THE ASTROPHYSICAL JOURNAL, 309:362–371, 1986 October 1

COMPTON SCATTERING IN STRONG MAGNETIC FIELDS

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Received 1985 December 27; accepted 1986 March 21

ABSTRACT

We have derived the relativistic cross section for Compton scattering by electrons in strong magnetic fields. The derivation assumes that the electrons are initially in their lowest Landau levels, but considers transitions to arbitrary Landau levels in the final state. The results confirm and extend earlier work which has treated only transitions to the lowest or first excited Landau levels. For the teragauss field strengths expected in neutron star magnetospheres, the relative rates for excited state transitions are found to be significant, especially for incident photon energies several times the cyclotron frequency. Since these transitions must result in the rapid emission of one or more cyclotron photons as well as the Compton-scattered photon, the scattering process actually becomes a photon "splitting" mechanism which acts to soften hard photon spectra, and also provides a specific mechanism for populating higher Landau levels in the electron distribution function. The results should be significant for models of gamma-ray bursters and pulsating X-ray sources, although it appears likely that in these applications numerical evaluation of the cross section will be necessary for reliable calculations of radiative transfer and emission spectra.

Subject headings: magnetic fields — radiation mechanisms — relativity

I. INTRODUCTION

The scattering of photons by electrons in strong magnetic fields has been a subject of great interest since the discovery of radio pulsars and cyclotron lines in X-ray pulsars provided evidence that magnetic fields of order 10^{12} G are present in nature. It has been recognized for some time that, while the presence of an external magnetic field causes significant deviations from classical Thomson scattering (e.g., Canuto, Lodenquai, and Ruderman 1971; Ventura 1979), relativistic effects can be at least as important in fields approaching the critical value, $B_{\rm cr} = 4.413 \times 10^{13}$ G, at which $h\nu_B = mc^2$, where ν_B is the cyclotron frequency. The relativistic regime, where the recoil of the electron becomes important, is reached roughly when $(h\nu/mc^2)\gamma \approx 1$. In the case of nonrelativistic electrons ($\gamma \approx 1$) and photon energy expressed in multiples n of the cyclotron frequency in fields of strength B, this condition takes the form $n(B/B_{\rm cr}) \approx 1$. Accurate treatment of Compton scattering near the surface of strongly magnetized neutron stars therefore requires a fully quantum mechanical relativistic calculation of the cross section.

Most calculations of the cross section to date have been done in the nonrelativistic regime (Canuto *et al.*; Borner and Mészáros 1979*a*, *b*; Ventura 1979). These results have been useful in studying the approximate effects of angle, frequency, and polarization dependence of the magnetized cross sections, but they do not include the excitation of electrons to higher Landau states or the resonances at higher harmonics, effects which require a relativistic treatment. Herold (1979) has presented relativistic calculations of the magnetic Compton cross section for electrons both initially and finally in the ground state ($j = 0$). Melrose and Parle (1983*b*) confirmed Herold's result and also gave results for the case where the electron is excited to the $j = 1$ Landau state.

Scattering events which leave the electron in excited Landau states could be important in applications to astrophysical sources. Such events will result in a soft scattered photon, which may have $\nu \ll \nu_B$, and at least one cyclotron or synchrotron photon, produced as the electron immediately decays from its excited state. Bussard, Mészáros, and Alexander (1985) point out that scattering by photons just above the cyclotron frequency is a significant source of soft photons, possibly more important than bremsstrahlung for the production of the low-energy continuum in X-ray pulsars. This process, as well as the related two-photon decay of electrons from excited Landau states (which also produces soft photons), will also affect the transfer and production of cyclotron photons (Kirk, Melrose, and Peters 1984). These scattering events may, in addition, be an important mechanism for populating Landau states if the density of photons above the cyclotron frequency is high enough. This mechanism has already been invoked in some models of gamma-ray bursts (e.g., Hameury *et al.* 1985), where large densities of high-energy (> 100 keV) photons are expected.

In this paper, we present fully relativistic calculations of the Compton scattering cross section which includes excitations of the electron to an arbitrary Landau state. As in previous treatments, we make the assumption that the electron is initially in the ground state, which is reasonable in view of the extremely rapid synchrotron transition rates from the excited states. Section II outlines the derivation of the cross section, and § III shows results of numerical calculations for a range of relevant field strengths and photon energies. Although the formula for the cross section is given for an arbitrary Lorentz frame, we will specialize to the electron rest frame ($p = 0$) to simplify the numerical calculations.

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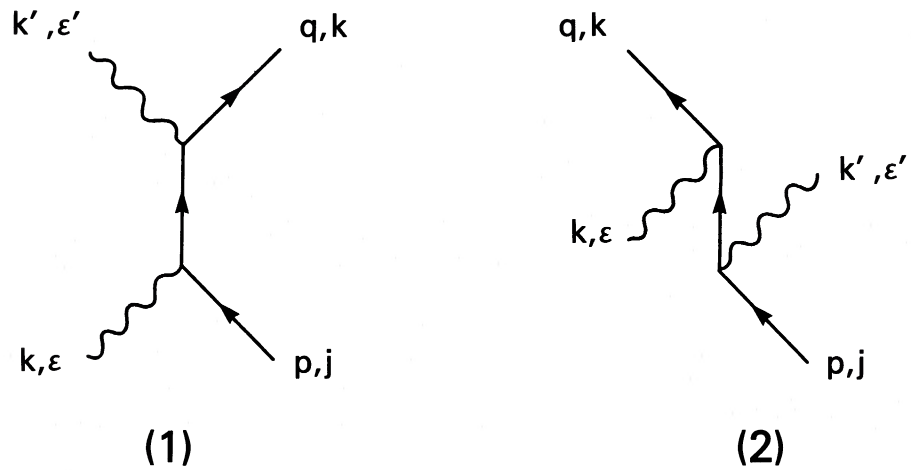


FIG. 1.—Feynman diagrams for Compton scattering. Solid lines represent electron wave functions in a uniform magnetic field.

II. DERIVATION OF CROSS SECTION

The S-matrix element for Compton scattering, denoted schematically by the pair of Feynman diagrams in Figure 1, may be written in the form

$$S_{fi} = -ie^2 \int d^4x' \int d^4x[\bar{\psi}_f(x')\gamma_\mu A_2^\mu(x')G_F(x'-x)\gamma_\nu A_1^\nu(x)\psi_i(x) + \bar{\psi}_f(x')\gamma_\mu A_1^\mu(x')G_F(x'-x)\gamma_\nu A_2^\nu(x)\psi_i(x)] , \tag{1}$$

where both the electron wave functions $\psi_i(x)$, $\psi_f(x)$ and the electron propagator $G_F(x'-x)$ satisfy the Dirac equation for motion in a constant, uniform magnetic field $\boldsymbol{B} = B\hat{z}$ (for such solutions see Johnson and Lippmann 1949; Melrose and Parle 1983*a*). Natural units, $\hbar = c = 1$, are used here, and in general this section follows the notational conventions of Bjorken and Drell (1964).

The particular choices for the wave functions and propagators used below have already been described by Daugherty and Bussard (1980) in dealing with the problem of pair annihilation in strong fields. Here it will be sufficient to note that the labels (j, p) refer, respectively, to the Landau level and longitudinal momentum of the incident electron, while (l, q) denote the same quantities for the scattered electron. The incident electron energy is then given by

$$E(j, p) = [p^2 + m^2(1 + 2jB/B_{\rm cr})]^{1/2} , \tag{2}$$

where $B_{\rm cr} = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G (the critical field strength). A similar expression with (l, q) holds for the final electron energy. It will be convenient to write simply E_j and E_l in the following, except for the lowest Landau levels which will be denoted by E_p and E_q, respectively.

The incident and scattered photon momentum four-vectors are denoted by $k^\mu = (\omega, \boldsymbol{k})$ and $k'^\mu = (\omega', \boldsymbol{k}')$, while the corresponding polarization three-vectors are written as $\boldsymbol{\epsilon}_i$ and $\boldsymbol{\epsilon}'_i$, $i = 1, 2$. Linear polarizations identical to those used by Daugherty and Bussard (1980) are assumed for both the incident and scattered photons, except for incidence along the field direction ($\theta = 0$), where circular polarizations are the appropriate choice. In this respect our treatment follows Herold (1979).

A straightforward derivation involving integration over δ-functions leads directly to the "energy-denominator" form of the S-matrix element:

$$S_{fi} = \frac{-ie^2}{2L^3}\left(\frac{L}{2\pi}\right)^2 \frac{eB}{\sqrt{\omega\omega'}} 2\pi\delta(E_j + \omega - E_l - \omega') \sum_{n=0}^{\infty} \sum_{i=1}^{2} \int dA \int dP[T_n^{(1)} + T_n^{(2)}] , \tag{3}$$

where

$$T_n^{(1)} = \frac{1}{(E_j + \omega - E_n)}\left[\int d^3x e^{-i\boldsymbol{k}'\cdot\boldsymbol{x}} u_l^{\dagger(s)}(\boldsymbol{x})M'u_n^{(i)}(\boldsymbol{x})\right]\left[\int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} u_n^{\dagger(i)}(\boldsymbol{x})Mu_j^{(r)}(\boldsymbol{x})\right]$$
$$+ \frac{1}{(E_j + \omega + E_n)}\left[\int d^3x e^{-i\boldsymbol{k}'\cdot\boldsymbol{x}} u_l^{\dagger(s)}(\boldsymbol{x})M'v_n^{(i)}(\boldsymbol{x})\right]\left[\int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} v_n^{\dagger(i)}(\boldsymbol{x})Mu_j^{(r)}(\boldsymbol{x})\right] , \tag{4a}$$

$$T_n^{(2)} = \frac{1}{(E_j - \omega' - E_n)}\left[\int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} u_l^{\dagger(s)}(\boldsymbol{x})Mu_n^{(i)}(\boldsymbol{x})\right]\left[\int d^3x e^{-i\boldsymbol{k}'\cdot\boldsymbol{x}} u_n^{\dagger(i)}(\boldsymbol{x})M'u_j^{(r)}(\boldsymbol{x})\right]$$
$$+ \frac{1}{(E_j - \omega' + E_n)}\left[\int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} u_l^{\dagger(s)}(\boldsymbol{x})Mv_n^{(i)}(\boldsymbol{x})\right]\left[\int d^3x e^{-i\boldsymbol{k}'\cdot\boldsymbol{x}} v_n^{\dagger(i)}(\boldsymbol{x})M'u_j^{(r)}(\boldsymbol{x})\right] . \tag{4b}$$

The labels (1) and (2) correspond to the first and second diagrams of Figure 1, respectively. Here $u_j^{(i)}(\boldsymbol{x})$ and $v_j^{(i)}(\boldsymbol{x})$ describe electron and positron spinor wave functions, where in general $i = 1, 2$ labels the "spin" state and j denotes the Landau level (see eq. [2]). The indices r and s in equation (4) specify the initial and final electron spin states. The quantities n, i, P, and A describe the intermediate electron state (the propagator) as follows: n denotes the Landau level, i labels the spin state, P is the longitudinal component of momentum, and A is the x-coordinate of the orbit center, which is an eigenvalue for this choice of wave functions. (The orbit-center eigenvalues for the incident and final electrons will be denoted by a and b, respectively.) The quantities M and M' are the polarization matrices of the incident and scattered photons, where in general $M = \boldsymbol{\epsilon} \cdot \boldsymbol{\alpha}$ (with $\boldsymbol{\alpha}$ the standard Dirac matrices). The quantities L and T are the dimensions of the "box" normalization four-volume L^3T. It should be noted that the form for $T_n^{(2)}$ may be obtained directly from $T_n^{(1)}$ by the crossing-symmetry replacements

$$\omega \leftrightarrow -\omega' , \qquad \boldsymbol{k} \leftrightarrow -\boldsymbol{k}' , \qquad \boldsymbol{\epsilon} \leftrightarrow \boldsymbol{\epsilon}' . \tag{5}$$

In the following, it is assumed that the electron is initially in the ground state ($j = 0$), but that the final-state Landau level may range over $l = 0, 1, 2, \ldots$ (each l-value except for 0 corresponds to a double degeneracy, reflecting two possible "spin" states with the same principal energy eigenvalue). The excited final states must, of course, decay rapidly via the emission of one or more cyclotron-synchrotron photons, so that in these cases the Compton process actually becomes a multiple-photon emission mechanism.

By the standard manipulations, described for example in Bjorken and Drell (1964), the norm of the S-matrix element may be rewritten as

$$\begin{aligned} |S_{fi}|^2 = {} & \frac{2\pi^5\alpha^2}{\omega\omega'}\left(\frac{T}{L^8}\right)\frac{(E_p+m)}{2E_p}\frac{(E_l+m)}{2E_l}\,\delta(E_p+\omega-E_l-\omega')\delta(p+k_z-q-k_z')\delta[k_y-k_y'-eB(a-b)] \\ & \times \exp\left(-\frac{k_\perp^2+k_\perp'^2}{2eB}\right)\left|\sum_{n=0}^{\infty} F_n^{(1)}e^{i\Phi_1}+F_n^{(2)}e^{i\Phi_2}\right|^2 , \end{aligned} \tag{6}$$

where the quantities $F_n^{(i)}$ (given explicitly below) correspond to the two diagrams of Figure 1 and are related by the crossing symmetry given by equation (5). The common phase factors Φ_1 and Φ_2 are given by

$$\Phi_1 = -\Phi_2 = \frac{k_\perp k_\perp'}{2eB}\sin(\phi-\phi') . \tag{7}$$

The three remaining δ-functions express conservation of total energy and z-momentum, as well as the relation between the y-momenta of the photons and the initial and final x-coordinates a and b of the electron orbit centers. (Note that the orbit-center eigenvalues for these wave functions could equivalently be regarded as eigenvalues of the y-momenta.) Since each Landau level above the ground state is doubly degenerate, there are distinct expressions for each $F_n^{(i)}$ which describe the "spin-flip" (final spin-up) and "no spin-flip" (final spin-down) cases. (Here the results depend on the choice of wave functions, as noted by Melrose and Parle 1983*a*, *b*; see their work for a different choice of wave functions. Any applications to radiation transfer problems must, of course, sum over final spin states to remove this dependence.) In an arbitrary Lorentz frame the full expressions for $F_n^{(i)}$ are quite cumbersome, and these have been deferred to the Appendix. At least some simplification results from specializing to the frame in which the electron is initially at rest, in which case the two expressions for $F_n^{(i)}$ may be written as

1. "No spin flip" case:

$$\begin{aligned} F_n^{(1)} = {} & \frac{1}{(2m\omega+k_\perp^2-2neB)(2m+\omega-\omega')}\,[\![[\omega(2m+\omega-\omega')+k_z(k_z-k_z')]\Lambda_{l,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^* \\ & + [\omega(2m+\omega-\omega')-k_z(k_z-k_z')]\Lambda_{l,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_+ \\ & + \sqrt{2neB}\,(k_z-k_z')[\Lambda_{l,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^* + \Lambda_{l,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_+] \\ & + \sqrt{2leB}\,\{k_z[\Lambda_{l-1,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_- + \Lambda_{l-1,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^*] \\ & + \sqrt{2neB}\,[\Lambda_{l-1,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_- - \Lambda_{l-1,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^*]\}]\!] . \end{aligned} \tag{8a}$$

2. "Spin-flip" case:

$$\begin{aligned} F_n^{(1)} = {} & \frac{1}{(2m\omega+k_\perp^2-2neB)(2m+\omega-\omega')}\,[\![-\sqrt{2leB}\,\{k_z[\Lambda_{l,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^* - \Lambda_{l,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_+] \\ & + \sqrt{2neB}\,[\Lambda_{l,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^* + \Lambda_{l,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_+]\} \\ & + [-\omega(2m+\omega-\omega')+k_z(k_z-k_z')]\Lambda_{l-1,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_- \\ & + [\omega(2m+\omega-\omega')+k_z(k_z-k_z')]\Lambda_{l-1,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^* \\ & + \sqrt{2neB}\,(k_z-k_z')[\Lambda_{l-1,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_- - \Lambda_{l-1,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^*]]\!] , \end{aligned} \tag{8b}$$

where

$$\beta_i = -i\,\frac{k_x+ik_y}{\sqrt{eB}} , \qquad \beta_f = i\,\frac{k_x'+ik_y'}{\sqrt{eB}} , \qquad \epsilon_\pm = \epsilon_x \pm i\epsilon_y ,$$

and

$$\Lambda_{l,m}(\beta) = (-i)^{G-S}\left(\frac{S!}{G!}\right)^{1/2} 2^{-(G+S)/2}(\beta^*)^l \beta^m \left(\frac{|\beta|^2}{2}\right)^{-S} L_S^{G-S}\left(\frac{|\beta|^2}{2}\right), \qquad G = \max(l, m), \qquad S = \min(l, m). \tag{9}$$

The associated Laguerre polynomials $L_n^\alpha(x)$ are defined according to the conventions of Abramowitz and Stegun (1965).

The energy-momentum δ-functions remaining in the squared S-matrix element imply that the scattered photon energy (in an arbitrary Lorentz frame) may be expressed as

$$\omega' = \frac{1}{\sin^2\theta'}\{(E_p - p\cos\theta') + \omega(1 - \cos\theta\cos\theta') - [(E_p - p\cos\theta')^2 + 2\omega(E_p\cos\theta' - p)(\cos\theta' - \cos\theta)$$
$$+ \omega^2(\cos\theta' - \cos\theta)^2 + 2leB\sin^2\theta']^{1/2}\}. \tag{10}$$

(The scattered electron momentum is then immediately given by $q = p + k - k'$.) It is worthwhile to point out that equation (10) is an asymptotic kinematical relation, valid in the limit of "infinitely rigid" field lines which allow violation of transverse momentum conservation. For this reason there is no dependence on B, and hence no way to recover the usual free-space kinematical relations from equation (10) in the weak-field limit.

With the above forms for the norm of the S-matrix element, the differential Compton scattering cross section (in the electron rest frame) is found to be

$$\frac{d\sigma}{d\Omega'} = \frac{r_e^2 m^2}{2}\frac{\omega'}{\omega}\frac{(2m + \omega - \omega')}{(m + \omega - \omega')} e^{-(k_\perp^2 + k_\perp'^2)/2eB}\left|\sum_{n=0}^{\infty}[F_n^{(1)}e^{i\Phi_1} + F_n^{(2)}e^{i\Phi_2}]\right|^2. \tag{11}$$

For the special case $l = 0$, this result confirms the expressions found earlier by Herold (1979), with the exception of an apparent typographical error in the denominator of his equation (8). The $l = 0$ result has already been confirmed by Melrose and Parle (1983*a*, *b*), who also considered the case $l = 1$. However, as mentioned above, these authors have used different spin-state wave functions.

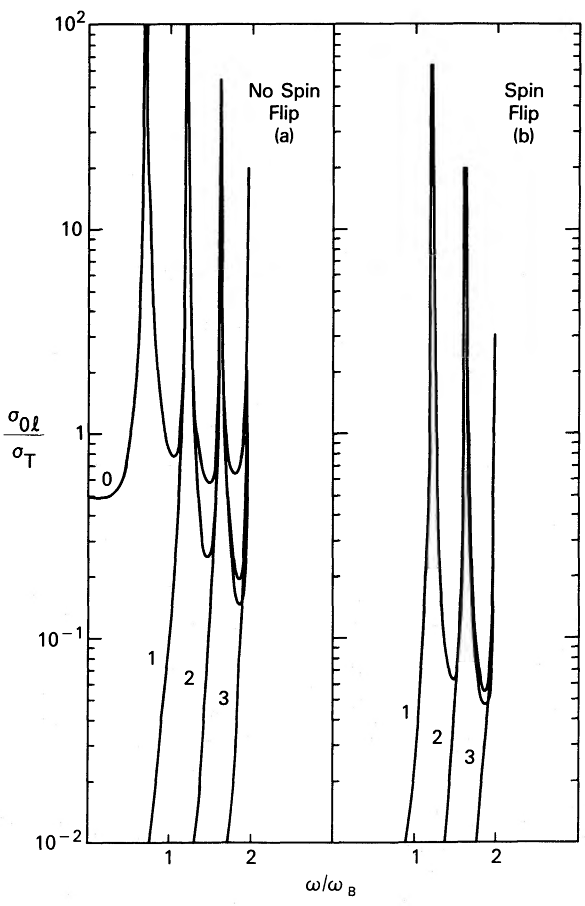


FIG. 2.—Magnetic Compton cross section σ_{0l} for scattering from the ground state into state l (in units of the Thomson cross section) as a function of incident photon energy ω (in units of the cyclotron energy $\omega_B = eB/m$) for $B = B_{cr}$ and incident photon angle $\theta = 90°$. Labels refer to final state principal quantum number l.

III. NUMERICAL RESULTS

The astrophysical applications of the cross section given by equation (11) are in many cases (e.g., gamma-ray burst sources) well outside the nonrelativistic Thomson regimê, so approximate expressions (see, e.g., Herold 1979) are useful for comparison with classical results but of limited use in quantitative models. In these regimes it is necessary to evaluate the cross sections numerically. Figures 2–5 show the results of such numerical calculations for field strengths $B = B_{cr}$, $B = 0.1B_{cr}$, and $B = 10^{12}$ G, from nonrelativistic energies to several times the cyclotron frequency $\omega_B = eB/m$ for each field strength. The cross sections have been integrated over final photon direction, summed over final polarization, and averaged over initial photon polarization. The incident photons have polar angle θ (the cross section depends only on $\phi - \phi'$ and is thus independent of ϕ after integration over final photon angle). The " spin flip " and " no spin flip " contributions to the cross section are plotted separately.

These plots confirm and extend the numerical results of Herold (1979) for transitions to the ground state, and show the relative significance of higher state transitions as functions of photon energy and incident angle. As noted by Herold, the three major relativistic corrections to the ground-state cross section are the energy shift of the resonances, the appearance of resonances at higher harmonics, and the decrease below the Thomson value at $\omega > \omega_B$. Resonances occur at energies

$$\begin{aligned} \omega_{\rm res}^n &= [(m^2 + 2neB\sin^2\theta)^{1/2} - m]/\sin^2\theta\,, \qquad &\theta \neq 0°\;; \\ \omega_{\rm res} &= eB/m\,, &\theta = 0°\;; \end{aligned} \tag{12}$$

where the denominator of the $F^{(1)}$ terms in equation (8) are equal to zero. The decrease below Thomson of the ground-state cross section is seen here to occur because of the increasing contributions of the excited-state transitions at higher photon energies. The sum over all final l states should add up to a total cross section near the Thomson value (in the nonrelativistic regime). As B decreases (and the classical limit is approached), the $l = 0$ cross section makes a greater contribution to the total cross section, and therefore stays closer to the Thomson value at high energies.

The cross section has a strong dependence on angle of the incident photon. Maxima in σ_{0l} at $\omega = \omega_{\rm res}^l$ appear at all angles except $\theta = 90°$. These events are really absorptions rather than scatterings, because the final photon energy ω' vanishes. When $\omega' = 0$, the resonant $[F^{(1)}]$ part of the cross section vanishes because all the terms contain $\Lambda_{l,n}(\beta_f) \propto (k')^l$. The nonresonant $[F^{(2)}]$ part includes

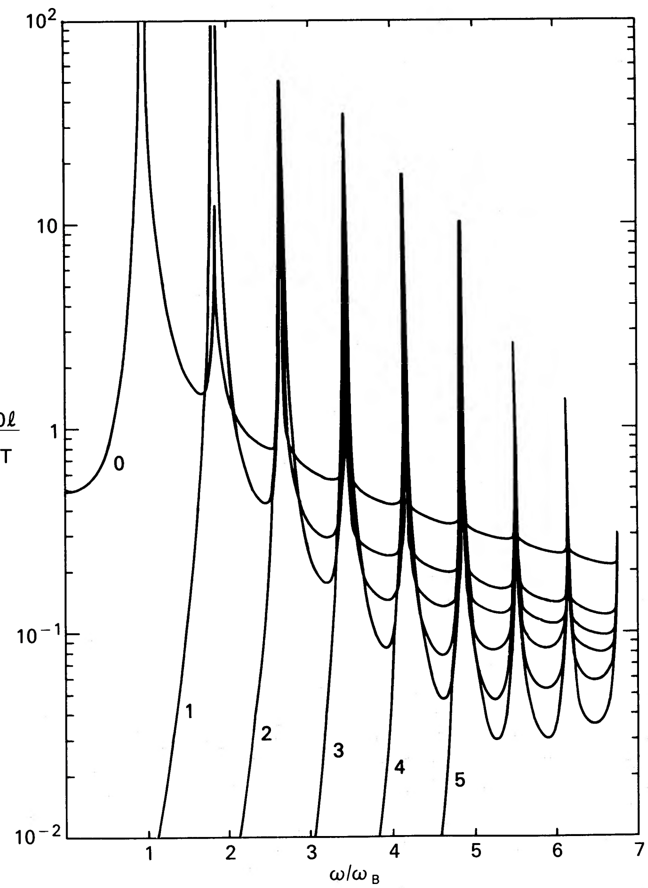


FIG. 3*a*

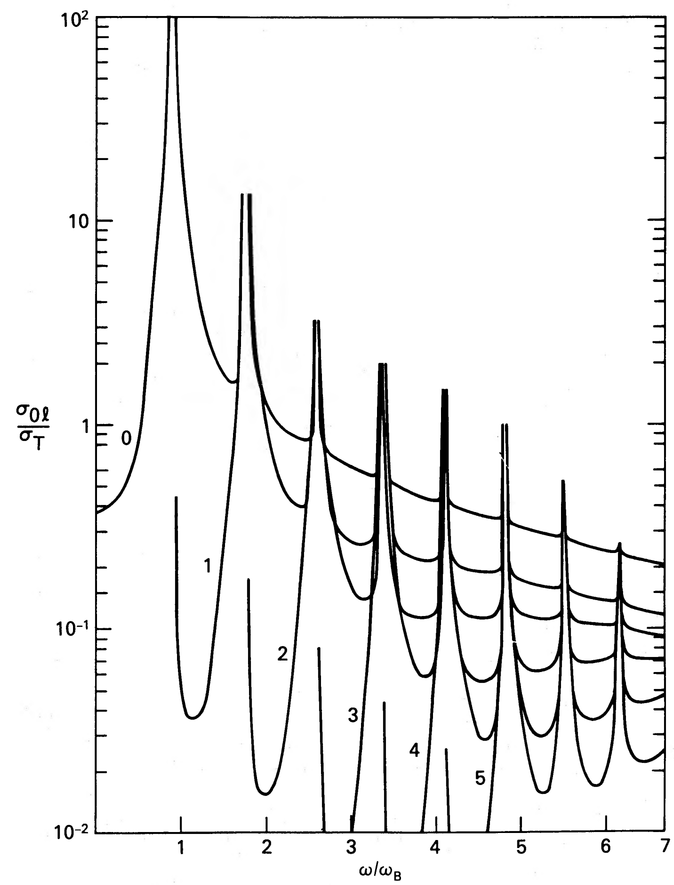


FIG. 3*b*

FIG. 3.—Magnetic Compton cross section σ_{0l} for scattering from the ground state into state l (in units of the Thomson cross section) as a function of incident photon energy ω (in units of the cyclotron energy $\omega_B = eB/m$) in the " no spin flip " case for $B = 0.1B_{cr}$ and incident photon angles (*a*) $\theta = 90°$, (*b*) $\theta = 60°$, (*c*) $\theta = 30°$, and (*d*) $\theta = 0°$. Labels refer to final state principal quantum number l.

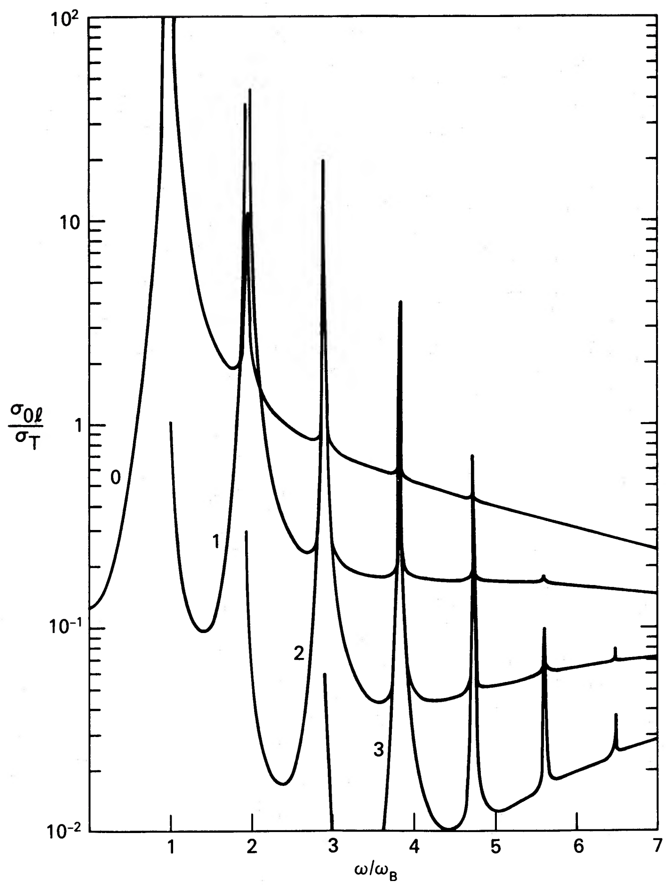


FIG. 3c

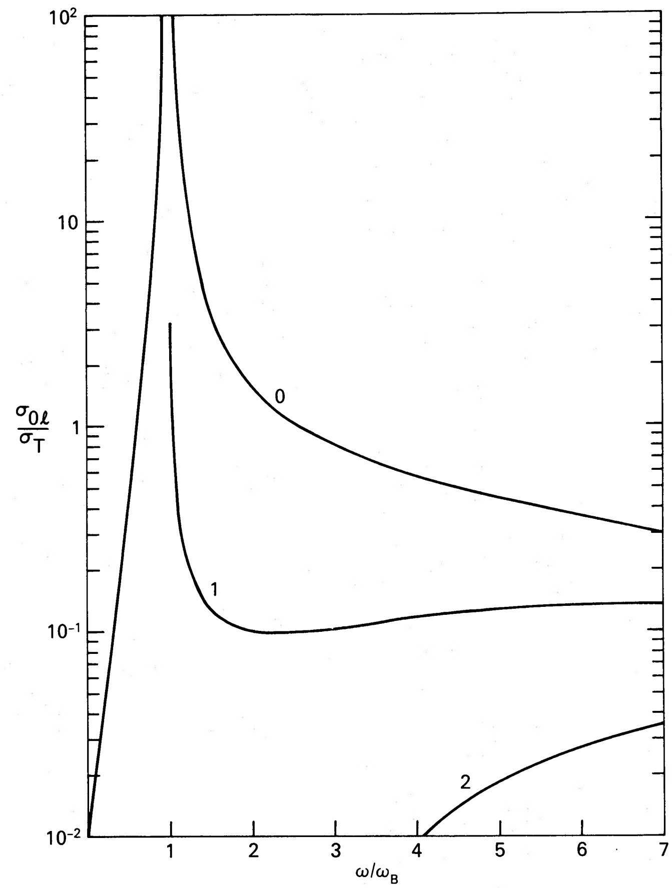


FIG. 3d

terms depending on k_z, which are nonzero for $\theta \neq 0°$. Resonances at higher harmonics become narrower and weaker as θ (and $k_\perp$) decreases, disappearing altogether at $\theta = 0°$, where there is no contribution from $n > 1$ terms for $k_\perp = 0$. Scattering into higher states is also relatively less important as θ decreases. Thus, the $\theta = 0°$ case is closest to the nonrelativistic, classical limit.

The "spin flip" contribution is down by a factor of roughly B/B_{cr} from the "no spin flip" case for scattering into the same l level, but the transitions to excited states are seen to make a larger relative contribution. In fact, the "spin flip" transitions to higher states become more probable than those to the first excited state at higher energies. In contrast, the "no spin flip" transitions to higher states, except near resonances, never cross over the ground-state transitions, but approach to within a factor of 2 at the high field strengths.

The figures show that, while the transitions to excited states are not dominant at most energies, they become the dominant scattering mode near the harmonics, even at field strengths as low as 10^{12} G. Scattering into state l is the largest contribution to the total cross section near the resonance at $\omega_{\rm res}^{l+1}$. Thus, electrons scattering with high-energy photons of sufficient density could populate the higher Landau levels.

The expression for the cross section given by equation (11) is not valid at energies above pair production threshold, $2m/\sin\theta$, where $F_n^{(2)}$ has resonances due to a real positron in the intermediate state of diagram (2) in Figure 1. In order to evaluate the scattering cross section at these energies correctly, the propagator must include real as well as virtual intermediate states. However, for photon energies above threshold, magnetic pair production (a first-order process, the cross section for which is given in Daugherty and Harding 1983) will be dominant over scattering at high field strengths. In the plots of the Compton cross section for $B = B_{cr}$, pair production threshold appears at around the third harmonic, and we do not evaluate the cross section above this point.

IV. DISCUSSION

An immediate conclusion to be drawn from these results is that the nonresonant cross section for scattering to higher states becomes comparable to the ground-state cross section for energies of several cyclotron frequencies in teragauss fields. In fact, the Comptonization of incident high-energy photons may prove an important mechanism for maintaining a significant population of electrons in excited Landau levels. Scatterings which leave the electron in an excited state effectively split the energy of the incident photon into two or more photons of lower energy. The presence of the magnetic field therefore softens the scattered photon spectrum. Near the cyclotron harmonics, the transitions to higher states are dominant, and the cross section for these events can exceed Thomson by several orders of magnitude. Compton scattering of photons near the cyclotron harmonics $\omega \sim l\omega_B$ which excite electrons into state l could also be an efficient source of soft photons, since the scattered photons in this case have energies less than the width of the resonances.

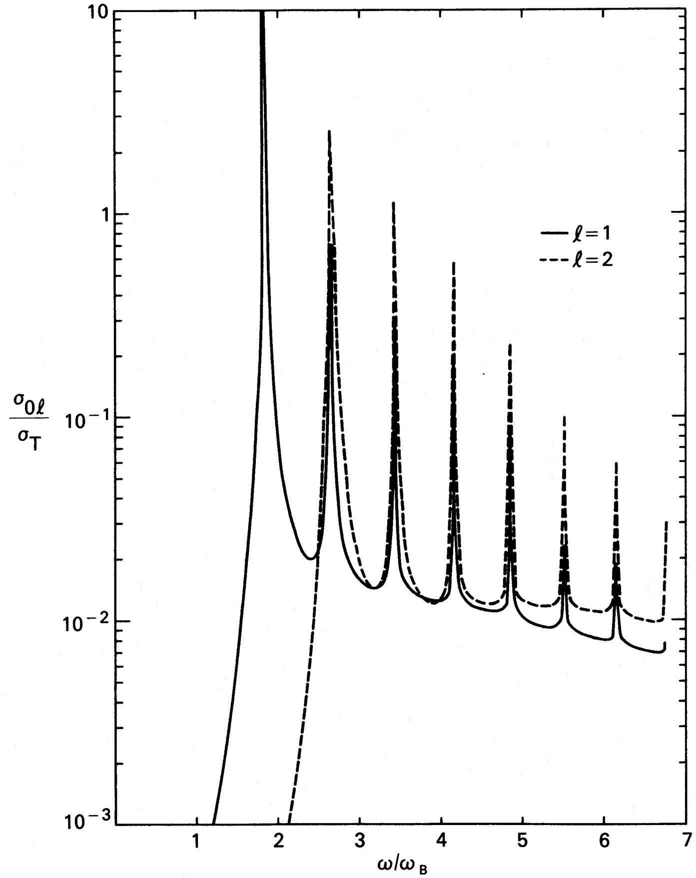


FIG. 4*a*

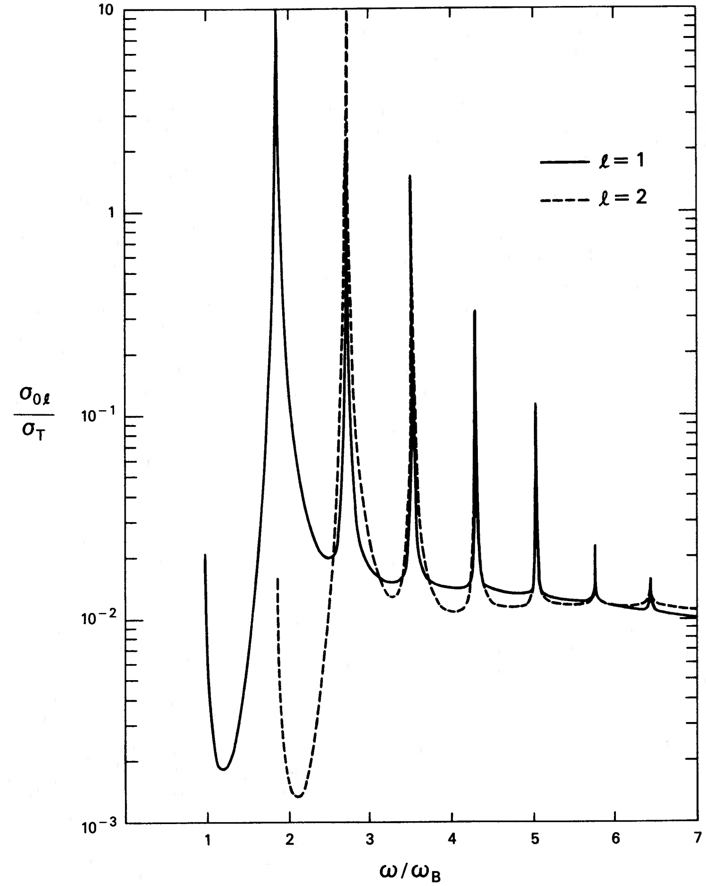


FIG. 4*b*

FIG. 4.—Magnetic Compton cross section σ_{0l} for scattering from the ground state into state l (in units of the Thomson cross section) as a function of incident photon energy ω (in units of the cyclotron energy $\omega_B = eB/m$) in the "spin flip" case for $B = 0.1B_{cr}$ and incident photon angles (*a*) $\theta = 90°$, (*b*) $\theta = 60°$, (*c*) $\theta = 30°$, and (*d*) $\theta = 0°$.

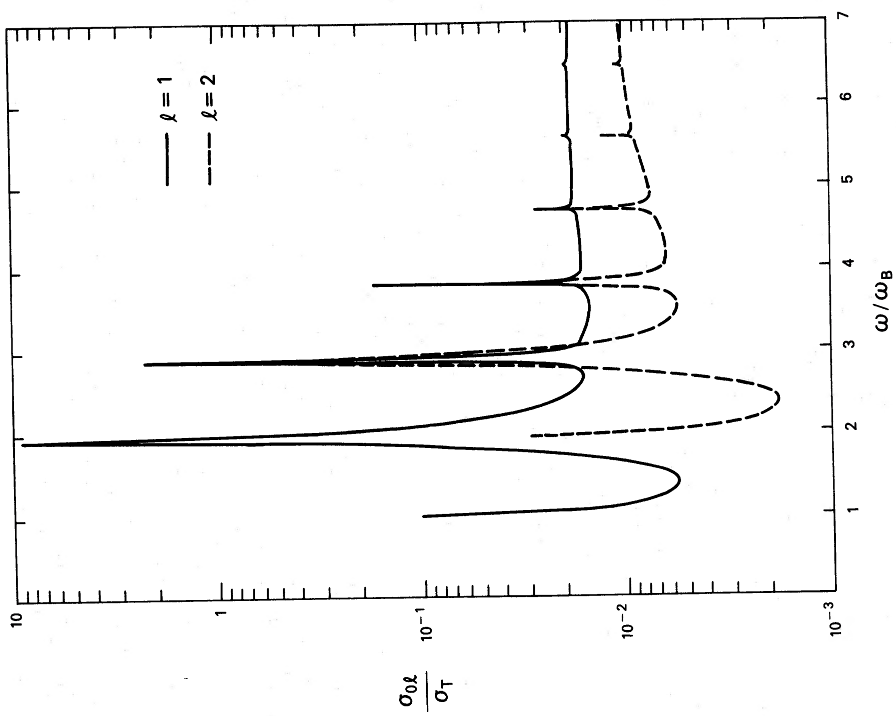
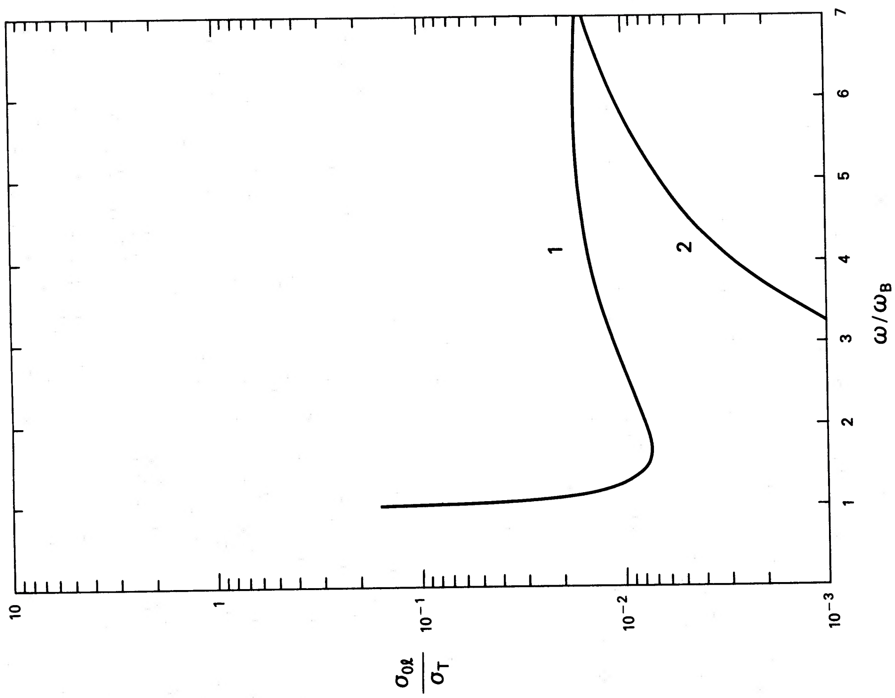

$\ell = 1$
$\ell = 2$
$\frac{\sigma_{0\ell}}{\sigma_T}$
ω/ω_B
10
1
10^{-1}
10^{-2}
10^{-3}
1 2 3 4 5 6 7

FIG. 4c


1
2
$\frac{\sigma_{0\ell}}{\sigma_T}$
ω/ω_B
10
1
10^{-1}
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10^{-3}
1 2 3 4 5 6 7

FIG. 4d

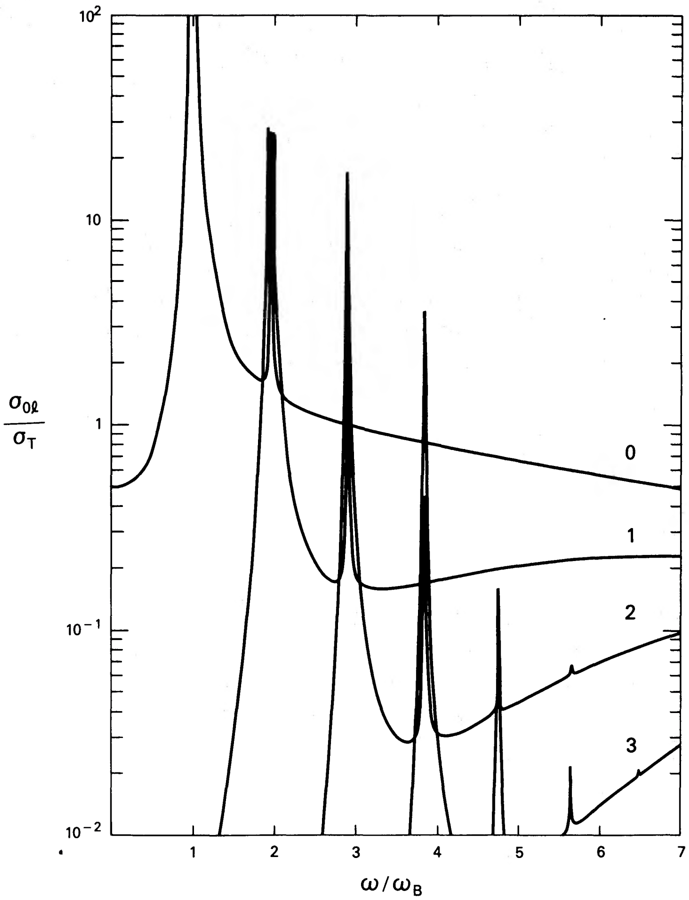

FIG. 5*a*

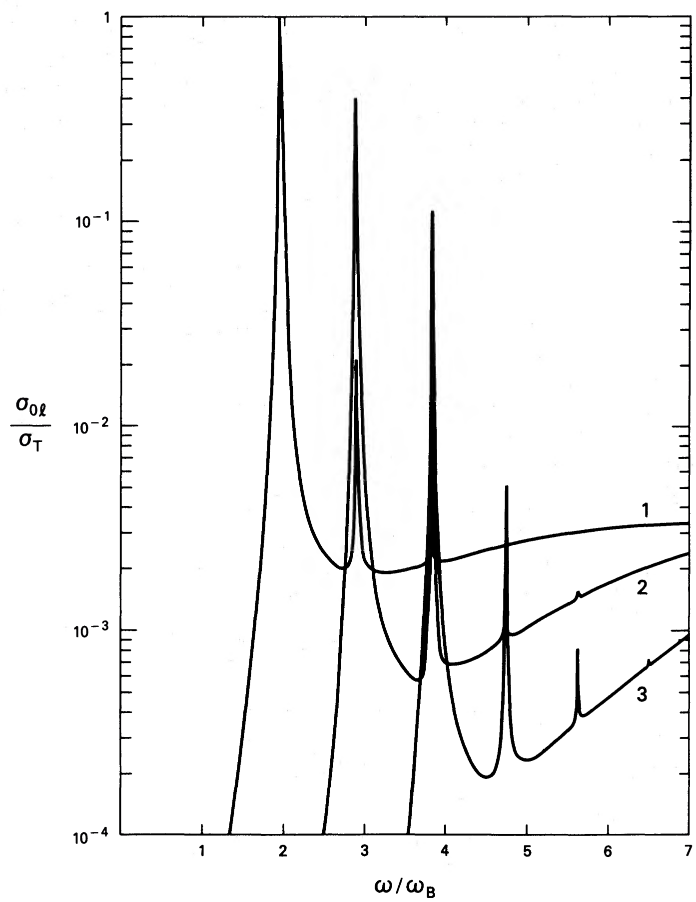

FIG. 5*b*

FIG. 5.—Magnetic Compton cross section σ_{0l} for scattering from the ground state into state l (in units of the Thomson cross section) as a function of incident photon energy ω (in units of the cyclotron energy $\omega_B = eB/m$) for $B = 10^{12}$ G and incident photon angle $\theta = 90°$. Labels refer to final state principal quantum number l. (*a*) "no spin flip"; (*b*) "spin flip."

Applications of the results in this paper to astrophysical sources most likely will require numerical treatment. Approximate expressions for the cross section can be derived in several limits, but many of the interesting effects discussed above cannot be well approximated over a useful parameter range. For example, Herold (1979) was able to derive a nonrelativistic expression for the cross section for scattering into the ground state by ignoring all terms in the intermediate sum except for $n = 0$ and $n = 1$. We have analyzed the relative contribution of intermediate states in the cross section for scattering into excited state l and have found that terms as high as $n = l$ and $n = l + 1$ make significant contributions. Therefore, the prescription used by Herold for the ground state cannot be carried over to derive approximate expressions for scattering into excited states. Alternatively, an asymptotic expression for σ_{0l} may be derived in the limit $l \gg 1$, but its applicability would be restricted to high photon energies which, for many field strengths of interest, would exceed pair production threshold. Fortunately, numerical calculation of the differential cross section, required for the majority of applications, is not overly time-consuming.

In sources with high magnetic fields, synchrotron transitions would need to be included in a self-consistent way to determine the steady-state population of electrons in higher Landau states which could result from scattering. If the excited-state population is found to be significant relative to the population in the ground state, then it would be necessary to extend calculations of the magnetized cross section to include scatterings between excited states.

We would like to thank F. C. Jones for programming advice and R. W. Bussard for useful discussions. J. K. D. acknowledges funding in part by the Research Corporation.

APPENDIX

The full expressions for the functions $F_n^{(i)}$ of the S-matrix element for electrons initially in the ground state, valid in an arbitrary Lorentz frame ($p \neq 0$), may be written as follows:

"No spin flip" case:

$$F_n^{(1)} = \frac{1}{[(E_p+\omega)^2 - E_n^2]}\left[\left[\left\{(E_p+\omega)\left[1+\frac{pq}{(E_p+m)(E_l+m)}\right] - m\left[1-\frac{pq}{(E_p+m)(E_l+m)}\right]\right.\right.\right.$$
$$+ k_z\left(\frac{p}{E_p+m}+\frac{q}{E_l+m}\right)\Bigg\}\Lambda_{l,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^* + \left\{(E_p+\omega)\left[1+\frac{pq}{(E_p+m)(E_l+m)}\right]\right.$$
$$- m\left[1-\frac{pq}{(E_p+m)(E_l+m)}\right] - k_z\left(\frac{p}{E_p+m}+\frac{q}{E_l+m}\right)\Bigg\}\Lambda_{l,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_+$$
$$+ \sqrt{2neB}\left(\frac{p}{E_p+m}+\frac{q}{E_l+m}\right)[\Lambda_{l,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^* + \Lambda_{l,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_+]$$
$$+ \frac{\sqrt{2leB}}{(E_l+m)}\left\{\left[(E_p+m+\omega)\left(\frac{p}{E_p+m}\right)+k_z\right]\Lambda_{l-1,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_-\right.$$
$$- \left[(E_p+m+\omega)\left(\frac{p}{E_p+m}\right)-k_z\right]\Lambda_{l-1,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^*$$
$$\left.\left.+ \sqrt{2neB}\,[\Lambda_{l-1,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_- - \Lambda_{l-1,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^*]\right\}\right]\!\!\Bigg] . \tag{A1}$$

"Spin flip" case:

$$F_n^{(1)} = \frac{1}{[(E_p+\omega)^2 - E_n^2]}\left[\left[\frac{-\sqrt{2leB}}{(E_l+m)}\left\{\left[(E_p+m+\omega)\left(\frac{p}{E_p+m}\right)+k_z\right]\Lambda_{l,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^*\right.\right.\right.$$
$$+ \left[(E_p+m+\omega)\left(\frac{p}{E_p+m}\right)-k_z\right]\Lambda_{l,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_+$$
$$\left.+ \sqrt{2neB}\,[\Lambda_{l,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^* + \Lambda_{l,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_+]\right\}$$
$$+ \left\{m\left[1+\frac{pq}{(E_p+m)(E_l+m)}\right] - (E_p+\omega)\left[1-\frac{pq}{(E_p+m)(E_l+m)}\right]\right.$$
$$- k_z\left(\frac{p}{E_p+m}-\frac{q}{E_l+m}\right)\Bigg\}\Lambda_{l-1,n}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z(\epsilon'^*)_- - \left\{m\left[1+\frac{pq}{(E_p+m)(E_l+m)}\right]\right.$$
$$- (E_p+\omega)\left[1-\frac{pq}{(E_p+m)(E_l+m)}\right] + k_z\left(\frac{p}{E_p+m}-\frac{q}{E_l+m}\right)\Bigg\}\Lambda_{l-1,n-1}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-\epsilon_z'^*$$
$$\left.\left.- \sqrt{2neB}\left(\frac{p}{E_p+m}-\frac{q}{E_l+m}\right)[\Lambda_{l-1,n}(\beta_f)\Lambda_{n-1,0}(\beta_i)\epsilon_-(\epsilon'^*)_- - \Lambda_{l-1,n-1}(\beta_f)\Lambda_{n,0}(\beta_i)\epsilon_z\epsilon_z'^*]\right]\right] . \tag{A2}$$

The $F_n^{(2)}$ terms are obtainable from $F_n^{(1)}$ by the crossing symmetry replacements given in equation (5).

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