



Last week's practical

String does no work!

Outline for Today

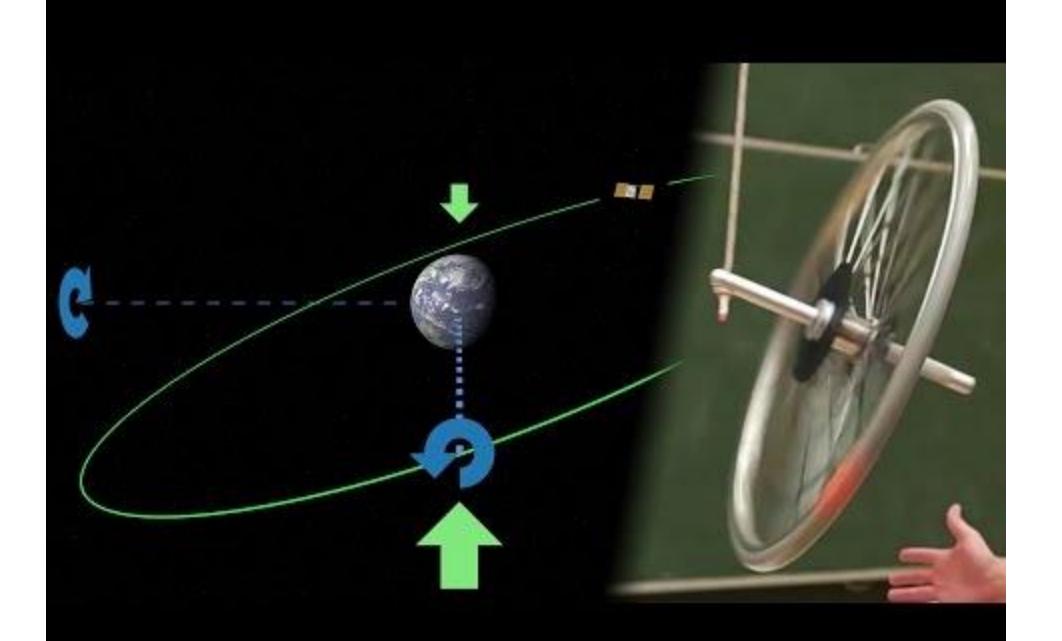
- 6:10 7:00 Practice problems
 - Do them
- 7:00 9:00 Practical Activities
 - Mechanics Module 3, Activity 17
 - Mechanics Module 6, Activities 2 & 3 (and 1 if you have time.)

Tutorial Problems

- 1. **Gyroscopic effects** are often taught purely as vector problems, but there is a very intuitive way of thinking about them...
- 2. What quantities are conserved during an orbit?
- 3. You can do this orbital manoeuvre in 2 thrusts, but it is probably easier to think of it as 4 thrusts, and then recognize that they can be grouped together...

Orbital speed is
$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$
 or $v = \sqrt{\frac{GM}{R}}$.
Escape speed is $\frac{1}{2}mv^2 = \frac{GMm}{R}$ or $v = \sqrt{\frac{2GM}{R}}$.

- 4. This is a cool problem, you can try it yourself using the metre stick in your supplies for today's practical!
 - a. Remember from high school physics that an object just begins to slip when the normal force is just small enough that $\mathbf{F}_s = \mathbf{F}_k$.
 - b. You can find the normal forces by considering torque.



Today's Tutorial Problems

- 1. The initial angular momentum of the wheel points north.
 - (a) The torque points west. The wheel will turn slightly northward.
 - (b) The torque points up. The wheel will lean slightly southward.
 - (e) The torque points down. The wheel will lean slightly northward.
- 2. Energy conservation gives

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

Angular momentum conservation gives

$$mr_1v_1 = mr_2v_2$$

Combining these two concepts and eliminating v_2 gives the required equation.

3. Ignore the Moon.

Orbital speed is
$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$
 or $v = \sqrt{\frac{GM}{R}}$.

Escape speed is
$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$
 or $v = \sqrt{\frac{2GM}{R}}$.

We need one impulse to leave Earth's orbit (get escape velocity). Orbital velocity is about 8 km/s and escape speed is about 11 km/s. So that is an impulse of $3 \times 10^6 \text{ kg m/s}$.

Then we're in orbit around the Sun. We need more impulse to get to into an elliptical orbit with short side at Earth's orbital radius and far end at Mars' orbital radius. Earth's orbital speed around the Sun is 30 km/s. From question 2 we need to get to a speed of $30 \times \sqrt{\frac{2r_2}{r_1+r_2}} = 33$ km/s. So that's an extra impulse of 3×10^6 kg m/s, for a total of 6×10^6 kg m/s.

To circular the orbit at Mars' distance from the Sun we need to know first that we arrive with $v_2 = v_1 \frac{r_1}{r_2} = 22 \text{ km/s}$ speed (from angular momentum conservation. Mars' orbital speed around the Sun is 24 km/s. So we need an impulse of 2×10^6 kg m/s to match orbits. But then we're going too fast to orbit Mars, so we need to reduce this impulse to go from Mars' escape speed of 5 km/s to its orbital speed of 3.5 km/s. That means this impulse should only be 5×10^5 kg m/s.

So we need an initial impulse of 6×10^6 kg m/s and a final impulse of 5×10^5 kg m/s to get from Earth orbit to Mars orbit in 2 thrusts.

4. When sliding, the kinetic friction (left hand) must remain less than the static friction (right hand). The transition happens when the two forces are equal, at which point the right hand exceeds the maximum allowed static friction so it starts slipping. At the instant of transition we have

$$f_k = f_s \to \mu_k n_L = \mu_s n_R$$

The normal forces are found from the torques. Pick the middle of the meter stick as the pivot point (so gravity exerts no torque), and defining x as the initial position of the right finger and y as the final position of the left finger (and measuring both from the zero point of the meter stick which is near the right finger) we find

$$n_L(y - 0.5) = n_R(0.5 - x)$$

Combining these two equations we find

$$y = 0.5(1 + \frac{\mu_k}{\mu_s}) - \frac{\mu_k}{\mu_s}$$

We can use either the slope or the y-intercept to solve the problem.

Using the slopes, the negative of the slope is the required ratio. I find the slope is between $\frac{0.78-0.58}{0-0.4} = -0.50$ and $\frac{0.76-0.56}{0.1-0.4} = -0.67$. This gives a ratio of about 0.58 ± 0.08 .

Using the intercept, the required ratio is 2b-1 where b in the intercept. I find the intercept if between 0.82 and 0.78. This gives a ratio between 0.64 and 0.56. Call it 0.60 ± 0.04 .

The values agree, though the intercept method seems to have smaller uncertainties.

Practical Activities

Today do:

- Mechanics Module 3, Activity 17
- Mechanics Module 6, Activities 2 & 3 (and 1 if you have time.)