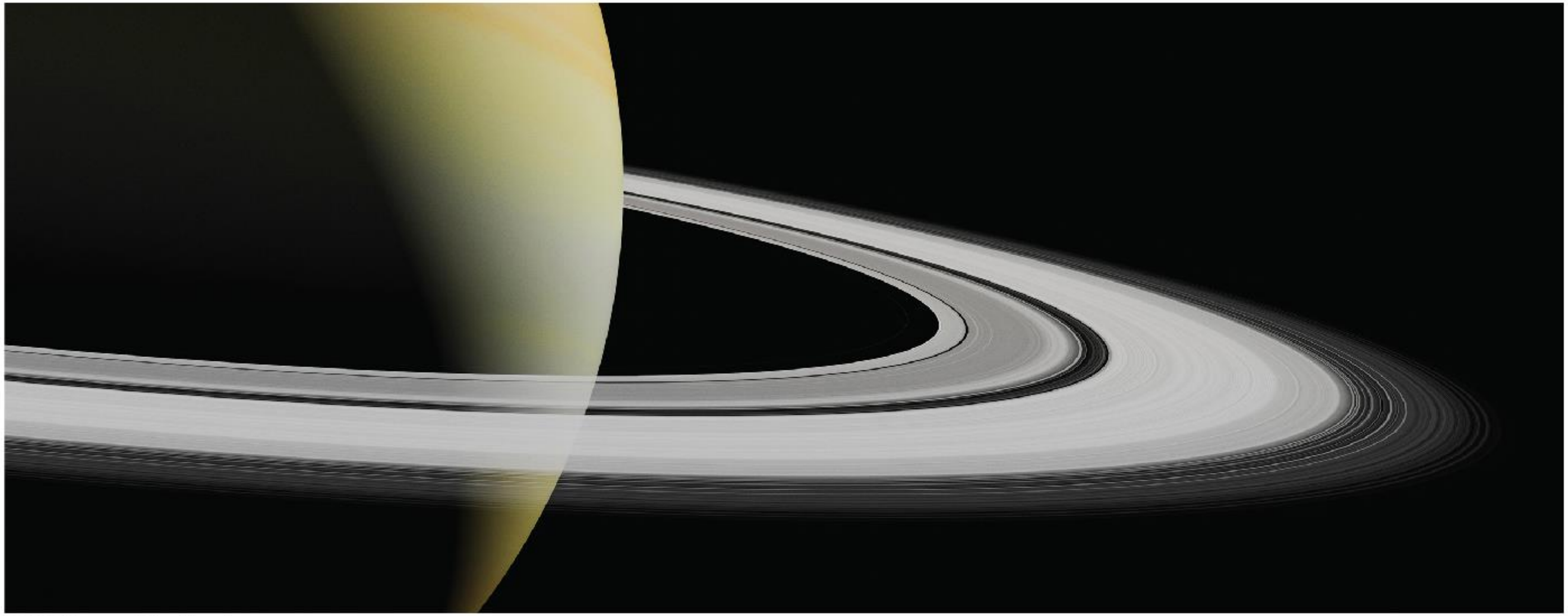
Chapter 13 – Newton’s theory of gravity

- Newton’s theory of gravity
- Kepler’s Laws
- Orbits



Newton's Theory of Gravity

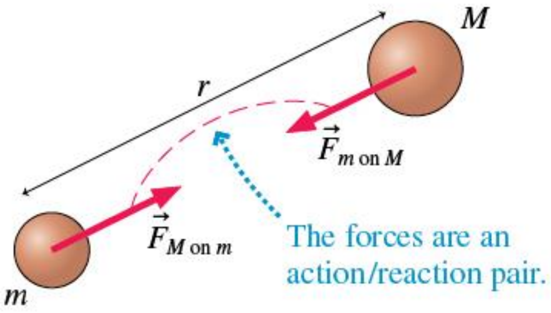
1. Two objects with masses M and m a distance r apart exert attractive **gravitational forces** on each other of magnitude

$$F_{M\text{ on }m} = F_{m\text{ on }M} = \frac{GMm}{r^2} \sim 1/r^2$$

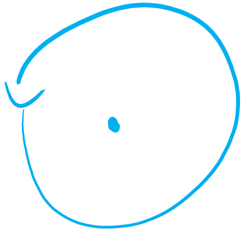
where the **gravitational constant** is $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$. Gravity is an **inverse-square** force.

2. Gravitational mass and inertial mass are equivalent.

3. Newton's three laws of motion apply to all objects in the universe.



Uniform Circular Motion



$$\frac{\cancel{m} v^2}{R} = \Sigma F = \frac{GM\cancel{m}}{R^2}$$

$$v^2 = GM/R$$

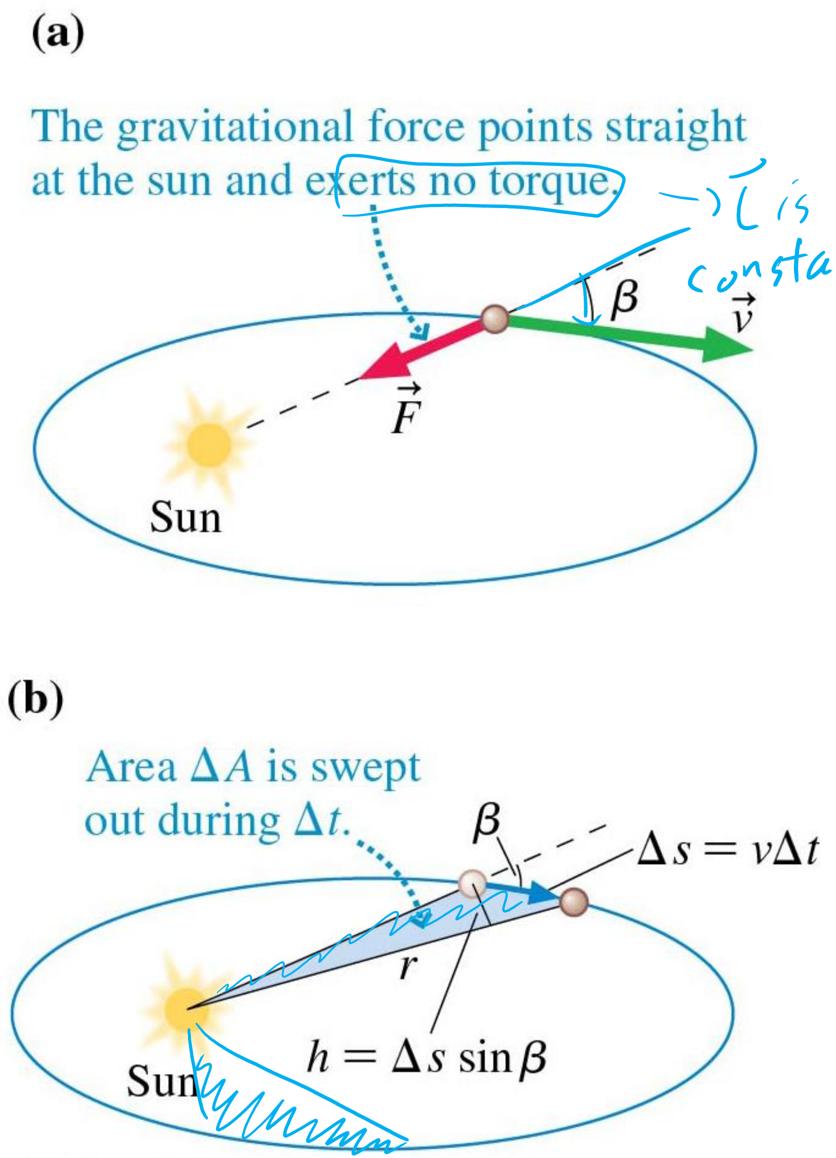
$$V = 2\pi R/T$$

Period of orbit

$$\frac{(2\pi)^2 R^2}{T^2} = GM/R$$

$$R^3 = \frac{GM}{4\pi^2} T^2$$

$$T^2 = \frac{4\pi^2}{GM} R^3$$



$\rightarrow \vec{L}$ is constant

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r \frac{\Delta s \sin\beta}{\Delta t}$$

Δt is small $\quad \Delta s \simeq v \Delta t$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{r \sin\beta}{\cancel{\Delta t}} v \cancel{\Delta t} \simeq \frac{1}{2} r v \sin\beta$$

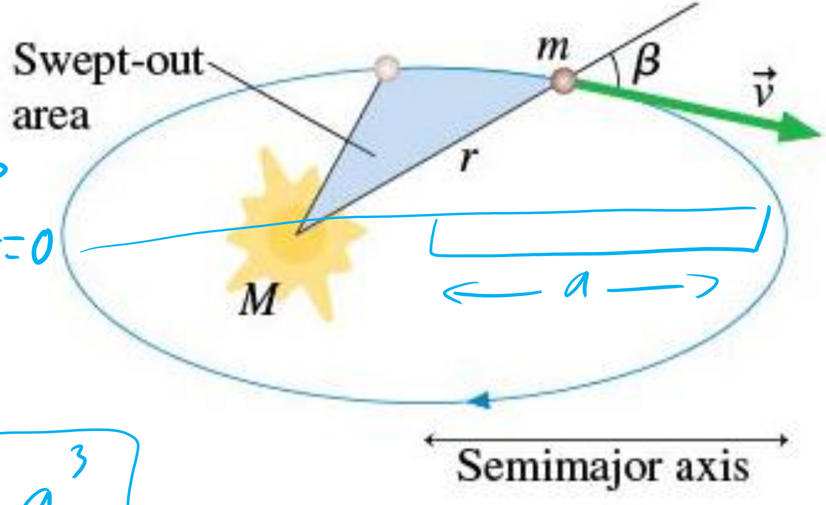
$$= \frac{1}{2} |\vec{r} \times \vec{v}|$$

$$m \frac{dA}{dt} \simeq \frac{1}{2} \vec{L}$$

Orbital motion of a planet (or satellite) is described by **Kepler's laws:**

1. Orbits are ellipses with the sun (or planet) at one focus.
2. A line between the sun and the planet sweeps out equal areas during equal intervals of time.
3. The square of the planet's period T is proportional to the cube of the orbit's semimajor axis.

$\frac{d\vec{L}}{dt}=0$



$\boxed{T^2 \propto a^3}$

Circular orbits are a special case of an ellipse. For a circular orbit around a mass M,

$$v = \sqrt{\frac{GM}{r}} \qquad \text{and} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Conservation of Angular Momentum

The angular momentum $L = mrv \sin\beta$ remains constant throughout the orbit. Kepler's second law is a consequence of this law.

Orbital Energetics

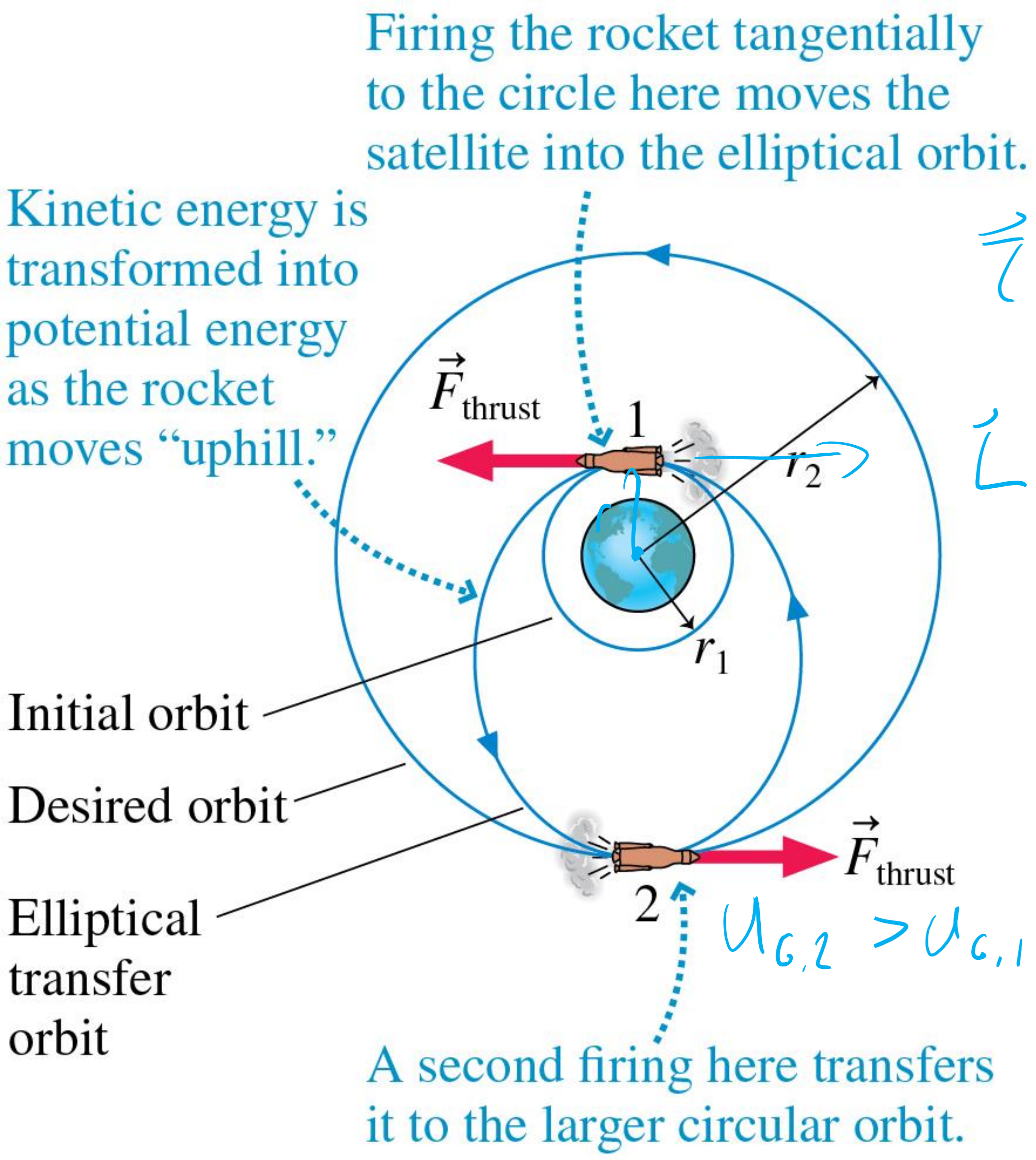
A satellite's mechanical energy $E_{\text{mech}} = K + U_{\text{G}}$ is conserved, where the gravitational potential energy is

$$\boxed{U_{\text{G}} = -\frac{GMm}{r}} \longrightarrow -\frac{dU}{dr} = F_G = -\frac{GMm}{r^2}$$

$$\frac{d}{dr}\frac{1}{r} = -\frac{1}{r^2}$$

For circular orbits, $K = -\frac{1}{2}U_{\text{G}}$ and $E_{\text{mech}} = \frac{1}{2}U_{\text{G}}$. Negative total energy is characteristic of a **bound system.**

$$E = K + U = -\frac{U}{2} + U = \frac{1}{2}U_G$$



$\vec{\tau} = \vec{r} \times \vec{F}$ ⊙ $\rightarrow$ $\frac{dL}{dt} > 0$

small thrust (impulse approx)

$\vec{L}$ ⊙

$p \sim$ constant

$\vec{L} = (\vec{p} \times \vec{r}) \times$ constant

$\Delta |\vec{p}| > 0$ speed up

$U_{G,2} > U_{G,1}$