# Game Theory

Game theory studies what agents should do in multi-agent settings.

**Framework**

Strategic form of a game:

- a finite set of agents $I = \{1, \dots, n\}$
- a set of actions $A_i$ for each agent $i \in I$
- a utility function $u_i$ for each agent $i \in I$

Each agent chooses an action without knowing what other agents choose.

**Strategy**

- pure strategy: one action
- stochastic (mixed) strategy: a mix of actions with probabilities

A **strategy profile** is an assignment of a strategy to each agent. If $\sigma$ is a strategy profile, let $\sigma_i$ be the strategy of agent i in $\sigma$, and let $\sigma_{-i}$ be the strategies of other agents. (i.e. $\sigma$ is $\sigma_i\sigma_{-i}$)

**Nash Equilibrium**

Let $\text{utility}(\sigma, i)$ be the expected utility of strategy profile $\sigma$ for agent i.

A best response for an agent i to the strategies $\sigma_{-i}$ of the other agents is a strategy $\sigma_i$ that satisfies:

$$\text{utility}(\sigma_i\sigma_{-i}) \geq \text{utility}(\sigma_i'\sigma_{-i}, i) \quad \textit{for all other strategies } \sigma_i'$$

A strategy profile $\sigma$ is a Nash equilibrium if, for each agent i, strategy $\sigma_i$ is a best response to $\sigma_{-i}$.

**Computing Nash Equilibrium**

Iterated elimination of strategies:

- eliminate any pure strategy dominated by another strategy
- if there are multiple strategies left with the same expected utility, then need a stochastic strategy

**Pareto Optimal**

An outcome is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off.