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Nuclear and Particle Physics Proceedings 273–275 (2016) 2342–2344

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Generation of cosmic magnetic fields in electroweak plasma

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Abstract

We study the generation of strong magnetic fields in magnetars and in the early universe. For this purpose we calculate the antisymmetric contribution to the photon polarization tensor in a medium consisting of an electron-positron plasma and a gas of neutrinos and antineutrinos, interacting within the Standard Model. Such a contribution exactly takes into account the temperature and the chemical potential of plasma as well as the photon dispersion law in this background matter. It is shown that a nonvanishing Chern-Simons parameter, which appears if there is a nonzero asymmetry between neutrinos and antineutrinos, leads to the instability of a magnetic field resulting to its growth. We apply our result to the description of the magnetic field amplification in the first second of a supernova explosion. It is suggested that this mechanism can explain strong magnetic fields of magnetars. Then we use our approach to study the cosmological magnetic field evolution. We find a lower bound on the neutrino asymmetries consistent with the well-known Big Bang nucleosynthesis bound in a hot universe plasma. Finally we examine the issue of whether a magnetic field can be amplified in a background matter consisting of self-interacting electrons and positrons.

Keywords: magnetic field, Chern-Simons theory, magnetar, early universe

The origin of magnetic fields (B fields) in some astrophysical and cosmological media is still a puzzle for the modern physics and astrophysics. There are multiple models for the generation of strong B fields in magnetars [1]. The observable galactic B field can be a remnant of a strong primordial B field existed in the early universe [2]. Recently the indication on the existence of the inflationary B field was claimed basing on the analysis of BICEP2 data [3]. In the present work we analyze the possibility for the strong B field generation in an electroweak plasma. First, we study the B field generation driven by neutrino asymmetries. Then, we apply our results for the description of strong B fields in magnetars and in the early universe. Finally, we analyze the evolution of a B field in a self-interacting electron-positron plasma.

To study the B field evolution we start with the analysis of the electromagnetic properties of an electroweak plasma consisting of electrons e^-, positrons e^+, neutrinos ν, and antineutrinos $\bar{\nu}$ of all types. These particles are supposed to interact in frames of the Fermi theory. This interaction is parity violating. Thus the photon polarization tensor $\Pi_{\mu\nu}$ acquires a contribution [4],

$$\Pi_{ij}(k) = \mathrm{i}\varepsilon_{ijn}k^n\Pi_2 + \ldots, \tag{1}$$

where $\Pi_2 = \Pi_2(k)$ is the new form factor, or the Chern-Simons (CS) parameter, we will be looking for and $k^\mu = (k_0, \mathbf{k})$ is the photon momentum. Here we adopt the notation of [5]

First, we will be interested in the contribution to Π_2 arising from the interaction of a e^-e^+ plasma with a $\nu\bar{\nu}$ gas. In this case the most general analytical expression for Π_2 can be obtained on the basis of the Feynman diagram shown in Fig. 1. We shall represent Π_2 as $\Pi_2 = \Pi_2^{(\nu)} + \Pi_2^{(\nu e)}$, where $\Pi_2^{(\nu)}$ is the contribution of only the neutrino gas and $\Pi_2^{(\nu e)}$ is the contribution of the e^-e^+ plasma with the nonzero temperature T and the chemi-

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http://dx.doi.org/10.1016/j.nuclphysbps.2015.09.385
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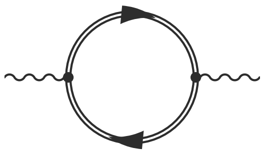
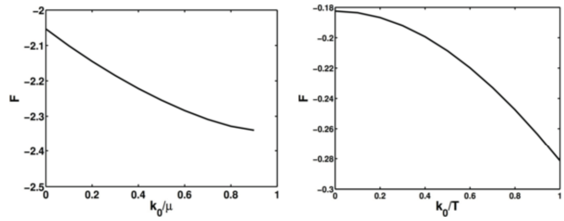


Figure 1: The Feynman diagram for the one loop contribution to the photon polarization tensor in case of a e^-e^+ plasma interacting with a $\nu\bar{\nu}$ gas. The electron propagators are shown as broad straight lines since they account for the densities of background ν and $\bar{\nu}$ [6].



(a) (b)

Figure 2: The function F versus k_0 for a e^-e^+ plasma interacting with a $\nu\bar{\nu}$ gas. (a) Degenerate relativistic plasma. (b) Hot relativistic plasma.

cal potential μ.

The expression for $\Pi_2^{(\nu)}$ can be obtained using the standard quantum field theory technique [6],

$$\Pi_2^{(\nu)} = V_5 \frac{e^2}{2\pi^2} \frac{k^2}{m^2} \int_0^1 \mathrm{d}x \frac{x(1-x)}{1 - \frac{k^2}{m^2} x(1-x)}. \qquad (2)$$

where e is the electron charge, m is the electron mass, $V_5 = (V_\mathrm{R} - V_\mathrm{L})/2$, and $V_\mathrm{R,L}$ are the potentials of the interaction of right and left chiral projections of the e^-e^+ field with the $\nu\bar{\nu}$ background. The explicit form of $V_\mathrm{R,L}$ can be found in [6].

The expression for $\Pi_2^{(\nu e)}$ can be obtained using the technique for the summation over the Matsubara frequencies [6],

$$\begin{aligned}\Pi_2^{(\nu e)} =& V_5 e^2 \int_0^1 dx \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\mathcal{E}_\mathbf{p}^3} \\ & \times \bigg\{ I_0^+ - (1-x) \bigg[\frac{1}{\mathcal{E}_\mathbf{p}^2} \Big(\mathbf{p}^2 \left[3 - 5x\right] \\ & - 3x \left[k^2 x(1-x) + m^2\right] \Big) \left(J_0^+ + J_0^-\right) \\ & - \frac{\beta k_0}{2} x(1-2x) \left(J_1^+ - J_1^-\right) - x \left(J_2^+ + J_2^-\right) \bigg] \bigg\}, \quad (3)\end{aligned}$$

where

$$\begin{aligned} I_0^+ =& \frac{1}{\exp[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} + \frac{\beta \mathcal{E}_\mathbf{p}}{2} \frac{1}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} \\ & + (\mu_+ \to -\mu_+), \\ J_0^+ =& \frac{1}{\exp[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} + \frac{\beta \mathcal{E}_\mathbf{p}}{2} \frac{1 + \frac{\beta \mathcal{E}_\mathbf{p}}{3} \tanh\left[\frac{\beta}{2}(\mathcal{E}_\mathbf{p} + \mu_+)\right]}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} \\ & + (\mu_+ \to -\mu_+), \\ J_1^+ =& \frac{1 + \beta \mathcal{E}_\mathbf{p} \tanh\left[\frac{\beta}{2}(\mathcal{E}_\mathbf{p} + \mu_+)\right]}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} - (\mu_+ \to -\mu_+), \\ J_2^+ =& \frac{1}{\exp[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} + \frac{\beta \mathcal{E}_\mathbf{p}}{2} \frac{1 - \beta \mathcal{E}_\mathbf{p} \tanh\left[\frac{\beta}{2}(\mathcal{E}_\mathbf{p} + \mu_+)\right]}{\cosh[\beta(\mathcal{E}_\mathbf{p} + \mu_+)] + 1} \\ & + (\mu_+ \to -\mu_+). \qquad (4)\end{aligned}$$

Here $\mathcal{E}_\mathbf{p} = \sqrt{\mathbf{p}^2 + M^2}$, $\beta = 1/T$, $\mu_+ = \mu + k_0 x$ and $M^2 = m^2 - k^2 x(1-x)$. To obtain $J_{0,1,2}^-$ in Eq. (3) we should replace $\mu_+ \to \mu_- = \mu - k_0 x$ in $J_{0,1,2}^+$ in Eq. (4). Note that in Eqs. (3) and (4) we assume that $k^2 < 4m^2$, i.e. no creation of e^-e^+ pairs occurs [7].

It is convenient to represent Π_2 as $\Pi_2 = 2\frac{\alpha_\mathrm{em}}{\pi} V_5 F$, where F is the dimensionless function and $\alpha_\mathrm{em} = \frac{e^2}{4\pi}$ is the fine structure constant. Using Eqs. (2)-(4), in Fig. 2 we show the behavior of F versus k_0 in relativistic plasmas. It should be noted that in the static limit $F(k_0 = 0) \neq 0$. To plot Fig. 2 we take into account the dispersion law of long electromagnetic waves in plasma $k^2 = k^2(T, \mu)$ [6] and the fact that an electron acquires the effective mass $m_\mathrm{eff}^2 = \frac{e^2}{8\pi^2}(\mu^2 + \pi^2 T^2)$ in a hot and dense matter [7]. As shown in [6], the nonzero $\Pi_2(0) = \Pi_2(k_0 = 0)$ results in the instability of a B field leading to the exponential growth of a seed field.

We can apply our results for the description of the B field evolution in a dense relativistic electron gas in a supernova explosion. It is known that, just after the core collapse, a supernova is a powerful source of ν_e whereas the fluxes of $\nu_{\mu,\tau}$ and $\bar{\nu}_{e,\mu,\tau}$ are negligible [8]. Thus $V_5 \neq 0$ and we get that $\Pi_2(0) = \frac{\sqrt{2}}{\pi} \alpha_\mathrm{em} G_\mathrm{F} n_{\nu_e} F(0) \neq 0$, where G_F is the Fermi constant and $|F(0)| \approx 2$, see Fig. 2(a), since electrons are degenerate. The magnetic diffusion time $t_\mathrm{diff} = \sigma \Pi_2^{-2}(0) \approx 2.3 \times 10^{-2}\,\mathrm{s}$ for $n_e = 3.7 \times 10^{37}\,\mathrm{cm}^{-3}$ and $n_{\nu_e} = 1.9 \times 10^{37}\,\mathrm{cm}^{-3}$ in the supernova core [6]. Here σ is the electron gas conductivity. Thus at $t \sim 10^{-3}\,\mathrm{s} \ll t_\mathrm{diff}$, when the flux of ν_e is maximal, no seed magnetic field dissipates. Therefore the neutrino driven instability can result in the growth of the B field. It should be noted that the scale of the B field turns out to be small $\Lambda \sim 10^{-3}\,\mathrm{cm}$. However, at later stages of the star evolution V_5 diminishes and Λ can be comparable with the magnetar radius. Thus our mechanism can be used to explain strong B fields of magnetars.

Now let us apply out results to study the B field

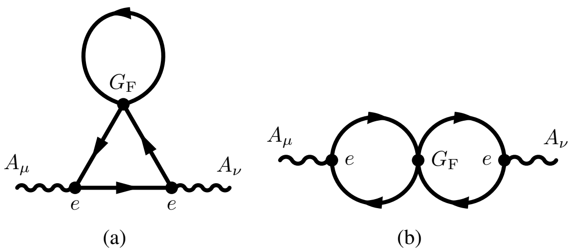


Figure 3: The Feynman diagrams contributing to the photon polarization tensor in case of a e^-e^+ self-interacting plasma. Here A_μ is the potential of the electromagnetic field.

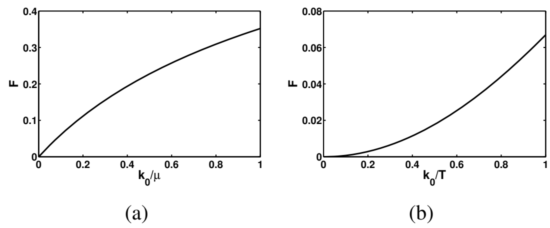


Figure 4: The function F versus k_0 for a e^-e^+ self-interacting plasma. (a) Degenerate relativistic plasma. (b) Hot relativistic plasma.

evolution in the primordial plasma. At the stages of the early universe evolution before the neutrino decoupling at $T > (2-3)\,\text{MeV}$, the e^-e^+ plasma is hot and relativistic. Assuming the causal scenario, in which $\Lambda < H^{-1}$, where H is the Hubble constant, we get that $|\xi_{\nu_e} - \xi_{\nu_\mu} - \xi_{\nu_\tau}| > 1.1 \times 10^{-6}\sqrt{g^*/106.75} \times (T/\text{MeV})^{-1}$, see [6], where $\xi_\alpha = \mu_\alpha/T$, g^* is the number of relativistic degrees of freedom, and μ_α is the chemical potential of neutrinos of the type $\alpha = \nu_e, \nu_\mu, \nu_\tau$. Here we use that $|F(0)| \approx 0.2$, see Fig. 2(b). Assuming that before the Big Bang nucleosynthesis at $T \sim (2-3)\,\text{MeV}$ all neutrino flavors equilibrate owing to neutrino oscillations $\xi_{\nu_e} \sim \xi_{\nu_\mu} \sim \xi_{\nu_\tau}$, we get the lower bound on the neutrino asymmetries, which is consistent with the well-known Big Bang nucleosynthesis upper bound on $|\xi_\alpha|$, see [9].

Finally, let us examine the issue of whether a B field can be amplified in a e^-e^+ plasma self-interacting within the Fermi model, i.e. when a $\nu\bar{\nu}$ gas is not present. In this case the contributions to Π_2 are schematically depicted in Fig. 3. The analytical expression for $\Pi_2^{(ee)}$ can be obtained analogously to the previous case [10],

$$\Pi_2^{(ee)} = \frac{\left(1 - 4\sin^2\theta_W\right)}{2\sqrt{2}} e^2 G_F (n_e - n_{\bar{e}}) \int_0^1 (1-x)\mathrm{d}x \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\mathcal{E}_\mathbf{p}^3} \Big\{ [J_2' - J_2''] - \left[J_0' - J_0''\right] \frac{1}{\mathcal{E}_\mathbf{p}^2} \times \left(\mathbf{p}^2 [3 - 2x] - 3\left[m^2(1+x) + k^2 x^2\right]\right) \Big\}, \quad (5)$$

where $n_{e,\bar{e}}$ are the electron and positron densities, θ_W is the Weinberg angle, and $J_{0,2}' = J_{0,2}^+$ in Eq. (4), with $\mu' = \mu_+$. The expressions for $J_{0,1}''$ can be obtained from $J_{0,2}'$ if we make the replacement $\mu' \to \mu'' = \mu + k_0(1-x)$ there. As in deriving of Eqs. (3) and (4), here we also assume that $k^2 < 4m^2$.

Let us express Π_2 in Eq. (5) as $\Pi_2 = \frac{\alpha_{\text{em}}}{\sqrt{2}\pi}\left(1 - 4\sin^2\theta_W\right) G_F (n_e - n_{\bar{e}}) F$, where F is the dimensionless function. We shall analyze this function in the static limit $k_0 \to 0$. We mention that, if we neglect k_0 in Eq. (5), then $J_{0,2}' = J_{0,2}''$ and $\Pi_2 \to 0$. The behavior of F for relativistic plasmas is shown in Fig. 4, where one can see that $\Pi_2(0) = 0$. In Fig. 4 we also account for the thermal corrections to the photon dispersion and to the electron mass. It means that a e^-e^+ plasma does not reveal the instability of a B field leading to its growth. Therefore, contrary to the claim of [5], one can use this mechanism for neither the explanation of strong B fields of magnetars nor the B field amplification in the early universe.

In conclusion we mention that we have derived the CS term Π_2 in an electroweak plasma consisting of e^- and e^+ as well as ν and $\bar{\nu}$ of all flavors. These particles are involved in the parity violating interaction. It makes possible the existence of a nonzero CS term. In case of a e^-e^+ plasma interacting with a $\nu\bar{\nu}$ background, the CS term is nonvanishing in the static limit when $k_0 = 0$. Therefore, a B field becomes unstable in this system. We have shown that a seed field can be exponentially amplified. This feature of an electroweak plasma in question can be used to explain strong B fields of magnetars and to study the evolution of a primordial B field. We have also demonstrated that there is no B field instability in a self-interacting e^-e^+ plasma.

I am thankful to the organizers of 37th ICHEP for the invitation and a financial support, to V.B. Semikoz for helpful discussions, and to FAPESP (Brazil) for a grant.

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