

CMPUT 307 or 414: Background Notes

2D and 3D Transforms, Homogeneous Transforms & Perspective Projection

Basic Transforms

- Translation
- Scaling
- Rotation
- Homogeneous Transform
- Perspective Projection

2D Translation

Let $P(x, y)$ \rightarrow original point, with coordinates x and y

Let $P'(x', y')$ \rightarrow point after applying transformation, when its coordinates change to x' and y'

In case of translation of the original point by t_x and t_y in the x and y direction respectively, we have the translation vector $T(t_x, t_y)$ applied to (x, y) to move it to (x', y') , such that:

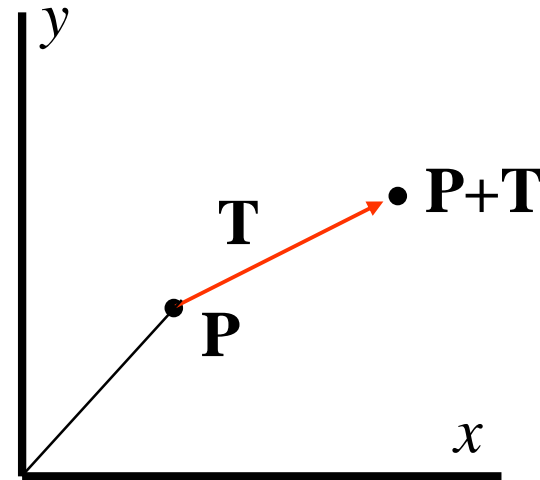
$$x' = x + t_x, \quad y' = y + t_y$$

2D Translation

Alternatively,

$\mathbf{P}' = \mathbf{P} + \mathbf{T}$, with

$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \mathbf{T} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

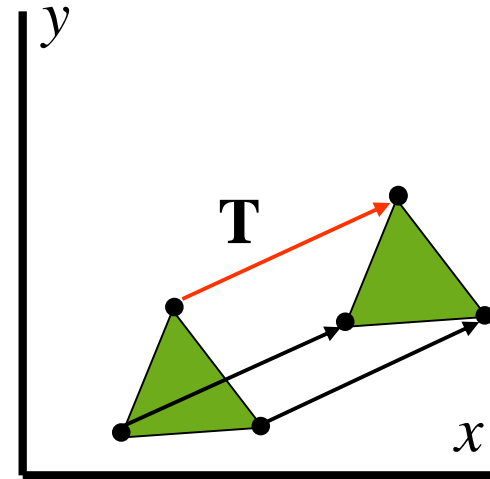


2D Translation

Translate polygon:

Apply the same operation on all points.

Works always, for all transformations of objects defined as a set of points.



2D Scaling

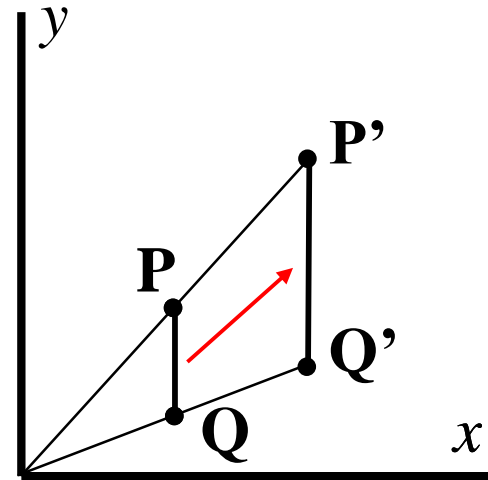
Scale with factor s_x and s_y :

$$x' = s_x x, \quad y' = s_y y$$

or

$\mathbf{P}' = \mathbf{S}\mathbf{P}$, with

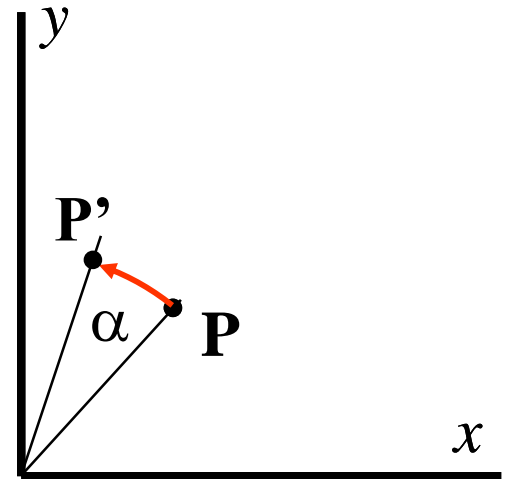
$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{S} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$



2D Rotation

Rotating by an angle α

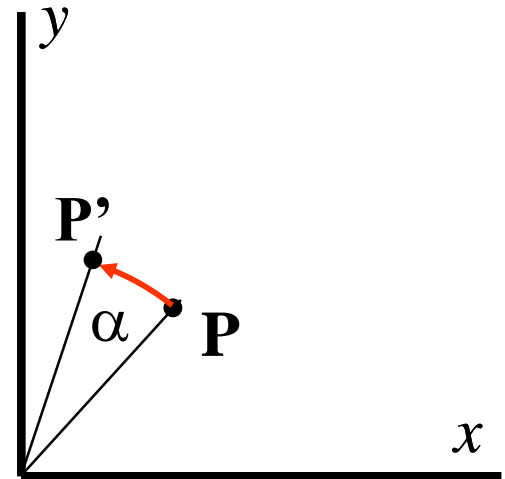
If coordinates of P are (x, y) , and coordinates of P' are (x', y') can you express the latter in terms of the former and the angle α ?



2D Rotation: by an angle α

$$x' = x \cos \alpha - y \sin \alpha$$

$$y' = x \sin \alpha + y \cos \alpha$$

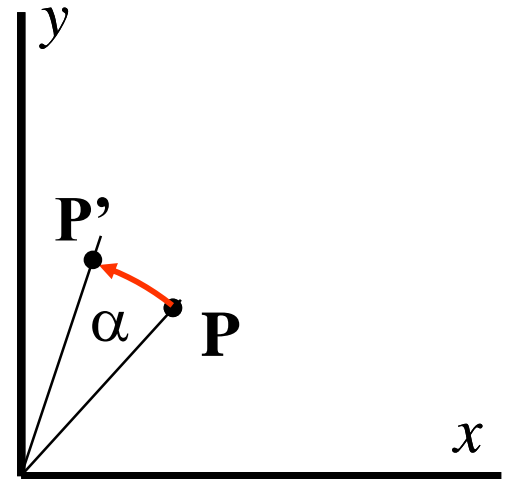


2D Rotation by an angle α

Alternatively,

$\mathbf{P}' = \mathbf{R}\mathbf{P}$, with

$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$



Series of Transforms

Translate with \mathbf{V} :

$$\mathbf{T} = \mathbf{P} + \mathbf{V}$$

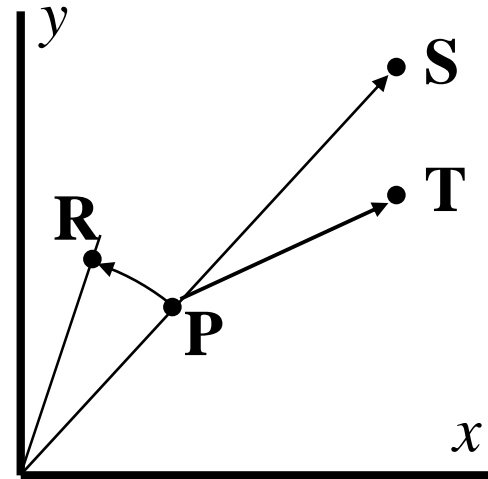
Scale with factor $s_x = s_y = s$:

$$\mathbf{S} = s\mathbf{P}$$

Rotate by an angle α :

$$R'_x = \cos \alpha P_x - \sin \alpha P_y$$

$$R'_y = \sin \alpha P_x + \cos \alpha P_y$$



Problems

-Inconvenient

-How to combine transforms ?

-Solution: Homogeneous !

Homogeneous coordinates

- Uniform representation of translation, rotation, scaling
- Uniform representation of points and vectors
- Compact representation of sequence of transformations

Homogeneous coordinates

-Add extra coordinate:

$$\mathbf{P} = (p_x, p_y, p_h) \quad \text{or}$$

$$\mathbf{x} = (x, y, h)$$

-Cartesian coordinates: divide by h

$$\mathbf{x} = (x/h, y/h)$$

-We take $h = 1$ for convenience

2D Homogeneous Translation

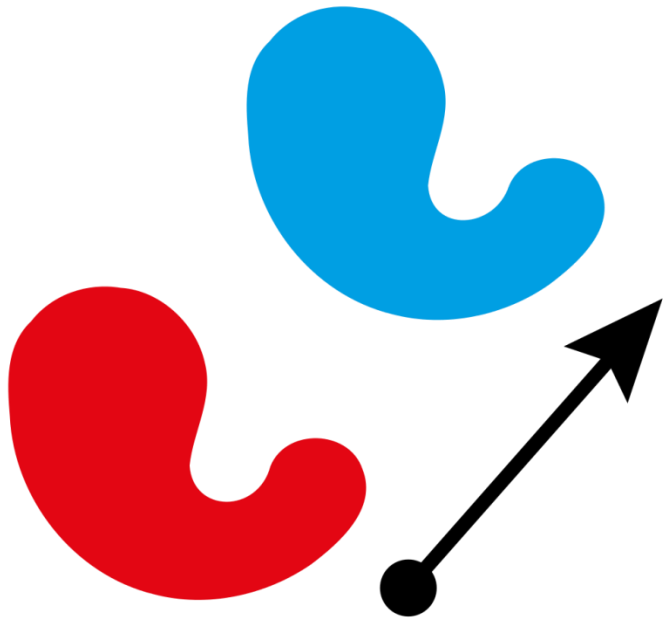
Translation :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{T}(t_x, t_y) \mathbf{P}$$

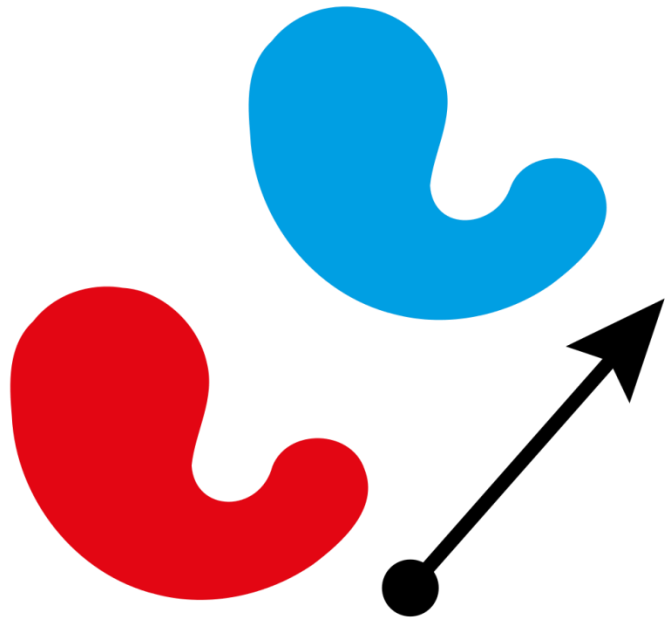
3D Translation



Move X by ΔX , Y by ΔY , and Z by ΔZ

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} X + \Delta X \\ Y + \Delta Y \\ Z + \Delta Z \end{bmatrix}$$

3D Homogeneous Translation (Using a Matrix Multiplication)



Translation Matrix:

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Homogeneous Translation

Question:

What is the matrix for Translation of (2,-3,2)?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2D Homogeneous Scaling

Scaling :

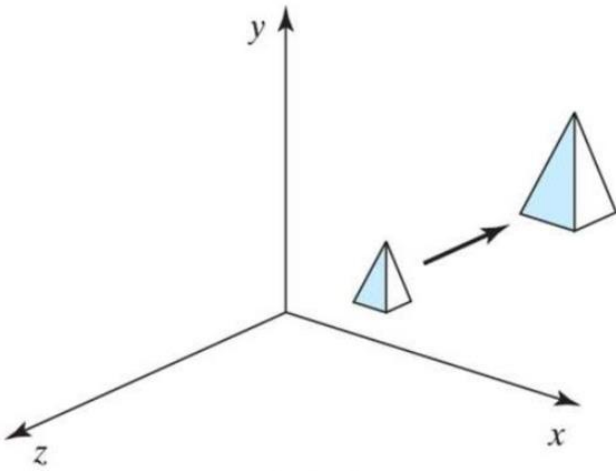
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{S}(s_x, s_y) \mathbf{P}$$

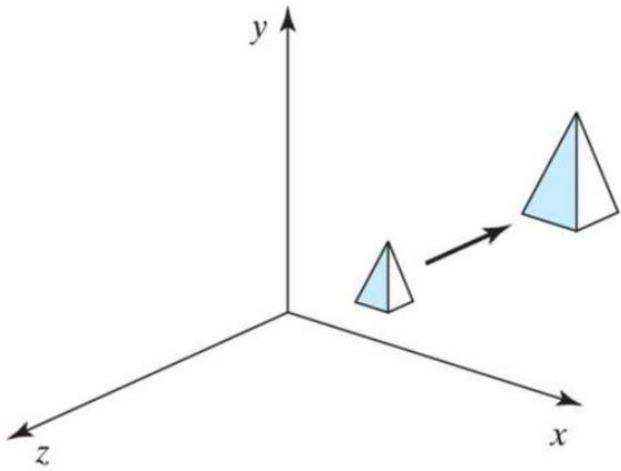
3D Scaling

Scale X by S_X , Y by S_Y , and Z by S_Z



$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} S_X & 0 & 0 \\ 0 & S_Y & 0 \\ 0 & 0 & S_Z \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D Homogeneous Scaling



Scaling Matrix:

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2D Homogeneous Rotation

Rotation :

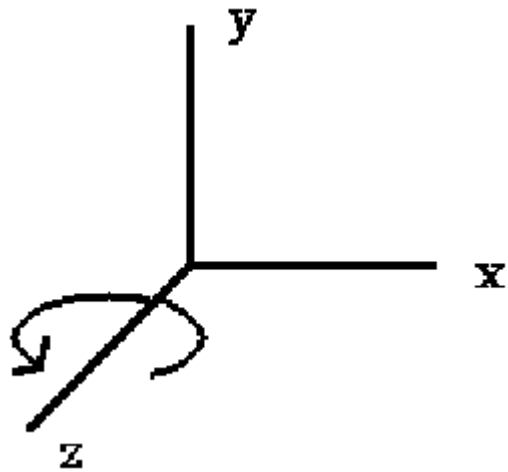
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{R}(\theta)\mathbf{P}$$

3D Homogeneous Rotation

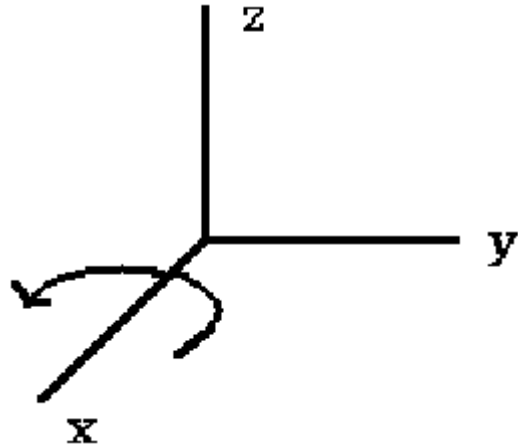
Z-Axis Rotation Matrix



$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Homogeneous Rotation

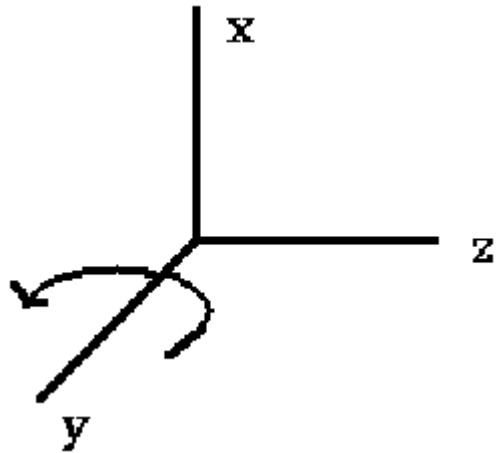
X-Axis Rotation Matrix



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Homogeneous Rotation

Y-Axis Rotation Matrix



$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Combining transformations

First transformation: $P' = M_1 P$

Second transformation: $P'' = M_2 P'$

How can we express the P'' in terms of a transformation on P ?

Combining transformations

Solution:

$$P'' = M_2(M_1P)$$

$$= M_2M_1P$$

$$= MP \quad \text{with } M=M_2M_1$$

Combining transformations: Example

$$\mathbf{P}' = \mathbf{T}(t_{1x}, t_{1y})\mathbf{P} \quad \text{first translation}$$

$$\mathbf{P}'' = \mathbf{T}(t_{2x}, t_{2y})\mathbf{P}' \quad \text{second translation}$$

t_{1x} is magnitude of translation in x direction (dx) for first translation

t_{1y} is magnitude of translation in y direction (dy) for first translation

(similarly t_{2x} and t_{2y} for second translation)

Express \mathbf{P}'' as a translation on \mathbf{P} .

Combining transformations: Example

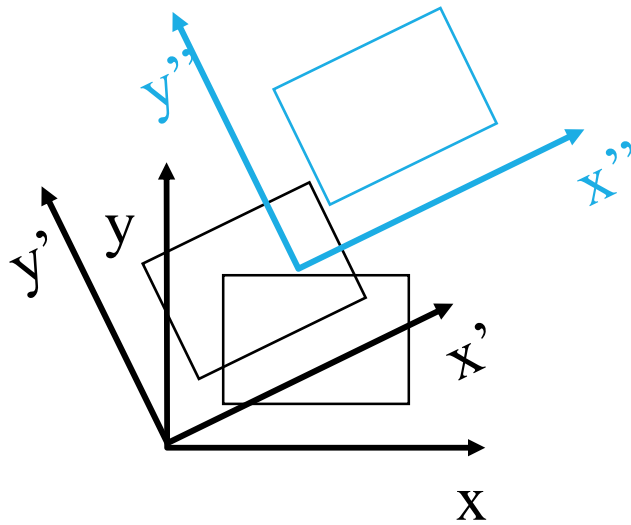
$$\mathbf{P}'' = \mathbf{T}(t_{2x}, t_{2y}) \mathbf{T}(t_{1x}, t_{1y}) \mathbf{P}$$

$$= \begin{pmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{pmatrix} \mathbf{P}$$

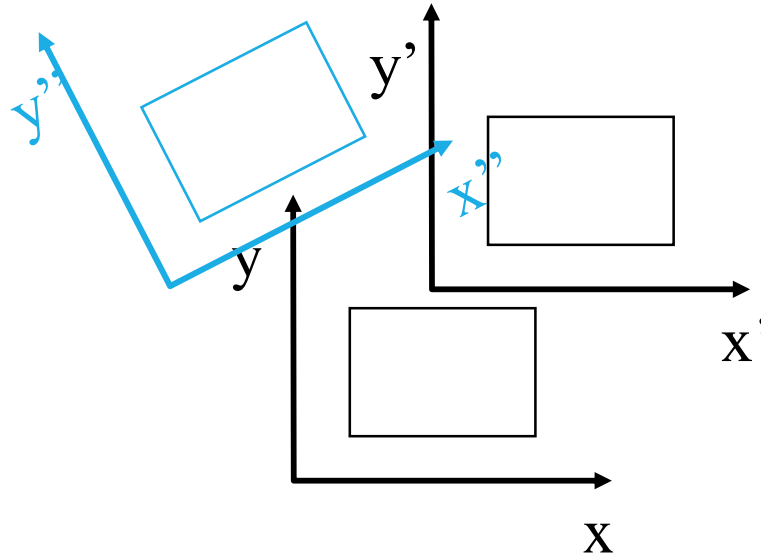
$$= \begin{pmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{pmatrix} \mathbf{P} = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y}) \mathbf{P}$$

Order of transformations

Rotation, translation...



Translation, rotation...



Matrix multiplication does **not** commute.

The order of transformations makes a difference!

Example 1

Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): ($\cos 30 = .86$ & $\sin 30 = 0.5$)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \begin{bmatrix} .86 & -0.5 & 0 & 0 \\ 0.5 & .86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 1

Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): ($\cos 30 = .86$ & $\sin 30 = 0.5$)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?] \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .86 & -0.5 & 0 & 0 \\ 0.5 & .86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 1

Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): ($\cos 30 = .86$ & $\sin 30 = 0.5$)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .86 & -0.5 & 0 & 0 \\ 0.5 & .86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?] \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 3

Scale (X by 0.5, Y by 1.5, and Z by 0.8), followed by rotate by 30 degree around Y-axis, followed by followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?][?] \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 3

Scale (X by 0.5, Y by 1.5, and Z by 0.8), followed by rotate by 30 degree around Y-axis, followed by followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?] \begin{bmatrix} 0.86 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0.86 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Example 3

Scale (X by 0.5, Y by 1.5, and Z by 0.8), followed by rotate by 30 degree around Y-axis, followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.86 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0.86 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Arbitrary Rotations

Rotating from one direction to another, e.g. change vector V_0 to V_1

-Angle between two vectors V_0 & $V_1 = ?$

-Direction normal to the two vectors = ?

Arbitrary Rotations

Angle θ between two vectors v_0 & v_1

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$

$$\cos\theta = \frac{v_0 \cdot v_1}{||v_0|| \cdot ||v_1||} = \frac{x_0 \cdot x_1 + y_0 \cdot y_1 + z_0 \cdot z_1}{||v_0|| \cdot ||v_1||}$$

$$||v_0|| = \sqrt{x_0^2 + y_0^2 + z_0^2} \quad ||v_1|| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

Arbitrary Rotations

Direction normal to two vectors

- Cross product of the two vectors

Once we find this Normal Axis of Rotation we can use the equation in the next slide to compute an arbitrary rotation

Arbitrary Rotations

Cross product of two vectors

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$

$$v_0 \times v_1 = (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1)$$

To Remember this Formula, Note X does not appear in X coordinate of the formula, Y does not appear in Y coordinate of the formula, and Z does not appear in Z coordinate of the formula.

Cross product of $(1,0,0) \times (0, 1, 0)$:

$$(0 \times 0 - 0 \times 1, 0 \times 0 - 1 \times 0, 1 \times 1 - 0 \times 0) = (0, 0, 1)$$

Arbitrary Rotation Matrix

Rotate by an angle θ around an arbitrary normalized direction (x, y, z)

$$\begin{bmatrix} x^2 + \cos \theta(1 - x^2) & xy(1 - \cos \theta) - z \sin \theta & xz(1 - \cos \theta) + y \sin \theta & 0 \\ xy(1 - \cos \theta) + z \sin \theta & y^2 + \cos \theta(1 - y^2) & yz(1 - \cos \theta) - x \sin \theta & 0 \\ xz(1 - \cos \theta) - y \sin \theta & yz(1 - \cos \theta) + x \sin \theta & z^2 + \cos \theta(1 - z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example A

How do we rotate the Vector (1, 1, 2) to the vector (1, 2, 1) ?

(1) Angle between the two vectors is:

$$\begin{aligned} & \text{Cos-1} \{ (1x1 + 1x2 + 2x1) / (\text{SQRT}(1x1 + 1x1 + 2x2) \times \text{SQRT}(1x1 + 2x2 + 1x1)) \} \\ & = \text{Cos-1} \{ 5 / (\text{SQRT}(6) \times \text{SQRT}(6)) \} = \text{Cos-1} \{ 5/6 \} = \text{Cos-1} \{ 0.83 \} = 33.56 \end{aligned}$$

(2) Cross product of the two vectors is:

$$(y_1z_2 - z_1y_2, z_1x_2 - x_1z_2, x_1y_2 - y_1x_2) = (1x2 - 2x1, 2x1 - 1x1, 1x2 - 1x1) = (-3, 1, 1)$$

Gives direction orthogonal to both vectors.

So, we need to Rotate by 33.56 degrees around the Normalized vector $(-3, 1, 1) / \text{SQRT}(11) = (-3, 1, 1) / 3.32 = (-0.9, .3, .3)$

Example B

How do we rotate the Vector (1, 0, -1) to the vector (1, 0, 1) ?

(1) Angle between the two vectors is:

$$\begin{aligned} & \cos^{-1} \left\{ \frac{(1 \times 1 + 0 \times 0 - 1 \times 1)}{(\sqrt{1 \times 1 + 0 \times 0 + (-1) \times (-1)}) \times \sqrt{1 \times 1 + 0 \times 0 + 1 \times 1}} \right\} \\ &= \cos^{-1} \left\{ \frac{0}{(\sqrt{2}) \times \sqrt{2}} \right\} = \cos^{-1} \{0\} = 90 \end{aligned}$$

(2) Cross product of the two vectors is:

$$(y_1 z_2 - z_1 y_2, z_1 x_2 - x_1 z_2, x_1 y_2 - y_1 x_2) = (0 \times 1 - (-1) \times 0, (-1) \times 1 - 1 \times 1, 1 \times 0 - 0 \times 1) = (0, -2, 0)$$

Gives direction orthogonal to both vectors.

So, we need to Rotate by 90 degrees around the Normalized vector $(0, -2, 0)/2 = (0, -1, 0)$,

Example A

How do we rotate the Vector (1, 1, 2) to the vector (1, 2, 1) ?

Solution:

1. Rotate by 33.56 degrees around the Normalized vector (-0.9, 0.3, 0.3)
2. Use the matrix below for performing the rotation

$$\begin{bmatrix} x^2 + \cos \theta (1 - x^2) & xy(1 - \cos \theta) - z \sin \theta & xz(1 - \cos \theta) + y \sin \theta & 0 \\ xy(1 - \cos \theta) + z \sin \theta & y^2 + \cos \theta (1 - y^2) & yz(1 - \cos \theta) - x \sin \theta & 0 \\ xz(1 - \cos \theta) - y \sin \theta & yz(1 - \cos \theta) + x \sin \theta & z^2 + \cos \theta (1 - z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example A

Final Answer ?

$$\begin{bmatrix} ? & ? & ? & 0 \\ ? & ? & ? & 0 \\ ? & ? & ? & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

0.968	-0.211	0.121	0
0.121	0.848	0.512	0
-0.211	-0.482	0.848	0
0	0	0	1

Example B

How do we rotate the Vector (1, 0, -1) to the vector (1, 0, 1) ?

Solution:

1. We need to Rotate by 90 degrees around the vector (0, -1, 0);
Normalized vector is (0, -1, 0); $\sin(\theta) = 1$, $\cos(\theta) = 0$
2. Use the below matrix for performing the rotation

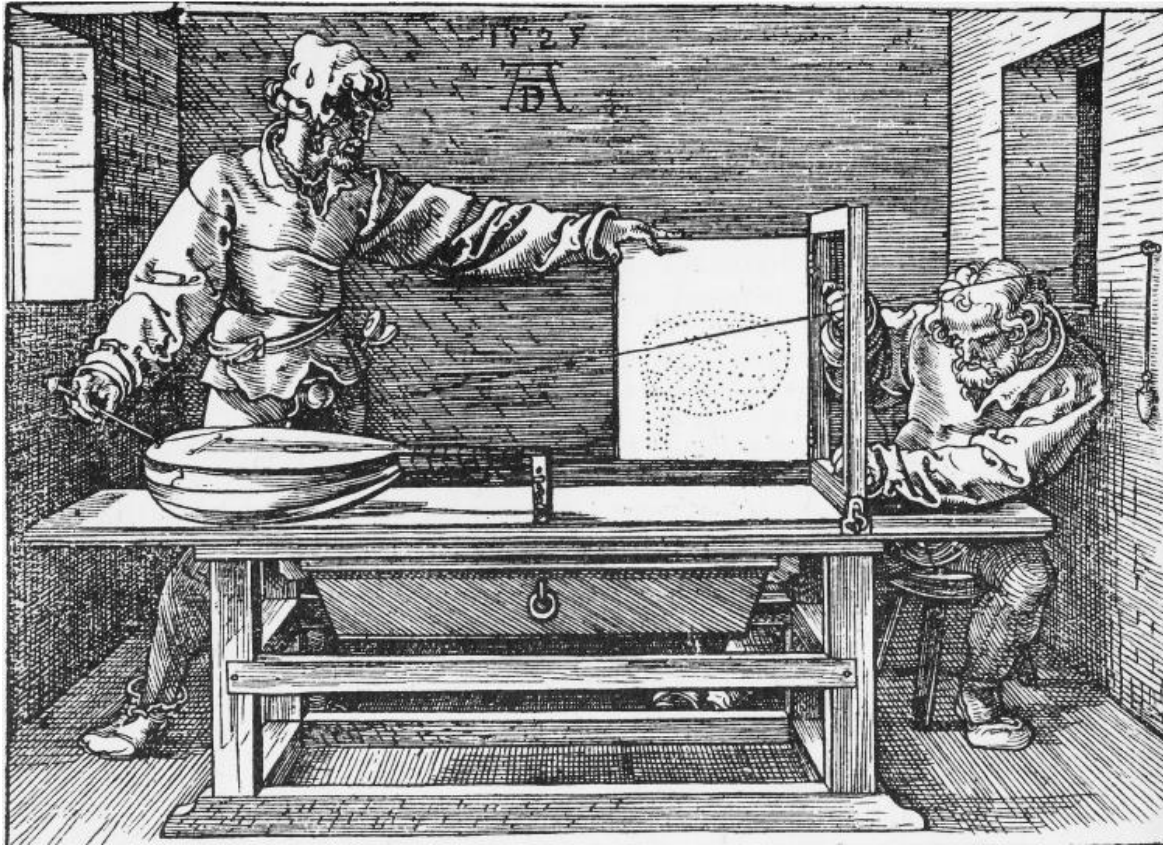
$$\begin{bmatrix} x^2 + \cos \theta (1 - x^2) & xy(1 - \cos \theta) - z \sin \theta & xz(1 - \cos \theta) + y \sin \theta & 0 \\ xy(1 - \cos \theta) + z \sin \theta & y^2 + \cos \theta (1 - y^2) & yz(1 - \cos \theta) - x \sin \theta & 0 \\ xz(1 - \cos \theta) - y \sin \theta & yz(1 - \cos \theta) + x \sin \theta & z^2 + \cos \theta (1 - z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example B

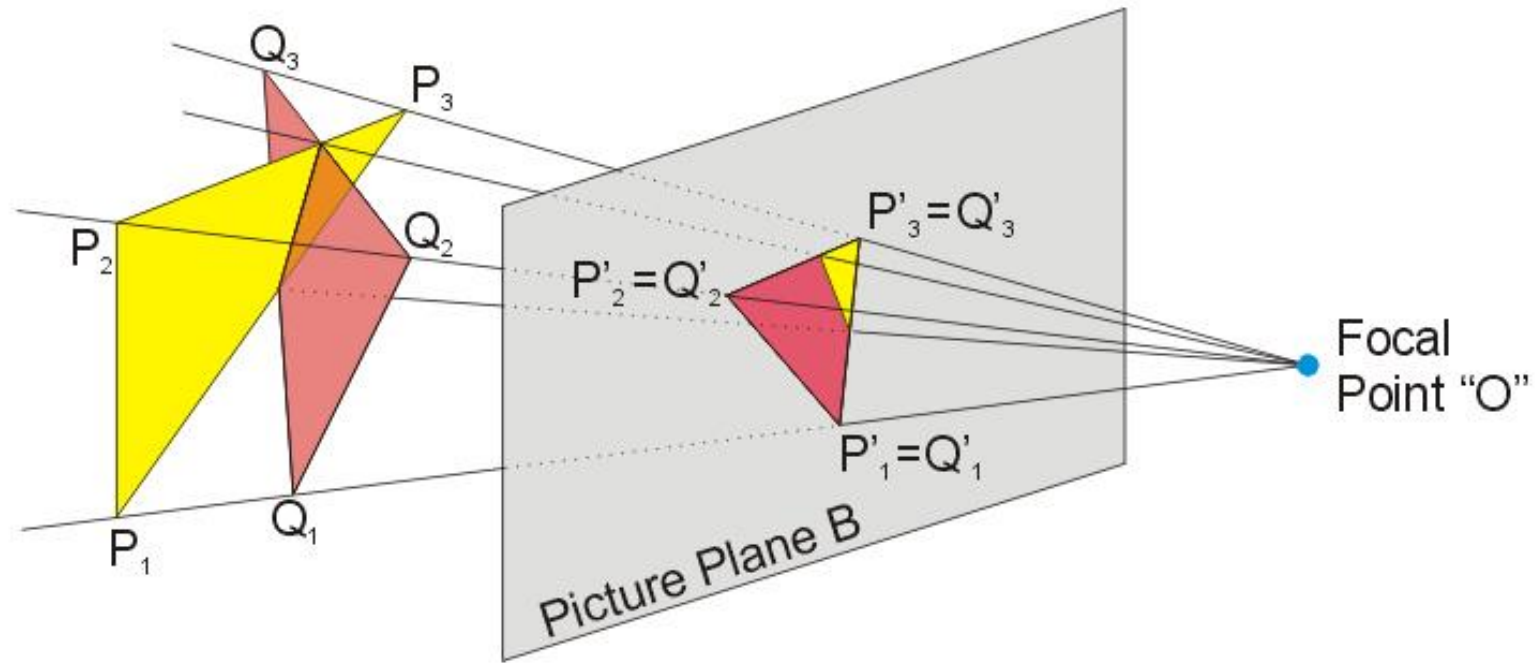
Final Answer:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

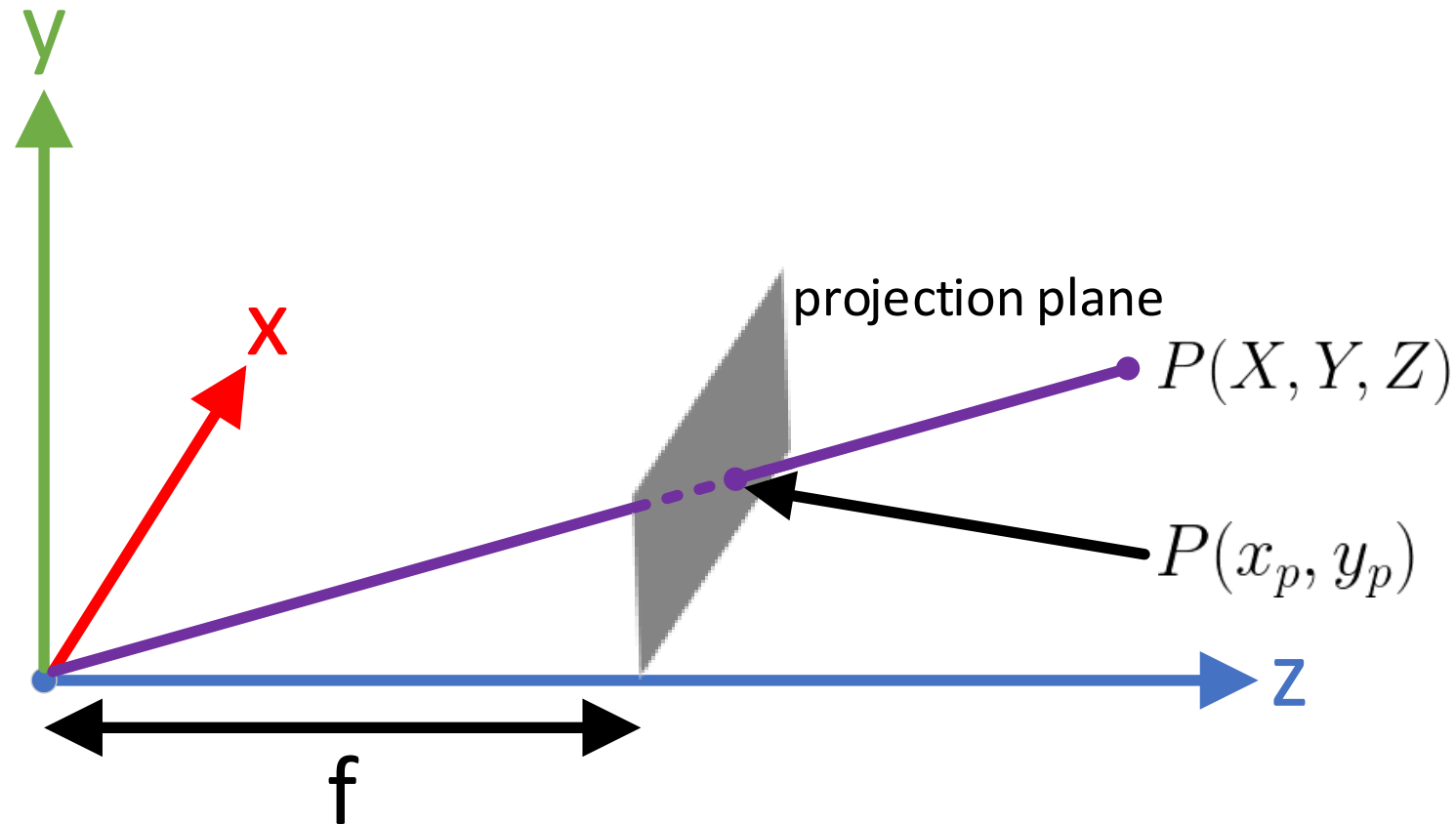
Perspective Projection



Perspective Projection



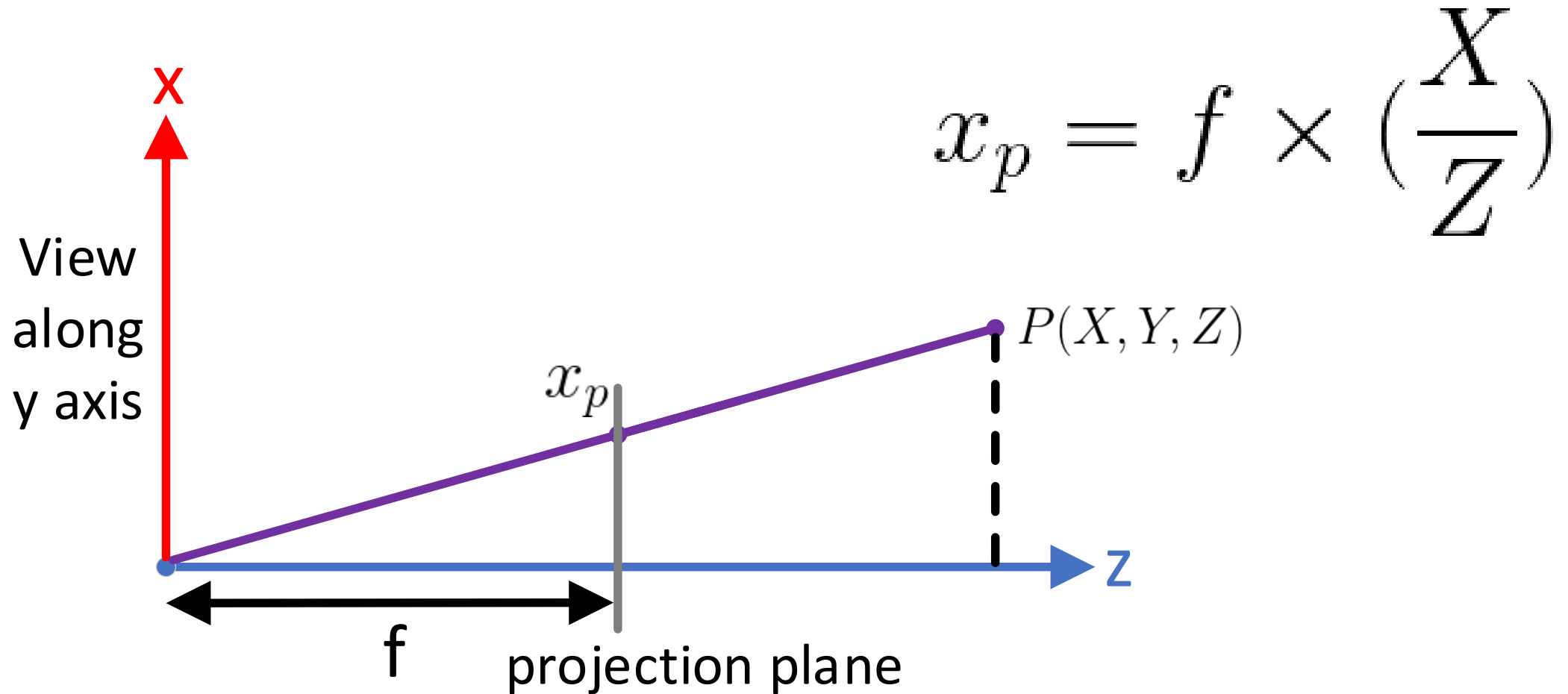
Mathematics of Perspective Projection



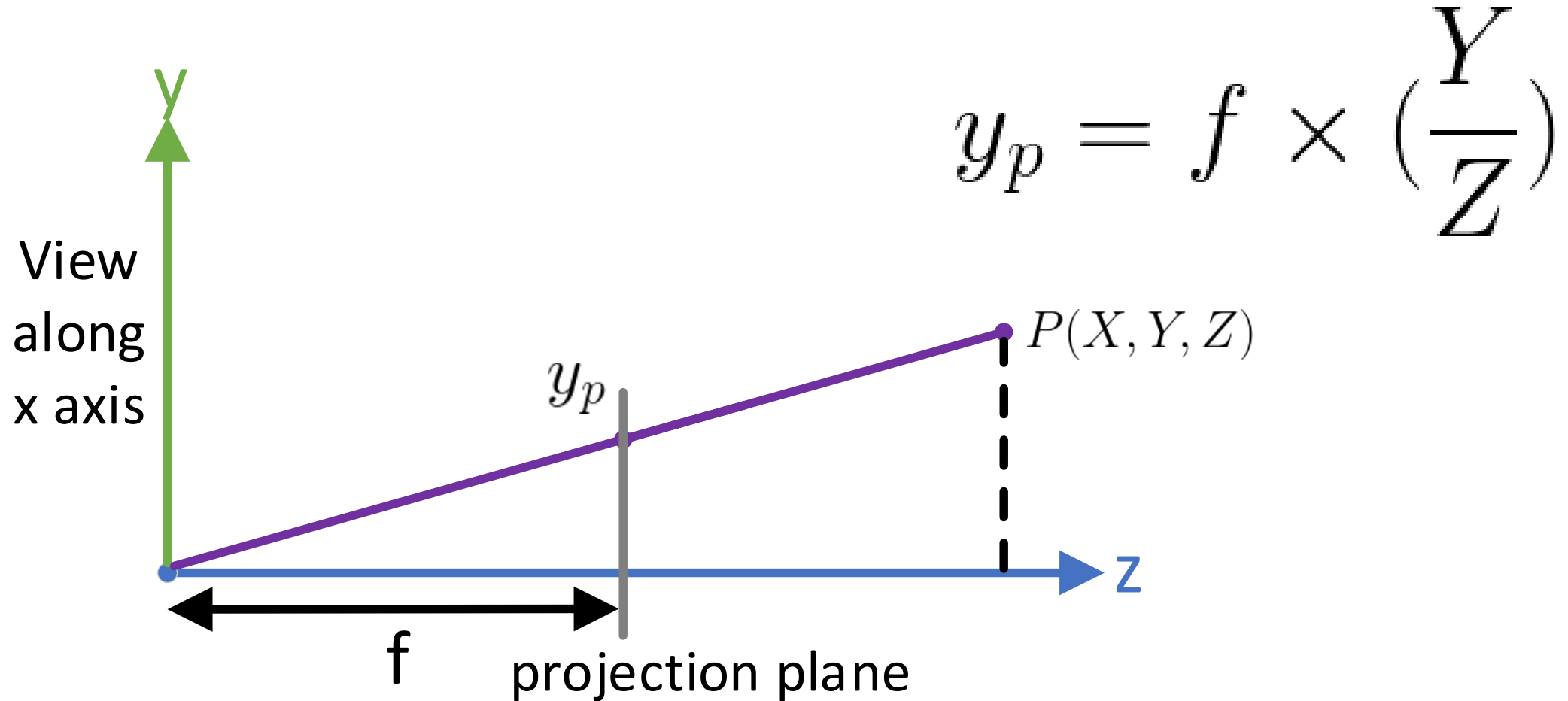
f: Focal Length

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \Rightarrow \begin{pmatrix} f \frac{X}{Z} \\ f \frac{Y}{Z} \\ f \end{pmatrix}$$

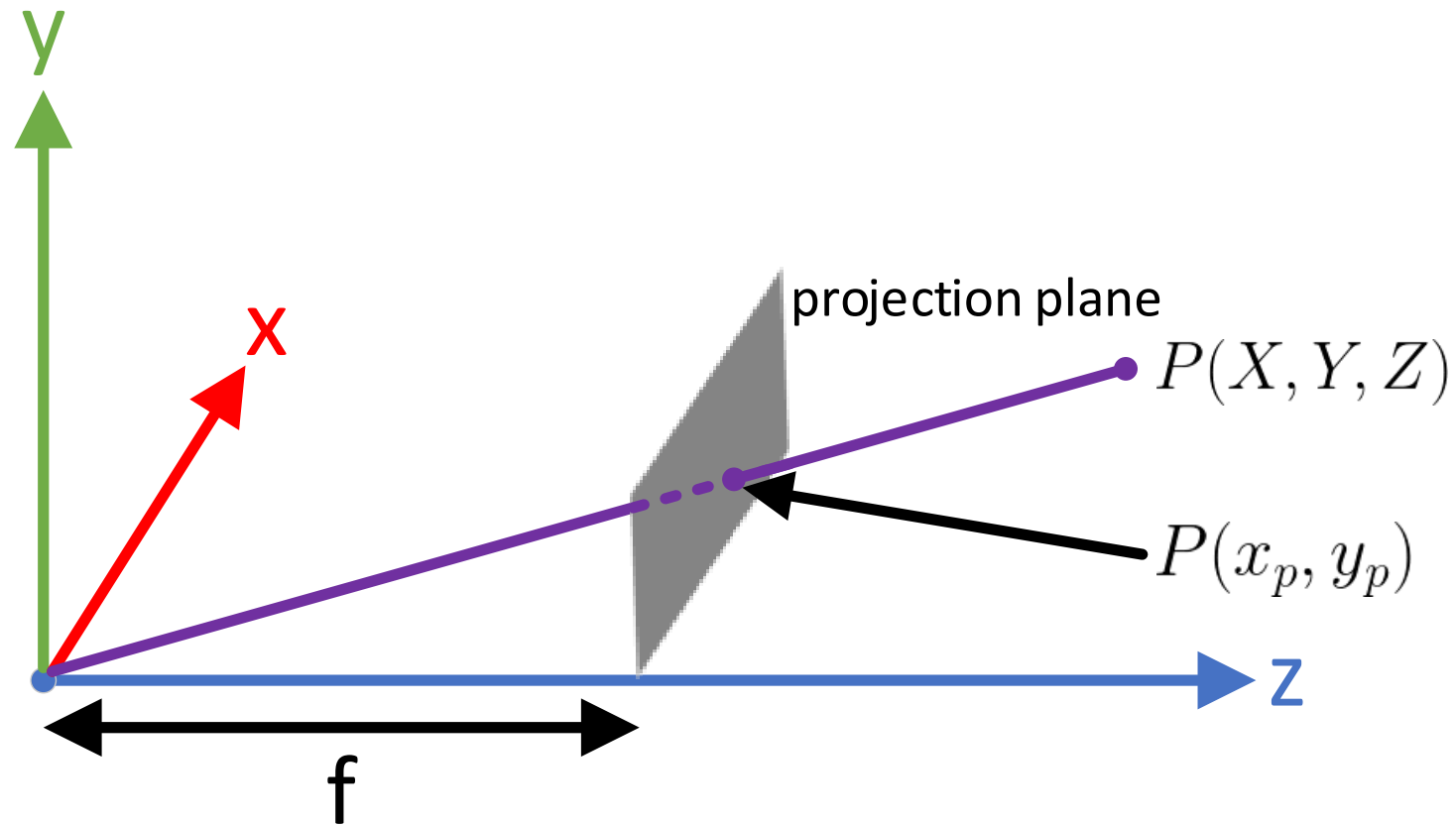
Mathematics of Perspective Projection



Mathematics of Perspective Projection



Mathematics of Perspective Projection



3D Coordinate to 2D
Coordinate:

$$\begin{array}{ccc} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} & \xrightarrow{\quad} & \begin{pmatrix} f \frac{X}{Z} \\ f \frac{Y}{Z} \\ f \end{pmatrix} \\ P(X, Y, Z) & & P(x_p, y_p) \end{array}$$