Nonlinear Dynamics and Chaos I

151-0532-00, Fall 2022

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Course outline

- **Basic facts about nonlinear systems**

  Existence, uniqueness, dependence on initial data

- **Near equilibrium dynamics**

  Linear and Lyapunov stability

- **Bifurcations of equilibria**

  Center manifolds, normal forms, elementary bifurcations

- **Nonlinear dynamical systems on the plane**

  Phase plane techniques, limit sets, limit cycles

- **Time-dependent dynamical systems**

  Floquet theory, Poincare maps, averaging methods, resonance

- **Chaotic dynamics**

  Homoclinic dynamics, Melnikov's method, attractors, Lyapunov exponents

- ***Recommended books:***
- **J. Guckenheimer & P. Holmes: *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields***
- ***F. Verhulst, Nonlinear Differential Equations and Dynamical Systems***
- ***V. I. Arnold, Ordinary Differential Equations***
- ***S. Strogatz, Nonlinear Dynamics and Chaos***

Logistics

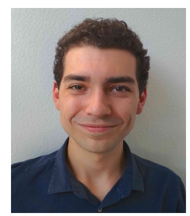
Course information on Moodle: https://moodle-app2.let.ethz.ch/course/view.php?id=18052

Prerequisites: Analysis and a basic course in differential equations.

Exam:
-- A written, two-hour exam in English (see the Moodle page for prior exams and solutions)
-- Time and location will be announced soon on the Moodle page

Problem sets:
- Will be discussed in problem sessions (watch for related announcements)
- You are strongly encouraged to solve the problems before the problem sessions (see the course Moodle page for the problems and their solutions)

In-class lectures: (no streaming or recording)
Tues. 16:15-17:45 Location: **HG D7.2**
Wed. 10:15-11:45 Location: **HG D7.1**



Problem sessions: (no streaming or recording)
Mon 17:15-18:00; Location: **NO C.44**
by Mr. Bálint Kaszás

Office hours: (no streaming or recording)
Thurs 16:00-17:30; Location: **LEE M.214**
by Mr. Joar Axås.

Motivation

- Complex systems can show simple behavior (after 605 trials)

Honda: The Cog

https://www.youtube.com/watch?v=_ve4M4UsJQo

Motivation

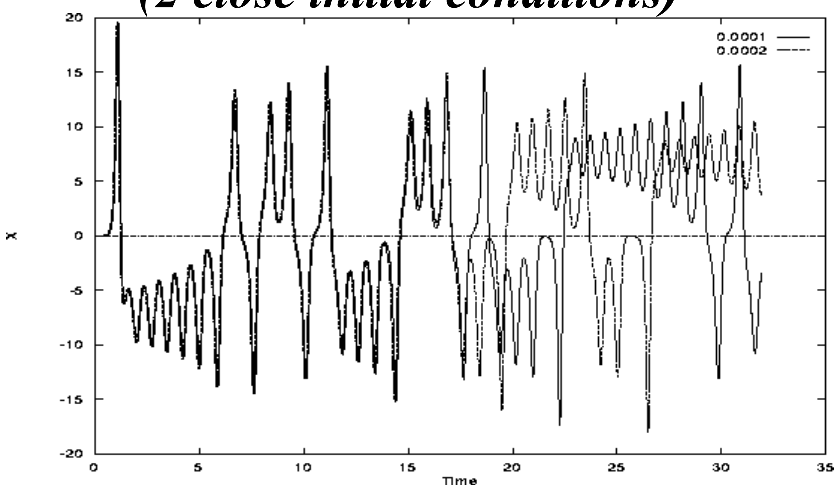
- Simple systems may show complex behavior (right away)

  A model for atmospheric convection (Lorenz [1963]):

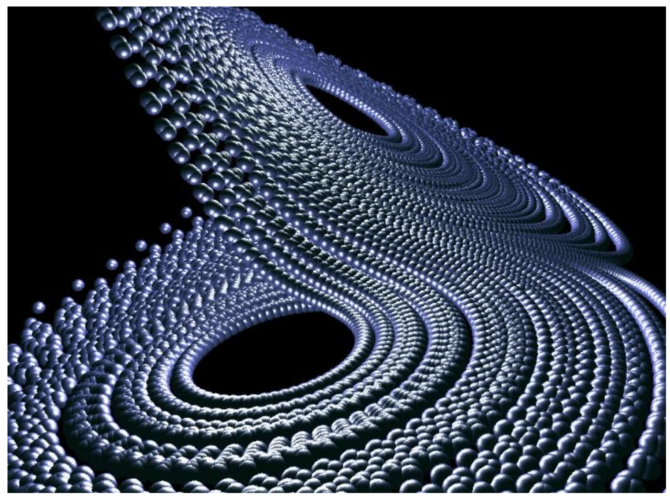
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y-x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

$(x, y, z):$ amplitudees of velocity modes

Time history of one variable (2 close initial conditions)



Trajectories in (x,y,z) space



Objectives

- Learn methods to analyze nonlinear dynamics without solving the underlying differential equations
- Develop intuition for geometry of nonlinear systems through numerical assignments
- Learn a few important techniques from applied mathematics
- Analyze applications from various areas of engineering and applied science