CSC110 Proof Practice Problems

David Liu and Mario Badr

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For each of the following statements, either prove or disprove them. Most can be proved/disproved using *only* definitions (e.g., of divisibility or prime numbers), though some require using other theorems we've learned about in class, like the Quotient-Remainder Theorem.

When using these problems as practice, do the following:

- Translate the statement into predicate logic.
- If disproving the statement, *negate* the statement using the standard negation rules.
- Set up the correct *proof header* based on the structure of the statement you're proving. This will help you get your proof organized in an automatic way, and is a great habit to get into when learning to write proofs!

1. Every integer divides itself.
2. Every integer is divisible by 1.
3. Every integer divides 0.
4. Every integer is divisible by 0.
5. There exists a non-zero integer that is divisible by every other integer.
6. For all integers a and b, if a divides b and b divides a then $a = b$.
7. For all integers a, b, and c, if a divides b and b divides c then a divides c.
8. For all integers a, b, and c, if a divides c then a divides b and b divides c.
9. For all integers a and b, if both are greater than 1 then ab is not prime.
10. For all integers a, $a^2 + a$ is divisible by 2. (Use the Quotient-Remainder Theorem to split up your proof into two cases based on the remainder when a is divided by 2.)

If you reach the end of this section, don't look for more problems—instead, *try to generate new ones yourself*! You can take any of the statements we've written here and try to modify them in a few different ways:

- Change a statement so that its truth value changes from True to False, or vice versa.
- *Generalize* a True statement so that it covers a broader range of cases but is still True.
- Or, try to generalize a True statement and see that the resulting statement is actually False, and disprove it!