CMPUT 307: 2D TEMPLATE MATCHING

Instructor: Dr. Anup Basu

Notes prepared by: Dr. Mehdi Faraji | Winter 2023

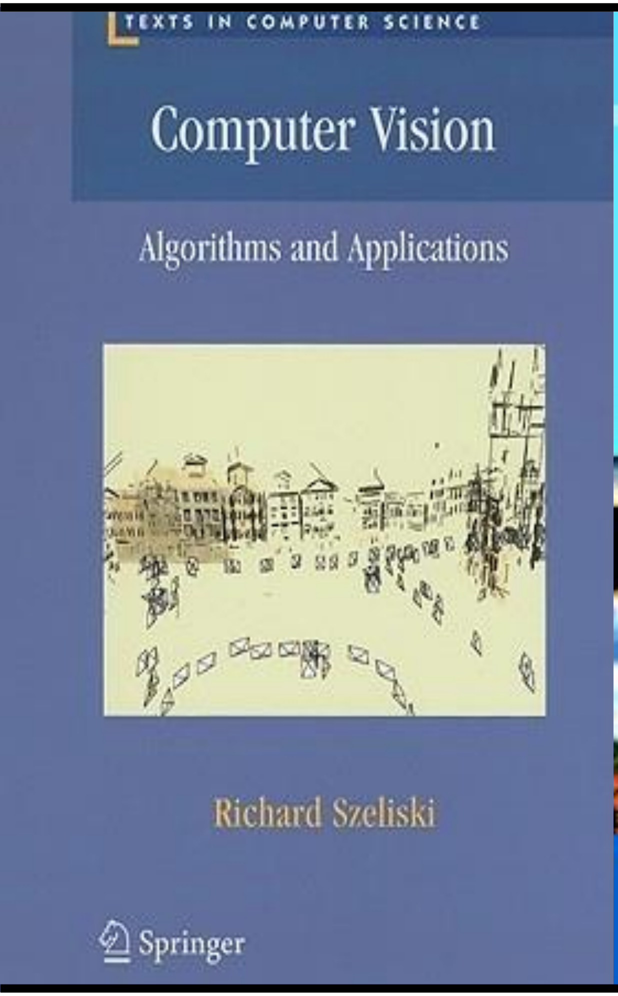
Department of Computing Science

UNIVERSITY OF ALBERTA

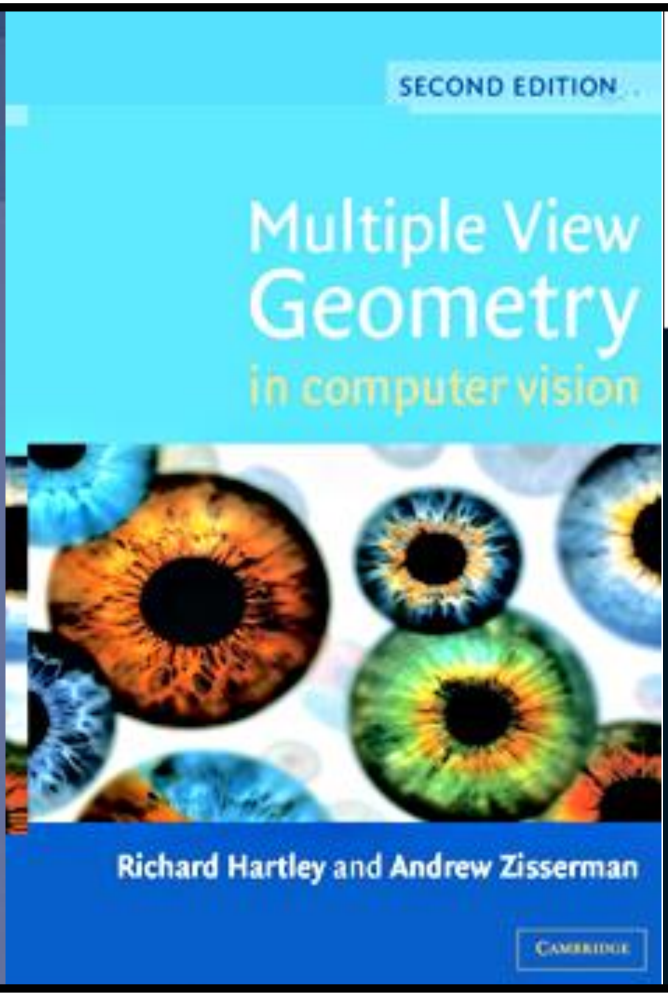
EDMONTON·ALBERTA·CANADA

References

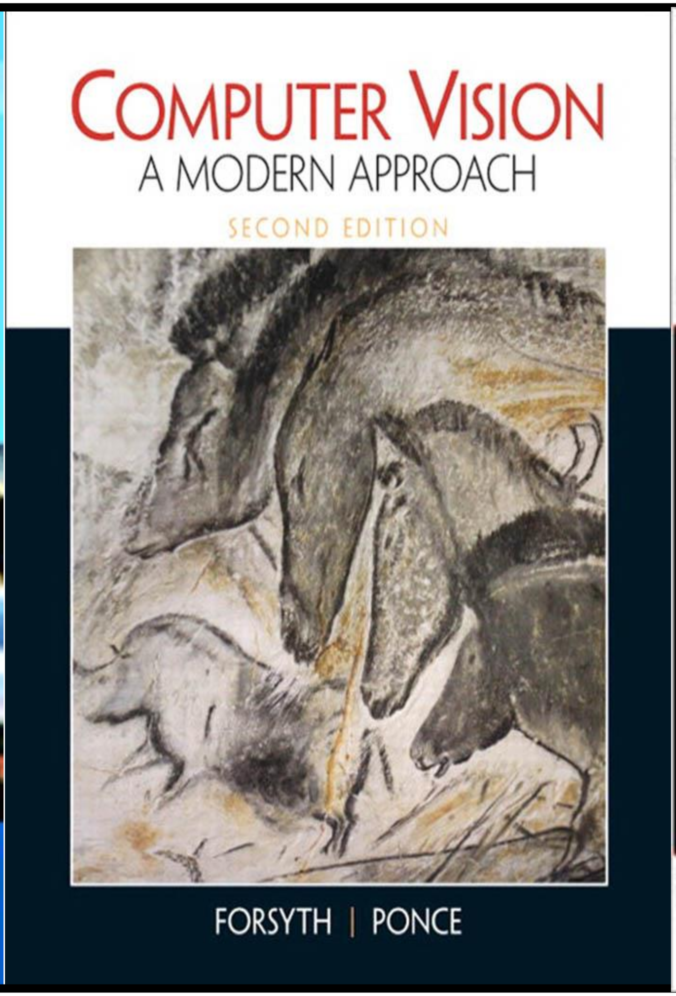
UNLESS OTHERWISE STATED, MATERIALS ON THIS SLIDE ARE TAKEN FROM THE FOLLOWING TEXTBOOKS



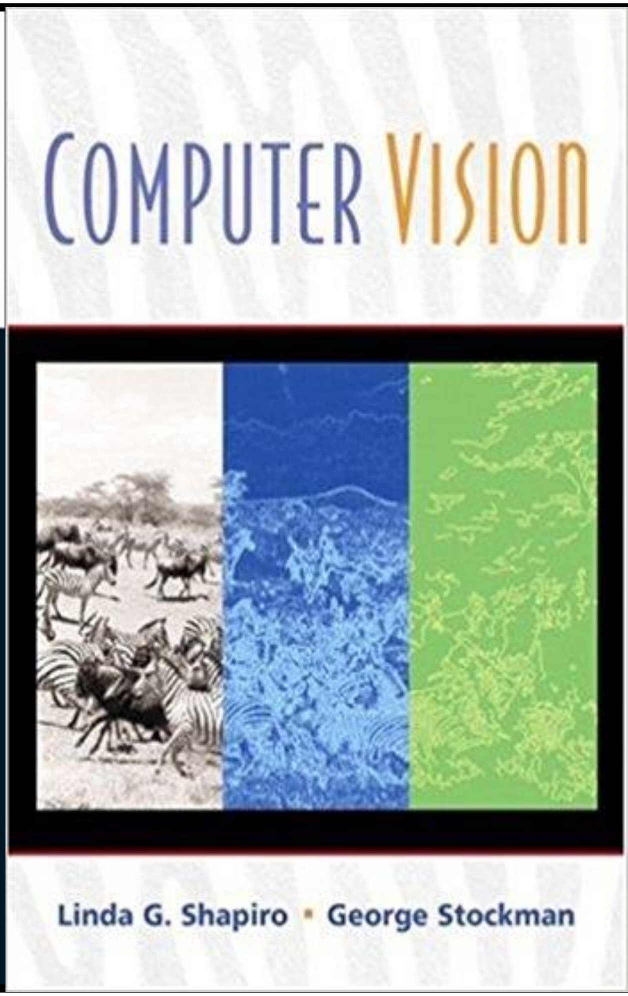
(Szeliski, 2010)



(Hartley & Zisserman, 2003)



(Forsyth & Ponce, 2011)



(Shapiro & Stockman, 2001)

Template Matching

- Sometimes we need to match (align) images
- How can you match these images?
  - Assuming that both images are available

Template Matching

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Naïve Way:

1. Compare all pixels' intensity of the first image with all intensities of the second image

Template Matching

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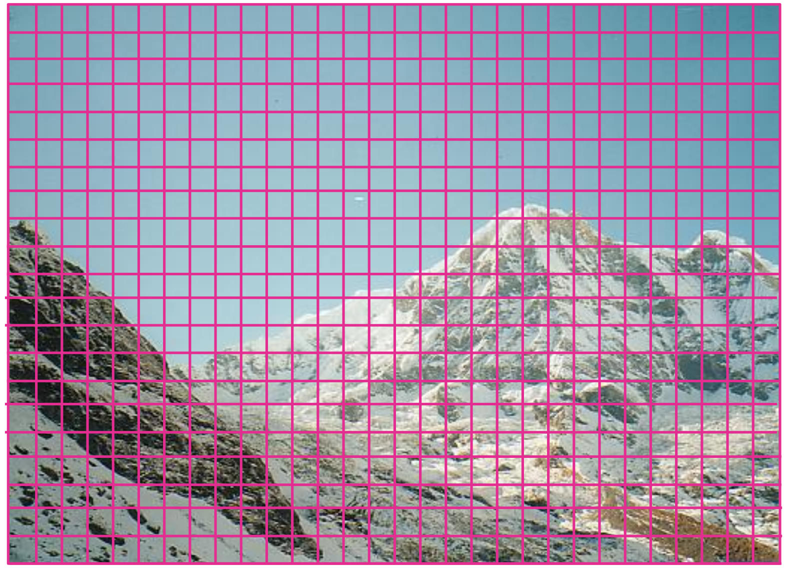
- How can you match these images?

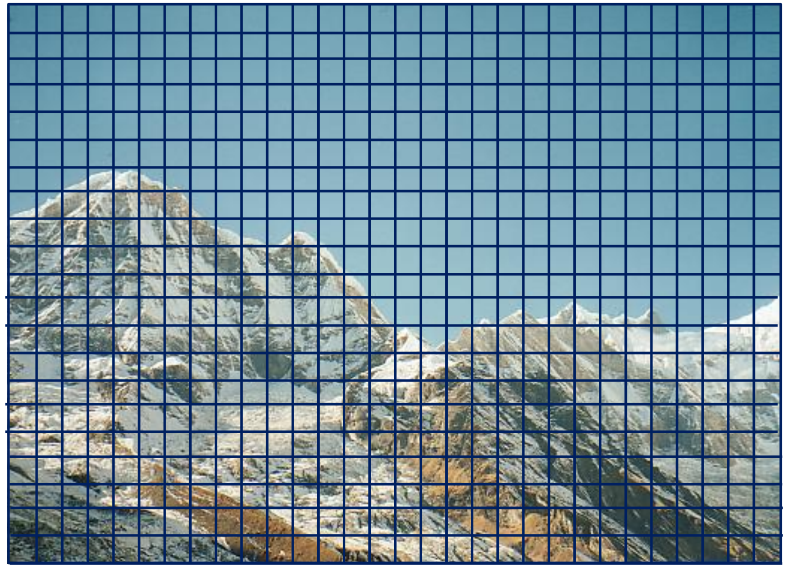
  Assuming that both images are available

Naïve Way:

1. Compare all pixels' intensity of the first image with all intensities of the second image

Will it work?





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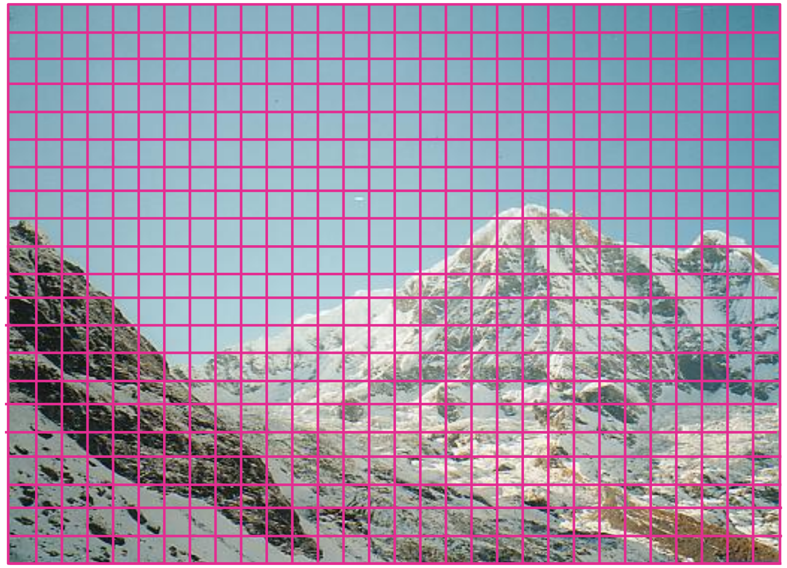
Naïve Way:

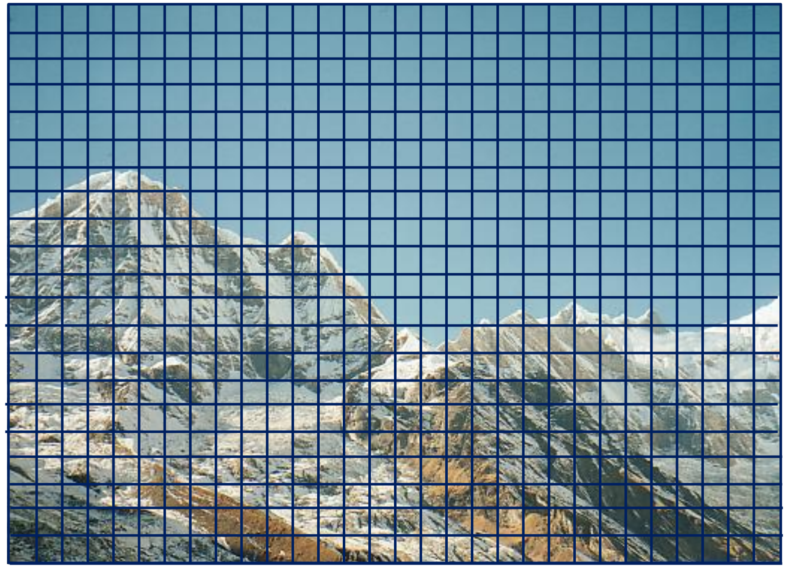
1. Compare all pixels' intensity of the first image with all intensities of the second image

Will it work?

Let's see…

1. Assume that it is a good idea → is it practical?





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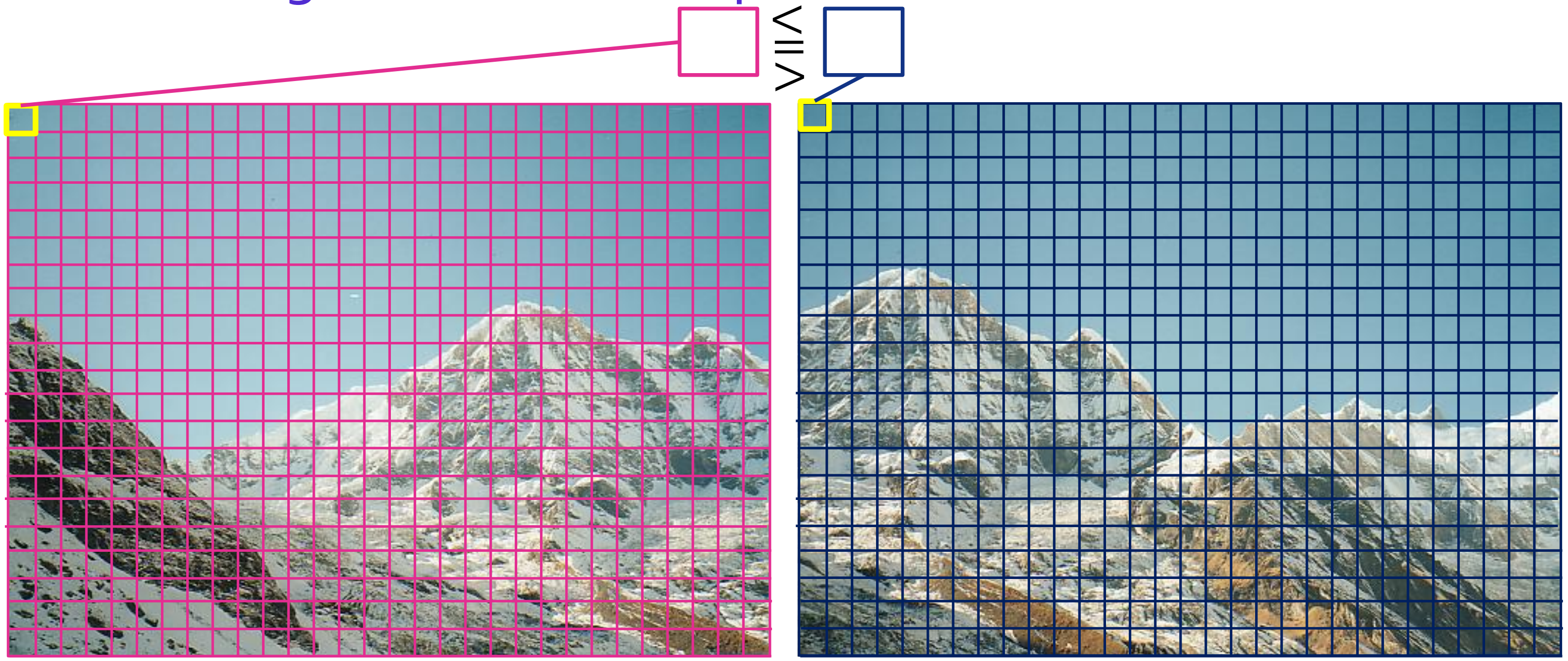

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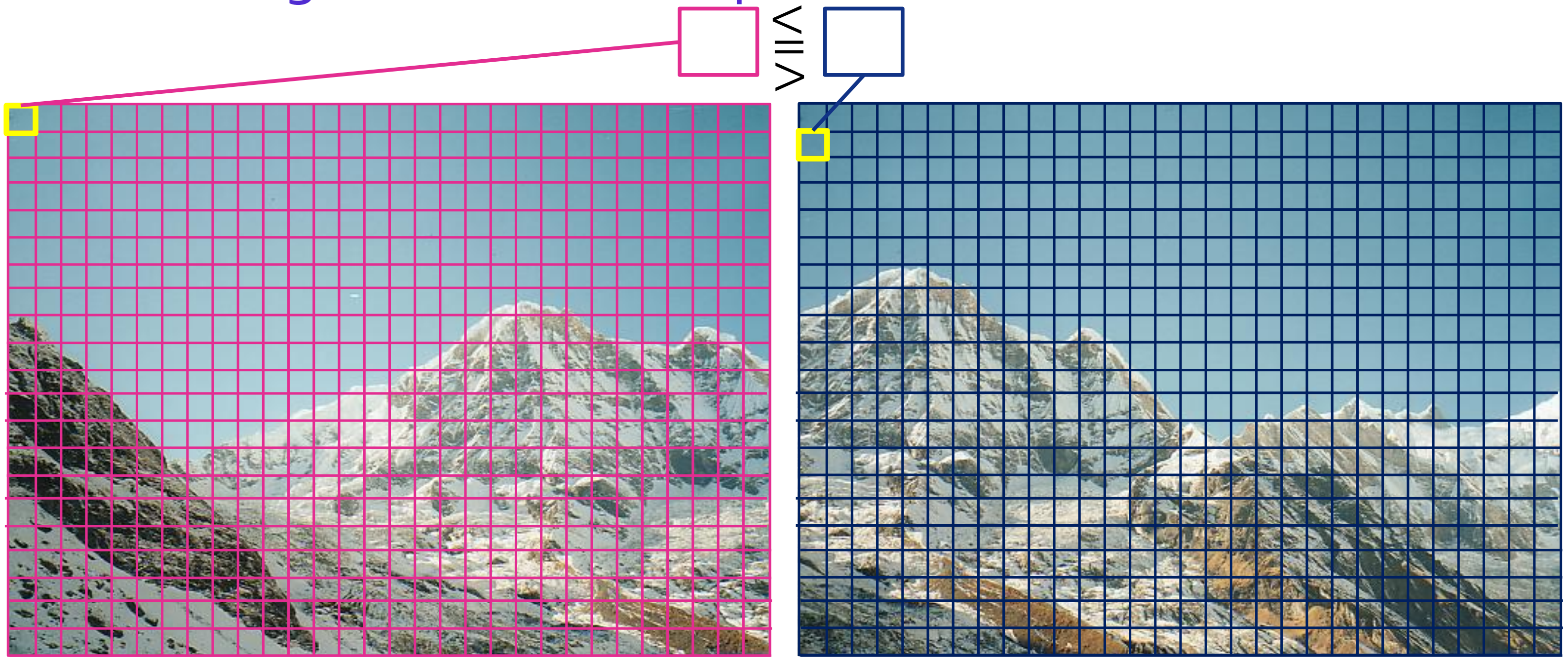

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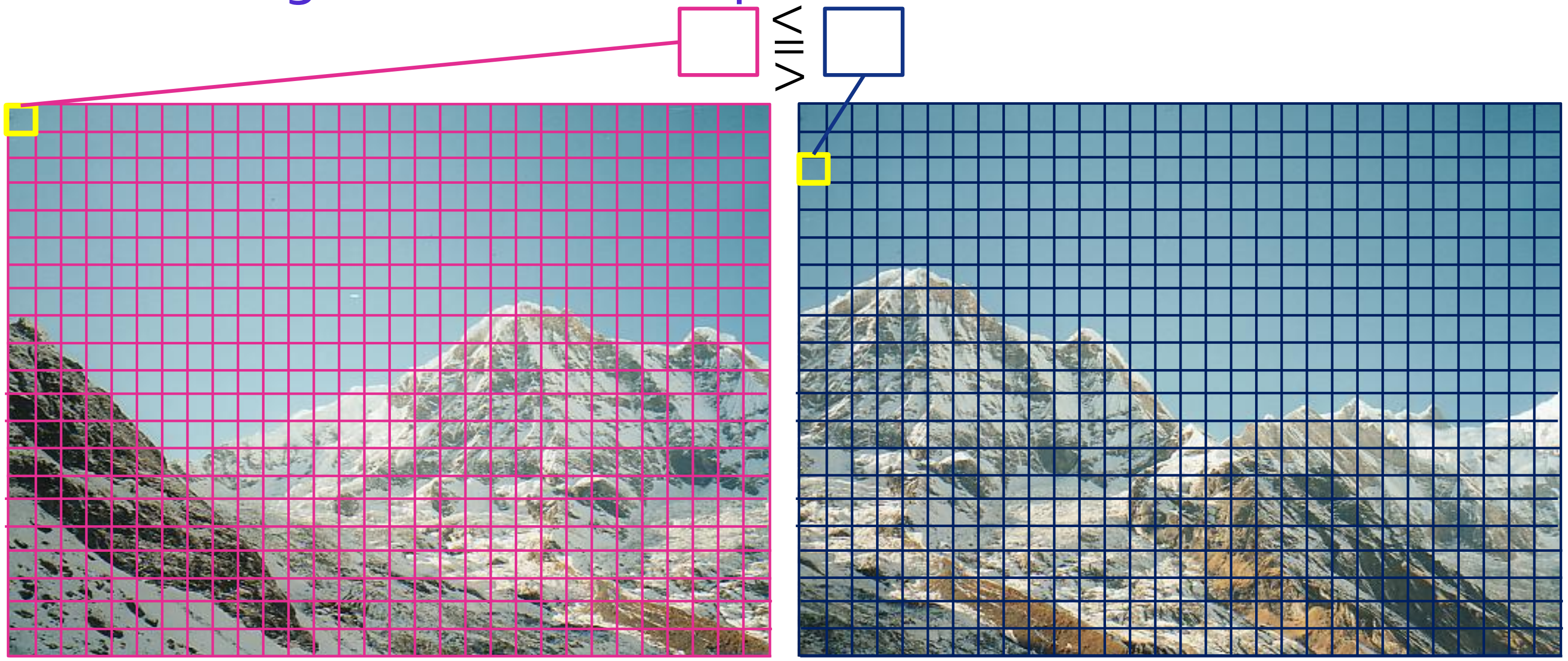

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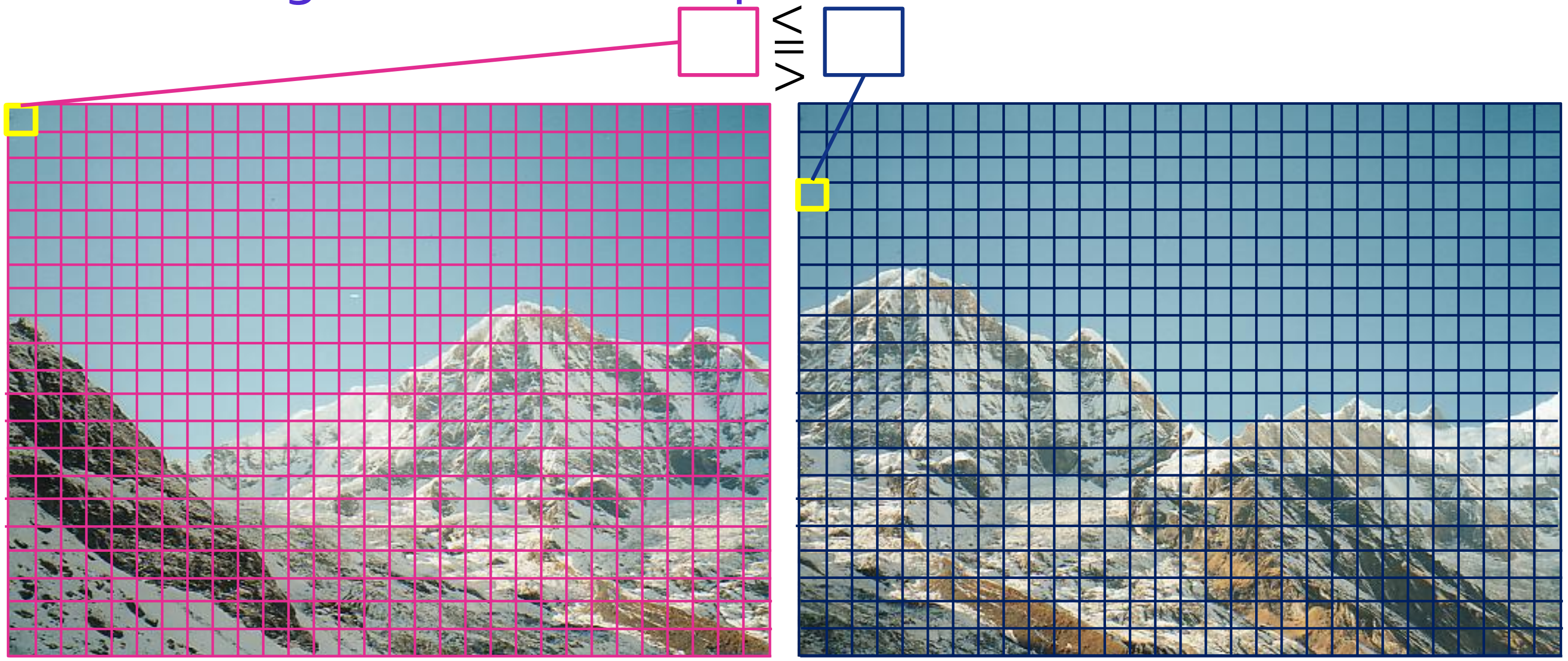

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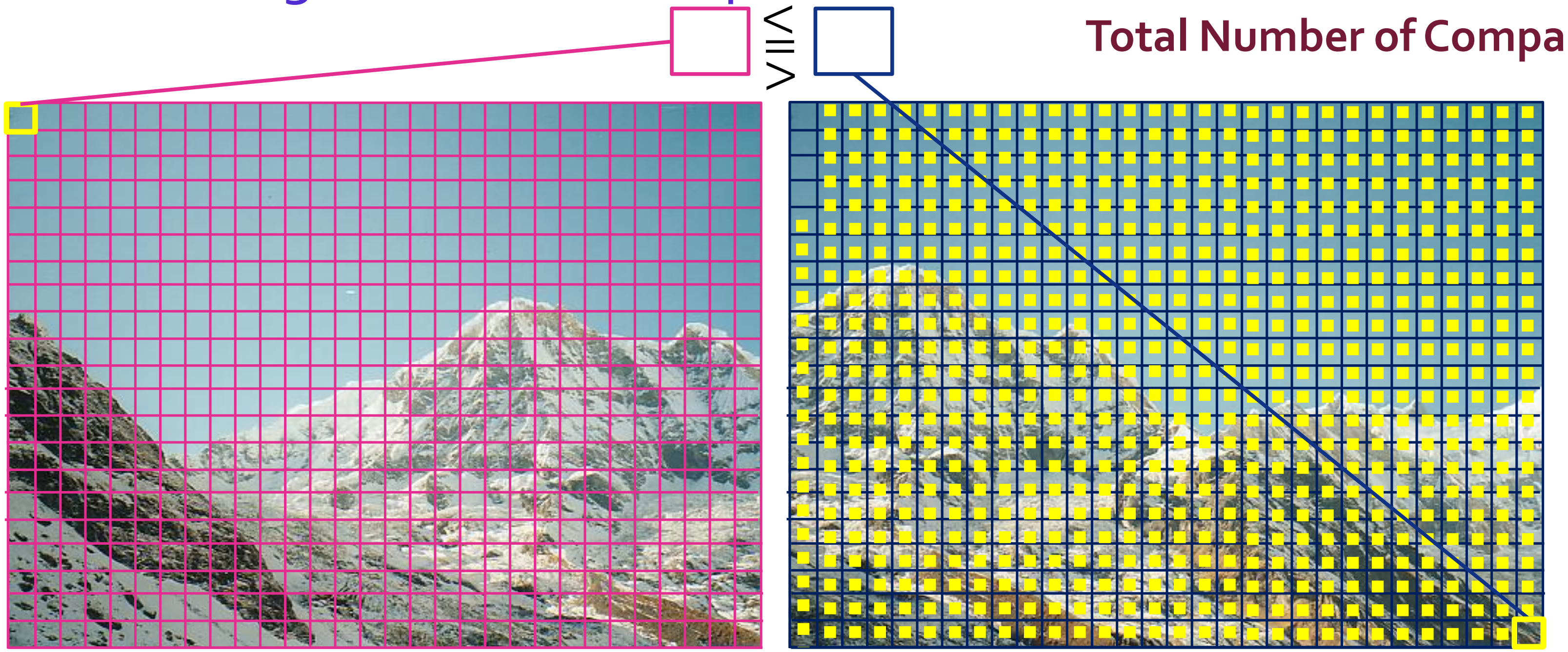


- If left image contains N_1 pixels and the right image consists of N_2 pixels:

For each pixel of the first image we need N_2 comparisons

Total Number of Comparisons= $N_1 N_2$

Template Matching

Naïve Way:

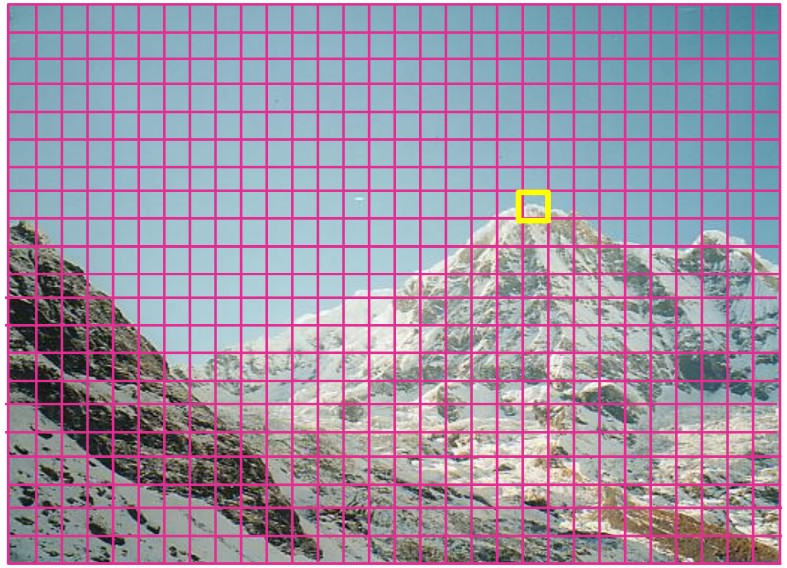
1. Compare all pixels' intensity of the first image with all intensities of the second image

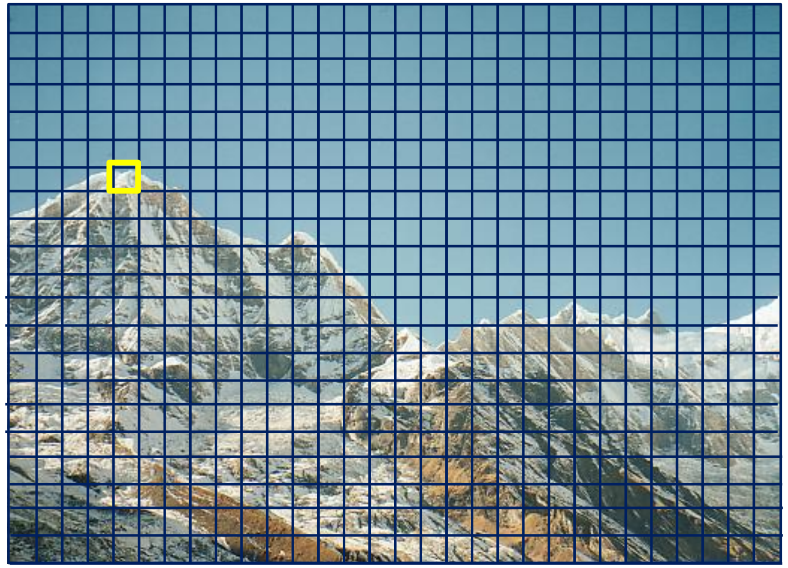
Will it work?

Let's see…

2. Let's figure out if we can correctly match two images.

For simplicity, we select a small patch of both images





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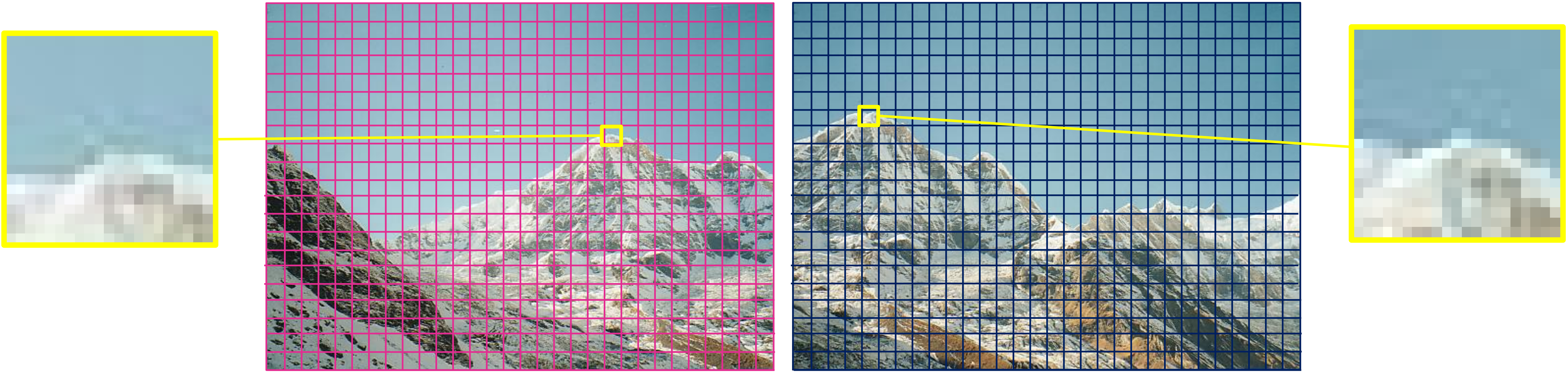

Template Matching

Naïve Way:

1. Compare all pixels' intensity of the first image with all intensities of the second image

Will it work?

Let's see…

2. Let's figure out if we can correctly match two images.

For simplicity, we select a small patch of both images

Notice that two patches are visually very similar

```
I1 = imread('Images\mountain1.png');
I2 = imread('Images\mountain2.png');

rad = 20;
croppedI1 = I1(123-rad:123+rad,325-rad:325+rad,:);
croppedI2 = I2(103-rad:103+rad,75-rad:75+rad,:);

imtool(rgb2gray(croppedI1));
imtool(rgb2gray(croppedI2));
```

Template Matching

179	179	178	178	178	177	177	178	178	178	177	177	176	176	177	177	177	176	176	176	175
179	179	179	178	178	178	177	178	178	178	177	177	177	176	177	177	177	177	176	176	176
180	179	179	179	178	178	178	179	179	178	178	177	177	177	178	178	177	177	177	176	176
180	180	179	179	179	178	178	179	179	179	178	178	177	177	178	178	178	177	177	177	176
180	180	180	179	179	179	179	180	179	179	178	178	178	178	179	178	178	178	177	177	177
181	180	180	180	179	179	179	180	180	179	179	178	178	178	179	179	178	178	178	177	177
181	181	180	180	179	179	179	179	180	181	179	178	178	179	180	179	179	177	177	176	177
181	181	182	181	180	179	179	177	178	182	184	178	176	180	183	181	179	178	175	173	177
177	179	180	182	182	182	185	186	174	177	185	180	173	175	175	177	179	181	179	176	173
182	183	183	183	183	182	181	181	184	180	178	180	183	182	179	178	176	174	175	177	178
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182	183	182	181	181	180	182	182	171	180	193	186	177	184	180	179	177	172	178	187	185
184	185	185	186	185	185	185	176	167	192	221	219	210	221	228	221	200	178	173	178	175
178	177	177	176	175	173	174	191	219	232	225	218	219	219	234	238	234	221	218	215	198
219	226	214	213	206	194	205	239	245	231	212	219	211	208	190	201	204	236	224	233	234
246	250	235	230	229	232	242	250	236	214	193	203	189	170	166	176	178	230	223	229	231
230	232	226	217	226	242	249	247	228	213	194	209	204	170	188	193	182	218	210	213	232
212	219	231	225	233	244	241	236	225	224	203	221	236	210	205	213	192	192	170	167	199
[illegible]	[illegible]	[illegible]	237	243	242	231	227	226	228	199	203	223	215	206	221	200	186	171	158	173
[illegible]	[illegible]	[illegible]	236	241	237	230	231	234	236	209	199	209	214	216	222	192	184	193	176	181
[illegible]	[illegible]	[illegible]	243	243	239	239	230	232	235	224	212	204	213	217	211	187	192	203	163	176

158	161	160	160	159	159	158	157	158	160	160	160	160	160	160	160	162	161	161	160	160
159	161	160	159	159	159	158	158	158	161	161	161	161	161	161	161	162	161	161	160	160
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167	164	163	162	160	161	162	164	161	163	170	165	160	165	167	163	162	162	162	163	163
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168	165	166	167	168	167	164	162	164	164	156	163	169	164	163	168	164	164	163	162	162
156	153	155	158	162	165	166	167	166	162	158	161	162	157	163	174	164	164	164	164	164
154	163	162	159	158	158	159	162	155	160	187	185	165	158	160	159	162	163	165	165	165
238	206	192	182	167	153	154	163	169	201	237	231	202	194	189	179	164	162	164	167	167
227	223	209	212	198	172	203	246	251	239	205	204	210	216	216	242	197	167	158	166	167
236	229	207	218	240	248	253	244	222	198	183	210	219	203	205	225	232	201	181	189	172
222	219	212	229	243	241	243	232	214	192	188	203	187	166	185	183	198	225	228	228	190
205	212	223	245	250	238	230	222	215	196	198	198	160	145	181	160	161	211	214	224	221
210	235	235	233	229	225	225	226	219	188	191	214	187	163	190	175	174	219	217	226	227
239	238	242	242	233	226	219	221	222	190	184	229	218	188	196	197	165	185	[illegible]	200	230
233	235	239	231	228	231	231	236	234	202	188	218	204	197	207	211	153	[illegible]	[illegible]	[illegible]	[illegible]
227	218	217	231	220	228	226	227	225	205	186	201	205	197	196	198	151	[illegible]	[illegible]	[illegible]	[illegible]
220	228	238	252	226	202	183	178	188	229	210	166	170	208	212	189	169	[illegible]	[illegible]	[illegible]	[illegible]

Using the toolbar button “inspect Pixel Values” in imtool, you can see the pixel values

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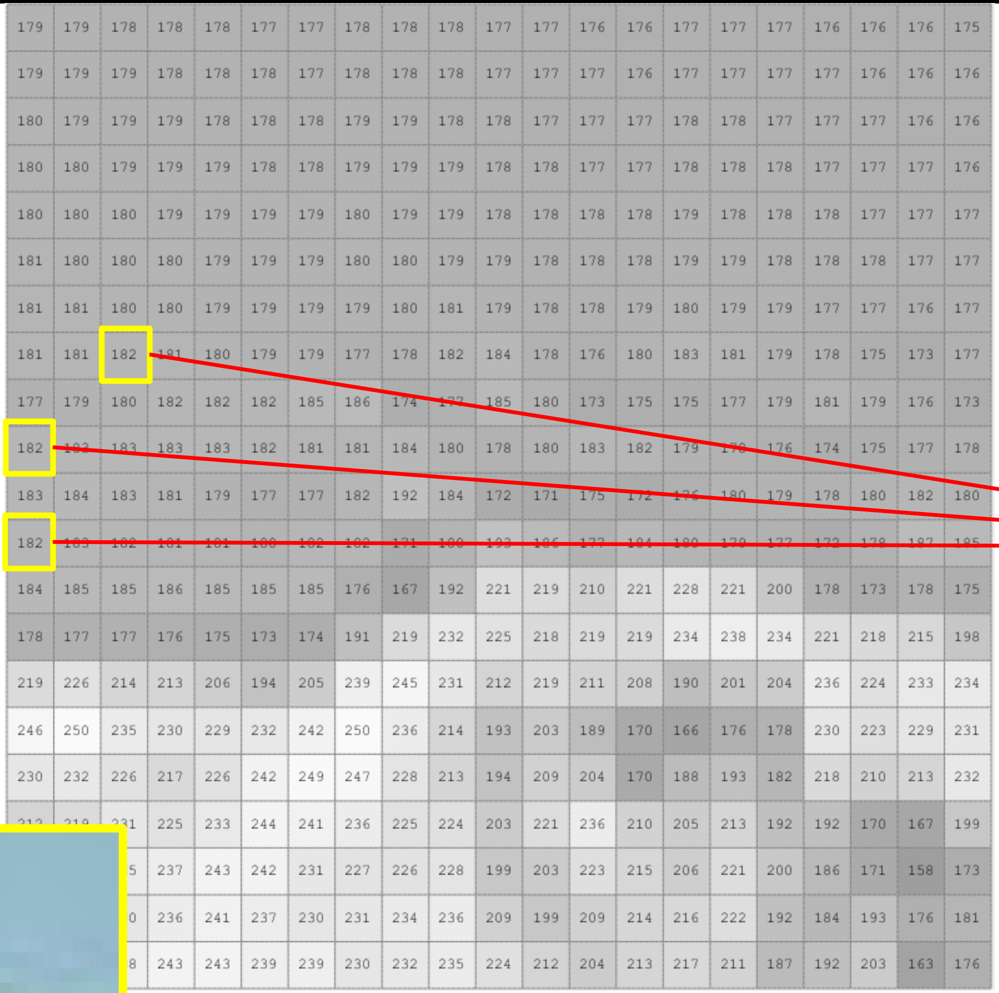
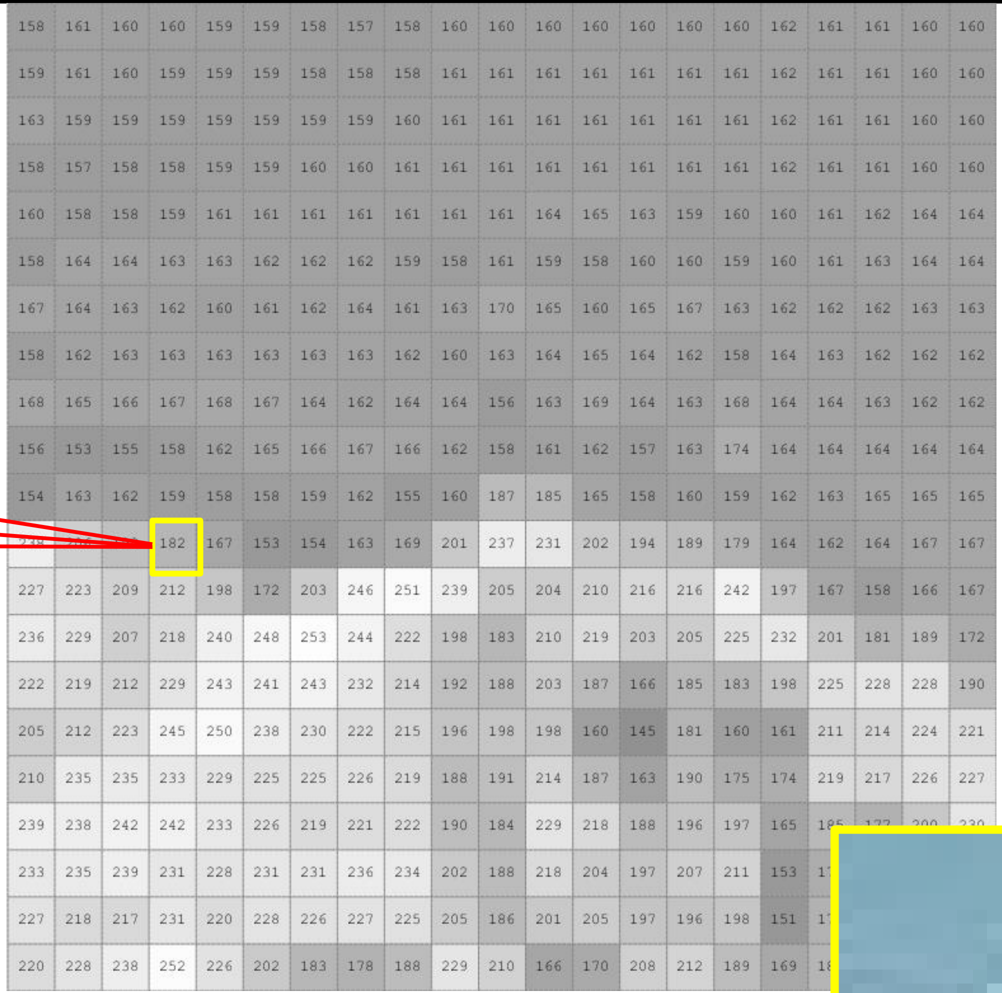
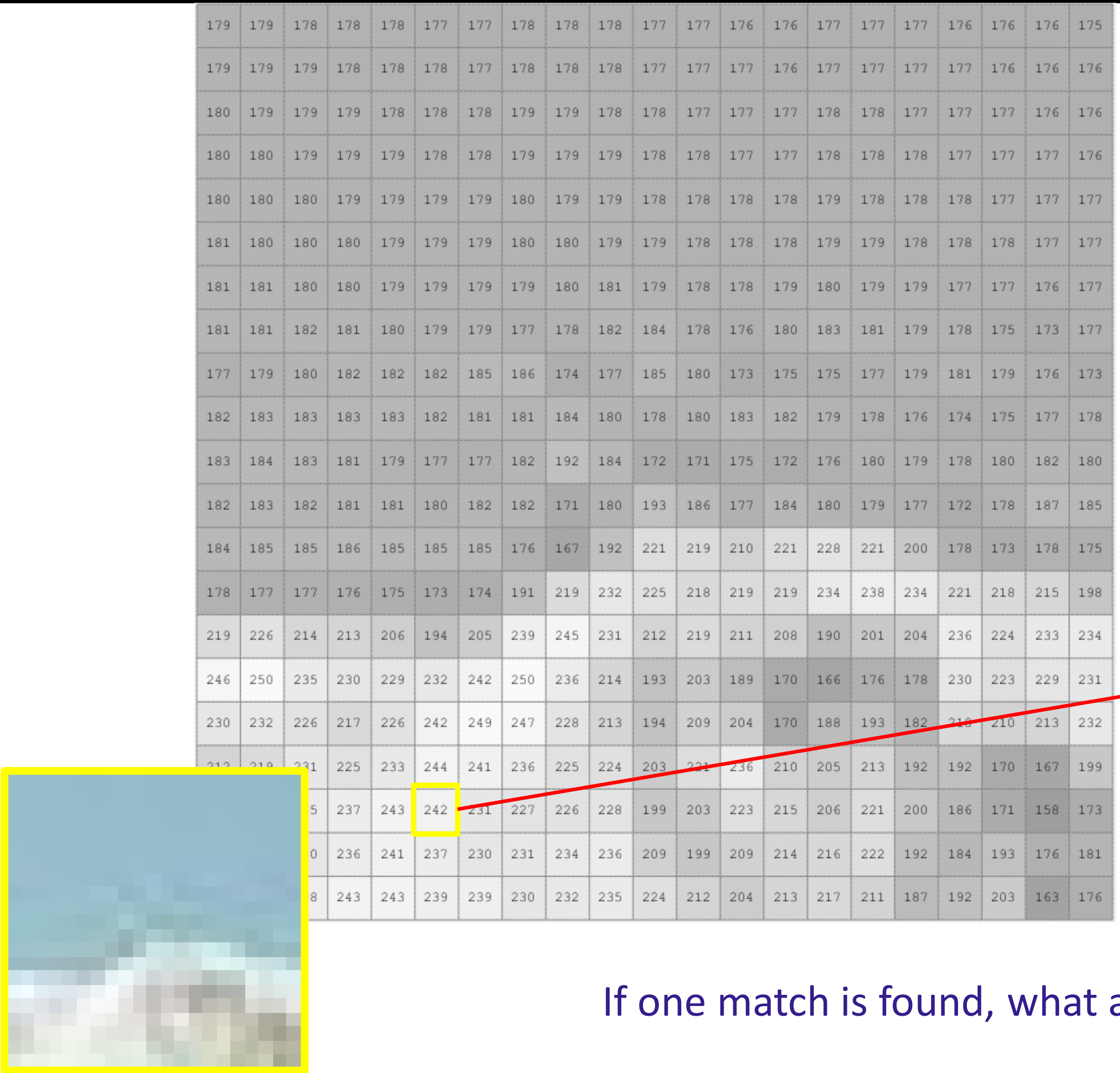
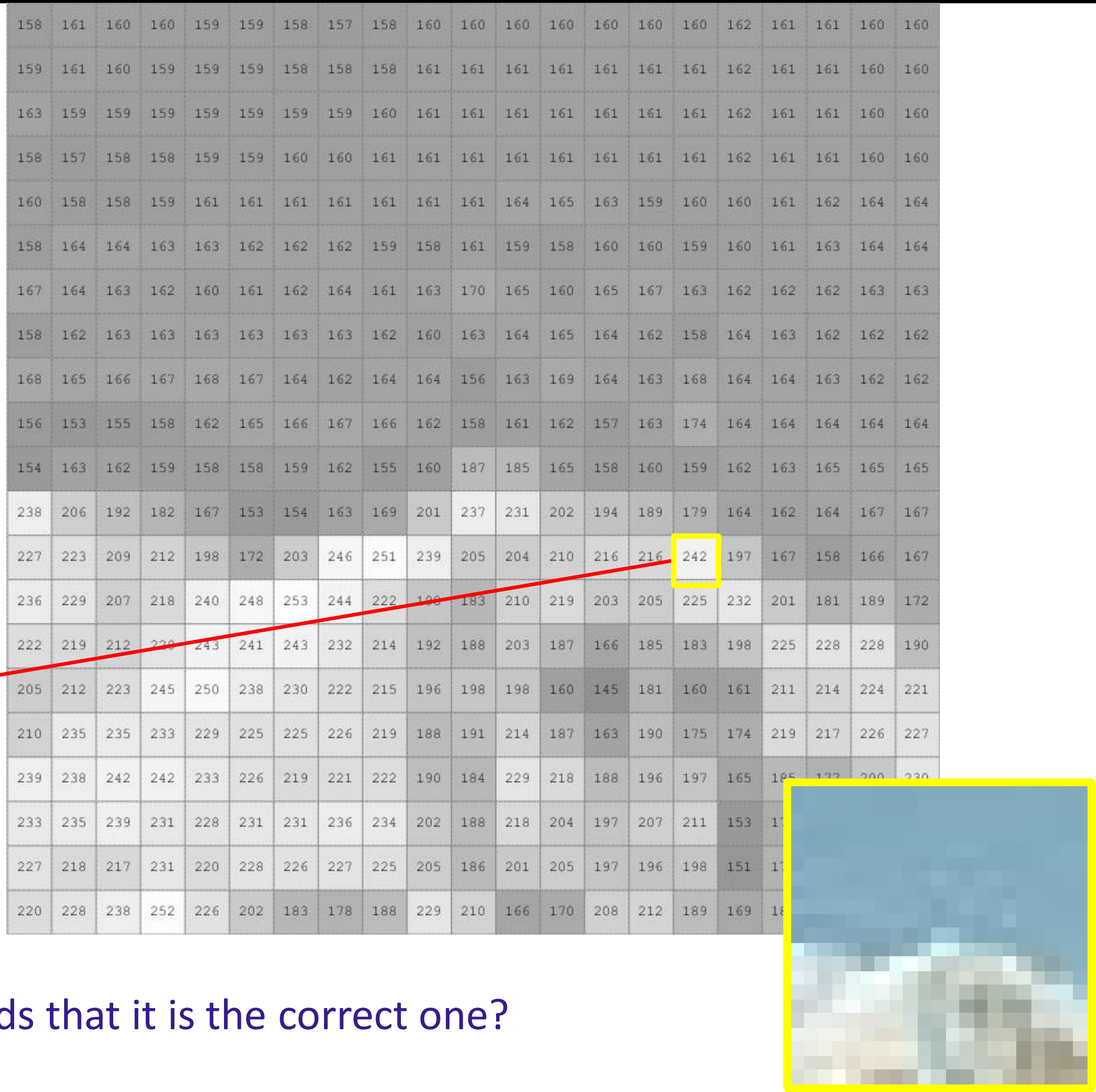




A lot of matches will be found. Which one is the correct one?

Template Matching





If one match is found, what are the odds that it is the correct one?

Template Matching

Naïve Way:

1. Compare all pixels' intensity of the first image with all intensities of the second image

Will it work?

Let's see…

No, It won't work! Bad Idea!

2. Let's divide the images into blocks and compare each block to the blocks of the other image.
   - This is a sample of exhaustive search for a specific block of the first image over all blocks of the second image
   - For doing this, we need a strategy to compare blocks
   - This approach sometimes is called Template Matching or Window Matching

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Block Comparisons

How can we find out which two blocks of pixels are similar?

Block (patch) Similarity Measures (Metrics)

To calculate a numerical value that indicates in what degree two blocks are similar, we can use:

1. Pixel-based distance

The simplest way is to directly calculate the distance between corresponding pixels of each block by subtracting them from each other. Assuming that both blocks have the same width (w) and height (h):

$$D(B_1, B_2) = \sum_{i=1}^{w} \sum_{j=1}^{h} |B_1(i,j) - B_2(i,j)|$$

Or Sum of Squared Differences

$$SSD(B_1, B_2) = \sum_{i=1}^{w} \sum_{j=1}^{h} (B_1(i,j) - B_2(i,j))^2$$

Having lower SSD score means higher similarity.

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Block Comparisons

How can we find out which two blocks of pixels are similar?

Block (patch) Similarity Measures

To calculate a numerical value that indicates in what degree two blocks are similar, we can use:

2. Cross Correlation

Calculate the Correlation score as follows:

$$C(B_1, B_2) = \sum_{i=1}^{w} \sum_{j=1}^{h} B_1(i,j) B_2(i,j)$$

Having higher correlation score means higher similarity.

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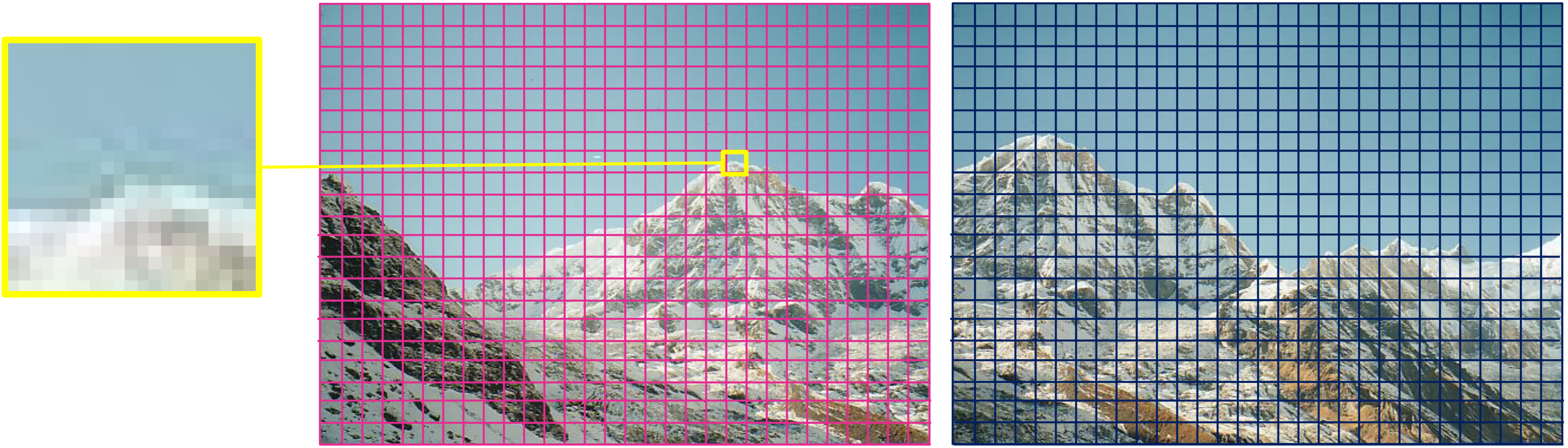
Block Comparisons

We search for the selected block from the left image in the right image using two similarity measures, namely the Pixel-based Distance and Correlation.

Let's code!



Template Matching

Block Comparisons

Let's code!

```
clc;
close all;
I1rgb = imread('Images\mountain1.png');
I2rgb = imread('Images\mountain2.png');

I1 = im2double(rgb2gray(I1rgb));
I2 = im2double(rgb2gray(I2rgb));

% rad is 10 so the width of the block is 2*10+1=21
rad = 10;
w = 2*rad+1; % here width = height
% Reading a block from the reference image
B1x = 325;
B1y = 123;
B1 = I1(B1y-rad:B1y+rad,B1x-rad:B1x+rad);
% Padding zeros to I2
I2 = padarray(I2,[rad,rad]);
[R2,C2] = size(I2);
C_Scores = zeros(R2,C2);
SSD_Scores = zeros(R2,C2);
```

Reading Images and converting to grayscale double matrices

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Block Comparisons

Let's code!

```
clc;
close all;
I1rgb = imread('Images\mountain1.png');
I2rgb = imread('Images\mountain2.png');

I1 = im2double(rgb2gray(I1rgb));
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B1 = I1(B1y-rad:B1y+rad,B1x-rad:B1x+rad);
% Padding zeros to I2
I2 = padarray(I2,[rad,rad]);
[R2,C2] = size(I2);
C_Scores = zeros(R2,C2);
SSD_Scores = zeros(R2,C2);
```

Selecting the a block from the reference image.
We know where the block is.
rad is the width of the block.

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Block Comparisons

Let's code!

```
clc;
close all;
I1rgb = imread('Images\mountain1.png');
I2rgb = imread('Images\mountain2.png');

I1 = im2double(rgb2gray(I1rgb));
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I2 = padarray(I2,[rad,rad]);
[R2,C2] = size(I2);
C_Scores = zeros(R2,C2);
SSD_Scores = zeros(R2,C2);
```

14	10	32	25
8	16	0	9
7	6	2	55
88	54	25	32

Image

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	14	10	32	25	0	0
0	0	8	16	0	9	0	0
0	0	7	6	2	55	0	0
0	0	88	54	25	32	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

After padding with 2 rows and 2 columns

To move a block on the second image, we need to pad the image with zeros.
Initialize two matrices for scores.

Block Comparisons

Let's code!

```
% Searching for B1 in I2
for x = rad +1:C2-rad
    for y = rad+1:R2-rad
        B2 = I2(y-rad:y+rad,x-rad:x+rad);
        SSD_Scores(y,x) = SSDScore(B1,B2);
        C_Scores(y,x) = CorrelationScore(B1,B2);
    end
end
% Converting scores to the original coords
SSD_Scores = SSD_Scores(rad+1:R2-rad,rad+1:C2-rad);
C_Scores = C_Scores(rad+1:R2-rad,rad+1:C2-rad);
I2 = I2(rad+1:R2-rad,rad+1:C2-rad);
[R2,C2] = size(I2);
% Finding the maximum score
[SSDmin,SSDminInd] = min(SSD_Scores(:));
[Cmx,CmxInd] = max(C_Scores(:));
[ySSD,xSSD] = ind2sub([R2,C2],SSDminInd);
[yC,xC] = ind2sub([R2,C2],CmxInd);
```

```
function SSD = SSDScore(B1,B2)
   SSD = sum((B1(:)-B2(:)).^2);
end
function C = CorrelationScore(B1,B2)
   C = sum(B1(:).*B2(:));
end
```

The main loop of the code that goes over all blocks of the second image and calculates the scores based on the SSD and Correlation metrics.

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Block Comparisons

Let's code!

```
% Searching for B1 in I2
for x = rad +1:C2-rad
    for y = rad+1:R2-rad
        B2 = I2(y-rad:y+rad,x-rad:x+rad);
        SSD_Scores(y,x) = SSDScore(B1,B2);
        C_Scores(y,x) = CorrelationScore(B1,B2);
    end
end
% Converting scores to the original coords
SSD_Scores = SSD_Scores(rad+1:R2-rad,rad+1:C2-rad);
C_Scores = C_Scores(rad+1:R2-rad,rad+1:C2-rad);
I2 = I2(rad+1:R2-rad,rad+1:C2-rad);
[R2,C2] = size(I2);
% Finding the maximum score
[SSDmin,SSDminInd] = min(SSD_Scores(:));
[Cmx,CmxInd] = max(C_Scores(:));
[ySSD,xSSD] = ind2sub([R2,C2],SSDminInd);
[yC,xC] = ind2sub([R2,C2],CmxInd);
```

```
function SSD = SSDScore(B1,B2)
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end
function C = CorrelationScore(B1,B2)
   C = sum(B1(:).*B2(:));
end
```

Converting the score matrices back to the original dimensions.

Block Comparisons

Let's code!

```
% Searching for B1 in I2
for x = rad +1:C2-rad
    for y = rad+1:R2-rad
        B2 = I2(y-rad:y+rad,x-rad:x+rad);
        SSD_Scores(y,x) = SSDScore(B1,B2);
        C_Scores(y,x) = CorrelationScore(B1,B2);
    end
end
% Converting scores to the original coords
SSD_Scores = SSD_Scores(rad+1:R2-rad,rad+1:C2-rad);
C_Scores = C_Scores(rad+1:R2-rad,rad+1:C2-rad);
I2 = I2(rad+1:R2-rad,rad+1:C2-rad);
[R2,C2] = size(I2);
% Finding the maximum score
[SSDmin,SSDminInd] = min(SSD_Scores(:));
[Cmx,CmxInd] = max(C_Scores(:));
[ySSD,xSSD] = ind2sub([R2,C2],SSDminInd);
[yC,xC] = ind2sub([R2,C2],CmxInd);
```

```
function SSD = SSDScore(B1,B2)
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end
```

Finding the most similar block by finding the maximum correlation score and minimum SSD score.

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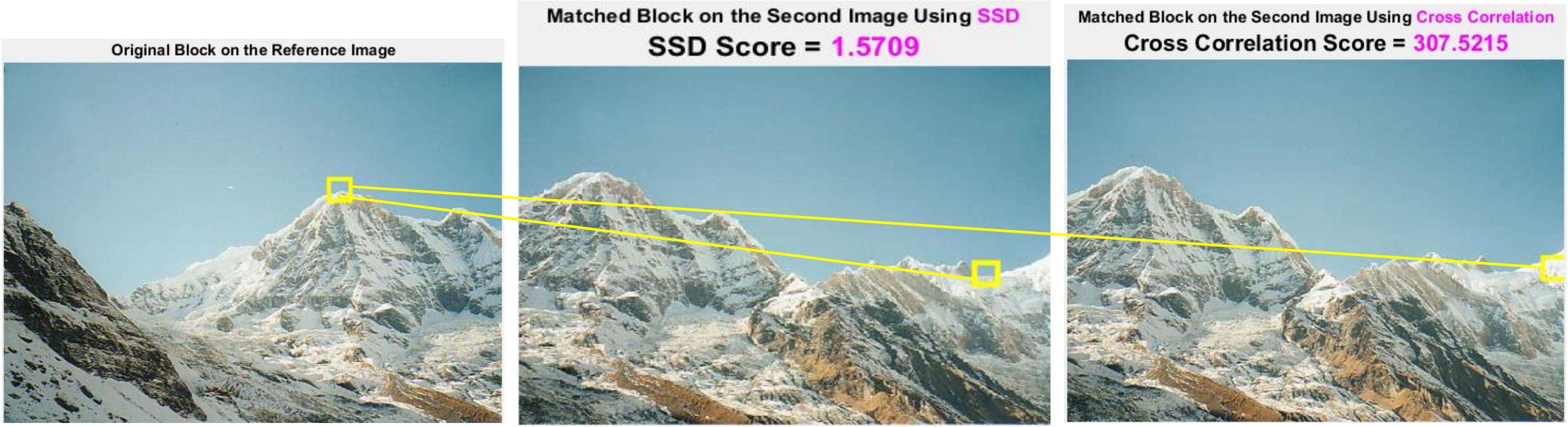
Block Comparisons

Let's run it!



Both measures failed! ☹

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Block Comparisons

Why did it fail?

1. Even if the images have been taken by the same model of camera, the gain and sensitivity still varies.
2. Overall radiance of surfaces varies in time due to air molecules, wind, clouds, … that creates images with different illumination level.
3. Noise plays a crucial role.

Block Comparisons

What is the solution?

Intensity Normalization

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MATH REVIEW: *Statistics – Standard Score*

- Normalization in Statistics conveys a range of meanings.
- Usually, we use normalization when we want to adjust vectors of values that are measured on different scales to a common scale.
- *Standard Score or z-score is a way of normalization when the populations are available.*
- *Z-score is a dimensionless quantity.*
- *We need to know the mean and standard deviation of the complete population.*
- For a raw vector x we can calculate the *z-score* as follows:

$$z = \frac{\mathbf{x} - \mu}{\sigma}$$

- $\mu = \frac{1}{n}\left(\sum_{i=1}^{n} x(i)\right)$ is the mean of the population
- $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x(i) - \mu)^2}$ is the standard deviation of the population that quantifies the amount of variation of elements of the vector x

Block Comparisons

What is the solution?

Intensity Normalization

If B is a block of image (matrix of intensities), we calculate the z-scores for each pixels as:

$$\mathrm{z}(x, y) = \frac{\mathbf{B}(x,y) - \mu}{\sigma}$$

The normalized scores are then calculated as:

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h}(\mathrm{z}_1(i,j) - \mathrm{z}_2(i,j))^2$$

Normalized Cross Correlation (NCC):

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h}\mathrm{z}_1(i,j)\ \mathrm{z}_2(i,j)$$

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Block Comparisons

- For simplicity of writing equations, we convert the matrices to vector, so we need only one summation notation.
- In Mathematics specially in Linear Algebra, this process is called Vectorization which converts a matrix into a column vector.

$$\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \begin{pmatrix} a & b & c & d & e & f & g & h & i \end{pmatrix}^T$$

Therefore, we can use only one $\sum$ in our equations to traverse all the element of the matrix.

$$\sum_{i=1}^{w} \sum_{j=1}^{h} \mathbf{B}(i,j) \rightarrow \sum_{i} \mathbf{B}(i)$$

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Block Comparisons

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mathbf{B}_1(i)-\mu_1}{\sigma_1}-\frac{\mathbf{B}_2(i)-\mu_2}{\sigma_2}\right)^2$$

Bring n inside the parenthesis:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\mathbf{B}_1(i)-\mu_1}{\sqrt{n}\sigma_1}-\frac{\mathbf{B}_2(i)-\mu_2}{\sqrt{n}\sigma_2}\right)^2$$

Insert the standard deviation equation:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\mathbf{B}_1(i)-\mu_1}{\sqrt{n}\sqrt{\frac{1}{n}\sum_{j=1}^{n}(\mathbf{B}_1(j)-\mu_1)^2}}-\frac{\mathbf{B}_2(i)-\mu_2}{\sqrt{n}\sqrt{\frac{1}{n}\sum_{j=1}^{n}(\mathbf{B}_2(j)-\mu_2)^2}}\right)^2$$

ns in the denominators are cancelled out.

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Block Comparisons

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\mathbf{B}_1(i) - \mu_1}{\sqrt{\sum_{j=1}^{n} (\mathbf{B}_1(j) - \mu_1)^2}} - \frac{\mathbf{B}_2(i) - \mu_2}{\sqrt{\sum_{j=1}^{n} (\mathbf{B}_2(j) - \mu_2)^2}} \right)^2$$

Substituting $\overline{\mathbf{B}} = \mathbf{B} - \mu$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_1(j)^2}} - \frac{\overline{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_2(j)^2}} \right)^2$$

Block Comparisons

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Bring n inside the parenthesis:

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \sum_{i=1}^{n}\left(\frac{\mathbf{B}_1(i) - \mu_1}{\sqrt{n}\sigma_1}\right)\left(\frac{\mathbf{B}_2(i) - \mu_2}{\sqrt{n}\sigma_2}\right)$$

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Block Comparisons

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MATH REVIEW: *Linear Algebra*

Vector

Vector is often used in Geometry and Physics to represent various quantities, such as force, speed, movement, acceleration and etc.

A vector has a:

- Length
- Direction

We show a vector by a **lower case boldfaced** character.

Examples.

$$\mathbf{v_1} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \text{ is a vector in } \Re^2$$

$$\mathbf{v_2} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} \text{ is a vector in } \Re^3$$

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MATH REVIEW: *Linear Algebra*

Vector

$$\mathbf{v}_{\boldsymbol{n}} = \begin{pmatrix} 2 \\ \vdots \\ -4 \end{pmatrix}_{1\times n}$$ is a column vector in $\Re^n$

Transpose of a Vector

A column vector can be expressed as a row vector by using the transpose:

$$\mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} \rightarrow \mathbf{v}^T = \begin{pmatrix} 2 & 0 & -4 \end{pmatrix}$$

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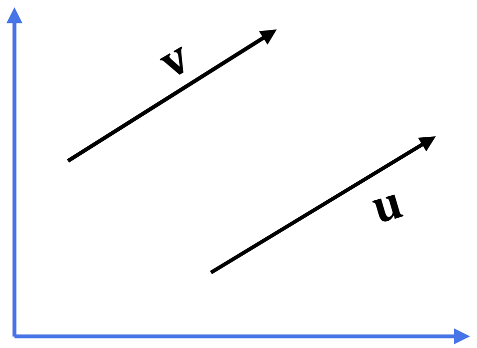
MATH REVIEW: *Linear Algebra* from: (Hughes & Foley, 2014)

Vector – How to think about vectors

- Usually students see vectors as arrows.
- What if we ask about these two vectors? Are these vectors equal?
- Although these arrows are in different places and are clearly different, in terms of vector representation, they are representing the same vector.



- Therefore, to have a better understanding about vectors, it is better to see them as a **displacement:**

  It represents an amount by which you must move to get from one place to another.

For example, to get from the point $(3,1)$ to point $(5,0)$ you must move by 2 in x direction and by -1 in y direction. So, this displacement is represented by vector $[2 \quad -1]^T$

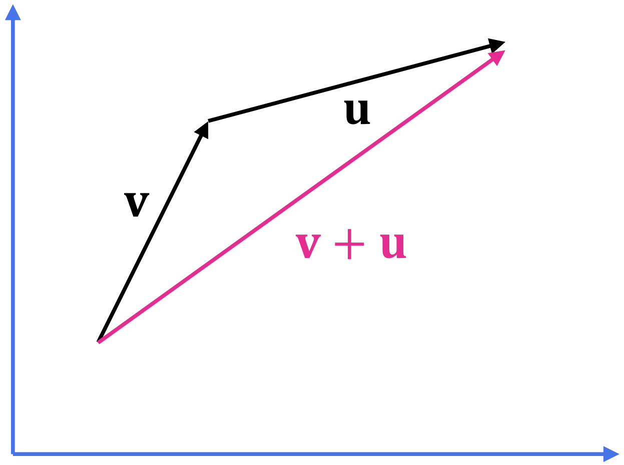
- Now, we can see that both vectors depicted in the figure are representing the same displacement, so they are equivalent.

MATH REVIEW: *Linear Algebra*

Vector Addition

$$\mathbf{v} + \mathbf{u} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} v_1 + u_1 \\ v_2 + u_2 \end{pmatrix}$$

Geometric representation



MATH REVIEW: *Linear Algebra*

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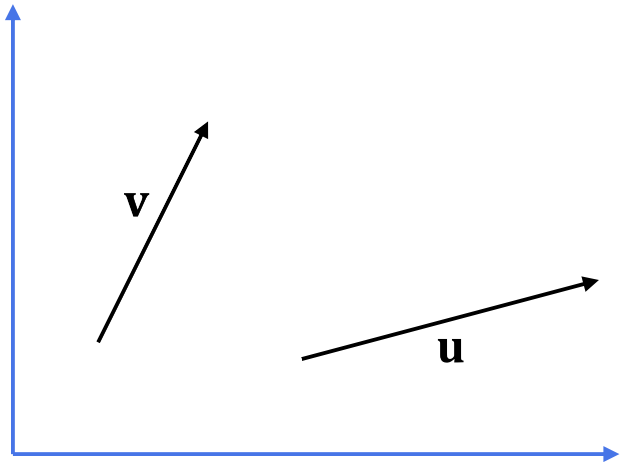
Geometric representation

If the vectors are not attached:

- Note that for vectors only length and direction properties are important.
- Therefore, we can shift vectors in a way that both start from the same location → Origin



MATH REVIEW: *Linear Algebra*

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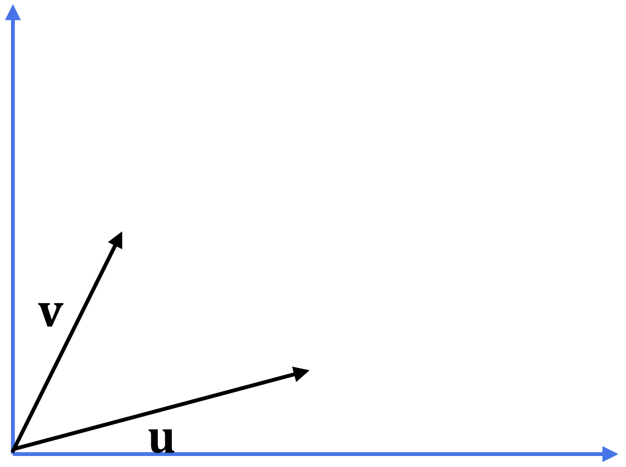

- Using the so-called parallelogram law we obtain the sum of two vectors.

MATH REVIEW: *Linear Algebra*

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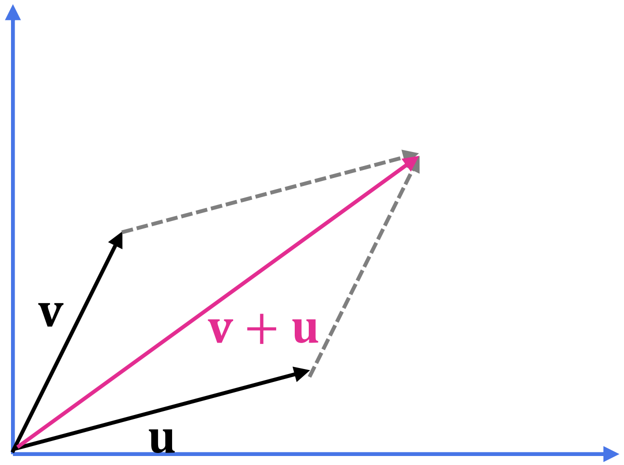

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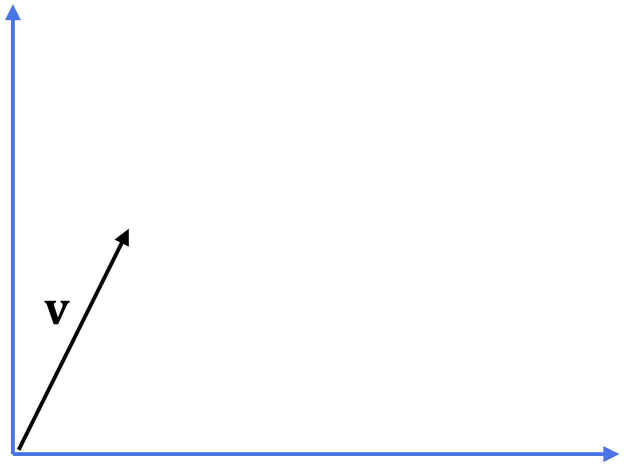
MATH REVIEW: *Linear Algebra*

Scalar multiplication of Vectors

Vectors can be multiplied by a number:

$$s\mathbf{v} = s\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} sv_1 \\ sv_2 \end{pmatrix}$$

Geometric representation



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MATH REVIEW: *Linear Algebra*

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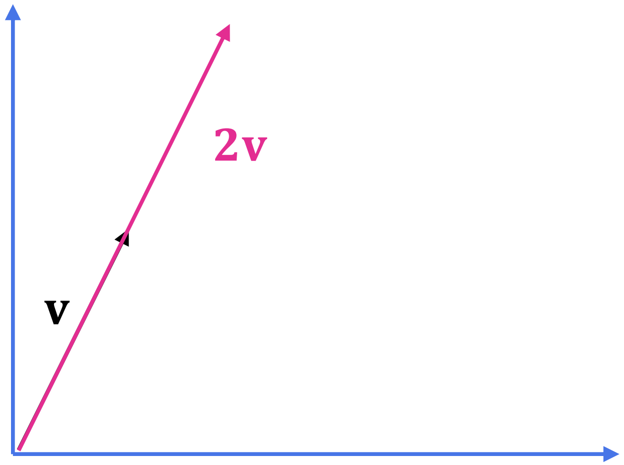


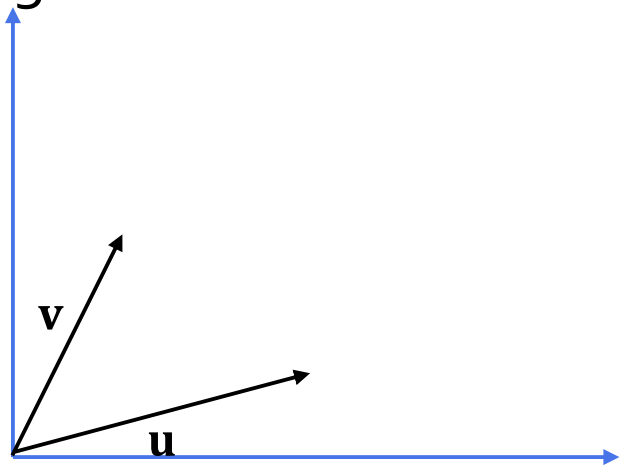
MATH REVIEW: *Linear Algebra*

Vector Subtraction

$$\mathbf{v} - \mathbf{u} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} v_1 - u_1 \\ v_2 - u_2 \end{pmatrix}$$

Geometric representation

We can add $(-\mathbf{u})$ to $\mathbf{v}$ using the parallelogram:



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MATH REVIEW: *Linear Algebra*

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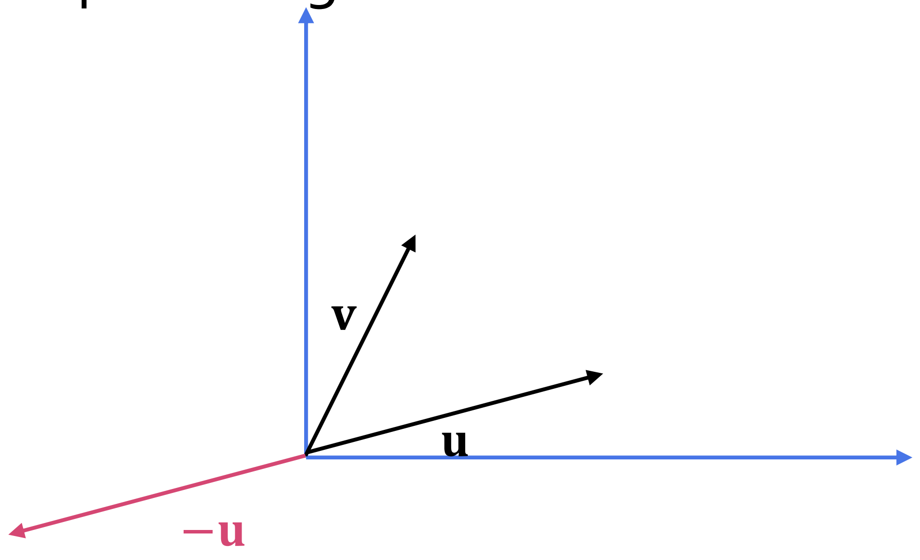

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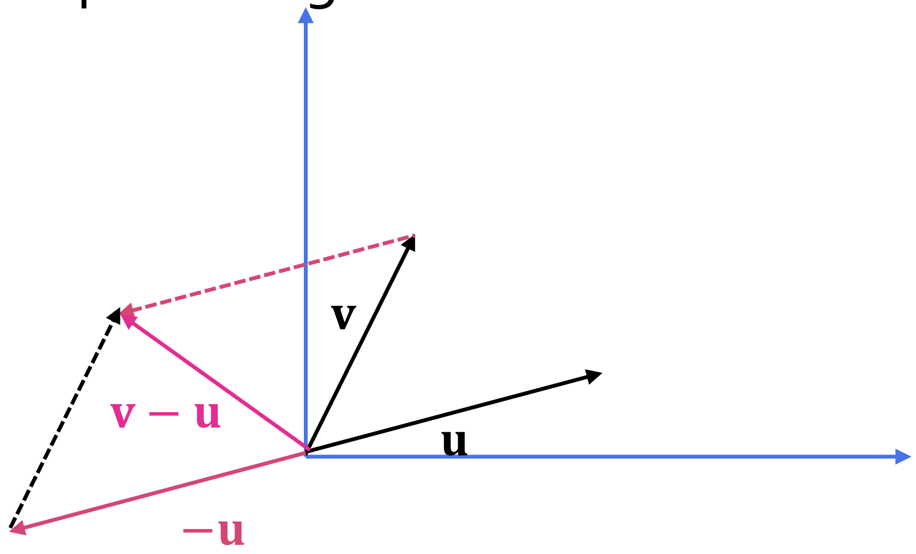
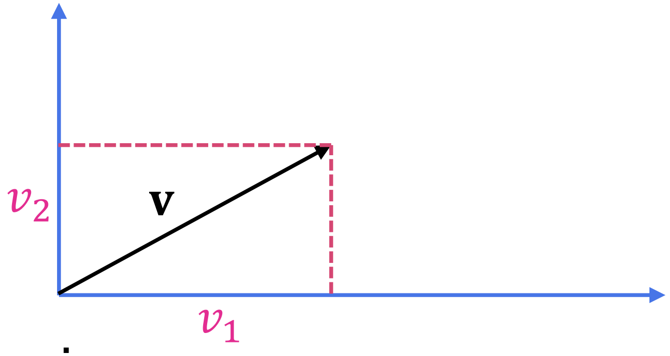


MATH REVIEW: *Linear Algebra*

Length of a Vector

The length of the vector $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ is calculated by the Pythagorean Theorem:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$$



- The length of the vector is also called the *norm* of the vector.
- It is denoted by double absolute values.
- If the vector has three components (vectors in 3D space) the norm is:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

- For n dimensional vectors:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

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MATH REVIEW: *Linear Algebra*

Length of a Vector

By definition, the norm satisfies the following conditions:

1. If $\mathbf{v} \neq 0$, $\|\mathbf{0}\| = 0 \rightarrow \|\mathbf{v}\| > 0$
2. For all scalar c and vectors $\mathbf{v}$, $\|c\mathbf{v}\| = |c|\|\mathbf{v}\|$
3. $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

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Length of a Vector - Recap

- There are numerous norms that are used in practice.
- In our work, the norm most often used is the so-called 2-norm, which, for a vector $\mathbf{v}$ in real $\Re^n$, space is defined as:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

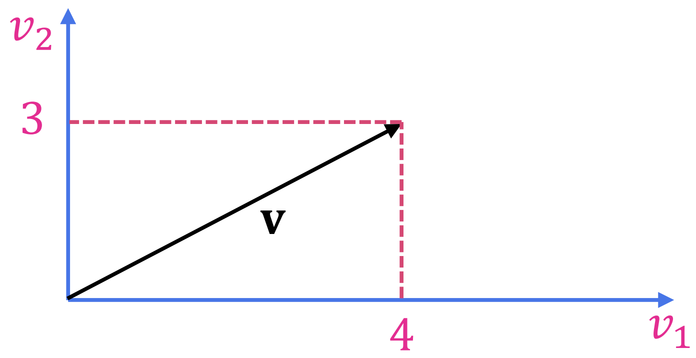
- Which is recognized as the Euclidean distance from the origin to point $\mathbf{v}$
- This gives the expression the familiar name Euclidean norm.
- The expression also is recognized as the length of a vector $\mathbf{v}$, with origin at point O.
- The norm also can be written as

$$\|\mathbf{v}\| = (\mathbf{v}^T\mathbf{v})^{\frac{1}{2}}$$

MATH REVIEW: *Linear Algebra*

Unit Vectors

- Unit vectors are vectors that their length is **1**.
- We denote the unit vector by a boldfaced lowercase letter with a hat $\hat{\mathbf{v}}$ (pronounced v hat)
- All non-zero vector can be decomposed into unit vectors
- For example, if we have a vector $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$
- We can say that $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 5 \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$



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MATH REVIEW: *Linear Algebra*

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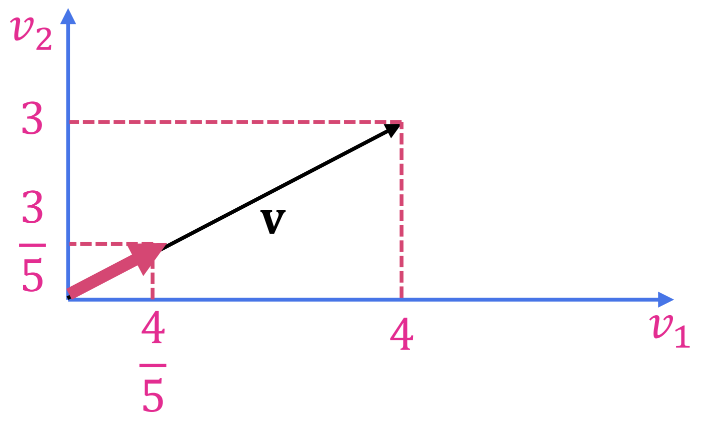

MATH REVIEW: *Linear Algebra*

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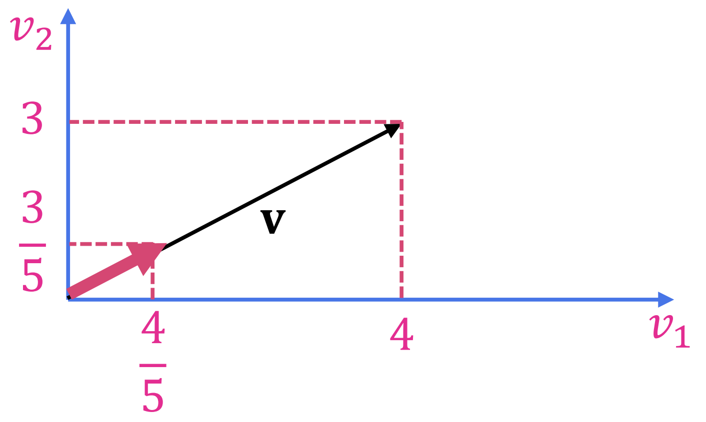
- What is the length of length vector $\hat{\mathbf{v}}$?



MATH REVIEW: *Linear Algebra*

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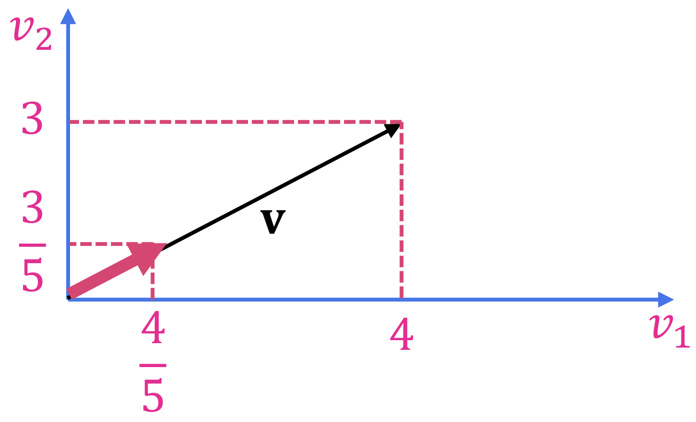

- What is the length of length vector $\hat{\mathbf{v}}$?

$$||\hat{\mathbf{v}}|| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\left(\frac{16}{25}\right) + \left(\frac{9}{25}\right)} = 1$$

MATH REVIEW: *Linear Algebra*

Unit Vectors

$$\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \rightarrow \|\mathbf{v}\| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 5 \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \rightarrow \hat{\mathbf{v}} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$$

- We can write the vector as a scalar product of its length and its unit vector
- Therefore, the unit vector can be obtained by:

$$\forall\, \mathbf{v} \neq 0 : \hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

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MATH REVIEW: *Linear Algebra*

Dot Product

- The Inner Product or Dot Product of two vectors is defined as:

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}_{1\times n} \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}_{1\times n}$$

$$\mathbf{v} \cdot \mathbf{u} = v_1 u_1 + v_2 u_2 + \cdots + v_n u_n = \sum_{i=1}^{n} v_i u_i$$

- The result of dot product of two vectors is a scalar.
- Dot Product is not defined, if the vectors have different number of components

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MATH REVIEW: *Linear Algebra*

Dot Product - Properties

- For all vectors $\mathbf{v}$, $\mathbf{u}$, and $\mathbf{w}$ of the same dimension and for all numbers c, we have:

Commutative:

$$\mathbf{v} \cdot \mathbf{u} = \mathbf{u} \cdot \mathbf{v}$$

Distributive over vector addition:

$$\mathbf{v} \cdot (\mathbf{u} + \mathbf{w}) = \mathbf{v} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{w}$$

Associative with respect to scalar multiplication:

$$\mathbf{v} \cdot (c\mathbf{u}) = (c\mathbf{v}) \cdot \mathbf{u} = c(\mathbf{u} \cdot \mathbf{v})$$

No cancellation:

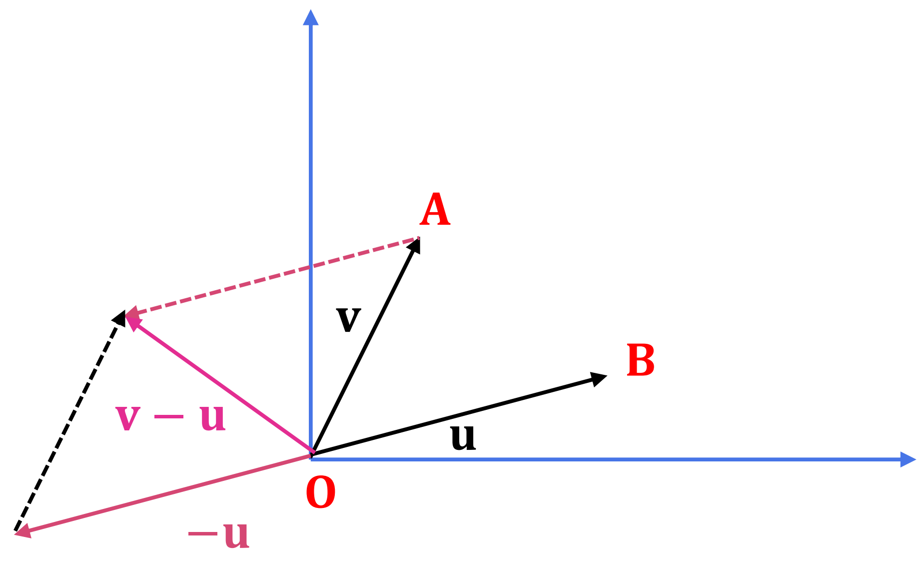
if $\mathbf{v} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{w}$ and $\mathbf{v} \neq \mathbf{0}$, you cannot cancel out $\mathbf{v}$ from both sides of the equation.

- Also, note that: $\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v}$

MATH REVIEW: *Linear Algebra*

Dot Product – Geometric Point of View

- The Distance between two vectors is obtained by finding the norm of their subtraction:
- Subtraction of the two vectors:



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MATH REVIEW: *Linear Algebra*

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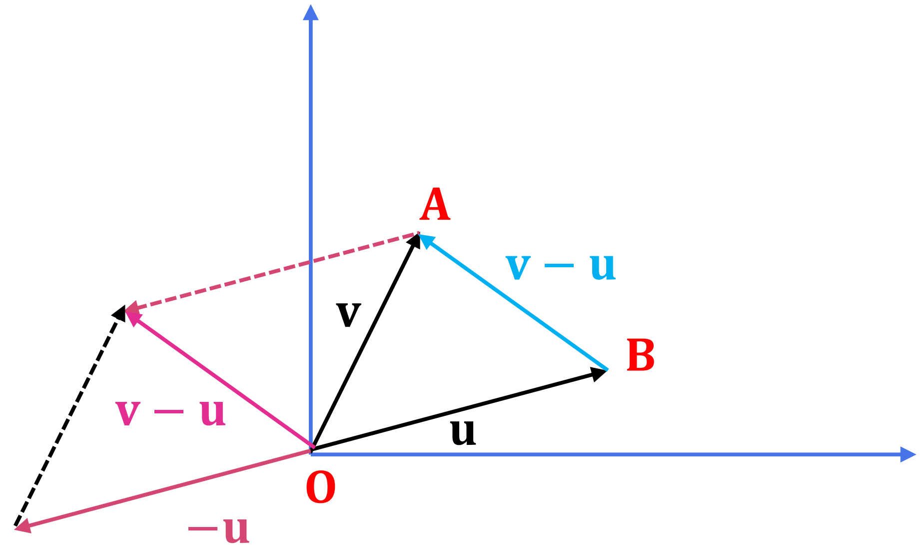

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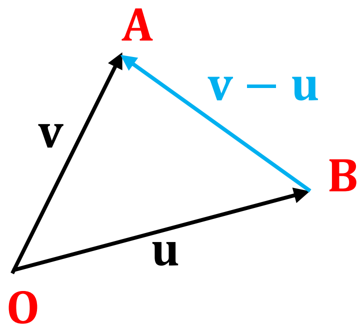

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MATH REVIEW: *Linear Algebra*

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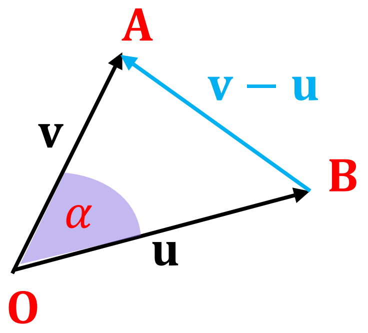
- **Cosine Law:**



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MATH REVIEW: *Linear Algebra*

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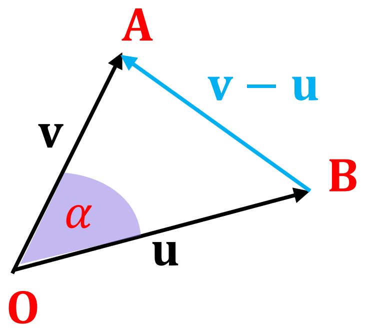
- Subtraction of the two vectors:
- **Cosine Law:**

$$AB^2 = OA^2 + OB^2 - 2\,.\,OA.\,OB.\cos(\alpha)$$

- $AB = \|\mathrm{v} - \mathrm{u}\|$
- $OA = \|\mathrm{v}\|$
- $OB = \|\mathrm{u}\|$



- Therefore:

$$\|\mathbf{v} - \mathbf{u}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\|\mathbf{v}\|\|\mathbf{u}\|cos(\alpha)$$

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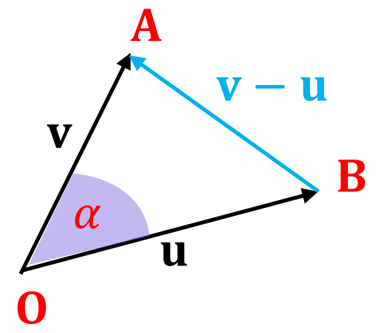
MATH REVIEW: *Linear Algebra*

Dot Product – Geometric Point of View



$$\|\mathbf{v}-\mathbf{u}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\|\mathbf{v}\|\|\mathbf{u}\|cos(\alpha) \qquad (1)$$

- From the properties of dot product, we know that $\|\mathbf{v}\|^2 = \mathbf{v}\cdot\mathbf{v}$

$$\rightarrow \|\mathbf{v}-\mathbf{u}\|^2 = (\mathbf{v}-\mathbf{u})\cdot(\mathbf{v}-\mathbf{u}) = \mathbf{v}\cdot\mathbf{v} - 2\mathbf{v}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{u}$$

$$\rightarrow \|\mathbf{v}-\mathbf{u}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\mathbf{v}\cdot\mathbf{u} \qquad (2)$$

- From (1) and (2) we have:

$$\|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\mathbf{v}\cdot\mathbf{u} = \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\|\mathbf{v}\|\|\mathbf{u}\|cos(\alpha)$$

- Therefore:

$$\mathbf{v}\cdot\mathbf{u} = \|\mathbf{v}\|\|\mathbf{u}\|cos(\alpha)$$

Block Comparisons

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\bar{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_1(j)^2}} - \frac{\bar{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_2(j)^2}} \right)^2$$

Normalized Cross Correlation (NCC):

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \sum_{i=1}^{n} \left(\frac{\bar{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_1(j)^2}} \right) \left(\frac{\bar{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_2(j)^2}} \right)$$

Where $\bar{\mathbf{B}} = \mathbf{B} - \mu$

Based on what we just learned in the Math Review:

Block Comparisons

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_1(j)^2}} - \frac{\overline{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_2(j)^2}} \right)^2$$

Normalized Cross Correlation (NCC):

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_1(j)^2}} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_2(j)^2}} \right)$$

Where $\overline{\mathbf{B}} = \mathbf{B} - \mu$

Based on what we just learned in the Math Review:

What is $\sqrt{\sum_{j=1}^{n} \overline{\mathbf{B}}_1(j)^2}$?

Block Comparisons

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\bar{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_1(j)^2}} - \frac{\bar{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_2(j)^2}} \right)^2$$

Normalized Cross Correlation (NCC):

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \sum_{i=1}^{n} \left(\frac{\bar{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_1(j)^2}} \right) \left(\frac{\bar{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_2(j)^2}} \right)$$

Where $\bar{\mathbf{B}} = \mathbf{B} - \mu$

Based on what we just learned in the Math Review:

$$\sqrt{\sum_{j=1}^{n} \bar{\mathbf{B}}_1(j)^2} = \|\bar{\mathbf{B}}_1\|$$

Block Comparisons

Replacing $||\overline{\mathbf{B}}_1||$ into the equations:

Normalized Sum of Squared Differences (NSSD):

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{||\overline{\mathbf{B}}_1||} - \frac{\overline{\mathbf{B}}_2(i)}{||\overline{\mathbf{B}}_2||} \right)^2$$

Normalized Cross Correlation (NCC):

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{||\overline{\mathbf{B}}_1||} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{||\overline{\mathbf{B}}_2||} \right)$$

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Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

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Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

$$\widehat{\mathbf{B}}_1 = \frac{\overline{\mathbf{B}}_1}{\|\overline{\mathbf{B}}_1\|}$$

Is the unit vector of $\overline{\mathbf{B}}$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i) \overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|}$$

Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

$$\widehat{\mathbf{B}}_1 = \frac{\overline{\mathbf{B}}_1}{\|\overline{\mathbf{B}}_1\|}$$

Is the unit vector of $\overline{\mathbf{B}}$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1, \mathbf{B}_2) = \left\|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\right\|^2}$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2}$$

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Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2) = \sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|}\right)^2$$

$$NCC(\mathbf{B}_1,\mathbf{B}_2) = \sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|}\right)\left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|}\right)$$

$$\widehat{\mathbf{B}}_1 = \frac{\overline{\mathbf{B}}_1}{\|\overline{\mathbf{B}}_1\|}$$

Is the unit vector of $\overline{\mathbf{B}}$

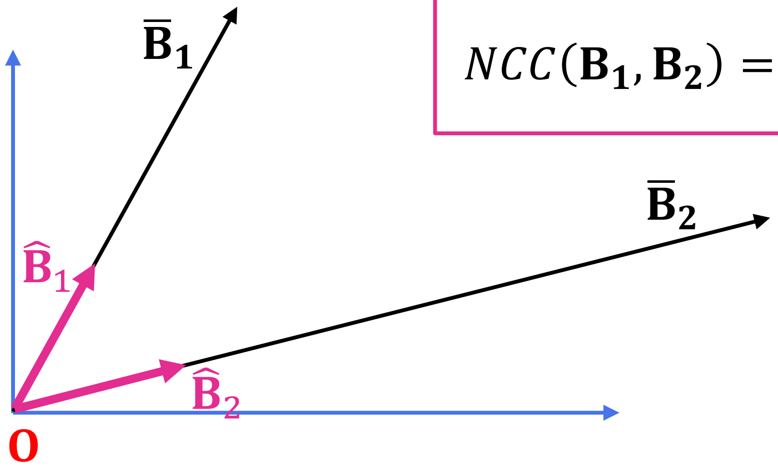
$$NCC(\mathbf{B}_1,\mathbf{B}_2) = \frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1,\mathbf{B}_2) = \left\|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\right\|^2}$$

$$\boxed{NCC(\mathbf{B}_1,\mathbf{B}_2) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2}$$

Suppose we have 2D vectors:



Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\bar{\mathbf{B}}_1(i)}{\|\bar{\mathbf{B}}_1\|}-\frac{\bar{\mathbf{B}}_2(i)}{\|\bar{\mathbf{B}}_2\|}\right)^2$$

$$NCC(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\bar{\mathbf{B}}_1(i)}{\|\bar{\mathbf{B}}_1\|}\right)\left(\frac{\bar{\mathbf{B}}_2(i)}{\|\bar{\mathbf{B}}_2\|}\right)$$

$$\hat{\mathbf{B}}_1=\frac{\bar{\mathbf{B}}_1}{\|\bar{\mathbf{B}}_1\|}$$

Is the unit vector of $\bar{\mathbf{B}}$

$$NCC(\mathbf{B}_1,\mathbf{B}_2)=\frac{\sum_{i=1}^{n}\bar{\mathbf{B}}_1(i)\bar{\mathbf{B}}_2(i)}{\|\bar{\mathbf{B}}_1\|\|\bar{\mathbf{B}}_2\|}$$

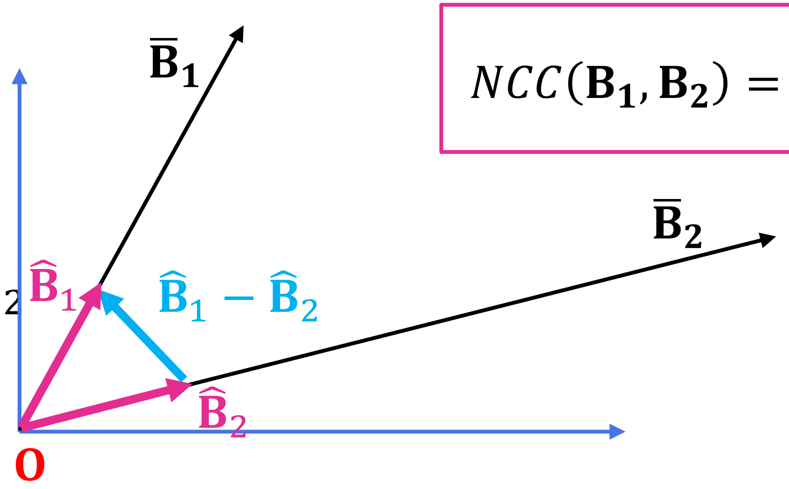
Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1,\mathbf{B}_2)=\|\hat{\mathbf{B}}_1-\hat{\mathbf{B}}_2\|^2}$$

$$\boxed{NCC(\mathbf{B}_1,\mathbf{B}_2)=\frac{\bar{\mathbf{B}}_1\cdot\bar{\mathbf{B}}_2}{\|\bar{\mathbf{B}}_1\|\|\bar{\mathbf{B}}_2\|}=\hat{\mathbf{B}}_1\cdot\hat{\mathbf{B}}_2}$$

Suppose we have 2D vectors:

- NSSD is the squared length (norm) of $\hat{\mathbf{B}}_1-\hat{\mathbf{B}}_2$



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Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

$$\widehat{\mathbf{B}}_1 = \frac{\overline{\mathbf{B}}_1}{\|\overline{\mathbf{B}}_1\|}$$

Is the unit vector of $\overline{\mathbf{B}}$

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i) \overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|}$$

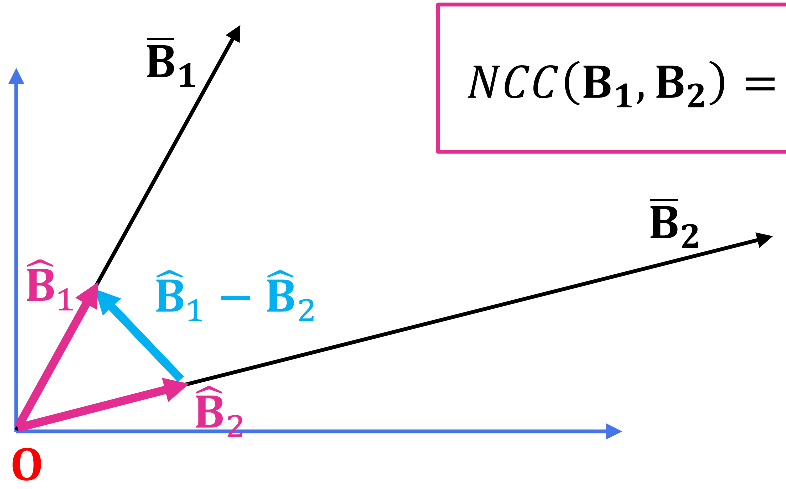
Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1, \mathbf{B}_2) = \left\|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\right\|^2}$$

$$\boxed{NCC(\mathbf{B_1}, \mathbf{B_2}) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2}$$

Suppose we have 2D vectors:

- What is the NCC?



Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

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$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i) \overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|}$$

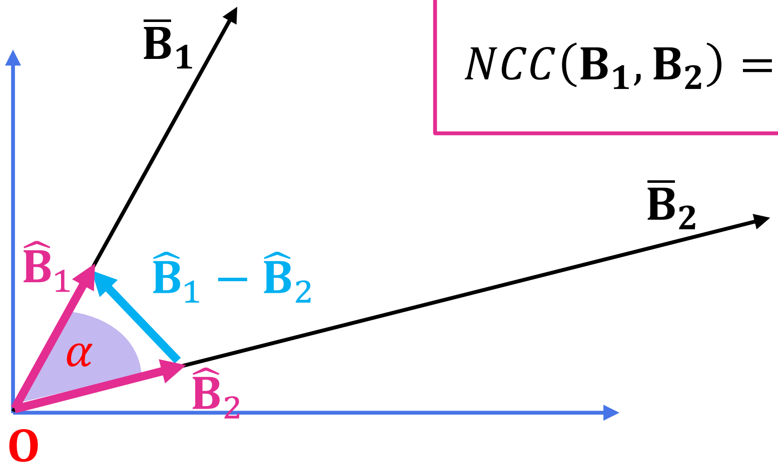
Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1, \mathbf{B}_2) = \|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\|^2}$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2}$$

Suppose we have 2D vectors:

- What is the NCC?
- Remember $\mathbf{v} \cdot \mathbf{u} = \|\mathbf{v}\| \|\mathbf{u}\| cos(\alpha)$?



Block Comparisons

A Geometric Interpretation:

$$NSSD(\mathbf{B_1},\mathbf{B_2}) = \sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|}\right)^2$$

$$NCC(\mathbf{B_1},\mathbf{B_2}) = \sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|}\right)\left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|}\right)$$

$$\widehat{\mathbf{B}}_1 = \frac{\overline{\mathbf{B}}_1}{\|\overline{\mathbf{B}}_1\|}$$

Is the unit vector of $\overline{\mathbf{B}}$

$$NCC(\mathbf{B_1},\mathbf{B_2}) = \frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

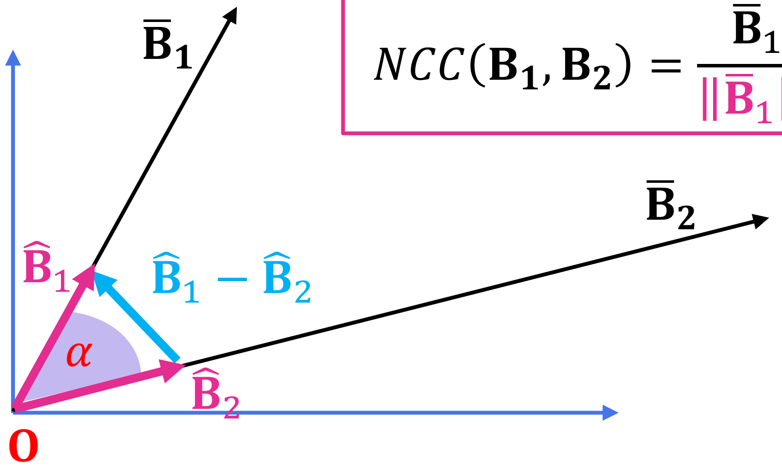
Based on the definition of dot product:

$$\boxed{NSSD(\mathbf{B}_1,\mathbf{B}_2) = \|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\|^2}$$

$$\boxed{NCC(\mathbf{B_1},\mathbf{B_2}) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2 = \cos(\alpha)}$$

Suppose we have 2D vectors:

- What is the NCC?
- Remember $\mathbf{v}\cdot\mathbf{u} = \|\mathbf{v}\|\|\mathbf{u}\|cos(\alpha)$?



Therefore, NCC is the cosine of the angle between the two vectors

Block Comparisons

A Geometric Interpretation:

$$\boxed{NCC(\mathbf{B_1}, \mathbf{B_2}) = \frac{\bar{\mathbf{B}}_1 \cdot \bar{\mathbf{B}}_2}{\|\bar{\mathbf{B}}_1\|\|\bar{\mathbf{B}}_2\|} = \hat{\mathbf{B}}_1 \cdot \hat{\mathbf{B}}_2 = \cos(\alpha)}$$

Therefore, NCC is the cosine of the angle between the two vectors

$$\boxed{-\mathbf{1} \leq NCC(\mathbf{B_1}, \mathbf{B_2}) = \cos(\alpha) \leq \mathbf{1}}$$

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Block Comparisons

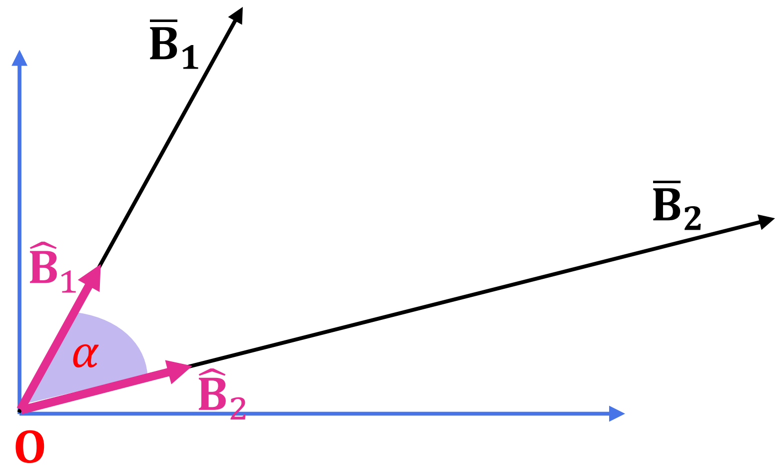
A Geometric Interpretation:

$$NCC(\mathbf{B_1},\mathbf{B_2}) = \frac{\bar{\mathbf{B}}_1 \cdot \bar{\mathbf{B}}_2}{\|\bar{\mathbf{B}}_1\|\|\bar{\mathbf{B}}_2\|} = \hat{\mathbf{B}}_1 \cdot \hat{\mathbf{B}}_2 = \cos(\alpha)$$

Therefore, NCC is the cosine of the angle between the two vectors

$$-\mathbf{1} \leq NCC(\mathbf{B_1},\mathbf{B_2}) = \cos(\alpha) \leq \mathbf{1}$$

- When NCC is close to **1**, the angle between two vectors is small which means that the two vectors are highly correlated.



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Block Comparisons

A Geometric Interpretation:

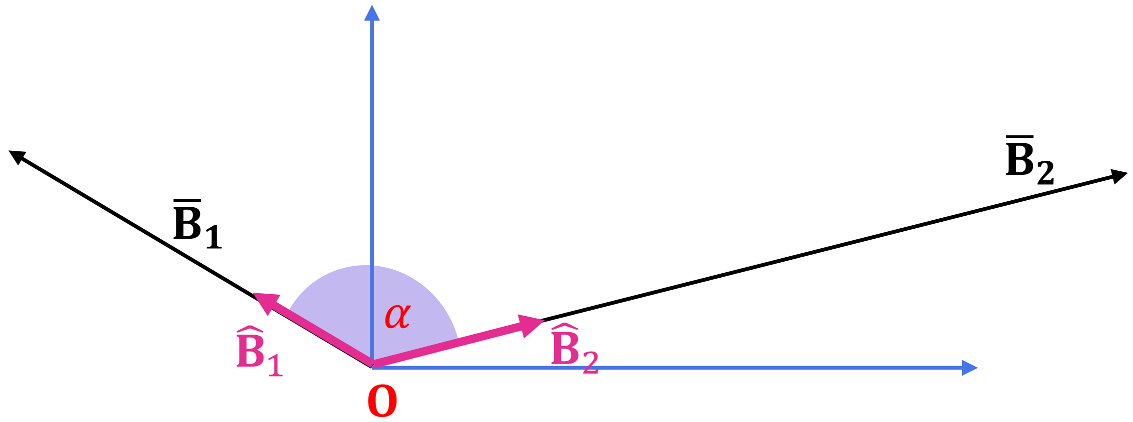
$$\boxed{NCC(\mathbf{B_1}, \mathbf{B_2}) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2 = \cos(\alpha)}$$

Therefore, NCC is the cosine of the angle between the two vectors

$$\boxed{-\mathbf{1} \leq NCC(\mathbf{B_1}, \mathbf{B_2}) = \cos(\alpha) \leq \mathbf{1}}$$

- When NCC is close to **-1**, the angle between two vectors is large which means that the two vectors are different and far from each other (negatively correlated).
- When NCC is close to ZERO,

the vectors are Orthogonal (uncorrelated).



Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} \right) \left(\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)$$

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Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\| \|\overline{\mathbf{B}}_2\|} \right)}$$

Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} \right)}$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2} - 2\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} + \frac{\overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2} \right)$$

Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} \right)}$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2} - 2\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} + \frac{\overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2} \right)$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2} + \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2} - 2\frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

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Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|} - \frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|} \right)^2$$

$$\boxed{NCC(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} \right)}$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \sum_{i=1}^{n} \left(\frac{\overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2} - 2\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} + \frac{\overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2} \right)$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2} + \frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2} - 2\frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \frac{\|\overline{\mathbf{B}}_1\|^2}{\|\overline{\mathbf{B}}_1\|^2} + \frac{\|\overline{\mathbf{B}}_2\|^2}{\|\overline{\mathbf{B}}_2\|^2} - 2\frac{\sum_{i=1}^{n} \overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

Block Comparisons

Algebraic relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\|\overline{\mathbf{B}}_1\|}-\frac{\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_2\|}\right)^2$$

$$\boxed{NCC(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}\right)}$$

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2}-2\frac{\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}+\frac{\overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2}\right)$$

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_1(i)^2}{\|\overline{\mathbf{B}}_1\|^2}+\frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_2(i)^2}{\|\overline{\mathbf{B}}_2\|^2}-2\frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\frac{\|\overline{\mathbf{B}}_1\|^2}{\|\overline{\mathbf{B}}_1\|^2}+\frac{\|\overline{\mathbf{B}}_2\|^2}{\|\overline{\mathbf{B}}_2\|^2}-2\frac{\sum_{i=1}^{n}\overline{\mathbf{B}}_1(i)\overline{\mathbf{B}}_2(i)}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|}$$

$$\boxed{NSSD(\mathbf{B}_1,\mathbf{B}_2)=2\big(1-NCC(\mathbf{B}_1,\mathbf{B}_2)\big)}$$

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Block Comparisons – Similarity Metrics Summary

- Standard Score or z-score ($z = \frac{x-\mu}{\sigma}$) is a way of normalization when the populations are available.
- Z-score is a dimensionless quantity.
- We showed that the normalized scores can be obtained using z-score normalization:

$$NSSD(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h}(z_1(i,j) - z_2(i,j))^2$$

$$NCC(B_1, B_2) = \frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h} z_1(i,j)\ \ z_2(i,j)$$

- Using a geometric interoperation, we proved that both NSSD and NCC can be viewed as some basic operations by the unit vectors of each image block.

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Block Comparisons – Similarity Metrics Summary

- In fact, NSSD can be calculated by the squared length (norm) of the difference between the unit vector of each block $\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2$:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = \left\|\widehat{\mathbf{B}}_1 - \widehat{\mathbf{B}}_2\right\|^2$$

- NCC can also be obtained by dot product of the unit vectors of each block which equals the cosine of the angle between two vectors:

$$NCC(\mathbf{B_1}, \mathbf{B_2}) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2 = \cos(\alpha)$$

- An inevitable corollary of this equivalence is that NCC is bounded by:
  - **-1** : meaning that the two vectors are negatively correlated (0: Orthogonal/Uncorrelated)
  - **1** : meaning that the two vectors are equal and highly correlate

- We learned that there is a relationship between NSSD and NCC:

$$NSSD(\mathbf{B}_1, \mathbf{B}_2) = 2\big(1 - NCC(\mathbf{B_1}, \mathbf{B_2})\big)$$

Block Comparisons – Similarity Metrics Summary - Let's code!

- We can write Matlab code in various ways:

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n}\overline{\mathbf{B}}_1(j)^2}}-\frac{\overline{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n}\overline{\mathbf{B}}_2(j)^2}}\right)^2$$

```matlab
function NSSD = NSSDScore(B1,B2)
% Calculates NSSD metric without any built-in function
   f = (B1(:)-mean(B1(:)));
   g = (B2(:)-mean(B2(:)));
   f_norm = sqrt(sum(f.*f));
   g_norm = sqrt(sum(g.*g));
   NSSD = sum( (f./f_norm - g./g_norm).^2);
end
```

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\left\|\widehat{\mathbf{B}}_1-\widehat{\mathbf{B}}_2\right\|^2$$

```matlab
function NSSD = NSSDScore_UV(B1,B2)
% Calculates NSSD metric based on the unit vector formulation
   f = (B1(:)-mean(B1(:)));
   g = (B2(:)-mean(B2(:)));
   NSSD = norm((f./norm(f)) - (g./norm(g)))^2;
end
```

$$NSSD(\mathbf{B}_1,\mathbf{B}_2)=\frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h}(z_1(i,j)-z_2(i,j))^2$$

```matlab
function NSSD = NSSDScore_ZS(B1,B2)
% Calculates NCC metric based on the z-scores
   z1 = zscore(B1(:));
   z2 = zscore(B2(:));
   % Since Matlab's std used the Corrected equations of
   %Standard Deviation, we should divide our summation by n-1
   NSSD = sum( (z1 - z2).^2)./(numel(B1)-1);
end
```

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Block Comparisons – Similarity Metrics Summary - Let's code!

- We can write Matlab code in various ways:

$$NCC(\mathbf{B_1},\mathbf{B_2}) = \sum_{i=1}^{n}\left(\frac{\overline{\mathbf{B}}_1(i)}{\sqrt{\sum_{j=1}^{n}\overline{\mathbf{B}}_1(j)^2}}\right)\left(\frac{\overline{\mathbf{B}}_2(i)}{\sqrt{\sum_{j=1}^{n}\overline{\mathbf{B}}_2(j)^2}}\right)$$

```
function NCC = NCCScore(B1,B2)
% Calculates NCC metric without any built-in function
   f = (B1(:)-mean(B1(:)));
   g = (B2(:)-mean(B2(:)));
   f = f ./ sqrt(sum(f.*f));
   g = g ./ sqrt(sum(g.*g));
   NCC = sum(f.*g);
end
```

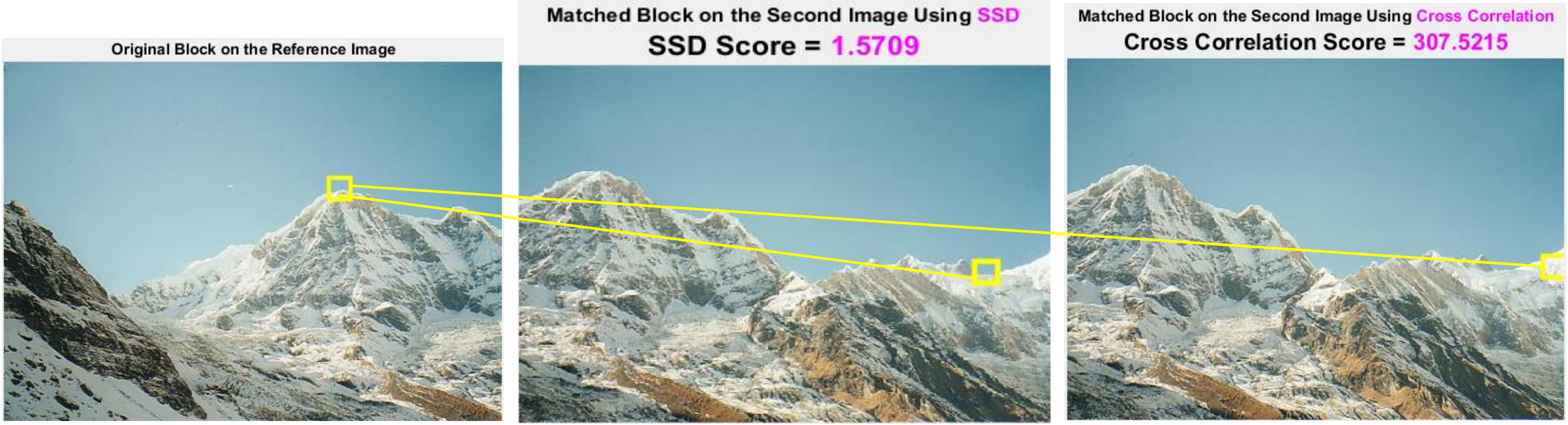
$$NCC(\mathbf{B_1},\mathbf{B_2}) = \frac{\overline{\mathbf{B}}_1 \cdot \overline{\mathbf{B}}_2}{\|\overline{\mathbf{B}}_1\|\|\overline{\mathbf{B}}_2\|} = \widehat{\mathbf{B}}_1 \cdot \widehat{\mathbf{B}}_2 = \cos(\alpha)$$

```
function NCC = NCCScore_UV(B1,B2)
% Calculates NCC metric based on the unit vector formulation
   f = (B1(:)-mean(B1(:)));
   g = (B2(:)-mean(B2(:)));
   NCC = dot(f./norm(f),g./norm(g));
end
```

$$NCC(\mathbf{B_1},\mathbf{B_2}) = \frac{1}{n}\sum_{i=1}^{w}\sum_{j=1}^{h} z_1(i,j)\ z_2(i,j)$$

```
function NCC = NCCScore_ZS(B1,B2)
% Calculates NCC metric based on the z-scores
   z1 = zscore(B1(:));
   z2 = zscore(B2(:));
   % Since Matlab's std used the Corrected equations of
   Standard Deviation, we should divide our summation by n-1
   NCC = sum(z1.*z2)./(numel(B1)-1);
end
```

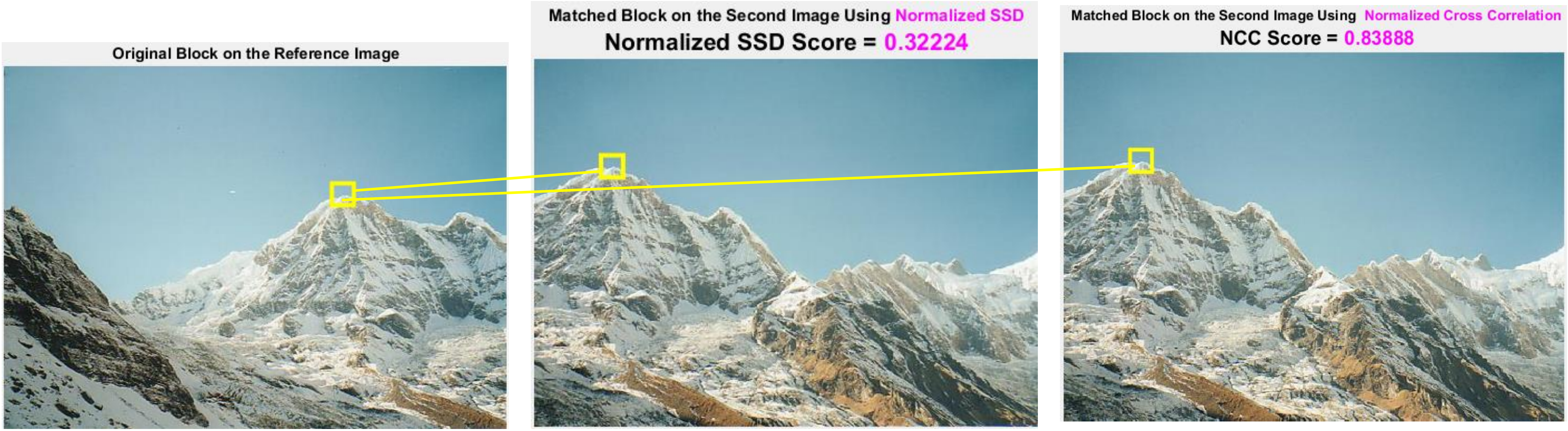
Block Comparisons



We saw that without normalization both methods failed!

Block Comparisons

Let's check the normalized versions! (m-file name: Unit_3_Block_Comparisons_Normalized_Scores.m)

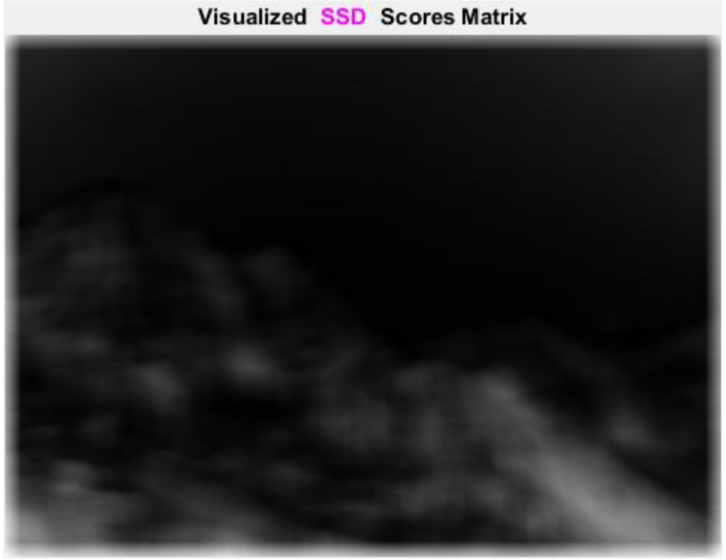


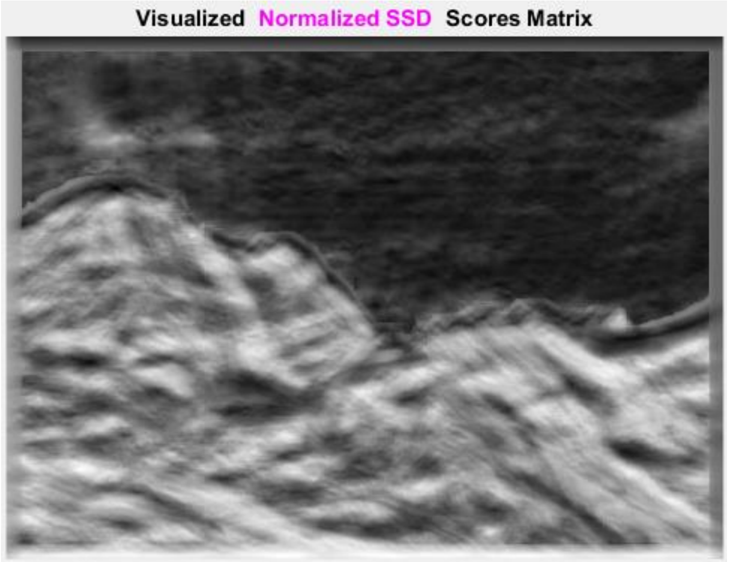
- In comparing blocks with NSSD we looked for minimum NSSD.
- However, when we use NCC we should find blocks with maximum NCC (higher correlation)

Block Comparisons

What do the score matrices looks like?

- Note that we have compared a block around every pixels in the second image with the template block of the first image, and store the NSSD and NCC in two different matrices.
- If we adjust the intensity range of the score matrices and display them, we obtain:





- We can see that scores obtained without doing normalizations (left image) are very blurry which demonstrate high uncertainty in the process of comparisons.

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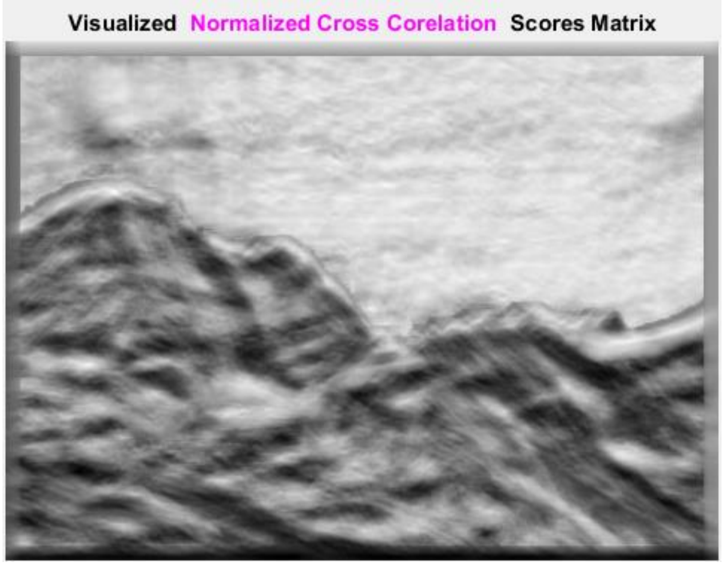


- We can see that scores obtained without doing normalizations (left image) are very blurry which demonstrate high uncertainty in the process of comparisons.

Block Comparisons

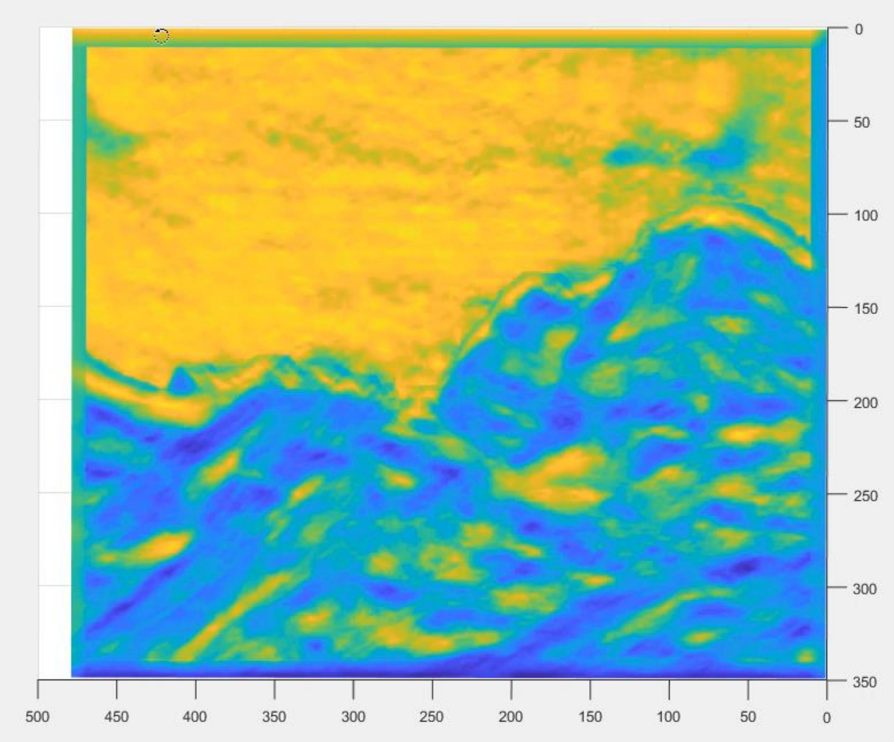
What do the score matrices looks like?

- We can also display the scores as a 3D surfaces:

Block Comparisons

What do the score matrices looks like?

- We can also display the scores as a 3D surfaces:

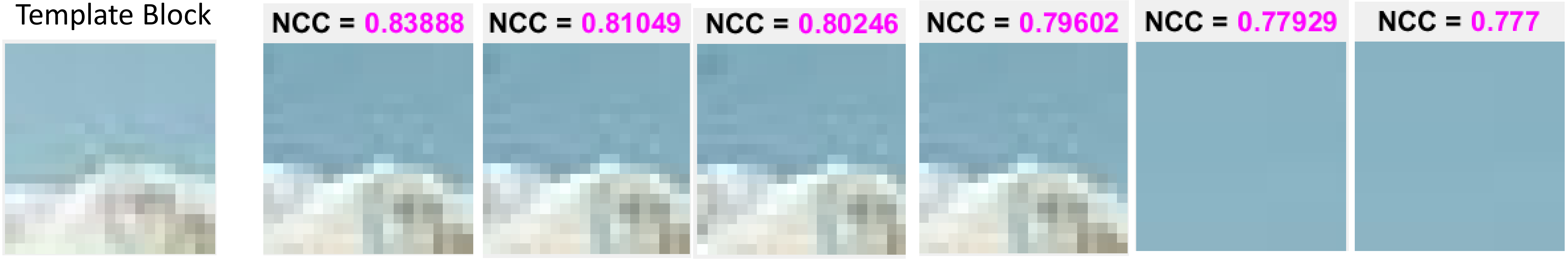


Template Matching

Block Comparisons – Highest Correlated Blocks

What are the highest correlated blocks?

Template Block



- We see the 6 highest correlated blocks.
- Since there is a large portion of sky in the template block, the last two blocks obtained a high correlation score.

Template Matching ---- 3D

Can the 2D techniques discussed so far be extended to 3D

YES

- 3D blocks are essentially a collection of 2D blocks.
- So we can create vectors that have more components & simply use the steps already decribed.

- An alternate approach can be using PCA to reduce 3D to 2D, and apply 2D techniques after that.