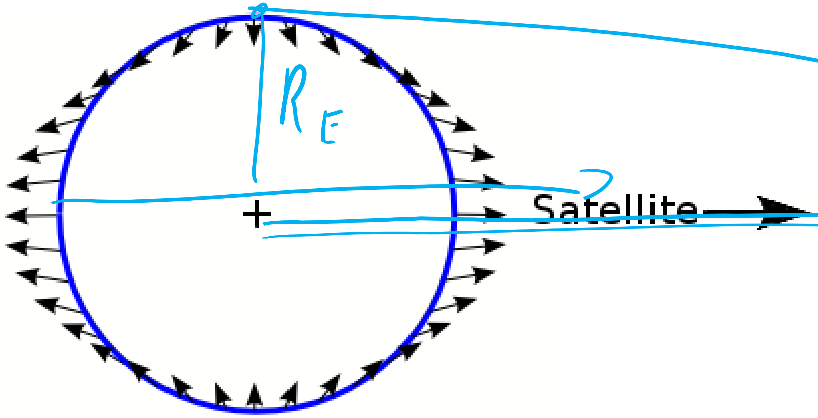
$\frac{R_E}{r} \ll 1$

Tidal forces

$(1+x)^n \approx 1+nx$

$(x \ll 1)$ Binomial approx



wikipedia

r

$$F_G = \frac{GMm}{r^2}$$

$$\Delta F = GMm\left(\frac{1}{r^2} - \frac{1}{(r+R_E)^2}\right)$$

$$= \frac{GMm}{r^2}\left[1 - \frac{1}{\left(1+\frac{R_E}{r}\right)^2}\right]$$

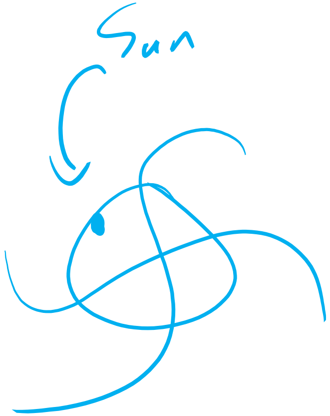
$(1+x)^n$ $n=-2$, $x=\frac{R_E}{r}$

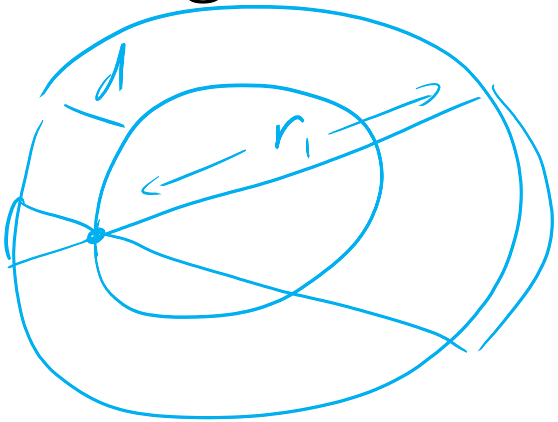
$$\Delta F = \frac{GMm}{r^2}\left[1-\left(1-2\frac{R_E}{r}\right)\right]$$

$$\boxed{\Delta F = \frac{2GMm}{r^3}R_E}$$

tidal force

Rotation curves of galaxies





$$r_2\theta = s_2 \qquad s_1 = r_1\theta$$

$$M = \pi s_2^2 d \qquad M = \pi s_1^2 d$$

$$F = \frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r}$$

$$F \sim \frac{M}{r_1^2} \sim \frac{4\pi r_1^2 d}{r_1^2} \sim \text{const}$$

$$v^2 = \frac{GM}{r}$$

Mass inside our orbit

$$v^2(r) = \frac{GM(r)}{r}$$

$$M(r) = 4\pi \int \rho(r)\, r^2\, dr$$

Team Up questions

$$v^2 = \frac{GM(r)}{r}$$

$$v^2 \propto \frac{M(r)}{r}$$

1 Sphere

$M(r) \propto r^3$

$$v^2 \propto \frac{r^3}{r} = r^2$$

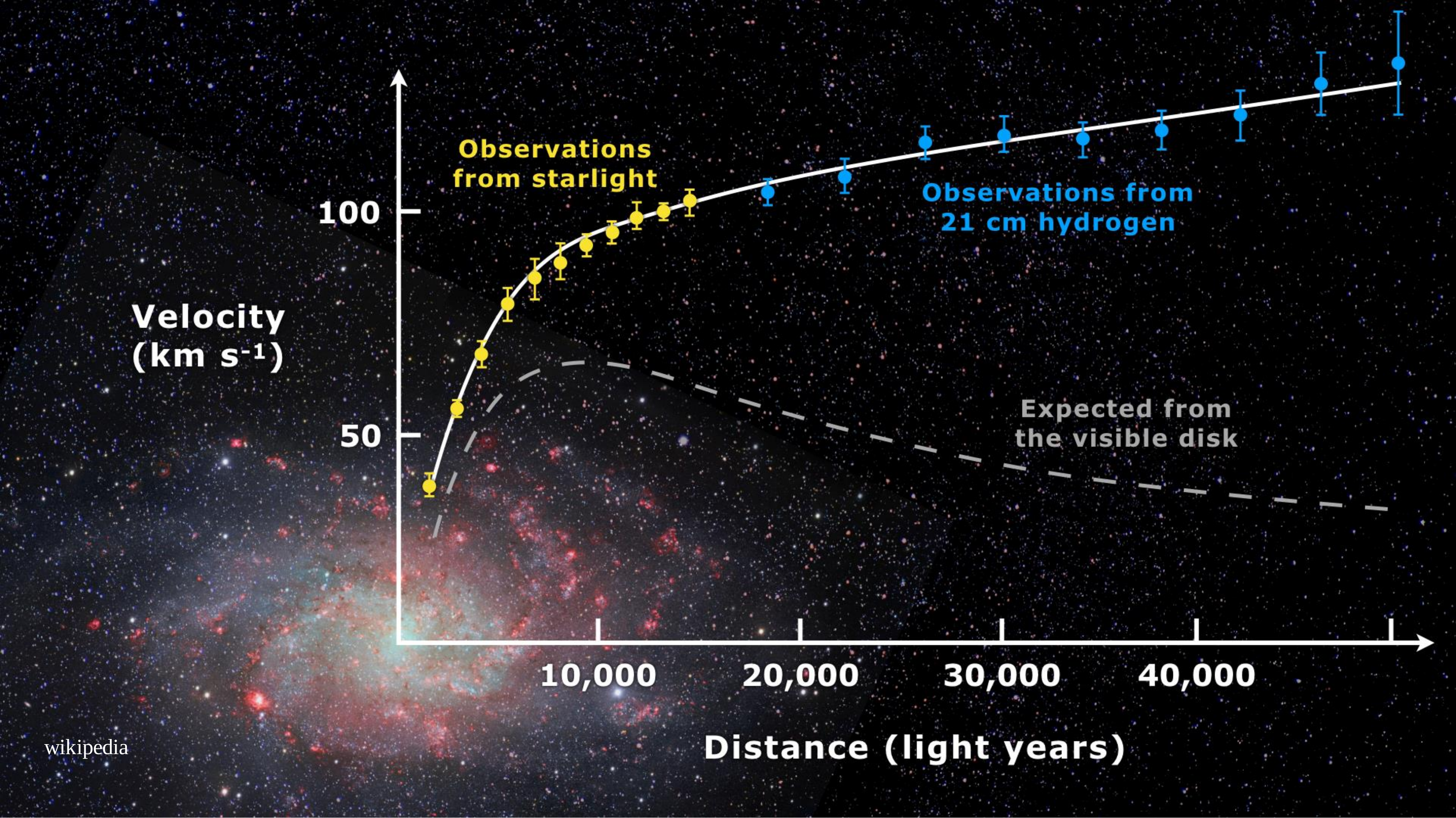
$v \propto r$

2 disk

$M(r) \propto r^2$

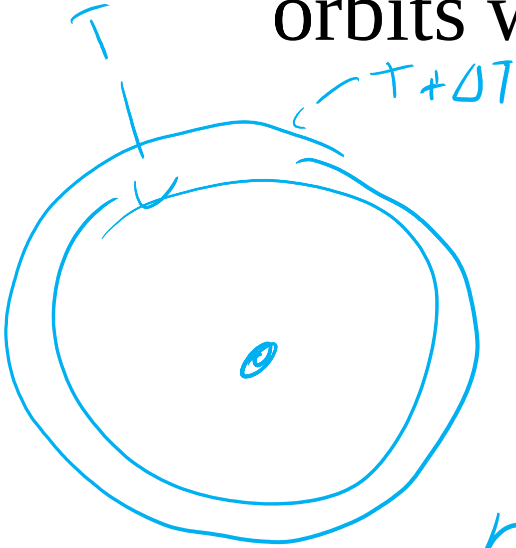
3 far away

$M \sim$ constant


Observations from starlight
Observations from 21 cm hydrogen
Expected from the visible disk
Velocity (km s-1)
100
50
10,000
20,000
30,000
40,000
Distance (light years)

wikipedia

Two satellites are in circular orbits around Earth with radii of 6700 km and 6701 km (from the center of the Earth). One day they pass each other (so only 1 km apart). How many orbits will they make before they repeat this?



$$T^2 = kR^3 \longrightarrow \Delta(T^2) = \Delta(kR^3)$$

$$2T\Delta T = 3R^2\Delta R\, k$$

$$T^2/R^3$$

$$2T\Delta T = 3R^2\Delta R \frac{T^2}{R^3}$$

$$\frac{2\Delta T}{T} = 3\frac{\Delta R}{R}$$

$$(n+1)T = n(T+\Delta T)$$

$$T = n\Delta T$$

$$n = \frac{T}{\Delta T} = \frac{2}{3}\frac{R}{\Delta R}$$

$$= \frac{2}{3}\frac{6700}{1}$$