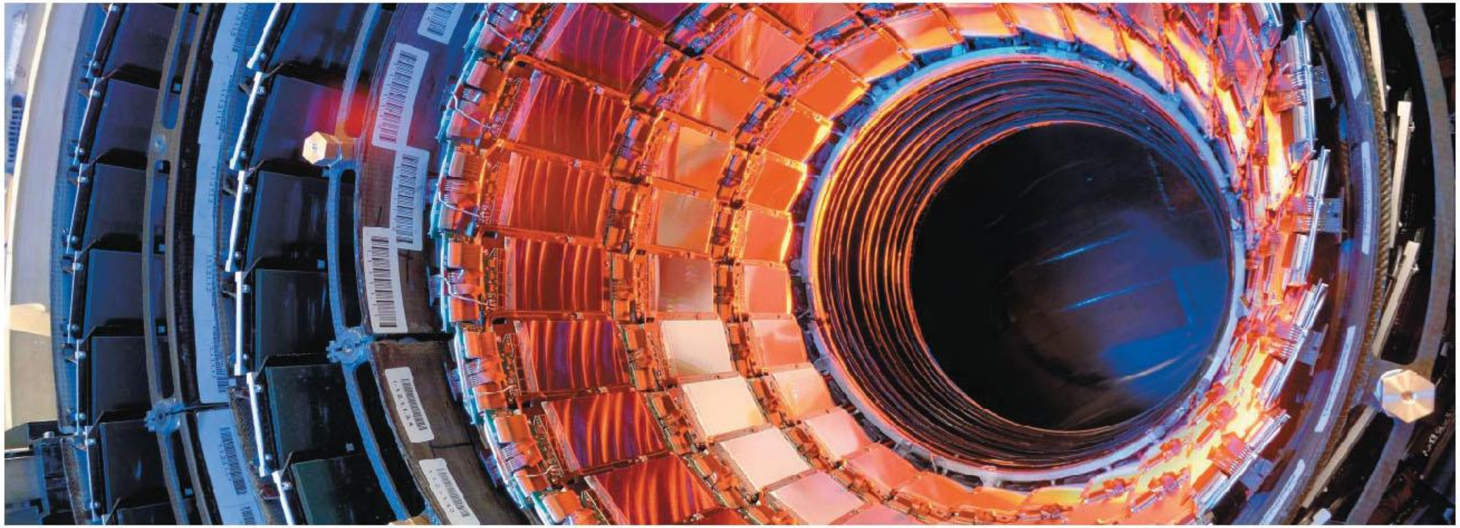
Chapter 36 – Relativity

- Reference frames, events, measurements, space-time diagrams
- Postulates of special relativity, impact on simultaneity
- Time dilation, space contraction, and Lorentz transformations
- Relativistic momentum and energy



Lorentz Transformations

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

unchanged

$v \to x$-direction

linear in x & t

depend v

$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

S & S'

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

Lorentz Transformations: space-time diagrams

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) = 0 \qquad t = \gamma(t' + vx'/c^2)$$

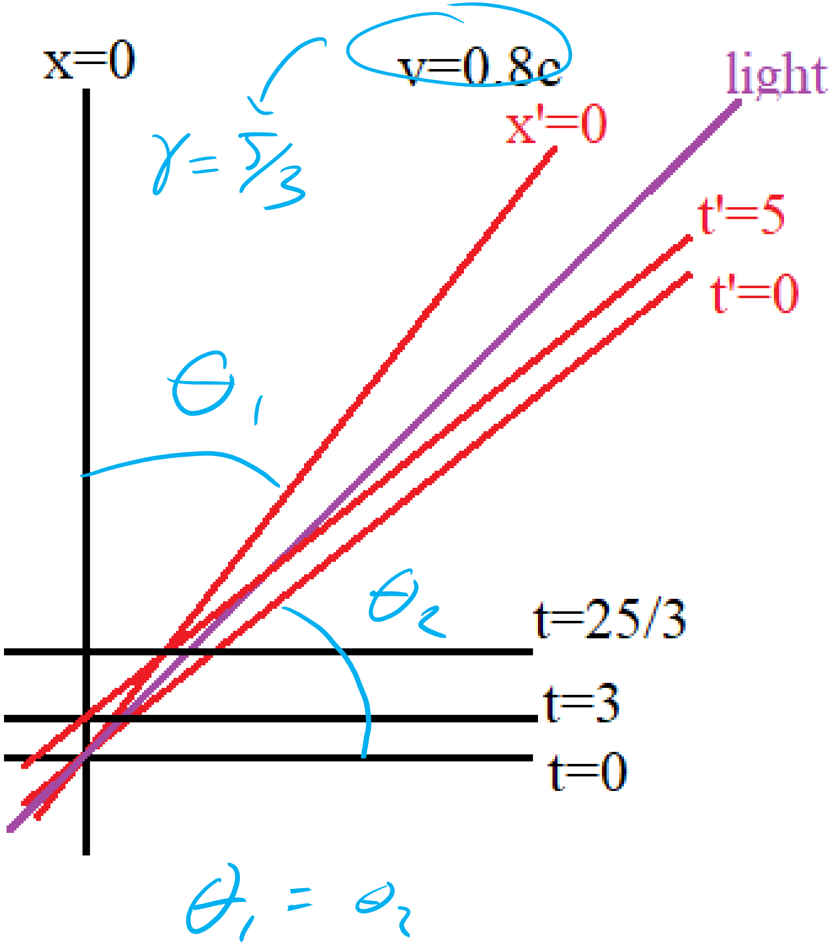
$x' = 0 \rightarrow \gamma(x - vt) = 0$

$\rightarrow x = vt$

$t = \frac{1}{v} x$

$t' = 0$

$\rightarrow t = vx/c^2$



Lorentz Transformations: simultaneous events in S

$v = 0.8c$

$\gamma = 5/3$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$\frac{5}{3}(L - v0)$

in S

A : $x = 0$, $t = 0$

B : $t = 0$, $x = L$

$\Delta t = 0 \qquad \Delta x = L$

A $\quad x' = 0$

in S' $\quad t' = 0$

B $\quad x' = \frac{5}{3}L$

$t' = \frac{5}{3}\left(0 - \frac{4}{5}L/c\right) = -\frac{4}{3}L/c$

$\Delta x' = \frac{5}{3}L \qquad \Delta t' = -\frac{4}{3}\frac{L}{c}$

$\Delta t' \neq 0 \rightarrow \Delta x' \neq$ length

Lorentz Transformations: proper time in S

$v = \frac{4}{5}c \quad \gamma = 5/3$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

in S

$\Delta x = 0 \rightarrow$ proper time

$A: x = 0,\ t = 0$

$B: x = 0,\ t = \tau$

in S'

A: $x' = 0$, $t' = 0$

B: $x' = -\frac{5}{3}\frac{4}{5}c\tau = -\frac{4}{3}c\tau$

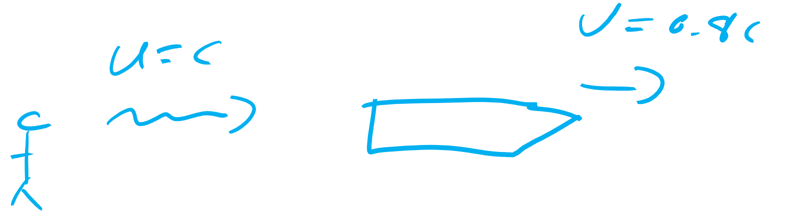
$t' = \frac{5}{3}\tau = \gamma\tau$

Lorentz Transformations: what if u = c

$v = 0.8c$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$u = c$



$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

$$u' = \frac{c - v}{1 - cv/c^2} = \frac{c\left(1 - \frac{v}{c}\right)}{1 - \frac{v}{c}}$$

$$= c$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

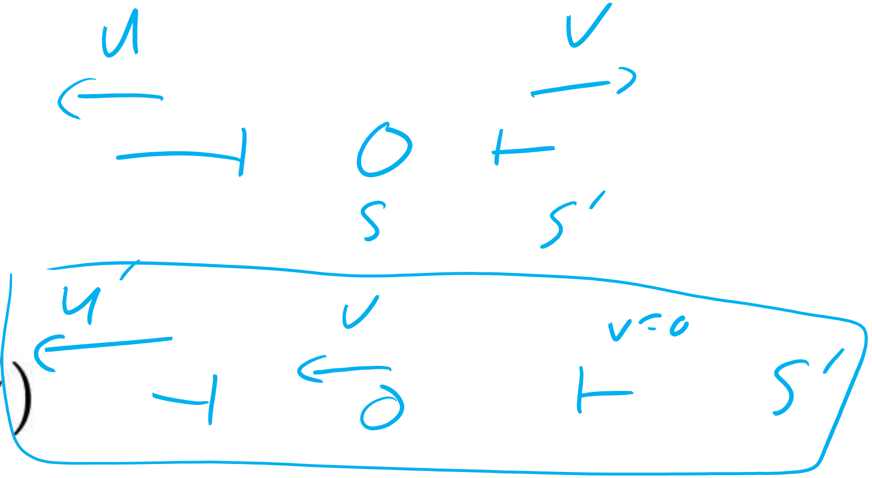
Team Up questions

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$



$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

→ ~ zero → $u' = u - v$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

We have 2 versions of what happened between event A "Ryan and Priya meet" and event B "Priya sees the front explosion". Ryan sees $\Delta x = 1.73$ m and $\Delta t = 11.55$ ns. Priya sees $\Delta x = 0$ and $\Delta t = 10$ ns. Find $c^2(\Delta t)^2-(\Delta x)^2$ for both.

$\boxed{ds^2 = c^2 dt^2 - dx^2 = ds'^2}$ → metric

$ds^2 = c(11.55)^2 - (1.73)^2 = 9$

$ds'^2 = c^2 (10\,ns)^2 - 0 = 9$

$f = 1 - \frac{2m}{r}$

$ds^2 = c^2 dt^2 f - \frac{1}{f}(dx^2 + dy^2 + dz^2)$

black hole