Unit 4 log derivative

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MAT137

Inverse functions

- Today we will discuss logarithmic differentiation.

Logarithmic differentiation

Often we come across exponentiated quantities such as $f(x) = x^x$.

$$(x^x)' \neq \underline{x\, x^{x-1}}$$

$$(e^{x \ln x})' = e^{x \ln x}\,\underline{(\overset{f}{x}\,\overset{g}{\ln x})'}$$

$$= x^x\left(x' \cdot \ln x + x(\ln x)'\right)$$

$$= x^x(\ln x + 1)$$

Logarithmic differentiation

Often we come across exponentiated quantities such as $f(x) = x^x$. To make our life easy with taking derivative, we use implicit differentiation:

$$y = f(x) \Rightarrow \ln y = \ln f(x) \Rightarrow y' = y\,(\ln f(x))'. \qquad (1)$$

$$(\ln y)' = \frac{1}{y}\, y'$$

Logarithmic differentiation

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So in the example, $y' = y(x \ln x)' = x^x(\ln x + 1)$.

$$y = x^x \iff \ln y = x \ln x$$

$$\iff \frac{y'}{y} = \ln x + 1 \iff \frac{d(x^x)}{dx} = x^x(\ln x + 1)$$

Logarithmic differentiation

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So in the example, $y' = y(x \ln x)' = x^x(\ln x + 1)$. In the case of different bases, useful formulas are

$$\log_a b = c \iff a^c = b. \quad (2)$$

and

$$\log_a x = \frac{\ln(b)}{\ln(a)} \log_b x = \frac{\log_b x}{\log_b(a)} \quad (3)$$

$$(\log_{10} x)' \neq \frac{1}{x} \qquad (2^x)' \neq 2^x$$

$$\left(\frac{\ln x}{\ln 10}\right)' = \frac{1}{\ln 10}\frac{1}{x} \qquad \left(e^{x\ln 2}\right)' = e^{x\ln 2}(x\ln 2)'$$

$$= 2^x \ln 2.$$

Computations - Exponentials and logarithms

Compute the derivative of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln x$
2. $f(x) = \pi^{\tan x}$
3. $f(x) = \ln\left[e^x + \ln \ln \ln x\right]$
4. $f(x) = \log_{10}(2x + 3)$

$\ln(a \cdot b) =$

1. $y = e^{\sin x + \cos x} \ln x$

$\Leftrightarrow \ln y = \ln(e^a \ln x)$

$= \ln e^a + \ln(\ln x)$

$= (\sin x + \cos x)\underbrace{\ln e}_{1} + \ln(\ln x)$

$\Leftrightarrow (\ln y)' = \frac{1}{y} \cdot y' = \cos x - \sin x + \frac{1}{\ln x} \cdot (\ln x)'$

$\Leftrightarrow \frac{d(f(x))}{dx} = e^{\sin x + \cos x} \ln x \left(\cos - \sin + \frac{1}{x \ln x}\right)$

2. $\pi^{\tan x} = y \Leftrightarrow \ln y = \tan x \cdot \ln \pi$

$\Leftrightarrow \frac{1}{y} y' = \ln(\pi) \sec^2(x)$

Chain rule

$\frac{1}{\pi^{\tan x}} (\pi^{\tan x})' = \ln \pi \sec^2 x$

3. $f(x) = \ln(\underbrace{e^x + \ln^{03}(x)}_{g})$

$\left(\ldots\right)'$

$$\frac{1}{g(x)} \cdot g'(x) = \frac{1}{g(x)} \left(e^x + \ln^{03} x \right)'$$

$$= \frac{1}{g(x)} \left(e^x + \ln(\ln \ln x) \right)'$$

$$= \frac{1}{e^x + \ln^{03} x} \left(e^x + \frac{1}{(\ln^{02} x) \ln x \cdot x} \right)$$

4. $$\left(\log_{10}(2x+3)\right)' = \left(\frac{\ln(2x+3)}{\ln 10}\right)'$$

$$= \frac{1}{\ln 10} \cdot \frac{1}{2x+3} \cdot \underbrace{(2x+3)'}_{=2}$$

Logarithm and Absolute Value

The function F is defined by the equation

$$F(x) = \ln|x|.$$

What is its derivative?

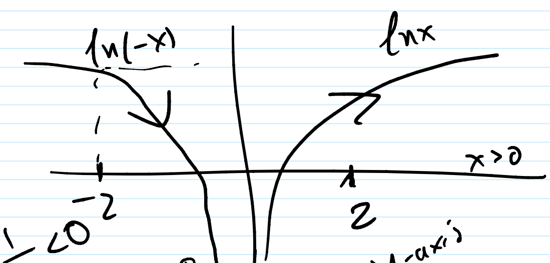
1. $F'(x) = \frac{1}{x}$
2. $F'(x) = \frac{1}{|x|}$
3. F is not differentiable

$$= \begin{cases} \ln x, & x > 0 \\ \underbrace{\ln(-x)}_{=\ln(h(x))}, & x < 0 \end{cases} \xRightarrow{\text{der.}} \begin{cases} \frac{1}{x}, & x > 0 \\ \frac{1}{-x}(-x)', & x < 0 \end{cases}$$

$$= \frac{1}{x}, \quad x \neq 0$$

$\frac{1}{x} < 0$

$\ominus\ x < 0$



reflect across y-axis

A different type of logarithm

Calculate the derivative of

$$f(x) = \log_{x+1}(x^2 + 1)$$

Note: This is a new function. We have not given you a formula for it yet, That is on purpose.

Hint: If you do not know where to start, remember the definition of logarithm:

$$\log_a b = c \iff a^c = b.$$

$$\log_a b = \frac{\ln b}{\ln a}$$

$$\left(\ln_{x+1}(x^2+1)\right)' = \left(\frac{\ln(x^2+1)}{\ln(x+1)}\right)'$$

$a = x+1$

$b = x^2+1$

Logarithmic differentiation

Calculate the derivative of

$$g(x) = x^{\tan x}$$

$y = x^{\tan x}$

$\iff \ln y = \tan x \ln x$

$$g' = \left(e^{\tan x \ln x}\right)'$$

$$\iff \quad = x^{\tan x} \cdot (\tan x \ln x)'$$

More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

$e^{\cos x \ln \sin x}$

$$= \underline{f_1(x)} + \underline{f_2(x)}$$

$$\left(\sin x^{\cos x}\right)' = (y)'$$

$$\iff \ln y = \cos x \ln(\sin x)$$

More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

What is wrong with this answer?

$$\begin{aligned}
\ln f(x) =& (\cos x)\ln(\sin x) + (\sin x)(\ln\cos x)\\
\frac{d}{dx}[\ln f(x)] =& \frac{d}{dx}[(\cos x)\ln(\sin x)] + \frac{d}{dx}[(\sin x)(\ln\cos x)]\\
\frac{f'(x)}{f(x)} =& -(\sin x)\ln(\sin x) + (\cos x)\frac{\cos x}{\sin x}\\
&+ (\cos x)\ln(\cos x) + (\sin x)\frac{-\sin x}{\cos x}
\end{aligned}$$

$$f'(x) = f(x)\left[-(\sin x)\ln(\sin x) + (\cos x)\ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x}\right]$$

Hard derivatives made easier

Calculate the derivative of

$$h(x) = \sqrt[3]{\frac{(\sin^6 x)\sqrt{x^7 + 6x + 2}}{3^x (x^{10} + 2x)^{10}}}$$

An Implicit Function

Find y' if $x^y = y^x$.