MAT137Y Tutorial 9 worksheet

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TOTAL POINTS

2 / 2

QUESTION 1

1 Q1+Q2+Q3(a) **2 / 2**

✓ **- 0 pts** **Correct / a few minor mistakes**

- 1 pts Major mistake(s) or some parts incomplete, but significant effort shown

- 2 pts Too many errors/insufficient progress

- 2 pts No TA signature

MAT 137
Tutorial #9– Inverse Trig functions
Nov 22-23, 2022
Due on Thursday, Nov 24 by 11:59pm via GradeScope

- **Submissions are only accepted by Gradescope**. Do not send anything by email. Late submissions are not accepted under any circumstance. Remember you can resubmit anytime before the deadline.
- **Submit your polished solutions using only this template PDF.** You will submit a single PDF with your full written solutions. If your solution is not written using this template PDF (scanned print or digital) then you will receive zero. Do not submit rough work. Organize your work neatly in the space provided.
- **Show your work and justify your steps** on every question, unless otherwise indicated. Put your final answer in the box provided, if necessary.

We recommend you write draft solutions on separate pages and afterwards write your polished solutions here. You must fill out and sign the academic integrity statement below; otherwise, you will receive zero.

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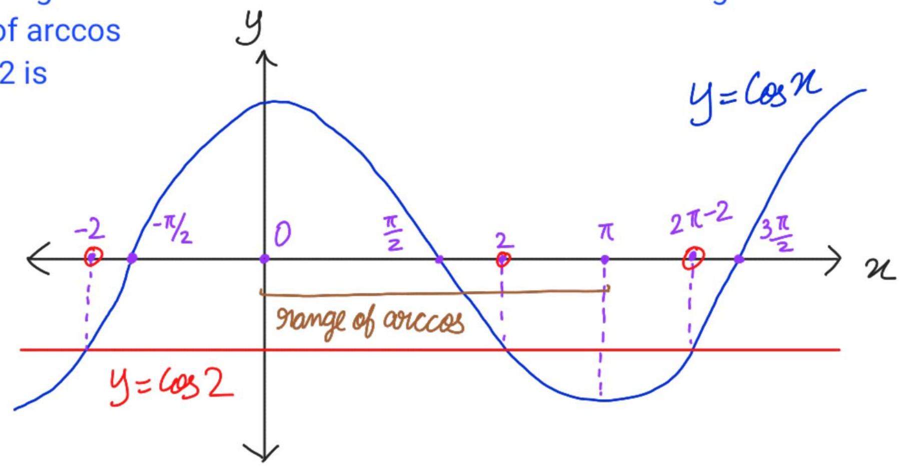
1. Compute:

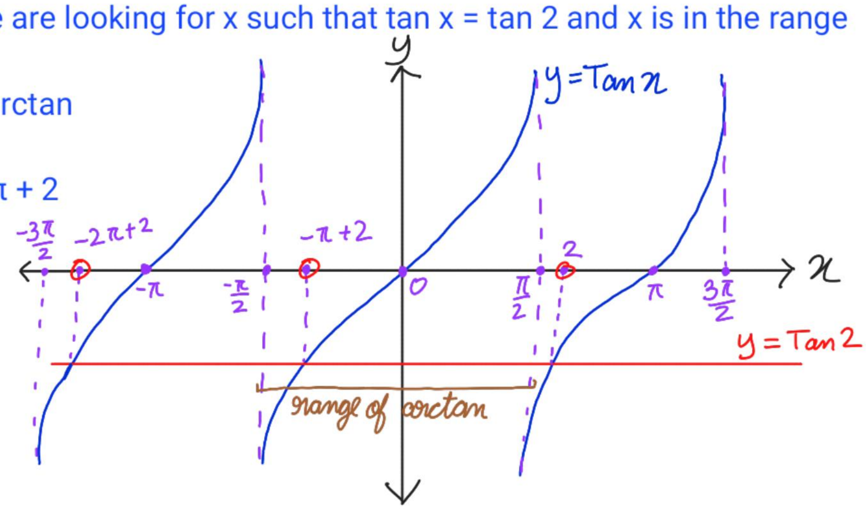
   (a) arccos(cos 2) Hint: How do we define arccos? What are the domain and the range of arccos? Let $x = \arccos(\cos 2)$. We are looking for x such that $\cos x = \cos 2$ and x is in the range of arccos.

arccos is defined as the inverse function of the restriction of cos to [0, π]. Its domain is [-1, +1] and range is [0, π]. Let x = arccos(cos 2), we are looking for x such that cos x = cos 2 and x is in the range of arccos.
From the graph we see that in the range of arccos the only value of x satisfying cos x = cos 2 is x = 2, thus the value of arccos(cos 2) = 2



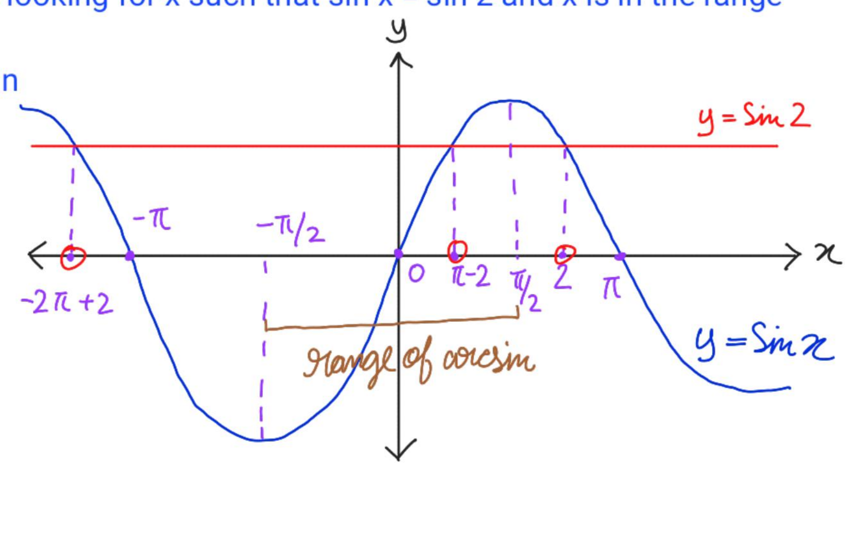
   (b) arctan(tan 2)

arctan is defined as the inverse function of the restriction of tan to [-π/2 , π/2]. Its domain is (-∞, +∞) and range is [-π/2 , π/2]. Let x = arctan(tan 2), we are looking for x such that tan x = tan 2 and x is in the range of arctan.
From the graph we see that in the range of arctan the only value of x satisfying tan x = tan 2 is x = -π + 2, thus the value of arctan(tan 2) = -π + 2



   (c) arcsin(sin 2)

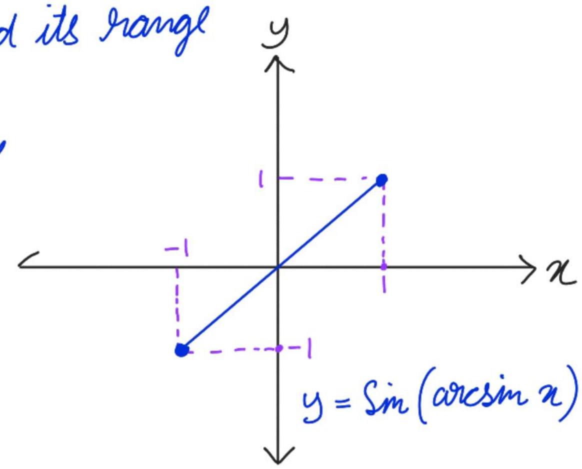
arcsin is defined as the inverse function of the restriction of sin to [-π/2 , π/2]. Its domain is (-1, +1) and range is [-π/2 , π/2]. Let x = arcsin(sin 2), we are looking for x such that sin x = sin 2 and x is in the range of arcsin.
From the graph we see that in the range of arcsin the only value of x satisfying sin x = sin 2 is x = π - 2, thus the value of arcsin(sin 2) = π - 2



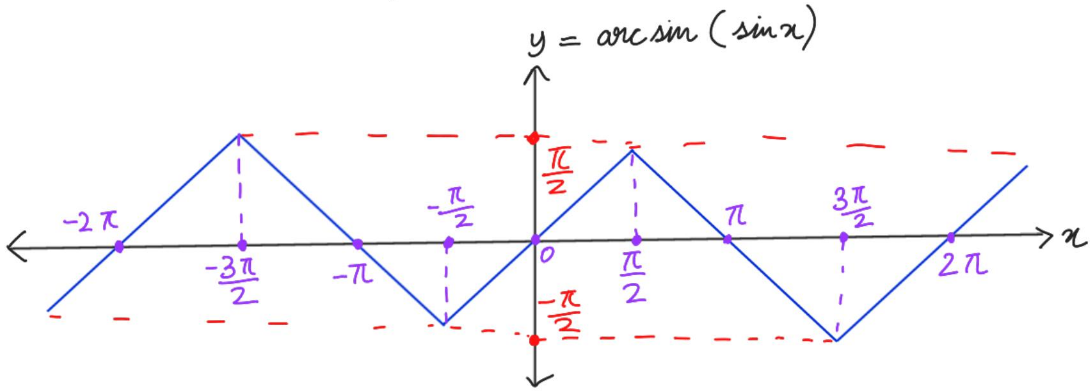
2. Sketch the graph of the following functions.

(a) $f(x) = \sin(\arcsin x)$ Hint: what is the domain of this function? what is the range of this function?

The domain of $f(x) = \sin(\arcsin x)$ is $[-1, 1]$ and its range is $[-1, 1]$. Since arcsin is the inverse of the restriction of sin to $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and the domain of arcsin is $[-1, 1]$, $y = \sin(\arcsin x) = x$



(b) $f(x) = \arcsin(\sin x)$ Hint: what is the domain of this function? what is the range of this function? Is this function periodic?



The domain of $f(x) = \arcsin(\sin x)$ is $\mathbb{R}$ and range is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. arc sin is the inverse of the restriction of Sin to $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ but the domain of Sin is $\mathbb{R}$ so $y = \arcsin(\sin x)$ forms a periodic function which repeats itself after an interval of 2π.

(c) $f(x) = \tan(\arctan x)$**(practice after tutorial)**

(d) $f(x) = \arctan(\tan x)$**(practice afte tutorial)**

Warning: The four graphs are all different.

3. We restrict the function $\sec x$ on $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ to get an one-to one function. We define the inverse of it as arcsec. The function arcsec has domain $(-\infty, -1] \cup [1, \infty)$. Its derivative is

$$\frac{d}{dx}\text{arcsec}x = \frac{1}{|x|\sqrt{x^2-1}}$$

Derive this formula in two different ways:

(a) First, prove the formula by using that $\text{arcsec}x = \arccos\frac{1}{x}$ and a formula $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$.

We know that, $\text{arc sec } x = \text{arc cos } \frac{1}{x}$, differentiating both sides w.r.t $x \rightarrow$

$$\frac{d}{dx}\text{arc sec } x = \frac{d}{dx}\text{arc cos }\frac{1}{x} = \frac{-1}{\sqrt{1-(\frac{1}{x})^2}} \times \left(\frac{-1}{x^2}\right) = \frac{1}{x^2\sqrt{\frac{1}{x^2}(x^2-1)}} = \frac{1}{x^2\left|\frac{1}{x}\right|\sqrt{x^2-1}}$$

Here absolute value comes from the square root, since it is always non-negative.

$$\Rightarrow \frac{d}{dx}\text{arc sec } x = \frac{1}{x^2\left|\frac{1}{x}\right|\sqrt{x^2-1}} = \frac{1}{|x|\sqrt{x^2-1}}$$

(b) Second, prove the formula by implicitly differentiating $\sec(\text{arcsec}x) = x$. Hint: What is the range of arcsec? What are the domain and range of $f(x) = \sec(\text{arcsec}(x))$? **We will not mark this one. However, we strongly recommend you to solve this question as a practice. It's a good practice for your PS3.**

In both cases, make sure you fully explain where the absolute value comes from. **More practice after the tutorial:**

4. Find formulas for the following expressions, using rational functions and roots (if necessary). Write the values of x for which is formula is valid.

(a) $\arccos(\sin x) =$

(b) $\sec(\arccos x) =$

(c) $\sin(\arccos x) =$

(d) $\sin(\arctan x) =$

(e) $\cot(\text{arcsec}x) =$