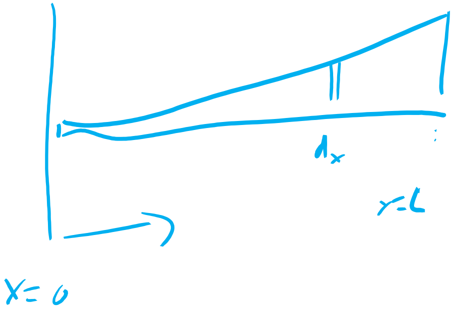
Chapter 12 – Rotation of a Rigid Body

- Centre of mass and moment of inertia
- Torque and cross product
- Rolling motion and rotational energy
- Angular momentum

Examples today

An object has length L and mass density $dm/dx = A\,x^2$. Find the x-component of its centre of mass.



$$\lambda(x) \equiv \frac{dm}{dx} = Ax^2 \longrightarrow dm = Ax^2\,dx$$

$$\int dm = M = \int_0^L Ax^2\,dx = A\int_0^L x^2\,dx = A\,\frac{1}{3}x^3\Big|_{x=0}^{x=L}$$

$$= A\,\frac{1}{3}\left(L^3 - 0^3\right)$$

$$M = \frac{1}{3}AL^3$$

$$\boxed{dm = \frac{3M}{L^3}x^2\,dx}$$

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\,\frac{3M}{L^3}x^2\,dx$$

$$= \frac{3}{L^3}\int_0^L x^3\,dx = \frac{3}{L^3}\,\frac{1}{4}x^4\Big|_{x=0}^{x=L}$$

$$= \frac{3}{4L^3}\left(L^4 - 0^4\right) = \frac{3}{4}L$$

An object has length L and mass density $dm/dx = A x^2$.
Find I around the centre of mass (around the y-axis).

$$I_{cm} = \int (x - x_{cm})^2 \, dm$$

$$I_{cm} = \int r^2 \, dm$$

$$= \int_0^L \left(x - \frac{3}{4}L\right)^2 \frac{3M}{L^3} x^2 \, dx$$

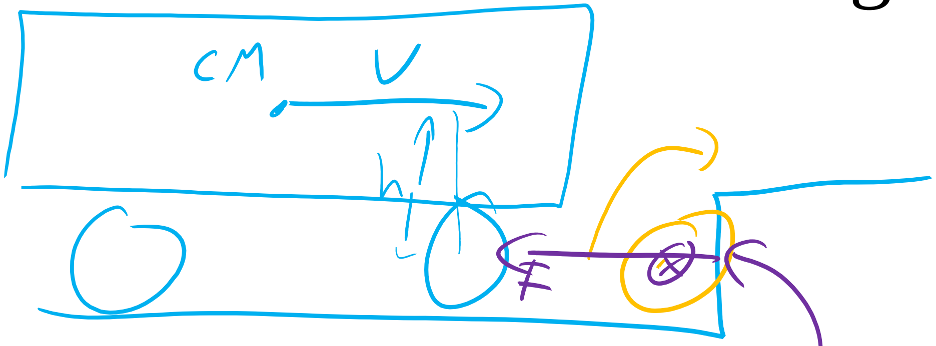
$$x_{cm} = \frac{3}{4} L$$

$$I_{cm} = \frac{3M}{L^3} \int_0^L \left(x^2 - \frac{3}{2} xL + \frac{9}{16} L^2\right) x^2 \, dx$$

$$I_{cm} = \frac{3}{80} ML^2$$

A child's wagon is rolling on the street when it hits the curb. ~~How fast must~~ the wagon go to flip onto the sidewalk?

$\overleftarrow{L}$



Where is the pivot point?

assume L_i → point particle

$L_i = mvh \quad (\vec{r} \times \vec{p}) \qquad h = r\sin\theta$

assume I_f is a rod $I \sim \frac{1}{3} m l^2$

after collision → conserve E

$\vec{\tau} = 0 \quad \therefore \quad \Delta \vec{L} = 0$

$L_i = L_f$

$\cancel{m} v h = \frac{1}{3} \cancel{m} l^2 \omega$

$E_i = E_f$

$\frac{1}{2} I \omega^2 = mgH$

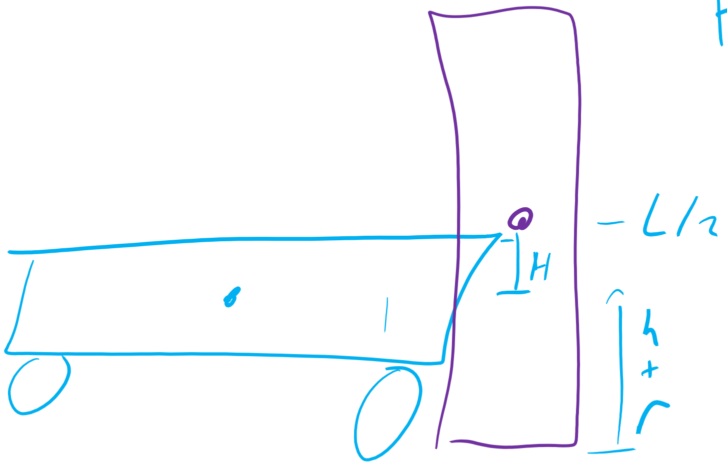
A child's wagon is rolling on the street when it hits the curb. How fast must the wagon go to flip onto the sidewalk?

$$H = \left(\frac{L}{2} - h - r\right)$$

$$\frac{1}{2} I \omega^2 = mgH$$

$$\frac{1}{2} I \omega^2 = mg\left(\frac{\ell}{2} - h - r\right)$$

$$\frac{1}{2} I \left(\frac{2vh}{\ell^2}\right)^2 = mg\left(\frac{\ell}{2} - h - r\right)$$



$$L \rightarrow \quad vh = \frac{1}{3} \ell^2 \omega$$

Team Up Questions

Counter steering

A bowling ball is thrown down the lane with zero initial spin. What will its final spin be? (Bowling lanes have small but non-zero kinetic friction.)