Chapter 12 – Rotation of a Rigid Body

- Centre of mass and moment of inertia
- Torque and cross product
- Rolling motion and rotational energy
- Angular momentum

$\theta = s/R$

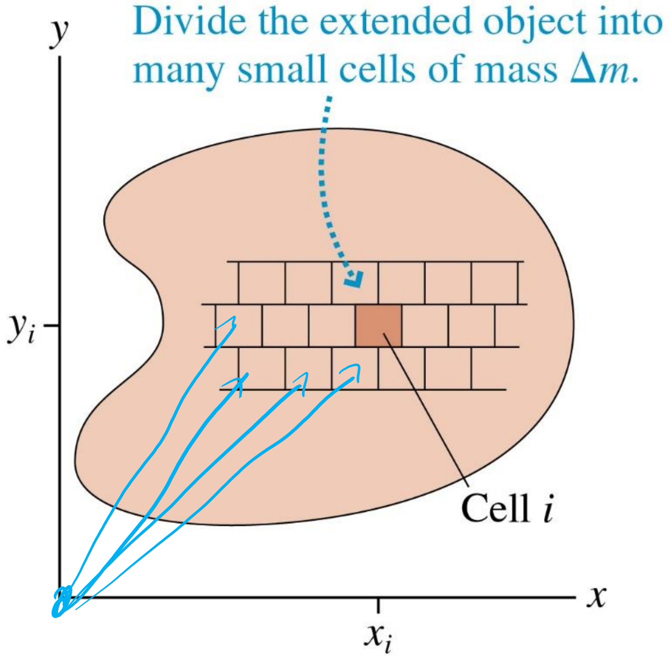
$\omega = v/R$

$\alpha = a/R$

A system of particles on which there is no net force undergoes unconstrained rotation about the **center of mass:**

$$x_{cm} = \frac{1}{M}\int x\,dm \qquad y_{cm} = \frac{1}{M}\int y\,dm$$

The gravitational torque on a body can be found by treating the body as a particle with all the mass M concentrated at the center of mass.

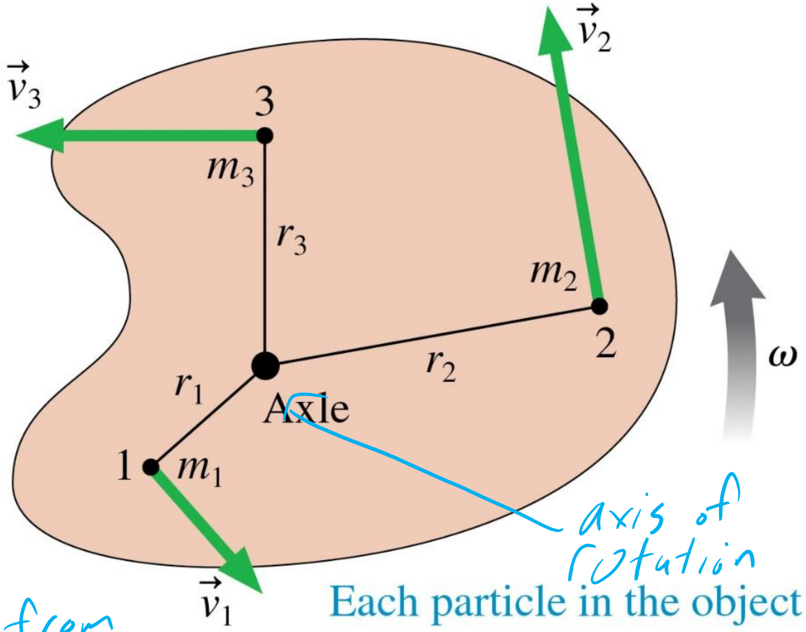


$$X_{cm} = \frac{1}{M}\sum_{i=1}^{N} x_i m_i \qquad y_{cm} = \frac{1}{M}\sum_{i=1}^{N} x_i m_i$$

$$M = \sum_{i=1}^{N} m_i$$

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum m_i (r_i \omega_i)^2 \qquad v = R\omega$$

$$= \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$



axis of rotation

Each particle in the object has kinetic energy as the object rotates.

The **moment of inertia**

(rotational inertia)

$$I = \boxed{\sum_i m_i r_i^2} = \int r^2 \, dm$$

distance from the axle

$$x_{cm} = \int x \, dm$$

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, I about a parallel axis distance d away is given by the **parallel-axis theorem:** $\boxed{I = I_{cm} + Md^2}$.

I_{cm} rotational $\qquad Md^2 \rightarrow$ point approx far away

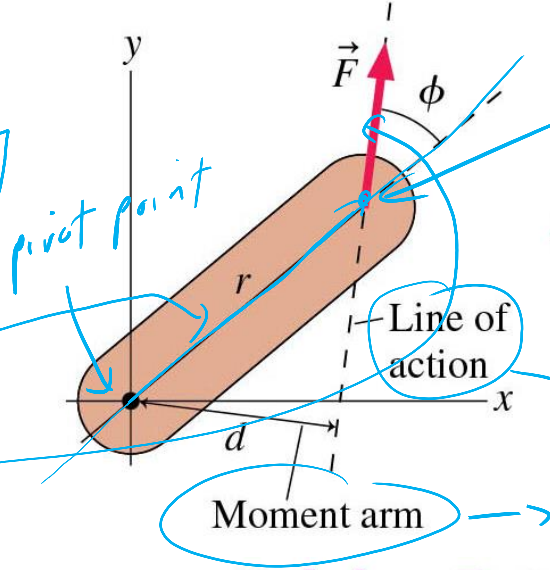
Torque is the rotational equivalent of force:

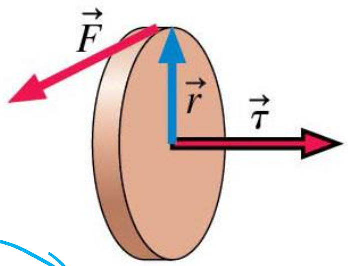
$$\boxed{\tau = rF\sin\phi = rF_t = dF}$$

The vector description of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$|\vec{r}||\vec{F}|\sin\phi$





$\vec{r} \parallel \vec{F} \rightarrow \sin 0 = 0$

$\vec{r} \cdot \vec{F}$ is how much of $\vec{r}$ is $\parallel$ $\vec{F}$?

$\vec{r} \times \vec{F}$ is how much of $\vec{r}$ is $\perp$ $\vec{F}$?

Solving Rotational Dynamics Problems

MODEL Model the object as a **rigid body.**

VISUALIZE Draw a pictorial representation.

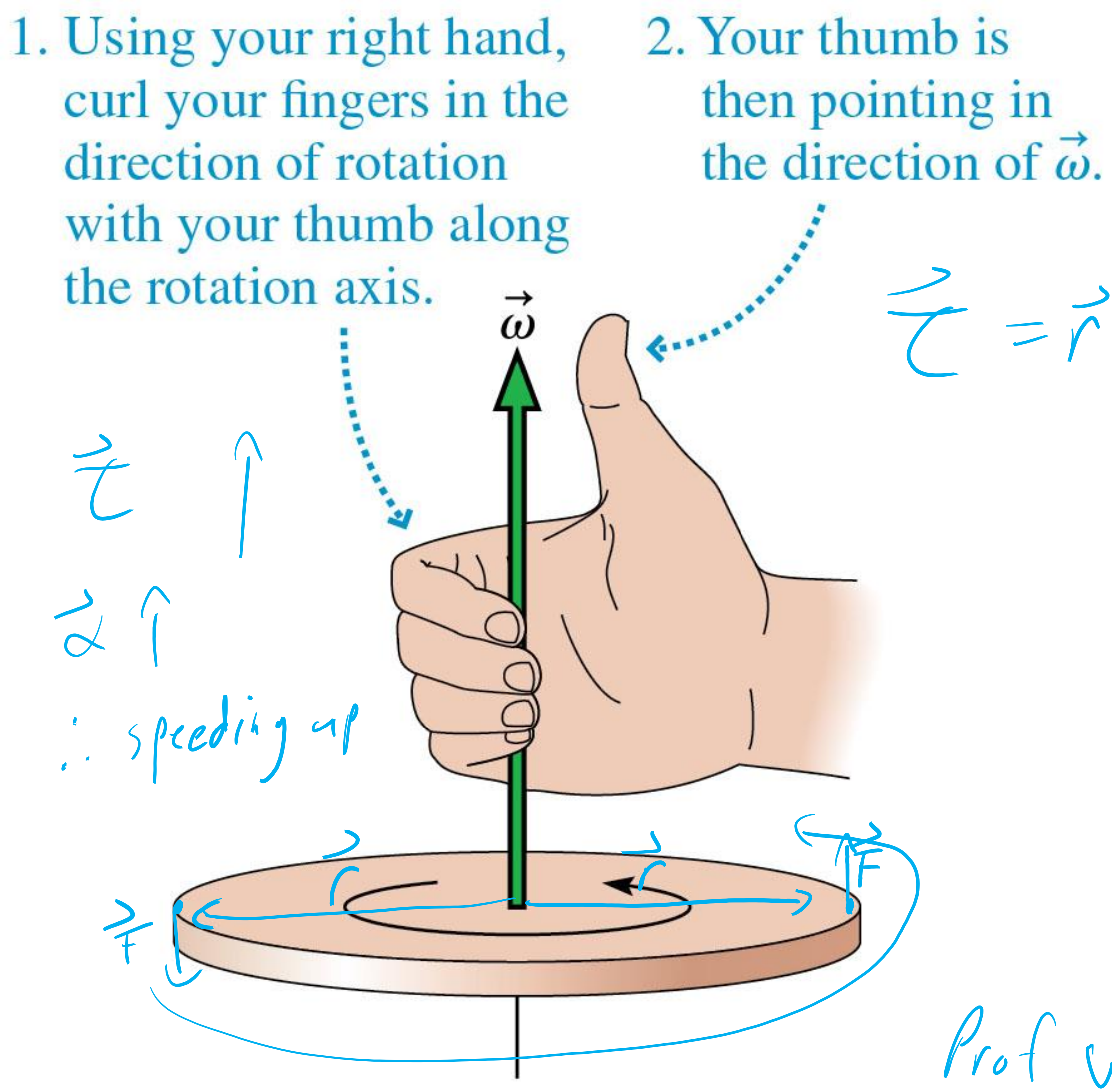
SOLVE Use **Newton's second law** for rotational motion:

$$\vec{\alpha} = \frac{\vec{\tau}_{\text{net}}}{I}$$ like $\vec{a} = \frac{\vec{F}_{net}}{m}$

Use rotational kinematics to find angles and angular velocities.

REVIEW Is the result reasonable?

$\vec{\tau} = \vec{r} \times \vec{F}$

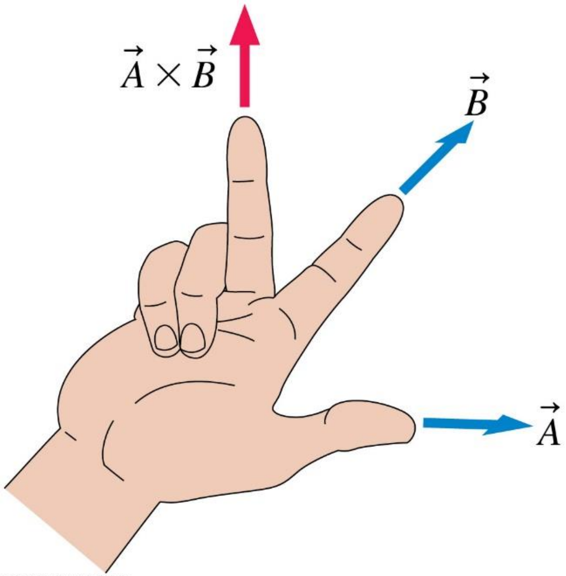


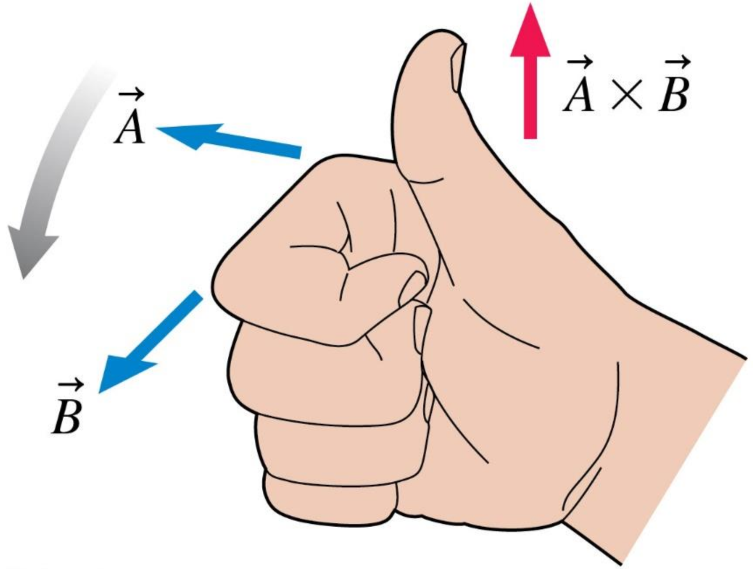
$\vec{\tau}$ ↑

$\vec{\alpha}$ ↑

∴ speeding up

Prof Wilson approved





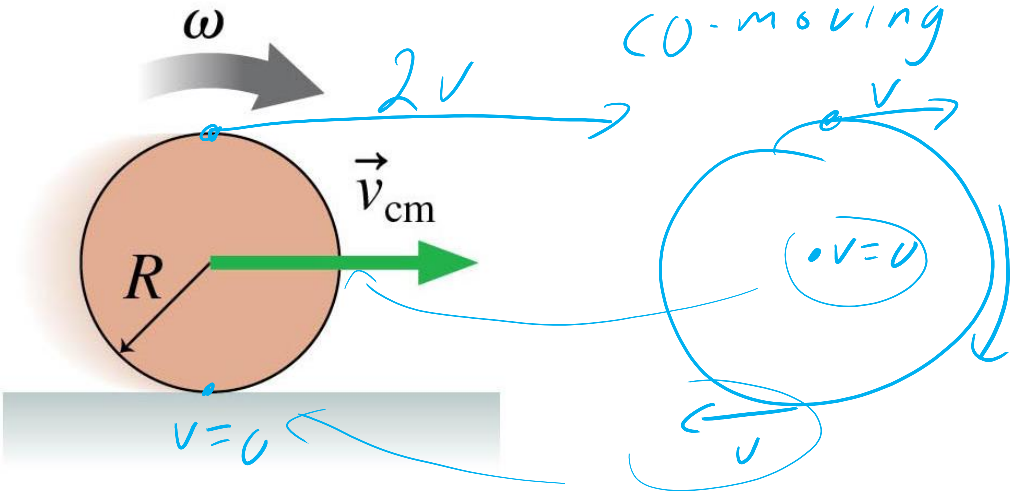
For an object that rolls without slipping

$$\boxed{v_{cm} = R\omega}$$

$$K = K_{rot} + K_{cm}$$

co-moving spin

point-like



Energy is conserved for an isolated system.

- Pure rotation $E_{mech} = K_{rot} + U_G = \frac{1}{2}I\omega^2 + Mgy_{cm}$ rare
- Rolling $E_{mech} = K_{rot} + K_{cm} + U_G = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}^2 + Mgy_{cm}$

Angular velocity $\vec{\omega}$ points along the rotation axis in the direction of the right-hand rule.

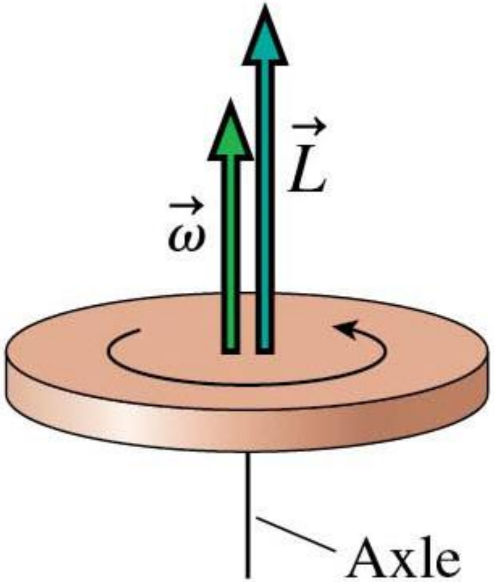
For a rigid body rotating about a fixed axle, the angular momentum is $\boxed{\vec{L} = I\vec{\omega}}$. $\longleftrightarrow$ $\vec{p} = m\vec{v}$

I: 3x3 matrix

Newton's second law is $\dfrac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}}$.

$$\updownarrow$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$



$m \longleftrightarrow I$

$\vec{v} \longleftrightarrow \vec{\omega}$

$\vec{a} \longleftrightarrow \vec{\alpha}$

$\vec{p} \longleftrightarrow \vec{L}$

$\vec{F} \longleftrightarrow \vec{\tau}$