HIGH PERFORMANCE COMPUTING for SCIENCE & ENGINEERING (HPCSE) I

TUTORIAL 05: MC integration, Accept-Reject Sampling

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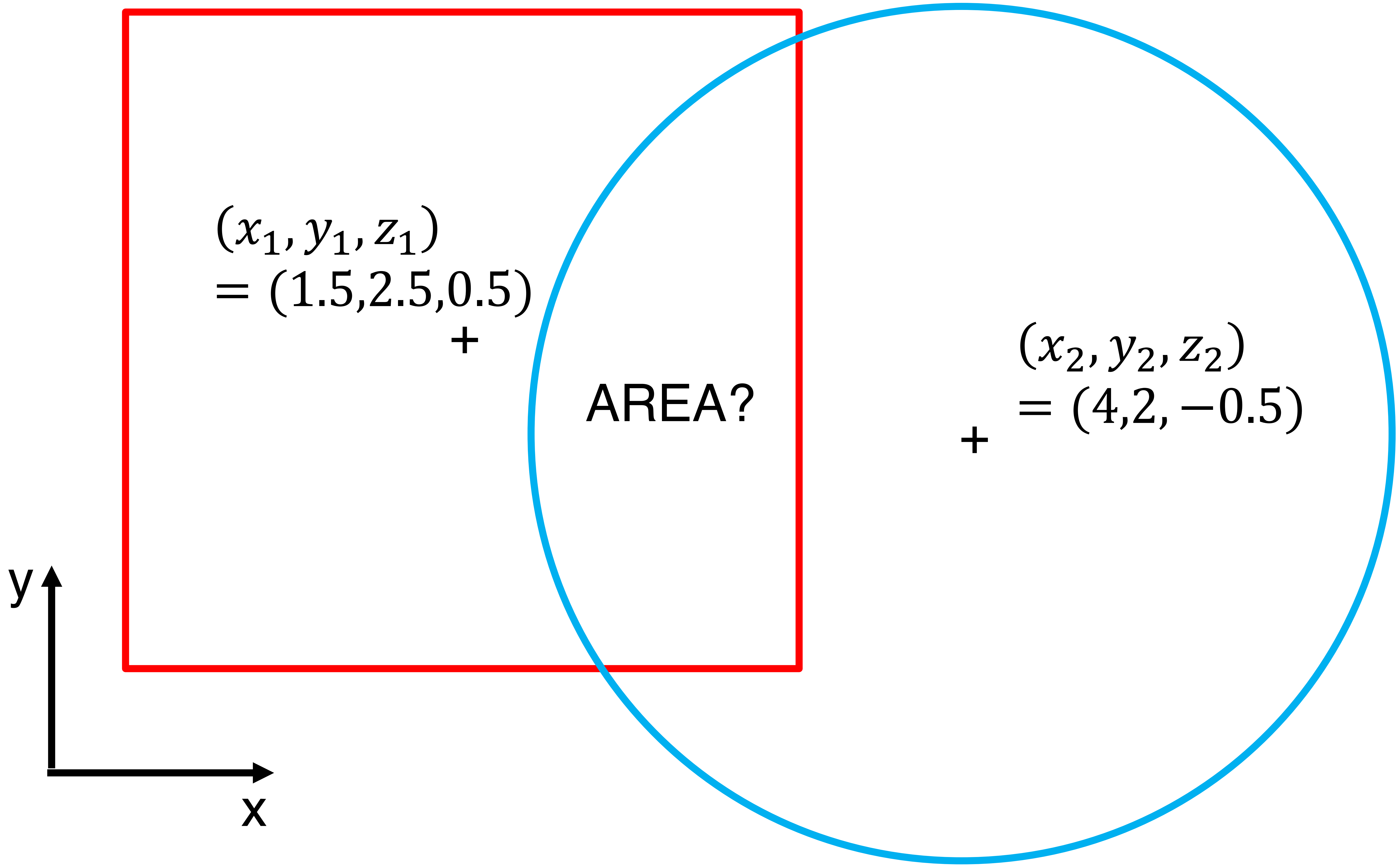
Outline

- Monte-Carlo integration
- Accept-Reject sampling
- Uniform distribution samples in a circle

3D Monte-Carlo integration

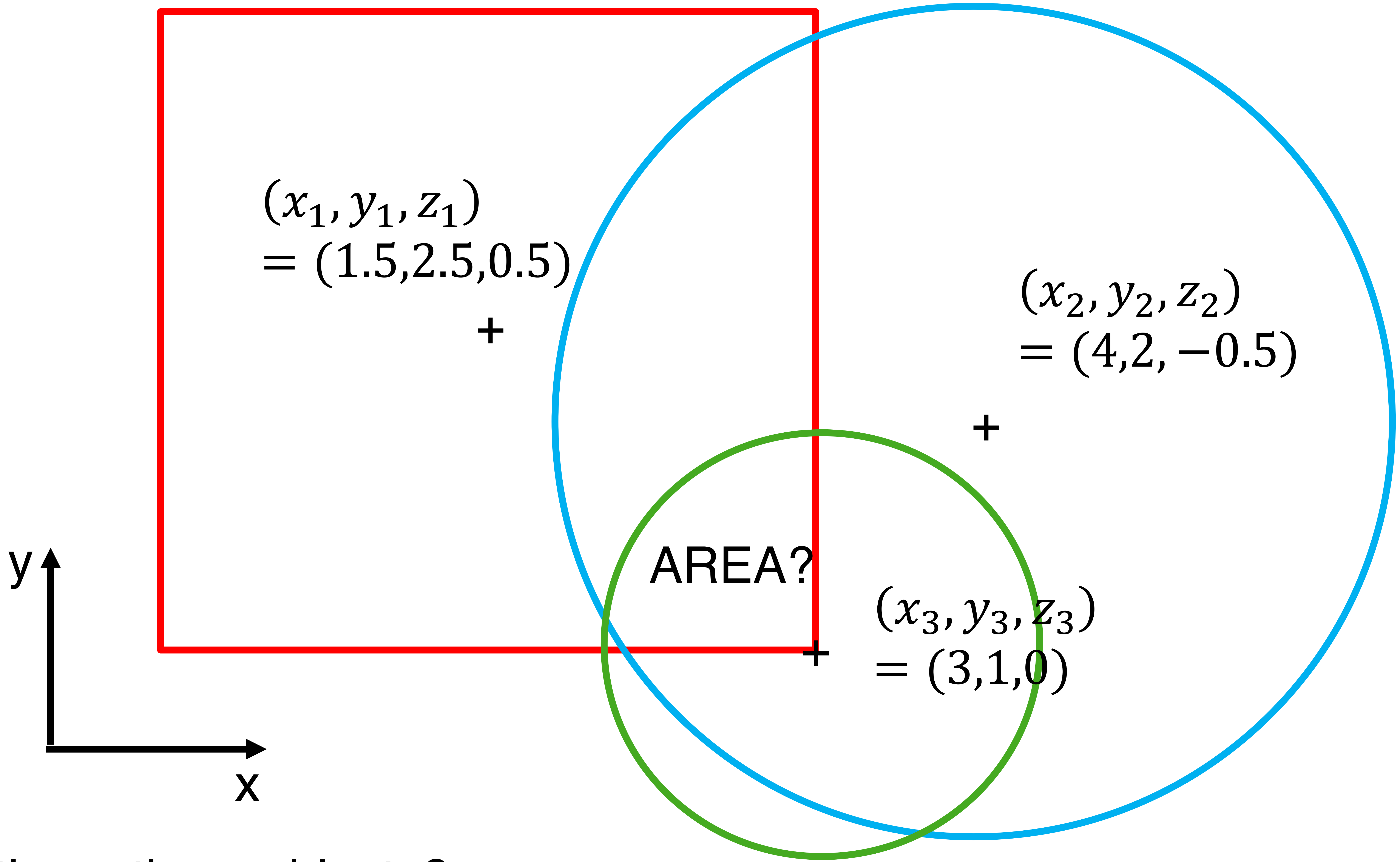
- square side length = 3
- circle radius = 2



What is the intersecting are of these three objects?

3D Monte-Carlo integration

- square side length = 3
- circle radius = 2
- circle radius = 1



What is the intersecting are of these three objects?

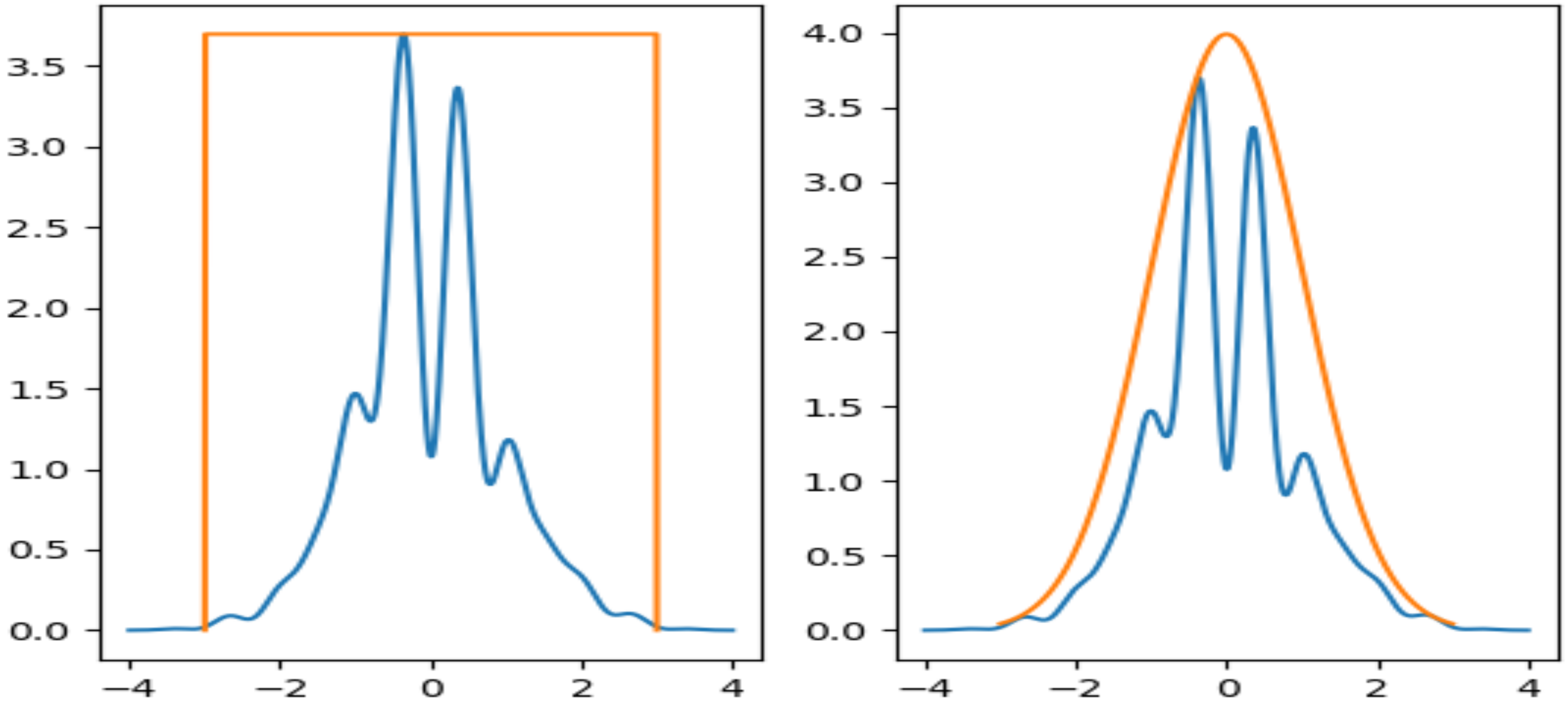
Accept-Reject Sampling

Assume we want to integrate a function in 1-dimension $f\colon \Omega \subseteq \mathbb{R} \to I \subseteq \mathbb{R}$

1. Find simple distribution $h(x)$, define λ s.t. $f(x) \le \lambda h(x)$
2. Draw sample $\mathbf{x}^{(i)}$ from $h(\mathbf{x}^{(i)})$ and $u^{(i)} \sim \mathcal{U}_{[0,1]}$ for $i = 1, \dots, n$
3. Accept, if $u^{(i)} < f(\mathbf{x}^{(i)})/\lambda h(\mathbf{x}^{(i)})$, otherwise reject the sample
4. Compute Monte Carlo Estimate of the Integral $I \approx |\Omega_{\lambda h(x)}| \dfrac{\#accepted\ samples}{n}$

We first do sample from $\mathrm{h}(\mathrm{x}) \sim \mathcal{U}[-3,3]$

Then $h(x) \sim N(0,1) \in [-3,3]$



Uniform distribution samples in a circle

- We look at two different ways to create random samples:

1. Accept-Reject Sampling:

$$x, y \sim \mathcal{U}[-R, R]$$
accept samples if:
$$\sqrt{x^2 + y^2} \leq R$$

2. Using polar coordinates:

$$r \sim \mathcal{U}[0, R] \;\&\; \phi \sim \mathcal{U}[0, 2\pi]$$
$$x = r * \cos(\phi)$$
$$y = r * \sin(\phi)$$

sampling u[0,r^2]
then sqrt(u) when draw this points.