THE NOT-SO-HARMLESS AXION

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Cosmological aspects of a very weakly interacting axion are discussed. A solution to the problem of domain walls discussed by Sikivie is mentioned. Demanding that axions do not dominate the present energy density of the universe is shown to give an upper bound on the axion decay constant of at most 10^{12} GeV.

It has been suggested that the strong *CP* problem may be solved by extending the Peccei–Quinn idea [1] to grand unified theories [2–4]. In other words, one should require of a grand unified theory that it possesses a U(1) symmetry broken explicitly only by anomalies. Moreover, this symmetry should be broken spontaneously at the unification scale. In such a theory, the *CP* violating angle, θ, becomes dynamical; it is the would-be Goldstone boson (axion) of this spontaneously broken symmetry. QCD gives rise to a potential for this angle with a minimum very near the origin. The axion itself receives a tiny mass as a result of this symmetry-violating potential. Its interactions are extremely weak, having a strength inversely proportional to f_A, the axion decay constant. In particular, this axion would not appear in experiments nor would it play any role in astrophysical environments such as stars.

However, this lack of interaction raises concerns of a cosmological nature. For temperatures much higher than QCD scales, we expect θ-dependent effects to fall as some large power of T [5]. Thus, at the grand-unified scale, the Peceei–Quinn symmetry will be an essentially exact symmetry. At temperatures above f_A, this symmetry will be unbroken; as the temperature is lowered, spontaneous symmetry breaking will occur, and θ will take some random value on the interval $[0, 2\pi]$ (actually, on a somewhat smaller interval; see below).

At temperatures well below the QCD scale, we can calculate the axion potential using current algebra arguments. For definiteness, we focus on the model of ref. [2], in which the Peccei–Quinn symmetry is broken by the expectation value of a singlet field (with Peccei–Quinn charge 1)

$$\langle\varphi\rangle = 2^{-1/2} f_A\, e^{i\theta} .$$

Following ref. [6], it is natural to work with the anomaly-free, partially conserved current (specializing for simplicity to the case of two light quarks; inclusion of the strange quark is straightforward)

$$\tilde{j}_\mu = j_\mu^{PQ} - [N/(1+Z)]\,(\bar{u}\gamma_\mu\gamma_5 u + Z\bar{d}\gamma_\mu\gamma_5 d) ,$$

where $Z = m_u/m_d$. We must also include the current of axial isospin,

$$j^3_{\mu A} = \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$$

Following a method discussed by Sikivie, we write the expectation values of fermion bilinears as

$$\langle\bar{u}_L u_R\rangle = |\langle\bar{u}_L u_R\rangle| \exp\{i\{\alpha + [N/(1+Z)]\,\theta\}\} ,$$

$$\langle\bar{d}_L d_R\rangle = |\langle\bar{d}_L d_R\rangle| \exp\{i\{-\alpha + [NZ/(1+Z)]\,\theta\}\} .$$

Then the effective potential for α and θ is obtained by replacing fields by their expectation values (using $\langle\bar{u}u\rangle \approx \langle\bar{d}d\rangle$)

$$V = |\langle\bar{u}u\rangle| \, [m_u \cos(\alpha + [N/(1+Z)]\theta) + m_d \cos(ZN\theta/(1+Z) - \alpha)] \,. \tag{1}$$

Here αf_π is the pion field and θf_A is the axion field. Expanding to second order in the π^0 and axion fields, the cross terms cancel (this is the reason for the choice of Z) we find the usual results:

$$f_\pi^2 m_\pi^2 = (m_u + m_d)\langle\bar{u}u\rangle \,,$$

$$f_A^2 m_A^2 = [Z/(1+Z)^2] N^2 m_\pi^2 f_\pi^2 \,.$$

As Sikivie has emphasized, this potential has a discrete Z_N symmetry. It corresponds to the transformation

$$\theta \to \theta + 2\pi/N \,, \quad \alpha \to \alpha + 2\pi - 2\pi/(1+Z)$$

Modulo this discrete transformation, the minimum of the potential lies at $\theta = 0$ (it will, of course, acquire a tiny non-zero value once we include *CP* violating effects). However, the question arises, how quickly will θ approach its minimal value? In particular, since the axion is so weakly coupled to matter, there is virtually no damping term in the equation of motion for θ. If we neglect, for a moment, the expansion of the universe, θ obeys the equation (keeping, for simplicity, only the quadratic term in the potential),

$$\ddot{\theta} + \Gamma\dot{\theta} + m^2\theta = 0 \tag{2}$$

The only damping comes from the decay of the axion to two photons. The axion lifetime in vacuum is 10^{58} yr. In the early universe (essentially for any temperature greater than about 10 eV) in fact, it is not clear that the axion can decay at all! The reason is that for these temperatures the Debye frequency of the plasma is larger than the axion mass. In any case, Γ is *extremely* tiny. Thus if we neglect the expansion of the universe, θ simply rolls back and forth in the potential, eq. (1), essentially forever.

Of course, the universe does expand, and this will induce a significant decrease in θ with time. At temperatures small compared to QCD scales, the generally covariant version of eq. (2), in a Robertson–Walker metric, is

$$\ddot{\theta} + 3(\dot{R}/R)\dot{\theta} + m^2\theta = 0 \,. \tag{3}$$

This equation has the solution, for a radiation-dominated universe ($\dot{R}/R = 1/2t$)

$$\theta = t^{-1/4}[aJ_{1/4}(mt) + bJ_{-1/4}(mt)] \underset{t \gg m^{-1}}{\sim} \theta_0 (mt)^{-3/4} \sin mt \,, \tag{4}$$

For a matter-dominated universe ($\dot{R}/R = 2/3t$)

$$\theta = t^{-1/2}[a'J_{1/2}(mt) + b'J_{-1/2}(mt)] \underset{t \gg m^{-1}}{\sim} \theta_0 (mt)^{-1} \sin mt \tag{5}$$

The coefficients of the Bessel functions in these equations depend on the initial conditions which we impose. The asymptotic behavior, however, is not particularly sensitive to these conditions. The question of appropriate initial conditions will be described below, when we describe the axion potential at high temperatures.

The axion mass, of course, does not simply turn on at the phase transition, but presumably falls off, at high temperatures, as some power of the temperature. The θ-dependence of the free energy arising from instantons can, in fact, be calculated at high temperature and will fall as a high power of T [5]. For example, for three light quarks, instanton calculations give a result for the free energy

$$F \approx -C(\pi T)^{-8} \Lambda_{PV}^9 m_u m_d m_s \, [4\pi^2/g^2(T)]^6 \cos N\theta \,, \tag{6}$$

Here the constant, C, is obtained from a numerical integration of an expression for the instanton density due to Gross et al. [5]. We have made the estimate

$$C = 2.4 \times 10^{-2} \,. \tag{7}$$

Λ_{PV} is the QCD scale factor appropriate to the Pauli–Villars scheme. From ref. [7], we know $\Lambda_{PV} \approx \Lambda_{\overline{MS}}$. Also, in obtaining the expression (6), it should be noted that we have neglected the scale dependence of the $(1/g^2)^6$ factor in the integration over scale sizes.

Of course, at low temperatures, many effects (apart from instantons) presumably conspire to give the θ-dependence of the vacuum energy, and these presumably also fall as some power of T. It is possible that even at large temperatures these effects are the dominant ones. Thus to obtain an idea of what θ may do in the early universe not too deeply tied to our prejudices from instanton physics, we take

$$m_A^2(T) = \tilde{m}^2 (T/\tilde{T})^{-a} , \quad (8)$$

and consider a range of plausible values for $\tilde{m}$, $\tilde{t}$ and a, including the values obtained from instanton computations. Note that eq. (8) is written in this form for purposes of motivation; obviously only the product $\tilde{m}^2\tilde{T}^a$ is physically meaningful. We expect $\tilde{T}$ to be some characteristic QCD scale, and m to be some fraction of the zero-temperature axion mass.

The equation of motion for θ follows from conservation of energy at constant entropy. In a radiation-dominated universe one obtains

$$\ddot{\theta} + (3/2t)\dot{\theta} + \tilde{m}^2 [T(t)/\tilde{T}]^{-a} = 0 . \quad (9)$$

Our previous remarks about the Peccei–Quinn phase transition suggest that at some very small time, t_0, corresponding to a temperature well above QCD scales (the precise meaning of "well-above" will become clear shortly) we should set $\theta = \theta_0$, $\dot{\theta}_0 = 0$. The temperature in eq. (9) depends upon the time through the relation

$$T = (M_P/2\gamma t)^{1/2} , \quad (10)$$

where above QCD scales, for three light quarks,

$$\gamma \approx 13 .$$

This equation has the solution, with these initial conditions

$$\theta = \theta_0 (t/\tilde{t})^{-1/4} J_\nu [(t/\tilde{t})^\beta] \, \Gamma(\nu + 1) 2^\nu$$
$$\underset{t \gg \tilde{t}}{\approx} 2^\nu \Gamma(\nu + 1)(2/\pi)^{1/2} \theta_0 (t/\tilde{t})^{-(\beta+1/2)/2}$$
$$\times \cos[(t/\tilde{t})^\beta - \tfrac{1}{2}\pi\nu - \tfrac{1}{4}\pi] , \quad (11)$$

where

$$\beta = \tfrac{1}{4}a + 1 , \quad 2\beta\nu = \tfrac{1}{2}$$

$$\tilde{t} = [\beta^{-1} (\tilde{m}^2 \tilde{T}^a)^{1/2} (M_P/2\gamma)^{-a/4}]^{-1/\beta} . \quad (12)$$

Note that this time corresponds to a temperature which can be computed from eq. (10). $\tilde{t}$ can be thought of as a characteristic time for the motion of θ. In particular, we should choose $t_0 \ll \tilde{t}$.

We are now in a position to discuss the cosmological development of θ. First, we can see that θ today is extremely small, no matter what we assume for a, $\tilde{T}$, $\tilde{m}$, the location and nature of the phase transition, etc. If $\theta \sim 1$ at $T = 100$ MeV, then $\theta \approx 10^{-18}$ today. Certainly, then, we do not need to worry about the neutron electric dipole moment.

Also, in this picture, we can discuss a serious concern raised by Sikivie [8]. He points out that, since the Z_N symmetry respected by QCD is spontaneously broken by $\langle\phi\rangle$, one expects the formation of domain walls. This leads to conflicts with standard cosmology. One way out of this problem is to choose the fermion content so that the left-over discrete symmetry is trivial [‡1]. This has been discussed by Sikivie and by Georgi and Wise [9]. An alternative possibility is suggested by the picture above of the time evolution of θ. In the early universe scenario discussed by Guth [10] and refined by Linde [11] and Albrecht and Steinhardt [12], the universe goes through a period of exponential expansion at temperatures below the grand unified mass. We live, essentially, in one of the causal regions of this early universe and hence in one domain. In this region, θ will take a single value, which will evolve in time according to the equations of motion we have discussed earlier. [One can easily check, from eq. (3), that θ is virtually constant during the period of exponential expansion.] Thus a period of exponential expansion can solve the domain wall problem, provided that the Peccei–Quinn transition occurs at high enough temperatures, and that one does not reheat the universe above this temperature.

A more serious concern is the energy density associated with the axion field, at the time of helium synthesis and today (this point has been emphasized to us by L. Abbott and P. Sikivie). The pressure and energy density of the axion field obey an "equation of state" appropriate to non-relativistic matter, for $t \gg m^{-1}$. This can be seen directly by substituting our solutions, eqs. (4) and (5), into the expressions for the energy density, ρ. and pressure, p, and averaging over a period, m^{-1}

$$\rho = \tfrac{1}{2} f_A^2 (\dot{\theta}^2 + m_A^2 \theta^2)$$
$$\underset{mt \gg 1}{\approx} (mt)^{-3/2} , \quad \text{radiation domination,}$$
$$\underset{mt \gg 1}{\approx} (mt)^{-2} , \quad \text{matter domination ,} \quad (13)$$

‡1 This can also be achieved by choosing the Peccei–Quinn charge of ϕ to be N.

$$p = \tfrac{1}{2} f_A^2 (\dot{\theta}^2 - m_A^2 \theta^2)$$

$$\underset{mt \gg 1}{\approx} 0 . \qquad \text{(13 cont'd)}$$

Thus, if at some time ρ is large compared to the energy density of radiation, the universe will expand as if matter dominated. Note that, in either case $\rho \sim R^{-3}$ for large times.

Suppose that the potential is given by the zero temperature expression, eq. (1), below some temperature T_c (presumably near the chiral symmetry violating phase transition). If the universe has been expanding adiabatically since that time, then the ratio of ρ to the entropy density is a constant. If we calculate the entropy at T_c appropriate to a quark–gluon plasma ‡2, tje requirement that the universe be radiation dominated at the time of helium synthesis gives

$$\theta^2(T_c)/T_c^3 \lesssim 30 \text{ GeV}^{-3} \qquad (14)$$

From the limits on the present energy density of the universe, and the present temperature of the cosmic radiation background we obtain the stronger limit

$$\theta^2(T_c)/T_c^3 \lesssim 10^{-4} \text{ GeV}^{-3} . \qquad (15)$$

If we take $T_c = 0.1$ GeV, we obtain, from the latter equation

$$\theta^2(T_c) \lesssim 10^{-7} , \qquad (16)$$

whereas if we take the much more optimistic value, $T_c = 1$ GeV, the limit is

$$\theta^2(T_c) \lesssim 10^{-4} . \qquad (17)$$

We can calculate $\theta^2(T_c)$ from our solution of eq. (9). As a pessimistic estimate, we can use the formula for the axion mass implied by instantons. We will see later that more optimistic assumptions for $\widetilde{m}$, $\widetilde{T}$, T_c, and a do not drastically alter the situation.We consider the case of three light quarks (this will be seen to be self-consistent), and we will, as stated earlier, neglect the logarithmic variation of the $g^{-2}(T)$ terms, taking these to be simply g^{-2} (1 GeV). Of course, the instanton calculation cannot be trusted down to arbitrarily low temperatures, since the free energy diverges. We will make the ad hoc assumption that the temperature, T_c, is just the temperature where formula (6) gives the zero temperature result, eq. (1). One can check that the final results are not too sensitive to this choice. In any case, these assumptions yield (taking $\Lambda_{\overline{\text{MS}}} = 0.150$ GeV)

$$a = 8, \qquad \widetilde{m}^2 \widetilde{T}^8 f_A^2 = 5.8 \times 10^{-12} \, (\text{GeV})^{12}$$

$$T_c = 0.089 \, \text{GeV}, \quad \tilde{t} = 5 \times 10^{18} \text{GeV}^{-1} (f_A/10^{15} \text{GeV})^{1/3}$$

Plugging this into our solution, we find that we need (assuming $\theta_0 \sim 1$)

$$f_A \lesssim 10^{11} \text{ GeV} \qquad (18)$$

in order to satisfy the bound, eq. (15).

As a more optimistic extreme, we can take

$$\widetilde{m}^2 = 0.1 \, m_A^2 , \quad a = 2 ,$$

$$\widetilde{T} = 1 \text{ GeV} , \qquad T_c = 1 \text{ GeV} . \qquad (19)$$

This yields the bound

$$f_A \lesssim 10^{12} \text{ GeV} . \qquad (20)$$

So it appears that the estimate from instantons is not likely to be off by more than a factor of 10.

So we see that the current mass density and photon temperature severely constrain the value which θ can take at the grand unified phase transition. The invisible axion idea thus appears to be in serious trouble. In particular, in conventional grand unified theories satisfying the bounds on f_A described above requires a fine-tuning of one part in 10^6–10^{10}. Thus the strong *CP* problem, or the problem of the fine-tuning of θ, has simply been replaced by a fine-tuning of other parameters.

A few solutions to this dilemma may be imagined but none, at present, look particularly attractive. The most obvious is a process occurring after T_c which significantly increases the entropy (by a factor of order 10^4). An example is the decay of the long-lived gravitinos in certain supersymmetric theories [12]. However, no particularly satisfactory model of this type yet exists. Moreover, such a process would also dilute the baryons, which will be difficult to reconcile with current ideas concerning baryon number generation.

‡2 This is an optimistic estimate. A pessimist would calculate the entropy relevant to a gas of pions and nucleons. The truth is probably somewhere in between, and depends on the details of the true QCD equation of state at and around the chiral symmetry breaking transition. These considerations, however, can change our results only by small factors.

Finally, apart from fine-tuning, there is another disturbing feature of an axion with decay constant less than 10^{12} GeV. As mentioned earlier, the domain wall problem might be solved by a period of exponential expansion. However, in order to generate baryons, this period must be followed by significant reheating of the universe, almost certainly to temperatures above 10^{12} GeV, i.e. above the temperature of the new Peccei–Quinn transition. So the domain wall problem discussed by Sikivie will still be with us.

All of this adds up to a troubling picture. For the moment, at least, the strong *CP* puzzle remains a puzzle.

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