Hedging using Futures

Long & Short Hedges

- Long future hedge: when you want to purchase an asset in the future and want to lock in the price
- Short future hedge: when you want to sell an asset in the future and want to lock in the price

Basis

- Basis = Spot price (S) – Future price (F)
- Long hedge:
  F_1 : Future price when hedge is set up
  F_2 : Future price when asset is purchased
  S_2 : Spot price when asset is purchased
  b_2 : Basis when asset is purchased

Cost of asset	S_2
Gain of future	$F_2 - F_1$
Net amount paid	$S_2 - (F_2 - F_1) = F_1 + b_2$

- Short hedge:
  F_1 : Future price when hedge is set up
  F_2 : Future price when asset is purchased
  S_2 : Spot price when asset is purchased
  b_2 : Basis when asset is purchased

Price of asset	S_2
Gain of future	$F_1 - F_2$
Net amount paid	$S_2 + (F_1 - F_2) = F_1 + b_2$

- Since b_2 is uncertain at time t_1, this is called the basis risk

Cross Hedging: When there is no future contract on the asset being hedging, choose the contract whose future price is the most correlated with the asset price

Optimal Hedge Ratio

- $h^* = \rho \frac{\sigma_S}{\sigma_F}$
  where σ_S is the standard deviation of Δ_S, the change in spot price
  σ_F is the standard deviation of Δ_F, the change in future price
  ρ is the correlation between Δ_S and Δ_F
- Similar to option delta

Optimal Number of Contracts

Q_a: size of position being hedged (unit)

Q_F: size of one future contract (unit)

N^*: Optimal number of future contracts for hedging

$$N^* = \frac{h^* Q_a}{Q_F}$$

Tailing the Hedge

When futures are used for hedging, a small adjustment, known as tailing the hedge, can be made to adjust for the daily settlement. In practice, this means

$$N^* = \frac{h^* V_a}{V_F}$$

where V_a, V_F are the dollar values of the position and one future contract.

If the forward contract is used, there is no daily settlement and the previous equation should be used.

Why Hedge Equity Returns?

- May want to be out of the market for a while; Hedging avoids selling and repurchasing costs
- Suppose portfolio beta is 1.0, but we are confident that the portfolio will outperform the market in both good and bad times, then hedging ensures that the portfolio return is risk-free rate plus excess return over the market (i.e. Locking in the benefits of stock picking)
- Can also change the beta of the portfolio without changing the stocks that make up the portfolio

Stack and Roll

- A stack hedge is a position concentrated in one specific future contract month as opposed to a string of contracts that extend out the maturity of the swap
- Reduce liquidity risk (only trade on-the-run futures contracts)
- Change in term structure from contango and backwardation, or vice versa, can put stack and roll hedge at a great risk