



(d) [2 marks] Given the following code:

```
def my_function(n: int) -> None:
    """Precondition: n >= 2"""
    i = 4
    while i < n * n:
        i += 7
```

Find a formula for the *exact number of iterations* that this while loop runs, in terms of the input value  $n$ . You will need to use one of the floor or ceiling functions to ensure your expression is an integer.

Show your work (do not just write a final expression).

The assignment ' $i = 4$ ' takes one step. After ' $k$ ' iterations of while loop,

$$i_k = 4 + 7k \text{ for this to be } \geq n^2, \quad 4 + 7k \geq n^2 \Rightarrow k \geq \frac{n^2 - 4}{7}.$$

To make it an integer, the while loop stops after  $\lceil \frac{n^2 - 4}{7} \rceil$  iterations and each iteration takes one step. Thus the total number of steps in this function are  $1 + \lceil \frac{n^2 - 4}{7} \rceil(1) = 1 + \lceil \frac{n^2 - 4}{7} \rceil$ .

The exact number of iteration that this while loop runs is  $\lceil \frac{n^2 - 4}{7} \rceil$ .

(e) [2 marks] Complete the body of the function below so that its running time is  $\Theta(n \log n)$ . Only add code to the locations indicated by the boxes. Your code should use only variables, arithmetic and comparison operations, and (re)assignment statements. You may **not** call any functions (e.g., `math.log`).

No explanation is necessary; just write the code.

```
def f(n: int) -> None:
    """Precondition: n >= 0"""
    for _ in range(0, n):
        a = 1
```

while  $a < n$  : # Fill in the while loop condition

$a = a * 2$

# Fill in the loop body (with one or more statements)