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# Resonant axion-photon conversion in magnetized plasma

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It is demonstrated that the invisible axion, and its relatives (familon, etc.), can be converted via resonance to the photon in a strongly magnetized plasma.

The axion,[1] postulated to solve a major theoretical problem of the standard model (violation of $CP$ invariance at the strong-interaction level), has been elusive despite intensive experimental efforts.[2] A variant of axion models predicts a very weakly interacting pseudoscalar particle: the invisible axion,[3] which is essentially undetectable in laboratories, but in principle can have important astrophysical[4] and cosmological consequences.[5] So far only bounds on the Peccei-Quinn symmetry-breaking scale $f_A$ have been derived from astrophysical and cosmological arguments: $4\times 10^9 < f_A < 2\times 10^{12}$ GeV in the standard model.

An interesting feature of the invisible-axion model is the presence of an induced axion-photon-photon coupling in the form of $ca\mathbf{E}\cdot\mathbf{B}$ ($a=$axion field, $\mathbf{E}=$ electric field, $\mathbf{B}=$magnetic field), where the coupling $c$ is determined[6] by the current-algebra method and found to be fairly insensitive to details of the model, roughly of the order of $\alpha/f_A$. This term has been used in a proposal[7] for experimental search of the halo axion presumed to explain the dark matter in our Galaxy. Indeed, some experiments of this kind are already under way.[8]

In this work we shall point out that the same vertex of $a\gamma\gamma$ gives rise to interesting oscillation and conversion phenomena between the axion and the photon under a slowly varying magnetic field. In a magnetized plasma a resonant conversion, very similar to the Mikheyev-Smirnov-Wolfenstein (MSW) effect[9] for neutrinos, can occur. A nonresonant conversion of the axion into an x ray in the magnetosphere of a neutron star has been considered by Morris,[10] who discussed a possibility of deriving a new limit of $f_A$, but the resonant conversion proposed below has never been considered elsewhere. Needless to say, our process depends solely on the presence of $a\gamma\gamma$ term, and may also be applied to various kinds of pseudoscalars similar to the invisible axion.

After submitting the original version of this work, we learned that Raffelt and Stodolsky[11] have independently obtained a similar result, including QED higher orders neglected in our previous treatment. In this modified version we shall also include the QED effect, whose numerical importance somewhat differs from Ref. 11.

Consider an effective Lagrangian of the axion-photon system in a magnetized plasma under a static magnetic field $\mathbf{B}_0$:

$$L=\tfrac{1}{2}\mathbf{E}\cdot\epsilon\cdot\mathbf{E}-\tfrac{1}{2}\mathbf{B}^2+\tfrac{1}{2}(\partial_0 a)^2-\tfrac{1}{2}(\nabla a)^2-\tfrac{1}{2}m^2a^2 -ca\mathbf{E}\cdot\mathbf{B}_0 \ . \tag{1}$$

The axion parameters, the mass $m$ and the coupling $c$, are determined by the current algebra to yield[6] for six quark flavors

$$m=0.72\times 10^{-4}\ \text{eV}\ f_{12}^{-1}\ , \tag{2a}$$

$$c=1.0\times 10^{-14}\ \text{GeV}^{-1}\ \delta f_{12}^{-1}\ , \tag{2b}$$

with $f_{12}\equiv f_A/10^{12}$ GeV. The axion model of Dine, Fischler, and Srednicki[3] (DFS) gives $\delta=1$, while hadronic axion models[3,6] yield model-dependent factors of $\delta=10^{-2}-10$. The dielectric tensor $\epsilon$ in a cold plasma is diagonal in an orthonormal frame in which the $z$ axis is parallel to $\mathbf{B}_0$, and other bases of $e_\pm=(e_x\pm ie_y)/\sqrt{2}$ are taken:[12,13]

$$\epsilon_\pm=1-\frac{\omega_p^2}{\omega^2}\frac{1}{1\pm\sqrt{u}}+\frac{2\alpha}{45\pi}\left[\frac{B_0}{B_{\text{cr}}}\right]^2 ,$$
$$\epsilon_z=1-\frac{\omega_p^2}{\omega^2}+\frac{7\alpha}{90\pi}\left[\frac{B_0}{B_{\text{cr}}}\right]^2 , \tag{3}$$

$$\omega_p^2=4\pi\alpha N_e/m_e\sim[37\ \text{eV}(N_e/10^{24}\ \text{cm}^{-3})^{1/2}]^2\ , \tag{4a}$$

$$u=(\omega_B/\omega)^2, \omega_B=\sqrt{\alpha}B_0/m_e\sim 12\ \text{keV}\ B_{12}\ , \tag{4b}$$

$$B_{\text{cr}}=m_e^2/\sqrt{4\pi\alpha}\sim 4.4\times 10^{13}\ \text{G}\ , \tag{4c}$$

with $\omega$ the photon energy, $N_e$ the electron number density, and $B_{12}=B_0/10^{12}$ G. The last term in (3) arises from the nonlinear Euler-Heisenberg Lagrangian[13] due to

QED higher orders in vacuum.

It is a straightforward matter to derive equations of motion from (1). In terms of the Fourier components $\propto \exp[-i\omega(t - n\hat{\mathbf{k}}\cdot\mathbf{x})]$

$$\epsilon\cdot\mathbf{E} - n\hat{\mathbf{k}}\times\mathbf{B} = c\mathbf{B}_0 a \ , \tag{5a}$$

$$-\omega^2(1-n^2)a + m^2 a = -c\mathbf{B}_0\cdot\mathbf{E} \ , \tag{5b}$$

where $n(\omega)$ is the index of refraction and $\hat{\mathbf{k}}$ the unit propagation vector. We assume here that the magnetic field $\mathbf{B}_0$ is constant everywhere: a reasonable approximation in a slowly varying magnetic field. Using the Gauss-law-type constraint $\hat{\mathbf{k}}\cdot\epsilon\cdot\mathbf{E} = c\hat{\mathbf{k}}\cdot\mathbf{B}_0 a$, derivable from (5a), one obtains a dispersion relation for the refractive index from the determinant formula:

$$\frac{(n^2-n_+^2)(n^2-n_-^2)(n^2-n_a^2)}{n^2(n^2-n_0^2)} = \xi \ , \tag{6}$$

where $n_\pm^2$ are photon refractive indices with the coupling term ($\propto c$) switched off. The mode with $n_+$ is called the ordinary mode, and the one with $n_-$ is called the extraordinary mode in the literature.[12] In the absence of the coupling $\xi = 0$ in (6), and the system decouples into the photon modes ($n_\pm^2$) and the axion mode ($n_a^2$).

For propagation of the wave along a direction perpendicular to the magnetic field, the polarization field $E_\pm$ orthogonal to $\mathbf{B}_0$ decouples from $E_z$ and $a$. Coupled field components yield two eigenmodes with refractive indices given by

$$n_{1,2}^2 = \frac{1}{2}\left[\epsilon_z + 1 - \frac{m^2}{\omega^2}\right] \pm \frac{1}{2}\left[\left[\epsilon_z - 1 + \frac{m^2}{\omega^2}\right]^2 + 4\left[\frac{cB_0}{\omega}\right]^2\right]^{1/2} . \tag{7}$$

Even for a small coupling $cB_0$ a resonant mixing of modes may occur if the unmixed system develops a degeneracy. In our problem at hand this takes place at a particular location satisfying $n_a^2 = n_+^2$ or $n_-^2$. For the perpendicular propagation given by (7) this degeneracy condition, $\epsilon_z = 1 - m^2/\omega^2$, leads to

$$\omega_p^2 = m^2 + \frac{7\alpha}{90\pi}\left[\frac{B_0}{B_{cr}}\right]^2 \omega^2 . \tag{8}$$

For a small magnetic field this equation gives a resonance condition, $\omega_p^2 = m^2$, implying that

$$N_e = \frac{m^2 m_e}{4\pi\alpha} \sim 3.8\times10^{12}\ \mathrm{cm}^{-3}\ f_{12}^{-2} . \tag{9}$$

The resonance condition (9) can be understood as follows. In an isotropic plasma with $\omega \gg \omega_B$, the refractive index changes by an amount $\delta n = 2\pi N_e f(\omega)/\omega^2$, which yields $-2\pi N_e\alpha/(m_e\omega^2)$, taking the Thomson value for the forward-scattering amplitude. Equating the phase factor $\omega\delta n$ to the axion phase $-m^2/(2\omega)$ gives the energy-independent relation (9) for the degeneracy.

Near the resonance the level crossing phenomenon occurs in much the same way as in the MSW mechanism.[9] We may approximate the analysis of the general problem by restricting ourselves to the relevant two levels, very similar to the case of the perpendicular propagation. It is thus justified to replace (6) by a simpler relation such as $(n^2-n_-^2)(n^2-n_a^2) \simeq \xi^2$. This two-level system can fully be analyzed by elementary methods, leading to two phase eigenstates defined by $|X\rangle = \cos\theta|-\rangle - \sin\theta|a\rangle$ and $|Y\rangle = \sin\theta|-\rangle + \cos\theta|a\rangle$, with the mixing angle $\theta$ given by $\tan 2\theta = 2\xi/(n_a^2 - n_-^2)$. Suppose then that the axion denoted by $|a\rangle$ is created with an energy $\omega$, at some point in compact objects, and propagates in a surrounding magnetized plasma region characterized by an electron density $N_e(r)$, slowly decreasing as it advances. At the resonant density $N_e$ satisfying (8) the mixing with the photon mode becomes maximal, $\theta = \pi/4$, and the wave gradually gets dominated by the photon component, eventually leaving the plasma at $B_0 = 0$. The state initially created at $\theta \sim 0+$ thus emerges as another state at $\theta \sim \pi/2$: conversion of $|a\rangle \to |-\rangle$.

The picture of the resonant conversion presented above holds under three premises: (1) small mixing at production site of the axion, (2) subsequent adiabatic change of the electron density, and (3) absence of incoherent scattering of transformed photons. We shall discuss these in sequence. The small mixing at production is ensured by $\xi < \frac{1}{2}|n_a^2 - n_-^2|$. Using formulas valid for $\omega \gg \omega_p$, one obtains a restriction roughly like (with $\omega_{\mathrm{keV}} = \omega/\mathrm{keV}$)

$$B_{12}\omega_{\mathrm{keV}} < 20 f_{12}^{-1}\delta^{-1} . \tag{10}$$

Adiabatic conversion takes place only if the rate of change of the mixing angle $\theta$ is small compared to the oscillation frequency: $|d\theta/dr| < \frac{1}{2}\omega|n_X - n_Y|$. A better quantitative estimate of the adiabaticity is given by multiplying both sides of this equation by $2/\pi$, because the inverse of this combination at the resonance is precisely the factor that appears in the exponent giving the transition probability between adiabatic states: the Landau-Zener formula.[14] This condition yields

$$\left|\frac{d}{dr}\ln N_e\right|^{-1} > \frac{7\alpha}{45\pi^2}\omega(cB_{\mathrm{cr}})^{-2} , \tag{11}$$

for the field configuration orthogonal to the wave propagation and for a large magnetic field of $B_{12}\omega_{\mathrm{keV}} f_{12} > 2\times10^{-4}$. Numerically this gives a length scale, $\sim 300$ km, for $\omega = 1$ keV and $c^{-1} = 3\times10^9$ GeV relevant to the bound on hadronic axions.[15] For a small magnetic field of $B_{12}\omega_{\mathrm{keV}} f_{12} < 2\times10^{-4}$, the adiabatic condition (11) should be replaced by

$$\left|\frac{d}{dr}\ln N_e\right|^{-1} > \frac{2}{\pi\omega}\left[\frac{m}{cB_0}\right]^2 \sim 0.8\times10^4\ \mathrm{km}\ \omega_{\mathrm{keV}}^{-1} B_{12}^{-2}\delta^{-2} . \tag{12}$$

Coherence of the wave propagation is crucial in our discussion. If an incoherent interaction between the photon and the plasma medium, such as Thomson scattering, occurs, the resonant conversion and subsequent emergence of the photon out of the plasma will not take place as described here. This coherence gives an additional constraint that does not have to be considered in the case of the neutrino oscillation. The Thomson scattering gives a bound on the scale height:

$$\left| \frac{d}{dr} \ln N_e \right|^{-1} < 1.5\times 10^7 \text{ km}(N_e/10^{12} \text{ cm}^{-3})^{-1} . \quad (13)$$

A place one can think of for immediate application of the idea presented here is the magnetosphere near the surface of a neutron star, in which a strong magnetic field $\sim 10^{12}$ G, is available. The plasma condition surrounding an isolated neutron star is not well known, however. If one takes as an example the model of Goldreich and Julian,[16] the plasma density $N_e \sim 7\times 10^{10} B_{12}$ cm$^{-3}\times$ (pulsar period in s)$^{-1}$. In this case it is difficult to satisfy the resonance condition (8) for a realistic set of parameters such as periods ($\gtrsim 1$ ms) and x-ray energies ($\gtrsim 1$ keV). We have not found any other compact stars that easily realize the resonant axion-photon conversion.

In conclusion we formulated the problem of the resonant axion-photon conversion in a magnetized plasma, and clarified under what conditions the process is realized.

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