

Unit6 Related rates

Friday, November 25, 2022 10:06 AM

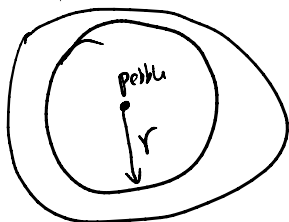
unit6_Rel...

MAT137

- By December 8, we will cover: Rolle's theorem, L'Hopital and finally Curve sketching.
- Today we will go over Related rates word problems.
- Problem set 3 is due Nov.24.

Lake ripple

We drop a pebble into a lake. It produces a circular ripple. When the radius is 2 meters and is increasing at a rate of 10cm/s, at what rate is the area increasing?



Model
variables: $r(t)$ radius

$$A(t) = \pi r^2(t)$$

area

$$\text{Find } \frac{dA}{dt} = ?$$

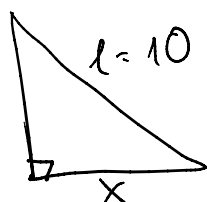
$$\frac{dA}{dt} = 2r\pi \frac{dr}{dt}$$
$$= 2 \cdot 2 \cdot \pi \cdot 0.1 = 0.4\pi \text{ m}^2/\text{s}$$

Sliding ladder

A ten-meter long ladder is leaning against a vertical wall and sliding. The top end of the ladder is 8 meters high and sliding down at a rate of 1 meter per second. At which rate is the bottom end sliding?

Model

$$h = 8$$



decreasing

$$\frac{dh}{dt} = -1$$

$$\frac{dx}{dt} = ?$$

Calculus

$$l^2 = h^2 + x^2$$

Calculus

$$l^2 = h^2 + x^2$$

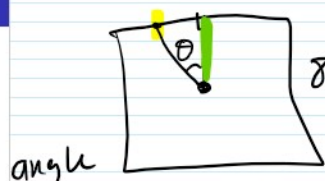
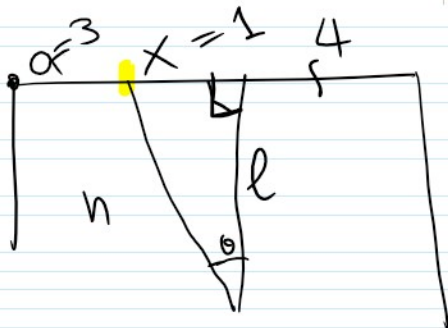
$$\frac{d}{dt} 0 = 2 \cdot h \dot{h} + 2x \dot{x}$$

$$\Rightarrow \frac{dx}{dt} = \frac{-h \dot{h}}{x} = \frac{-8 \cdot (-1)}{6} = \frac{4}{3} \text{ m/s.}$$

$$x = \sqrt{l^2 - h^2} = \sqrt{10^2 - 8^2} = 6$$

Math party

- The MAT137 TAs wanted to rent a disco ball for their upcoming party.
- However, since they are poor, they could only afford a flashlight. At the party, one TA is designated the "human disco ball".
- The TA stands in the center of the room pointing the flashlight horizontally and spins at 3 revolutions per second. (Yes, they are that fast. Ask your TA to demonstrate if you don't believe me!)
- The room is square with side length 8 meters. At which speed is the light from the flashlight moving across the wall when it is 3 meters away from a corner?



angle θ 3.360 degrees around.

\Rightarrow for each 1s moved $3 \cdot 2\pi$ radians

$$\frac{d\theta}{dt} = 6\pi$$

$$\frac{dx}{dt} = ?$$

Revolution
↓
angle

fix reference line
wrt to which we measure angle

Not $\sin \theta$! B/c $\sin \theta = \frac{x(t)}{h(t)}$
and they both vary.

$$\tan(\theta(t)) = \frac{x(t)}{h(t)} = \frac{x(t)}{4}$$

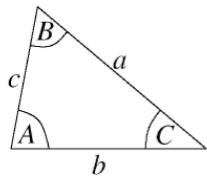
$$\Leftrightarrow \frac{1}{(\cos^2(\theta(t)))} \dot{\theta} = \frac{\dot{x}}{4}$$

$$\dot{x} = 4 \cdot 6\pi \cdot \frac{1}{(\cos(\theta))^2} = \frac{24\pi}{(\frac{4}{h})^2}$$

$$h = \sqrt{x^2 + l^2} = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$= \frac{51\pi}{2} \text{ m/s}$$

Cosine and sine laws



Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

(for finding sides)

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

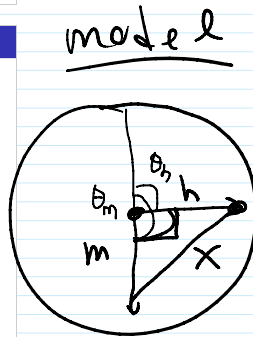
Sleepy ants

Two ants are taking a nap. The first one is resting at the tip of the minute hand of a cuckoo clock, which is 25 cm long. The second one is resting at the tip of the hour hand, which is half the length. At what rate is the distance between the two ants changing at 3:30?

$$\frac{d\theta_m}{dt} = ?$$

$$\frac{dx}{dt} = ?$$

$$\frac{d\theta_h}{dt} = ?$$



The kite

Mary Poppins is flying a kite. The kite is 21 meters above the ground and it is being blown horizontally by the wind at 2 m/s. Mary's hands are 1 meter above the ground. Right now 30 meters of string are out. At what rate is the string being released from Mary's hands?

Coffee

A coffee filter is shaped like an inverted cone. It has a radius at the top of 4 cm and it is 6 cm in height. Coffee flows out of at the bottom at a rate of $2\text{ cm}^3/\text{s}$. If the filter begins completely filled, how fast is the coffee level decreasing after 30 seconds?

Hint: the cone has volume $V = \frac{\pi r^2 h}{3}$ for radius r and height h and compute the ratio of $\frac{r}{h}$.