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Tutorial Session : N-body solver with MPI

In this task you will implement an N-body solver describing a system of positively charged inertialess particles with MPI parallelization. The provided skeleton code

- seeds N particles on a unit circle

$$\boldsymbol{x}_i = (x_i, y_i) = \big(\cos\frac{2\pi i}{N},\ \sin\frac{2\pi i}{N}\ \big), \qquad i = 0, \ldots, N-1;$$

  distributing them over P ranks such that each rank takes N/P particles (assume N is divisible by P);

- performs multiple time steps calling functions `Step()` and `PrintStat()`;

- writes the particle positions to files `init.dat` and `final.dat`.

a) Implement `Step()` which computes the forces and advances the particles

$$\boldsymbol{f}_i = \sum_{\substack{j=0 \\ j\neq i}}^{N-1} \frac{\boldsymbol{x}_i - \boldsymbol{x}_j}{|\boldsymbol{x}_i - \boldsymbol{x}_j|^3},$$
$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \Delta t\, \boldsymbol{f}_i^t,$$

where $|\boldsymbol{x}| = \sqrt{x^2 + y^2}$ and Δt is a given time step.

Each rank can make $\mathcal{O}(P)$ MPI calls[1] and use $\mathcal{O}(N/P)$ bytes of memory. Your solution will be given more points for overlapping communication and computation. You may use `make` to build the executable, `make run` to run it on 2 processors and `make plot` to create `image.png` with particle positions as shown in Figure 1.

[1] Notation $p = \mathcal{O}(q)$ means that $p < Mq$ for a large enough constant M.

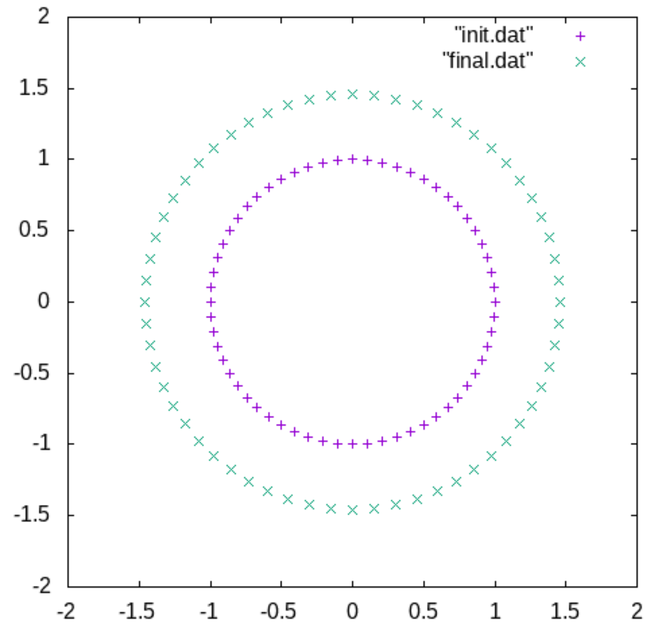

Figure 1: Expected output `image.png` produced by `make plot`.

b) Implement `PrintStat()` which computes and prints the mean and variance of the radial distance $r_i = |\boldsymbol{x}_i|$

$$\mu = \frac{1}{N} \sum_{i=0}^{N-1} r_i,$$

$$\sigma^2 = \frac{1}{N} \sum_{i=0}^{N-1} r_i^2 - \mu^2.$$

Expected output after 10 time steps:

```
mean=1          var=0
mean=1.0672     var=6.6613e-16
mean=1.1261     var=6.6613e-16
mean=1.1791     var=0
mean=1.2274     var=-8.8818e-16
mean=1.272      var=8.8818e-16
mean=1.3135     var=-2.2204e-16
mean=1.3524     var=4.4409e-16
mean=1.3891     var=4.4409e-16
mean=1.4239     var=1.3323e-15
mean=1.4571     var=0
```

The values may depend on P and the implementation but the absolute difference should be below 10^{-3} for the mean and 10^{-14} for the variance.