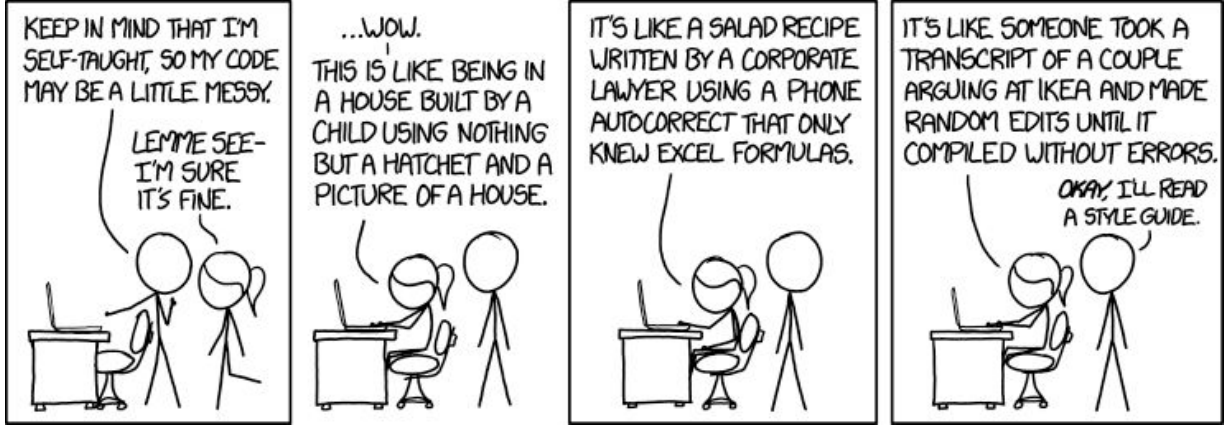
PHY151

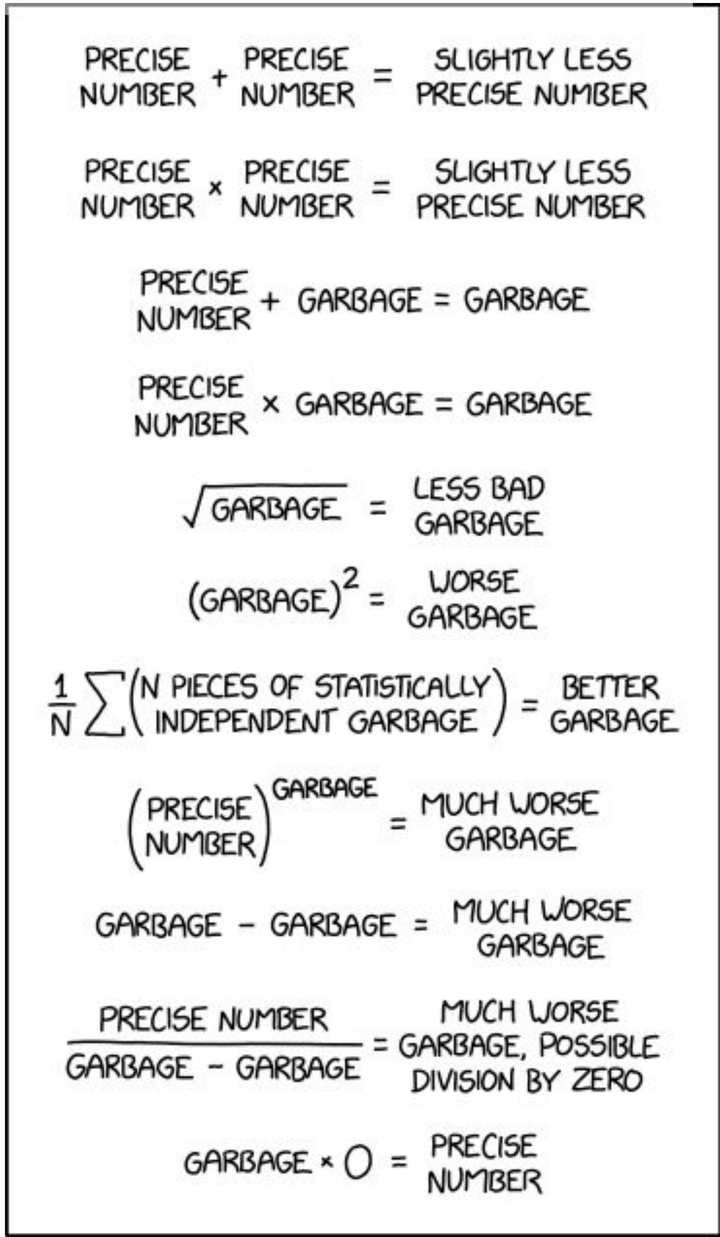
Practical

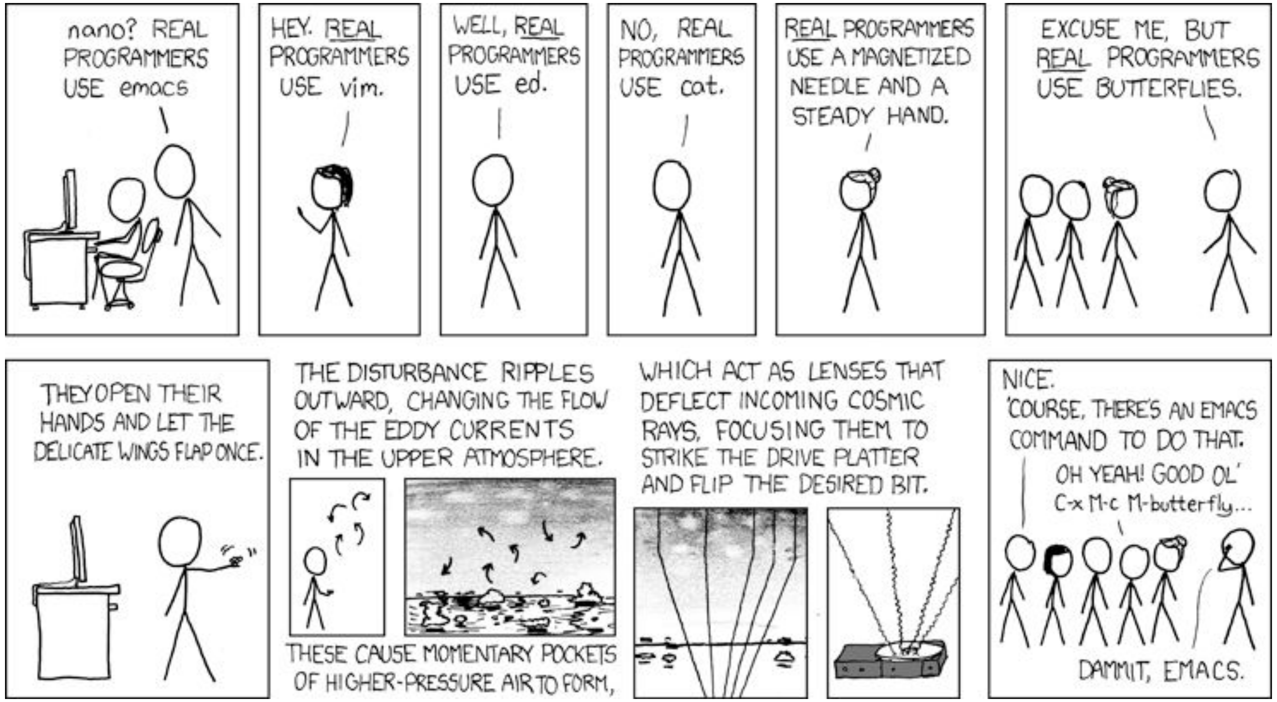
4

Even if you're self taught and know everything, it's still good to see how other people approach the same problems.



I went looking for jokes about uncertainties, but this is actually kinda useful





Emacs, vim, etc. are text editors that you can edit code in. Today we'll be using Spyder, a development environment with more features specific to python but less versatile than text editors.

Outline for Today

- First 50 minutes: Practice problems
  - Prof. Wilson has written 4 problems similar to those on tests. Please work ***together*** on these ***(not for marks)!***
- Final 2 hours: working on the Practical Activities of the week.
  - Write-ups in the TERM booklets **for marks**
  - Freefall with Python, Activities A & B (questions 1-8, and 9 if you have time). You can do activities C & D after if you like, but there are no bonus marks for these so make sure A & B are completely done before you try them.
  - Print out your code and plots (not tables of data though). Make sure your code is explained.

Today’s Tutorial Problems

1. If you know the force applied at the end of a rope, what does it tell you about the tension throughout the rope?

2. If you have two objects connected by fixed ropes, they must move with the same velocity and acceleration. Similarly, if objects are connected by pulleys, their velocities and accelerations will be related. This is what Brian is referring to as a “constraint”, you should start the problem by identifying it.

3. Nathan will describe on a board.

4. L is the length of rope above the lower pulley. If the mass is lowered by 1 cm, this length must decrease by 1 cm since the rope can’t stretch, meaning dL/dy=1.

Today’s Tutorial Problems

1. The rope pulls up on the chair. The rope also pulls up on the painter’s hands due to Newton’s third law (the painter is pulling down on the rope). Assume a massless rope and a massless, frictionless pulley.

   $ma_y = 2T - mg$

   $T = \frac{m}{2}(a+g) = (40)(9.8+0.2) = 40\,\mathrm{N}$

2. Acceleration constraint: $2a_x = -a_y$ if x is positive to the right and y is positive up.

   Assume massless rope and massless, fricitonless pulley.

   3 kg object: $m_3 a_x = 2T$ or $a_x = 2T/m_3$

   1 kg object: $m_1 a_y = T - m_1 g$ or $a_y = T/m_1 - g$

   $T/m_1 - g = -2(2T/m_3)$ or $T = g/(1/1 + 4/3) = 3g/7$

   So the acceleration we want is $a_x = \frac{2}{3}\frac{3g}{7} = \frac{2}{7}g = 2.8\,\mathrm{m/s^2}$

3. The acceleration of an Atwood machine is
$a = g\frac{m_2-m_1}{m_2+m_1}$ where we assume $m_2 > m_1$ and the heavier mass accelerates down.

From kinematics we have $d = \frac{1}{2}at^2$ or $t^2 = \frac{2d}{a}$

Combining, we get $t^2 = \frac{2d}{g}\frac{m_2+m_1}{m_2-m_1}$

There are two easy ways to find g from this. One is to find the y-intercept ($m_1 = 0$) which should be $\frac{2d}{g}$.

The other is to find the slope for $m_1 \ll m_2$. The derivative of t^2 is
$\frac{d}{dm_1}(t^2) = \frac{2d}{g}\frac{2m_2}{(m_2-m_1)^2}$
which for small m_1 is $\frac{4d}{m_2 g}$.

So either estimate the slope or the intercept (or both) to find g given the knowns (d is 0.3 m and m_2 is 1 kg).

If I take the two left-most data points, draw a straight line through them, and find the slope, it looks to be about $\frac{0.16-0.05}{0.8-0} = 0.138\,\frac{\mathrm{s}^2}{\mathrm{kg}}$ which gives $g = \frac{4d}{m_2}\frac{1}{0.138\,\mathrm{s}^2/\mathrm{kg}} = 8.7\,\mathrm{m/s}^2$. Given that the data looks like the slope increases with mass, 8.7 is the smallest value.

Note that with the errorbars, I can guess a slope as small as 0.1 and as large as 0.2 which means g is between $6\,\mathrm{m/s}^2$ and $12\,\mathrm{m/s}^2$

The intercept looks to be around $0.05\,\mathrm{s}^2$ which gives $g = \frac{2d}{0.05} = 12\,\mathrm{m/s}$.

Note that with the errorbars, I can guess an intercept between 0.04 and 0.06, which means g is between $10\,\mathrm{m/s}^2$ and $15\,\mathrm{m/s}^2$

The two methods have about the same uncertainties, so I might want to average the two results. The average is $10.4\,\mathrm{m/s}^2$. Final uncertainty is probably around $2\,\mathrm{m/s}^2$. So my final result should be stated as $10 \pm 2\,\mathrm{m/s}^2$.

Note: on a test you should only use one of these two methods unless you have lots of extra time.

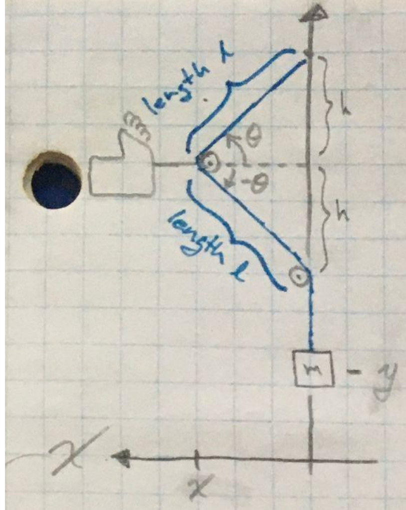
Chain rule should cancel out that 2 with a ½ from the power on x^2+C^2

That's a mistake and even if it had been written correctly, also a pretty significant piece of reasoning to just sweep under the rug

4. $L = 2\sqrt{x^2 + C^2}$ where C is a constant, equalt to half the vertical distance between the pulley and the top anchor point of the string.

$\frac{dL}{dy} = 1 = \frac{d}{dy} 2\sqrt{x^2 + C^2} = \frac{2}{\sqrt{x^2+C^2}} \frac{d}{dy}(x^2) = \frac{2x}{\sqrt{x^2+C^2}} \frac{dx}{dy} \boxed{= 2\cos\theta}$

Therefore the acceleration constraint is $\frac{dy}{dx} = 2\cos\theta$ which means for every mm that the foot moves right, the mass moves $2\cos\theta$ mm down. If the angle is 60 degrees, this is a 1:1 constraint. For small angles, the constraint is 2:1 as we might expect. At the angle goes to 90 degrees, small foot movements (x) have almost no impact on the height of the mass (y).



We define L as $2l$. Since the rope can't stretch, any decrease in L is accounted for by lowering in the block so $\Delta L = \Delta y$, or in calculus terms

$$\frac{dL}{dy} = 1$$

Since we have right triangles, we also know that

$$L = 2\sqrt{x^2+h^2} \qquad \tan\theta = \frac{h}{x}$$

Plugging this form for L into the first equation gives

$$\frac{dL}{dy} = \frac{d}{dy} 2\sqrt{x^2+h^2} \quad \leftarrow \text{note that } h \text{ is fixed so } \frac{dh}{dy} = 0$$

$$1 = 2 \cdot \frac{1}{2}(x^2+h^2)^{-1/2} \frac{d}{dy}(x^2+h^2)$$

$$1 = \frac{2x}{\sqrt{x^2+h^2}} \frac{dx}{dy}$$

Multiplying both sides by dy/dt we can see how this is actually a <u>constraint</u> relating the velocities of the foot $\frac{dx}{dt}$ and the mass $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{2x}{\sqrt{x^2+h^2}} \frac{dx}{dt}$$

$\leftarrow$ note that it's really more of a velocity constraint than acceleration, you'd need to take d/dt on both sides to get the latter, but your prof doesn't seem to care.

If we plug in $x = h\cot\theta$ we see that this can be written more simply

$$\frac{dy}{dx} = \frac{2h\cot\theta}{\sqrt{h^2\cot^2\theta + h^2}} = \frac{2\cot\theta}{\sqrt{\cot^2\theta + 1}}$$

over the interval $\theta \in (0, \pi)$ this equal to

$$\left.\frac{dy}{dx}\right|_{\theta\in(0,\pi)} = 2\cos\theta$$

we're only interested in $\theta \in (0, \pi/2]$ so this range is sufficient for our purposes.

because of some trig identities. Plot the thing with the cotangents in Desmos if you want to convince yourself of this relationship.

This means that if you move the foot by ~1mm to the left, then the mass drops by about $(2\cos\theta) \times (1\text{m})$ upward.

General Notes on Coding

- The command `from numpy import *` should generally be avoided. While it should work for you today, the NumPy module contains thousands of functions, so when you `import *`, this tells your code to recognize all of their names. This could lead to issues should you write a variable or function with a similar name.
- `import numpy as np`
- `x = np.linspace(0,1,10)`

Today's Practical

- Write-ups in the TERM booklets **for marks**
- Freefall with Python, Activities A & B (questions 1-8, and 9 if you have time). You can do activities C & D after if you like, but there are no bonus marks for these so make sure A & B are completely done before you try them.
- Print out your code and plots (not tables of data though). Make sure your code is explained.

- **Cite lab manual** or write out complete procedure.
- **Save yourself time**: no need to print raw data or write out the questions. You can also submit scrap pieces of paper so you can all be working rather than person B sitting and waiting for person A to finish writing their bit before person B can add theirs
- **REFLECT ON RESULTS**