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Photon damping in a strongly magnetized plasma

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Abstract. The process of propagation of an electromagnetic wave in a strongly magnetized, charge-symmetric plasma is investigated. Taking into account the change in the dispersion properties of a photon in a magnetic field and plasma, it was found that, as well as the case of a pure magnetic field, the process of photon damping in a magnetized plasma has a nonexponential character. It is shown that the effective absorption width of a photon is significantly smaller in comparison with the results known in the literature.

1. Introduction

The problem of the propagation of electromagnetic fields in an active medium arises during the consideration of many physical phenomena. In this paper, the objects with scale fields of the so-called critical value $B_e = m^2/e \simeq 4.41 \times 10^{13}$ G of particular interest [1]. Recent observations allow, in particular, to identify some astrophysical objects, such as soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs), with magnetars [1]. According to the currently best known model (see, for example, [2]) in the vicinity of such objects a strong magnetic field exists reaching a value of $10^{15} - 10^{16}$ G. In addition, analysis of the emission spectrum of some of these objects indicates the presence in their vicinity a relatively hot and dense electron-positron plasma [3] with temperature of $T \sim 1$ MeV. These are precisely the conditions in which the consideration of the process of photon decay due to the reactions of absorption of a photon by an electron (positron), $\gamma e^\pm \to e^\pm$, and the creation of e^+e^- - pairs, $\gamma \to e^+e^-$, which are important in astrophysics magnetized neutron stars [4,5], is of particular interest. It should be noted that the expression for the decay width in the limit of a strongly magnetized plasma contains singularities of the root type at points of a cyclotron resonances. As emphasized in [6], this fact indicates the impossibility of interpreting the given decay width calculated by the theory perturbations near cyclotron resonances as the damping coefficient. In this case, the primary for finding the damping coefficient is the time dependence of the photon wave function in the presence of a magnetic field and plasma.

In the present work, we consider the photon decay as a result of processes $\gamma e^\pm \to e^\pm$ and $\gamma \to e^+e^-$ in a strongly magnetized plasma, $eB \gg T^2$, with the temperature of $T \sim 1$ MeV and chemical potential $\mu = 0$. We apply the method used in the field theory for finite temperatures and in plasma physics, see, for example, [7]. It consists of finding a retarded solution of the

[1] We use the natural units $c = \hbar = k = 1$, m is the electron mass, $e > 0$ is the elementary charge.

electromagnetic field equation in the presence of an external source, taking into account the polarization of the vacuum in a magnetized plasma.

2. Photon propagation in the magnetized medium

To describe the evolution of the electromagnetic wave $\mathcal{A}_\alpha(x)$, $x_\mu = (t, \mathbf{x})$ in time, we use the technique detailed in [8]. We consider the linear response of the system ($\mathcal{A}_\alpha(x)$ and vacuum polarized in a magnetized plasma) to an external source, which is adiabatically switched on at $t = -\infty$ and at time $t = 0$ turns off. At $t > 0$, the electromagnetic wave will propagate independently. For this, the source function should be selected in the form:

$$\mathcal{J}_\alpha(x) = j_\alpha \, e^{i\,\mathbf{kx}} \, e^{\varepsilon t} \, \theta(-t), \;\; \varepsilon \to 0^+. \tag{1}$$

Here $j_\alpha = (0, \mathbf{j})$, $\mathbf{j} \cdot \mathbf{k} = 0$ is the current conservation condition. In additional, for simplicity, we consider the evolution of a monochromatic wave.

The dependence of $\mathcal{A}_\alpha(x)$ on time is determined by the equation

$$(g_{\alpha\beta} \, \partial_\mu^2 - \partial_\alpha \partial_\beta) \, \mathcal{A}_\beta(x) + \int d^4x' \, \mathcal{P}_{\alpha\beta}(x - x') \, \mathcal{A}_\beta(x') = \mathcal{J}_\alpha(x), \tag{2}$$

where $\mathcal{P}_{\alpha\beta}(x - x')$ is the polarisation operator in the magnetized plasma, $q^\mu = (q_0, \mathbf{k})$ is the four-momentum of the photon.

We have notice, that in general case of arbitrary magnetized plasma the photons have an elliptical polarization. But in a strongly magnetized, charge symmetric plasma ($\mu = 0$), the physical polarization vectors of the photons

$$\varepsilon_\alpha^{(1)}(q) = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}} \tag{3}$$

are exactly the same as in pure magnetic field [9]. However, it should be emphasized that this coincidence is approximate to within $O(1/eB)$ and $O(\alpha^2)$ accuracy.

Here the four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\, 2\}$-subspace and the Minkowski $\{0,\, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along third axis; $(ab)_\perp = (a\varphi\varphi b) = a_\alpha \varphi_\alpha^{\,\rho} \varphi_{\rho\beta} b_\beta$, $(ab)_\parallel = (a\tilde{\varphi}\tilde{\varphi} b) = a_\alpha \tilde{\varphi}_\alpha^{\,\rho} \tilde{\varphi}_{\rho\beta} b_\beta$. The tensors $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor, correspondingly.

The solution of this equation for photons of the $\lambda = 1, 2$ modes can be represented as:

$$\mathcal{A}_\alpha^\lambda(x) = V_\alpha^{(\lambda)}(0, \mathbf{x}) \, ReF^{(\lambda)}(t)\,, \tag{4}$$

where

$$V_\alpha^{(\lambda)}(0, \mathbf{x}) = 2\, e^{i\,\mathbf{kx}} \, \varepsilon_\alpha^{(\lambda)} \, (\varepsilon^{(\lambda)} j)\,. \tag{5}$$

The function $F^{(\lambda)}(t)$ can be represented in the form of two terms

$$F^{(\lambda)}(t) = F_{pole}^{(\lambda)}(t) + F_{cut}^{(\lambda)}(t), \tag{6}$$

The first term is determined by the residue at the point $q_0 = \omega$, which is solution of the dispersion equation $q^2 - \mathcal{P}^{(\lambda)}(q) = 0$, in the kinematic region, where the value of the photon polarization operator, $\mathcal{P}^{(\lambda)}(q)$, is real.

The second term determines the dependence of the electromagnetic field on time in the region between the cyclotron resonances thresholds and has the form of a Fourier integral:

$$F_{cut}^{(\lambda)}(t) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} F_{cut}^{(\lambda)}(q_0) e^{-iq_0 t}, \tag{7}$$

$$F_{cut}^{(\lambda)}(q_0) = \frac{2\,\theta(q_0 - 2m_e)\, I^{(\lambda)}}{q_0\left([q_0^2 - \mathbf{k}^2 - R^{(\lambda)}]^2 + [I^{(\lambda)}]^2\right)}, \tag{8}$$

where $R \equiv Re\mathcal{P}^{(\lambda)}(q_0)$ is the real part of polarization operator, $I \equiv -Im\mathcal{P}^{(\lambda)}(q_0 + i\varepsilon)$ is the imaginary part of polarization operator.

The imaginary part of polarization operator can be obtained from the photon absorption coefficient

$$W_{abs}^{(\lambda)} = W_{\gamma^{(\lambda)} \to e^+e^-} + W_{\gamma^{(\lambda)} e^\pm \to e^\pm}\,. \tag{9}$$

Taking into account the photon radiation processes, the equation (9) can be presented in the following form [6,10,11]:

$$Im\mathcal{P}^{(\lambda)} = -2q_0[1 - exp(-q_0/T)]W_{abs}^{(\lambda)}\,. \tag{10}$$

The values $W_{\gamma^{(\lambda)} e^\pm \to e^\pm}$ for $\lambda = 1, 2$ can be obtained from the paper [8] and represented in the form:

$$W_{\gamma^{(1)} e^\pm \to e^\pm} = \frac{\alpha eB}{2q_0} \sum_{\ell=0}^{\infty} \sum_{n=n_0}^{\infty} \sum_{\epsilon=\pm1} \frac{f_{E_\ell^\epsilon}(1 - f_{E_\ell^\epsilon + q_0})}{\sqrt{(M_n^2 - M_\ell^2 - q_\parallel^2)^2 - 4q_\parallel^2 M_\ell^2}} \times \tag{11}$$
$$\times \left\{ [2eB(n+\ell) - q_\parallel^2](I_{n,\ell-1}^2 + I_{n-1,\ell}^2) - 8eB\sqrt{\ell n} I_{n,\ell-1} I_{n-1,\ell} \right\},$$

$$W_{\gamma^{(2)} e^\pm \to e^\pm} = \frac{\alpha eB}{2q_0} \sum_{\ell=0}^{\infty} \sum_{n=n_0}^{\infty} \sum_{\epsilon=\pm1} \frac{f_{E_\ell^\epsilon}(1 - f_{E_\ell^\epsilon + q_0})}{\sqrt{(M_n^2 - M_\ell^2 - q_\parallel^2)^2 - 4q_\parallel^2 M_\ell^2}} \times \tag{12}$$
$$\times \left\{ \left[\frac{(2eB(n-\ell))^2}{q_\parallel^2} - 2eB(n+\ell) - 4m^2 \right] (I_{n,\ell}^2 + I_{n-1,\ell-1}^2) - 8eB\sqrt{\ell n} I_{n,\ell} I_{n-1,\ell-1} \right\},$$
$$E_\ell^\epsilon = \frac{1}{2q_\parallel^2} \left[q_0 \left(M_n^2 - M_\ell^2 - q_\parallel^2 \right) + \epsilon k_z \sqrt{\left(M_n^2 - M_\ell^2 - q_\parallel^2 \right)^2 - 4q_\parallel^2 M_\ell^2} \right],$$

where $M_\ell = \sqrt{m^2 + 2eB\ell}$, $f_{E_\ell} = \{\exp[E_\ell/T] + 1\}^{-1}$, $I_{n,\ell} \equiv I_{n,\ell}\left(q_\perp^2/(2eB)\right)$,

$$I_{n,\ell}(x) = \sqrt{\frac{\ell!}{n!}}\, \mathrm{e}^{-x/2} x^{(n-\ell)/2} L_\ell^{n-\ell}(x)\,,$$
$$I_{\ell,n}(x) = (-1)^{n-\ell} I_{n,\ell}(x)\,, \quad n \geq \ell\,, \tag{13}$$

and $L_n^k(x)$ is the generalized Laguerre polynomials,

$$n_0 = \ell + \left[\frac{q_\parallel^2 + 2M_\ell\sqrt{q_\parallel^2}}{2eB} \right], \tag{14}$$

$[x]$ is the integer part of x.

The values $W_{\gamma^{(\lambda)}\to e^+e^-}$ can be obtained from (11) and (12) by using the crossing symmetry.

The real part of the polarization operator can be reconstructed from its imaginary part using the dispersion relation with one subtraction:

$$\mathcal{P}^{(\lambda)}(t) = \int\limits_0^\infty \frac{Im(\mathcal{P}^{(\lambda)}(t'))\,dt'}{t'-t-io} - \mathcal{P}^{(\lambda)}(0)\,, \qquad t = q_0^2\,. \tag{15}$$

Expressions (7) – (10), taking into account (15), solve the problem of finding the time dependence of the photon wave function in the presence of strongly magnetized plasma. Strictly speaking, due to the threshold behavior of the Fourier transform $F^{(\lambda)}_{cut}(q_0)$ character of time decay of the function $F^{(\lambda)}_{cut}(t)$, and hence the wave function $\mathcal{A}^{(\lambda)}_\mu(t)$, differs from the exponential one. However, during a certain characteristic time interval ($\sim [W^{(\lambda)}_{abs}]^{-1}$), the dependence of the wave function can be approximately described as exponentially decaying harmonic oscillations

$$\mathcal{A}^{(\lambda)}_\mu(t) \sim e^{-\gamma^{(\lambda)}_{\rm eff}\,t/2}\cos(\omega_{\rm eff}t + \phi_0)\,. \tag{16}$$

Here $\omega_{\rm eff}$ and $\gamma^{(\lambda)}_{\rm eff}$ is the effective frequency and ratio photon absorption of the λ mode, respectively, which should be found using equations (7) - (9) for each value of the impulse $\mathbf{k}$, which determines the effective dispersion law of a photon in the region of its instability.

3. Numerical analysis

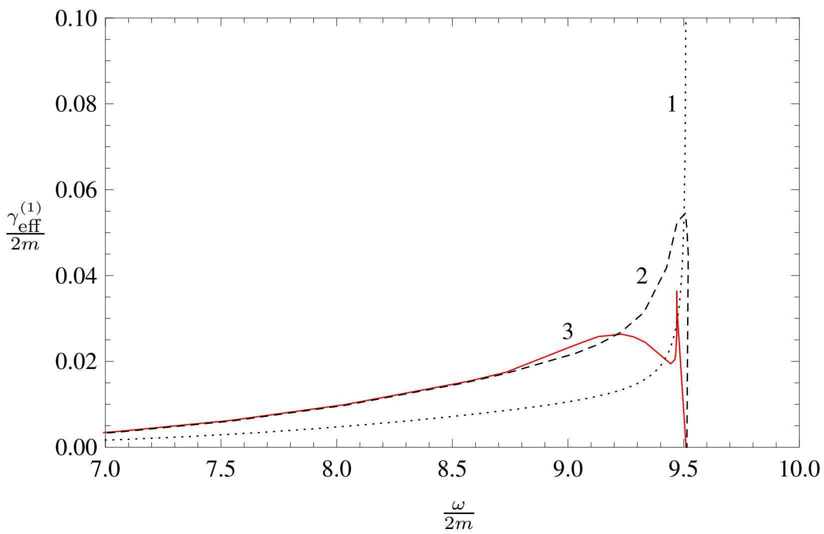

Figure 1. Frequency dependence of the decay width of the photon of mode 1 in the near-thresholds regions at $B = 200B_e$, $T = 1$ MeV and $\mu = 0$. Line *1* - coefficient of the photon absorption $W^{(1)}_{abs}$, calculated in the tree approximation and containing root singularities; line *2* - decay width obtained from the complex solution of the dispersion equation on the second Riemannian sheet [6]; line *3* matches the width decay $\gamma^{(1)}_{\rm eff}$ calculated based on the approximation (16).

An important role in astrophysical applications is played by the quantity $\gamma_{\rm eff}$, which determines the intensity of absorption of γ quanta in a magnetic field due to the processes

$\gamma \to e^+e^-$ and $\gamma e^\pm \to e^\pm$. Usually in astrophysics, an expression for the absorption coefficient containing the root singularities is used (see, for example, [5,12]). Our analysis shows (see figure 1 and figure 2), that the calculation of the absorption coefficient taking into account the nonexponential character damping, leads to the final expression for the absorption coefficient of a photon in the vicinity of resonances $q_0^2 = (\sqrt{m^2 + 2eB} \pm m)^2$.

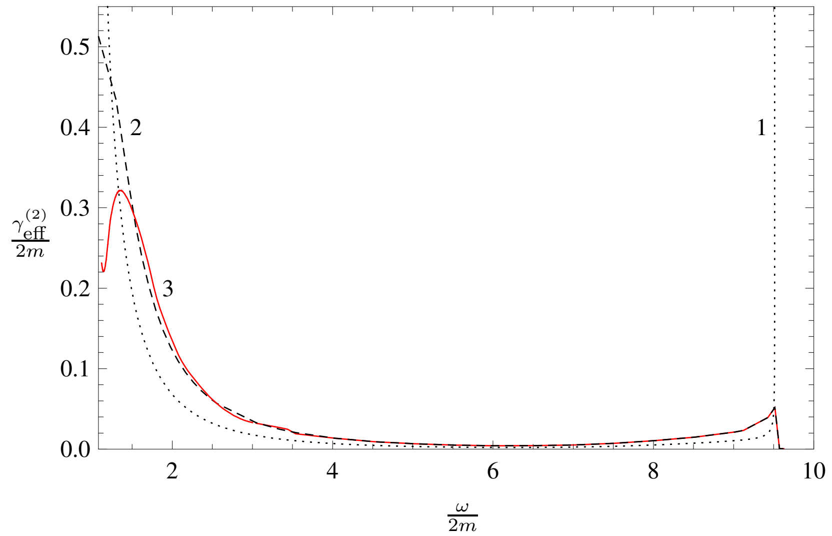

Figure 2. Frequency dependence of the decay width of the photon of mode 2 in the near-thresholds regions for the same parameters and notation as those in figure 1.

4. Conclusion
The process of propagation of an electromagnetic wave in a highly magnetized, charge-symmetric plasma is investigated. Taking into account the change in the dispersion properties of a photon in a magnetic field and plasma, it has been established that, similar to the case of a pure magnetic field, the process of photon decay in a magnetized plasma has a nonexponential character. It is shown that the effective absorption width of a photon is significantly smaller in comparison with the results known in the literature.

Acknowledgments
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