Assignment - 5.

Q1. Find the mean and variance of binomial distribution.

Ans. Let X be the random variable following binomial distribution with parameters n and p.

$$X \sim B(n, p)$$

First, we find the mean or expectation of X.

$$E(X) = \sum x P_r (X = x)$$

Thus:

$$E(X) = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k}$$

By definition of Binomial distribution

$$p + q = 1$$

$$E(X) = \sum_{k=1}^{n} k \binom{n}{k} p^k q^{n-k}$$

$$\left[\because k = 0 \quad k \left(\frac{n}{k}\right) p^k q^{n-k} = 0 \right]$$

Roll No........................Scholar No...........................

$$E(X) = \sum_{k=1}^{n} n \binom{n-1}{k-1} p^k q^{n-k}$$

Factors of Binomial coefficient:

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

$$E(X) = np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{(n-1)-(k-1)}$$

taking ~~out~~ out 'np' and using $(n-1)$

$$= np \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \quad \begin{bmatrix} m = n-1 \\ j = k-1 \end{bmatrix}$$

$$\boxed{E(X) = np}$$

Now, we find variance

$$E(X^2) = \sum x^2 Pr(X = x)$$

$$E(X^2) = \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k}$$

$$= \sum_{k=0}^{n} nk \binom{n-1}{k-1} p^k q^{n-k}$$

$$= np \sum_{k=1}^{n} k \binom{n-1}{k-1} p^{k-1} q^{(n-1)-(k-1)}$$

$$= np \sum_{j=0}^{m} (j+1) \binom{m}{j} p^j q^{m-j}$$

$$= np \left(\sum_{j=0}^{m} j \binom{m}{j} p^j q^{m-j} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np \left(\sum_{j=0}^{m} m \binom{m-1}{j-1} p^j q^{m-j} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np \left((n-1) p \sum_{j=1}^{m} \binom{m-1}{j-1} p^{j-1} q^{(m-1)-(j-1)} + \sum_{j=0}^{m} \binom{m}{j} p^j q^{m-j} \right)$$

$$= np((n-1)p + 1)$$
$$= n^2p^2 + np(1-p)$$

$$var(X) = E(X^2) - (E(X))^2$$
$$= n^2p^2 + np(1-p) - (np)^2$$
$$= np(1-p)$$

$$\boxed{Var(X) = np(1-p)} \text{ or } \boxed{Var(X) = npq}$$

Roll No.........................Scholar No.............................

Q2. Find the SD of binomial distribution

Solⁿ Variance $= \left(\sum_r r^2 \cdot P(r)\right) - \text{mean}^2$

$$= \sum_r [r(r-1)+r]\, {}^nC_r\, p^r q^{n-r} - (np)^2$$

$$= n(n-1)p^2 \left\{ \sum_r {}^{n-2}C_{r-2}\, p^{r-2} q^{n-r} \right\} + np - (np)^2$$

$$= n(n-1)p^2 (q+p)^{n-2} + np - n^2p^2$$

By Binomial Theorem

$$(a+b)^n = \sum_{k=0} {}^nC_k\, a^n b^{n-k}$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p) \qquad \{p+q=1\}$$

Variance $= npq$

Standard Deviation $= (\text{Variance})^{1/2}$

$$= (npq)^{1/2}$$

$$\boxed{\text{Standard Deviation} = \sqrt{npq}}$$