

Example of multiplying two 2-digit numbers ($n = 2$)

$$\begin{array}{r} 99 \\ 99 \\ \hline \end{array}$$

$$n = 2$$

Step 1. Generating Partial Products

c_1	c_2	0
	9	9
	9	9

Step 1. Generating Partial Products

$$\begin{array}{r} c_1 \quad 8 \quad 0 \\ \quad 9 \quad 9 \\ \quad 9 \quad 9 \\ \hline \quad \quad 1 \end{array}$$

Multiplications: 1, Additions: 0

Step 1. Generating Partial Products

A multiplication problem is shown. The multiplicand 880 is written in orange above the multiplier 11. A horizontal blue line separates the multiplicand from the multiplier. The partial products 99 and 99 are written in black below the line. A light blue oval highlights the two 9s in the first partial product (99).

$$\begin{array}{r} 880 \\ 11 \\ \hline 99 \\ 99 \end{array}$$

Multiplications: 2, Additions: 0

8	8	0
	9	9
	9	9
<hr/>		
	1	1

Step 1. Generating Partial Products

8	8	0
0	1	1
<hr/>		

Multiplications: 2, Additions: 0

Step 1. Generating Partial Products

8	8	0
	9	9
	9	9
<hr/>		
	1	1

8	8	0
0	1	1
<hr/>		
8	9	1

Multiplications: 2, Additions: 3

Step 1. Generating Partial Products

$$\begin{array}{r} 99 \\ 99 \\ \hline 891 \end{array}$$

Multiplications: 2, Additions: 3

Step 1. Generating Partial Products

c_1	c_0	0	
		9	9
		9	9
<hr/>			
	8	9	1

Multiplications: 2, Additions: 3

Step 1. Generating Partial Products

$$\begin{array}{r} c_1 \quad 8 \quad 0 \\ \quad 9 \quad 9 \\ \quad 9 \quad 9 \\ \hline 8 \quad 9 \quad 1 \\ \quad 1 \end{array}$$

Multiplications: 2, Additions: 3

Multiplications: 1, Additions: 0

Step 1. Generating Partial Products

8	8	0	
		9	9
		9	9
<hr/>			
8	9	1	
1	1		

Multiplications: 2, Additions: 3

Multiplications: 2, Additions: 0

Step 1. Generating Partial Products

$$\begin{array}{r} 880 \\ \quad 99 \\ \hline 891 \\ 11 \end{array}$$

$$\begin{array}{r} 880 \\ 011 \\ \hline \end{array}$$

Multiplications: 2, Additions: 3

Multiplications: 2, Additions: 0

Step 1. Generating Partial Products

$$\begin{array}{r}
 880 \\
 99 \\
 \hline
 891 \\
 11
 \end{array}$$

$$\begin{array}{r}
 880 \\
 011 \\
 \hline
 891
 \end{array}$$

Multiplications: 2, Additions: 3

Multiplications: 2, Additions: 3

Step 1. Generating Partial Products

		9	9
		9	9
	<hr/>		
	8	9	1
8	9	1	

Multiplications: 2, Additions: 3

Multiplications: 2, Additions: 3

$$n = 2$$

		9	9	
		9	9	
<hr/>				
	8	9	1	← Partial product 1 took: (Multiplications: 2, Additions: 3)
8	9	1	×	← Partial product 2 took: (Multiplications: 2, Additions: 3)
<hr/>				

$$n = 2$$

Step 2. Adding Partial Products

```
 $p \leftarrow 0$   
for  $i = 0$  to  $n - 1$  do  
     $p \leftarrow p + a \cdot b_i \cdot B^i$   
end for
```

$$n = 2$$

Step 2. Adding Partial Products

```

p ← 0
for i = 0 to n − 1 do
    p ← p + a · bi · Bi
end for

```

		9	9
		9	9
<hr/>			
	8	9	1
8	9	1	×
<hr/>			

$$p = 0$$

$$n = 2$$

Step 2. Adding Partial Products

```

p ← 0
for i = 0 to n − 1 do
    p ← p + a · bi · Bi
end for

```

		9	9
		9	9
<hr/>			
	8	9	1
8	9	1	×
<hr/>			

$$p = 0$$

i = 0

$p = p + 891 = 891$

1 summation of 2+1 digits

$$n = 2$$

Step 2. Adding Partial Products

```

 $p \leftarrow 0$ 
for  $i = 0$  to  $n - 1$  do
     $p \leftarrow p + a \cdot b_i \cdot B^i$ 
end for

```

$$\begin{array}{r}
 99 \\
 99 \\
 \hline
 891 \\
 891\times \\
 \hline
 \end{array}$$

This digit does not incur any operation

$$p = 0$$

$$i = 0 \quad p = p + 891 = 891$$

2 summations of 2+1 digits

$$i = 1 \quad p = 0891 + 8910 = 9801$$

For multiplying two n-digit numbers

Computing n partial products take n multiplications and n+1 additions

$$n(n + (n+1)) = n(2n+1) = 2n^2 + n$$

Addition of the aligned partial products take n summations of n+1 digits

$$n(n+1) = n^2 + n$$

Total operations

$$= 3n^2 + 2n$$