

# CMPUT 307: Hints related to Quiz 3

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Computing Surface Normal  
for a Triangle/Face + the  
Equation of the Plane  
containing the Triangle

# Surface Normal

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Given 3 vertices of a triangle/face NOT on a Straight Line:

1. We can compute TWO Vectors from the 3 Points (Vertices).
2. Given 2 vectors the direction Orthogonal to both of them can be computed as the Cross Product of the 2 vectors; this direction is the surface normal.

# Computing Cross Product

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Cross product of two vectors

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$

$$v_0 \times v_1 = (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1)$$

To Remember this Formula, Note X does not appear in X coordinate of the formula, Y does not appear in Y coordinate of the formula, and Z does not appear in Z coordinate of the formula.

Cross product of  $(1,0,0) \times (0, 1, 0)$ :

$$(0 \times 0 - 0 \times 1, 0 \times 0 - 1 \times 0, 1 \times 1 - 0 \times 0) = (0, 0, 1)$$

# Equation of Plane containing a triangle

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Surface Normal you just computed is Orthogonal to ANY vector on the plane containing the triangle. So we can consider the following steps:

1. Make length of Surface Normal  $(a, b, c) = 1$ , by dividing by its current length. This is necessary to confirm with the Formula for QEM.
2. Then consider any of the 3 points of the triangle, say  $(x_0, y_0, z_0)$ .
3. For any point  $(x, y, z)$  on the plane containing the triangle,  $(x - x_0, y - y_0, z - z_0)$  is a vector on the plane. The inner product of this vector with the surface normal is ZERO, since the two vectors are orthogonal.
4. Thus,  $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ . This gives us the equation of the plane containing the triangle/face.

Example: Quiz 3, F1: (v1, v3, v4)

$$v1 = (3, 2, 5) \quad v3 = (1, 3, 3) \quad v4 = (1, 1, 2)$$

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$$(v1 - v3) \times (v1 - v4) = (2, -1, 2) \times (2, 1, 3) = (-5, -2, 4)$$

Making the length of this vector 1, Surface Normal  $(a, b, c) = (-0.75, -0.3, 0.6)$

Equation of Plane containing (v1, v3, v4) is:

$$a(x - x_4) + b(y - y_4) + c(z - z_4) = 0$$

$$\rightarrow -0.75(x - 1) - 0.3(y - 1) + 0.6(z - 2) = 0$$

$$\rightarrow -0.75x - 0.3y + 0.6z + 0.15 = 0$$