Impossibility of the strong magnetic fields generation in an electron-positron plasma

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(Dated: July 29, 2018)

We examine the issue whether a magnetic field can be amplified in a background matter consisting of electrons and positrons self-interacting within the Fermi model. For this purpose we compute the antisymmetric contribution to the photon polarization tensor in this matter having nonzero temperature and chemical potential. It is shown that this contribution is vanishing in the static limit. Then we study a particular case of a degenerate relativistic electron gas present in a magnetar. We demonstrate that a seed magnetic field is attenuated in this case. Thus, contrary to the recent claim, we show that there is no magnetic field instability in such a system, which can lead to the magnetic field growth. Therefore recently proposed mechanism cannot be used for the explanation of strong magnetic fields of magnetars.

PACS numbers: 11.10.Wx, 11.15.Yc, 97.60.Gb, 97.10.Ld

Recently the new mechanism of the strong magnetic fields generation in various astrophysical and cosmological plasmas was proposed in Ref. [1]. It is based on the nonzero Chern-Simons (CS) parameter in the Standard Model plasma consisting of all kinds of neutrinos, charged leptons, and quarks. This CS parameter results in the instability of magnetic fields (B-fields), which, in its turn, leads to the growth of a seed magnetic field. As predicted in Ref. [1], there is a very strong magnetic field amplification in case of the electron-positron (e^-e^+) self-interacting plasma, which can be used to explain strong magnetic fields of magnetars [2]. We show, basing on the explicit calculation of the CS parameter in this system using the imaginary time perturbation theory, that this result of Ref. [1] is invalid.

In our work we shall examine the evolution of a B-field in an isotropic e^-e^+ plasma where macroscopic fluxes are absent. For this purpose we shall calculate one of the photon form factors contributing to the photon polarization tensor $\Pi_{\mu\nu}(x)$. In an isotropic medium $\Pi_{\mu\nu} = \int d^4x e^{ikx} \Pi_{\mu\nu}(x)$ has the form,

$$\Pi_{\mu\nu} = \left(g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)\Pi_1 + i\varepsilon_{\mu\nu\alpha0}k^{\alpha}\Pi_2 + \frac{k_{\mu}k_{\nu}}{k^2}\Pi_3 \quad (1)$$

where $\Pi_{1,2,3} = \Pi_{1,2,3}(k)$ are the form factors of a photon, $k^{\mu} = (k^0, \mathbf{k})$ is the photon momentum, $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is the metric tensor in Minkowski space, and $\varepsilon_{\mu\nu\alpha\beta}$ is the absolute antisymmetric tensor having $\varepsilon^{0123} = +1$. In Eq. (1) we adopt the notation for the photon form factors from Ref. [1]. Note that a

more general form of $\Pi_{\mu\nu}$, which also includes the contributions of nonzero macroscopic plasma flows, is given in Ref. [3].

It should be noted that Π_1 in Eq. (1) has a nonzero value even for purely virtual electrons and positrons. It describes the vacuum polarization in QED. The generation of the plasmon mass in a QED plasma takes place if $\Pi_3 \neq 0$. Thus $\Pi_{1,3} \neq 0$ in case of a parity conserving interaction. We shall study the contribution of a parity violating interaction to $\Pi_{\mu\nu}$. That is why we shall concentrate on the analysis of Π_2 , which is absent in the QED case. We also mention that, if $\Pi_2 \neq 0$, the effective Lagrangian of the electromagnetic field acquires a CS term, $\mathcal{L}_{\text{CS}} = \Pi_2(\mathbf{A} \cdot \mathbf{B})$, where \mathbf{A} is the vector potential and $\mathbf{B} = (\nabla \times \mathbf{A})$ is the magnetic field.

Following Ref. [1], let us examine the generation of a CS term in an e^-e^+ plasma, with particles in this plasma self-interacting in frames of the Fermi model. Denoting the electron-positron field as a bispinor ψ , we get the Lagrangian of this system, which also includes the interaction of ψ with the external electromagnetic field $A^{\mu} = (A^0, \mathbf{A})$, in the form [4],

$$\mathcal{L}_{\rm I} = -e\bar{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{G_{\rm F}}{\sqrt{2}}\bar{\psi}\gamma_{\alpha}\left(g_{V} - g_{A}\gamma^{5}\right)\psi$$
$$\times \bar{\psi}\gamma^{\alpha}\left(g_{V} - g_{A}\gamma^{5}\right)\psi,\tag{2}$$

where $\gamma^{\mu}=(\gamma^0, \boldsymbol{\gamma})$ are the Dirac matrices, $\gamma^5=\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, e is the electron's electric charge, G_F is the Fermi constant, $g_V=-\frac{1}{2}+2\sin^2\theta_W$ and $g_A=-\frac{1}{2}$ are the vector and axial constants of the Fermi interaction, and θ_W is the Weinberg angle. The electroweak interaction in Eq. (2) is parity violating. Thus one expects that Π_2 in Eq. (1) could be nonzero.

The contributions to $\Pi_{\mu\nu}$, potentially containing an-

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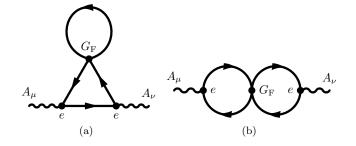


FIG. 1. Schematic representation of the contributions to $\Pi_{\mu\nu}$ in Eq. (3). Straight lines stay for ψ .

tisymmetric terms, are schematically shown in Fig. 1. These terms are quadratic in e and linear in G_F . However, since the Lagrangian in Eq. (2) includes the self-interaction, one should better rely on the expansion of the S-matrix, $S = \mathcal{T} \left[\exp \left(i \int d^4 x \mathcal{L}_I \right) \right]$, where the symbol \mathcal{T} stays for the time ordering. Averaging the aforementioned terms over the state corresponding to the vacuum of ψ and taking into account the definition of $\Pi_{\mu\nu}(x)$, $\langle S \rangle_0 = \frac{i}{2} \int d^4 x d^4 y A^{\mu}(x) \Pi_{\mu\nu}(x-y) A^{\nu}(y)$, we get that

$$\Pi_{\mu\nu} = \sqrt{2}e^{2}G_{F} \int \frac{d^{4}pd^{4}q}{(2\pi)^{8}} \left\{ \operatorname{tr} \left[\gamma_{\mu}S_{F}(k+p)\gamma_{\nu}S_{F}(p)\Gamma_{\alpha}S_{F}(p) \right] \cdot \operatorname{tr} \left[\Gamma^{\alpha}S_{F}(q) \right] \right. \\
\left. - \operatorname{tr} \left[\gamma_{\mu}S_{F}(k+p)\Gamma_{\alpha}S_{F}(k+p)\gamma_{\nu}S_{F}(p) \right] \cdot \operatorname{tr} \left[\Gamma^{\alpha}S_{F}(q) \right] \\
+ \operatorname{tr} \left[\gamma_{\mu}S_{F}(k+p)\Gamma_{\alpha}S_{F}(p) \right] \cdot \operatorname{tr} \left[\gamma_{\nu}S_{F}(q)\Gamma^{\alpha}S_{F}(k+q) \right] \\
+ \operatorname{tr} \left[\gamma_{\mu}S_{F}(k+p)\Gamma_{\alpha}S_{F}(k+q)\gamma_{\nu}S_{F}(q)\Gamma^{\alpha}S_{F}(p) \right] \right\}, \tag{3}$$

where $S_{\rm F}(k) = \int d^4x e^{ikx} S_{\rm F}(x) = (m-k)^{-1}$ is the Fourier transforms of the vacuum electron propagator $S_{\rm F}(x-y) = i \left\langle \mathcal{T} \left[\psi(x) \bar{\psi}(y) \right] \right\rangle_0$ and m is the electron mass. In Eq. (3) we define $\Gamma_{\alpha} = \gamma_{\alpha} \left(g_V - g_A \gamma^5 \right)$ for brevity.

Taking the antisymmetric part of $\Pi_{ij} = i\varepsilon_{ijn}k^n\Pi_2$ in Eq. (3) we obtain Π_2 in Eq. (1). First we calculate Π_2 for purely virtual ψ 's. Using the standard methods of QFT we get that $\Pi_2 = 0$ in this case. Thus there is no ambiguity in the CS term determination, mentioned in Ref. [5], which can appear in a Fermi-like theory with a parity violation.

To get the contribution to Π_2 from a e^-e^+ plasma with the nonzero temperature T and the chemical potential μ we make the following replacement in Eq. (3) [6]: i $\int \frac{dp_0}{2\pi} \to T \sum_n$, where $p_0 = (2n+1)\pi T i + \mu$ and $n=0,\pm 1,\pm 2,\ldots$ One can show that the contribution to Π_2 arising from the two last lines in Eq. (3) cancel each other now. The remaining nonzero contribution can be derived using the standard technique for the summation over the Matsubara frequencies (see, e.g., Ref. [7]),

$$\Pi_{2} = \frac{\left(1 - 4\sin^{2}\theta_{W}\right)}{2\sqrt{2}} e^{2} G_{F} \left(n_{e} - n_{\bar{e}}\right)
\times \int_{0}^{1} (1 - x) dx \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{\mathcal{E}_{\mathbf{p}}^{3}}
\times \left\{ \left[J'_{1} - J''_{1}\right] - \left[J'_{0} - J''_{0}\right] \right.
\left. \times \frac{3}{\mathcal{E}_{\mathbf{p}}^{2}} \left[\mathbf{p}^{2} \left(1 - \frac{2}{3}x\right) - m^{2}(1 + x) - k^{2}x^{2}\right] \right\}, (4)$$

where $\mathcal{E}_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + M^2}$, $M^2 = m^2 - k^2 x (1 - x)$, and

$$J_0' = \frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}} + \mu')] + 1} + \frac{\beta\mathcal{E}_{\mathbf{p}}}{2} \frac{1 + \frac{\beta\mathcal{E}_{\mathbf{p}}}{3} \tanh\left[\frac{\beta}{2}(\mathcal{E}_{\mathbf{p}} + \mu')\right]}{1 + \cosh[\beta(\mathcal{E}_{\mathbf{p}} + \mu')]} + (\mu' \to -\mu'),$$

$$J_{1}' = \frac{1}{\exp[\beta(\mathcal{E}_{\mathbf{p}} + \mu')] + 1} + \frac{\beta\mathcal{E}_{\mathbf{p}}}{2} \frac{1 - \beta\mathcal{E}_{\mathbf{p}} \tanh\left[\frac{\beta}{2}(\mathcal{E}_{\mathbf{p}} + \mu')\right]}{1 + \cosh[\beta(\mathcal{E}_{\mathbf{p}} + \mu')]} + (\mu' \to -\mu').$$
 (5)

Here $\beta=1/T$ and $\mu'=\mu+k_0x$. The expressions for $J_{0,1}''$ are analogous to those in Eq. (5) if we make the replacement $\mu'\to\mu''=\mu+k_0(1-x)$. The electron and positron number densities are defined as $n_{e,\bar{e}}=\int \frac{\mathrm{d}^3p}{(2\pi)^3}(\exp[\beta(E_{\mathbf{p}}\mp\mu)]+1)^{-1}$, where $E_{\mathbf{p}}=\sqrt{\mathbf{p}^2+m^2}$ is the electron energy. To derive Eqs. (4) and (5) we suppose that $k^2<4m^2$, i.e. no creation of e^-e^+ pairs occurs [8].

It is worth mentioning that the first two lines in Eq. (3) correspond to Fig. 1(a) and the last two lines in Eq. (3) – to Fig. 1(b). One can see that the first two lines in Eq. (3) have opposite signs as a consequence of the anticommutativity of ψ . This fact is important for the cancelation of divergencies in Π_2 . The direct calculation shows that the contribution to Π_2 from the last two lines in Eq. (3) equals zero for both virtual and real ψ . This fact also results from the possibility to cut the graph in Fig. 1(b) by a vertical line passing through the central vertex into two identical parts which are linear in momentum.

Let us express Π_2 in Eq. (4) as $\Pi_2 = \frac{\alpha_{\rm em}}{\sqrt{2\pi}} \left(1 - 4\sin^2\theta_W\right) G_{\rm F} \left(n_e - n_{\bar e}\right) F$, where $\alpha_{\rm em} = \frac{e^2}{4\pi}$ is the fine structure constant and F is the dimensionless

function. We shall analyze this function in the static limit $k_0 \to 0$. We mention that, if we neglect k_0 in Eq. (5), $J'_{0,1} = J''_{0,1}$ and $\Pi_2 \to 0$.

To study more carefully the behavior of Π_2 in the static limit we shall consider the case of a degenerate relativistic electron gas inside a magnetar, where $\mu \gg (m,T)$. The dispersion law for long waves with $k_0 \gg |\mathbf{k}|$ in this background matter reads $k^2 \approx \omega_p^2$ [7], where $\omega_p^2 = \frac{4}{3\pi}\alpha_{\rm em}\mu^2$ and $k_0 \approx 0.06\mu \ll \mu$. Thus k_0/μ is the small parameter

and hence can be neglected.

Considering the limit $T/\mu \to 0$ in Eqs. (4) and (5) and using the results of Ref. [7], we get F in the explicit form,

$$F = \int_0^1 (1 - x) dx \left[Z^2 (I_2' - I_2'') - 2(1 - x)(I_1' - I_1'') - (I_0' - I_0'') \right], \tag{6}$$

where

$$I'_{0} = \left\{ \left[2(1-x) - 3\frac{Z^{2}}{\mathcal{M}'^{2}} \right] \left(1 - \frac{\tilde{M}^{2}}{\mathcal{M}'^{2}} \right)^{1/2} + \left[2\left(1 - \frac{x}{3} \right) + \frac{Z^{2}}{\mathcal{M}'^{2}} \left(1 - 2\frac{\tilde{M}^{2}}{\mathcal{M}'^{2}} \right) \right] \left(1 - \frac{\tilde{M}^{2}}{\mathcal{M}'^{2}} \right)^{-1/2} \right\}$$

$$\times \theta(\mathcal{M}' - \tilde{M})$$

$$I'_{1} = \left\{ \ln \left(\frac{\mathcal{M}' + \sqrt{\mathcal{M}'^{2} - \tilde{M}^{2}}}{\tilde{M}} \right) + \frac{1}{\mathcal{M}'\tilde{M}^{2}} \left[\left(\mathcal{M}'^{2} - \tilde{M}^{2} \right)^{3/2} - \mathcal{M}'^{2}\sqrt{\mathcal{M}'^{2} - \tilde{M}^{2}} \right] \right\} \theta(\mathcal{M}' - \tilde{M}),$$

$$I'_{2} = \frac{\left(\mathcal{M}'^{2} - \tilde{M}^{2} \right)^{3/2}}{\mathcal{M}'^{3}\tilde{M}^{2}} \theta(\mathcal{M}' - \tilde{M}),$$

$$\mathcal{M}' = 1 + \frac{k_{0}}{\mu}x, \quad \mathcal{M}'' = 1 + \frac{k_{0}}{\mu}(1 - x), \quad \tilde{M} = \frac{1}{\mu}\sqrt{m_{\text{eff}}^{2} - k^{2}x(1 - x)},$$

$$Z^{2} = \tilde{M}^{2} \left(1 - \frac{2}{3}x \right) + \frac{1}{\mu^{2}} \left[m_{\text{eff}}^{2}(1 + x) + k^{2}x^{2} \right],$$

$$(7)$$

where $\theta(z)$ is the Heaviside step function. The expressions for $I_{0,1,2}''$ can be obtained by replacing $\mathcal{M}' \to \mathcal{M}''$ in $I_{0,1,2}'$ in Eq. (7). In Eqs. (6) and (7) we take into account that an electron acquires the effective mass $m_{\text{eff}}^2 = \frac{e^2}{8\pi^2}\mu^2$ in case of $\mu \gg m$ as found in Ref. [8].

Basing on Eqs. (6) and (7), in Fig. 2 we plot the function F for a relativistic degenerate electron gas. One can see that $F \to 0$ and thus $\Pi_2 \to 0$ for small k_0 in agreement with Eqs. (4) and (5).

Note that the nonzero $\Pi_2(0)=\Pi_2(k_0=0)$ can potentially generate the instability of a large scale seed B-field resulting in its exponential growth (see, e.g., Ref. [7]). Longitudinal plasmons, contributing to the B-field growth, can be created in this case. However, as results from Fig. 2, in our situation $\Pi_2(0)=0$ and $\frac{d\Pi_2}{dk_0}(k_0=0)\neq 0$. To study the B-field evolution in this case we should analyze the modified Maxwell equations

$$i(\mathbf{q} \times \mathbf{B}) = -i\omega \mathbf{E} + \sigma \mathbf{E} + \mathbf{j}_5, \quad i(\mathbf{q} \times \mathbf{E}) = i\omega \mathbf{B}, \quad (\mathbf{q} \cdot \mathbf{B}) = 0,$$
(8)

where σ is the plasma conductivity, ${\bf E}$ is the electric field, and

$$\mathbf{j}_5 = \Pi_2(\omega)\mathbf{B} = -\mathrm{i}\zeta\omega\mathbf{B}.\tag{9}$$

Here ζ is the constant parameter, which can be obtained

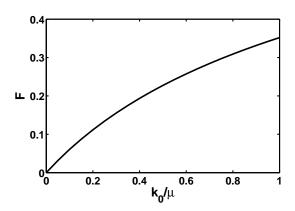


FIG. 2. The function F versus k_0 for a degenerate relativistic electron gas.

using the definition of F as

$$\zeta = i \frac{\alpha_{\rm em}}{\sqrt{2}\pi} \left(1 - 4\sin^2\theta_W \right) \frac{G_F(n_e - n_{\bar{e}})}{\mu} \left. \frac{\mathrm{d}F}{\mathrm{d}x} \right|_{x=0}. \quad (10)$$

In Eqs. (8) and (9) we use the Fourier representation of the electromagnetic field $\sim e^{-\mathrm{i}\omega t + \mathrm{i}\mathbf{q}\mathbf{r}}$. It should be noted that ζ in Eq. (10) is purely imaginary as predicted in Ref. [9] for a CP even Lagrangian, cf. Eq. (2).

Using the magnetohydrodynamic approximation,

$$\sigma \gg \omega, \quad |\mathbf{q}||\mathbf{B}| \gg \omega |\mathbf{E}|, \tag{11}$$

and assuming that

$$|\mathbf{B}| \gg -\mathrm{i}\zeta |\mathbf{E}|,$$
 (12)

on the basis of Eqs. (8) and (9), we get the equation for the B-field evolution.

$$\mathbf{q}^2 \mathbf{B} = i\sigma \omega \mathbf{B} - \zeta^2 \omega^2 \mathbf{B}. \tag{13}$$

Using Eq. (13), we obtain the dispersion relation,

$$\omega = \frac{\mathrm{i}}{2\zeta^2} \left(\sigma \mp \sqrt{\sigma^2 + 4\zeta^2 \mathbf{q}^2} \right). \tag{14}$$

One can see that, at any values of ζ , σ , and $|\mathbf{q}|$, $\mathrm{Im}(\omega) \leq 0$ in Eq. (14). Thus there is no *B*-field growth in the system in question. A seed magnetic field is attenuated instead.

Let us discuss the approximations made in deriving of Eq. (13). First we note that $-\mathrm{i}\zeta\ll 1$ in the case of an electron plasma component in a magnetar. In this situation $n_{\bar{e}}=0$. On the basis of Fig. 2 as well as Eqs. (6) and (7), we get that $\frac{\mathrm{d}F}{\mathrm{d}x}|_{x=0}\approx 0.5$. The chemical potential of a relativistic degenerate electron gas is $\mu=(3\pi n_e)^{1/3}$. The electron density in a neutron star is maximal just after the core collapsing stage $n_e\sim 10^{37}\,\mathrm{cm}^{-3}$ [7]. At the subsequent stages of the magnetar evolution n_e diminishes and reaches only a few percent of the neutron density. Assuming that $n_e=10^{37}\,\mathrm{cm}^{-3}$ in Eq. (10), we get that $-\mathrm{i}\zeta\approx 4.5\times 10^{-11}\ll 1$.

We shall study the situation when in Eq. (13) the *B*-field attenuation is small, i.e. $\sigma \ll -\mathrm{i}\zeta |\mathbf{q}|$. In this case the dispersion relation in Eq. (14) reads, $|\mathbf{q}| = -\mathrm{i}\zeta\omega$. Therefore the condition in Eq. (12) is automatically satisfied if the magnetohydrodynamic approximation in Eq. (11) is valid. Indeed $|\mathbf{B}| \gg \frac{\omega}{|\mathbf{q}|} |\mathbf{E}| = \frac{\mathrm{i}}{\zeta} |\mathbf{E}| \gg -\mathrm{i}\zeta |\mathbf{E}|$, since $-\mathrm{i}\zeta \ll 1$.

If we discuss a situation opposite to that in Eq. (11), i.e. assume that $\omega \gg \sigma$, one can obtain from Eqs. (8) and (9) that **B** as well as ω and **q** obey the following equations:

$$\mathbf{B} \pm \mathrm{i} \left(\mathbf{e}_q \times \mathbf{B} \right) = 0, \quad \omega^2 = |\mathbf{q}|^2 \left(\frac{1}{\chi \epsilon} \pm \mathrm{i} \frac{\zeta \omega}{\epsilon |\mathbf{q}|} \right), \quad (15)$$

where $\mathbf{e}_q = \mathbf{q}/|\mathbf{q}|$ is the unit vector. In Eq. (15) we also restore the nonzero permittivity ϵ and permeability χ . Eq. (15) describes the birefringence of electromagnetic

waves [9]. However, the amplitude of the B-field is constant, i.e. again no instability occurs.

Now let us make some general comments on the method adopted for the calculation of Π_2 . The electromagnetic vertex γ^{μ} can get radiative corrections in the presence of e^-e^+ plasma with finite temperature and chemical potential. These corrections were studied in frames of QED in Ref. [10]. The correction to γ^{μ} is additive and proportional to $\alpha_{\rm em} \approx 1/137$. Taking into account that Π_2 in Eq. (4) is also proportional to $\alpha_{\rm em}$, we get that the corresponding correction to Π_2 is $\sim 10^2$ times smaller than the leading term in Eq. (4).

In the imaginary time perturbation theory used in our work the electron propagator $S_{\rm F}(k)$ is unchanged. Nevertheless, since we account for the dispersion relation of a plasmon $k^2 = k^2(T,\mu)$ in matter, we have to take into account the radiative correction to the electron mass $m \to m_{\rm eff}(T,\mu)$. Firstly, as shown in Refs. [7, 8], $m_{\rm eff}^2$ and k^2 are of the same order of magnitude. Therefore one should take them into account simultaneously in $\mathcal{E}_{\bf p}$ in Eqs. (4) and (5). Secondly, to avoid the plasmon decay into e^-e^+ pairs [8], we should guarantee that $k^2 < 4m_{\rm eff}^2$. One can check that it is the case for a degenerate electron gas in a magnetar.

In conclusion we notice that we have explicitly demonstrated that a medium consisting of electrons and positrons self-interacting within the Fermi model, cf. Eq. (2), does not reveal an instability of a B-field leading to its growth. We have studied a particular case of a relativistic degenerate electrons inside a magnetar and showed that, using this mechanism, one cannot explain the amplification of a B-field of a protostar to the values observed in a magnetar contrary to the claim of Ref. [1].

We should note that the instability of the B-field does not exist in a hot relativistic e^-e^+ plasma, with $T\gg(m,\mu)$, either. To analyze this case one should take into account that $k^2\approx\frac{4\pi}{9}\alpha_{\rm em}T^2$, $k_0\approx 0.1T\ll T$, and $m^2\to m_{\rm eff}^2=\frac{e^2}{8}T^2$ (see Refs. [7, 8]) in Eqs. (4) and (5). It means that one cannot use this mechanism for the description of the B-field amplification in the early universe at $m\ll T(\ll m_\mu)$, where m_μ is the muon mass.

ACKNOWLEDGMENTS

I am thankful to V. B. Semikoz for helpful comments, to FAPESP (Brazil) for a grant, and to Y. S. Kivshar for the hospitality at ANU where a part of the work was made.

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