MAT137Y Tutorial 10 worksheet

Shivesh Prakash, Raffaele Dengler, Fahia Mohamed, Kaamil Shaikh

TOTAL POINTS

2 / 2

QUESTION 1

1 Q1+Q2 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** No signature
- **- 1 pts** Mistake
- **- 2 pts** Multiple mistakes or no effort
- **- 2 pts** Blank

MAT 137
Tutorial #9– MVT and Related Rates
Nov 29-30, 2022
Due on Thursday, Dec 1 by 11:59pm via GradeScope

- **Submissions are only accepted by** Gradescope. Do not send anything by email. Late submissions are not accepted under any circumstance. Remember you can resubmit anytime before the deadline.
- **Submit your polished solutions using only this template PDF.** You will submit a single PDF with your full written solutions. If your solution is not written using this template PDF (scanned print or digital) then you will receive zero. Do not submit rough work. Organize your work neatly in the space provided.
- **Show your work and justify your steps** on every question, unless otherwise indicated. Put your final answer in the box provided, if necessary.

We recommend you write draft solutions on separate pages and afterwards write your polished solutions here. You must fill out and sign the academic integrity statement below; otherwise, you will receive zero.

Academic integrity statement

I confirm that:

- I have read and followed the policies described in the Policies and FAQ.
- I understand that the collaboration of this worksheet is only among this group. I have not violated this rule while writing this worksheet.
- I understand the consequences of violating the University's academic integrity policies as outlined in the Code of Behaviour on Academic Matters. I have not violated them while writing this assessment.

By signing this document, I agree that the statements above are true.

First Name	Last Name	UofT email	signature
SHIVESH	PRAKASH	SHIVESH.PRAKASH@MAIL.UTORONTO.CA	[illegible]
Kaamil Saveed	Shaikh	kaamil.shaikh@mail.utoronto.ca	[illegible]
Raffaele	Dengler	r.dengler@mail.utoronto.ca	[illegible]
Fahia	Mohamed	fahia.mohamed@mail.utoronto.ca	[illegible]

TA name: AUSTIN TA signature: [illegible]

1. Let $a < b$. Let f and g be functions defined on $[a, b]$. f and g are continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b) = 0$, show that $f'(c) + f(c)g'(c) = 0$ for some $c \in (a, b)$.

(a) Consider $h(x) = f(x)e^{g(x)}$. What are $h(a)$ and $h(b)$?

Since $f(a) = f(b) = 0$

$h(a) = f(a) \cdot e^{g(a)} = 0$ and $h(b) = f(b) \cdot e^{g(b)} = 0$

(b) Can we say h is continuous on $[a, b]$ and differentiable on (a, b)? If so, prove it.

Since g is continuous on [a, b], differentiable on (a, b) and e^x is continuous and differentiable everywhere, thus $e^{g(x)}$ is continuous on [a, b] and differentiable on (a, b). Furthermore, f is continuous on [a, b] and differentiable on (a, b), thus $h(x) = f(x).e^{g(x)}$ is continuous on [a, b] and differentiable on (a, b).

(c) Compute $h'(x)$.

$h(x) = f(x) \cdot e^{g(x)}$ Differentiating with respect to x on both sides and using chain rule

$h'(x) = f'(x) \cdot e^{g(x)} + f(x) \cdot g'(x) e^{g(x)} = e^{g(x)} \left[f'(x) + f(x) \cdot g'(x) \right]$

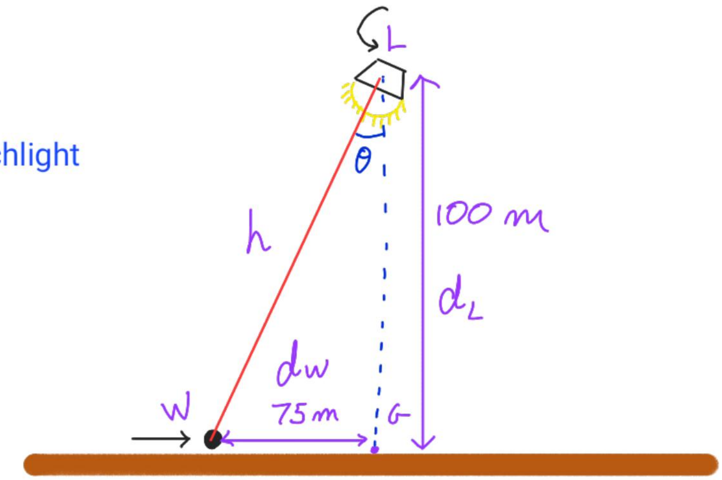
(d) Apply the Rolle's Theorem and prove $f'(c) + f(c)g'(c) = 0$ for some $c \in (a, b)$.

Observe that h is continuous on [a, b] and differentiable on (a, b). Furthermore h(a) = h(b) = 0, applying Rolle's theorem on h we say that $\exists$ c $\in$ (a, b) such that h'(c) = 0

$\Rightarrow h'(c) = e^{g(c)} \left[f'(c) + f(c) \cdot g'(c) \right] = 0$

Since $e^x \neq 0$ for any real x, thus f'(c) + f(c).g'(c) = 0 for some c $\in$ (a, b).

2. A woman walks along a straight path at a speed of 2 m/s. A searchlight is located on the ground 100 meters from the path and is kept focused on her. At what rate is the searchlight rotating when the woman is 75 meters from the point on the path closest to the searchlight? Hint: draw the scenario first.



In the diagram,
W marks the current position of the woman
L shows the location of the searchlight
G represents the point on ground closest to the searchlight
Distance d_L = 100m is the distance between L and G
Distance d_w = 75m is the distance between W and G
Angle θ is the angle between LW and LG
h = $\sqrt{d_w^2 + d_L^2}$ from Pythagoras theorem

Observe that,
d_L is constant and fixed at 100m, so $\frac{d}{dt}(d_L) = 0$

d_w decreasing at the rate of 2 m/s, so $\frac{d}{dt}(d_w) = -2$ m/s

We are required to find the rate of rotation of searchlight or rate of change of θ or $\frac{d}{dt}(\theta)$

From the triangle LGW, tan θ = $\frac{d_w}{d_L}$

Differentiating this equation with respect to time t, using quotient rule and chain rule of differentiation:

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{d_w}{d_L}\right) \Rightarrow \sec^2\theta \cdot \frac{d}{dt}(\theta) = \frac{d_L \cdot \frac{d}{dt}(d_w) - d_w \cdot \frac{d}{dt}(d_L)}{(d_L)^2}$$

Substituting values of $\frac{d}{dt}(d_w)$ and $\frac{d}{dt}(d_L)$ from our previous calculations and secθ = $\frac{\sqrt{d_w^2 + d_L^2}}{d_L}$:

$$\left(\frac{d_w^2 + d_L^2}{\cancel{d_L^2}}\right)\frac{d}{dt}(\theta) = \frac{d_L \cdot \frac{d}{dt}(d_w) - d_w \cdot \frac{d}{dt}(d_L)}{\cancel{(d_L)^2}} \Rightarrow (75^2 + 100^2)\frac{d}{dt}(\theta) = 100(-2) - 75(0)$$

$$\Rightarrow \frac{d}{dt}(\theta) = \frac{-200}{15625} = -0.0128 \text{ rad/s}$$

The negative sign indicates that the angle θ is decreasing, thus the rate of change of θ or rate of rotation of searchlight is 0.0128 rad/s in the anti clockwise direction.