CSC110 Lecture 9: Programming and Proofs

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Exercise 1: Practice with proofs

Definition. Let $n, d \in \mathbb{Z}$. We say that d **divides** n, or n **is divisible by** d, when there exists a $k \in \mathbb{Z}$ such that $n = dk$.

Using the symbols of predicate logic, we can define divisibility as follows:

$$d \mid n : "\exists k \in \mathbb{Z},\ n = dk" \quad \text{where } n, d \in \mathbb{Z}$$

1. Consider the following statement.

$$\forall n, d, a \in \mathbb{Z},\ d \mid n \Rightarrow d \mid an$$

   a. Rewrite this statement in symbolic logic, but with the definition of divisibility expanded.

   b. Prove this statement.

2. Consider this statement:

$$\forall n, d, a \in \mathbb{Z},\ d \mid an \Rightarrow d \mid a \lor d \mid n$$

   This statement is *False*, so here you'll disprove it.

   a. First, write the negation of this statement. You might need to review the negation rules in the Course Notes Section 3.2.

   b. Prove the negation of the statement. (By proving the statement's negation is True, you'll prove that the original statement is False.)

Additional Exercises

1. Prove the following statement, which extends the first statement in Exercise 1.

$$\forall n, m, d, a, b \in \mathbb{Z},\ d \mid n \land d \mid m \Rightarrow d \mid (an + bm)$$

2. *Disprove* the following statement, which is very similar to the one we proved in the second part of today's lecture.

$$\forall p \in \mathbb{Z},\ Prime(p) \Leftrightarrow \big(p > 1 \land (\forall d \in \mathbb{N},\ 2 \le d < \sqrt{p} \Rightarrow d \nmid p)\big).$$