Unit5 local extremum

Sunday, November 13, 2022 2:06 PM

unit5_local...

MAT137

- By December 8, we will cover: Rolle's theorem, L'Hopital and finally Curve sketching.
- Today we will go over local max and min.
- Problem set 3 has been posted a week ago and it is due Nov.24.

Definition of Local extremum

Local min and max

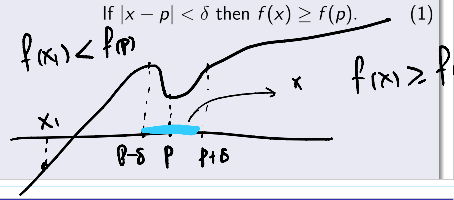
Consider function $f : [a, b] \to \mathbb{R}$.

- We say that $p \in [a, b]$ corresponds to a *local minimum* of f is there is a small enough $\delta > 0$ such that for $x \in [a, b]$,

$$\text{If } |x - p| < \delta \text{ then } f(x) \geq f(p). \quad (1)$$

$f(x_1) < f(p)$

$f(x) \geq f(p)$



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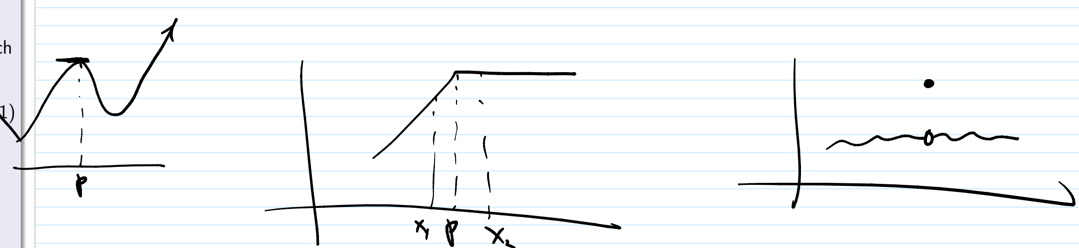
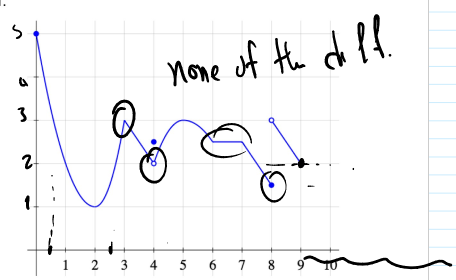
$$\text{If } |x - p| < \delta \text{ then } f(x) \leq f(p). \quad (2)$$



$f(x_1) \leq f(p) = f(x_2)$

Definition of local extremum

Find local and global extrema of the function with this graph:



none of the diff.

1) At $p_1 = 0$ we have glob max.

b/c $x \in D \Rightarrow f(x) \leq f(p_1)$.

2) At $p_2 = 2$ global min

b/c $x \in D \Rightarrow f(x) \le f(p_1)$.

2) At $p_2 = 2$ global min

b/c $x \in D \Rightarrow f(x) \ge f(p_2)$

3) At $p_3 = 3$ local max

take $\delta \le 2$

$|x - p_3| \le \delta \Rightarrow f(x) \le f(p_3)$.

4) At $p_4 = 4.5$ local max

Take $\delta \le \frac{1}{2}$

$|x - p_4| \le \delta \Rightarrow f(x) \le f(p_4)$.

5) At $p_5 = 5$ local max

Take $\delta \le 4$

$|x - p_5| \le \delta \Rightarrow f(x) \le f(p_5)$

6) all the $y \in [6, 7)$ are local mins with $\alpha \, \delta \le 7 - y$

7) At $x = 7$ we have local max

and take $\delta \le 1$

8) At $x = 8$ local min

$\delta \le 8 - 2.5 = 5.5$

9) At $x = 9$ local min

take $\delta < 1$

$|x - 9| \le \delta \Rightarrow f(x) \ge f(9)$.

Extremum and derivative

As we saw above we can have local max/min even if not differentiable.

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Criterion for local max/min/saddle

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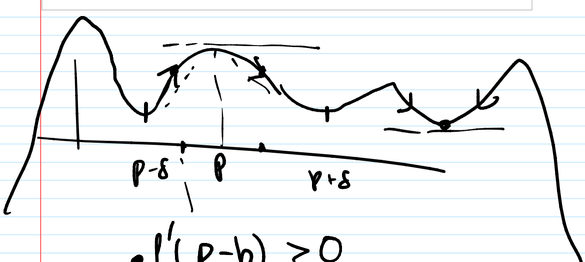
Criterion for local max/min/saddle

Suppose $f'(p) = 0$. If for all small enough $h > 0$

- $f'(p-h) \underline{<} 0$ and $f'(p+h) > 0$, then p is a local min.
- $f'(p-h) > 0$ and $f'(p+h) < 0$, then p is a local max.
- $f'(p-h) \cdot f'(p+h) > 0$, then p is a local saddle.



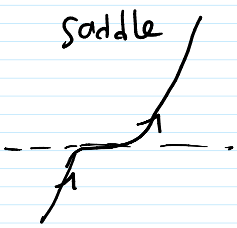
$\cdot f'(p-h) > 0$

$\cdot f'(p+h) < 0$

x^3 saddle

$\underline{f' > 0}$

$\underline{3x^2 > 0}$



Where is the maximum?

We know the following about the function h:

- The domain of h is $(-4, 4)$.
- h is continuous on its domain.
- h is differentiable on its domain, except at 0.
- $h'(x) = 0 \iff x = -1$ or 1.

What can you conclude about the maximum of h?

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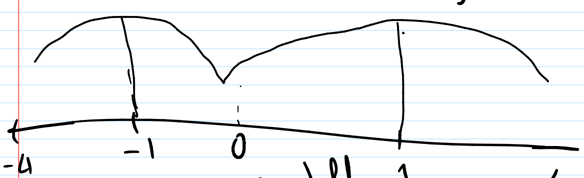
What can you conclude about the maximum of h?

1. h has a maximum at $x = -1$, or 1.
2. h has a maximum at $x = -1, 0$, or 1.
3. h has a maximum at $x = -4, -1, 0, 1$, or 4.
4. None of the above.

3. This is out b/c the domain is open $(-4, 4)$



so ~~not~~ diff at $x=0$ but it is local max.

What can you conclude?

We know the following about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.

What can you conclude about $f'(0)$? Prove it.

Hint: Sketch the graph of f. Looking at the graph, make a conjecture.
To prove it, imitate the proof of the Local EVT from Video 5.3.

Fractional exponents

Let $g(x) = x^{2/3}(x-1)^3$.

Find local and global extrema of g on $[-1, 2]$.

Trig extrema

Let $f(x) = \dfrac{\sin x}{3 + \cos x}$.

Find the maximum and minimum of f.