CSC110 Fall 2022 Assignment 2: Logic, Constraints, and Wordle!

Shivesh Prakash

October 11, 2022

Part 1: Conditional Execution

Complete this part in the provided `a2_part1_q1_q2.py` and `a2_part1_q3.py` starter files. Do **not** include your solutions in this file.

Part 2: Proof and Algorithms, Greatest Common Divisor edition

1. As stated and proved in Lecture 9, for all positive integers `n` and `d`, if `d | n` then $d \leq n$. So no common divisor of `m` and `n` can be greater than either `m` or `n`. Since $m \leq n$, no common divisor can be greater than `m`. Therefore, we use `range(1, m + 1)` to find possible common divisors in this approach.

2. As stated and proved in Lecture 9, every integer is divisible by 1. So the set `common_divisors` can never be empty it always contains `1`. Also range(1, m + 1) always includes `1`. Therefore we can safely use the `max()` function on set `common_divisors` without checking whether it is empty or not.

3. *Proof.* To prove: $\forall nmd \in Zd \mid m \cap m \neq 0 \implies (d \mid n \iff d \mid (n\%m))$

   Let $n, m, d \in$ Z.

   To prove the implication, we assume $d \mid m \cap m \neq 0$ to be true. Now to prove the "if and only if" statement I will first prove $d \mid n \implies d \mid (n\%m)$. Based on Quotient-Remainder theorem, n on division by m gives: $n = qm + r$, where q is the quotient and r is the remainder. Rearranging this equation we get $r = n - qm$, which is of the form $an + bm$ from the given property. The given property states: $\forall n, m, d, a, b \in Z, d \mid n \cap d \mid m \implies d \mid (an + bm)$. Clearly d divides n and m so d divides $r = n - qm$, using the given property with $a = 1andb = -1$. Therefore $d \mid n \implies d \mid (n\%m)$ is proven, let us now prove $d \mid (n\%m) \implies d \mid n$. To prove this let us assume d divides the remainder obtained when n is divided by m, that is, d divides $r = n - qm$. Rearranging this equation we get $n = r + qm$, which is of the form $an + bm$ from the given property. Clearly d divides m and r so d divides $n = r + qm$, using the given property with $a = 1andb = q$. Therefore $d \mid (n\%m) \implies d \mid n$ and $(d \mid n \iff d \mid (n\%m)$ is proven. Thus, $\forall n, m, d \in Z, d \mid m \cap m \neq 0 \implies (d \mid n \iff d \mid (n\%m)$ is proven to be true.

   □

4. If n divides m, then m itself is the greatest common divisor since no divisor of m can be greater than m. If m does not divide n, it will not show up in common_divisors so m can be removed from the range of possible_divisors. So m is returned if m divides m and m is removed from the range of possible_divisors in the else part of the function. Thus, the final code is as follows –

```python
def gcd(n: int, m: int) -> int:
    """Return the greatest common divisor of m and n.

    Preconditions:
    - 1 <= m <= n
    """
    r = n % m

    if r == 0:
        return m
    else:
        possible_divisors = range(1, m)
        common_divisors = {d for d in possible_divisors if divides(d, n) and divides(d, m)}
        return max(common_divisors)
```

Part 3: Wordle!

Complete this part in the provided `a2_part3.py` starter file. Do **not** include your solutions in this file.