

Question 1

$$\begin{array}{l} \text{a. } 2x - 5y = 27 \\ 3x + 4y = 6 \end{array}$$

$$\begin{array}{l} \left[\begin{array}{cc|c} 2 & -5 & 27 \\ 3 & 4 & 6 \end{array} \right] \xrightarrow{2R_1 - R_2 \rightarrow R_1} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 3 & 4 & 6 \end{array} \right] \\ \xrightarrow{R_2 - 3R_1 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 0 & 46 & -138 \end{array} \right] \xrightarrow{\frac{1}{46}R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 0 & 1 & -3 \end{array} \right] \\ \xrightarrow{R_1 + 14R_2 \rightarrow R_1} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & -3 \end{array} \right] \\ \therefore x = 6, y = -3 \end{array}$$

Question 1

$$\begin{array}{l} \text{a. } 2x - 5y = 27 \\ 3x + 4y = 6 \end{array}$$

$$\begin{array}{l} \left[\begin{array}{cc} 2 & -5 \\ 3 & 4 \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right] = \left[\begin{array}{c} 27 \\ 6 \end{array} \right] \\ \left[\begin{array}{cc|c} 2 & -5 & 27 \\ 3 & 4 & 6 \end{array} \right] \xrightarrow{2R_1 - R_2 \rightarrow R_1} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 3 & 4 & 6 \end{array} \right] \\ \xrightarrow{R_2 - 3R_1 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 0 & 46 & -138 \end{array} \right] \xrightarrow{\frac{1}{46}R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -14 & 48 \\ 0 & 1 & -3 \end{array} \right] \\ \xrightarrow{R_1 + 14R_2 \rightarrow R_1} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & -3 \end{array} \right] \\ \therefore x = 6, y = -3 \end{array}$$

b. $-0.5x + 3y = 16$
 $0.2x + 5y = 37$

$$\begin{bmatrix} -0.5 & 3 \\ 0.2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 37 \end{bmatrix}$$

$$\begin{bmatrix} -0.5 & 3 & | & 16 \\ 0.2 & 5 & | & 37 \end{bmatrix} \xrightarrow{-2R_1 \rightarrow R_1} \begin{bmatrix} 1 & -6 & | & -32 \\ 0.2 & 5 & | & 37 \end{bmatrix}$$

$$\xrightarrow{R_2 - 0.2R_1 \rightarrow R_2} \begin{bmatrix} 1 & -6 & | & -32 \\ 0 & 6.2 & | & 43.4 \end{bmatrix} \xrightarrow{\frac{5}{31}R_2 \rightarrow R_2} \begin{bmatrix} 1 & -6 & | & -32 \\ 0 & 1 & | & 7 \end{bmatrix}$$

$$\xrightarrow{R_1 + 6R_2 \rightarrow R_1} \begin{bmatrix} 1 & 0 & | & 10 \\ 0 & 1 & | & 7 \end{bmatrix}$$

$$\therefore x = 10, y = 7$$

c. $-3x - 5y = -40$

$$8x + 2y = -18$$

$$\begin{bmatrix} -3 & -5 \\ 8 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -40 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -5 & | & -40 \\ 8 & 2 & | & -18 \end{bmatrix} \xrightarrow{-3R_1 - R_2 \rightarrow R_1} \begin{bmatrix} 1 & 13 & | & 138 \\ 8 & 2 & | & -18 \end{bmatrix}$$

$$\xrightarrow{R_2 - 8R_1 \rightarrow R_2} \begin{bmatrix} 1 & 13 & | & 138 \\ 0 & -102 & | & -1122 \end{bmatrix} \xrightarrow{-\frac{1}{102}R_2 \rightarrow R_2} \begin{bmatrix} 1 & 13 & | & 138 \\ 0 & 1 & | & 11 \end{bmatrix}$$

$$\xrightarrow{R_1 - 13R_2 \rightarrow R_1} \begin{bmatrix} 1 & 0 & | & -5 \\ 0 & 1 & | & 11 \end{bmatrix}$$

$$\therefore x = -5, y = 11$$

$$(d) \begin{aligned} 2x + 5y - 3z &= -8 \\ 7x + 0.5y + 2z &= 56 \\ 3x - 7y - 0.5z &= 33 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & -3 & x \\ 7 & \frac{1}{2} & 2 & y \\ 3 & -7 & -\frac{1}{2} & z \end{array} \right] = \left[\begin{array}{c} -8 \\ 56 \\ 33 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 3 & -8 \\ 7 & \frac{1}{2} & 2 & 56 \\ 3 & -7 & -\frac{1}{2} & 33 \end{array} \right] \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 7 & \frac{1}{2} & 2 & 56 \\ 3 & -7 & -\frac{1}{2} & 33 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & -17 & \frac{25}{2} & 84 \\ 3 & -7 & -\frac{1}{2} & 33 \end{array} \right] \xrightarrow{R_2 - 7R_1 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & -17 & \frac{25}{2} & 84 \\ 0 & -\frac{29}{2} & 4 & 45 \end{array} \right] \xrightarrow{R_3 - 3R_1 \rightarrow R_3} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & -17 & \frac{25}{2} & 84 \\ 0 & 0 & 4 & 45 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & -17 & \frac{25}{2} & 84 \\ 0 & -\frac{29}{2} & 4 & 45 \end{array} \right] \xrightarrow{-\frac{1}{17}R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & 1 & -\frac{25}{34} & -\frac{84}{17} \\ 0 & -\frac{29}{2} & 4 & 45 \end{array} \right] \xrightarrow{-\frac{1}{17}R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & -4 \\ 0 & 1 & -\frac{25}{34} & -\frac{84}{17} \\ 0 & 0 & 4 & 45 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{23}{68} & \frac{142}{17} \\ 0 & 1 & -\frac{25}{34} & -\frac{84}{17} \\ 0 & 0 & 4 & 45 \end{array} \right] \xrightarrow{R_3 + \frac{29}{2}R_2 \rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{23}{68} & \frac{142}{17} \\ 0 & 1 & -\frac{25}{34} & -\frac{84}{17} \\ 0 & 0 & 0 & -\frac{453}{68} - \frac{453}{17} \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{23}{68} & \frac{142}{17} \\ 0 & 1 & -\frac{25}{34} & -\frac{84}{17} \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{-\frac{68}{453}R_3 \rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{R_1 - \frac{23}{68}R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -\frac{25}{34} - \frac{84}{17} \\ 0 & 0 & 1 & 4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{R_2 + \frac{25}{34}R_3 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

$$\therefore x = 7, y = -2, z = 4$$

Question 2

$$A = \begin{pmatrix} 5 & -4 & 1 \\ -1 & 3 & -1 \\ -2 & -2 & 1 \end{pmatrix}$$

$$\left[\begin{array}{ccc|ccc} 5 & -4 & 1 & 1 & 0 & 0 \\ -1 & 3 & -1 & 0 & 1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-R_2 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 5 & -4 & 1 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 & -1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & -1 & 0 \\ 5 & -4 & 1 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - 5R_1 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & -1 & 0 \\ 0 & 11 & -4 & 1 & 5 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 + 2R_1 \rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & -1 & 0 \\ 0 & 11 & -4 & 1 & 5 & 0 \\ 0 & -8 & -3 & 0 & -2 & 1 \end{array} \right] \xrightarrow{\frac{1}{11}R_2 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & -1 & 0 \\ 0 & 1 & -\frac{4}{11} & \frac{1}{11} & \frac{5}{11} & 0 \\ 0 & -8 & -3 & 0 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 + 3R_2 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{11} & \frac{3}{11} & \frac{4}{11} & 0 \\ 0 & 1 & -\frac{4}{11} & \frac{1}{11} & \frac{5}{11} & 0 \\ 0 & -8 & -3 & 0 & -2 & 1 \end{array} \right] \xrightarrow{R_3 + 8R_2 \rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{11} & \frac{3}{11} & \frac{4}{11} & 0 \\ 0 & 1 & -\frac{4}{11} & \frac{1}{11} & \frac{5}{11} & 0 \\ 0 & 0 & \frac{1}{11} & \frac{8}{11} & \frac{18}{11} & 1 \end{array} \right]$$

$$\xrightarrow{11R_3 \rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{11} & \frac{3}{11} & \frac{4}{11} & 0 \\ 0 & 1 & -\frac{4}{11} & \frac{1}{11} & \frac{5}{11} & 0 \\ 0 & 0 & 1 & 8 & 18 & 11 \end{array} \right] \xrightarrow{R_1 + \frac{1}{11}R_3 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & -\frac{4}{11} & \frac{1}{11} & \frac{5}{11} & 0 \\ 0 & 0 & 1 & 8 & 18 & 11 \end{array} \right]$$

$$\xrightarrow{R_2 + \frac{4}{11}R_3 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 3 & 7 & 4 \\ 0 & 0 & 1 & 8 & 18 & 11 \end{array} \right]$$

$$A^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 4 \\ 8 & 18 & 11 \end{pmatrix}$$

Question 4

$$\begin{bmatrix} 2 & 5 & -2 & 4 \\ 5 & -2 & 3 & -1 \\ 3 & -4 & -5 & 7 \\ -7 & 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -5 \\ 7 \end{bmatrix}$$

$$A \cdot x = b$$

$$|A| = \begin{vmatrix} 2 & 5 & 0 & -2 & 4 \\ 5 & -2 & 3 & -1 \\ 3 & -4 & -5 & 7 \\ -7 & 4 & -2 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= (2)[(-2)[(-5)(1) - (-2)(7)] - (3)[(-4)(1) - (4)(7)] + (-1)[(-4)(-2) - (4)(-5)]] - (5)[(5)[(-5)(1) - (7)(-2)] \\
 &\quad - (3)[(3)(1) - (-7)(7)] + (-1)[(3)(-2) - (-5)(-7)]] + (-2)[(5)[(-4)(1) - (4)(7)] - (-2)[(3)(1) - \\
 &\quad (7)(-7)] + (-1)[(3)(4) - (-4)(-7)]] - (4)[(5)[(-4)(-2) - (-5)(4)] - (-2)[(3)(-2) - (-7)(-5)] \\
 &\quad + (3)[(3)(4) - (-7)(-4)]]] \\
 &= 100 - (-350) + 80 - 40 \\
 &= 490
 \end{aligned}$$

$$A_1 = \begin{pmatrix} 2 & 5 & -2 & 4 \\ 3 & -2 & 3 & -1 \\ -5 & -4 & -5 & 7 \\ 7 & 4 & -2 & 1 \end{pmatrix}$$

$$|A_1| = \begin{vmatrix} 2 & 5 & -2 & 4 \\ 3 & -2 & 3 & -1 \\ -5 & -4 & -5 & 7 \\ 7 & 4 & -2 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= (2)[(-2)[(-5)(1) - (-2)(7)] - (3)[(-4)(1) - (4)(7)] + (-1)[(-4)(-2) - (4)(-5)]] - (5)[(3)[(-5)(1) - (-2)(7)] \\
 &\quad - (3)[(-5)(1) - (-7)(7)] + (-1)[(-5)(-2) - (-5)(7)]] + (-2)[(3)[(-4)(1) - (7)(4)] - (-2)[(-5)(1) - (7)(7)] \\
 &\quad + (-1)[(-5)(4) - (-4)(7)]] - (4)[(3)[(-4)(-2) - (-5)(4)] - (-2)[(-5)(-2) - (-7)(-5)] + (3)[(-5)(4) - \\
 &\quad (7)(-4)]]]
 \end{aligned}$$

$$\begin{aligned}
 &= (2)(-18 - (-96) + (-28)) - (5)(27 - (-162) + (45)) + (-2)(-96 - 108 + (-8)) - 4(84 \\
 &\quad - (-90) + 24)
 \end{aligned}$$

$$= 100 - 720 + 424 - 792$$

$$= -988$$

$$A_2 = \begin{pmatrix} 2 & 2 & -2 & 4 \\ 5 & 3 & 3 & -1 \\ 3 & -5 & -5 & 7 \\ -7 & 7 & -2 & 1 \end{pmatrix}$$

$$|A_2| = \begin{vmatrix} 2 & 2 & -2 & 4 \\ 5 & 3 & 3 & -1 \\ 3 & -5 & -5 & 7 \\ -7 & 7 & -2 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= (2) [(3)[(-5)(1) - (-2)(7)] - (3)[(-5)(1) - (7)(7)] + (-1)[(-5)(-2) - (-5)(7)]] - \\
 &\quad (2) [(5)[(-5)(1) - (-2)(7)] - (3)[(3)(1) - (-7)(7)] + (-1)[(3)(-2) - (-5)(-7)]] + \\
 &\quad (-2) [(5)[(-5)(1) - (7)(7)] - (3)[(3)(1) - (7)(-7)] + (-1)[(3)(7) - (-7)(-5)]] - \\
 &\quad (4) [(5)[(-5)(-2) - (7)(-5)] - (3)[(3)(-2) - (-5)(-7)] + (3)[(3)(7) - (-5)(-7)]] \\
 &= (2)(27 + 162 - 45) - (2)(45 - 156 + 41) + (-2)(-270 - 156 + 14) - (4)(225 \\
 &\quad + 123 - 42) \\
 &= 288 - (-140) + 824 - 1224 \\
 &= 28
 \end{aligned}$$

$$A_3 = \begin{pmatrix} 2 & 5 & 2 & 4 \\ 5 & -2 & 3 & -1 \\ 3 & -4 & -5 & 7 \\ -7 & 4 & 7 & 1 \end{pmatrix}$$

$$|A_3| = \begin{vmatrix} 2 & 5 & 2 & 4 \\ 5 & -2 & 3 & -1 \\ 3 & -4 & -5 & 7 \\ -7 & 4 & 7 & 1 \end{vmatrix}$$

$$\begin{aligned}
 &= (2) [(-2)[(-5)(1) - (7)(7)] - (3)[(-4)(1) - (7)(4)] + (-1)[(-4)(7) - (4)(-5)]] - (5)[(5) \\
 &\quad [(-5)(1) - (7)(7)] - (3)[(3)(1) - (-7)(7)] + (-1)[(3)(7) - (-7)(-5)]] + (2)[(5)[(-4)(1) - \\
 &\quad (4)(7)] - (-2)[(3)(1) - (-7)(7)] + (-1)[(3)(4) - (-7)(-4)]] - (4)[(5)[(-4)(7) - (-5)(4)] - \\
 &\quad (-2)[(3)(7) - (-7)(-5)] + (3)[(3)(4) - (-7)(-4)]] \\
 &= 2(108 - (-96) + 8) - 5(-270 - 156 + 14) + 2(-160 - (-104) + 16) - 4(-40 - 28 + \\
 &\quad (-48)) \\
 &= 424 - (-2060) + (-80) - (-464) \\
 &= 2868
 \end{aligned}$$

$$A_4 = \begin{pmatrix} 2 & 5 & -2 & 2 \\ 5 & -2 & 3 & 3 \\ 3 & -4 & -5 & -5 \\ -7 & 4 & -2 & 7 \end{pmatrix}$$

$$|A_4| = \begin{vmatrix} 2 & 5 & -2 & 2 \\ 5 & -2 & 3 & 3 \\ 3 & -4 & -5 & -5 \\ -7 & 4 & -2 & 7 \end{vmatrix}$$

$$\begin{aligned}
 &= (2) [(-2)[(-5)(7) - (-2)(-5)] - (3)[(-4)(7) - (4)(-5)] + (3)[(-4)(-2) - (4)(-5)]] - \\
 &\quad (5) [(5)[(-5)(7) - (-2)(-5)] - (3)[(3)(7) - (-7)(-5)] + (3)[(3)(-2) - (-7)(-5)]] + \\
 &\quad (-2)[(5)[(-4)(7) - (4)(-5)] - (-2)[(3)(7) - (-7)(-5)] + (3)[(3)(4) - (4)(-7)]] - \\
 &\quad (2)[(5)[(-4)(-2) - (-5)(4)] - (-2)[(3)(-2) - (-7)(-5)] + (3)[(3)(4) - (-7)(-4)]] - \\
 &= (2)(90 - (-24) + 84) - (5)(-225 - (-42) + (-123)) + (-2)(-40 - 28 + (-48)) - (2)(140 \\
 &\quad - 82 + (-48))
 \end{aligned}$$

$$= 396 - (-1530) + 232 - 20$$

$$= 2138$$

$$\lambda_1 = \frac{\det(A_1)}{\det(A)} = \frac{-988}{490} = -2.0163$$

$$\lambda_2 = \frac{\det(A_2)}{\det(A)} = \frac{28}{490} = 0.0571$$

$$\lambda_3 = \frac{\det(A_3)}{\det(A)} = \frac{2868}{490} = 5.8531$$

$$\lambda_4 = \frac{\det(A_4)}{\det(A)} = \frac{2138}{490} = 4.3633$$

$$\therefore \lambda_1 = -2.0163, \lambda_2 = 0.0571, \lambda_3 = 5.8531, \lambda_4 = 4.3633$$

Question 5

i. Let Munich = x , Venice = y , Barcelona = z

$$200x + 150y + 185z = 1500$$

$$170x + 150y + 190z = 1300$$

$$x + y - z = 0$$

$$\left[\begin{array}{ccc|c} 200 & 150 & 185 & 1500 \\ 170 & 150 & 190 & 1300 \\ 1 & 1 & -1 & 0 \end{array} \right]$$

Question 6

$$i. Ali \rightarrow 50 + x_1 = x_2 + x_3$$

$$Raju \rightarrow x_2 + 12 = x_1 + x_5$$

$$Han \rightarrow x_1 + x_3 = 2x_2 + 30$$

$$Chin \rightarrow x_5 + 30 = x_1 + x_4$$

$$Abu \rightarrow 2x_2 + x_4 = 50 + 12$$

$$x_1 - x_2 - x_3 = -50$$

$$-x_1 + x_2 - x_5 = -12$$

$$x_1 - 2x_2 + x_3 = 30$$

$$-x_1 - x_4 + x_5 = -30$$

$$2x_2 + x_4 = 62$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & 0 & -50 \\ -1 & 1 & 0 & 0 & -1 & -12 \\ 1 & -2 & 1 & 0 & 0 & 30 \\ -1 & 0 & 0 & -1 & 1 & -30 \\ 0 & 2 & 0 & 1 & 0 & 62 \end{array} \right]$$