1 Temporal Logic

Exercise 1

1. is Fp true if p is true in the current state and only in this one ?
2. is Gp true if p is false in the current state and true in every other ?
3. is pUq true if p is false and q true in the current state ?
4. is pUq true if q is always false and p always true ?

Exercise 2
Draw some tree path to illustrate the properties EX, AX, EU, AU.
Exercise 3
Express the following properties :

1. All states satisfy p.
2. p is reachable by a path on which q is always true.
3. Whatever the state, we always can come to a state, where p is true
4. No deadlock.

Exercise 4
We consider the following coffee machine :

— the machine serves two types of drinks : large coffees, costing 40 cents each, and small coffees, costing 30 cents each.
— the machine accepts only 10 cents and 20 cents coins (call the actions `Pay10` and `Pay20`).
— the user must give the exact change.
— The controller keeps in its memory the sum of money entered by the user.
— As soon as the sum reaches 30 cents, the controller switches to the state `choice`, and the action `smallChoice` is available. The user is also allowed to input 10 more cents in the machine.
— As soon as the sum reaches 40 cents, the action `largeChoice` becomes available.
— When the user has made his choice, the controller, the machine takes the money and serves the drink (states `smallServe` and `largeServe`).
— When the drink is served, the controller comes back to its initial state (action `served`).
— As long as the user has not chosen the drink, it is possible to cancel the order (action `cancel`). The machine give the money back and the controller gets back to its initial state.

1. draw a state machine modelling the controller.
2. draw the corresponding transition system.
3. We denote by `state=E` the property "the machine controller is in state E". We can also use the notations sum $\leq$ S, sum $\geq$ S and sum $\neq$ S.
   (a) Translate in CTL* the following natural language formulas :
   — "The machine always goes back to its initial state"

— "After having served a small coffee, the machine immediately goes back to its initial state"
— "when the machine has received 30 cents, it can serve a large coffee"
— as long as the machine did not received at least 30 cents, it does not serve any coffee.

(b) Translate in natural language the following CTL formulas :
— $\neg$ E F G (state=init)
— A G (state=choice $\Rightarrow$ X(sum=0))
— A (state=init) U (state=choice)
— E (X(sum=0) $\wedge$ F(state=largeServe))

(c) among the four formulas above, which one is verified by the coffee machine ?