Topology Connectivity Analysis of Internet Infrastructure Using Graph Spectra

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Abstract—**Understanding and modelling the Internet has been a major research challenge in part due to the complexity of the interaction among its protocols and in part due to multilevel, multidomain topological structure. It is therefore crucial to properly analyse each structural level of the Internet to gain a better understanding, as well as to improve its resilience properties. In this paper, first we present the physical and logical topologies of two ISPs and compare these topologies with the US interstate highway topology by using graph metrics and then using the normalised Laplacian spectrum. Our results indicate that physical network topologies are closely correlated with the motorway transportation topology. Finally, we study the spectral properties of various communication networks and observe that the spectral radius of the normalised Laplacian matrix is a good indicator of graph connectivity when comparing different size and order graphs.**

Keywords-**Internet resilience; critical infrastructure; Internet modelling; multilevel analysis; graph spectrum; bipartite graph**

I. INTRODUCTION AND MOTIVATION

The Internet has become a critical infrastructure and its modelling has been a major research effort with some controversial findings [1]–[3]. The primary focus has been on the logical aspects of the topology, since tools were developed to collect, measure, and analyse IP-layer properties of the Internet (e.g. Rocketfuel [4]). On the other hand, physical topologies provide the necessary connectivity to higher layers; thus defining physical connectivity is a major research challenge [5], [6]. Previously, we observed that the link connectivity of the physical topologies appear visually correlated with other critical infrastructures such as motorways and railways [7].

In this paper, we intend to provide insight into the evolution of the communication networks by generating the topology of the US interstate highway system and analysing its graph properties against the physical fibre topology and PoP (point of presence) level topology of two ISPs (Internet service providers). Previous studies analysed US interstate highways [8], Dutch roads [9], road networks of California, Pennsylvania, and Texas [10], US and European road networks [11], European railways [12], and Indian railways [13].

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In this paper, first we present our ongoing efforts towards making the physical network and transportation network topologies available. Next, we analyse network characteristics of these different critical infrastructures both in terms of graph metrics and graph spectrum.

The second major contribution of this paper is to analyse the spectrum of the normalised Laplacian matrix for resilience properties. We show that traditional graph metrics are not sufficient for a comparative analysis of graphs with different numbers of nodes and links. Furthermore, we observe that the spectral radius of the normalised Laplacian matrix may be the ideal measurement for comparing connectivity of graphs with different order and size.

The rest of the paper is organised as follows: We present brief background on graph spectra in Section II. The dataset for the communication and transportation topologies we use in this study is presented in Section III. The evaluation of graphs using metrics and spectrum is presented in Section IV. We correlate graph diversity with the spectral radius of the normalised Laplacian matrix in Section V. Finally, we summarise our findings as well as propose future work in Section VI.

II. BACKGROUND AND RELATED WORK

Let $G = (V, E)$ be an unweighted, undirected graph with n vertices and l edges. Let $V = \{v_1, v_2, \ldots, v_n\}$ denote the vertex set and $E = \{e_1, e_2, \ldots, e_l\}$ denote the edge set. The connections between its nodes can be represented by several methods including an adjacency matrix, incidence matrix, Laplacian matrix, and normalised Laplacian matrix [14], [15]. $A(G)$ is the symmetric adjacency matrix with no self-loops where $a_{ii} = 0$, $a_{ij} = a_{ji} = 1$ if there is a link between $\{v_i, v_j\}$, and $a_{ij} = a_{ji} = 0$ if there is no link between $\{v_i, v_j\}$. The Laplacian matrix of G is: $L(G) = D(G) - A(G)$ where $D(G)$ is the diagonal matrix of node degrees, $d_{ii} = deg(v_i)$. Given degree of a node is $d_i = d(v_i)$, the normalised Laplacian matrix $\mathcal{L}(G)$ can be represented:

$$\mathcal{L}(G)(i,j) = \begin{cases} 1, & \text{if } i = j \text{ and } d_i \neq 0 \\ -\dfrac{1}{\sqrt{d_i d_j}}, & \text{if } v_i \text{ and } v_j \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

Let M be a symmetric matrix and I be the identity matrix of order n. Then, *eigenvalues* (λ) and the *eigenvector* ($\mathbf{x}$) of M satisfy $M\mathbf{x} = \lambda\mathbf{x}$. In other words, eigenvalues are the roots of the characteristic polynomial, $\det(M-\lambda I) = 0$ for $\mathbf{x} \neq 0$. The set of eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ together with their multiplicities (number of occurrences of an eigenvalue λ_i) define the *spectrum* of M. Spectral graph theory has been extensively covered in several monographs [14]–[18]. The spectrum of the AS-level topology of the Internet has been analysed based on the k largest values of the adjacency matrix [19]. The IP-level topology of the Internet has also been investigated and its Laplacian spectrum compared against synthetically generated topologies [20]. The normalised Laplacian spectrum of AS-level topologies has been shown to differ significantly from that of synthetically generated topologies [21]. Recently, a weighted spectral distribution metric has been proposed and shown that synthetically generated graphs can be fine-tuned using spectral properties [22]. While previous studies utilised graph spectra to analyse logical level topologies, in this study we focus on physical networks and how they relate to each other structurally.

III. TOPOLOGICAL DATASET

We study real networks (i.e. transportation and communication) that are geographically located within the continental United States. Therefore, we only include the 48 contiguous US states, the District of Columbia, and exclude Hawaii, Alaska, and other territories. Furthermore, we have developed the KU-TopView (KU Topology Visualiser) [7] using the Google Map API and JavaScript to visually present these topological maps. Unlike other visualisation tools, KU-TopView makes raw data conveniently available in the universal form of an adjacency matrix along with the node coordinates. We have made these topologies publicly available [23].

A. Transportation Network

We have generated the interstate highway topology to represent the transportation network. Our starting point is the American Association of State Highway and Transportation Officials (AASHTO) data, which lists *control cities* and their sequential listing along each interstate highway. A control city is a major population center or destination on or near the interstate highway system determined by each state [24]. However, while building the transportation topology, we realised that the existing list of control cities are not sufficient to represent the graph accurately. For example, there is no control city at some interchanges between interstate highways. Therefore, we add 6 additional cities in those cases after verifying the crossing on Google Maps. There are also a few newer highways that are not listed in the 2001 AASHTO document that we add to reflect current connectivity (e.g. I-335 Kansas Turnpike, I-86 East, I-97, I-68, I-495 in NY). This US highway graph with 400 nodes, 540 links, and an average degree of 2.7 is shown in Figure 1. We note that in a previous study of US interstate highway system, the authors used GIS (geographic information system) databases from the year 2000

(unfortunately there is no reference to the source of data nor is the topology publicly available), and the resulting interstate highway network consisted of 935 nodes, 1337 links with an average degree of 2.86 [8].
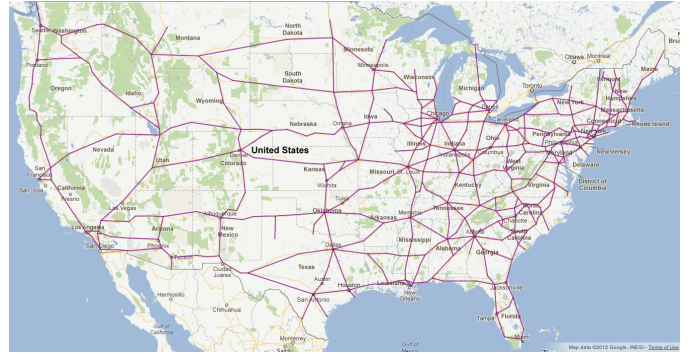

Fig. 1. US interstate highways

B. Communication Networks

The Internet is a complex and large-scale network for which collective analysis is non-trivial. Therefore, we restrict our study to include PoP-level and physical fibre topologies of individual service providers. We use Rocketfuel-inferred AT&T and Sprint PoP-level topologies [4] to study logical level topologies as shown in Figure 2. We note that international links, as well as links crossing over Pacific and Atlantic Oceans, are removed intentionally to compare the PoP-level topologies against the interstate highway topology. The original PoP-level topological data from Rocketfuel is used to compare graph diversity and spectral radius in Section V.
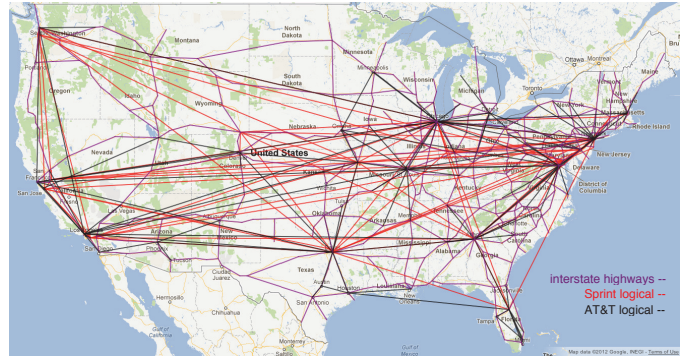

Fig. 2. Logical topologies overlaid on highways

We use a US *long-haul fibre-optic routes* map data to generate physical topologies [25]. In this map, US fibre-optic routes cross cities throughout the US and each ISP has a different coloured link to differentiate between them. We project the cities to be physical node locations and connect them based on the map, which is sufficiently accurate for national scale. We use this data to generate adjacency matrices for each individual ISP as shown in Figure 3. Initial visual inspection suggests that the physical fibre topologies are similar to interstate highway topology. The relation of the fibre topology and other physical
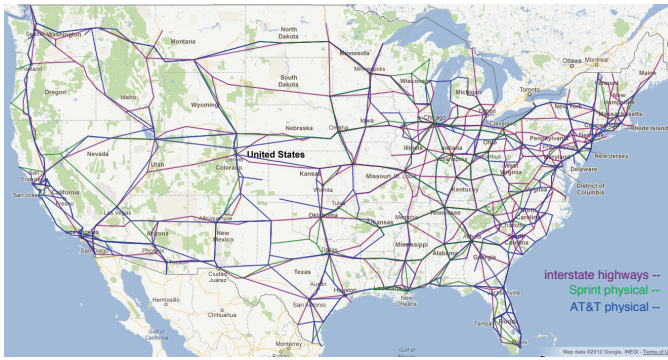
Fig. 3.   Physical topologies overlaid on highways

infrastructures was stated before [7], [26]; however, to best of our knowledge, we are not aware any work that correlates these different infrastructures rigorously.

IV. TOPOLOGY ANALYSIS

Although topology viewing is a powerful tool, it does not suffice for rigorous analysis of topologies [27]. We therefore calculate the graph metrics of regular networks (shown in Table I) and critical infrastructures as shown in Table II using the Python NetworkX library [28].

A. Metrics Analysis

Some of the well-known metrics provide insight on a variety of graph properties, including distance, degree of connectivity, and centrality. Network diameter, radius, and average hop count provide distance measures [5]. Betweenness is the number of shortest paths through a node or link and provides a centrality or importantness measure [29]. Clustering coefficient is a centrality measure of how well a node's neighbours are connected [5]. Closeness centrality is the inverse of the sum of shortest paths from a node to every other node [30], [31]. Assortativity provides a measure of degree variance in a network [32]. Algebraic connectivity, $a(G)$, is the second smallest eigenvalue of the Laplacian matrix [33]. For the graphs of the *same order*, algebraic connectivity provides a very good measure of how well the graph is connected and it indicates robustness of networks against node and link failures [34]–[36].

We start our metrics-based analysis on six baseline topologies: star, linear, tree, ring, grid, and full mesh. We investigate the effect of an increase in the size and order from $n = 10$ to $n = 100$ for the baseline topologies as shown in Table I. Since some metrics yield the same values for graphs of the same order (e.g. average degree for star, linear, tree), and others yield the same values for graphs of differing sizes and orders (e.g. same $a(G)$ for 10 node linear and 100 node grid), relying on a single metric for graph analysis is clearly not sufficient.

We also investigate graph properties of the two ISP networks, which include PoP-level and fibre-link level topologies, as well as the US interstate highway graph as shown in Table II. The metrics for the *logical* topologies of Sprint and AT&T differ from the *physical* topologies of the Sprint

and AT&T. In general, AT&T topologies have more nodes and links compared to Sprint topologies. Physical topologies have more nodes and links compared to logical topologies for each backbone provider with very different characteristics. The maximum degree of each provider's physical topology is less than that of its corresponding logical topology. This is due to the ability of logical topologies to arbitrarily overlay virtual links and a number of degree two intermediate nodes needed for accurate geographic representation. Average degree values for the logical topologies are greater than those of the fibre layer topologies. Both physical topologies have a network diameter of 37, which is an order of magnitude greater than the network diameter of the logical topologies. Similarly, the network radii of the physical topologies are an order of magnitude greater than the logical topologies. The average hop counts of the physical topologies are greater than those of the logical topologies. Betweenness values also differ for physical and logical topologies, showing a difference of an order of magnitude higher for physical topologies.

From a distance metrics perspective, clearly physical topologies have higher values. This is expected since physical topologies have more nodes with low degree. We observe that degree based metric values also differ between physical and logical topologies. This can be attributed to ease of connecting nodes on a logical topology, whereas physical connections require fibre to be installed physically between nodes. From the centrality metrics perspective, we can see that the physical topologies are not as clustered and the degree distributions are more homogeneous. We can also see that US highway graph metrics are closer to those of the physical topologies. This is not surprising: since both the US highway system and the physical layer of the Internet are physical infrastructures rather than logical overlays, they frequently share the same right-of-way. Collective analysis of graph metrics provides a good indication of resilience of different topologies; however, it is difficult to infer sensible conclusions about the structure of a network or how similar two different networks are. Therefore, we redirect our attention to the spectra of these graphs.

B. Spectrum Analysis

The normalised Laplacian spectrum provides insight into the structure of networks that are different in size and order. The eigenvalues of the normalised Laplacian reside in the $[0, 2]$ interval and take values $\{0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n\}$. The algebraic multiplicity of $\lambda = 0$ indicates the number of connected components. Furthermore, similar matrices may have similar eigenvalues and multiplicity. The spectrum of $\mathcal{L}(G)$ is symmetric about 1. A large algebraic multiplicity for the eigenvalue $\lambda = 1$ may indicate *duplications* in a network [37]. In other words, two separate nodes $\{u, v\}$ might have all or many of their neighbours being same. The presence of many small eigenvalue multiplicities may indicate that there are many components within a graph and these components are loosely connected to each other [37]. An eigenvalue of 2 indicates the graph is bipartite; eigenvalues close to 2 indicates the graph is close to being a bipartite graph [37]. A bipartite

TABLE I
TOPOLOGICAL CHARACTERISTICS OF BASELINE NETWORKS

Network Topology	Star	Linear	Tree	Ring	Grid	Mesh	Star	Linear	Tree	Ring	Grid	Mesh
Number of nodes	10	10	10	10	10	10	100	100	100	100	100	100
Number of links	9	9	9	10	13	45	99	99	99	100	180	4950
Maximum degree	9	2	3	2	3	9	99	2	3	2	4	99
Average degree	1.8	1.8	1.8	2	2.6	9	1.98	1.98	1.98	2	3.6	99
Degree assortativity	-1	-0.13	-0.53	1	0.28	1	-1	-0.01	-0.34	1	0.57	1
Node closeness	0.58	0.29	0.37	0.36	0.44	1	0.51	0.03	0.13	0.04	0.15	1
Clustering coefficient	0	0	0	0	0	1	0	0	0	0	0	1
Algebraic connectivity	1	0.1	0.18	0.38	0.38	10	1	0.001	0.01	0.004	0.1	100
Network diameter	2	9	5	5	5	1	2	99	12	50	18	1
Network radius	1	5	3	5	3	1	1	50	6	50	10	1
Average hop count	1.8	3.67	2.82	2.78	2.3	1	1.98	33.67	7.8	25.3	6.67	1
Node betweenness (max)	36	20	26	8	11	0	4851	2450	3068	1201	616	0
Link betweenness (max)	9	25	24	13	12	1	99	2500	2496	1250	341	1

Network Topology	Sprint Physical	Sprint Logical	AT&T Physical	AT&T Logical	US Highways
Number of nodes	263	28	361	107	400
Number of links	311	76	466	140	540
Maximum degree	6	14	7	23	7
Average degree	2.37	5.43	2.58	2.62	2.7
Degree assortativity	-0.17	-0.23	-0.16	-0.4	0.11
Node closeness	0.07	0.48	0.08	0.3	0.08
Clustering coefficient	0.03	0.41	0.05	0.09	0.05
Algebraic connectivity	0.0053	0.6844	0.0061	0.1324	0.0059
Network diameter	37	4	37	6	40
Network radius	19	2	19	3	21
Average hop count	14.78	2.19	13.57	3.38	13.34
Node betweenness (max)	11159	100	15970	2168	22798
Link betweenness (max)	9501	27	14270	661	18585

graph is a graph in which its vertex set can be divided into two groups in such a way that there will be no edges between the vertices in each group.

The PMFs (probability mass functions) of the normalised Laplacian eigenvalue multiplicities for baseline topologies (star, linear, ring, tree, grid, full mesh) of order $n = 100$ is shown in Figure 4. Since most of the eigenvalues have very small multiplicities, the distribution of multiplicities has a floor that is too noisy to be able to gather useful information. Therefore, we use the CDFs (cumulative distribution functions) of the eigenvalue multiplicities for these baseline topologies as shown in Figure 5. The star topology has its eigenvalues fixed $\{0 = \lambda_1 \leq 1 = \lambda_2 = \ldots = \lambda_{n-1} \leq \lambda_n = 2\}$. The spectrum of a full mesh looks similar to a star, except that it does not have an eigenvalue of 2 and the eigenvalues are fixed at 1.0101 (we comment on that later). An interesting observation is that the spectrum of these two baseline topologies look very similar. Indeed, at a micro level we can think of each individual node in a mesh as a star. Furthermore, the algebraic connectivity of a star is 1 [33]. However, since node centrality measures are largest for a star topology, the central node in a star can be the target of an attack or the single point of failure from a network engineering perspective. The spectrum of linear and ring topologies look almost identical, since a ring has an additional link compared to a linear topology, and both linear and ring topologies have the lowest algebraic

connectivity values. Multiplicities of tree and grid topologies lie somewhere between the two extremes of mesh and linear. We also observe that since a Manhattan grid is a combination of linear topologies, its spectrum looks closer to a linear topology.

We show CDFs of eigenvalue multiplicities of five different *complete* graphs in Figure 6. The eigenvalues of a n order complete graph are: $\{0 = \lambda_1 \leq \frac{n}{n-1} = \lambda_2 = \ldots = \lambda_n\}$. The multiplicity of the eigenvalue equal to $n/(n-1)$ for complete graphs is $n-1$. Moreover, as the order of the graph approaches infinity, the eigenvalues will converge to a value of 1 since $\lim_{n\to\infty} \frac{n}{n-1} = 1$. However, eigenvalues λ_2 through λ_n are never exactly equal to 1 in a *finite* full mesh topology. Furthermore, the algebraic connectivity is equal to the order of a complete graph $a(G) = n$.

The PMFs of eigenvalues of real networks are shown in Figure 7; however, as with the baseline graph spectrum comparison, its floor is noisy. Therefore, we plot the CDFs of eigenvalues of real networks as shown in Figure 8. Clearly, the spectra of the logical and physical topologies differ. Furthermore, the spectra of the physical topologies resemble the spectra of the US interstate highway graph. This confirms our supposition that the properties of networks are similar since fibre is laid along right-of-ways, such as highways. The algebraic multiplicity for the eigenvalue $\lambda = 1$ is largest for the AT&T logical topology, indicating that this topology contains
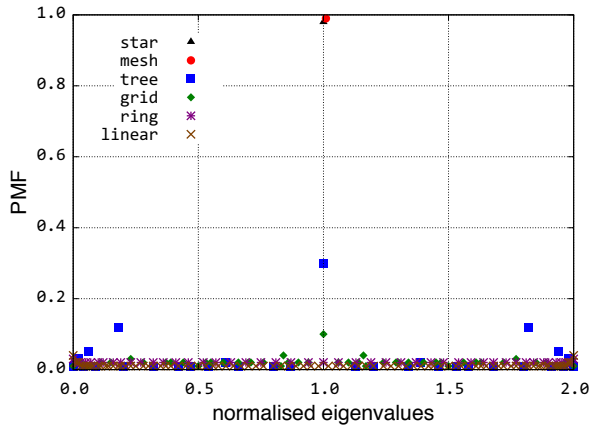
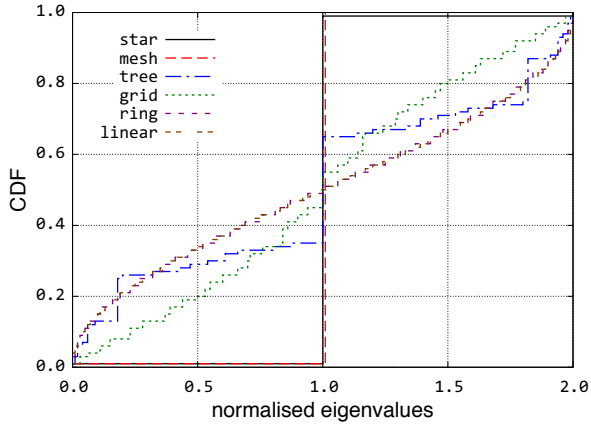Fig. 4. PMF of eigenvalues of baseline topologies, $n = 100$

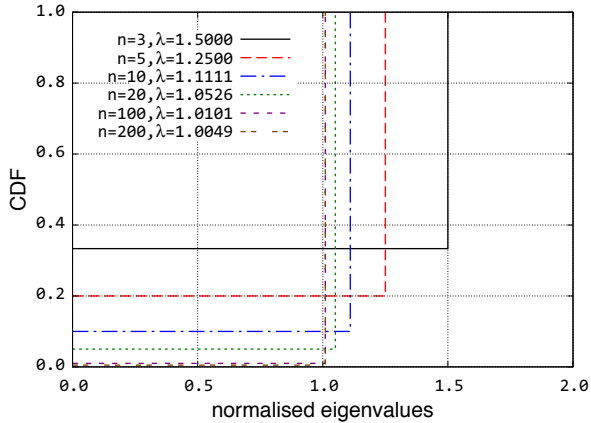Fig. 5. CDF of eigenvalues of baseline topologies, $n = 100$

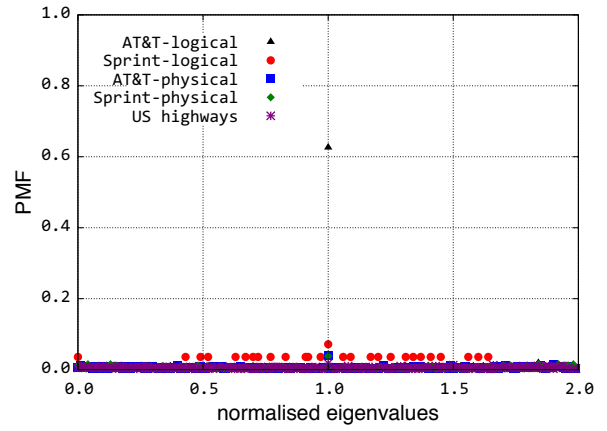Fig. 6. CDF of eigenvalues of complete graphs
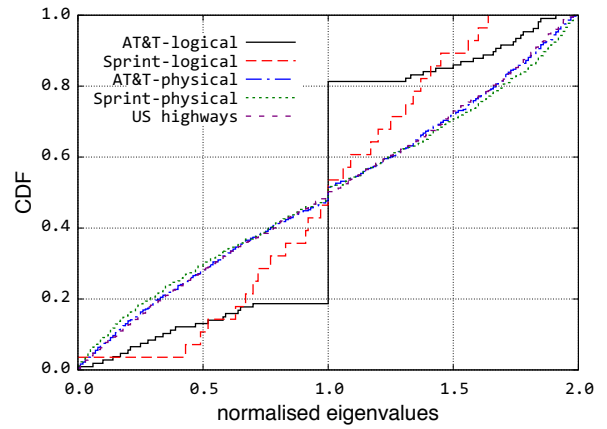

Fig. 7. PMF of eigenvalues of real networks

Fig. 8. CDF of eigenvalues of real networks

V. SPECTRAL RADIUS AND CONNECTIVITY

Previously, we presented how physical communication topologies match the structure of the motorways using normalised Laplacian spectra. In this section, we present our observations on evaluation of a graph by its normalised Laplacian spectrum. We investigate 13 communication networks with a variety of structural properties that we studied in our path diversity metric study [30], [31]. The CDFs of eigenvalue multiplicities of communication networks are shown in Figure 9. The eigenvalues of the 13 topologies are symmetric about 1, with Telstra having the largest multiplicity corresponding to the eigenvalue $\lambda = 1$. AboveNet has the smallest of the 13 largest eigenvalues, and the AT&T physical topology has the largest of the 13 largest eigenvalues.

cTGD (compensated total path diversity) is a heuristic metric that predicts survivability of a topology and indicates the ability to construct *diverse paths* through a network that do not share fate in node or link [30], [31]. The 13 topologies we studied with their cTGD rankings are shown in Table III. The $a(G)$ (algebraic connectivity) of these topologies and their rankings is shown in columns 3 and 4. Algebraic connectivity is the second smallest eigenvalue of the Laplacian matrix and is well-suited for measuring graph connectivity and for

the largest number of node duplications. In other words, this topology has the most star-like components, as is evident by visually inspecting it on KU-TopView [23]. The largest eigenvalues indicate to what degree a graph is bipartite [37]. The largest eigenvalues of the physical topologies and the largest eigenvalues of the interstate motorways graph are the eigenvalues closest to 2. Hence, the physical topologies and the motorways topology are the most nearly bipartite graphs.

TABLE III
GRAPH RANKING COMPARISON FOR PATH DIVERSITY AND SPECTRAL PROPERTIES

| Network	cTGD	cTGD Rank	$a(G)$	$a(G)$ Rank	$\lambda = 1$ Multiplicity	Multiplicity Rank	$\rho(\mathcal{L})$	$\rho(\mathcal{L})$ Rank	$\rho(\mathbf{L})$	$\rho(\mathbf{L})$ Rank	$\rho(\mathbf{A})$	$\rho(\mathbf{A})$ Rank
Level 3	0.4494	1	0.9518	3	0.0755	9	1.5033	2	43.0326	13	24.4635	13
AboveNet	0.4386	2	0.9645	2	0.0455	10	1.4978	1	18.1283	8	9.1761	10
Exodus	0.3617	3	1.0083	1	0.0455	11	1.7408	4	14.2244	6	5.9927	6
EBONE	0.3113	4	0.6477	4	0.1429	6	1.7335	3	12.6298	5	6.2532	7
Tiscali	0.2641	5	0.5255	5	0.2353	3	1.7470	5	23.3563	10	9.5895	11
Sprint	0.2407	6	0.3817	6	0.1136	7	1.7853	6	15.9659	7	8.7908	9
Verio	0.2009	7	0.2448	8	0.1967	4	1.8463	7	31.4149	12	12.4096	12
VSNL	0.1783	8	0.3402	7	0.1429	5	1.9053	9	4.6597	1	2.3799	1
GÉANT2 Phys.	0.1668	9	0.1515	9	0.0882	8	1.8518	8	10.3580	4	4.0859	4
AT&T	0.1446	10	0.1324	10	0.6296	2	1.9127	10	24.2950	11	6.9552	8
Telstra	0.0941	11	0.0454	11	0.7931	1	1.9797	11	19.1441	9	4.6868	5
AT&T Phys.	0.0348	12	0.0061	12	0.0388	12	1.9892	13	8.3783	3	3.7058	3
Sprint Phys.	0.0307	13	0.0053	13	0.0380	13	1.9839	12	7.1893	2	2.9316	2
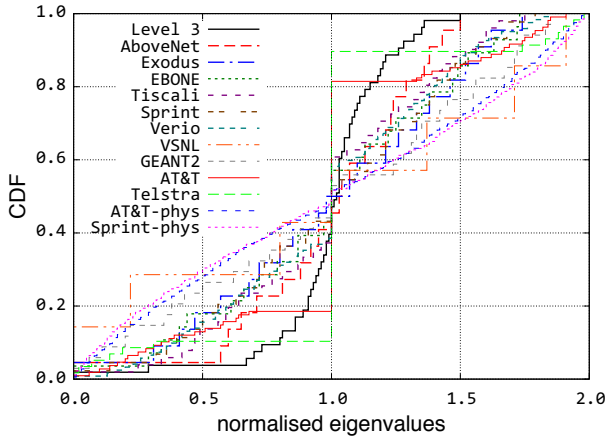
Fig. 9. CDF of eigenvalues of communication networks

comparing the connectivities of graphs with the *same vertex set* [33]. Since we are examining graphs of different orders, $a(G)$ cannot be used to effectively compare these topologies; however, the overall rankings of the $a(G)$ closely match the cTGD rankings.

Next, we consider the $\lambda = 1$ multiplicities and rank them as shown in columns 5 and 6 in Table III. Multiplicities of $\lambda = 1$ indicate node duplications [37]. For example, a star graph has $n - 2$ multiplicities at eigenvalue 1, in which $n - 1$ pendants are connected to central node and each pendant has the same neighbour. In regards to the topologies, Telstra and AT&T have the highest $\lambda = 1$ multiplicities at 79% and 63% respectively. Indeed, when we visually check these two topologies in KU-TopView [23], they have individual star components connected to each other. AT&T and Sprint physical topologies are the lowest on ranking, since they follow a grid-like infrastructure.

Finally, we consider the spectral radius of these 13 topologies. The spectral radius ρ is the absolute value of the maximum eigenvalue, $\rho = |\lambda_{\max}|$. We calculate the spectral radius of the normalised Laplacian matrix $\rho(\mathcal{L})$, Laplacian matrix $\rho(L)$, and adjacency matrix $\rho(A)$. We rank the spectral radius in increasing order next to the eigenvalues as shown in the last six columns of Table III. The ranking of the spectral radius of the normalised Laplacian closely matches the cTGD ranking, but some adjacent ranks are swapped compared to the cTGD and $a(G)$ rankings. In a normalised Laplacian spectrum, an eigenvalue of 2 indicates the graph is *bipartite* and closer the eigenvalues to 2 indicates the graph is closer being a bipartite graph [37]. Among the 13 topologies we study, AboveNet with 22 nodes and 80 links has the lowest spectral radius; Level 3 has second lowest spectral radius with 53 nodes and 456 links. The physical topologies of Sprint and AT&T rank last among 13 topologies with normalised Laplacian spectral radii very close to 2. Ranking of the spectral radius of adjacency and the Laplacian matrices are somewhat close to each other. For example, while Level 3 has the highest spectral radius for its adjacency and the Laplacian matrices, VSNL has the lowest spectral radius for adjacency and the Laplacian representations. On the other hand, the spectral radius of the adjacency and the Laplacian matrices are not close to cTGD or $a(G)$ and the rankings do not seem to follow any pattern.

We also calculate the spectral gap, which is the difference between the largest and the second largest eigenvalues for adjacency, Laplacian, and normalised Laplacian matrices. We do not observe any obvious pattern among the calculated spectral gap values.

VI. CONCLUSIONS AND FUTURE WORK

Understanding the evolution of networks is crucial for rigorous analysis and modelling of the Internet. We presented the US interstate highway topology and compared its graph metric characteristics with those of physical and logical communication network topologies. Our results indicate the motorway graph is highly correlated with the physical fibre network. Despite of many statistical studies on modelling the Internet, we can show analytically that Internet evolution relies heavily on other critical infrastructures. In addition to statistical methods, dependency between critical infrastructures should be considered when modelling the Internet. By comparing the normalised Laplacian spectra and visual representations of these topologies, we have shown that fibre topologies we obtained are representative of the two commercial ISPs.

Secondly, we studied 13 communication networks along with the characteristics of their normalised Laplacian spectra. Our initial observation was that the spectral radius of the normalised Laplacian matrix is a good connectivity indicator of graphs with different size and order. We will later study how graph bipartiteness relates to the connectivity of a graph. Our future work will include investigating spectral properties of synthetically generated graphs. Future work will consist of generating railways topologies and analysing multilevel properties of these critical infrastructures.

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