IIIT Vadodara
Autumn 2018-19
TE3 Computer Vision
Lab-1: Fundamentals of numerical/error analysis in linear inverse systems

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Problem 1. (i) Solve the following set of linear equations:
2.0x1 + 2.0x2 = 6.0 (1)
2.0x1 + 2.0005x2 = 6.001 (2)
Now consider this: the coefficient x2 is changed to 2.001 in equation (2) due to limited (finite) precision of the computer while all other entries remain the same.
(ii) Find the new solution to the set of equations.
(iii) From this find the relative error in the coefficient matrix and the corresponding relative errors in the obtained solutions. (iv) Using this set of relative errors conclude on whether this problem is ill-posed or better-posed. Note that the set of linear equations can be written in matrix form Ax = b, where A is the coefficient matrix.
Find analytical answers and compare them with the practical re- sults.

Solution

```python
import numpy as np
a = [[2 , 2] ,[2 , 2.0005]]
b = [[6], [6.001]]
solution = np.dot(np.linalg.inv(a),b)
#solution 1
print(solution)
a1 = [[2,2] , [2 , 2.001]]
solution2 = np.dot(np.linalg.inv(a1),b)
print(solution2)
# Solution 2
relative_error_coeff = (np.linalg.norm(np.array(a)-
    np.array(a1)))/np.linalg.norm(a)
print(relative_error_coeff)
relative_error_solution = (np.linalg.norm(np.array(solution) -
    np.array(solution2)))/np.linalg.norm(solution)
print(relative_error_solution)
print("Cond A", np.linalg.cond(a))
print("Cond A1",np.linalg.cond(a1))
print("Sensitivity being ",relative_error_solution/relative_error_coeff)
#Comparing the condition number we can say that they are ill posed problems
#Also a small error in A creates a large error in the soltion, x ( by the errors
    found in the above code)
```

Output

```
[[ 1.]
 [ 2.]]
[[ 2.]
 [ 1.]]
0.000124992187256
0.632455532034
('Cond A', 16002.000187502217)
('Cond A1', 8002.0003750284477)
('Sensitivity being ', 5059.9605136827286)
```

As we can see that changing the coefficients of one of the variables with a very small amount have a drastic amount of effect on the output. By checking the Conditioning constant of both the variables we can say that the given problem is ill-posed. By ill posed we mean, by changing the input by a small amount the output changes drastically. In the ouput the first vector is for the solution of unchanged equation, and then when we change the equation we get the second vector as the output.

Then the third line contains the relative error of the inputs between the two images, while the fourth carries the same for the outputs. As we can see here the error in output is many-folds greater then the error in the input. Then in the next line we get the condition matrix of A and in the next line we get the condition matrix of the output. Finally we calculate the Sensitivity of the Equations. Thus conclude about the ill-posedness of the equations.