Contents

1 Introduction

The main objective of this assignment is to get familiar with Deterministic Finite Automata (DFA) and Non-Deterministic Finite Automata (NFA).

2 Exercise 1: Describing NFAs

We recognize the language of M_1 following the oricess of DFA to Generalized NFA conversion.

Step 1 We add extra start state and accept state. The automata is in Figure 1.

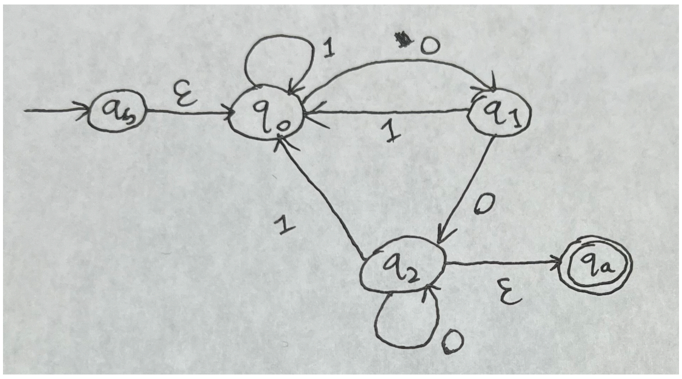


Figure 1: M_1 with extra start and accept state

Then, we eliminate all the states (except for the start and the accept state) in order.

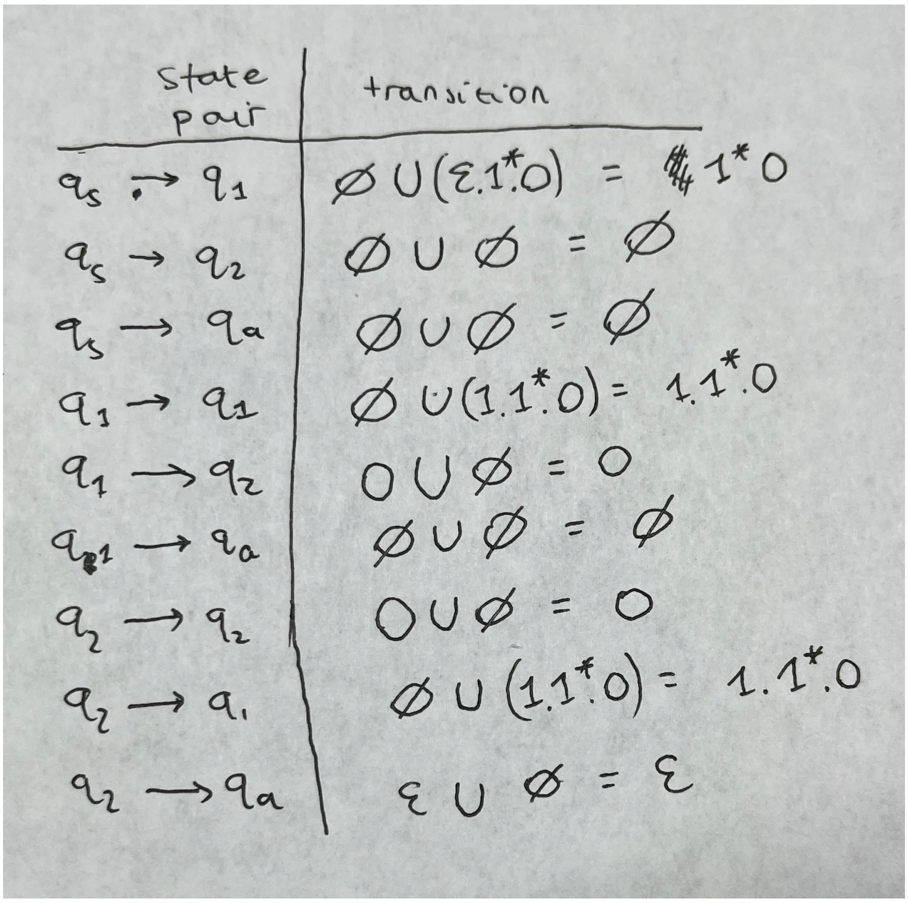
Step 2 We eliminate the state q_0. To do so, we use the transition table in Figure 2. With this transition table, we present the same automata without state q_0 as is shown in Figure 3.

Step 3 We eliminate the state q_1. To do so, we use the transition table in Figure 4. With this transition table, we present the same automata without state q_1 as is shown in Figure 5.

Step 4 Finally, we eliminate q_2. The final equivalent automata is presented in Figure 6. The regular language it recognizes is also shown in Figure 6.

The automata M_1 recognizes the following language in plain English:

$$L = \{\omega \text{ in}\{0,1\}^* : \omega \text{ ends with at least 2 consecutive zeros}\}$$



State pair	transition
$q_s \to q_1$	$\emptyset \cup (\varepsilon.1^*.0) = 1^*0$
$q_s \to q_2$	$\emptyset \cup \emptyset = \emptyset$
$q_s \to q_a$	$\emptyset \cup \emptyset = \emptyset$
$q_1 \to q_1$	$\emptyset \cup (1.1^*.0) = 1.1^*.0$
$q_1 \to q_2$	$0 \cup \emptyset = 0$
$q_1 \to q_a$	$\emptyset \cup \emptyset = \emptyset$
$q_2 \to q_2$	$0 \cup \emptyset = 0$
$q_2 \to q_1$	$\emptyset \cup (1.1^*.0) = 1.1^*.0$
$q_2 \to q_a$	$\varepsilon \cup \emptyset = \varepsilon$

Figure 2: Transition table to eliminate q_0

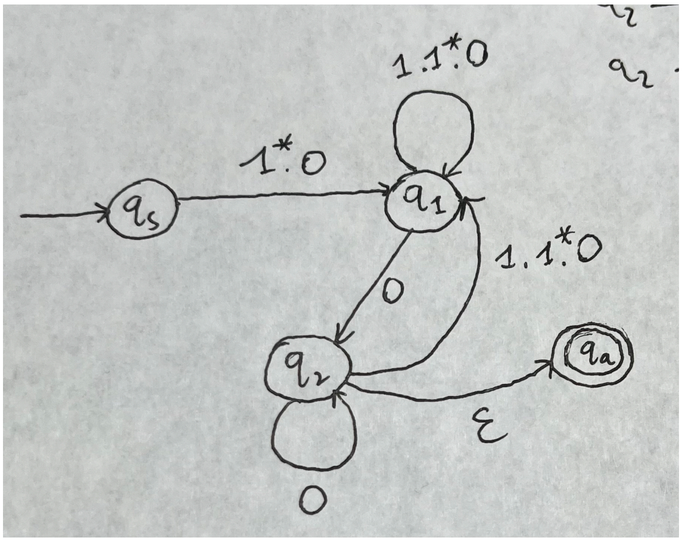
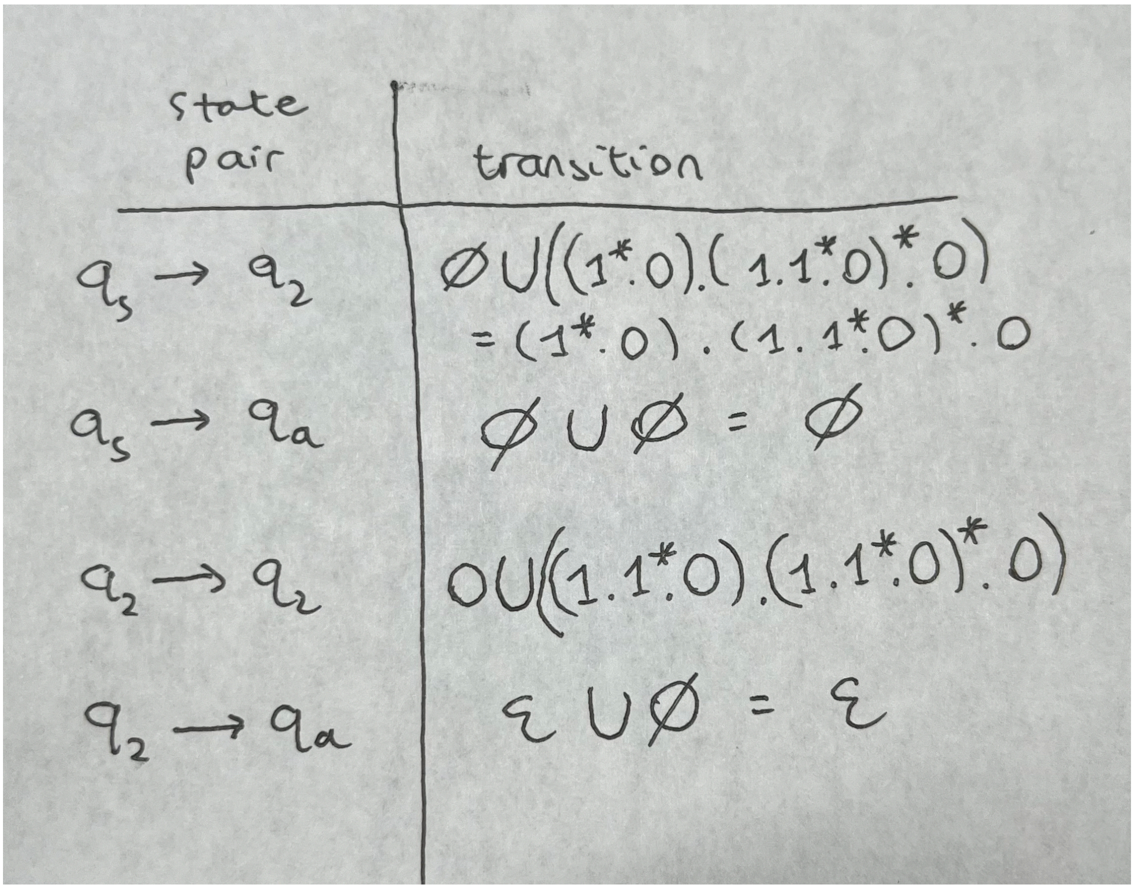


Figure 3: M_1 after elimination of q_0



state pair	transition
$q_s \to q_2$	$\emptyset \cup ((1^*.0).(1.1^*.0)^*.0)$ $= (1^*.0).(1.1^*.0)^*.0$
$q_s \to q_a$	$\emptyset \cup \emptyset = \emptyset$
$q_2 \to q_2$	$0 \cup ((1.1^*.0).(1.1^*.0)^*.0)$
$q_2 \to q_a$	$\varepsilon \cup \emptyset = \varepsilon$

Figure 4: Transition table to eliminate q_1

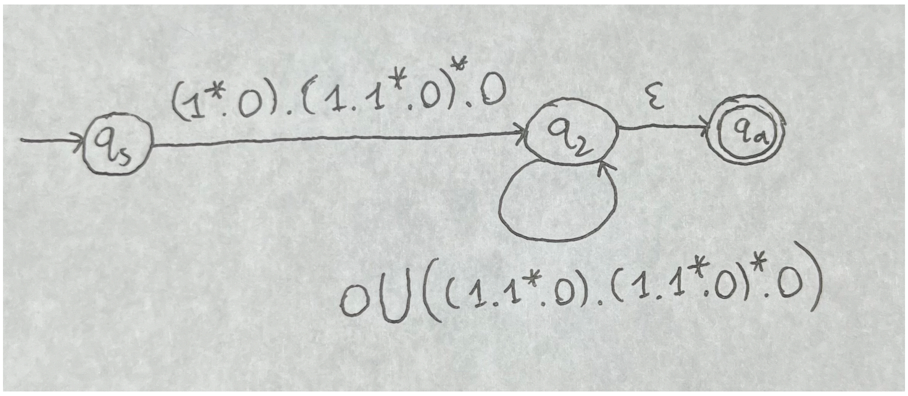


Figure 5: M_1 after elimination of q_0 and q_1

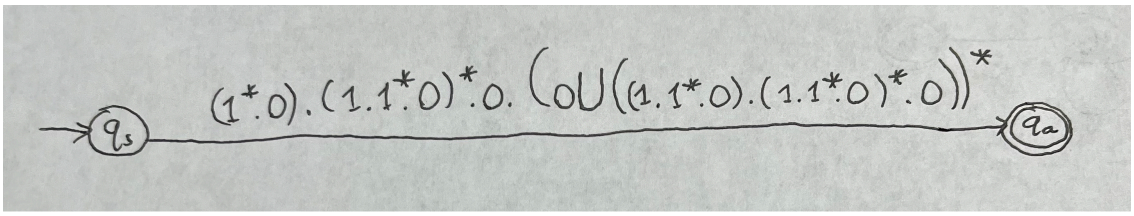


Figure 6: M_1 after elimination of q_0, q_1, and q_2

3 Exercise 2: Building NFAs

First of all, we translate the requirement from regular expression to plain English, and we define this language as concatenation of 3 different languages L_A, L_B, and L_C, as is shown in Figure 7.

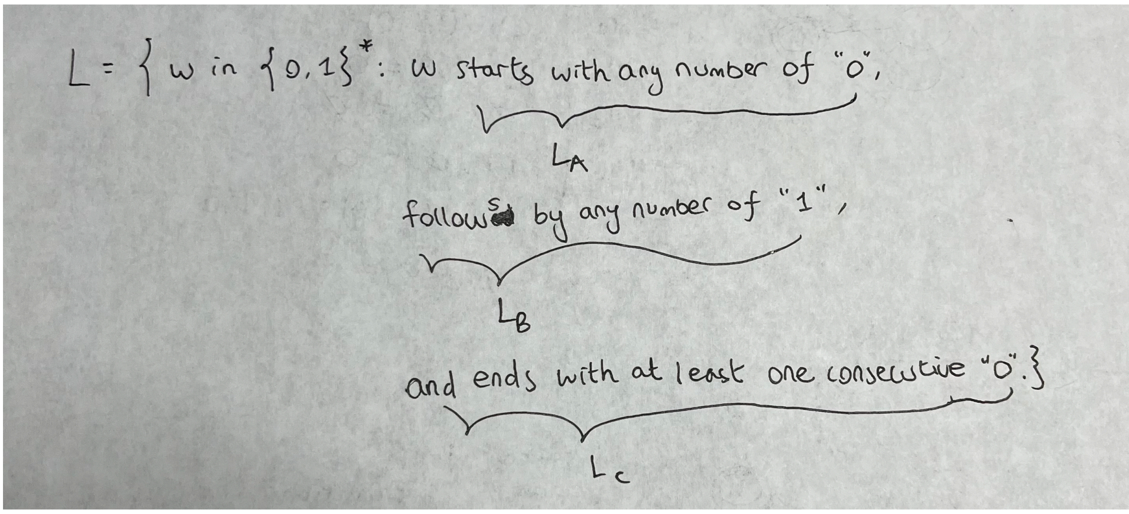


Figure 7: Description of the language $0^*1^*0^+$ in plain English

Before we do concatenation, we draw the following automatas M_A, M_B, and M_C in Figure 8, corresponding respectively to the languages L_A, L_B, and L_C.

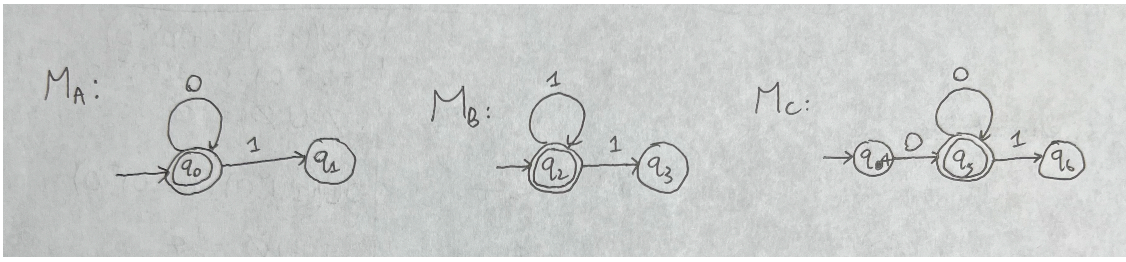


Figure 8: Automatas of sub-languages L_A, L_B, and L_C

Using the rule of building NFA from concatenation of DFAs, we add σ transitions and define the final NFA in Figure 9.

4 Exercise 3: Describing NFAs

We give the following formal definition, in Figure 10, of this automata:

We propose the path in Figure 11 to show the acceptance of 'aab'. In the figure, we propose the following sequence of 5 states:

$$r_0 = q_0, r_1 = q_1, r_2 = q_2, r_3 = q_3, r_4 = q_4$$

We propose the transitions in Figure 12, following which could fully demonstrate the acceptance of 'aab' in a formal way.

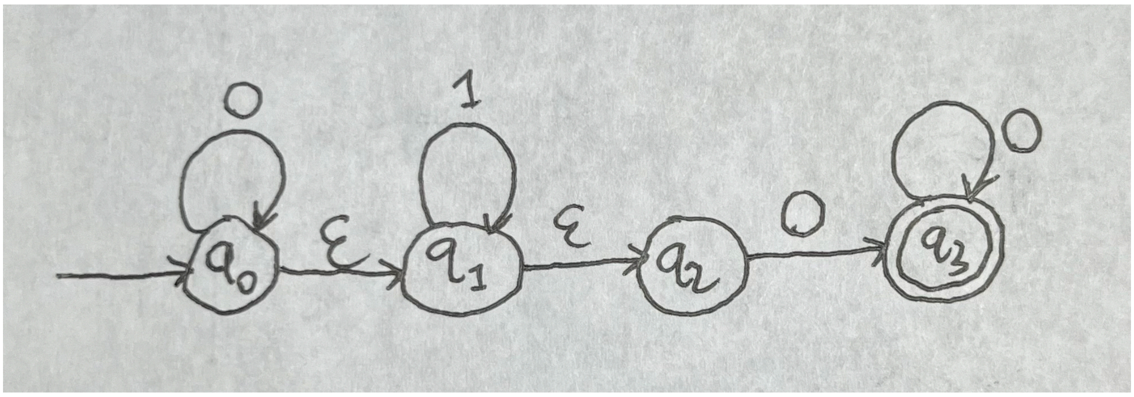


Figure 9: Automata representing the regular language $0^*1^*0^+$

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{a, b\}$
- δ: $\delta(q_0, a) = \{q_0, q_1, q_2\}$ $\delta(q_0, b) = \emptyset$ $\delta(q_0, \varepsilon) = \emptyset$

  $\delta(q_1, a) = \emptyset$ $\delta(q_1, b) = \emptyset$ $\delta(q_1, \varepsilon) = \{q_0\}$

  $\delta(q_2, a) = \emptyset$ $\delta(q_2, b) = \{q_0, q_1\}$ $\delta(q_2, \varepsilon) = \emptyset$
- $q_0 \in Q$ is the start state
- $F = \{q_1\} \subseteq Q$ is the set of accept states

Figure 10: Formal definition of automata in Exercise 3

$$q_0 \xrightarrow{w_1 = a} q_1 \xrightarrow{w_2 = \varepsilon} q_0 \xrightarrow{w_3 = a} q_2 \xrightarrow{w_4 = b} q_1$$

Figure 11: Path of demonstration of acceptance of 'aab'

$$r_0 = q_0$$
$$r_1 = q_1 \in \delta(r_0, a) = \{q_0, q_1, q_2\}$$
$$r_2 = q_0 \in \delta(r_1, \varepsilon) = \{q_0\}$$
$$r_3 = q_2 \in \delta(r_2, a) = \{q_0, q_1, q_2\}$$
$$r_4 = q_1 \in \delta(r_3, b) = \{q_0, q_1\} \text{ AND } q_1 \in F = \{q_1\}$$

Figure 12: Transitions in detail of path of acceptance of 'aab'

5 Exercise 4: Subset Construction

To turn the NFA in Exercise 3 into DFA, we do the following steps.

First, before we draw it, we formally define the new DFA in Figure 13.

Then, we draw the DFA in Figure 14, according to the transitions we listed before.

Next, we find the states without incoming arrows, as in Figure 15.

Finally, we eliminate the states not in use shown in Figure 15, and re-draw the DFA with actually functioning states. The final DFA is shown in Figure 16.

6 Exercise 5: Intersection

We use induction to demonstrate that a set of regular languages is closed under the intersection operation.

Initialization Assuming that A and B are two regular languages sharing the same alphabet, we construct the NFA with formal definition shown in Figure 17 to recognize the language of $A \cap B$, which is thus the prove that two languages are closed under the intersection operation.

Iteration Suppose C is another regular language. We use the initialized $A \cap B$ as a whole as one single language, so we are safe to say the the $C \cap (A \cap B)$ (which is exactly the set of languages A, B, and C) is also a regular language.

Conclusion By iteratively intersecting new regular languages with the formal ones starting from the case of initialization, we can conclude that the set of any number (greater than 2) of regular languages is closed under the intersection operation.

7 Conclusion

In this assignment, we learned about DFAs and NFAs.

$M' = \{Q', \Sigma', \delta', q_0', F'\}$ where

- Q' = powerset of Q = powerset of $\{q_0, q_1, q_2\}$
  $= \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$
- $\Sigma' = \Sigma = \{a, b\}$
- $q_0' = E(\{q_0\}) = \{q_0\}$
- $F' = \{s \in Q' \mid s \text{ contains elements from } F\} = \{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$
- δ' defined as:

① $\delta'(\{q_0\}, a) = E(\delta_{NFA}(q_0, a)) = \{q_0, q_1, q_2\}$

② $\delta'(\{q_0\}, b) = E(\delta_{NFA}(q_0, b)) = \emptyset$

③ $\delta'(\{q_1\}, a) = E(\delta_{NFA}(q_1, a)) = \emptyset$

④ $\delta'(\{q_1\}, b) = E(\delta_{NFA}(q_1, b)) = \emptyset$

⑤ $\delta'(\{q_2\}, a) = E(\delta_{NFA}(q_2, a)) = \emptyset$

⑥ $\delta'(\{q_2\}, b) = E(\delta_{NFA}(q_2, b)) = E(\{q_0, q_1\}) = \{q_0, q_1\}$

⑦ $\delta'(\{q_0, q_1\}, a) = E(\delta_{NFA}(q_0, a) \cup \delta_{NFA}(q_1, a)) = E(\{q_0, q_1, q_2\} \cup \emptyset) = \{q_0, q_1, q_2\}$

⑧ $\delta'(\{q_0, q_1\}, b) = E(\delta_{NFA}(q_0, b) \cup \delta_{NFA}(q_1, b)) = E(\emptyset \cup \emptyset) = \emptyset$

⑨ $\delta'(\{q_0, q_2\}, a) = E(\delta_{NFA}(q_0, a) \cup \delta_{NFA}(q_2, a)) = E(\{q_0, q_1, q_2\} \cup \emptyset) = \{q_0, q_1, q_2\}$

⑩ $\delta'(\{q_0, q_2\}, b) = E(\delta_{NFA}(q_0, b) \cup \delta_{NFA}(q_2, b)) = E(\emptyset \cup \{q_0, q_1\}) = \{q_0, q_1\}$

⑪ $\delta'(\{q_1, q_2\}, a) = E(\delta_{NFA}(q_1, a) \cup \delta_{NFA}(q_2, a)) = E(\emptyset \cup \emptyset) = \emptyset$

⑫ $\delta'(\{q_1, q_2\}, b) = E(\delta_{NFA}(q_1, b) \cup \delta_{NFA}(q_2, b)) = E(\emptyset \cup \{q_0, q_1\}) = \{q_0, q_1\}$

⑬ $\delta'(\{q_0, q_1, q_2\}, a) = E(\delta_{NFA}(q_0, a) \cup \delta_{NFA}(q_1, a) \cup \delta_{NFA}(q_2, a))$
$= E(\{q_0, q_1, q_2\} \cup \emptyset \cup \emptyset) = \{q_0, q_1, q_2\}$

⑭ $\delta'(\{q_0, q_1, q_2\}, b) = E(\delta_{NFA}(q_0, b) \cup \delta_{NFA}(q_1, b) \cup \delta_{NFA}(q_2, b))$
$= E(\emptyset \cup \emptyset \cup \{q_0, q_1\}) = \{q_0, q_1\}$

Figure 13: Formal definition of DFA

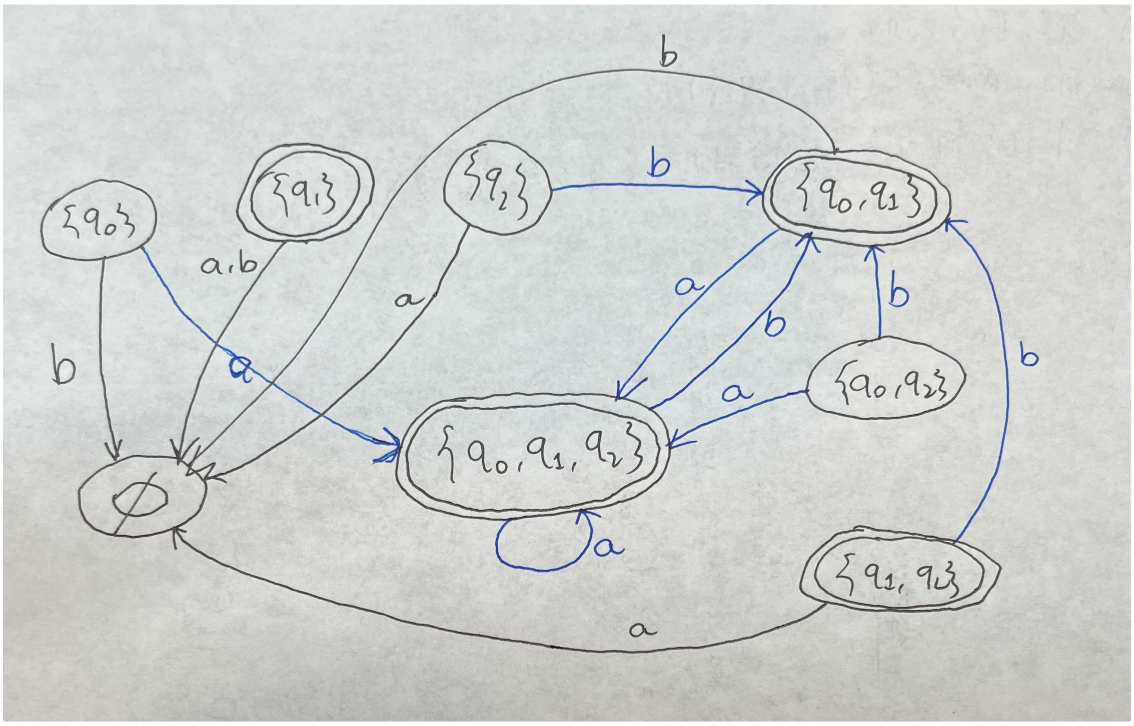


Figure 14: Form of DFA according to the formal definition

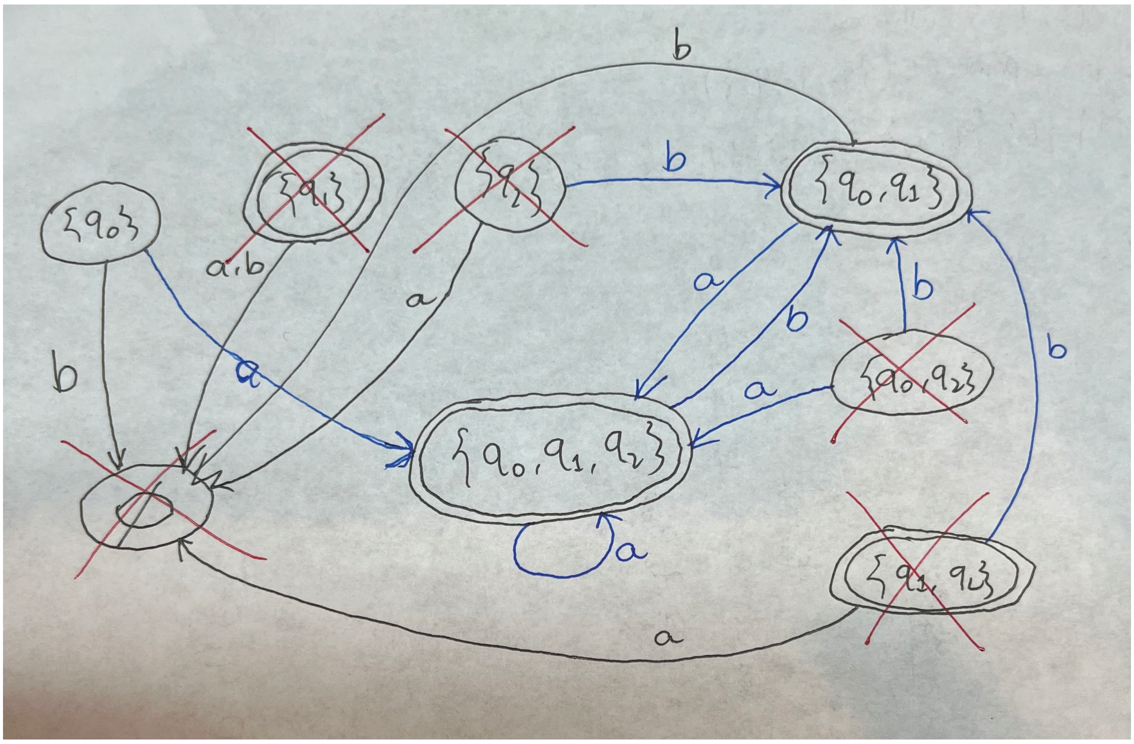


Figure 15: States not in use of DFA

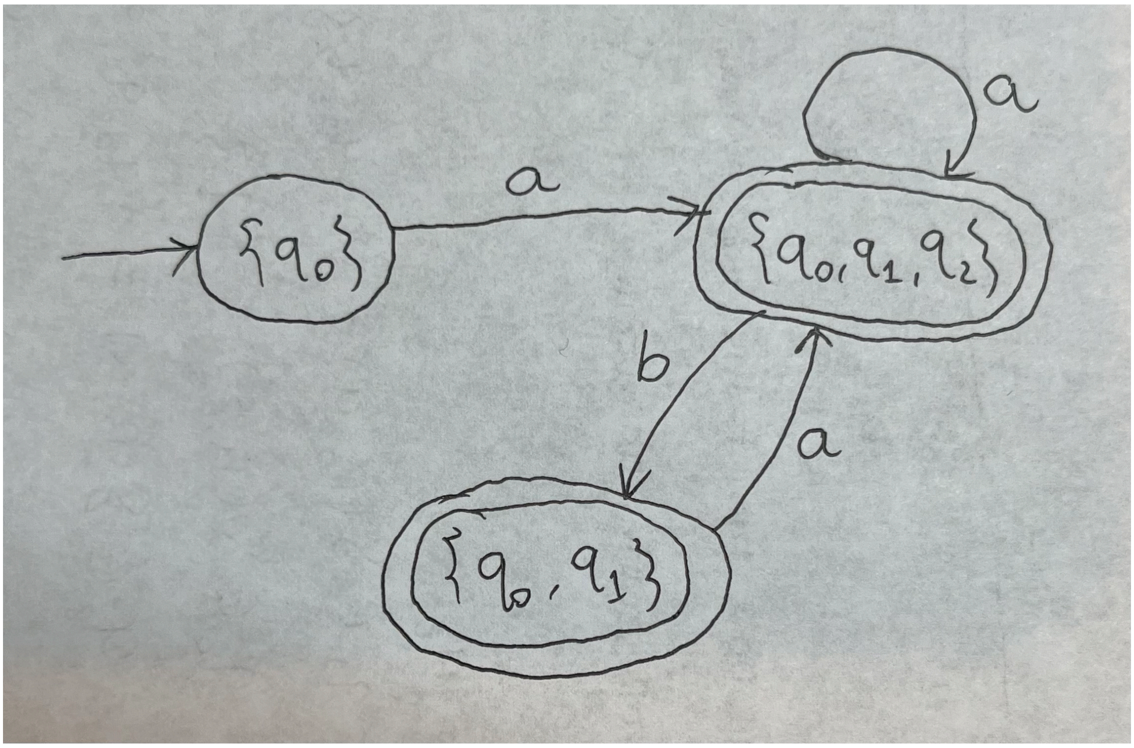


Figure 16: Final simplified DFA

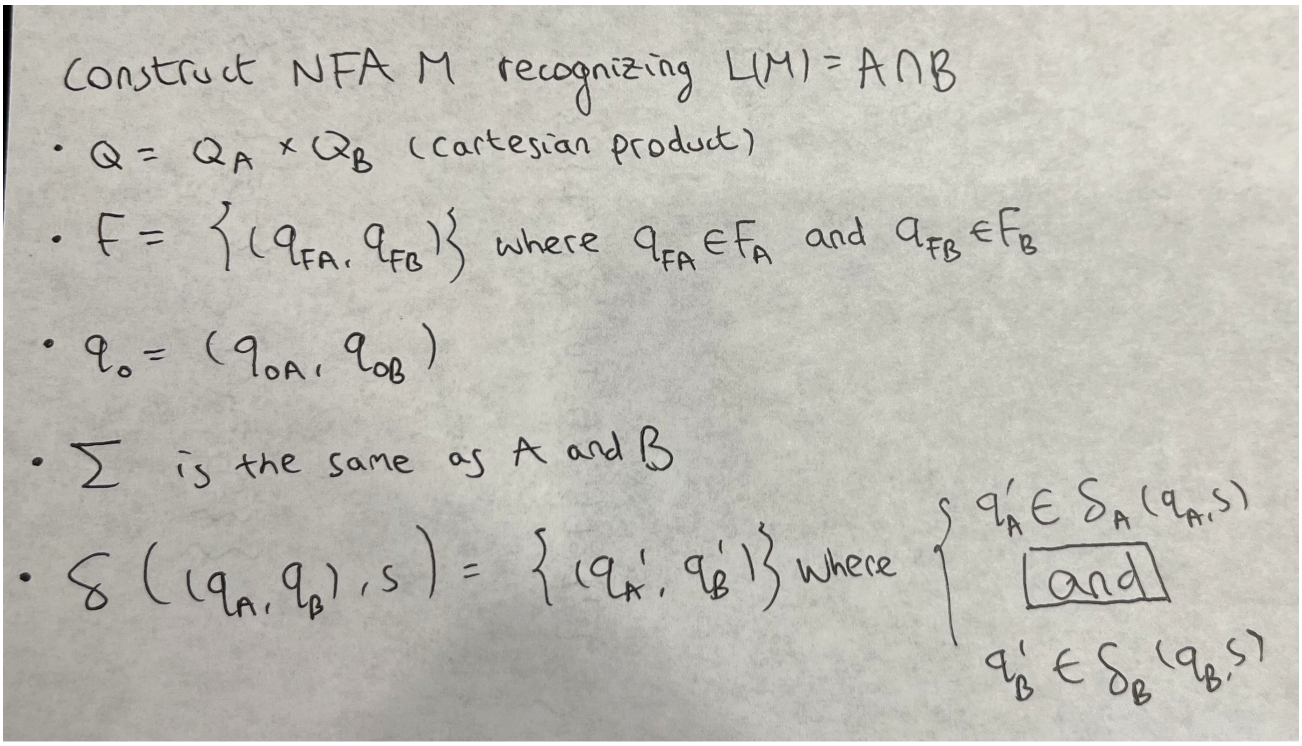


Figure 17: Formal definition of NFA recognizing $A \cap B$