Friday’s class

- I have an all-day academic meeting
- Prof. Jason Harlow will step in (please be nice!)
- TeamUp will start 2 hours early
  - I hear there’s a climate strike during class, feel free to finish TeamUp early and go to the strike

Summary
General Principles

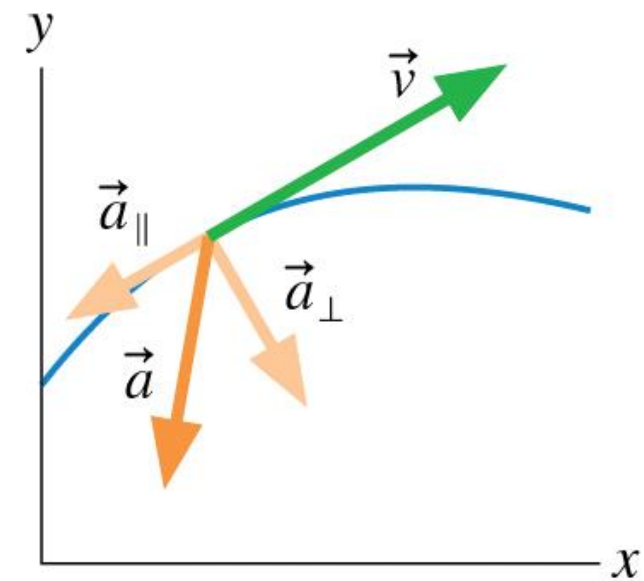
The instantaneous velocity

$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The instantaneous acceleration is

$$\vec{a} = d\vec{v}/dt$$



$\vec{a}_{\parallel}$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_{\perp}$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.

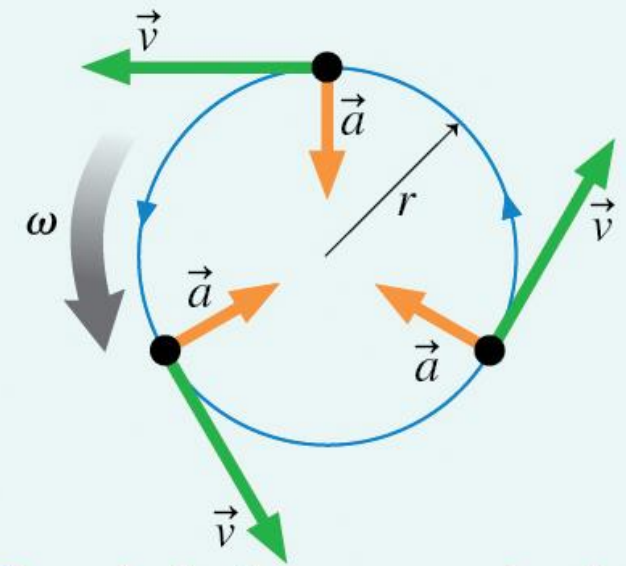
1 rotation = 2π radians (360°)

MODEL 4.2

Uniform circular motion

For motion with constant angular velocity ω.

- Applies to a particle moving along a circular trajectory at constant speed or to points on a solid object rotating at a steady rate.
- Mathematically:
  - The tangential velocity is $v_t = \omega r$.
  - The centripetal acceleration is v_t^2/r or $\omega^2 r$.
  - ω and v_t are positive for ccw rotation, negative for cw rotation.
- Limitations: Model fails if rotation isn't steady.



The velocity is tangent to the circle. The acceleration points to the center.

Exercise 20

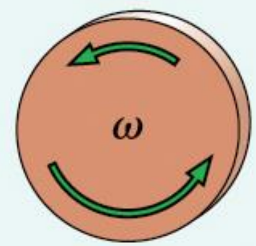
$\alpha = \frac{d\omega}{dt}$ $\quad\quad$ $\omega = \frac{d\theta}{dt}$

MODEL 4.3

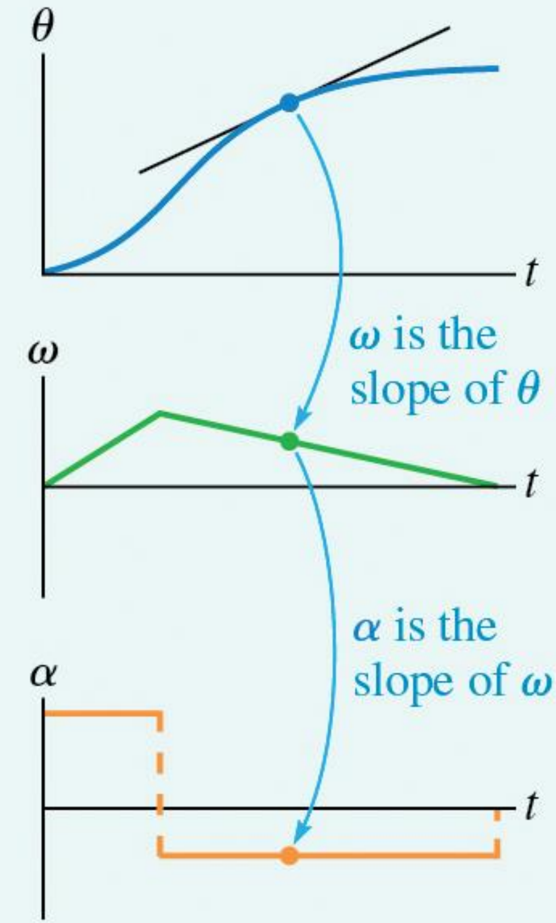
Constant angular acceleration

For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.



- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
  - Analogs: $s \rightarrow \theta \quad v_s \rightarrow \omega \quad a_s \rightarrow \alpha$



Rotational kinematics		Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$		$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$\xrightarrow{\times r}$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\,\Delta\theta$		$v_{fs}{}^2 = v_{is}{}^2 + 2a_s\,\Delta s$

$\theta = s/r \quad\quad \omega = v/r \quad\quad \alpha = a/r$

Team Up Questions

$$a = a_r = \frac{v^2}{r} = \omega^2 r$$

$$\alpha = 2.5$$

$$\omega_f = 5.0$$

A wheel starts from rest and speeds up with constant angular acceleration. What is the acceleration as a function of time for an object on the rim of the wheel?

Vector

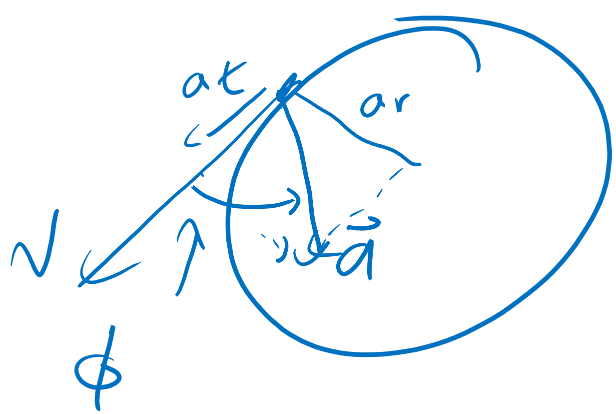
$$\vec{a} = a_\perp + a_\parallel = a_r + a_t$$

$$a_t = \alpha r$$

$$a_r = \omega^2 r = (\alpha t)^2 r$$

$$\omega(t) = \cancel{\omega_i}^{\,0} + \alpha t$$

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{\left((\alpha t)^2 r\right)^2 + (\alpha r)^2}$$

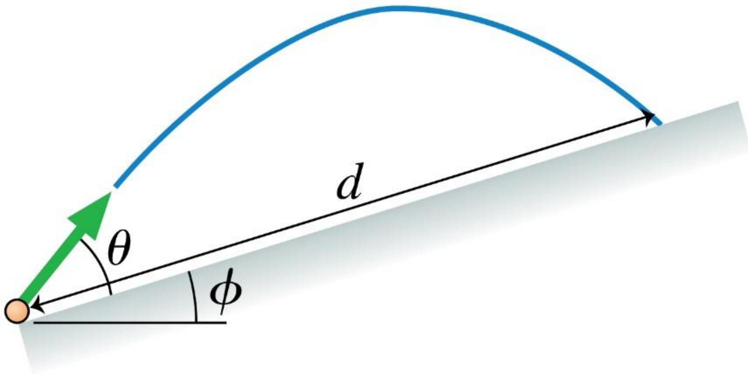


$$\tan\phi = \left(\frac{a_t}{a_r}\right)^{-1}$$

$$\tan\phi = \frac{\alpha r}{(\alpha t)^2 r} = \frac{1}{\alpha t^2}$$

$$\phi = \operatorname{atan}\left(\frac{1}{\alpha t^2}\right)$$

What's the best angle (to get the farthest distance) to throw a rock if you're at the bottom of a hill of constant slope?



$d(\theta) = ?$

$\max \rightarrow \frac{d}{d\theta} d(\theta) = 0$