Algorithm & Data Structure Analysis

Lecture 1: Growth of Function

Appetizer

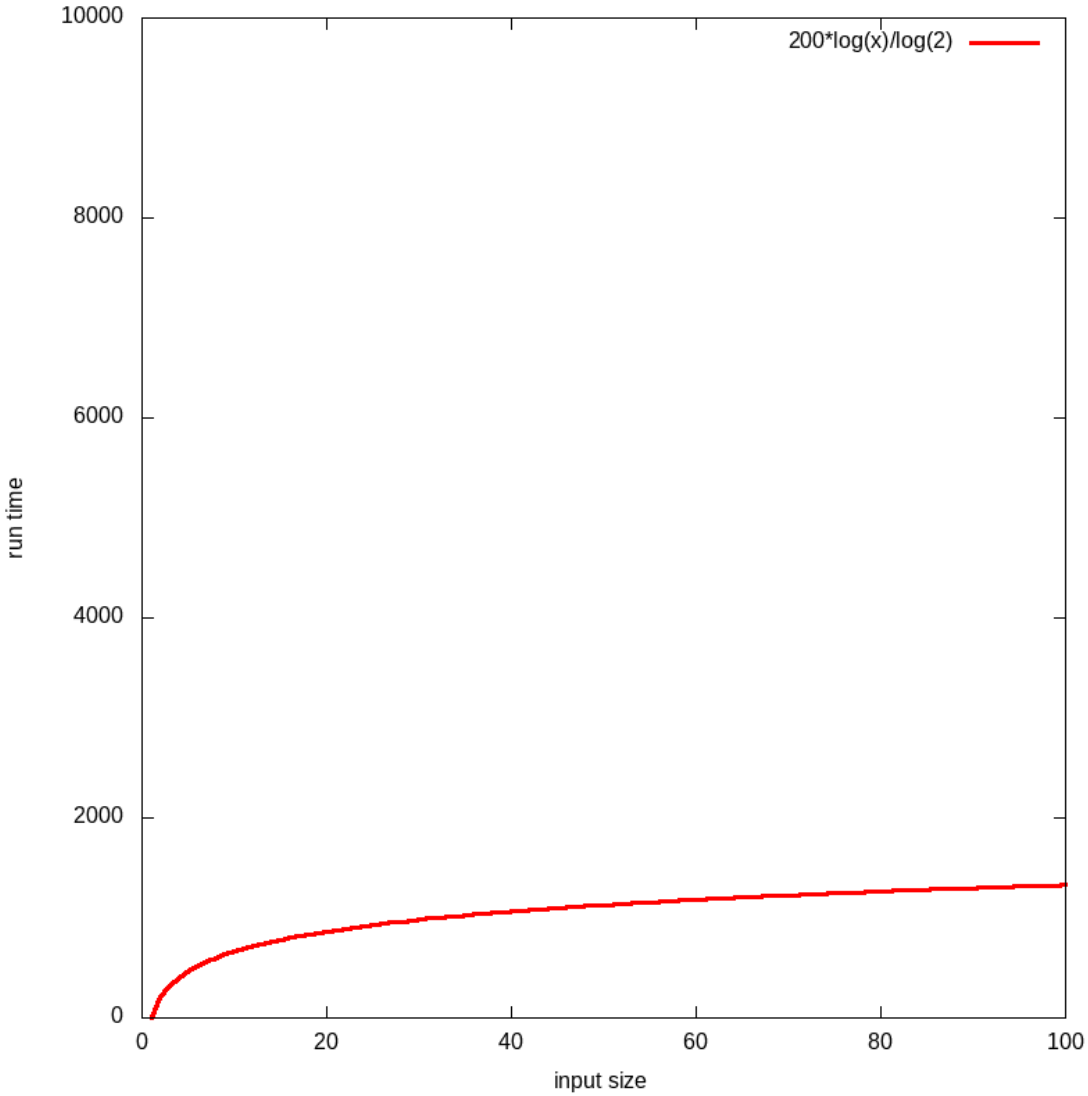
Sorting: Why should I care?

Which one do you prefer?

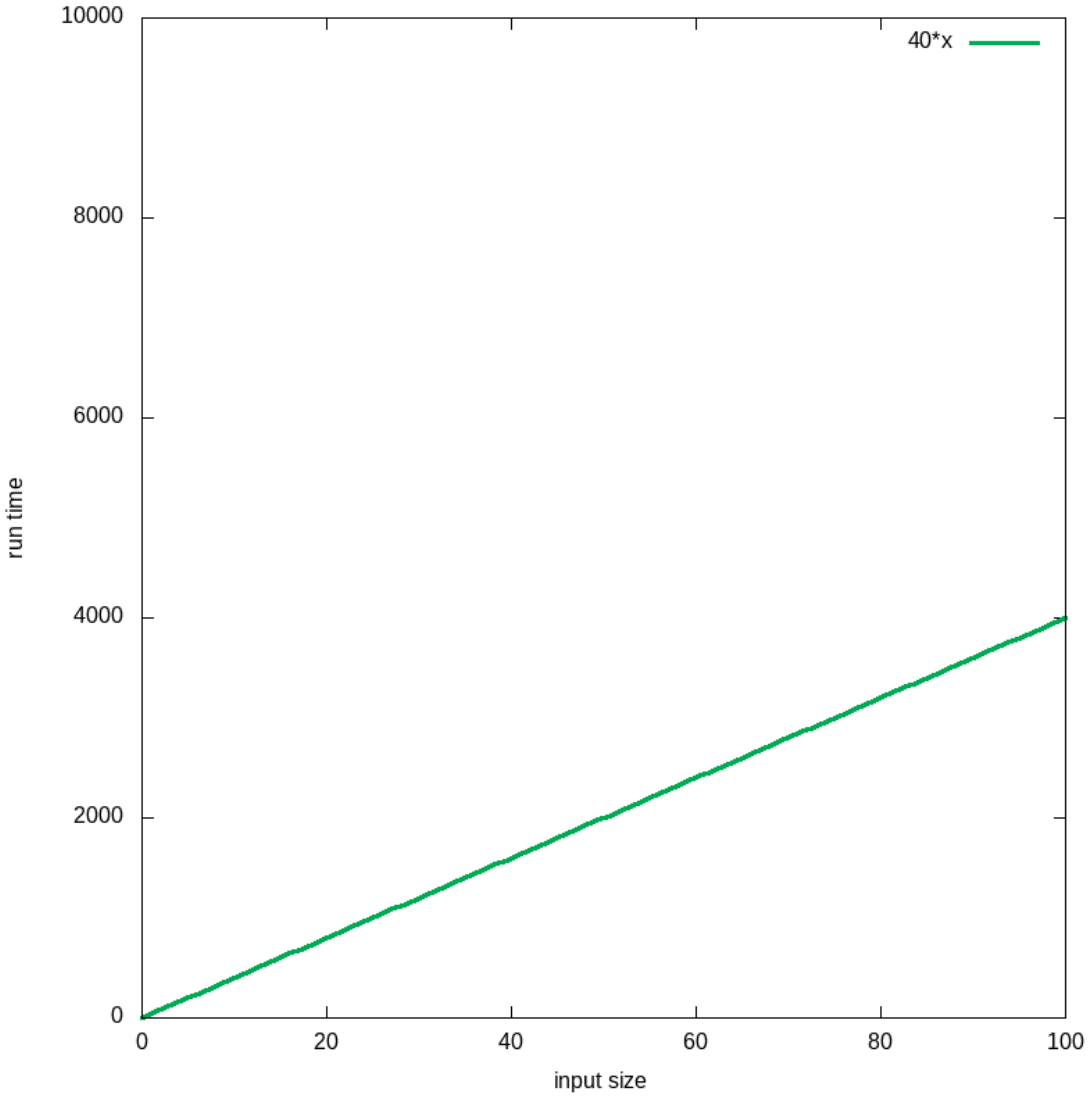
Measuring efficiency

- Let n be the input size
- Runtime
- Memory usage
- (usually interested in large n)

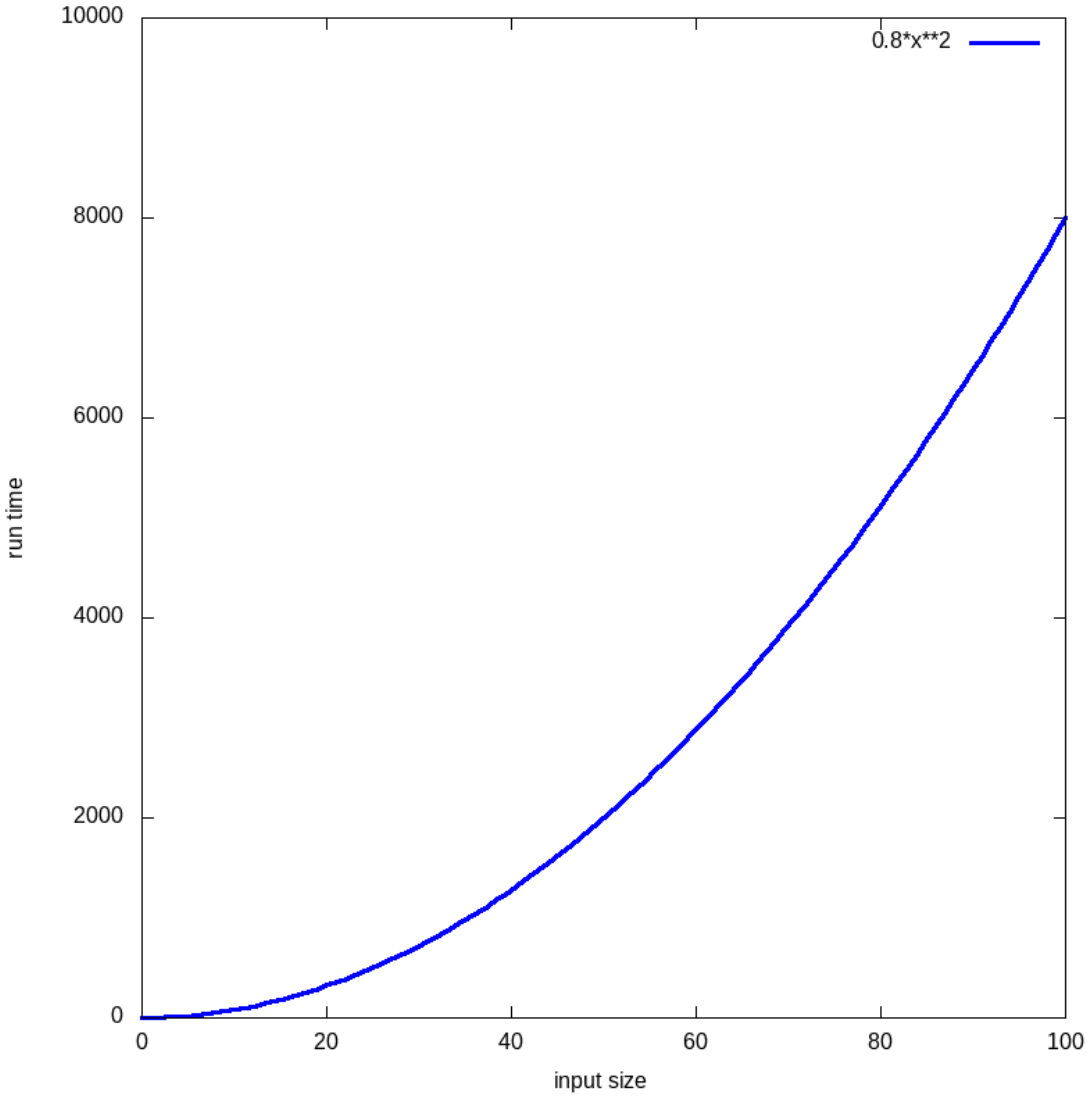
Logarithmic runtime



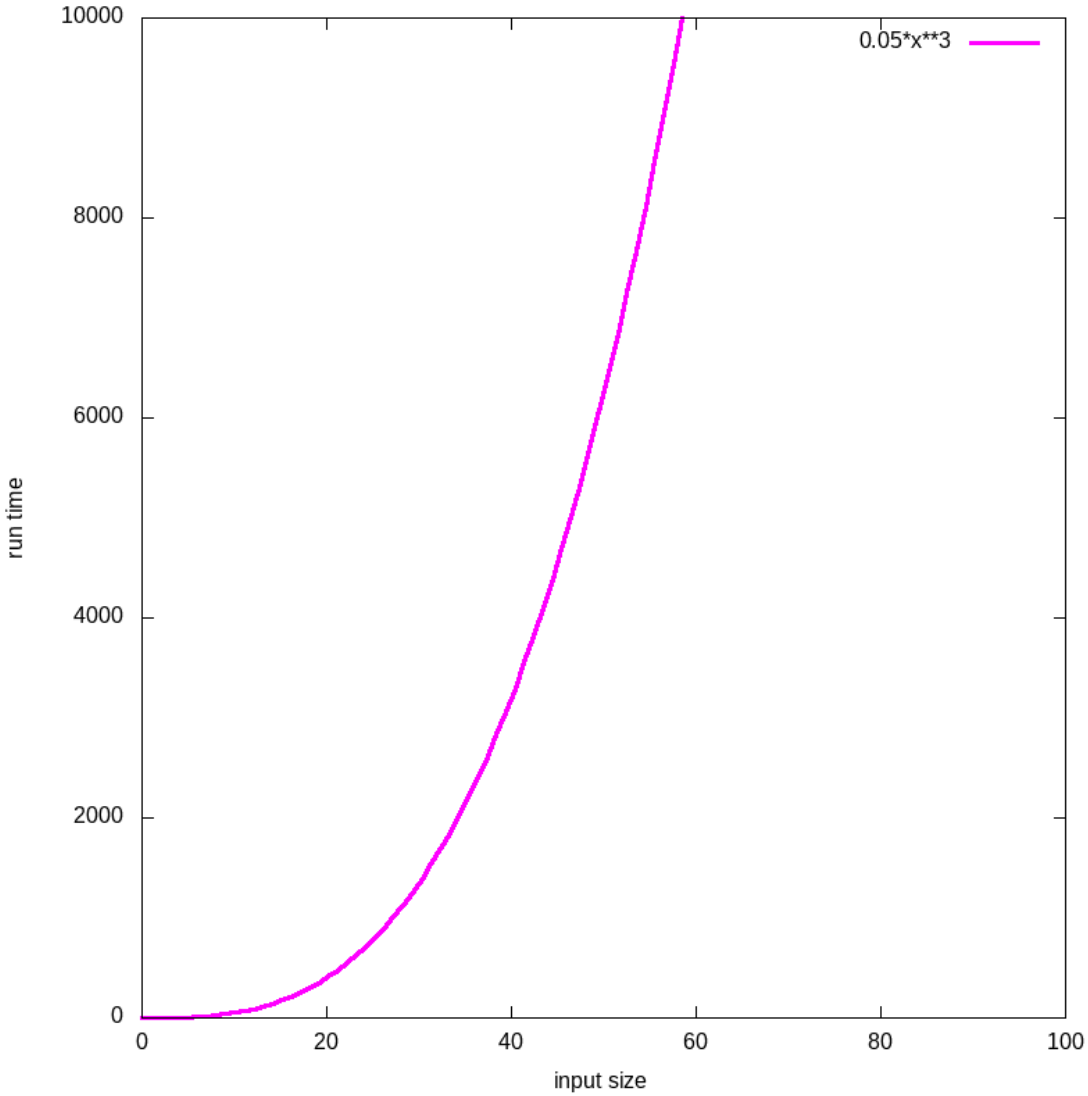
Linear runtime



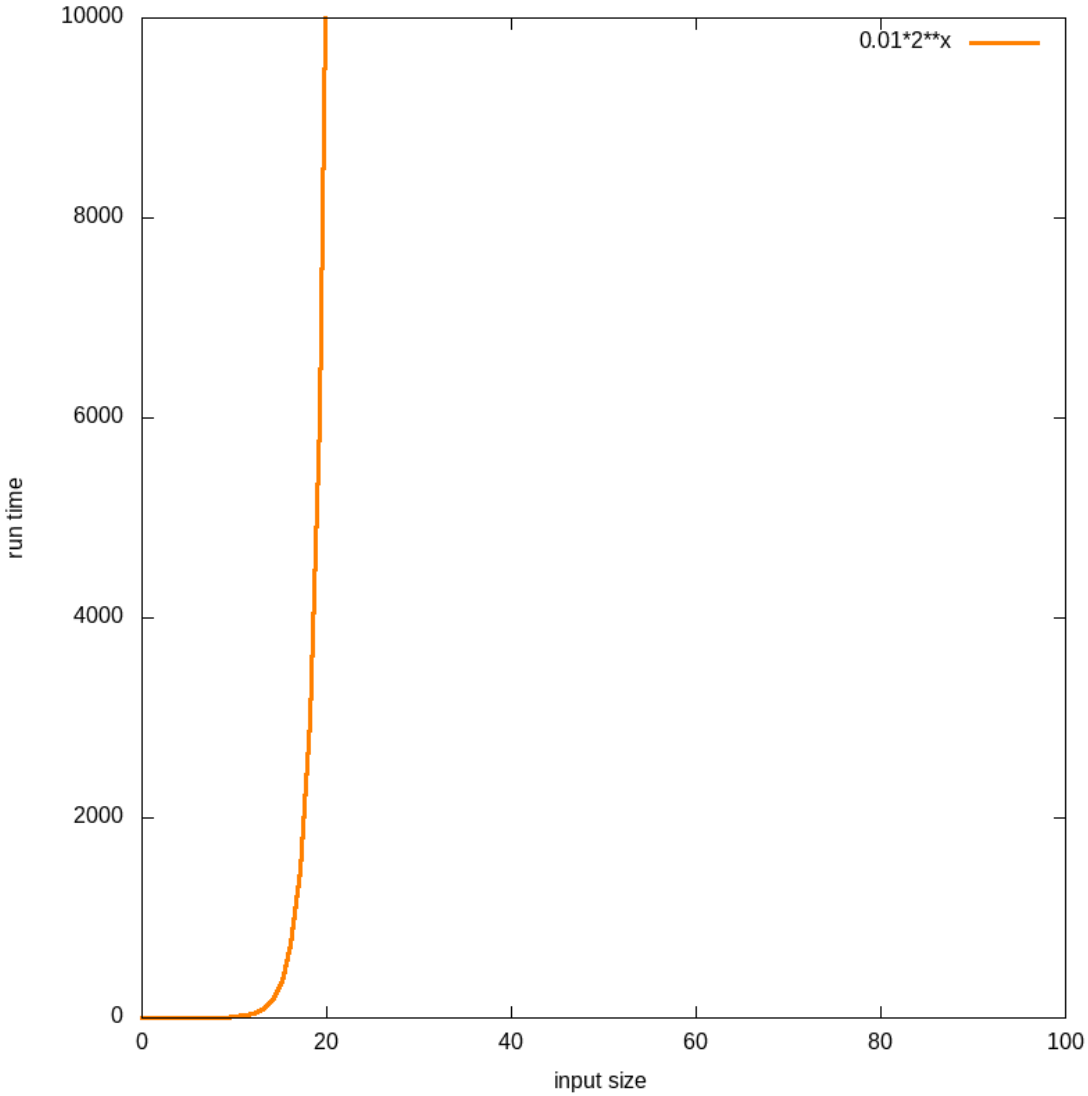
Quadratic runtime



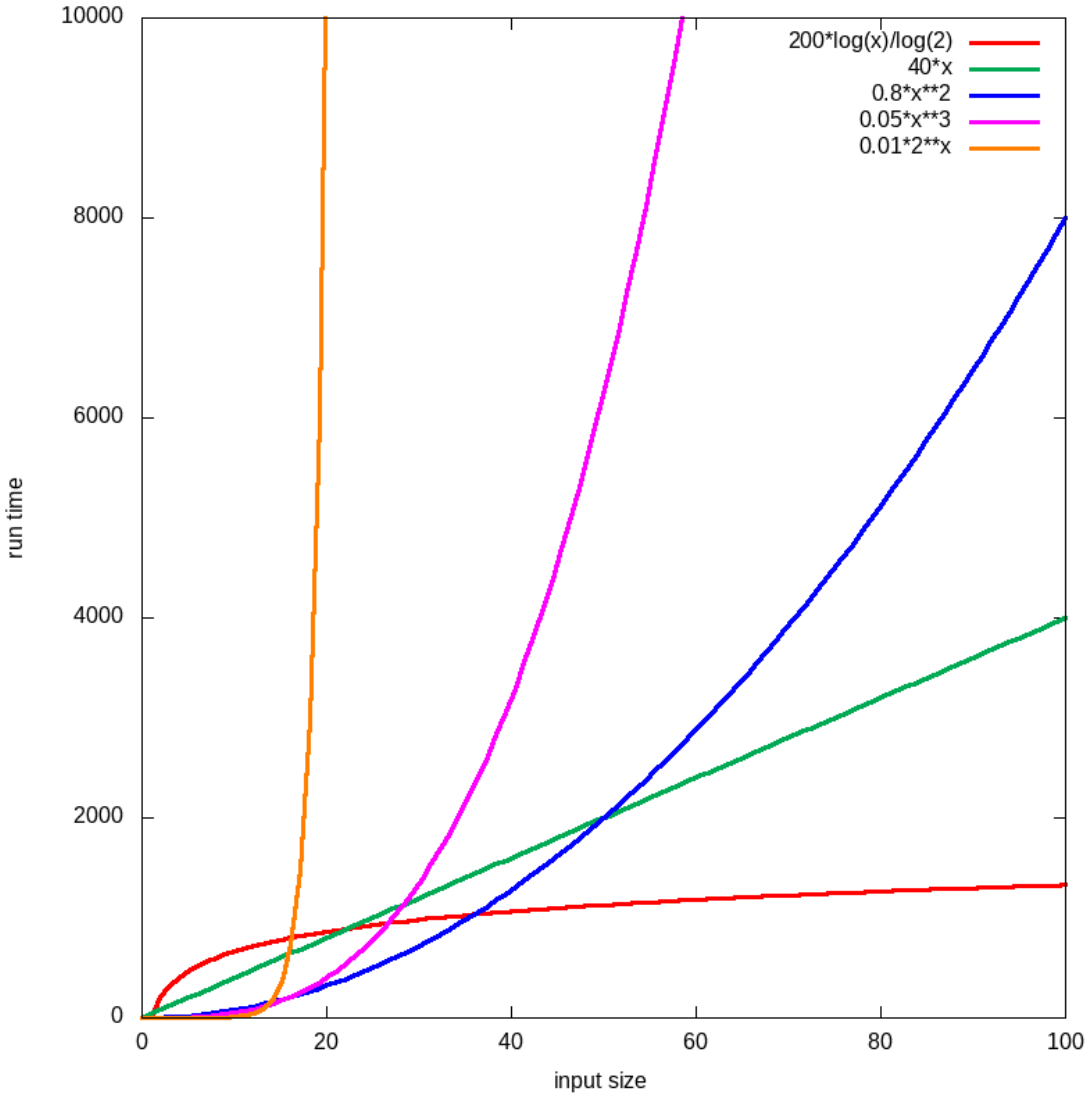
Cubic runtime



Exponential runtime



Growth of function



Asymptotic notations

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Examples

$5n$

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$5n \quad : O(n), \Omega(n), \Theta(n), o(n \log n), \omega(\sqrt{n})$

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$n^2 - n \log n$

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$n^2 - n \log n$: $O(n^2), \Omega(n^2), \Theta(n^2), o(n^3), \omega(n \log n)$

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$100n$

Examples

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$n^2 - n \log n$: $O(n^2), \Omega(n^2), \Theta(n^2), o(n^3), \omega(n \log n)$

$100n$: $O(n^2), \Omega(\sqrt{n}), \Theta(n), o(n \log n), \omega(\sqrt{n})$

True of false

$$5n \log n \in O(n \log n)$$

True of false

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True of false

$5n \log n \in O(n \log n)$ True

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True of false

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$5n \log n + n^2 \in O(n \log n)$

True of false

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$5n \log n \in \Omega(n^2)$	False
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$5n \log n + n^2 \in O(n \log n)$	False

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$5n \log n + n^2 \in O(n \log n)$	False
$5n \log n + n^2 \in O(n^2)$	True

Summary

- Efficiency of data structures and algorithms
- Growth function given input size
- Asymptotic behavior and complexity classes
- Reading: Introduction to Algorithms
  - Chapter 3.1: Growth of Functions

Next lecture

- Integer arithmetics
- Reading: Algorithms and Data Structures
  - Chapter 1.1: Addition
  - Chapter 1.2: Multiplication: The School Method