PHY151 Practical Questions for Sept 19 to 23

1. A car makes a round trip of 37 km travelling at a constant speed of 45 km/hr. A heavy truck makes the same trip travelling at 30 km/hr on the way there but much faster on the way back due to having offloaded the cargo (in basically zero time). What minimum return speed does the truck need to travel at in order to beat the car back to the starting point?

2. Water is dripping from the roof of a building. You see the drips fall passed your window. Your window is 1.5 m tall and you measure that each drop of water takes, on average, 0.13 seconds to fall passed your window (from top to bottom of your window). How high above the bottom of your window is the roof?

3. A person runs 100.0 meters in 10.0 seconds. During the first 4.00 seconds their acceleration is

$$a(t) = a_0 \left(1 - \frac{t}{4\,\mathrm{s}}\right)$$

   where a_0 is an unknown constant. After 4.00 seconds the acceleration is zero. Find the top speed of the runner.

4. Modelling question: A cheetah has a top speed of 28 m/s. It is hunting a gazelle, which has a top speed of 25 m/s. However, the cheetah can only maintain top speed for about 15 seconds, whereas the gazelle can run for much longer. How close must the cheetah get to the gazelle while hiding in order to catch the gazelle?

Answers:

1. $t_{t,1} + t_{t,2} = 2t_c \rightarrow \frac{d}{v_{t,1}} + \frac{d}{v_{t,2}} = \frac{2d}{v_c}$

   $v_{t,2} = \frac{v_{t,1}v_c}{2v_{t,1}-v_c} = 90\,\mathrm{km/h}$

2. Set the roof at y=0 and make down positive. The drops fall as

   $y(t) = \frac{1}{2}gt^2$

   where $t = 0$ at the instant each drop begins its fall. Let h_1 be the height from the roof to the top of your window and h_2 be to the bottom of the window (so we want h_2). Let t_1 be the time the water passes the top of your window and t_2 be the time the water passes the bottom of your window. We know

   $\Delta y = y(t_2) - y(t_1) = 1.5\,\mathrm{m}$ and $\Delta t = t_2 - t_1 = 0.13\,\mathrm{s}$

   so we can find

   $y(t_2) - y(t_2 - \Delta t) = \Delta y = \frac{1}{2}g(t_2^2 - (t_2 - \Delta t)^2) = \frac{1}{2}g(2t_2\Delta t - (\Delta t)^2)$

   $t_2 = \frac{2\Delta y/g + (\Delta t)^2}{2\Delta t} = 1.24\,\mathrm{s}$

   So we have the desired height as

   $h_2 = \frac{1}{2}gt_2^2 = 7.6\,\mathrm{m}$

3. $v_{max} = \frac{1}{2}a_0(4\,\mathrm{s})$

   $v(t) = \frac{1}{2}v_{max}(t - t^2/8)$ for $t < 4$, else $v(t) = v_{max}$ (Note: I've dropped the units here.)

   $100\,\mathrm{m} = \frac{v_{max}}{16}\int_0^4 (8t - t^2)\,dt + v_{max}(6\,\mathrm{s})$

   $100\,\mathrm{m} = v_{max}(\frac{8}{3} + 6)(1\,\mathrm{s})$

   $v_{max} \simeq 11.5\,\mathrm{m/s}$

4. Here's one possible attempt, others are fine:

   They have almost the same top speed, so assume they have the same acceleration and the same time to top speed. That means we can ignore the acceleration phases. Assume they both take 3 seconds to hit top speed.

   Now we only need to worry about the difference in top speed and the time delay between when the cheetah starts to run and when the gazelle realizes it's in danger and also starts to run. Let's assume that time delay is two seconds.

   Ignoring the 3-second acceleration phases, the cheetah gets 2 seconds at top speed plus 10 seconds with a small (3 m/s) speed advantage (3+2+10=15 seconds total). That gives us a distance of

   $d = (28\,\mathrm{m/s})(2\,\mathrm{s}) + (3\,\mathrm{m/s})(10\,\mathrm{s}) = 86\,\mathrm{m}$

   Given the assumptions made, one significant figure is all we can really justify, so call it about 90 meters.