# CMPUT 307 or 414: Background Notes

## 2D and 3D Transforms, Homogeneous Transforms & Perspective Projection

# Basic Transforms

-Translation

-Scaling

-Rotation

-Homogeneous Transform

-Perspective Projection

# 2D Translation

Let P (x, y) ➔ original point, with coordinates x and y

Let P' (x', y') ➔ point after applying transformation, when its coordinates change to x' and y'

In case of translation of the original point by $t_x$ and $t_y$ in the x and y direction respectively, we have the translation vector T ($t_x$, $t_y$) applied to (x, y) to move it to (x', y'), such that:

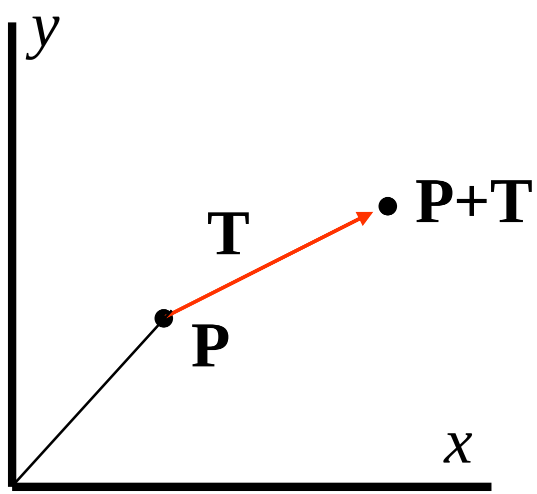
$x' = x + t_x, \quad y' = y + t_y$

# 2D Translation

Alternatively,

$$\mathbf{P}' = \mathbf{P} + \mathbf{T},\ \text{with}$$

$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \mathbf{T} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

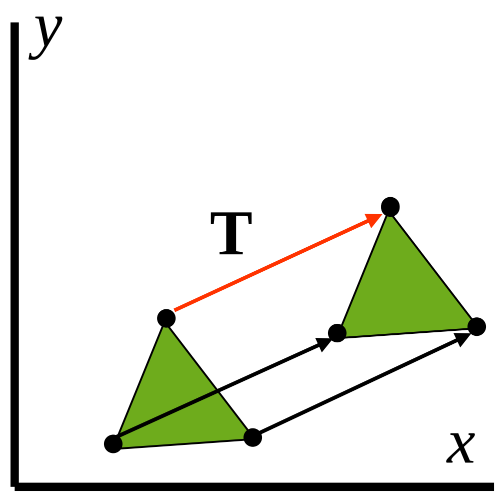


# 2D Translation

Translate polygon:

Apply the same operation on all points.

Works always, for all transformations of objects defined as a set of points.

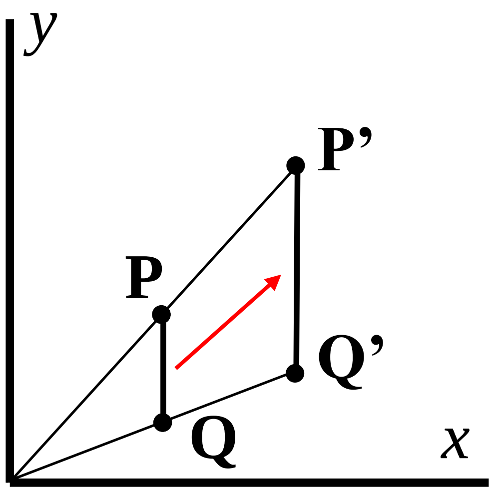


# 2D Scaling

Scale with factor $s_x$ and $s_y$:

$x' = s_x\, x,\quad y' = s_y\, y$

or



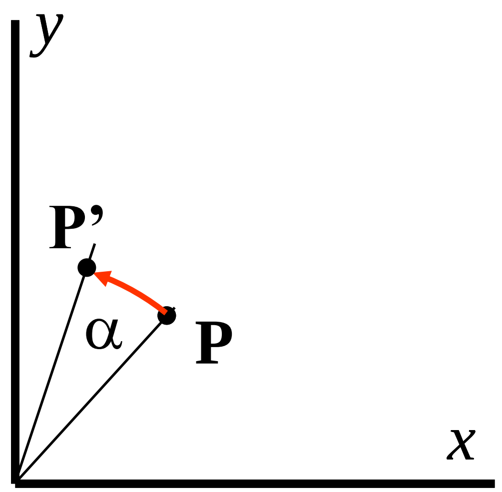

$$\mathbf{P}' = \mathbf{S}\mathbf{P},\ \text{with}$$

$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{S} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$

# 2D Rotation

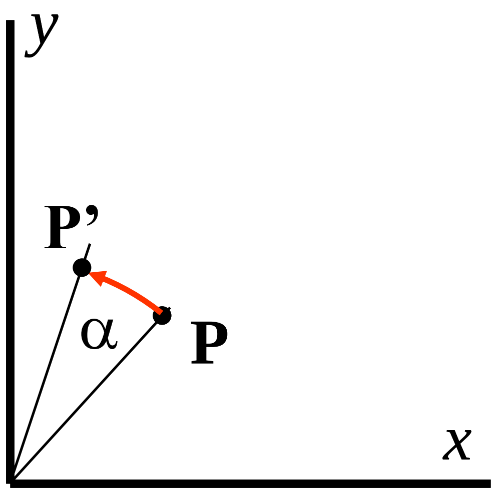
## Rotating by an angle α

If coordinates of P are (x, y), and coordinates of P’ are (x’, y’) can you express the latter in terms of the former and the angle α ?



# 2D Rotation: by an angle α

$$x' = x\cos\alpha - y\sin\alpha$$
$$y' = x\sin\alpha + y\cos\alpha$$

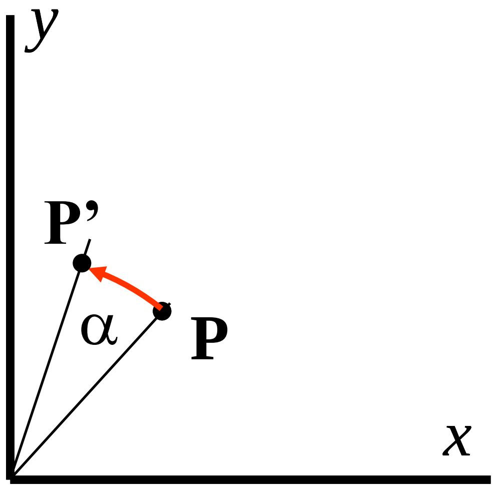


# 2D Rotation by an angle α

Alternatively,

$$\mathbf{P}' = \mathbf{R}\mathbf{P}, \quad \text{with}$$

$$\mathbf{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$



# Series of Transforms

Translate with **V**:

$\mathbf{T} = \mathbf{P} + \mathbf{V}$

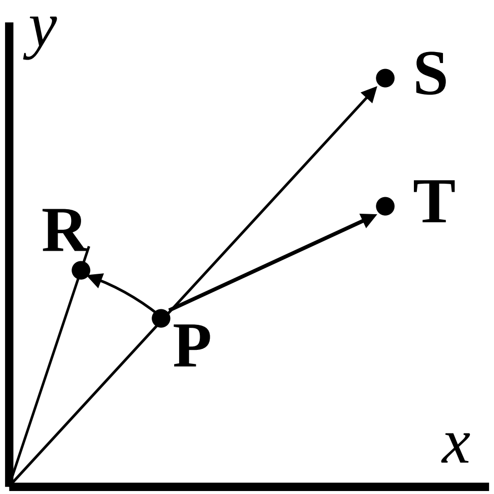
Scale with factor $s_x = s_y = s$:

$\mathbf{S} = s\mathbf{P}$

Rotate by an angle $\alpha$:

$R'_x = \cos\alpha\, P_x - \sin\alpha\, P_y$

$R'_y = \sin\alpha\, P_x + \cos\alpha\, P_y$



# Problems

-Inconvenient

-How to combine transforms ?

-Solution: Homogeneous !

# Homogeneous coordinates

-Uniform representation of translation, rotation, scaling

-Uniform representation of points and vectors

-Compact representation of sequence of transformations

# Homogeneous coordinates

-Add extra coordinate:

$\mathbf{P} = (p_x, p_y, p_h)$ or

$\mathbf{x} = (x, y, h)$

-Cartesian coordinates: divide by $h$

$\mathbf{x} = (x/h, y/h)$

-We take $h = 1$ for convenience

# 2D Homogeneous Translation

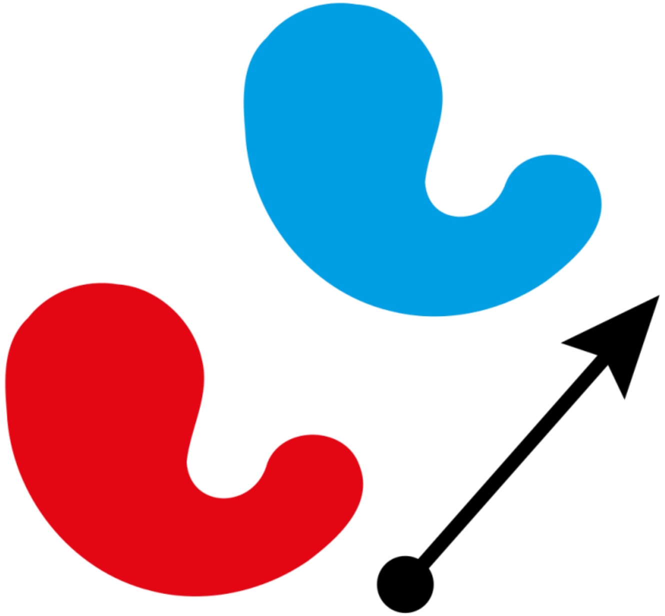
Translatio n :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{T}(t_x, t_y)\mathbf{P}$$

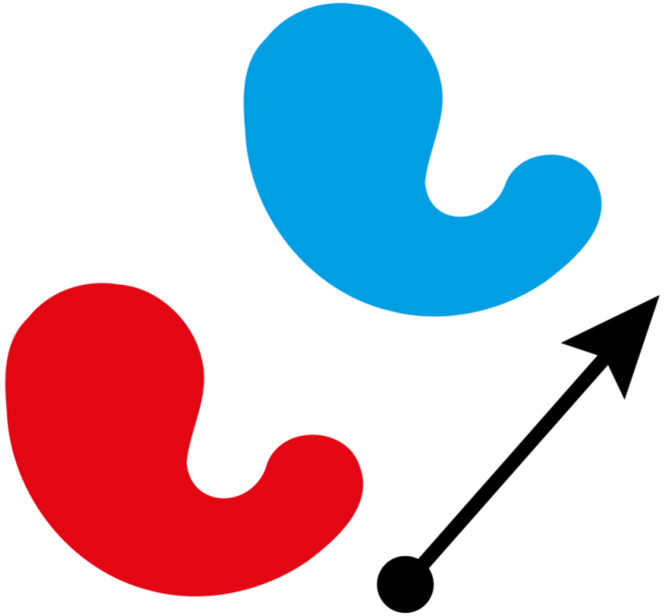
# 3D Translation



**Move X by ΔX, Y by ΔY, and Z by ΔZ**

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} X + \Delta X \\ Y + \Delta Y \\ Z + \Delta Z \end{bmatrix}$$

# 3D Homogeneous Translation (Using a Matrix Muliplication)



Translation Matrix：

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D Homogeneous Translation

Question:

What is the matrix for Translation of (2,-3,2)?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 2D Homogeneous Scaling

Scaling :

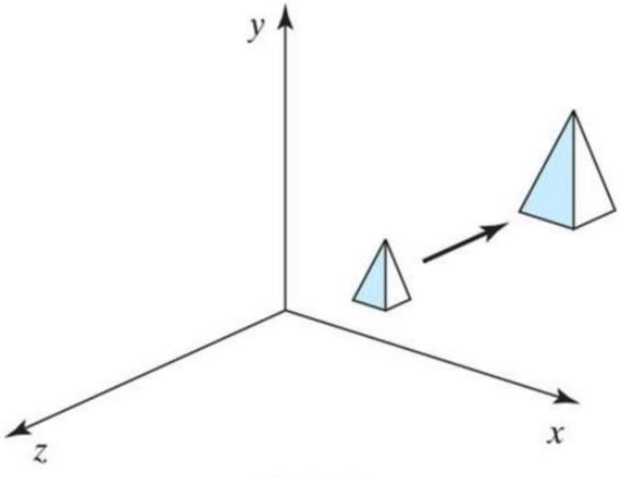
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{S}(s_x, s_y)\mathbf{P}$$

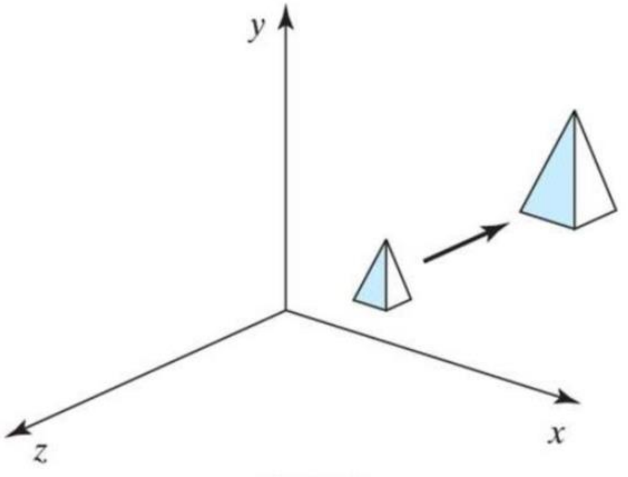
# 3D Scaling

**Scale X by $S_X$, Y by $S_Y$, and Z by $S_Z$**




$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} S_X & 0 & 0 \\ 0 & S_Y & 0 \\ 0 & 0 & S_Z \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

# 3D Homogeneous Scaling




Scaling Matrix：

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 2D Homogeneous Rotation

Rotation :

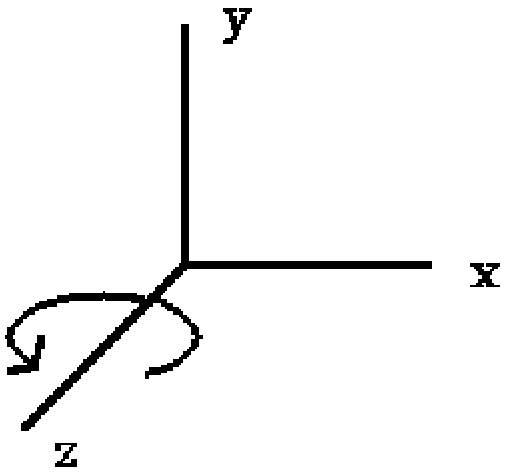
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{R}(\theta)\mathbf{P}$$

# 3D Homogeneous Rotation

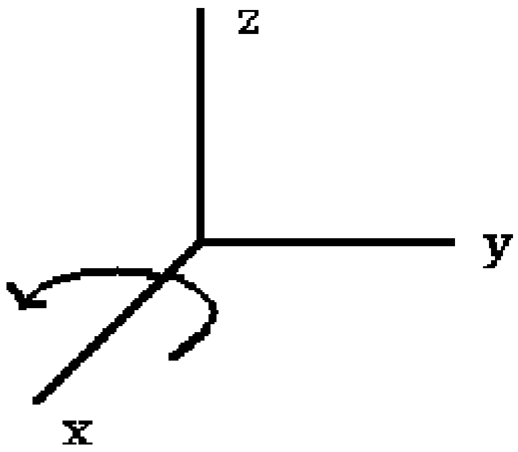
Z-Axis Rotation Matrix




$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D Homogeneous Rotation

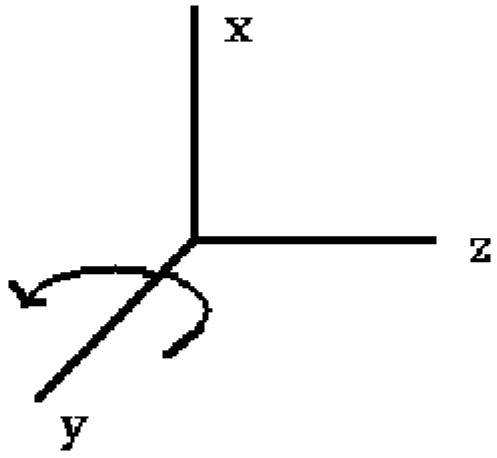
X-Axis Rotation Matrix




$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D Homogeneous Rotation

Y-Axis Rotation Matrix




$$\begin{bmatrix} cos\theta & 0 & sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Combining transformations

First transformation: $P' = M_1P$

Second transformation: $P'' = M_2P'$

How can we express the $P''$ in terms of a transformation on $P$ ?

# Combining transformations

Solution:

$P'' = M_2(M_1P)$

$= M_2M_1P$

$= MP$ with $M=M_2M_1$

# Combining transformations: Example

$$\mathbf{P}' = \mathbf{T}(t_{1x}, t_{1y})\mathbf{P} \qquad \text{first translation}$$

$$\mathbf{P}'' = \mathbf{T}(t_{2x}, t_{2y})\mathbf{P}' \qquad \text{second translation}$$

$t_{1x}$ is magnitude of translation in x direction (dx) for first translation
$t_{1y}$ is magnitude of translation in y direction (dy) for first translation

( similarly $t_{2x}$ and $t_{2y}$ for second translation )

Express P'' as a translation on P.

# Combining transformations: Example

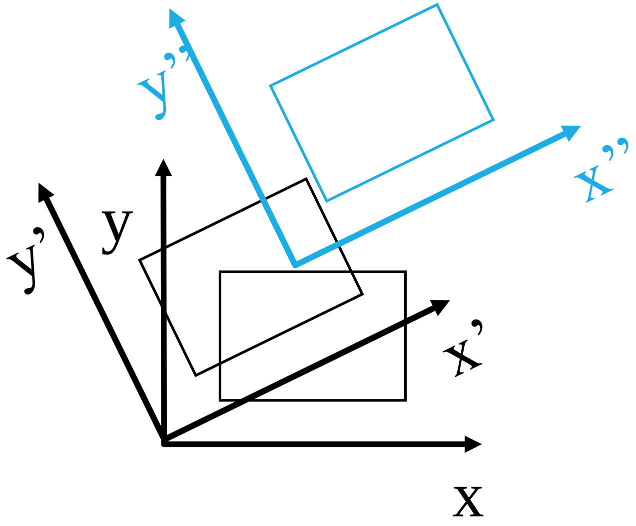
$$\mathbf{P}'' = \mathbf{T}(t_{2x}, t_{2y})\mathbf{T}(t_{1x}, t_{1y})\mathbf{P}$$

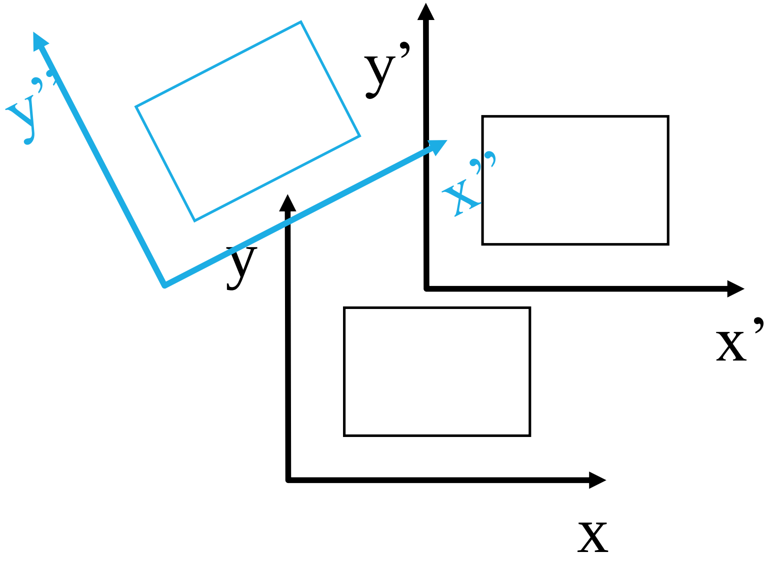
$$= \begin{pmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{pmatrix} \mathbf{P}$$

$$= \begin{pmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{pmatrix} \mathbf{P} \qquad = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})\,\mathbf{P}$$

# Order of transformations

*Rotation, translation…* *Translation, rotation…*




Matrix multiplication does **not** commute.

The order of transformations makes a difference!

# Example 1

Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?][?]\begin{bmatrix} .86 & -0.5 & 0 & 0 \\ 0.5 & .86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Example 1

Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

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Rotate by 30 degree around Z-axis, followed by scaling (x by 0.5, Y by 1.5, and Z by 0.8), followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .86 & -0.5 & 0 & 0 \\ 0.5 & .86 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [??] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?]\begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Example 2

Rotate by 60 degree around X-axis, followed by translation by (20, 10, -20), followed by scaling (x by 1.5, Y by 0.5, and Z by 1): (cos 60 = .5 & sin 60 = 0.86)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -0.86 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Example 3

Scale (X by 0.5, Y by 1.5, and Z by 0.8), followed by rotate by 30 degree around Y-axis, followed by followed by translation by (10, -10, 20): (cos 30 = .86 & sin 30 = 0.5)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = [?][?]\begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.86 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0.86 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} .5 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Arbitrary Rotations

Rotating from one direction to another, e.g. change vector V0 to V1

-Angle between two vectors V0 & V1 = ?

-Direction normal to the two vectors = ?

# Arbitrary Rotations

Angle $\theta$ between two vectors V0 & V1

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$

$$cos\theta = \frac{v_0 \cdot v_1}{||v_0|| \cdot ||v_1||} = \frac{x_0 \cdot x_1 + y_0 \cdot y_1 + z_0 \cdot z_1}{||v_0|| \cdot ||v_1||}$$

$$||v_0|| = \sqrt{x_0^2 + y_0^2 + z_0^2} \quad ||v_1|| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

# Arbitrary Rotations

Direction normal to two vectors
-Cross product of the two vectors

Once we find this Normal Axis of Rotation we can use the equation in the next slide to compute an arbitrary rotation

# Arbitrary Rotations

Cross product of two vectors

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$
$$v_0 \times v_1 = (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1)$$

To Remember this Formula, Note X does not appear in X coordinate of the formula, Y does not appear in Y coordinate of the formula, and Z does not appear in Z coordinate of the formula.

Cross product of (1,0,0) x (0, 1, 0):
(0x0 – 0 x1, 0 x 0 – 1 x 0, 1 x 1 – 0 x 0) = (0, 0, 1)

# Arbitrary Rotation Matrix

Rotate by an angle θ around an arbitrary normalized direction (x, y, z)

$$\begin{bmatrix} x^2+\cos\theta(1-x^2) & xy(1-\cos\theta)-z\sin\theta & xz(1-\cos\theta)+y\sin\theta & 0 \\ xy(1-\cos\theta)+z\sin\theta & y^2+\cos\theta(1-y^2) & yz(1-\cos\theta)-x\sin\theta & 0 \\ xz(1-\cos\theta)-y\sin\theta & yz(1-\cos\theta)+x\sin\theta & z^2+\cos\theta(1-z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Example A

## How do we rotate the Vector (1, 1, 2) to the vector (1, 2, 1) ?

(1) Angle between the two vectors is:
Cos-1 {(1x1+1x2+2x1)/(SQRT(1x1+1x1+2x2)xSQRT(1x1+2x2+1x1))}
= Cos-1 {5/(SQRT(6)xSQRT(6))} = Cos-1 {5/6} = Cos-1 {0.83}=33.56

(2) Cross product of the two vectors is:
(y1z2 – z1y2, z1x2 – x1z2, x1y2 – y1x2) = (1x1 – 2 x2, 2x1 – 1x1, 1x2 – 1x1) = (-3,1,1)
Gives direction orthogonal to both vectors.

So, we need to Rotate by 33.56 degrees around the Normalized vector (-3, 1, 1)/SQRT(11) = (-3, 1, 1)/3.32 = (-0.9, .3, .3)

# Example B

## How do we rotate the Vector (1, 0, -1) to the vector (1, 0, 1) ?

(1) Angle between the two vectors is:
Cos-1 {(1x1+0x0 -1x1)/(SQRT(1x1+0x0+(-1)x(-1))xSQRT(1x1+0x0+1x1))}
= Cos-1 {0/(SQRT(2)xSQRT(2))} = Cos-1 {0} = 90

(2) Cross product of the two vectors is:
(y1z2 – z1y2, z1x2 – x1z2, x1y2 – y1x2) = (0x1 – (-1) x0, (-1)x1 – 1x1, 1x0 – 0x1) = (0,-2,0)
Gives direction orthogonal to both vectors.

So, we need to Rotate by 90 degrees around the Normalized vector (0, -2, 0)/2 = (0, -1, 0),

# Example A

How do we rotate the Vector (1, 1, 2) to the vector (1, 2, 1) ?

Solution:

1. Rotate by 33.56 degrees around the Normalized vector (-0.9, 0.3,0.3)
2. Use the matrix below for performing the rotation

$$\begin{bmatrix} x^2+\cos\theta(1-x^2) & xy(1-\cos\theta)-z\sin\theta & xz(1-\cos\theta)+y\sin\theta & 0 \\ xy(1-\cos\theta)+z\sin\theta & y^2+\cos\theta(1-y^2) & yz(1-\cos\theta)-x\sin\theta & 0 \\ xz(1-\cos\theta)-y\sin\theta & yz(1-\cos\theta)+x\sin\theta & z^2+\cos\theta(1-z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Example A

Final Answer ?

$$\begin{bmatrix} ? & ? & ? & 0 \\ ? & ? & ? & 0 \\ ? & ? & ? & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

| 0.968 | -0.211 | 0.121 | 0 |
|---|---|---|---|
| 0.121 | 0.848 | 0.512 | 0 |
| -0.211 | -0.482 | 0.848 | 0 |
| 0 | 0 | 0 | 1 |

# Example B

How do we rotate the Vector (1, 0, -1) to the vector (1, 0, 1) ?

Solution:

1. We need to Rotate by 90 degrees around the vector (0, -1, 0);

   Normalized vector is (0, -1, 0); sin(theta) = 1, cos(theta) = 0

2. Use the below matrix for performing the rotation

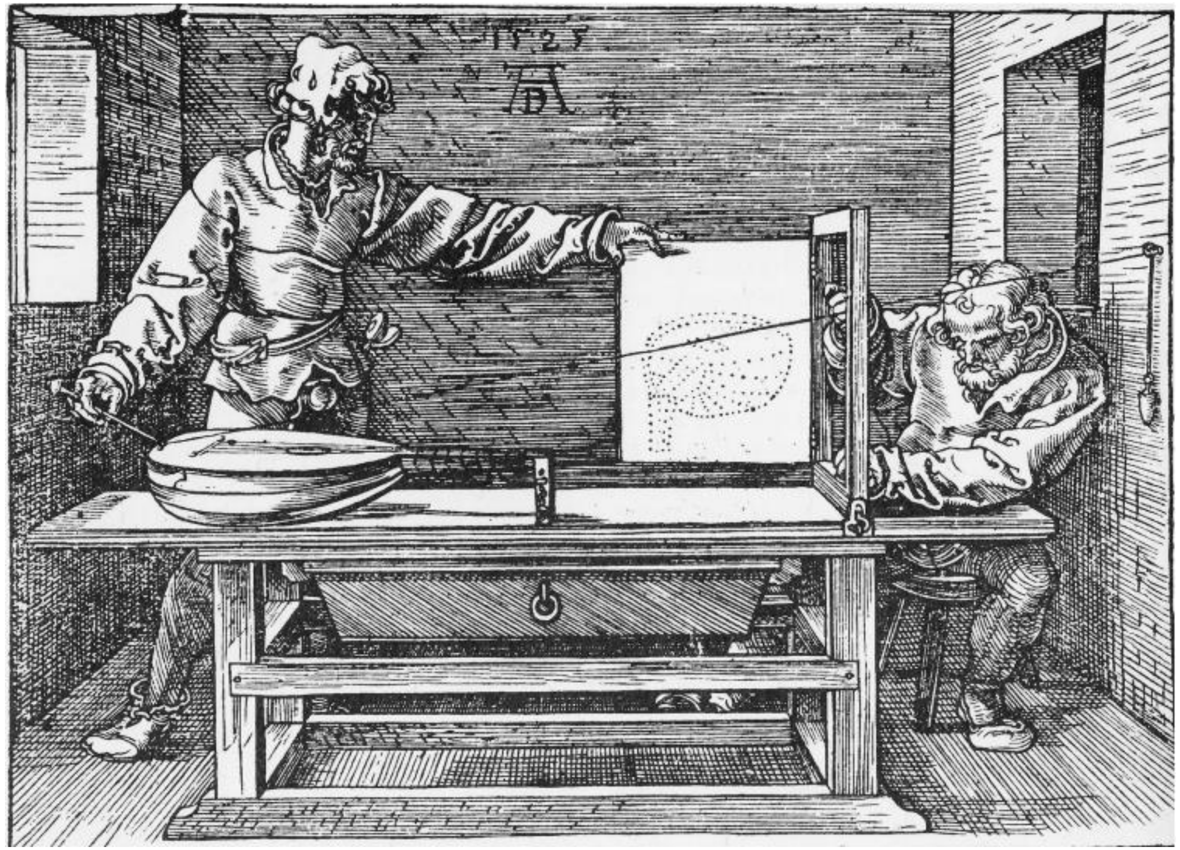
$$\begin{bmatrix} x^2+\cos\theta(1-x^2) & xy(1-\cos\theta)-z\sin\theta & xz(1-\cos\theta)+y\sin\theta & 0 \\ xy(1-\cos\theta)+z\sin\theta & y^2+\cos\theta(1-y^2) & yz(1-\cos\theta)-x\sin\theta & 0 \\ xz(1-\cos\theta)-y\sin\theta & yz(1-\cos\theta)+x\sin\theta & z^2+\cos\theta(1-z^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Example B

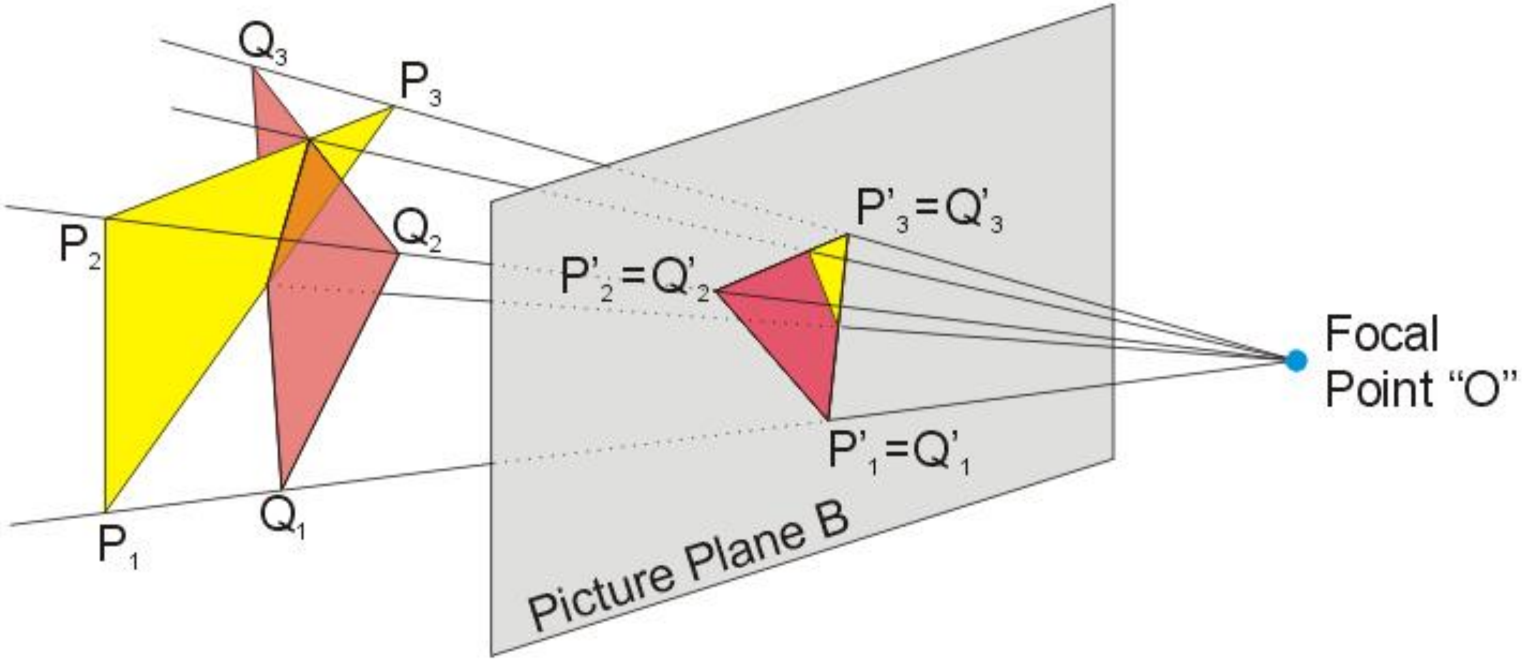
Final Answer:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

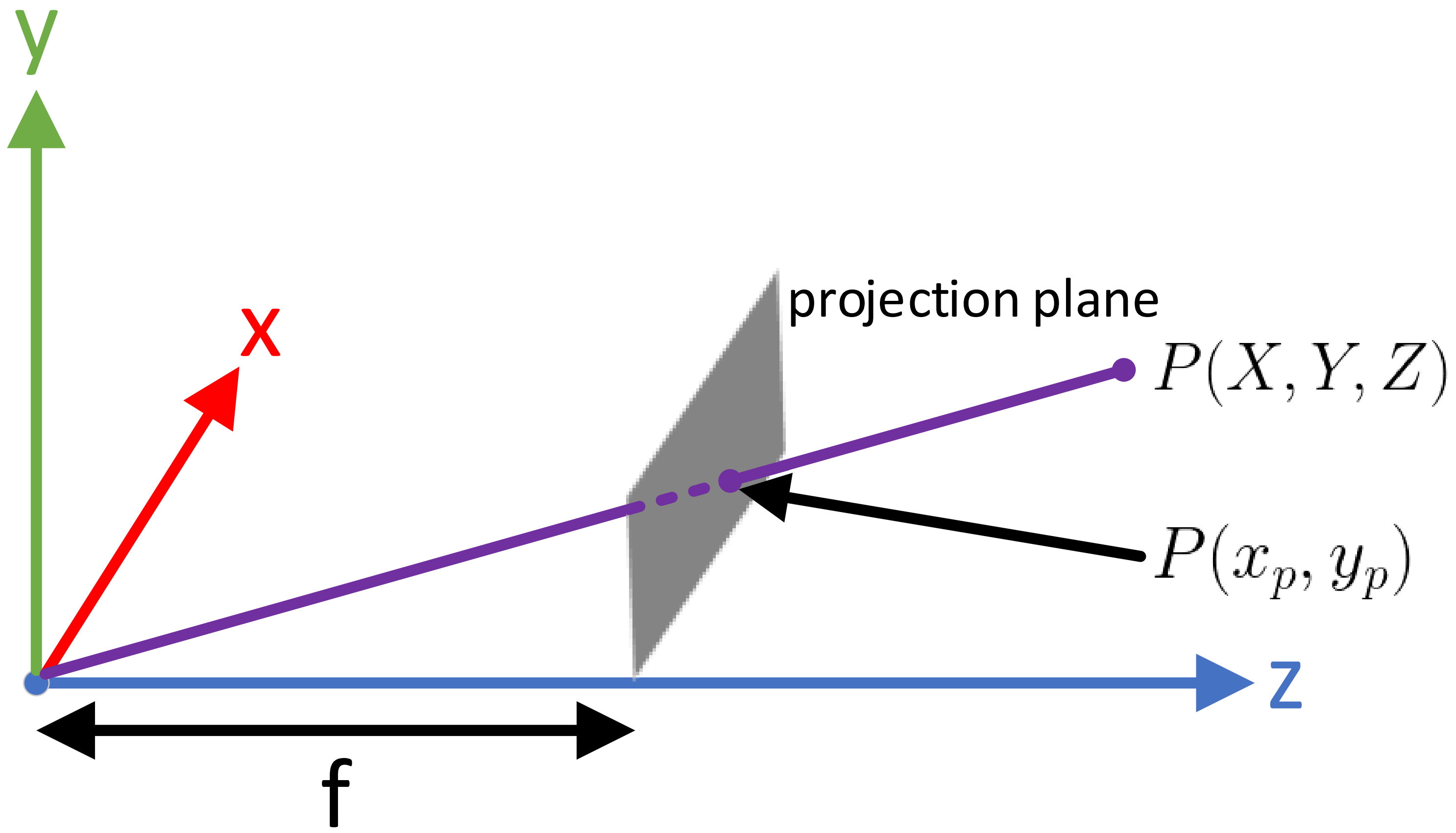
# Perspective Projection



# Perspective Projection



# Mathematics of Perspective Projection

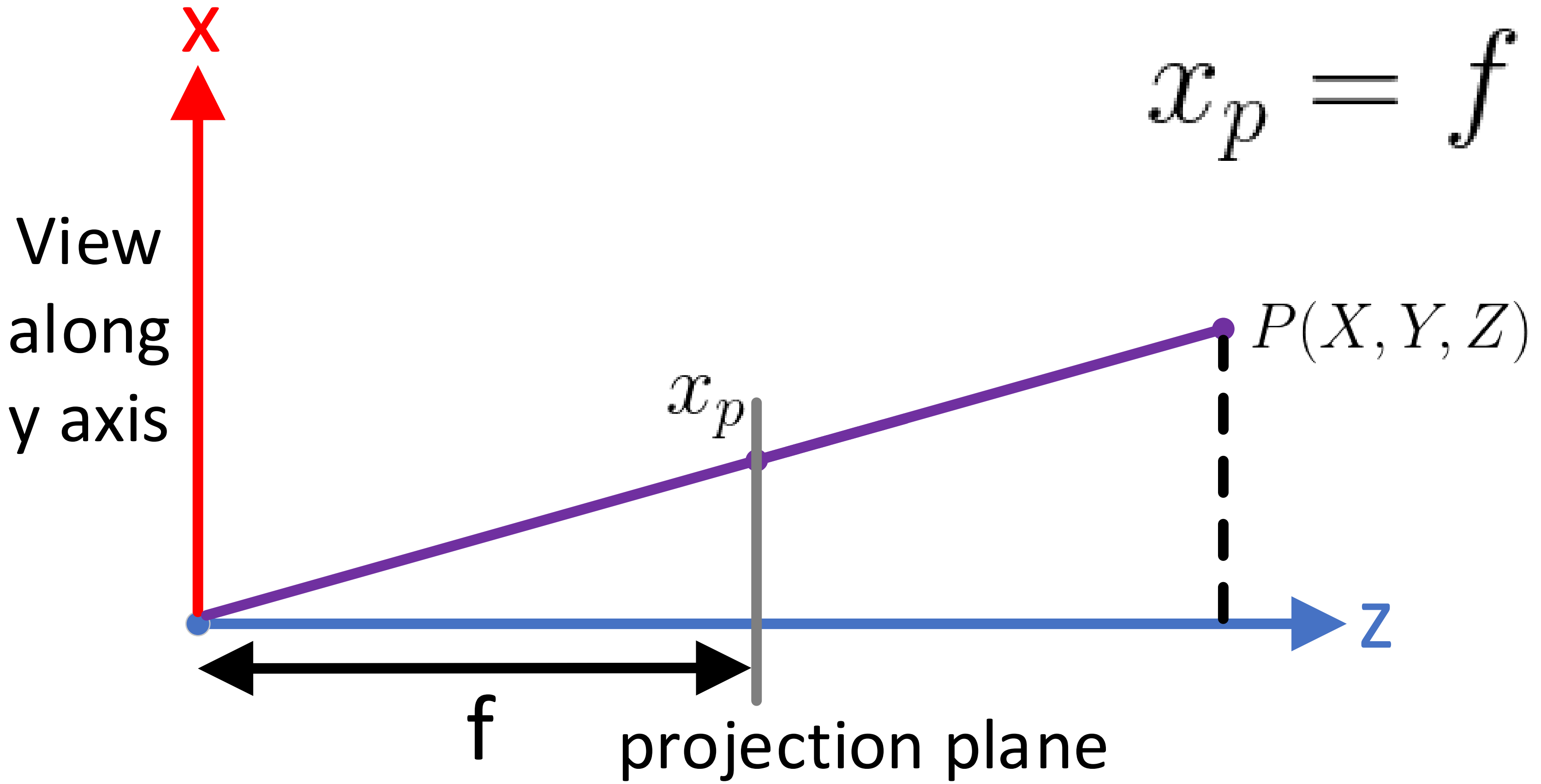



f: Focal Length

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \Rightarrow \begin{pmatrix} f\frac{X}{Z} \\ f\frac{Y}{Z} \\ f \end{pmatrix}$$

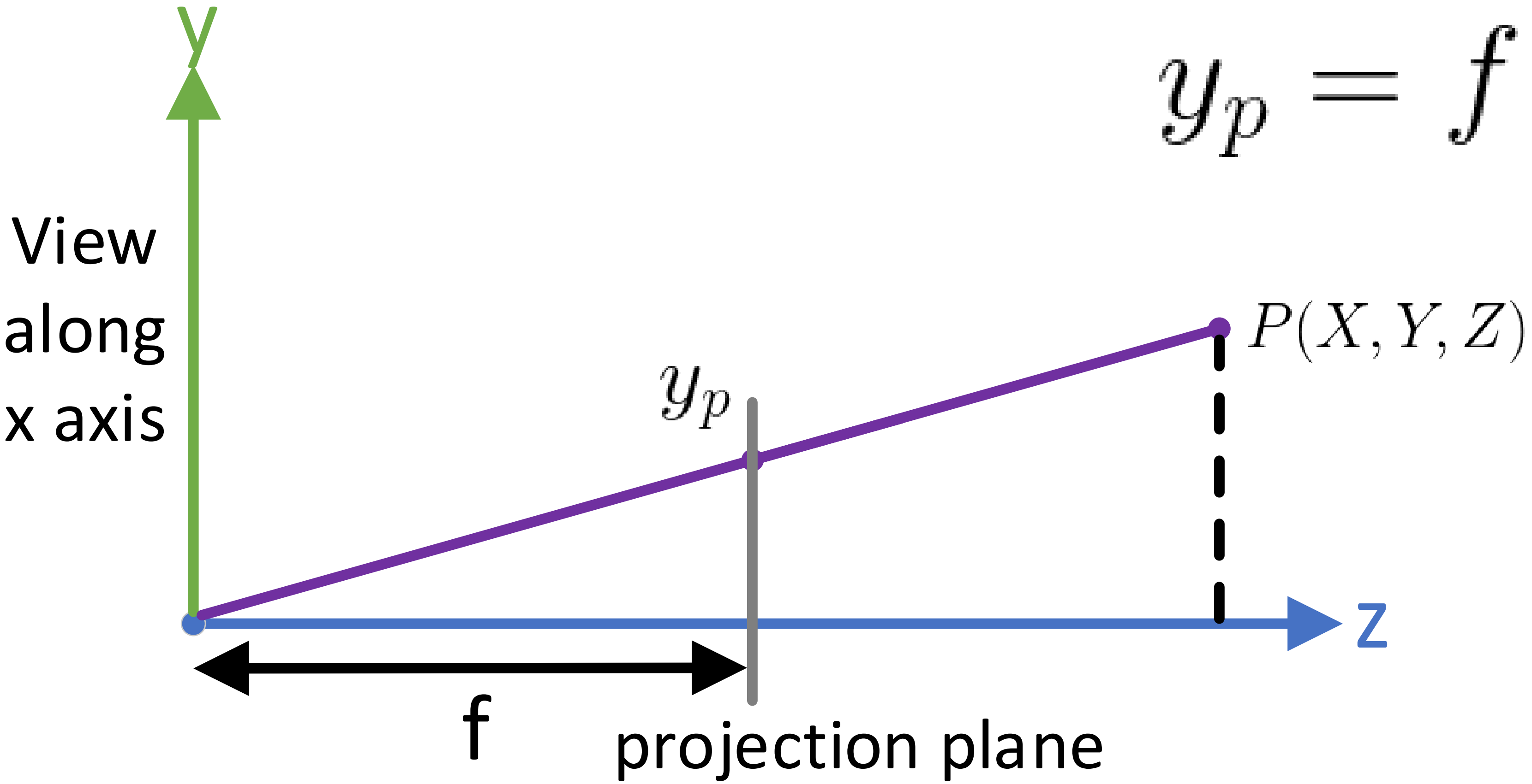
# Mathematics of Perspective Projection

$$x_p = f \times (\frac{X}{Z})$$



# Mathematics of Perspective Projection

$$y_p = f \times (\frac{Y}{Z})$$



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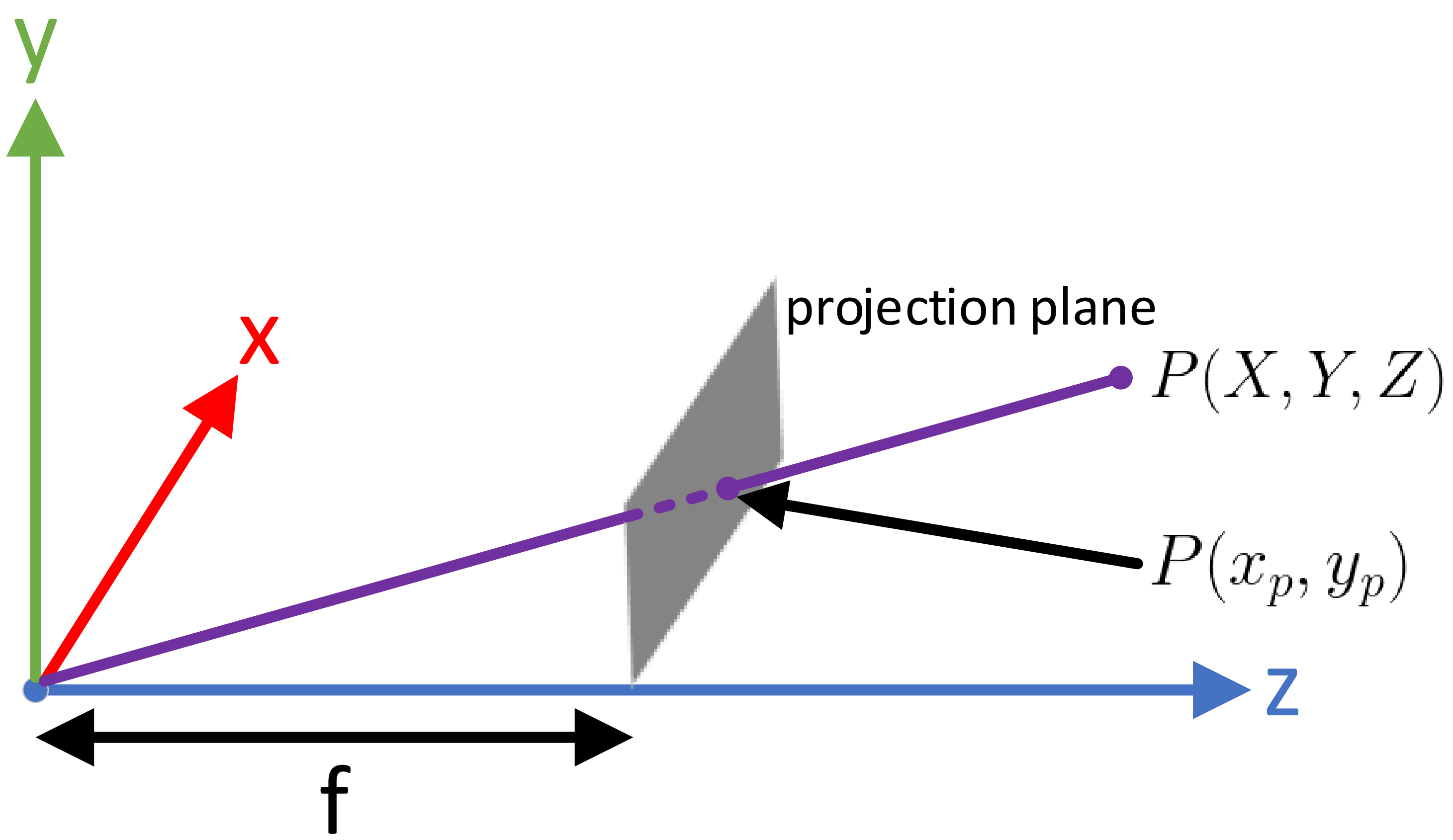


3D Coordinate to 2D Coordinate：

$$\underset{P(X,Y,Z)}{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}} \Rightarrow \underset{P(x_p,y_p)}{\begin{pmatrix} f\frac{X}{Z} \\ f\frac{Y}{Z} \\ f \end{pmatrix}}$$