MAT137Y Tutorial 8 Worksheet

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TOTAL POINTS

2 / 2

QUESTION 1

1 Q1+Q2 **2 / 2**

✓ **- 0 pts No major flaws.**

- 1 pts Major flaws present in the solutions, however there's significant progress and work in the worksheet. The solutions have demonstrated some degree of understanding and mastery.

- 2 pts Too many errors / No effort shown

- 2 pts No TA signature

- 1 pts incomplete

- 2 pts blank

MAT 137
Tutorial #8– Inverse function and computation of derivatives
Nov 15-16, 2022
Due on Thursday, Nov 17 by 11:59pm via GradeScope

- **Submissions are only accepted by** Gradescope. Do not send anything by email. Late submissions are not accepted under any circumstance. Remember you can resubmit anytime before the deadline.
- **Submit your polished solutions using only this template PDF.** You will submit a single PDF with your full written solutions. If your solution is not written using this template PDF (scanned print or digital) then you will receive zero. Do not submit rough work. Organize your work neatly in the space provided.
- **Show your work and justify your steps** on every question, unless otherwise indicated. Put your final answer in the box provided, if necessary.

We recommend you write draft solutions on separate pages and afterwards write your polished solutions here. You must fill out and sign the academic integrity statement below; otherwise, you will receive zero.

Academic integrity statement

I confirm that:

- I have read and followed the policies described in the Policies and FAQ.
- I understand that the collaboration of this worksheet is only among this group. I have not violated this rule while writing this worksheet.
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1. Let f and g be two functions with domain $\mathbb{R}$ and codomain $\mathbb{R}$. Therefore, if we say $f = g$, it means that

$$\forall x \in \mathbb{R}, f(x) = g(x).$$

We define a new definition here. We say a function f is *reliable* when

"For every two functions g and h, $f \circ g = f \circ h \implies g = h$."

(a) Write down the two equivalent definitions of one-to-one function. f is a one-to-one function with domain $\mathbb{R}$ if

(1) $\forall\ x, y \in \mathbb{R},\ x \neq y \Rightarrow f(x) \neq f(y)$

or

(2) $\forall\ x, y \in \mathbb{R},\ f(x) = f(y) \Rightarrow x = y$

(b) Prove that if a function f is one-to-one, then it is reliable.

Since f is one-to-one on $\mathbb{R}$, $\forall\ x, y \in \mathbb{R},\ f(x) = f(y) \Rightarrow x = y$

WTS: $(\forall\ x, y \in \mathbb{R},\ f(x) = f(y) \Rightarrow x = y) \Rightarrow (\forall\ g, h,\ f \circ g = f \circ h \Rightarrow g = h)$

For any function g defined, $\forall\ a \in$ Domain of g, $g(a) \in \mathbb{R}$

For any function h defined, $\forall\ b \in$ Domain of h, $h(b) \in \mathbb{R}$

Therefore, $\forall\ p \in$ (Domain of g) $\cap$ (Domain of h)

Assuming f is one-to-one,

fix $x = g(p)$

fix $y = h(p)$

Therefore since $f(x) = f(y) \Rightarrow x = y$

$$f(g(p)) = f(h(p)) \Rightarrow g(p) = h(p)$$

Since $\forall p \in$ (Domain of h) $\cap$ (Domain of g), $f(g(p)) = f(h(p)) \Rightarrow g(p) = h(p)$,

Therefore $f \circ g = f \circ h \Rightarrow g = h$

Thus if f is one-to-one then it is reliable. $\blacksquare$

(c) Prove that a function f is NOT one-to-one, then it is NOT reliable.

WTS: $(\exists\, x, y \in \mathbb{R}, f(x) = f(y)$ and $x \neq y) \Rightarrow (\exists\, g, h$ s.t. $f \circ g = f \circ h$ and $g \neq h)$

Assume f is not one-to-one

Therefore, $\exists\, x, y \in \mathbb{R}$, $f(x) = f(y)$ and $x \neq y$

Take $x, y \in \mathbb{R}$ that satisfy the above condition for the function f.

Let g be a constant function, st. $\forall\, a \in R$, $g(a) = x$

Let h be a constant function, st. $\forall\, b \in R$, $h(b) = y$

Observe that since $x \neq y$, h and g are different functions, so $h \neq g$

Let $p \in R$

By our assumption that f is not one-to one and the definitions of g and h, $f(g(p)) = f(h(p))$

Since this is true for all p, $f \circ g = f \circ h$

Therefore $f \circ g = f \circ h$ and $g \neq h$

Thus, $(f(x) = f(y)$ and $x \neq y) \Rightarrow (f \circ g - f \circ h$ and $g = h)$

$\rightarrow$ if f is not one-to-one, then f is not reliable

■

2. Find the tangent line to $f(x) = x^{\sin x} + (\sin x)^x + \ln(2x - \pi + 1)$ at $x = \frac{\pi}{2}$.

Substituting $x = \frac{\pi}{2}$ in $f(x) \to$

$$f\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^1 + (1)^{\pi/2} + \ln(\pi - \pi + 1) = \frac{\pi}{2} + 1$$

Thus the point is $\left(\frac{\pi}{2}, \frac{\pi}{2} + 1\right)$.

Differentiating $f(x)$ with respect to $x \to$

$$f'(x) = \frac{d}{dx}\left(x^{\sin x}\right) + \frac{d}{dx}\left(\sin x^x\right) + \frac{2}{2x - \pi + 1}$$

Let $y_1 = x^{\sin x}$

$$\ln|y_1| = \sin x \cdot \ln|x|$$

$$\frac{1}{y_1} \cdot \frac{dy_1}{dx} = \frac{\sin x}{x} + \cos x \ln|x|$$

$$\Rightarrow \frac{d}{dx}\left(x^{\sin x}\right) = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln|x|\right)$$

Let $y_2 = (\sin x)^x$

$$\ln|y_2| = x \ln|\sin x|$$

$$\frac{1}{y_2}\frac{dy_2}{dx} = \ln|\sin x| + \frac{x \cdot \cos x}{\sin x}$$

$$\Rightarrow \frac{dy_2}{dx} = (\sin x)^x\left(\ln|\sin x| + x \cdot \cot x\right)$$

$$\Rightarrow f'(x) = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln|x|\right) + (\sin x)^x\left(\ln|\sin x| + x \cdot \cot x\right) + \frac{2}{2x - \pi + 1}$$

Now, $f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\left(\frac{2}{\pi} + 0\right) + 1\left(\ln(1) + 0\right) + \frac{2}{1} = 3$

Thus slope of tangent to $f(x)$ at $\left(\frac{\pi}{2}, \frac{\pi}{2} + 1\right)$ is 3.

Thus the tangent is $\to$ $\left(y - \frac{\pi}{2} - 1\right) = 3\left(x - \frac{\pi}{2}\right)$

$$\Rightarrow \boxed{y = 3x - \pi + 1}$$