CMPUT 307: Hints related to Quiz 3

Computing Surface Normal for a Triangle/Face + the Equation of the Plane containing the Triangle

Surface Normal

Given 3 vertices of a triangle/face NOT on a Straight Line:

1. We can compute TWO Vectors from the 3 Points (Vertices).
2. Given 2 vectors the direction Orthogonal to both of them can be computed as the Cross Product of the 2 vectors; this direction is the surface normal.

Computing Cross Product

Cross product of two vectors

$$v_0 = (x_0, y_0, z_0), v_1 = (x_1, y_1, z_1)$$
$$v_0 \times v_1 = (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1)$$

To Remember this Formula, Note X does not appear in X coordinate of the formula, Y does not appear in Y coordinate of the formula, and Z does not appear in Z coordinate of the formula.

Cross product of (1,0,0) x (0, 1, 0):
(0x0 – 0 x1, 0 x 0 – 1 x 0, 1 x 1 – 0 x 0) = (0, 0, 1)

Equation of Plane containing a triangle

Surface Normal you just computed in Orthogonal to ANY vector on the plane containing the triangle. So we can consider the following steps:

1. Make length of Surface Normal (a, b, c) = 1, by dividing by its current length. This is necessary to confirm with the Formula for QEM.
2. Then consider any of the 3 points of the triangle, say (x0, y0, z0).
3. For any point (x, y, z) on the plane containing the triangle, (x – xo, y – y0, z – z0) is a vector on the plane. The inner product of this vector with the surface normal is ZERO, since the two vectors are orthogonal.
4. Thus, a(x – x0) + b(y – y0) + c(z – z0) = 0. This gives us the equation of the plane containing the triangle/face.

Example: Quiz 3, F1: (v1, v3, v4)
v1 = (3,2,5) v3 = (1, 3, 3) v4 = (1, 1, 2)

(v1 – v3) X (v1 – v4) = (2, -1, 2) X (2, 1, 3) = (-5, -2, 4)

Making the length of this vector 1, Surface Normal (a, b, c) = (-0.75, -0.3, 0.6)

Equation of Plane containing (v1, v3, v4) is:

$a(x – x4) + b(y – y4) + c(z- z4) = 0$

➔ $-0.75(x – 1) -0.03(y – 1) + 0.6(z – 2) = 0$

➔ $-0.75x -0.3y + 0.6z + 0.15 = 0$