Chapter 4 – Kinematics in 2D

- Mathematics
- Projectile motion
- Relative motion
- Circular motion (uniform, then nonuniform)

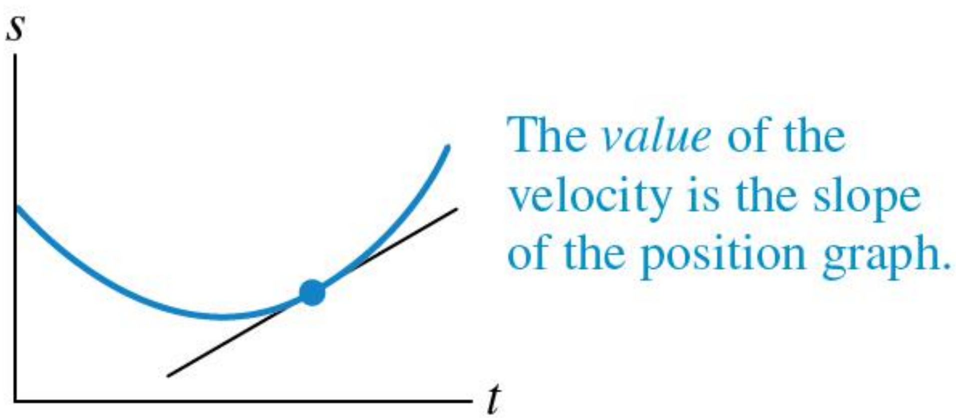
Kinematics in 2D: Mathematics

$$\vec{r} = x\hat{i} + y\hat{j}$$

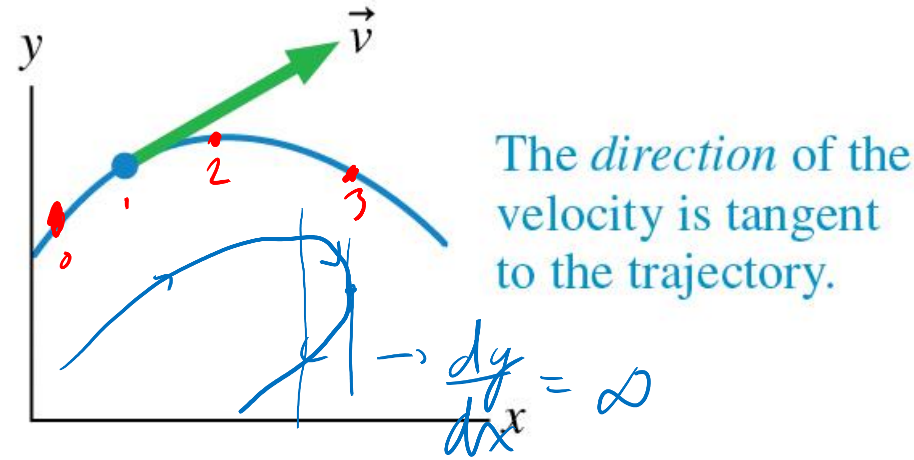
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

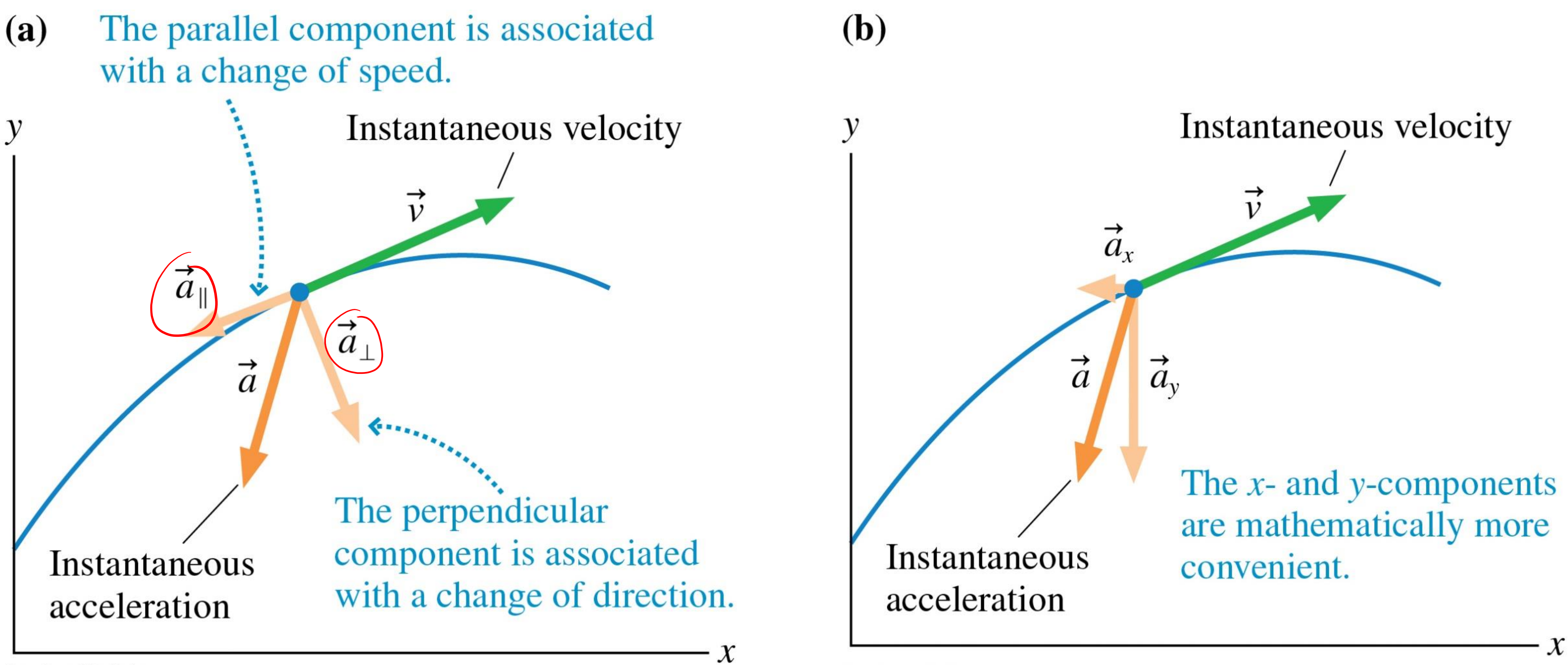
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

Position-versus-time graph



Trajectory





A car goes down a hill, through a valley, then up a hill on the other side of the valley.

constant $\vec{a}$

$\vec{a} \parallel \vec{v}$

speeding up

1D

2D

changing $\vec{a}$

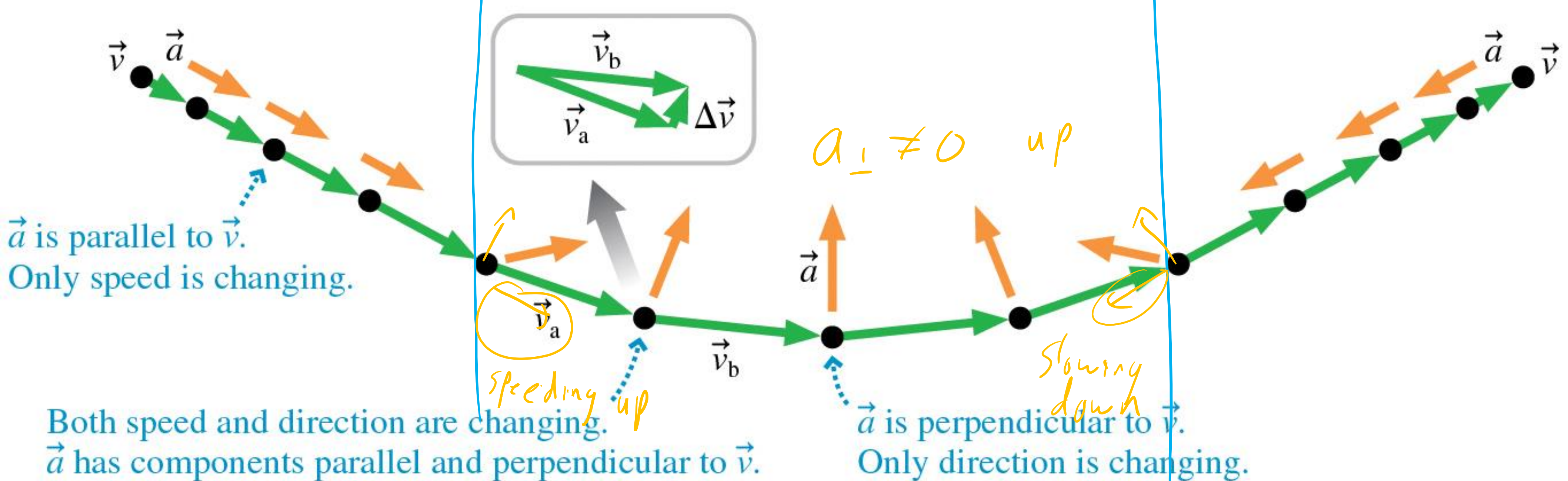
$\vec{a} \nparallel \vec{v} \to$ 2D

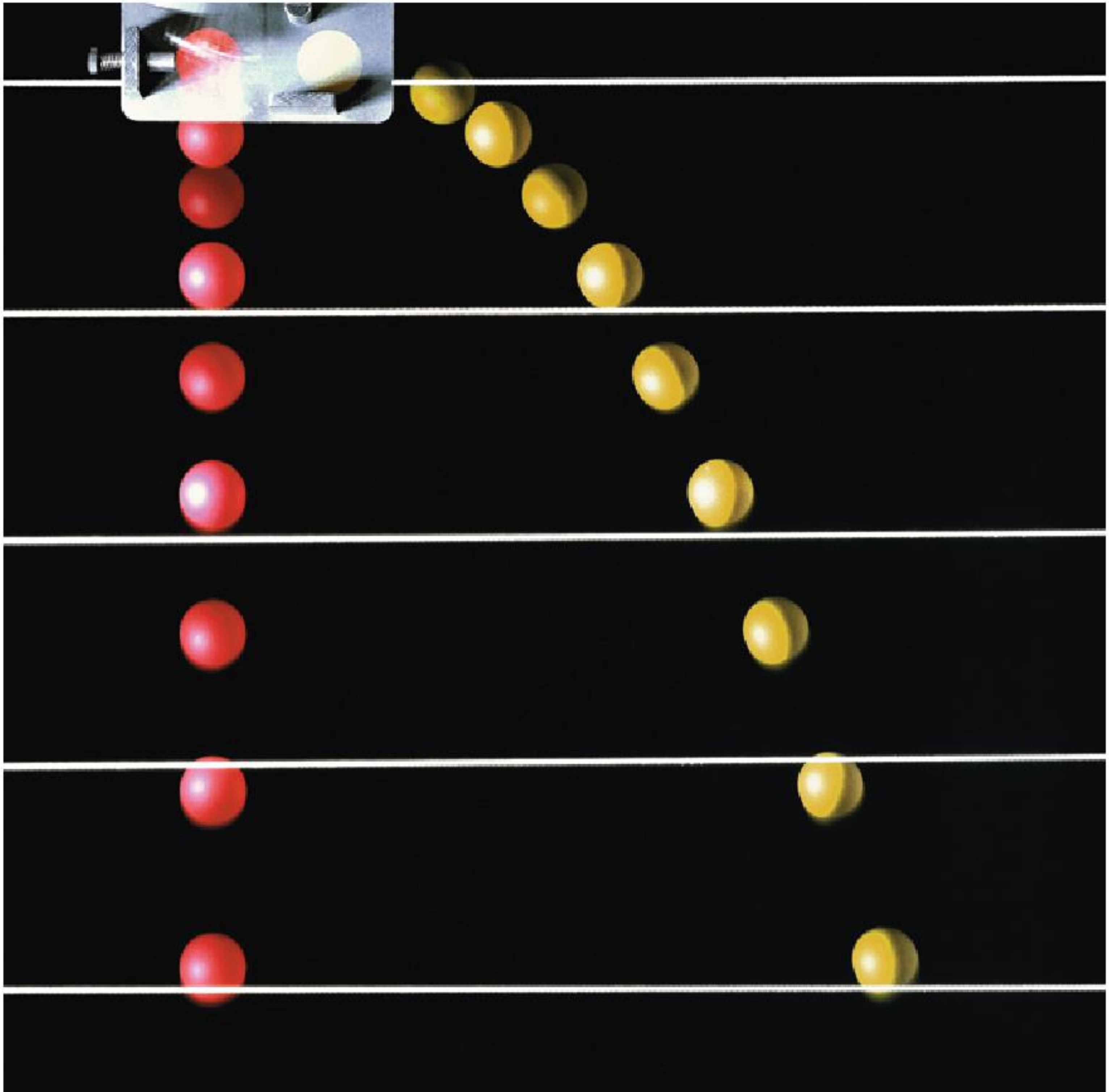
constant $\vec{a}$

$\vec{a} \parallel \vec{v}$

slowing down

1D



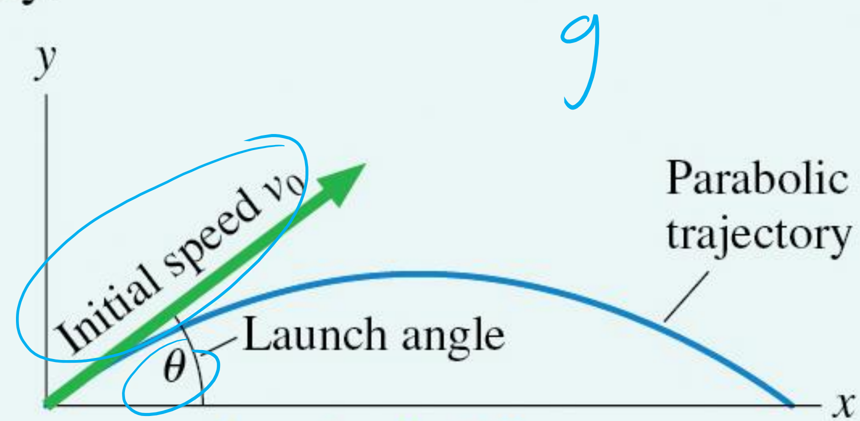


MODEL 4.1

Projectile motion

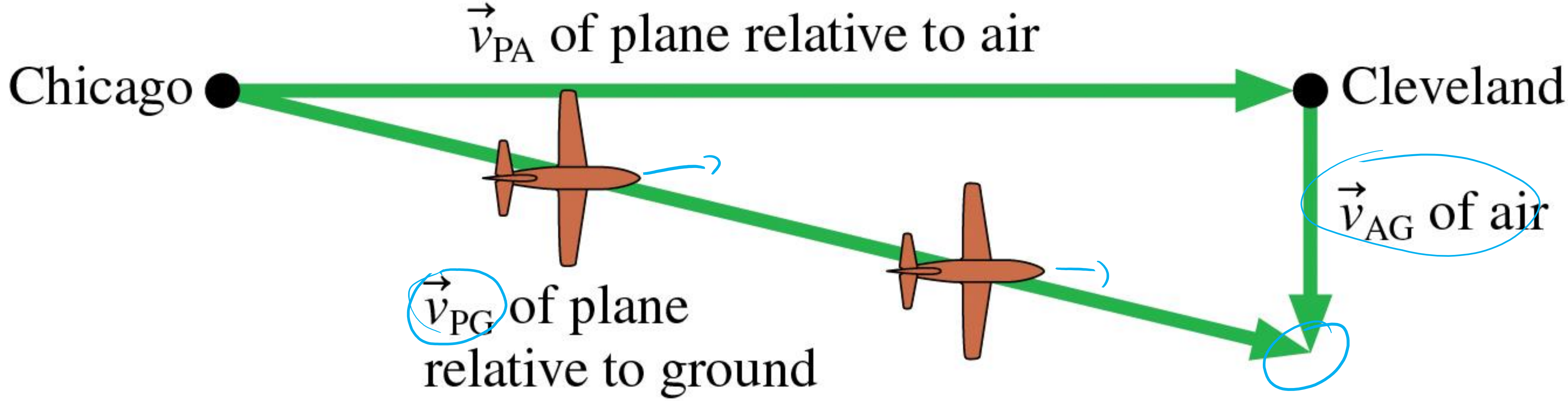
For motion under the influence of only gravity.

- Model the object as a particle launched with speed v_0 at angle θ:
- Mathematically:
  - **Uniform motion** in the horizontal direction with $v_x = v_0 \cos\theta$.
  - **Constant acceleration** in the vertical direction with $a_y = -g$.
  - Same Δt for both motions.
- Limitations: Model fails if air resistance is significant.



A projectile follows a parabolic trajectory.

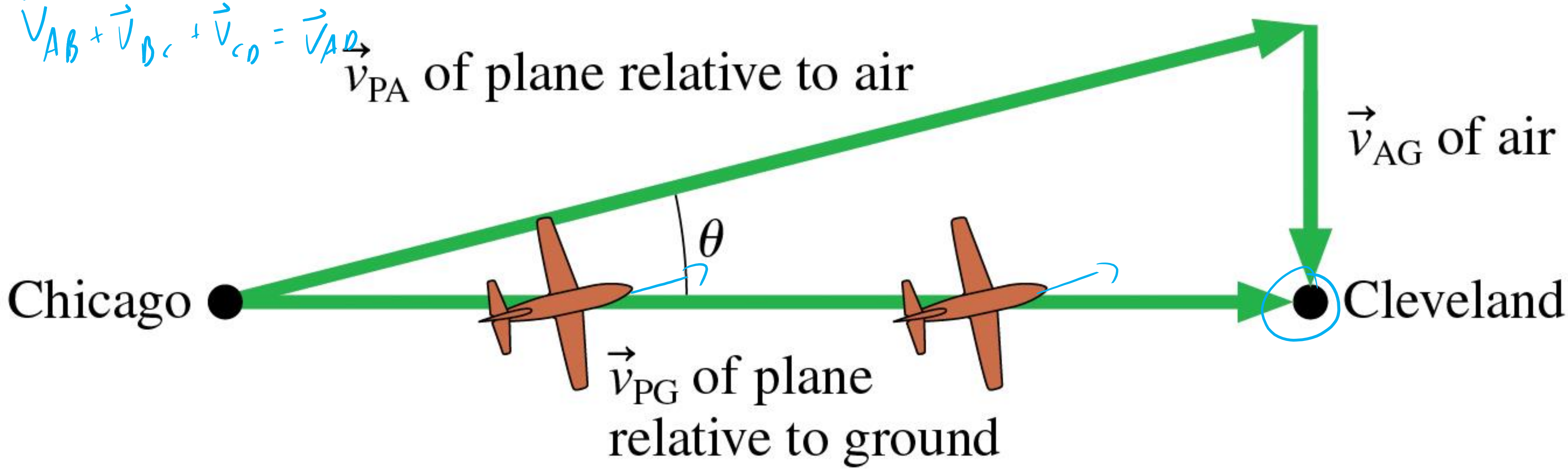
Exercise 9

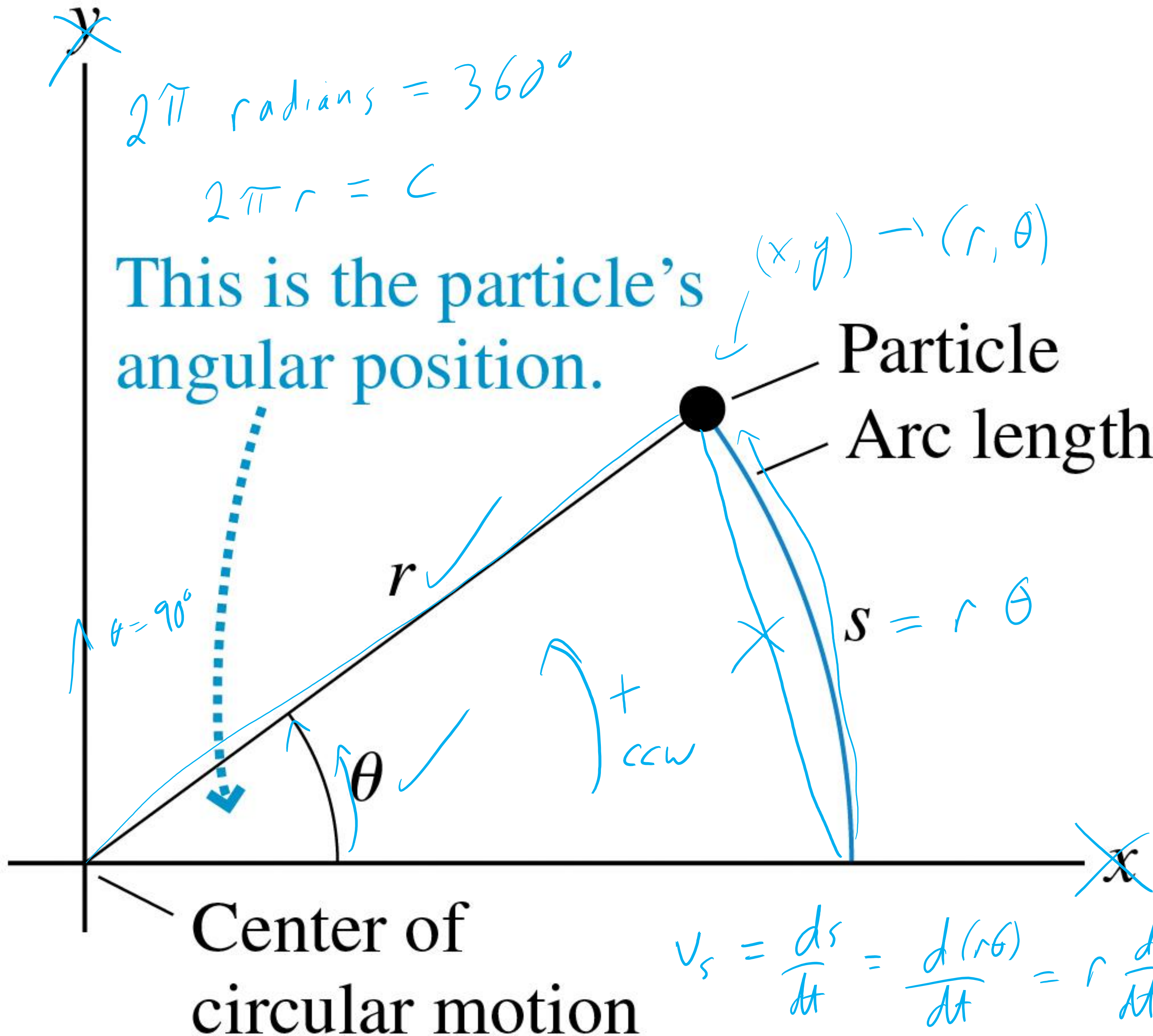


$$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$$

$$\vec{V}_{AB} = -\vec{V}_{BA}$$

$$\vec{V}_{AB} + \vec{V}_{BC} + \vec{V}_{CD} = \vec{V}_{AD}$$





2π radians $= 360°$

$2\pi r = C$

$(x, y) \rightarrow (r, \theta)$

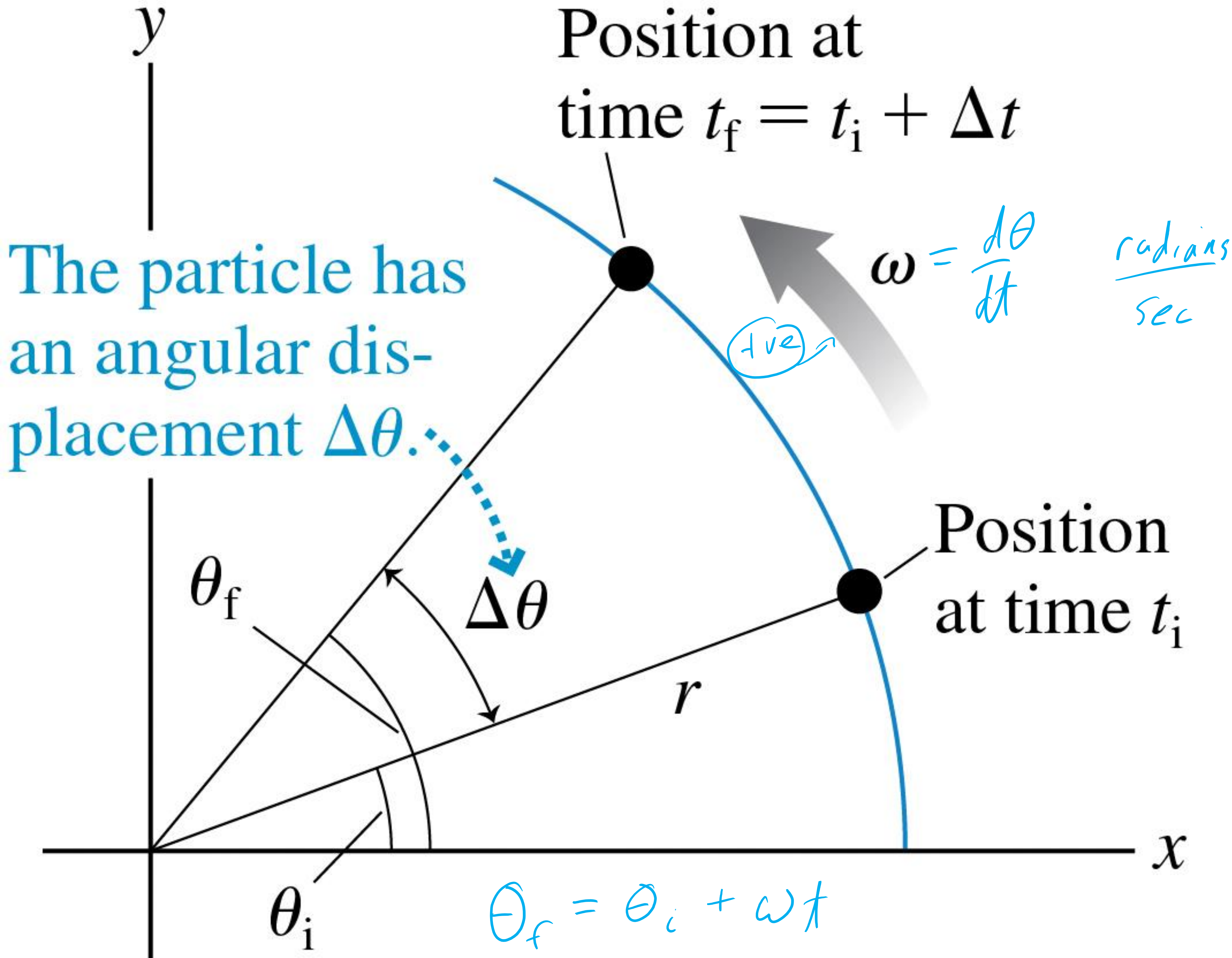
$\theta = 90°$

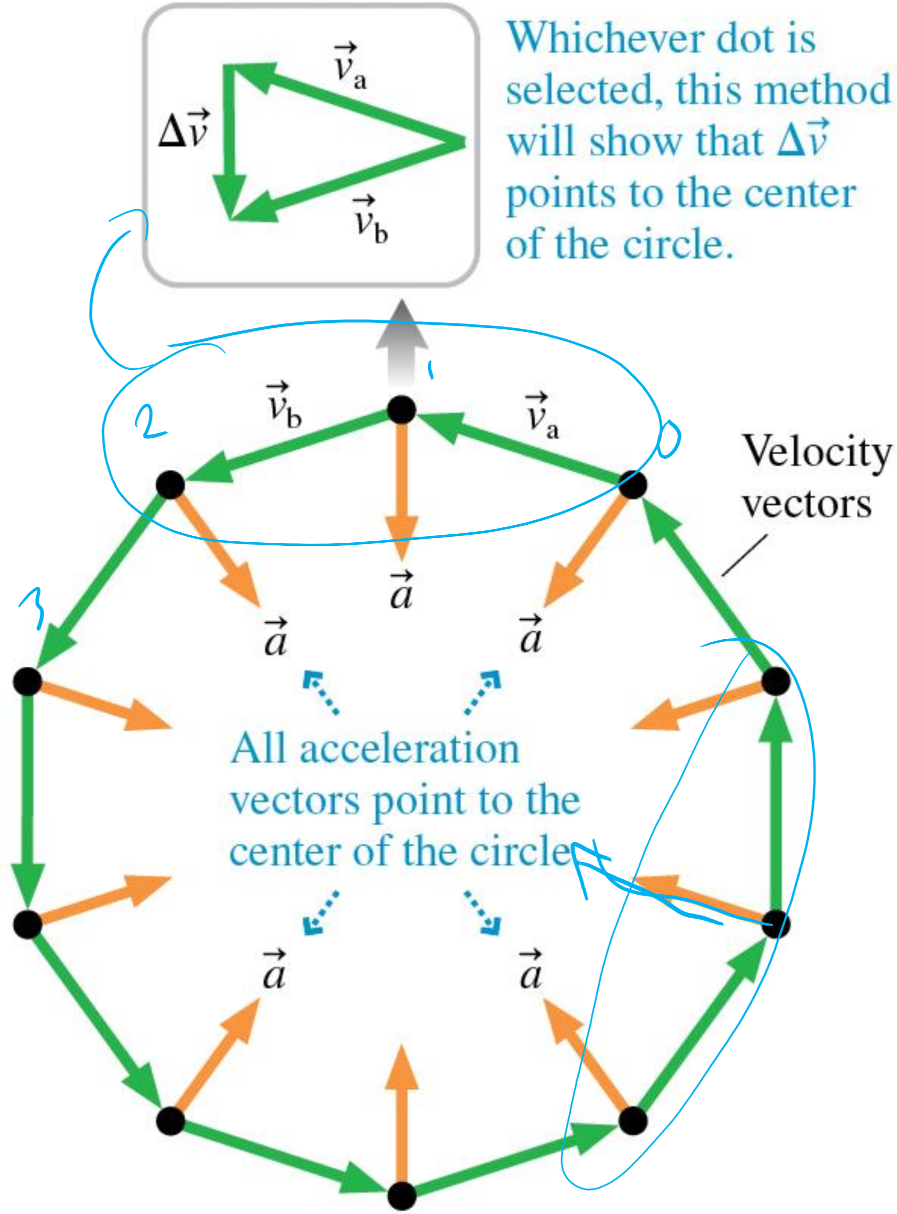
$+$ ccw

$s = r\theta$

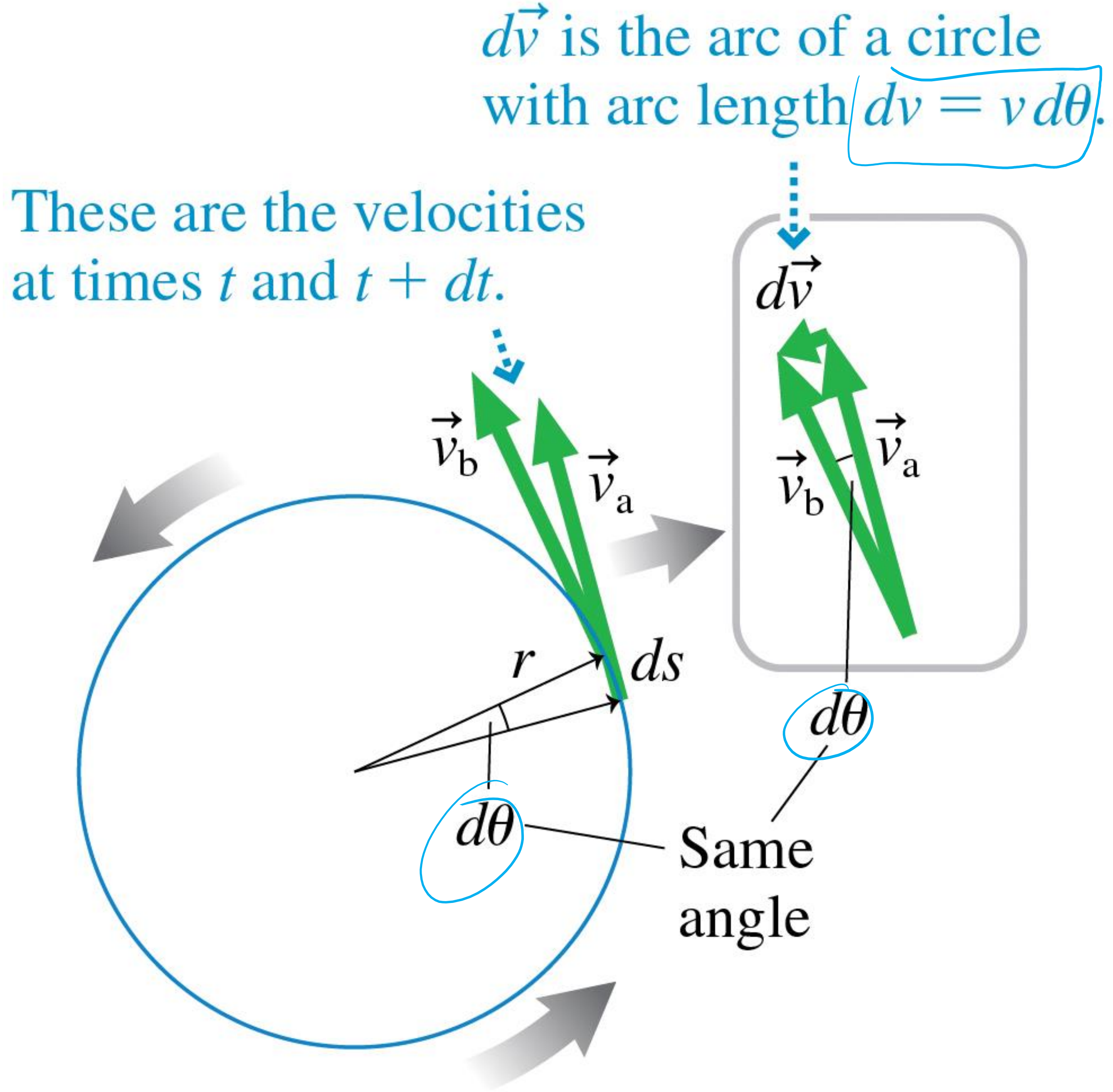
$\omega = \frac{d\theta}{dt}$

$v_s = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega$


y
Position at time $t_f = t_i + \Delta t$
The particle has an angular displacement $\Delta\theta$.
$\omega = \frac{d\theta}{dt}$
$\frac{\text{radians}}{\text{sec}}$
+ve
$\Delta\theta$
θ_f
Position at time t_i
r
x
θ_i
$\theta_f = \theta_i + \omega t$



Maria's acceleration is an acceleration of changing direction, not of changing speed.



$$|\vec{a}| = \frac{dv}{dt} = \frac{v\,d\theta}{dt} = v\omega = v\frac{v}{r} = \frac{v^2}{r}$$

$$|\vec{a}| = \omega^2 r$$

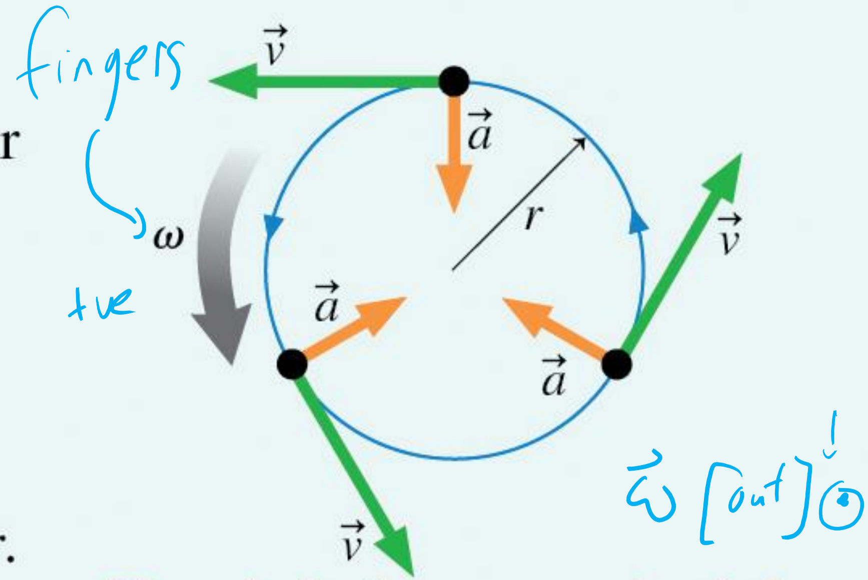
MODEL 4.2

Uniform circular motion

For motion with constant angular velocity ω.

- Applies to a particle moving along a circular trajectory at constant speed or to points on a solid object rotating at a steady rate.
- Mathematically:
  - The tangential velocity is $v_t = \omega r$.
  - The centripetal acceleration is v_t^2/r or $\omega^2 r$.
  - ω and v_t are positive for ccw rotation, negative for cw rotation.
- Limitations: Model fails if rotation isn't steady.

$\vec{v} \perp \vec{a}$



The velocity is tangent to the circle. The acceleration points to the center.

[in]

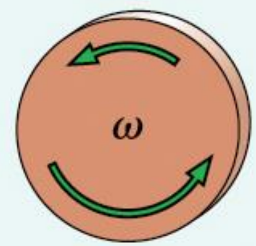
Exercise 20

MODEL 4.3

Constant angular acceleration

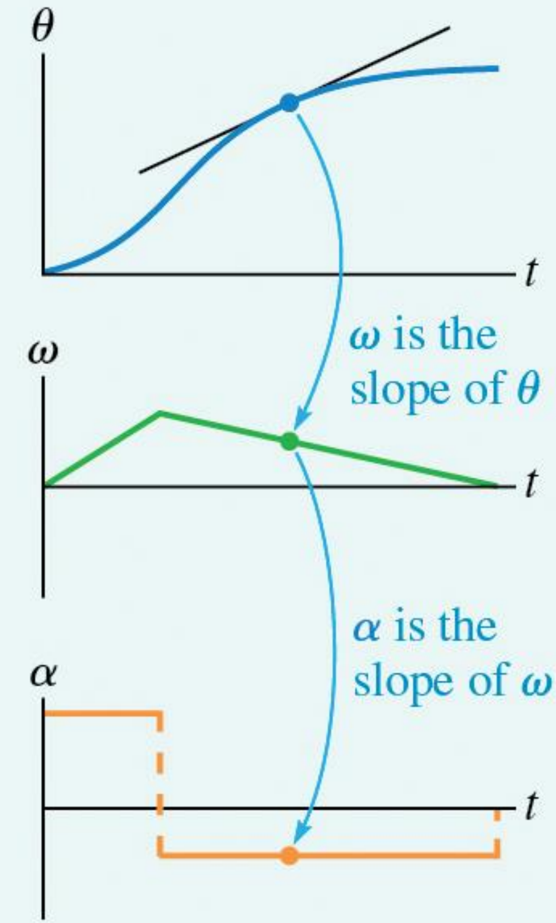
For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.



- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
  - Analogs: $s \rightarrow \theta \quad v_s \rightarrow \omega \quad a_s \rightarrow \alpha$

$\alpha = \dfrac{d\omega}{dt}$



Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\,\Delta\theta$	$v_{fs}{}^2 = v_{is}{}^2 + 2a_s\,\Delta s$

$v_s = \omega r$

$s = \theta r \qquad a = \alpha r$