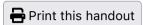
CSC110 Lecture 9: Programming and Proofs



Exercise 1: Practice with proofs

Definition. Let $n, d \in \mathbb{Z}$. We say that d divides n, or n is divisible by d, when there exists a $k \in \mathbb{Z}$ such that n = dk.

Using the symbols of predicate logic, we can define divisibility as follows:

$$d\mid n: "\exists k\in \mathbb{Z}, \ n=dk" \quad ext{where } n,d\in \mathbb{Z}$$

1. Consider the following statement.

$$orall n,d,a\in\mathbb{Z},\ d\mid n\Rightarrow d\mid an$$

a. Rewrite this statement in symbolic logic, but with the definition of divisibility expanded.

b. Prove this statement.

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$$\Rightarrow (\exists k_1 \in \mathbb{Z}, n = dk_1)$$
Let $n, d, a \in \mathbb{Z}$.

Assume the is $a \in \mathbb{Z}$ that is such that $n = dk_1$. We need to show $\exists k_1 \in \mathbb{Z}$, and dk_1 .

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2. Consider this statement:

$$\forall n, d, a \in \mathbb{Z}, \ d \mid an \Rightarrow d \mid a \lor d \mid n$$

This statement is *False*, so here you'll disprove it.

a. First, write the negation of this statement. You might need to review the negation rules in the Course Notes Section 3.2 (https://www.teach.cs.toronto.edu/~csc110y/fall/notes/03-logic/02-predicate-logic.html#manipulating-negation).

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b. Prove the negation of the statement. (By proving the statement's negation is True, you'll prove that the original statement is False.)

Let N = A d = 6 a = 3

We want to true (because of 3) a setting for 11, dρ εξξ that satisfies each of dan, dx α, d λο22-09-27, 1:15 p.m.

Then d (an)

d Xa \sink d 7a

d Xn \sink d 7n

(and apply realt from slider from gives an upper bond on divisors)

Additional Exercises

1. Prove the following statement, which extends the first statement in Exercise 1.

$$orall n, m, d, a, b \in \mathbb{Z}, \; d \mid n \wedge d \mid m \Rightarrow d \mid (an + bm)$$

2. *Disprove* the following statement, which is very similar to the one we proved in the second part of today's lecture.

$$orall p \in \mathbb{Z}, \; Prime(p) \Leftrightarrow ig(p > 1 \land (orall d \in \mathbb{N}, \; 2 \leq d < \sqrt{p} \Rightarrow d
mid p)ig).$$

Hint: the change is from < WP

to < NP

so prove by considering a number p

that is not prime and whose square

root is an integer.