

Chapter 13 – Newton's theory of gravity

- Newton's theory of gravity
- Kepler's Laws
- Orbits



Newton's Theory of Gravity

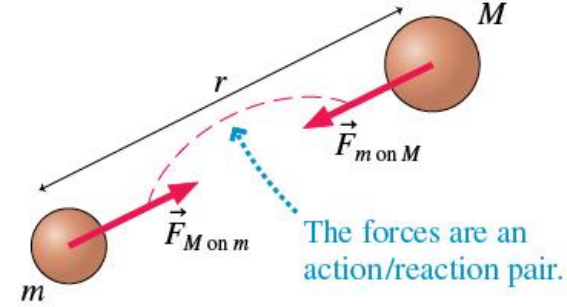
- Two objects with masses M and m a distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{M \text{ on } m} = F_{m \text{ on } M} = \frac{GMm}{r^2} \sim 1/r^2$$

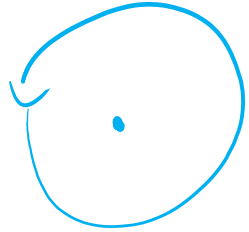
where the **gravitational constant** is $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$. Gravity is an **inverse-square** force.

- Gravitational mass and inertial mass are equivalent.
- Newton's three laws of motion apply to all objects in the universe.

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Un. form Circular motion



$$\frac{mv^2}{R} = \sum F = \frac{GMm}{R^2}$$

$$v^2 = GM/R$$

$$v = 2\pi R/T \quad \leftarrow \text{Period of orbit}$$

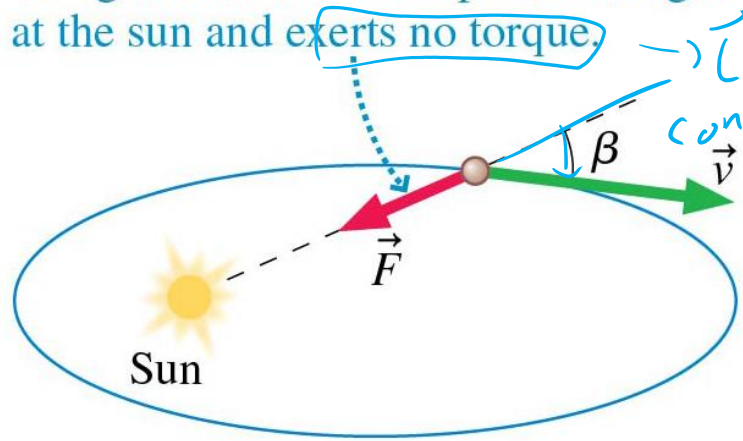
$$(2\pi)^2 \frac{R^2}{T^2} = GM/R$$

$$R^3 = \frac{GM}{4\pi^2} T^2$$

$$T^2 = \frac{4\pi^2}{GM} R^3$$

(a)

The gravitational force points straight at the sun and exerts no torque.



\vec{L} is constant

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r \frac{\Delta s \sin \beta}{\Delta t}$$

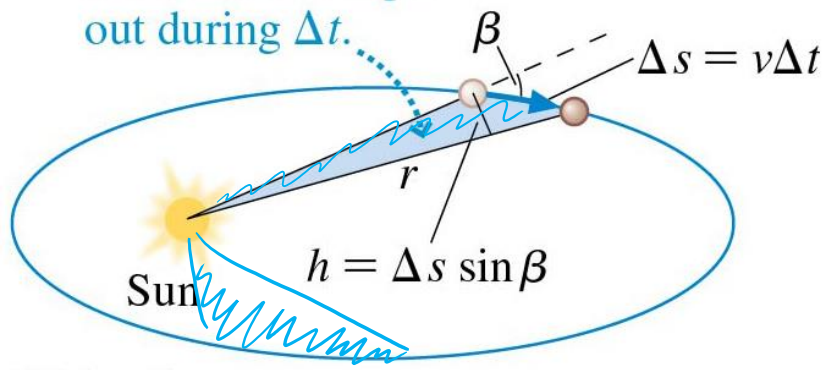
Δt is small $\Delta s \approx v \Delta t$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{r \sin \beta}{\Delta t} v \Delta t = \frac{1}{2} r v \sin \beta = \frac{1}{2} |\vec{r} \times \vec{v}|$$

$$m \frac{dA}{dt} \sim \frac{1}{2} \vec{L}$$

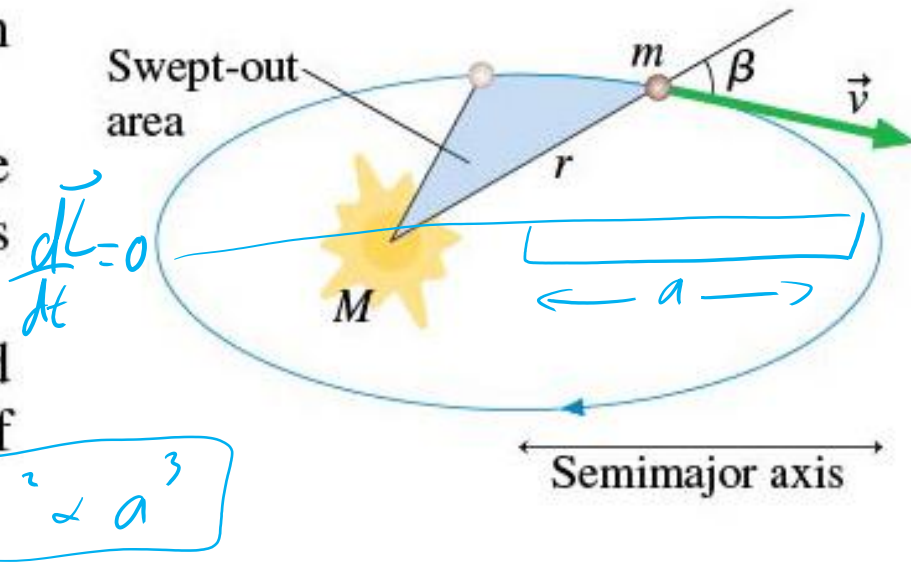
(b)

Area ΔA is swept out during Δt .



Orbital motion of a planet (or satellite) is described by **Kepler's laws**:

1. Orbits are ellipses with the sun (or planet) at one focus.
2. A line between the sun and the planet sweeps out equal areas during equal intervals of time.
3. The square of the planet's period T is proportional to the cube of the orbit's semimajor axis.



Circular orbits are a special case of an ellipse. For a circular orbit around a mass M ,

$$v = \sqrt{\frac{GM}{r}} \quad \text{and} \quad T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

Conservation of Angular Momentum

The angular momentum $L = mrv \sin \beta$ remains constant throughout the orbit. Kepler's second law is a consequence of this law.

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Orbital Energetics

A satellite's mechanical energy $E_{\text{mech}} = K + U_G$ is conserved, where the gravitational potential energy is

$$U_G = -\frac{GMm}{r} \rightarrow -\frac{dU}{dr} = F_G = -\frac{GMm}{r^2}$$

For circular orbits, $K = -\frac{1}{2}U_G$ and $E_{\text{mech}} = \frac{1}{2}U_G$. Negative total energy is characteristic of a **bound system**.

$$E = K + U \\ = -\frac{U}{2} + U = \frac{1}{2}U$$

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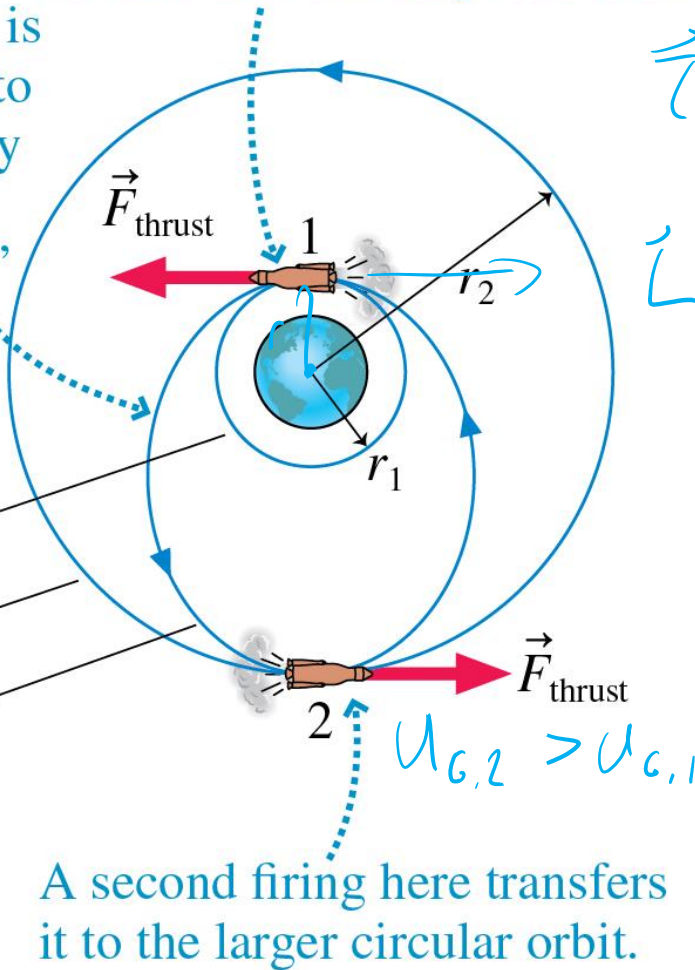
Firing the rocket tangentially to the circle here moves the satellite into the elliptical orbit.

Kinetic energy is transformed into potential energy as the rocket moves "uphill."

Initial orbit

Desired orbit

Elliptical transfer orbit



A second firing here transfers it to the larger circular orbit.

$$\vec{L} = \vec{r} \times \vec{F} \quad \odot \rightarrow \frac{dL}{dt} > 0$$

$$L \odot$$

small thrust (impulse approx)

$p \sim \text{constant}$

$$\vec{L} = (\vec{p} \times \vec{r}) \quad \times \text{ constant}$$

$$\Delta |\vec{p}| > 0 \quad \text{speed up}$$

$$U_{G,2} > U_{G,1}$$