Synchrotron radiation

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REVIEWS OF TOPICAL PROBLEMS

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Contents

Abstract. The classical and quantum theories of synchrotron radiation (SR) are reviewed. A concise history of the development and discovery of SR, and its fundamental properties, are given. The emphasis is placed on the quantum effects: quantum fluctuations of trajectories of electrons, and radiative polarisation of electrons and positrons in storage rings. The theories of undulator radiation and radiation in a short magnet are discussed in brief. Experimental investigations of synchrotron radiation and its applications in physical experiments are reviewed.

1. Introduction

In 1947, for the first time in history, Floyd Haber — a young staff member in the laboratory of Professor Pollock — observed radiation emitted by electrons as they moved circularly in the magnetic field of the chamber of an accelerator. This occurred during the adjustment of a cyclic accelerator-synchrotron, which accelerated electrons up to 100 MeV [1, 2]. The radiation was observed as a bright luminous patch on the background of the chamber of the synchrotron. It was clearly visible in daylight. In this way 'electronic light' was experimentally seen for the first time — radiation emitted by relativistic electrons having a large centripetal acceleration. The radiation was called synchrotron radiation (SR)† since it was observed for the first time in a synchrotron.

It is hard to overestimate the importance of SR in our days: interest is growing incessantly since the radiation features a rare combination of fundamental properties and important scientific and technical applications.

It was sheer accident that the SR was observed: the opaque metallised cover of the chamber was removed to perform an adjustment and this allowed the light to be seen outside the chamber.

The discovery and first observations of the synchrotron radiation were dramatic; its properties seemed mysterious and unusual at the initial stage of investigations. However, a number of theoretical studies on the emission of a relativistic accelerating electron had been carried out long before the experiment described above.

The first steps in this direction were taken by Lienard (1898) and Heaviside (1902) [4]. They extended the familiar Larmor formula for the plane power of a nonrelativistic electron,

$$W = -\frac{\partial E}{\partial t} = \frac{2e^2\,\dot{v}^2}{3c^3}\,, \tag{1.1}$$

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†There is another name in the literature — the magnetic breaking radia-tion. This term is common in astrophysical problems (see Ref. [3]).

to a high-velocity particle. In modern notation it takes the form

$$W = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{\mathrm{d}p_\mu}{\mathrm{d}\tau}\frac{\mathrm{d}p_\mu}{\mathrm{d}\tau}\right) = \frac{2}{3}\frac{e^2\gamma^6}{c}\left[\dot{\boldsymbol{\beta}}^2 - (\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}})^2\right]$$

(p_μ is the four-dimensional impulse, $\mathrm{d}\tau = \mathrm{d}t/\gamma$ is the intrinsic time, $\gamma = E/mc^2$, and $\boldsymbol{\beta} = v/c$). Lienard turned his attention to the fast growth of losses in the energy of an electron describing a circle ($\boldsymbol{\beta} \perp \dot{\boldsymbol{\beta}}$) of radius R:

$$W = \frac{2}{3}\frac{e^2c}{R^2}\beta^4\gamma^4 . \qquad (1.2)$$

The growth was proportional to the fourth power of the energy.

Subsequently Schott (1907) made an interesting detailed study of the radiation an electron emits as it follows a circular path [5]. Schott's objective was to explain the discrete nature of atomic spectra. Based on early models of the atom and especially on the Saturnian model in which electrons in an atom move about the positive charge in circles similar to the rings of Saturn, Schott made an attempt to calculate the spectrum and the distribution of spatial radiation of electrons in an atom by the strict methods of classical electrodynamics. He reckoned the spectral theory to be the most important issue of the theory of matter since he believed it to be the way to the working model of the atom.

The consistency and elegance of the Schott theory are admirable. However, his attempts to explain the atomic radiation within the scope of classical physics could not have met with success. For this reason Schott's work was only of academic interest for 40 years and was virtually forgotten. Their relavance was discovered in new circumstances 40 years later when the issue of an emitting charge moving in a macroscopic trajectory arose.

Of primary interest was the emission of accelerating electrons in a magnetic field. In 1939 I Ya Pomeranchuk established the radiative 'ceiling' for the energy of electrons in his attempt to determine the maximal energy the cosmic charged particles could possess at the Earth's surface due to radiative losses in the Earth's magnetic field [6]. By means of this estimation, the maximal energy was then predicted for a betatron — an induction accelerator, in which electrons move in a magnetic field which builds up in time and is virtually homogeneous along the trajectory of the particle (D D Ivanenko and I Ya Pomeranchuk) [7]. The existence of radiative losses in the energy of an electron in the magnetic field of an accelerator was soon verified in experiments conducted by Blewett (1946, [8]). He found that electrons moved in decreasing orbits as their energies increased: the particles moved in a converging spiral and ceased to accelerate because of a loss in energy to radiation (note that the energy, radius of orbit, and magnetic field strength are related by the equation $\beta E = eHR$, see Ref. [25]).

Blewett's experiments could be considered to be a proof of the actual existence of radiation from relativistic charges and this radiation could even be called the betatron radiation. However, attempts to visually — directly — observe this radiation did not meet with success: the search for radiation in the microwave range (dipole radiation) was a total failure. This exceptional situation — the energy losses of electrons was surely observed while the radiation itself was elusive — dramatically showed that large radiative losses alone did not uncover the fundamental features of this extraordinary phenomenon.

Having studied theoretically the spectral distribution of the radiation power emitted by a circularly moving relativistic electron, L A Artsimovich and I Ya Pomeranchuk found out that the maximal power fell not on the fundamental frequency (as would be the case for a dipole radiation), but on its higher harmonics: $\omega \sim \omega_0\gamma^3$ [see Eqn (2.1) below]. For electrons of energy 80 – 100 MeV, the radiation ought to be observed not in the microwave range but in the radiation range of higher multifields, i.e., in the visible range (1945, Ref. [9]; see also Ref. [10]). This was revealed in an experiment on the synchrotron in the USA [1, 2].

It was shown in Ref. [9] that the angular distribution of the power of synchrotron radiation is highly anisotropic — it is concentrated in a slender cone of angle $\delta\psi \sim 1/\gamma$ in the orbital plane of revolution of the electron and is directed forward in parallel to its motion. The theoretical study of the coherence of radiation [9] (this is of especial interest for the radiation of a cluster of electrons in a betatron, when they fill almost all the orbit) showed that the coherence could manifest itself at the lowest frequencies only because of fluctuations of the current density in a beam for $\gamma \gg 1$ — far from the maximum of the spectral distribution of the power.

Thus, the qualitative description of the properties of synchrotron radiation were known before it was observed for the first time. However, as is noted above, it was discovered by sheer accident.

The discovery of electronic light in the synchrotron stimulated further investigations of the SR properties and, first of all, analysis of the spectral and angular distribution of the radiation power. This was a complicated problem since the Schott formulas [5] were inconvenient for describing the radiation spectrum of a relativistic electron when it involved the higher harmonics of the frequency of the circular revolution of the electron. The conventional approach of expanding the series in terms of multifields was not applicable to the analysis of the radiation. Then the asymptotic problem in the radiation spectrum of a relativistic electron arose for a large relativistic factor $\gamma \gg 1$.

V V Vladimirskii [11] successfully applied Airy functions, which had been studied thoroughly by V A Fok, to describe the radiation spectrum of an electron moving in a magnetic field. The asymptotical formulas for the spectral composition of synchrotron radiation were independently determined in several theoretical studies [11 – 15]. They opened up a possibility for an experimental verification. Experiments demonstrated a good agreement with the theory in the visible range [16], the vacuum ultraviolet range [17], and the x-ray range [18].

The classical theory of SR was then contributed to by investigations of the polarisation features of SR [19]. It was established, for example, that SR is elliptically polarised in general and it is linearly polarised when observed in the direction close to the orbital plane of revolution. The first observations of the linear polarisation were made in the initial studies [2] but the polarisation features of SR were investigated in detail by the staff members of the Physical Department of the MSU (Moscow State University) on the synchrotron in the FIAN (Physical Institute of the Academy of Sciences) and showed definite agreement with the theory [20] (see also Ref. [21]).

Numerous investigations shaped the classical theory of synchrotron radiation to perfection, and the theory was included in a variety of monographs [3, 22–25] and courses [14, 26, 27]. Synchrotron radiation became important for astrophysics, in the analysis of nonthermal cosmic radiation. The Swedish scientists Alfven and Herlofson suggested in 1950 [28] that the nonthermal radiation of our galaxy could be explained through the mechanism of SR. In Russia this problem was addressed at the same time by Ginzburg, Syrovatskii [29], and Shklovskii [30]. The recognition of the importance of synchrotron radiation, the development of its theory, and experiments stipulated the outstanding advances in radioastronomy.

In the last few years the problem has acquired a new and important feature — synchrotron radiation is used extensively in scientific research. In parallel with our widening knowledge of the nature of the phenomenon, the objective of accelerators and storage units of electrons has changed recently — they have become a major source of synchrotron radiation which has taken up an independent position in experimental physics.

The physics of undulator radiation — the radiation of relativistic electrons as they move in a periodic outer field — is of utmost importance in experimental applications of SR. Undulator radiation, which Ginzburg predicted in 1947 [31], has the same origin as SR and is similar to it in many aspects. It has attracted considerable attention lately.

Although the theory of SR seemed to be complete, it turned out that the electronic light possessed a variety of fine and interesting properties which the classical theory of an accelerating charge did not describe: the physical nature of SR turned out to be richer and the quantum theory had to be applied for its comprehensive description [22, 24, 25].

The quantum theory of SR [22, 24] helps to explain the discrete nature of the radiation and its influence on the trajectory of the particle (the recoil effect). As the theory shows [32, 67], this influence manifests itself in the quantum widening of the trajectory of an electron — the particle is involved in a peculiar Brownian movement and quantum fluctuations of the trajectory are macroscopic in nature. The latter fact turned out to be important in the engineering of accelerator design and storage of electrons.

The quantum theory also made it possible to investigate the SR emitted by a polarised electron and to investigate the contribution of the spin of a particle to the radiation power. The analysis of the spin evolution during synchrotron radiation revealed the effect of the polarisation of radiation of electrons and positrons in storage rings [33]. This effect is of special interest in connection with the problem of how to create a beam of relativistic particles with an oriented spin.

It should be noted that storage rings in which there was a possibility to compensate for radiative energy losses became a unique laboratory for studying quantum effects, since electrons could circulate for tens of hours under such conditions, with the average energy remaining constant. The quantum effects in synchrotrons were also verified experimentally [27]. Thus, now both classical and quantum theories of SR are complete and reliable.

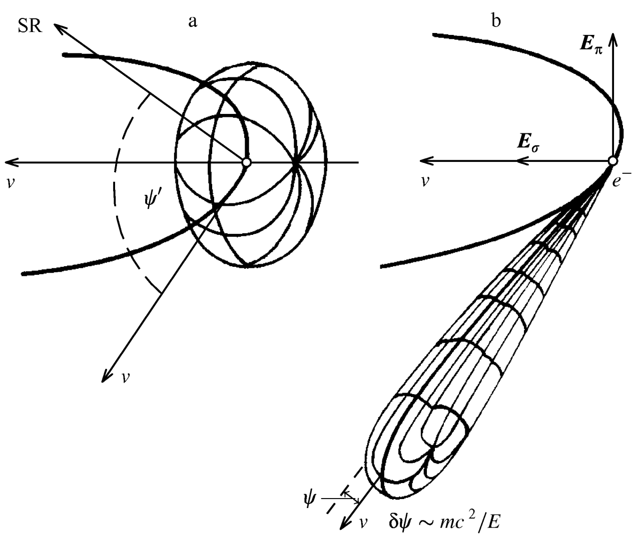


Figure 1. Angular distribution of synchrotron radiation: (a) nonrelativistic motion, $\beta \ll 1$; (b) relativistic motion, $\beta \to 1$.

2. Review of classical theory

2.1 Features of the radiation from a relativistic electron. Genetic relationship of synchrotron and undulator radiation

First of all, let us consider how the angular distribution of the radiative power and its spectral composition changes in the case of a relativistic particle. As is known (Larmor), the spatial distribution of the radiation power of a nonrelativistic electron can be described by a toroid (Fig. 1a) and, moreover, the radiation peaks in the direction of the outer magnetic field when the angle ψ' measured from the velocity vector of the particle is close to 2π.

If the relativistic velocity of the electron is $\beta = v/c \to 1$, i.e. $\gamma = E/mc^2 \gg 1$, then the toroid is strongly deformed because of the Doppler effect and is elongated in a cone the axis of which is directed in parallel with the velocity of the particle (Fig. 1b).

It is convenient to introduce the relativistic transformation of angles. Let ψ' be the angle in the system of coordinates in which the electron is at rest. Then the angle ψ from which the radiation is observed in the laboratory system of coordinates is found by means of the aberration formula:

$$\sin\psi = \frac{(1-\beta^2)^{1/2}\sin\psi'}{1+\beta\cos\psi'}\,.$$

Setting $\psi' = \pi/2$ for which the dipole radiation peaks, we obtain

$$\sin\psi \cong \delta\psi = (1-\beta^2)^{1/2} = \gamma^{-1} = \frac{mc^2}{E}\,.$$

Thus, synchrotron radiation has a pronounced 'gun' effect: it is directed ahead in parallel to the motion of an electron and is concentrated in a slender cone of angle $\delta\psi \sim \gamma^{-1}$ (Fig. 1b).

The peculiar features of the spectral composition of SR can also be readily explained. As a consequence of the gun effect typical of a single relativistic electron, the observer registers the radiation as a short impulse when the needle-shaped ray passes through the point of observation (Fig. 2). Let the efficient length of the arc along which the radiation occurs be $l = R\delta\psi$. The time $\tau' = l/c$ for which the electron

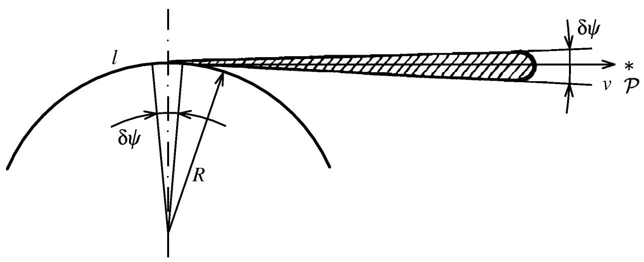


Figure 2. Scheme of observation of synchrotron radiation at a point $\mathcal{P}$.

travels this distance is equal to the duration of the burst of radiation. However the electromagnetic wave is late at the point $\mathcal{P}$ and the duration of the impulse is

$$\Delta t = (1 - \boldsymbol{\beta}\cdot\boldsymbol{n})\tau' \cong \tau'\gamma^{-2}, \quad |\boldsymbol{n}| = 1 .$$

in the laboratory system of coordinates.

However a short-time impulse of radiation is inconsistent with a narrow spectrum of frequencies (see Ref. [117]). As is well known in radiotelegraphy, a short signal always has a wide spectrum.

Thus, a wave packet arrives at the point of observation; and the intervals Δt and $\Delta\omega$ which characterise the duration of the approximate transmitted signal and frequency composition of the spectrum are related by the equation $\Delta\omega\Delta t \cong 1$. The observer will register a bunch of harmonics of the spectrum, including those of the order of the critical frequency $\omega_{\text{cr}} \approx \Delta\omega = c\gamma^2/l$.

The two frequency spectra are realisable, depending on the nature of the motion of a particle in the magnetic field.

(a) *Synchrotron radiation.* Since the length of the arc along which the radiation is emitted is equal to $l = R\delta\psi = R\gamma^{-1}$ in this case, the critical frequency $\omega_{\text{cr}} \approx \Delta\omega = c\gamma^3/R = \omega_0\gamma^3$. Thus, the spectrum involves the higher harmonics of the fundamental frequency proportional to γ^3 (see Ref. [9]).

(b) *Undulator radiation.* Another mode of radiation of a relativistic particle is admissable when the radiation is observed immediately from along the whole trajectory (see Fig. 3). In this case, $\Delta\omega = c\gamma^2/l_0$—the maximum falls on the fundamental and turns out to be proportional to γ^2, as a result of the relativistic multiplication of frequency (see Ref. [10]); l_0 is the length of the characteristic period of the undulator.

(c) *Radiation in a short magnet.* Of interest is the motion of an electron in an arc of a circle when $\delta\psi \ll \gamma^{-1}$. In this case, the radiation of an electron beam in a short magnet is 'white noise' [34] ranging from zero up to the frequency $\omega_{\text{cr}} = \beta c\gamma^2/l_0$. The spectral properties of this radiation differ greatly from those of synchrotron radiation, especially in the low-frequency range. Note that the angular distribution of radiation power is the same for synchrotron radiation as well as for undulator radiation, and also for the motion in an arc of a circle in a short magnet.

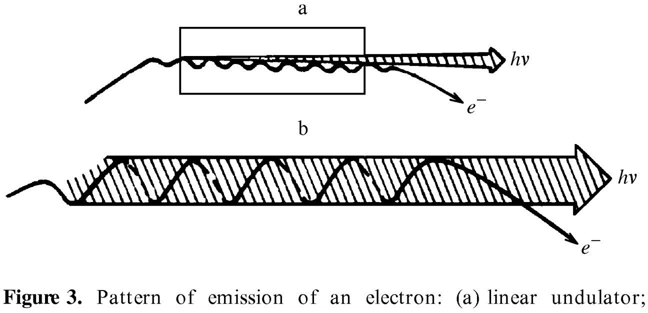


Figure 3. Pattern of emission of an electron: (a) linear undulator; (b) spiral undulator.

Thus, the above analysis shows that all of the three kinds of radiation an electron emits as it moves in a magnetic field are in the form of genetic relations.

2.2 Schott's formula, polarisation, and the angular distribution of radiation power

As is noted above, the problem of radiation from a relativistic charge moving in a circular orbit was solved for the first time by Schott. He derived a familiar formula for the spectral–angular distribution of radiation power

$$W = -\frac{\partial E}{\partial t} = \sum_{\nu=1}^{\infty} \oint \mathrm{d}\Omega W(\nu, \theta) ,$$

where the differential radiation power is

$$W(\nu, \theta) = \frac{e^2 c\beta^2 \nu^2}{2\pi R^2} \left[\cot^2\theta J_\nu^2(\nu\beta\sin\theta) + \beta^2 J_\nu'^2(\nu\beta\sin\theta)\right]. \tag{2.1}$$

Here, ν is the number of the harmonic of radiation, $\omega = \nu\omega_0$, $\omega_0 = e_0 cH/E$, $e = -e_0$, $e_0 > 0$ is the charge of an electron, and J_ν and J_ν' are the Bessel function and its derivative with respect to $[\nu\beta(\sin\theta)]$. Schott's formula is the exact solution to the equations of classical electrodynamics for the radiation of a nonrelativistic electron moving in a circle of radius R.

In applications of the theory of SR the polarisation properties of the radiation are of extreme importance. Therefore, first of all I shall dwell on the extended Schott formula, which accounts for the polarised radiation [19]. In order to describe a linearly polarised radiation, two perpendicular unit vectors $\boldsymbol{e}_\sigma$ and $\boldsymbol{e}_\pi$ are introduced. They are orthogonal to the wave vector $\boldsymbol{n}^0$

$$\boldsymbol{n}_\sigma = \frac{\boldsymbol{n}^0 \times \boldsymbol{j}}{|\boldsymbol{n}^0 \times \boldsymbol{j}|}, \quad \boldsymbol{e}_\pi = \boldsymbol{n}^0 \times \boldsymbol{e}_\sigma , \tag{2.2}$$

where $\boldsymbol{j} = \boldsymbol{H}/H$ is the vector parallel to the outer field. The components σ and π of the linear polarisation are characterised by the direction of the vector of the electric radiation field (Fig. 1b): $\boldsymbol{E}_\sigma$ lies in the orbital plane of revolution and is directed along the radius to the centre, and the vector $\boldsymbol{E}_\pi$ is nearly parallel to the outer magnetic field since $\boldsymbol{n}^0$ is nearly perpendicular to $\boldsymbol{H}$ in the relativistic case.

Then the extended Schott formula (2.1) is [19, 24]

$$W_{\sigma,\pi}(\nu, \theta) = \frac{e^2 c\beta^2 \nu^2}{2\pi R^2} \left[l_\sigma \beta J_\nu'(\nu\beta\sin\theta) + l_\pi \cot\theta J_\nu(\nu\beta\sin\theta)\right]^2 . \tag{2.3}$$

In this case the power of the σ-component of the linear polarisation of radiation is obtained by putting $l_\sigma = 1$, $l_\pi = 0$ (l_σ and l_π are introduced to simplify the notation) and the choice $l_\sigma = 0$, $l_\pi = 1$ corresponds to the π-component. Finally, by introducing the vector

$$\boldsymbol{e}_\pm = \frac{1}{\sqrt{2}}\left(\boldsymbol{e}_\sigma \pm \mathrm{i}\boldsymbol{e}_\pi\right) ,$$

to describe the circulation polarisation, we obtain that the choice of $l_\sigma = l_\pi = 1/\sqrt{2}$ corresponds to the clockwise circular polarisation and $l_\sigma = -l_\pi = 1/\sqrt{2}$ to the counterclockwise polarisation. The total radiation power is a sum of polarisations: $W = W_\sigma + W_\pi = W_1 + W_{-1}$. I draw the reader's attention to the fact that SR is fully polarised in the orbital plane of revolution of an electron since W_π vanishes for $\theta = \pi/2$. In the general case of observation, the synchrotron radiation has an elliptic polarisation the sign of which changes in going through the orbital plane of revolution of the particle.

The pronounced linear polarisation of SR becomes especially clear when one considers the total power of SR:

$$W_\sigma = \sum_{\nu=1}^{\infty} \oint \mathrm{d}\Omega W_\sigma(\nu, \theta) = (6+\beta^2)\,\frac{W}{8}\,,$$
$$W_\pi = \sum_{\nu=1}^{\infty} \oint \mathrm{d}\Omega W_\pi(\nu, \theta) = (2-\beta^2)\,\frac{W}{8}\,, \quad (2.4)$$

where $W = 2e^2 c\beta^4\gamma^4/3R^2$. In addition, $W_\sigma = 7W/8$ and $W_\pi = W/8$ in the ultrarelativistic case ($\beta \to 1$).

The polarisation of synchrotron radiation is of especial interest not only in connection with applications in the physical experiment under laboratory conditions, but also in astrophysical observations: the polarisation effect could be used as a decisive test for determining the nature of the radiation coming from extraterrestrial sources [28–30]. In particular, the study of electromagnetic radiation coming from the crab-like nebula — the gaseous envelope left after an exploded supernova — met with outstanding success. The synchrotron origin of the radiation of the crab-like nebula was established reliably on the basis of measurements of polarisation over the entire range of frequencies — from the rf to the optical, x-ray, and gamma radiation (from 10^7 to 10^{23} Hz) [37–39] (see Ref. [10]).

Then it was found that the crab-like nebula was not unique — there was a wide class of crab-like analogs, being remnants of supernovas, which were called plerions [40]. In line with the studies of polarisation, the radiation of plerions, the major sources of which are pulsating neutron stars or pulsars, was also established to be of synchrotron origin. Thus, synchrotron radiation holds a firm place in astrophysics, and the importance of its polarisation properties is quite obvious.

Note also that, as applied to astrophysics, it is worthwhile to extend Schott's formula (2.1) to an electron moving in an helical line, i.e. there is a component of the velocity vector not only in the direction perpendicular to the magnetic field $v_\perp = c\beta_\perp$, but also along the field $v_{||} = c\beta_{||}$. Since the synchrotron radiation power is invariant, it may be derived from the Schott formula (2.3) by means of the Lorentz transform that

$$W = \frac{e^2\omega^2}{c}\sum_{\nu=1}^{\infty}\nu^2\int_0^\pi \frac{\sin\theta\,\mathrm{d}\theta}{(1-\beta_{||}\cos\theta)^3}\left[l_\sigma\beta_\perp J'_\nu(x) + l_\pi\,\frac{\cos\theta-\beta_{||}}{\sin\theta}\,J_\nu(x)\right]^2, \quad (2.5)$$

where

$$x = \frac{\nu\beta_\perp\sin\theta}{1-\beta_{||}\cos\theta}$$

(see Refs [24, 3]) †.

Now I shall dwell briefly on the angular distribution of SR power and to this end sum up Eqn (2.3) over the indices of harmonics ν. This sum is calculated exactly [5, 24]:

$$W_i(\theta) = \sum_{\nu=1}^{\infty} W_i(\nu, \theta) = \frac{e^2 c\beta^4}{32\pi R^2}\,F_i(\theta)\,, \quad i = \sigma, \pi\,,$$

where

$$F_\sigma(\theta) = \frac{4+3\beta^2\sin^2\theta}{(1-\beta^2\sin^2\theta)^{5/2}}\,, \quad F_\pi(\theta) = \frac{\cos^2\theta(4+\beta^2\sin^2\theta)}{(1-\beta^2\sin^2\theta)^{7/2}}\,. \quad (2.6)$$

The needle-shaped character of the synchrotron radiation (the 'gun effect') becomes perfectly clear when the denominators of the above expressions are considered. By setting $\theta = \pi/2 + \delta\psi$, the denominators are transformed to

$$1-\beta^2\sin^2\theta = 1-\beta^2\cos^2\delta\psi \cong 1-\beta^2+(\delta\psi)^2\,.$$

Then it follows that $\delta\psi \sim (1-\beta^2)^{1/2} = \gamma^{-1}$ (see Fig. 1b) — the angle of the radiation cone is a very small quantity.

It is interesting to consider the angular distribution of radiation power in case of an ultrarelativistic electron when $1-\beta^2 \ll 1$. If we introduce the variable

$$\psi = \frac{\beta\cos\theta}{(1-\beta^2)^{1/2}} \cong \gamma\cos\theta\,,$$

and use Eqns (2.3) and (2.6), we get [24]‡

$$W_i(\theta) = \frac{ce^2\beta^4\gamma^5}{32\pi R^2}\oint f_i(\psi)\,\mathrm{d}\Omega\,,$$

where

$$f_i = \frac{7}{(1+\psi^2)^{5/2}}\,l_\sigma^2 + \frac{5\psi^2}{(1+\psi^2)^{7/2}}\,l_\pi^2 + \frac{64\psi}{\pi\sqrt{3}(1+\psi^2)^3}\,l_\sigma l_\pi\,. \quad (2.7)$$

The index i takes the values $i=\sigma$ ($l_\sigma = 1$, $l_\pi = 0$), $i = \pi$ ($l_\sigma = 0$, $l_\pi = 1$), and $i = \pm 1$ ($l_\pi = l_\sigma = 1/\sqrt{2}$, $l_\pi = -l_\sigma = 1/\sqrt{2}$). The plots of the functions $f_i(\psi)$ are presented in Fig. 4. It is very clear from the plots that there is a singularity in the angular distribution of the π-component of the linear polarisation: the component vanishes in the orbital plane of revolution of the electron ($\psi = 0$). Of

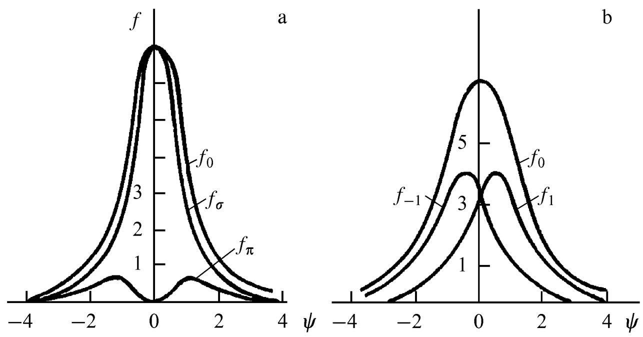


Figure 4. Linear (a) and circular (b) polarisations of synchrotron radiation as functions of the angle of radiation; f_0 is the sum of two components.

†Radiation intensity [quantity of energy an observer registers in unit time t: $\mathrm{d}I/\mathrm{d}\Omega = \mathrm{d}\mathcal{E}/\mathrm{d}\Omega\,\mathrm{d}t$)] should be distinguished from radiation power (energy a particle emits in a unit time t_r), $\mathrm{d}W/\mathrm{d}\Omega = \mathrm{d}\mathcal{E}/\mathrm{d}\Omega\,\mathrm{d}t_r$, where $\mathcal{E}$ is the energy of an electromagnetic wave; and the time intervals $\mathrm{d}t$ and $\mathrm{d}t_r$ are related by the equation $\mathrm{d}t = \mathrm{d}t_r(1-\boldsymbol{\beta}\cdot\boldsymbol{n})$ (see [3, 43]).

‡The simplest way to take the integral is to use Schott's formulaes in the ultrarelativistic approximation [see Eqn (2.14)].

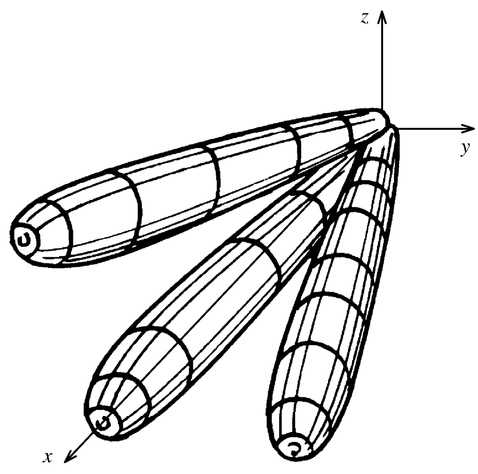


Figure 5. Instantaneous power distribution of the π-component of synchrotron radiation (the fourth peak is not shown).

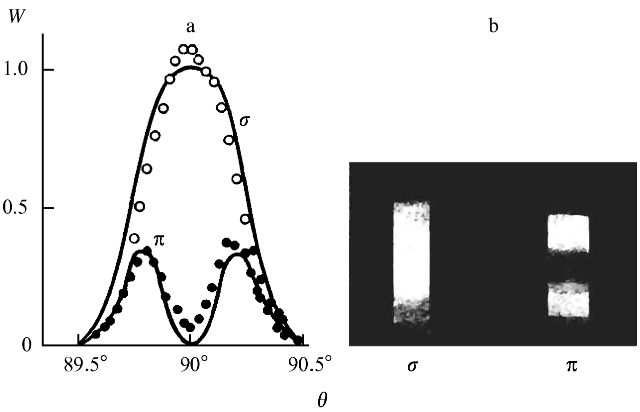


Figure 6. (a) Comparison of experimental (dots) and theoretical (lines) data for the angular distribution of SR power ($\lambda = 408$ nm, $E = 250$ MeV); (b) components of linear polarisation.

interest here are the angular distributions of polarisation components of radiation [35, 36]. Omitting the details of calculation, I wish to draw the reader's attention to the plots of the components of polarisation. The spatial distribution of the π-component presents most interesting and unexpected features (Fig. 5): it is characteristic of the π-component that four beams are symmetric about the velocity vector. The angular distribution of the power is averaged over the period of revolution of a moving electron, and two maxima are observed instead of four (Fig. 6) [36].

2.3 Spectral distribution of the power of synchrotron radiation. Ultrarelativistic approximation of Schott's formulas

The subject of this subsection is the spectral distribution of synchrotron radiation power. To this end Eqn (2.1) is integrated with respect to angles [5, 24, 25]. In this case

$$W(\nu) = \int_0^{\pi} \sin\theta \, d\theta \, W(\nu, \theta) = \frac{e^2 c\beta\nu}{R^2}\left[2\beta^2 J'_{2\nu}(2\nu\beta) - (1-\beta^2)\int_0^{2\nu\beta} J_{2\nu}(x)\, dx\right] . \quad (2.8)$$

This formula was also obtained by Schott [5]. However, difficult problems arose in applying his results to synchrotron radiation.

The point is that Schott considered the spectral distribution of radiation power in the context of the atomic model, i.e. as applied to micromotions, in which the radius of the orbit of an electron is of the order of the radius of the Bohr orbit. Synchrotron radiation manifests itself in macroscopic motions when the electron possesses an ultrarelativistic velocity $1-\beta^2 \ll 1$. Therefore, many features of synchrotron radiation, especially its spectral composition, could not be explained directly from the Schott formulas. But it is important that the formulas are exact. This made it possible to reveal their contents as applied to macroscopic motion. It should be stressed that they could not be applied directly to the theory of SR: the index of harmonics ν, which is very large in the case of macroscopic motions of an ultrarelativistic particle ($\nu \sim \gamma^3$), appears in the index as well as in the argument of Bessel functions.

The real progress in studies of synchrotron radiation was achieved once the ultrarelativistic approximations of Schott's formulas were obtained — the formulas were approximated by means of the Airy functions and their related modified Bessel functions or McDonald functions.

V V Vladimirskii was the first to describe the spectral–angular distribution of radiation power by means of the Airy functions [11]. The general idea of this description consists in approximating the Bessel functions of large index and argument $n \gg 1$, and $0 < x < 1$ (see Ref. [14]). It could be seen from the integral representation of the Bessel function,

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[i(n\varphi - x\sin\varphi)\right] d\varphi ,$$

that the integrand is a rapidly oscillating function for $n \gg 1$ and, therefore, only small values of the integration variable make noticeable contributions to the integral. By expanding the exponent in the integrand, in terms of powers of φ, and extending the limits of integration from $-\infty$ to ∞, we obtain

$$J_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left\{i\left[n\varphi - x\left(\varphi - \frac{\varphi^3}{6}\right)\right]\right\} d\varphi = \frac{1}{\sqrt{\pi}}\left(\frac{2}{n}\right)^{1/3} \Phi\left[\left(\frac{n}{2}\right)^{2/3}\varepsilon\right] , \quad (2.9)$$

since the integral converges rapidly. Here $\varepsilon = 1 - x^2/n^2$, and the Airy function is defined by the integral

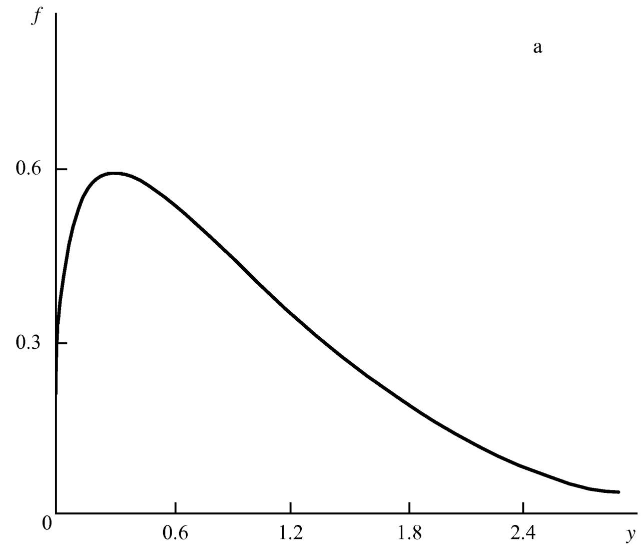
$$\Phi(z) = \frac{1}{\sqrt{\pi}}\int_0^{\infty} \cos\left(zt + \frac{t^3}{3}\right) dt . \quad (2.10)$$

Following the ideas cited above, Vladimirskii obtained an ultrarelativistic approximation for the spectral–angular distribution of the synchrotron radiation power [11]. His result can be presented in the form:

$$dW = \frac{2e^2\omega}{\pi\gamma^2}\, d\omega\, d\tau \left(\frac{\omega}{2\omega_c}\right)^{1/3}\left[\left(\frac{2\omega_c}{\omega}\right)^{2/3}\Phi'^2(z) + \tau^2\Phi^2(z)\right] , \quad (2.11)$$

where

$$\omega_c = \omega_0\gamma^3 , \quad \tau = \gamma\psi , \quad \psi = \gamma\cos\theta , \quad z = \left(\frac{\omega}{2\omega_c}\right)^{2/3}(1+\tau^2)$$



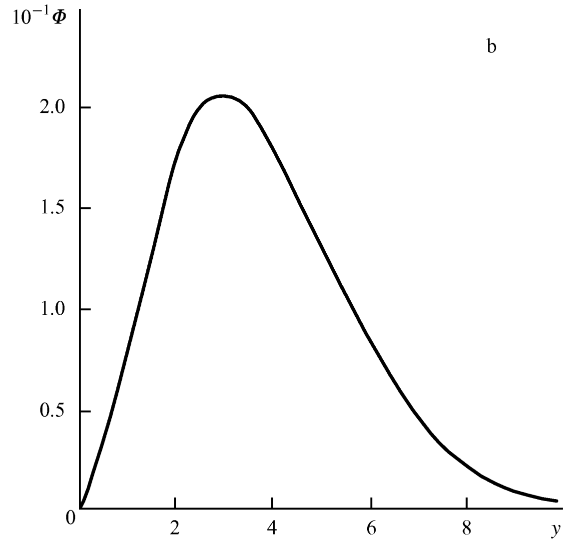


Figure 7. Universal curves for the spectral distribution of synchrotron radiation power [Eqn (2.16)] (a), and for the spectral radiation of a black body [Eqn (2.18)] (b).

(see also Refs [14, 43]). Here I have introduced other symbols for Airy functions and made a trivial transformation of variables.

As a result of the exact integration of Eqn (2.11) with respect to the angular variable by means of the tables of Airy functions composed by V A Fok, in Ref. [11] the spectral distribution of SR power was portrayed for the first time as a curve which has a maximum near $\omega \sim \omega_c$ (Fig. 7).

Another approach to the problem of approximation of Bessel functions was proposed by A A Sokolov [12] and is based on the Wentzel–Kramers–Brillouin method of quasiclassical quantum mechanics. In this method the solution is to be sought in the class of Bessel functions of an imaginary argument with a constant index. This problem was solved by D D Ivanenko and A A Sokolov in Ref. [13], where the Bessel function $J_n(x)$, $0 < x < n$ is expressed in terms of the McDonald function $K_{1/3}$ in the form

$$J_n(x) = \frac{\sqrt{\varepsilon}}{\pi\sqrt{3}} K_{1/3}\left(\frac{n}{3}\,\varepsilon^{3/2}\right), \quad \varepsilon = 1 - \frac{x^2}{n^2}\,. \qquad (2.12)$$

Then the spectral–angular distribution of the power of synchrotron radiation is presented in the form

$$W_{\sigma,\,\pi}(\nu,\,\theta) = \frac{e^2 c\beta^2\nu^2}{6\pi^3 R^2}\left[l_\sigma\beta\varepsilon K_{2/3}\left(\frac{\nu}{3}\,\varepsilon^{3/2}\right) + l_\pi \cot\theta\,\sqrt{\varepsilon}K_{1/3}\left(\frac{\nu}{3}\,\varepsilon^{3/2}\right)\right]^2, \qquad (2.13)$$

where $\varepsilon = 1 - \beta^2 \sin^2\theta$.

Since all the radiation is concentrated mainly near the orbital plane of revolution of the electron in the ultrarelativistic case, it is reasonable to introduce a small angle ψ between the orbital plane and the direction of radiation [see also Eqn (2.7)]:

$$\psi = \frac{\beta\cos\theta}{(1-\beta^2)^{1/2}} = \frac{\cos\theta}{\sqrt{\varepsilon_0}}\,, \quad \varepsilon_0 = 1 - \beta^2 = \frac{1}{\gamma}\,,$$

and then, taking into consideration that the peak of radiation falls on large harmonics for $\varepsilon_0 \ll 1$, the sum over ν can be replaced by the integral (the spectrum is close to a continuous one in the relativistic case). By introducing the new variable

$$y = \frac{2\nu\varepsilon_0^{3/2}}{3} = \frac{\omega}{\omega_{\rm cr}}\,,$$

the total radiation power can be presented in the form

$$W_{\sigma,\,\pi} = \frac{27}{16\pi^2}\,W^{\rm cl}\int_0^\infty y^2\,{\rm d}y\int_{-\infty}^{\infty}{\rm d}\psi\Big[l_\sigma(1+\psi^2)K_{2/3}(\eta) + l_\pi\,\psi\sqrt{1+\psi^2}\,K_{1/3}(\eta)\Big]^2\,, \qquad (2.14)$$

where

$$W^{\rm cl} = \frac{2}{3}\,\frac{e^2 c}{R^2}\,\gamma^4$$

is the total energy of radiation of an ultrarelativistic electron and $\eta = (y/2)\,(1+\psi^2)^{3/2}$. Then the degree of linear polarisation is determined by the relation

$$P = \frac{W_\sigma - W_\pi}{W_\sigma + W_\pi} = \frac{K_{2/3}^2(\eta) - \big[\psi^2/(1+\psi^2)\big]K_{1/3}^2(\eta)}{K_{2/3}^2(\eta) + \big[\psi^2/(1+\psi^2)\big]K_{1/3}^2(\eta)}\,.$$

Note that formula (2.14) is integrated with respect to the spectrum of the angular distribution of components of polarisation of SR and with respect to angles to the spectral composition of radiation [24, 27]:

$$W_{\sigma,\,\pi} = \frac{9\sqrt{3}}{16\pi}\,y\left[(l_\sigma^2 + l_\pi^2)\int_y^\infty K_{5/3}(x)\,{\rm d}x + (l_\sigma^2 - l_\pi^2)K_{2/3}(y)\right]. \qquad (2.15)$$

The spectral distribution of SR power summed up over polarisations takes the form [13, 15]

$$W = W^{\rm cl}\int_0^\infty f(y)\,{\rm d}y\,, \quad f(y) = \frac{9\sqrt{3}}{8\pi}\,y\int_y^\infty K_{5/3}(x)\,{\rm d}x\,, \qquad (2.16)$$

where f is a normalised function

$$\int_0^\infty f\,{\rm d}y = 1\,.$$

Note that formulas (2.11) and (2.14) are identical. It can readily be established, if one considers the relation between the Airy and McDonald function, that

$$\Phi(z) = \left(\frac{z}{3\pi}\right)^{1/2} K_{1/3}\left(\frac{2z^{3/2}}{3}\right) .$$

I shall describe in detail the spectral composition of synchrotron radiation. Strictly speaking, its spectrum is discrete — this is what attracted Schott's attention. However, it contains a large number of separate spectral lines very close to each other, so that SR possesses a nearly continuous spectrum in the case of an ultrarelativistic electron.

Let us consider the behaviour of the universal curve [Eqn (2.16)], not depending on the energy of an electron and normalised to unity. Since McDonald functions possess the asymptotics

$$\underset{y\to 0}{K_p}(y) \simeq 2^{p-1}\Gamma(p)y^{-p}, \qquad \underset{y\to\infty}{K_p}(y) \simeq \left(\frac{\pi}{2y}\right)^{1/2} e^{-y} ,$$

where $\Gamma(p)$ is a gamma function, the function $f(y)$ is (see Fig. 7a, Refs [24, 25], and also Ref. [12]) given by

$$f(y) = \begin{cases} 1.33y^{1/3}, & y \to 0, \\ 0.78\sqrt{y}\, e^{-y}, & y \to \infty . \end{cases} \tag{2.17}$$

The power peaks at $y = 1/3$, i.e., $\nu_{\max} = \gamma^3/2$. Varying the energy of an electron, one can cover the entire scale of electromagnetic waves — the spectrum of synchrotron radiation spans the gap between the infrared and rf ranges and the vacuum ultraviolet and x-ray ranges. Thus, synchrotron radiation is a unique source of electromagnetic waves, important as regards applications in the physical experiment.

Note also that the spectral distribution of SR power brings to mind the familiar Planck formula of the spectrum of radiation of a black body (Fig. 7b):

$$\rho_\omega = \frac{\pi^2}{15}\frac{(kT)^3}{\hbar^2 c^3}\,\Phi(y), \qquad \Phi(y) = \frac{15}{\pi^4}\frac{y^3}{e^y - 1} , \tag{2.18}$$

where the function $\Phi(y)$ is normalised to unity

$$\int_0^\infty \Phi\, dy = 1 ,$$

and $y = \omega/\omega_0$, $\omega_0 = kT/\hbar$. The comparison of the maxima of radiations,

$$\omega_{\max}^{\text{black}} = \frac{kT}{\hbar} = \omega_{\max}^{\text{syn}} = \frac{c}{R}\,\gamma^3 ,$$

shows that the synchrotron radiation of electrons of energy 1 GeV is similar to the radiation of a black body of the efficient (brightness) temperature $T \sim 10^7$ K. Another earthly source of such radiation could be a high-temperature plasma or a nuclear explosion.

2.4 Experimental research on synchrotron radiation

Theoretical studies stimulated experimental investigations of synchrotron radiation. After the first visual observation, Pollock's team conducted studies into the characteristics of SR in the visible range of wavelengths, which corresponds to the energy of electrons in the range 30 to 80 MeV [2]: in full accord with theoretical predictions, the synchrotron radiation was concentrated in a slender cone in the orbital plane of revolution of an electron and was observed as a dark red patch for electrons of energy 30 MeV and as a bright white-blue patch for electrons of energy 80 MeV. This luminescence exceeded daylight in terms of brightness. All experiments showed a good agreement with the theory. Ado and Cherenkov investigated the radiation in the visible range for the energies 150 and 200 MeV on the synchrotron in the FIAN and came to the same conclusion [16]. The electronic light offered a direct means of detecting a particle: it is emitted by the electron itself, which moves in a magnetic field in vacuum. The electron becomes luminous in a literal sense [13].

The experimental research of the properties of SR made rapid progress: in 1956 American physicists made systematic studies into radiation in the vacuum ultraviolet range on the synchrotron at Cornell University [17] and the radiation was investigated in the x-ray range for electrons of energy 4–6.3 GeV on the electron synchrotron DESY (in Germany) [18].

I shall describe briefly the experimental research of the polarisation of synchrotron radiation. The highly pronounced linear polarisation of SR was revealed in early observations in 1948 [2]. However, the first quantitative results were obtained much later. In 1956 a group of physicists from MSU performed the first systematic research into the linear polarisation of SR on the synchrotron in the FIAN [20]. Their results were corroborated in experiments on the synchrotron at Cornell University some time later [21]. At Cornell the elliptic polarisation was observed for the first time experimentally. Then the exhaustive research of elliptic polarisation was conducted on the synchrotron in the FIAN [41]; the polarisation of SR was also investigated in Frascati (Italy), in Germany (on DESY), and at the Tomsk Polytechnical Institute on the synchrotron 'Sirius' [42] for electrons of energy 1.5 GeV.

Amongst the variety of recent works, of special interest is the first measurement of the Stokes parameters which adequately describe the polarisation structure of synchrotron radiation [44]. The Stokes parameters comprise a complete description of polarisation:

$$P = W\begin{pmatrix} 1 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \frac{3}{32}\,W\begin{pmatrix} f_\sigma + f_\pi \\ f_\sigma - f_\pi \\ 0 \\ f_{\pm 1} \end{pmatrix} , \tag{2.19}$$

where the parameter p_3 characterises the average circular polarisation, and p_1 and p_2 are associated with the linear polarisation; W is the total radiation power. The Stokes parameters in Eqn (2.19) may be chosen as the functions f_σ, f_π, $f_{\pm 1}$ specified by means of formula (2.7). Note that here the Stokes parameter p_2 is zero since the principal axis of the ellipsoid of polarisation is assumed to refer to the plane of revolution of the electron, which is the plane of maximal linear polarisation.

Experimental measurements were performed on the synchrotron in the FIAN for electrons of energy 600 MeV [44]. The polarisation of SR was studied with consideration for betatron oscillations of an electron, owing to which the synchrotron radiation in the orbital plane of the electron is no longer fully linearly polarised.

Thus, the polarisation properties of synchrotron radiation are well studied both theoretically and experimentally by now.

2.5 Undulator radiation

Undulator radiation is attracting the attention of researchers more and more. This corresponds to the electromagnetic radiation of charged particles as they move in a periodic outer field. This radiation is due to the centripetal acceleration of particles when their trajectories bend. Undulator radiation (UR) is similar in nature to synchrotron radiation — they differ only in the effective length of the path over which radiation is emitted (see Figs 2 and 3). Similar to SR, undulator radiation has a highly pronounced angular trend: it is concentrated in a slender cone about the velocity vector of an electron and is directed forward in parallel to the motion of the particle. The polarisation properties of undulator radiation also similar to those of synchrotron radiation in many respects. I want to emphasise that the particle is a relativistic one in either case: the source of SR is a relativistic electron moving in a circular orbit, and UR is coupled tightly with the relativistic velocity of translational motion of a particle. Thus, undulator and synchrotron radiation have a close genetic relationship.

In 1947 Ginzburg suggested that relativistic electrons could radiate in periodic systems [10]. Studying the possibility of powerful and reliable generators in the microwave range, he considered the problem of the radiation of a fast charge in an electric field, which induces oscillations of the particle in the direction perpendicular to its forward motion.

The term undulator appeared for the first time in works of Motz [46, 47], who proposed to let electrons pass through a successive series of magnetic fields of different polarity (magnetic undulator). In 1953 SR was experimentally observed for the first time in the UHF range and in the optical range with the apparatus he created [48]. The electromagnetic radiation was generated by relativistic electrons which were passed through the undulator and accelerated beforehand by a linear accelerator.

Since electron beams were accelerated by a linear accelerator, the undulator had to be operated in an impulse mode; and since electrons passed through the magnetic field of the apparatus once only, the experimental results were not reliable. In 1960 Godwin came up with a proposal to place an undulator in a rectilinear gap between the synchrotron and the storage ring [49]. When electrons pass through the undulator repeatedly, the buildup of the beam over the rectilinear portion of the trajectory of particles significantly affects the spectrum of SR and can be used for generating intensive and nearly monochromatic fluxes of x-ray and ultraviolet radiation.

For the first time, the radiation from the undulator built in the chamber of an accelerator was observed by a group of experimentalists from the Physical Department of the MSU on the synchrotron 'Pakhra' (1 GeV) in FIAN [50].

In recent years, undulators have assumed an important and independent significance in connection with the project of a free-electron generator of coherent radiation. This stage in the advancement of technology is in essence the second birth of undulators, as the properties of coherent stimulated radiation (free-electron laser or FEL) make undulators such an important source of radiation that SR is relegated to the background†.

†It is not possible to review FELs and related problems in this paper. For more information on these topics, please refer to the work of Bessonov on UR and FEL, which has been published recently [52].

I shall now discuss briefly the properties of undulator radiation. There are two types of undulators: plain and spiral. In a plain undulator, the trajectories of particles are curved lines in a fixed plane, whereas in a spiral undulator the electrons move in a spiral (a spatial curve) — in both cases the radiation is emitted immediately along the whole trajectory of the particle. However, this does not hold for a wiggler — an undulator with a strong magnetic field.

The spiral undulator received wide acceptance in the experiment described in Ref. [51]. The magnetic field in a spiral undulator varies according to the law

$$\boldsymbol{H} = \left(H_0 \sin \frac{2\pi z}{\lambda_0}\,, \quad - H_0 \cos \frac{2\pi z}{\lambda_0}\,, \quad 0 \right) . \tag{2.20}$$

In such a magnetic field, electrons move in a spiral

$$\boldsymbol{r} = (R \cos \omega_0 t\,, \quad R \sin \omega_0 t\,, \quad \beta_{\|} c t)\;; \tag{2.21}$$

the radius of the spiral R is related to the transverse component of the velocity of a particle $v_\perp = c\beta_\perp$ by the equations

$$R = \frac{\beta_\perp c}{\omega_0}\,, \qquad \beta_\perp = \frac{eH_0\lambda_0}{2\pi mc^2\gamma}\,, \qquad \omega_0 = \frac{2\pi\beta_{\|} c}{\lambda_0}\,. \tag{2.22}$$

The longitudinal and transverse components of the velocity of an electron $\beta_{\|} c$ and $\beta_\perp c$ are related by the equation $\beta = (\beta_\perp^2 + \beta_{\|}^2)^{1/2}$. In what follows, the case of relativistic electrons when $\beta_{\|} \sim 1$, $\beta_\perp \ll 1$ is of primary interest. This definition has to be made more precise. On introducing the so-called undulator constant

$$K = \frac{\gamma\beta_\perp}{\beta_{\|}} = \frac{\lambda_0\gamma}{2\pi R} = \frac{eH_0\lambda_0}{2\pi mc^2}$$

into the expression for the longitudinal velocity $\beta_{\|} c$,

$$\beta_{\|} = \beta\left(1 - \frac{\beta_\perp^2}{\beta^2}\right)^{1/2} = \beta\left[1 - \left(\frac{K}{\gamma}\right)^2\right]^{1/2} . \tag{2.23}$$

Two types of undulators are distinguished depending on the value of the undulator constant K. If $K \ll 1$, then the resultant mode is said to be of the undulator type. The case of $K \geqslant 1$ corresponds to the wiggler mode — the mode of an undulator with a large magnetic field strength (of order of 50 kG). The approaches to these two cases are somewhat different.

Since, as follows from the foundations of electrodynamics, the radiation is fully specified when the trajectory of the charge is given, the above case of the radiation of an electron moving in a homogeneous magnetic field (motion in a spiral) is a good model for the problem of the radiation from an electron in a spiral undulator. Then the differential of the radiation power of a harmonic ν inside an elementary solid angle $\mathrm{d}\Omega$ is written according to Eqn (2.5) as

$$\frac{\mathrm{d}W_\nu}{\mathrm{d}\Omega} = \frac{e^2\omega^3\beta^2K^2}{2\pi c\gamma^2\nu\omega_0}\left[l_\sigma J'_\nu(x) + l_\pi \frac{\cos\theta - \beta_{\|}}{\beta_\perp \sin\theta} J_\nu(x)\right]^2 , \tag{2.24}$$

where

$$\omega = \frac{\nu\omega_0}{1 - \beta_{\|}\cos\theta}\,, \quad x = \frac{\nu\beta_\perp \sin\theta}{1 - \beta_{\|}\cos\theta} = \frac{K}{\gamma}\,\frac{\omega}{\omega_0}\,\sin\theta\,,$$

and l_σ and l_π serve to describe the polarisation properties of radiation, as described before.

Let us consider the frequency of undulator radiation given by Eqn (2.4). The quasiclassical spectrum of radiation of an undulator consists of harmonics $\omega = \nu\omega_1$ of the fundamental frequency $\omega_1 = \omega_0/(1 - \beta_{\|}\cos\theta)$. The undu-

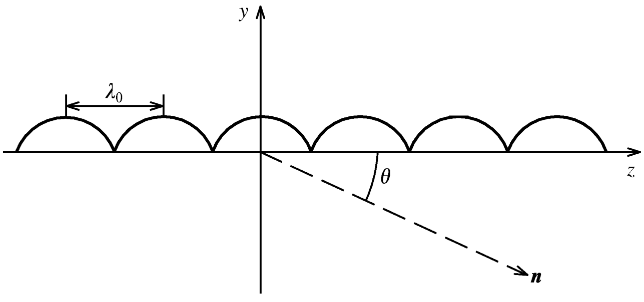


Figure 8. Pattern of electron radiation in an undulator.

lator radiation turns out to be coloured in various colours, which depend on the angle θ: the bright patch has all the colours of the rainbow (in the visible range) — from blue at the centre of the patch to the dark-red on its periphery. The fundamental frequency of radiation ω_1 peaks as $\theta \to 0$, i.e., when the radiation is observed along the axis of the undulator (Fig. 8). In this case, owing to the Doppler multiplication of frequencies

$$\omega_1 = \frac{\omega_0}{1-\beta_{\|}\cos\theta} = \frac{2\gamma^2\omega_0}{1+K^2+\gamma^2\theta^2} \tag{2.24a}$$

for small $\theta \to 0$ and

$$\omega_1 = \frac{2\gamma^2\omega_0}{1+K^2}\,, \qquad \lambda_1 = \frac{\lambda_0(1+K^2)}{2\gamma^2}$$

when the radiation is observed strictly along the axis. It can be seen from expression (2.24) for the radiation power that all harmonics vanish as $\theta \to 0$: the entire spectrum of UR consists of frequencies which are concentrated near the peak ω_1 [Eqn (2.24a)] (Fig. 9). In order to estimate the efficient wavelength of UR I will assume that $K = 1$, $\lambda_0 = 1$ cm, $\gamma = 10^4$ ($E \sim 1$ GeV). It turns out that the peak of undulator radiation corresponds to the wavelength $\lambda \sim 1$ Å. Thus the macroscopic undulator turns out to be similar to an atom: it has the capability of generating electromagnetic radiation in the visible and x-ray ranges of electromagnetic waves.

The angular distribution of the radiation power of an undulator will now be discussed. The analysis of the angular distribution of the power shows that almost all radiation of a spiral undulator is concentrated within an angle $\theta = \theta_0 + \delta\theta$, where $\delta\theta \sim 1/\gamma$ and $\sin\theta_0 = K/\gamma$. For $K < 1$ (undulator) the angle of the cone of radiation is found in the same way as in the case of SR. For large $K \geqslant 1$ (wiggler) the angular distribution is somewhat different (Fig. 10). However, the angular distribution of the radiation power of a relativistic charge is generally the same.

The polarisation properties of radiation of an electron in a spiral undulator are similar to the properties of synchrotron radiation. This similarity of polarisation properties follows from the solution to the problem on the polarised radiation the moving relativistic charges emit in the magnetic field of an arbitrary configuration (see Ref. [53]).

The angle θ in expression (2.24) is a small quantity when the undulator radiation is observed near the axis of the undulator. Therefore

$$\frac{\mathrm{d}W_\nu}{\mathrm{d}\Omega} = \frac{e^2\omega^3\beta^2K^2}{2\pi c\gamma^2\nu\omega_0}\,J_\nu'^2(x)\left(l_\sigma + l_\pi\,\frac{1+K^2-\gamma^2\theta^2}{1+K^2+\gamma^2\theta^2}\right)^2\,, \tag{2.25}$$

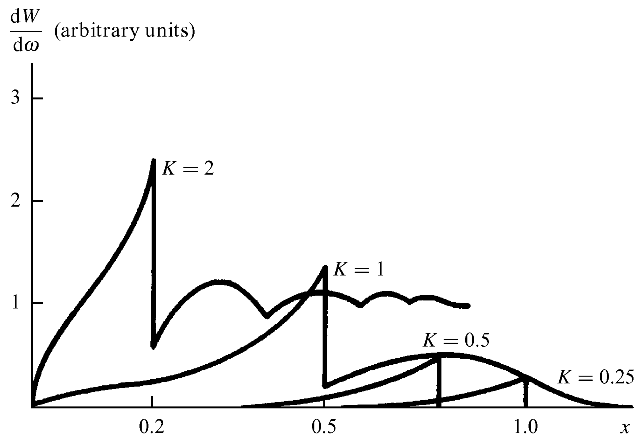


Figure 9. Spectral composition of undulator radiation for different values of the constant K; $x = \omega/2\gamma^2\omega_0$.

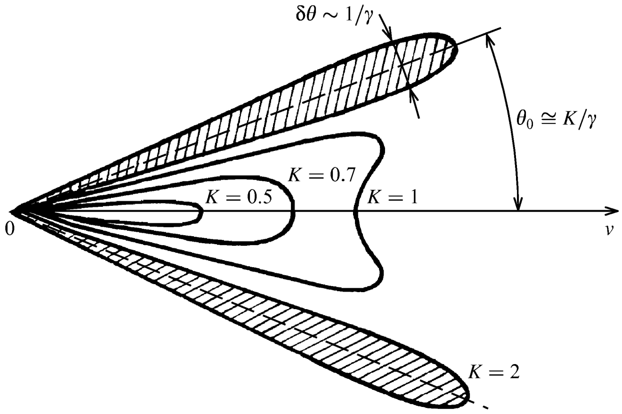


Figure 10. Polar diagram of the angular distribution of undulator radiation power for different values of the undulator constant K.

according to the asymptotics of Bessel functions of the small arguments. Hence, the polarisation of UR is strictly linear for angles $\theta = \sqrt{1+K^2}/\gamma$. Only the σ-component is emitted; this turns to be circularly polarised in a clockwise direction for angles $\theta = 0$ (the radiation is viewed along the axis of the undulator), and $l_\sigma = l_\pi = 1/\sqrt{2}$. The sign of the circular polarisation changes when the direction of current in the windings of a solenoid changes. Thus, a spiral undulator is a powerful source of circularly polarised radiation. Note that a plain undulator has distinct polarisation properties: the radiation turns out to be linearly polarised†, when it is viewed along the axis of the plain undulator. The generation of the circularly polarised radiation in a spiral undulator and linearly polarised radiation in a plain undulator opens up possibilities for applying UR in the physical experiment, since the problem is of polarisation control as well as of the source of polarised radiation.

I wish to draw the reader's attention to an important fact: an actual undulator has a finite length. Let the length of an undulator be $L = N\lambda_0$, where N is the number of periodic elements (see Fig. 8). Then the total energy of

†A detailed presentation of issues associated with undulators may be found in Refs [52, 54].

radiation of an electron in a finite undulator can be written in the form:

$$\frac{\mathrm{d}W^{\mathrm{UR}}}{\mathrm{d}\Omega\,\mathrm{d}\omega}=\frac{\beta^2e^2K^2\omega^2N^2}{\gamma^2c\omega_0^2}\left(\frac{\sin z}{z}\right)^2 J_\nu'^2(x)\times\left(l_\sigma+l_\pi\,\frac{1+K^2-\gamma^2\theta^2}{1+K^2+\gamma^2\theta^2}\right)^2, \tag{2.26}$$

where

$$\omega_0=2\pi\beta_{\|}\,\frac{c}{\lambda_0}\,,\qquad z=N\pi\left(\frac{\omega}{\omega_1}-\nu\right).$$

We come back to Eqn (2.25) as $N\to\infty$, as

$$\lim_{N\to\infty}\left(\frac{\sin z}{z}\right)^2=\frac{1}{N}\,\delta\left(\frac{\omega}{\omega_1}-\nu\right).$$

The radiation of an electron turns out to be strongly collimated in a finite undulator: the angular directivity of radiation is greatly amplified by a large number of periodic elements. In fact if one restricts oneself to the peak radiation at the fundamental frequency ($\nu=1$), and considers the undulator constant to be a small quantity ($K\ll 1$), then

$$z=N\pi\left[\frac{\omega(1+\gamma^2\theta^2)}{2\gamma^2\omega_0}-1\right]\cong N\pi\gamma^2\theta^2\,,$$

whence it follows that the radiation is almost concentrated in a slender cone of angle $\theta=1/\gamma\sqrt{N}$. This sharp directivity of UR is an important property of undulators, as regards their applications in physical experimentation.

The narrowing of the cone of radiation emitted by electrons in an undulator with a large number of periods ($N\sim 100$) is physically associated with the interference of electromagnetic waves. The cone of radiation of an electron which makes one oscillation has the angle $\delta\theta\cong 1/\gamma$. If the electron makes N phase-coherent oscillations, the radiation fields interfere and, as a result, the radiation is concentrated — the radiative cone is sharpened.

Undulator radiation has other advantages in comparison with SR when one considers the total energy of radiation. Let us consider the energy of radiation of an electron in an undulator when the UR is viewed along the axis of the undulator ($\theta=0$) at the fundamental frequency ($\omega=\omega_1$). Then it follows from Eqn (2.26) that

$$\left.\frac{\mathrm{d}W^{\mathrm{UR}}}{\mathrm{d}\omega\,\mathrm{d}\Omega}\right|_{\substack{\omega=\omega_1\\ \theta=0}}=\frac{2N^2e^2\gamma^2}{c}\left(\frac{K}{1+K^2}\right)^2.$$

The energy peaks for $K=1$ (i.e., the undulator is a wiggler) and

$$\frac{\mathrm{d}W^{\mathrm{UR}}_{\max}}{\mathrm{d}\omega\,\mathrm{d}\Omega}=\frac{N^2e^2\gamma^2}{2c}\,.$$

Let us compare the previous expression with the energy of the synchrotron radiation emitted by an electron in one turn, as it moves in the plane of revolution. According to Eqn (2.14)

$$\frac{\mathrm{d}W^{\mathrm{SR}}}{\mathrm{d}\omega\,\mathrm{d}\Omega}=\frac{3e^2\gamma^2}{4\pi^2c}\left[yK_{2/3}\left(\frac{y}{2}\right)\right]^2,$$

$y=\omega/\omega_{\mathrm{cr}}$, $\omega_{\mathrm{cr}}=3\omega_0\gamma^3/2$. The function $yK_{2/3}(y/2)$ reaches a maximum for $y\sim 1$. Then the expression for the SR power is

$$\frac{\mathrm{d}W^{\mathrm{SR}}_{\max}}{\mathrm{d}\omega\,\mathrm{d}\Omega}\cong\frac{3e^2\gamma^2}{4\pi^2c}\,.$$

Thus, if the number of periods of an undulator is $N\gg 1$ ($N\sim 100$), then its radiation power greatly exceeds the SR power. The concentration of the energy of radiation by means of a wiggler is the most important property of the undulator — this opens up new possibilities for applying SR as a tool in physical and technological research. Therefore the functions of the storage ring have changed and its primary objective is to obtain relativistic electrons which emit light by means of a wiggler.

Here we omit quantum effects in the radiation of a wiggler.

2.6 Radiation of an electron in a short magnet. Formation of synchrotron radiation

The radiation of electrons moving in systems of the 'short magnet' type has a number of peculiarities which are of practical importance as well as of theoretical interest. The simplest example of the motion of an electron in a short magnet is its motion in an arc of a circle, provided that the arc is small enough. In this example, all important properties of the radiation emitted by an electron when it moves in a short magnet of an arbitrary structure are apparent (see Refs [55, 56]).

Assume that an electron moves in a straight line with a constant velocity $v=c\beta$ in absolute value up to an instant of time. Then, under the action of external forces it describes an arc of a circle of radius R with an angle 2α, and again proceeds in a straight line (Fig. 11). The angle of deviation α is assumed to be small and $\alpha\ll 1/\gamma=mc^2/E$.

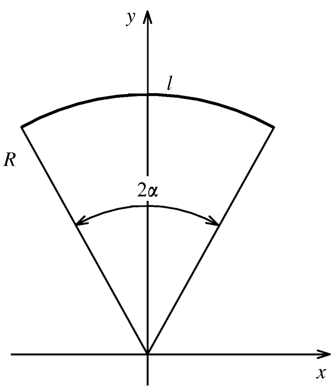


Figure 11. Motion of electrons in an arc of a circle.

Using the conventional methods of classical electrodynamics, we can present the spectral–angular distribution of the total energy $\mathcal{E}$ for the motion in question as follows:

$$\mathrm{d}\mathcal{E}=WTF\ \mathrm{d}q\,\mathrm{d}\Omega\,,\quad F=F_\sigma+F_\pi\,,\quad \mathrm{d}\Omega=\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \tag{2.27}$$

(see Ref. [56]), where $T=2\alpha/\omega_0$, $\omega_0=c\beta/R$, $q=\omega/\omega_0$, W is the synchrotron radiation power specified by means of formula (1.2). The function $F_{\sigma,\pi}$ is related to the polarisation components of radiation f_σ and f_π by the equation

$$F_{\sigma,\pi} = \frac{3}{16\pi^2\alpha\gamma^4}\left|f_{\sigma,\pi}\right|^2 , \tag{2.28}$$

where

$$f_\sigma = \int_{\varphi-\alpha}^{\varphi+\alpha} \frac{\cos x - \mu}{p^2(x)} \exp(-\mathrm{i}q\psi)\,\mathrm{d}x ,$$

$$f_\pi = \cos\theta \int_{\varphi-\alpha}^{\varphi+\alpha} \frac{\sin x}{p^2(x)} \exp(-\mathrm{i}q\psi)\,\mathrm{d}x ,$$

and $p(x) = 1 - \mu\cos x$, $\psi(x) = x - \mu\sin x$, $\mu = \beta\sin\theta$, $\gamma = E/mc^2$ is a relativistic factor. In these formulas ω is the radiation frequency and the angles θ, φ specify the direction in which radiation propagates.

The analysis of these formulas [56] shows that the general rules for the angular distribution of radiation power in a short magnet are the same as those for the synchrotron radiation. This may be verified by integrating Eqns (2.27) and (2.28) with respect to the spectrum: the radiation is directed ahead, in parallel to the motion of the charge and is concentrated in a slender cone of angle $\delta\theta \sim 1/\gamma$ (as is the case for SR). It is interesting to note that the formulas coincide with the expressions for the instantaneous distribution of synchrotron radiation power as $\alpha \to 0$.

The polarisation properties also coincide: Eqn (2.27) integrates with respect to angles and frequencies to

$$W_{\sigma,\pi} = \frac{\mathcal{E}}{T} = \int_0^\infty \mathrm{d}q \oint \mathrm{d}\Omega\, F_{\sigma,\pi} , \tag{2.29}$$

and

$$W_\sigma = \frac{6+\beta^2}{8}\,W , \qquad W_\pi = \frac{2-\beta^2}{8}\,W .$$

However, there are great distinctions between synchrotron radiation and the electron radiation in a short magnet. First of all, there is the way in which low-frequency radiation is generated as electrons move in an arc of a circle. If an electron moves in an arc of a small angle $\alpha \ll 1/\gamma$, then it follows from Eqn (2.28) that

$$F_{\sigma,\pi} = \frac{3A^2_{\sigma,\pi}}{\left[2\pi\gamma^2p^2(\varphi)\right]^2}\left\{\frac{\sin\left[\alpha qp(\varphi)\right]}{\alpha qp(\varphi)}\right\}^2 , \tag{2.30}$$

where $A_\sigma = \beta\sin\theta - \cos\varphi$, $A_\pi = \cos\theta\sin\varphi$. Clearly, the radiation peaks at $\omega = 0$ and the spectral–angular distribution decreases progressively to zero ($\propto \omega^{-2}$) as the frequency increases. Moreover, the effective width of the spectrum is

$$\Delta\omega = \frac{\pi\omega_0}{2\alpha(1-\beta^2)} = \frac{\pi c\beta\gamma^2}{l} , \tag{2.31}$$

where l is the length of the arc. Thus, the effective width of 'white noise' is $\Delta\omega$ in a 'short magnet'. If $l \cong 10-100$ cm, then the spectrum of the white noise extends from zero up to the ultraviolet range for an electron of energy 1 GeV. Note that the fundamental distinguishing feature of the radiation in a short magnet is that the radiation peaks at the zero frequency — these are the so-called 'strange electromagnetic waves' (see Refs [52, 54]). In this respect, the radiation in a short magnet differs from most processes of radiation in which the energy of radiation reduces to zero as the frequency of radiation tends to zero. For example, in the case of synchrotron radiation the power turns to zero as $\omega \to 0$ (see Fig. 7).

A closer analysis of Ref. [56] shows that the relative contribution of the low frequencies in radiation decreases on an increase in the angle of the arc, and the maximum in the spectrum shifts to short waves as α increases. It is interesting that there is no radiation of the 'zero' frequency when an electron moves in a circle ($\alpha = 2\pi$).

In connection with the peculiar features of the spectral composition of the radiation of a charge moving in an arc of a circle, it is worth noting the bending or magnetic drift radiation familiar in the astrophysical applications. It is emitted by a charge which moves along the lines of force of an inhomogeneous magnetic field. Omitting the detailed analysis of the radiation, we should note that, although the spectral composition of this radiation is usually thought to be analogous to the spectral composition of SR, it can be somewhat different in the low-frequency range since the arc in which an electron moves is finite.

Let us consider the radiation when an electron makes an integral number of turns and how the synchrotron radiation occurs. Let now $\alpha = N\pi$. In this case, following the approach discussed above [see Eqns (2.27), (2.28)], we have

$$\mathrm{d}\mathcal{E} = WTg_N(q)G\,\mathrm{d}q\,\mathrm{d}\Omega , \tag{2.32}$$

where the quantity G is associated with the Anger and Weber functions $\bar{J}_q(x)$ and $\bar{E}_q(x)$, $x = q\beta\sin\theta$. It is essential that the number of turns N made by the electron over a circular path appears only in the factor

$$g_N(q) = \frac{\sin^2(\pi N_q)}{N\sin^2(\pi q)} .$$

The rest of expression (2.32) is independent of N and coincides with the spectral–angular distribution of the radiation of an electron which makes one full turn. The full expression for function G may be found in Ref. [56] and also in Ref. [55].

Expression (2.32) makes it possible to trace how the spectrum of SR is formed as $N \to \infty$. On going over to the limit

$$\lim_{N\to\infty} g_N(q) = \sum_{n=-\infty}^{\infty} \delta(q-n) ,$$

we find that the Schott formula follows from Eqn (2.32) for a finite, but very large, number of turns [34]:

$$\mathrm{d}W = \lim_{N\to\infty}\frac{\mathcal{E}}{T} = \frac{e^2c\beta^2}{2\pi R^2}\sum_{n=1}^{\infty} n^2\Big[\beta^2 J_n'^2(n\beta\sin\theta) + \cot^2\theta J_n^2(n\beta\sin\theta)\Big] . \tag{2.33}$$

Anger functions convert into Bessel functions of integer indices and the spectral–angular distribution of radiation power is given by the familiar expressions which are true for synchrotron radiation.

2.7 Coherent synchrotron radiation of a cluster of electrons

The classical theory of SR developed as a theory of radiation of one electron moving in an angular trajectory in a homogeneous magnetic field. Although $10^{12}-10^{13}$ particles simultaneously emit radiation in accelerators and storage rings, and either fill the entire orbit (betatron) or are clustered in bunches (synchrotron, storage rings), the conclusions of the theory have been reliably verified in experiments. The interference of electromagnetic waves emitted by individual electrons can affect the total radiation power — coherent SR can be emitted.

The first person to consider the problem of coherent radiation was Schott himself in his early studies of the Saturnian model of an atom. The model presumed that a large number of electrons move simultaneously in a closed orbit. Studying the radiation of a cluster of electrons uniformly distributed along the circle, Schott came to the conclusion that the classical model of an atom is inconsistent, since the conclusions of the theory were contradictory to the observed data [5].

Much later, in connection with the advances in cyclic accelerators, the studies of coherence were resumed: of interest were the spectrum of synchrotron radiation and its power [58–60] and also the screening effect of the walls of the accelerator chamber [61, 62]. It was presumed that the radiation is emitted by separate clusters of particles of a finite expansion, with different values of the form factor which characterises the distribution of particles in the cluster.

Let us consider when the coherent radiation from electrons, which is distributed uniformly over the entire orbit, is possible. Let the ratio of the index of harmonics ν and the number of electrons N be equal to s. Then the radiation power differs from the radiation power of one electron $W(\nu)$ [Eqn (2.8)] in terms of the coherence factor S_N (see Refs [5, 25]):

$$W_N(\nu) = S_N W(\nu)\,, \quad S_N = N + \sum_{\substack{j=1,\, j'=1 \\ (j\neq j')}}^{N} \cos\left[\nu(\psi_j - \psi_{j'})\right]\,, \tag{2.34}$$

where ψ_j is the initial phase of the jth electron. If electrons are spread chaotically then $S_N = N$ and the radiation is coherent. If particles are distributed uniformly, then

$$W = \sum_s W_s\,,$$

where

$$W_s = \frac{e^2 c\beta N^3 s}{R^2}\left[2\beta^2 J'_{2sN}(2sN\beta) - (1-\beta^2)\int_0^{2sN\beta} J_{2sN}(x)\,\mathrm{d}x\right] \tag{2.35}$$

(see Ref. [5]). In the nonrelativistic approximation ($\beta \to 0$), the radiation peaks for one electron only ($N = 1$), since the contribution of the other particles is suppressed:

$$W_s \underset{\beta\to 0}{=} \frac{2e^2 c\beta^2 N^3 (N+1)}{R^2(2N+1)(2N)!}\,(N\beta)^{2N}\,.$$

In another—ultrarelativistic—approximation ($1-\beta^2 \ll 1$)

$$W_s = \frac{e^2 c\,\varepsilon_0 N^3 s}{\pi R^2\sqrt{3}}\int_\kappa^\infty K_{5/3}(x)\,\mathrm{d}x\,, \qquad \kappa = \frac{2}{3}\,Ns\varepsilon_0^{3/2}\,. \tag{2.36}$$

There are two limiting cases for the quantity κ. In the first, $\kappa \ll 1$. This corresponds to small concentrations of electrons and also relates to the long-wavelength part of the spectrum of SR since

$$\nu = sN \ll \frac{3}{2}\,\varepsilon_0^{-3/2} = \frac{3}{2}\,\gamma^3\,.$$

Then

$$W_s = \frac{3^{2/3} e^2 c\Gamma(2/3) N^2 (sN)^{1/3}}{\pi R^2\sqrt{3}}\,. \tag{2.37}$$

Thus, the coherence of SR is possible for low frequencies of radiation ($\omega \ll \omega_c$). In the second limiting case of the electron concentration N being equal to $(E/mc^2)^3$ (the short-wave part of the spectrum), all the radiation turns out to be suppressed to a large extent:

$$W_s = \frac{e^2 c\varepsilon_0^{1/4} N^3\sqrt{sN}}{\sqrt{2\pi}R^2}\exp\left(-\frac{2}{3}\,sN\varepsilon_0^{3/2}\right)^{1/2}\,. \tag{2.38}$$

The case in which electrons uniformly fill the orbit was discussed in connection with possible losses of the energy of particles in a betatron. Concerns were voiced that the radiation from the electrons would be suppressed in the betatron owing to its coherence. However, the analysis of the problem Artsimovich and Pomeranchuk carried out [9], and also the decisive experiment of Blewett, showed that the process is not coherent, at least at the peak of radiation. It was also noted [9] that fluctuations of the density prevent particles from filling the orbit regularly.

In contrast to the case in a betatron, electrons do not fill the entire orbit in a synchrotron (or in a storage ring) but move in separate bunches (see, for example, Ref. [25]). In this case, the coherence factor can be presented in the form

$$S_N = N + N(N-1)f_\nu\,.$$

The radiation is fully coherent for $f_\nu = 1$ and is incoherent for $f_\nu = 0$.

Assuming that electrons in a bunch are symmetric about a centre (at the zero azimuth), the function f_ν is given by [25]

$$f_\nu = \left[\int_{-\infty}^{\infty} w(\varphi)\cos(\nu\varphi)\,\mathrm{d}\varphi\right]^2\,, \tag{2.39}$$

where $w(\varphi)$ is the probability that electrons are in orbit in the angular range from φ to $\varphi + \mathrm{d}\varphi$. In particular, if the distribution follows the Gauss law, then

$$w(\varphi) = \frac{1}{\alpha\sqrt{\pi}}\exp\left(-\frac{\varphi^2}{\alpha^2}\right)\,,$$

$$f_\nu = \exp\left(-\frac{\nu^2\alpha^2}{2}\right)\,.$$

The SR power is the sum of the coherent and incoherent summands:

$$W_\nu = W^{\,\mathrm{incoh}}(\nu) + W^{\,\mathrm{coh}}(\nu) = W(\nu)N + W(\nu)N^2 f_\nu\,,$$

It is seen from the last formula that the maximal coherent radiation manifests itself in the range of wavelengths of the order of the size of the bunch, $\nu \sim 1/\alpha$ (the low-frequency range). The radiation power is integrated with respect to all harmonics to

$$W^{\,\mathrm{coh}} = \frac{e^2 cN^2(\sqrt{3}/\alpha)^{4/3}\Gamma^2(2/3)}{R^2\pi\sqrt{3}\times 2^{1/3}} = 0.56\,\frac{N^2 e^2 c}{\alpha^{4/3}R^2}\,, \tag{2.40}$$

whence it follows that the coherent radiation power is independent of the energy of a particle in the long-wave range of radiation of a bunch of an angular expansion α (see Refs [58, 60]). The recent observations of coherent synchrotron radiation [63, 112, 113] provide, however, reasons to further investigate the dependence of the phenomenon on the shape and sizes of a bunch (see Refs [64, 65, 111]).

Thus, the electron radiation is incoherent in the synchrotron at least in the high-frequency range — this is the reason why experimentalists are satisfied with the theory of radiation from one electron: the total power of SR is proportional to the number of electrons. However, the coherent radiation may be observed in the low-frequency range, especially in the rf range. This is the case when electrons move in bunches the sizes of which are comparable with the wavelength of radiation: the radiation can increase drastically in this case.

The coherent SR in the rf range aroused interest in connection with the observed rf radiation coming from pulsars [66]. The estimated radiation power suggests that the rf radiation is coherent and is emitted by clustered charges rotating near the surface of a pulsar. Owing to this relation, a storage ring in which electrons move in separate clusters is considered as the laboratory model of a radiating pulsar.

3. Review of the quantum theory of synchrotron radiation

3.1 Limits of the classical theory

The classical theory is based on the assumption that radiation is a continuous process. The initial estimates of the limits in which the classical theory is applicable validated that such an approach was reasonable for studying SR as a physical phenomenon up to very high energies, which would be unattainable in accelerators even in the foreseeable future. In fact, in general one would expect that the classical theory of radiation of a classical charge is valid until the energy of an emitted photon $\varepsilon_p = \hbar\omega = \hbar\nu\omega_0$ is small in comparison with the energy of the electron $\varepsilon_p \ll E$ [11] (see also Refs [15, 60]). Evaluating the energy of a photon at the peak of the radiation, one could obtain a criterion for the classical theory to be true for synchrotron radiation as an estimate for the energy $E \ll E_{1/2}$, where

$$E_{1/2} = mc^2\left(\frac{mcR}{\hbar}\right)^{1/2} \cong 10^9\,\mathrm{MeV}\,.$$

Then this criterion was corroborated by considerations of invariance: the SR power being an invariant should depend on invariant parameters only, one of which is the dynamic invariant

$$\chi = \frac{1}{mcH_0}\sqrt{-\left(F^{\mu\nu}p_\nu\right)^2} = \frac{H}{H_0}\,\frac{E}{mc^2} = \left(\frac{E}{E_{1/2}}\right)^2\,, \qquad (3.1)$$

where $F^{\mu\nu}$ is the tensor of the electromagnetic field, p_ν is the four-dimensional impulse, and H_0 is the Schwinger magnetic field.

However, it was then found out that the criterion $\chi \ll 1$ did not cover all discrete features of synchrotron radiation [32, 67]. It refers to the influence of discrete radiation on the trajectory of a moving particle.

Since the energy of an emitted photon is large enough at the maximum of the SR spectrum, $\varepsilon_p \sim \hbar\omega_0\gamma^3$, the number of photons an electron emits in a turn is finite and is equal to

$$N_{\mathrm{turn}} = \frac{W}{\varepsilon_\Phi} = \frac{e^2}{\hbar c}\,\frac{E}{mc^2}\,.$$

In order to make the role of discreteness of radiation more clear, it is worthwhile to find the length of the path (in cm) an electron travels without emitting a high-energy photon

$$\mathcal{L} = \frac{c\varepsilon_p}{W} = \frac{3}{2}\,\frac{\hbar^2}{me^2}\,\frac{H_0}{H} = \frac{34.9\times 10^4}{H}\,.$$

The previous expression depends solely on the magnetic field strength H, and it follows from it, for the values $\sim 10^4$ G typical of accelerators and storage rings, that one power photon is emitted on average over the path of $\sim$ 30 cm.

The discrete nature of radiation here is an important factor. It can tell on the trajectory of the particle, causing quantum fluctuations as a result of the recoil an electron experiences when it emits a photon (for details, see Section 4). Of special interest here is the motion of electrons in synchrotrons or storage rings. Since outer sources compensate for energy radiative losses, the quantum effects of fluctuations of the trajectory are observed when the radius is constant (on average). The storage ring opens up new possibilities for investigating quantum fluctuations of the trajectory of an electron experimentally.

If the invariant dynamic parameter χ takes a large value (high energies, an extremal field), this opens up a new area of physical phenomena — ultraquantum physics. In this region ($\chi \geqslant 1$), the classical theory is completely out of place; the strict quantum theory should be applied.

3.2 The method of exact solutions. Quantum states of an electron in a magnetic field

It proved worthwhile to develop the quantum theory of synchrotron radiation on the basis of quantum relativistic mechanics and quantum electrodynamics, applying the so-called 'method of exact solutions' [24, 25]. In this case the wave function which describes the quantum state of an electron obeys the Dirac equation:

$$\mathrm{i}\hbar\,\frac{\partial\boldsymbol{\Psi}}{\partial t} = \left[c\left(\boldsymbol{\alpha}\cdot\hat{\boldsymbol{P}}\right) + \rho_3 mc^2\right]\boldsymbol{\Psi}\,, \qquad (3.2)$$

where

$$\hat{\boldsymbol{P}} = -\mathrm{i}\hbar\nabla - \frac{e}{c}\left(\boldsymbol{A}^{\mathrm{ext}} + \boldsymbol{A}^{\mathrm{qu}}\right)\,,$$

refers to the outer magnetic field which is considered exactly, and $\boldsymbol{A}^{\mathrm{qu}}$ to the quantum transverse radiation field. Here α and ρ_3 are the Dirac matrices of four rows, and the wave function $\boldsymbol{\Psi}$ has four components (for details see Ref. [24]). Processes in which an electron in the bound state is involved when it interacts with the radiation field are considered by using the perturbation theory. In the perturbation theory, all expansions are made in terms of the full system of the exact solutions to the Dirac equation with an outer field (the Furry representation [68]).

Such an approach made it possible to predict and reveal a variety of new physical features of SR: quantum fluctuations of the trajectory of an electron when it moves in a cyclic accelerator and storage rings, the radiative polarisation effect for electrons and positrons, the peculiar features of synchrotron radiation in strong and very strong magnetic fields, and a number of others (see Refs [24, 25, 69]).

Before studying spin effects it is worthwhile to introduce the polarisation operator,

$$\hat{\boldsymbol{O}} = \rho_3\boldsymbol{\Sigma} + \frac{\rho_1 c\hat{\boldsymbol{P}}}{E} - \frac{\rho_3 mc^2\,\hat{\boldsymbol{P}}(\boldsymbol{\Sigma}\cdot\hat{\boldsymbol{P}})}{E(E+mc^2)}\,, \qquad (3.3)$$

which goes into the Pauli spin operator in the system in which the electron is at rest. This operator is a relativistic analog of the Pauli spin,

$$\hat{O}_i\hat{O}_j - \hat{O}_j\hat{O}_i = 2i\varepsilon_{ijk}\hat{O}_k\,, \quad \hat{O}_j^2 = 1\,, \tag{3.4}$$

and its component parallel to the magnetic field commutates with the Hamiltonian Eqn (3.2). This makes it possible to decouple the solution to the Dirac equation in spin states.

Given the homogeneous magnetic field $\boldsymbol{H} = (0, 0, H)$, and imposing the condition

$$\hat{O}_3\Psi = \zeta\Psi \tag{3.5}$$

on the wave function [Eqn (3.2)], we can introduce the 'fourth' quantum number $\zeta = \pm 1$. It characterises the transverse polarisation: $\zeta = 1$ in the forward direction and $\zeta = -1$ in the backward direction.

Note that the issue of whether the spin can be measured independently of the orbital motion of a particle is somewhat difficult in Dirac's relativistic theory since the electrons execute a 'twitching motion' (Zitterbewegung) owing to the interference of their charge-conjugate states. The result is that only the full moment is preserved in the Dirac theory. The unit polarisation operator [Eqn (3.3)] (the 'true' spin operator) opens up a possibility to measure the spin independently since it commutates with the Hamiltonian (see also Refs [24, 57]).

Omitting the details of solving the Dirac equation (3.2) (it is described in detail in Refs [24, 25]) I want only to emphasise that the quantum state of an electron in a magnetic field $\boldsymbol{H} = (0, 0, H)$ is specified by the set of four quantum numbers: n is the energy number, s is the radial number, k_3 is the projection of the impulse onto the direction of the field, and ζ is the projection of the spin onto the direction of the field. In this case, the energy of an electron takes the form:

$$E = \left(m^2c^4 + c^2\hbar^2k_3^2 + 2e_0Hc\hbar n\right)^{1/2}\,. \tag{3.6}$$

If an electron executes a macroscopic motion, then the energy number s takes a very large value and the energy spectrum is continuous. In the nonrelativistic approximation, formula (3.6) describes the familiar Landau levels and also involves the kinetic energy of the motion of an electron along the field. Note that the energy spectrum given by Eqn (3.6) is degenerate with respect to the spin and to the radial number s. Taking into consideration the anomalous magnetic moment eliminates the spin degeneration and the energy spectrum takes the form (see Ref. [70])

$$E = mc^2\left(\left(\frac{\hbar k_3}{mc}\right)^2 + \left\{\left[1 + \frac{H}{H_0}(2n+\zeta+1)\right]^{1/2} + \zeta\frac{H}{H_0}\frac{\mu-\mu_0}{2\mu_0}\right\}^2\right)^{1/2},$$

where

$$\mu = -\mu_0\left(1 + \frac{e^2}{2\pi\hbar c}\right); \quad \mu_0 = \frac{e_0\hbar}{2mc}\,. \tag{3.7}$$

The ground state $(n = 0,\ k_3 = 0)$ [see Eqn (3.6)] corresponds to the spin oriented in the opposite direction of the magnetic field $(\zeta = -1)$.

The quantum numbers n and s are related to the radius of the orbit of an electron and the quadratic fluctuation of the radius:

$$\langle r^2\rangle = \frac{n+s+1/2}{\gamma_0}\,, \quad \langle r^2\rangle - \langle r\rangle^2 = \frac{s+1/2}{2\gamma_0}\,,$$

$$\gamma_0 = \frac{e_0H}{2c\hbar}\,. \tag{3.8}$$

The electrons are localised near the orbital plane of revolution for an extremely strong magnetic field since

$$R = \sqrt{\frac{n}{\gamma_0}} = \frac{\hbar}{mc}\sqrt{\frac{nH_0}{H}}\,,$$

and the radius of the orbit is of the order of the Compton wavelength in weakly excited states as $H \to H_0$.

3.3 Quantum features of synchrotron radiation power

Restricting myself to the above remarks I move on to discuss synchrotron radiation. By the use of rather conventional methods [24, 35], the expression for the synchrotron radiation power can be obtained in the form

$$W = \frac{e^2c}{2\pi}\sum_{\nu,\,s',\,\zeta'}\int d^3\kappa\,\delta(\kappa - \kappa_{nn'})\Phi I_{ss'}^2(x)\,, \tag{3.9}$$

where

$$c\kappa_{hn'} = \frac{E_n - E_{n'}}{\hbar}\,, \quad x = \frac{\kappa^2\sin^2\theta}{4\gamma_0}\,, \quad \nu = n - n'\,.$$

It is taken into consideration in summing over k_3' that the component of the impulse parallel to the field is preserved. The radial factor $I_{ss'}^2(x)$ appearing in the formula for the radiation power is a Laguerre function, which is related to the Laguerre polynomials $Q_{s'}^{s-s'}(x)$ by the equation

$$I_{ss'}(x) = \frac{1}{\sqrt{s!s'!}}\exp\left(-\frac{x}{2}\right)x^{(s-s')/2}Q_{s'}^{s-s'}(x)\,. \tag{3.10}$$

The function Φ depends on the elements of the Dirac matrix and is expressed in terms of the Laguerre functions $I_{nn'}(x)$, which are approximated by McDonald functions $K_{1/3}$ by analogy with the classical theory of SR (see Refs [24, 35]). Thus, the integral under the summation sign in (3.9) is the power of radiation electrons emit in transitions $n \to n'$, $s \to s'$ with the spin flip $\zeta \to \zeta'$.

Making necessary manipulations, integrating with respect to angles and summing over the polarisation states of the radiation field, we find the following expression for the spectral distribution of the synchrotron radiation power [24, 27]:

$$W = W^{\text{cl}}\frac{9\sqrt{3}}{16\pi}\sum_{s'}\int_0^\infty\frac{y\,dy}{(1+\xi y)^4}I_{ss'}^2(x)F(y)\,, \tag{3.11}$$

where

$$F = \frac{1+\zeta\zeta'}{2}\left[2(1+\xi y)\int_y^\infty K_{5/3}(x)\,dx + \frac{1}{2}\xi^2y^2K_{2/3}(y) - \zeta(2+\xi y)\xi yK_{1/3}(y)\right] + \frac{1-\zeta\zeta'}{2}\xi^2y^2\left[K_{2/3}(y) + \zeta K_{1/3}(y)\right]\,.$$

Here the argument x of the function $I^2_{ss'}$ takes the form $v = n - n'$ upon changing the summation over y for integration with respect to the variable

$$x = \frac{\xi_1 y^2}{(1+\xi y)^2} \, .$$

It follows from Eqn (3.11) that the probability of spontaneous transitions of an electron is a function of two parameters†:

$$\xi = \frac{3}{2}\frac{H}{H_0}\frac{E}{mc^2} = \frac{3}{2}\chi = \frac{3}{2}\left(\frac{E}{E_{1/2}}\right)^2 , \quad E_{1/2} = mc^2\left(\frac{mcR}{\hbar}\right)^{1/2} , \tag{3.12}$$

$$\xi_1 = \frac{9}{8}\frac{H}{H_0}\left(\frac{E}{mc^2}\right)^4 = \frac{9}{8}\left(\frac{E}{E_{1/5}}\right)^2 , \quad E_{1/5} = mc^2\left(\frac{mcR}{\hbar}\right)^{1/5} .$$

We shall return to the parameter ξ_1 later. Now I want only to note that the SR power depends solely on one invariant parameter χ since the sum over the radial quantum number

$$\sum_{s'} I^2_{ss'}(x) = 1$$

is equal to unity. Thus, the limits in which the classical theory was predicted to be applicable [11] were strictly borne out by the quantum theory. Note that this refers, however, to the SR power only.

Expression (3.11) is exact: it allows any values of the parameter $\xi = 3\chi/2$, including $\chi \gg 1$, which are realisable in the physics of neutron stars where the magnetic field strength is close to the critical H_0. In addition, the formula for the radiation power involves the contribution of radiation which is accompanied by the flip of the spin (spin – flip transitions) when $\zeta' = -\zeta$. As follows from Eqn (3.11), the probabilities of such transitions are proportional to the square of the Planck constant: $\hbar^2$.

Since the SR power depends explicitly on the orientation of the spin of an electron, the radiation accompanied by the flip of a spin affects the orientation of the spin and stimulates the directed process of polarisation of the electron beam. In addition, there is a spin dependence in the formula for the SR power, which enters the terms which are independent of the flip of a spin. This is not only of theoretical interest. In fact, given a small invariant parameter ξ it follows from Eqn (3.11) that the spectral distribution of the synchrotron radiation power of a polarised electron beam takes the form

$$W^{\text{pol}} = W^{\text{cl}}\frac{9\sqrt{3}}{8\pi}\int_0^\infty \left[(1-3\xi y)\int_y^\infty K_{5/3}(x)\,dx - \zeta\xi y K_{1/3}(y)\right] y\,dy \, . \tag{3.13}$$

Note that Bordovitsyn [76] made a large advance in interpreting quantum corrections to the classical expression for the SR power [76]. He showed, in particular, that the quantum correction in formula (3.13) involves contributions from the interference of the radiation from electron, charge, and the spin magnetic moment of electron. Clearly, the difference between the expressions for the radiation power of polarised and nonpolarised (spin averaged) electron beams takes the form

$$W^{\text{pol}} - W^{\text{nonpol}} = -\zeta\xi W^{\text{cl}}\int_0^\infty \Phi^{\text{sp}}(y)\,dy \, ,$$

where

$$\Phi^{\text{sp}}(y) = \frac{9\sqrt{3}}{8\pi} y^2 K_{1/3}(y) \, .$$

Since this expression is related directly to the polarisation of an electron beam ζ, the difference $W^{\text{pol}} - W^{\text{nonpol}} = W^{\text{sp}}$ may be called the 'spin light'. Experimentally a nonpolarised electron beam can be obtained from a polarised electron beam by putting a depolariser in the chamber of a storage ring.

The radiation W^{sp} is linearly polarised — only the σ-component is emitted. The 'spin light' has a peculiar spectral distribution power: the maximum is shifted to the high-frequency range and is reached at $y = \omega/\omega_{\text{cr}} \cong 1.6$, whereas the maximum in the SR spectrum falls on $y = 0.3$ (here $\omega_{\text{cr}} = 3\omega_0\gamma^3/2$).

In contrast to the SR spectrum (see Eqn 2.17) the spectral distribution of the 'spin light' power has the form:

$$\Phi^{\text{sp}}(y) = \frac{9\sqrt{3}}{8\pi}\begin{cases} 2^{-2/3}\Gamma(1/3)y^{5/3} , & y \to 0 , \\ \sqrt{\pi/2}\,y^{3/2}\,e^{-y} , & y \to \infty . \end{cases}$$

The associated curves are shown in Fig. 12. The ratio of the radiation power of the 'spin light' and the classical expression has the form:

$$\Delta W^{\text{sp}}(y) = \frac{W^{\text{sp}}(y)}{W^{\text{cl}}(y)} = |\zeta|\xi y K_{1/3}(y)\left[\int_y^\infty K_{5/3}(x)\,dx\right]^{-1}$$

$$\cong |\zeta|\xi y \, , \quad y \gg 1 \, .$$

Here $|\zeta|$ characterises the degree of polarisation of an electron beam in the combined quantum state: $|\zeta| = P$.

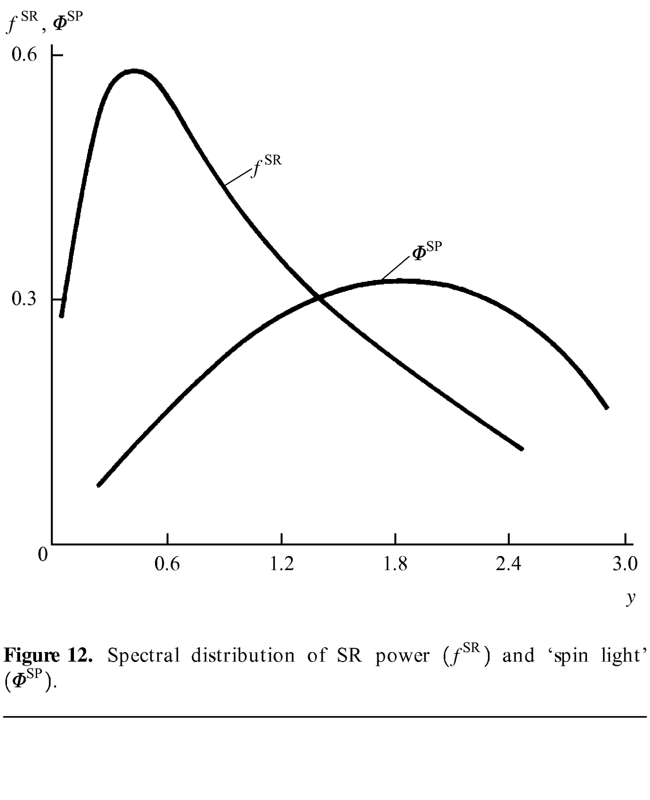


Figure 12. Spectral distribution of SR power (f^{SR}) and 'spin light' (Φ^{SP}).

†The probability of spontaneous transitions can be found by dividing the integrand in Eqn (3.11) by the energy of a quantum of the electromagnetic field $\hbar\omega$.

Thus, this offers a new possibility for visual observation of the polarisation characteristics of an electron beam by determining the SR power at a fixed spectral frequency (see Ref. [72]). The experiment which was performed in the Institute of Nuclear Physics, Siberian Branch of USSR Academy of Science [72], can be considered as a first visual observation of radiation which is directly associated with electron spin.

Further, the expression for the SR power can be found by summing up over the polarisation states of an electron:

$$W = W^{\mathrm{cl}} \frac{9\sqrt{3}}{8\pi} \int_0^{\infty} \frac{y\,\mathrm{d}y}{(1+\xi y)^3} \left[\int_y^{\infty} K_{5/3}(x)\,\mathrm{d}x + \frac{\xi^2 y^2}{1+\xi y} K_{2/3}(y) \right] . \quad (3.14)$$

The SR power which is uniformly applicable for any values of the parameter ξ was calculated exactly by Bagrov [73]. Here I cite two limiting cases in the form of asymptotic expansions. First, let us consider the case of a small invariant parameter; $\xi \ll 1$, i.e., the quantum corrections to the classical radiation formula are

$$W = W^{\mathrm{cl}} \left[1 - \frac{55\sqrt{3}}{24} \xi + \frac{64}{3} \xi^2 + \ldots \right] . \quad (3.15)$$

This correction was found up to a linear term in Ref. [74] and was then corroborated by Schwinger for spinless particles [75].

In the second limiting case, $\xi \gg 1$ (high energies, extremely large magnetic fields), the formula for the SR power differs drastically from the classical one (the ultraquantum limit)

$$W^{\mathrm{UQ}} = W^{\mathrm{cl}} \left[\frac{2^{8/3}}{9} \Gamma\left(\frac{2}{3}\right) \right] \xi^{-4/3} , \qquad \xi \gg 1 \quad (3.16)$$

[24, 77, 79]†. In the ultraquantum limit the principal term of the radiation power is of quantum nature. Therefore transition to the classical approximation is impossible.

It is characteristic that in the ultrarelativistic limit the spectrum is terminated at the frequency $\omega_{\max}^{\mathrm{UQ}} = E/\hbar \ll \omega_{\max}^{\mathrm{cl}}$, and does not reach the classical critical frequency

$$\omega_{\max}^{\mathrm{cl}} = \frac{c}{R} \left(\frac{E}{mc^2} \right)^3 .$$

This follows immediately from the formula for the classical frequency of radiation (see Ref. [35]):

$$\omega_{\max} = \frac{3}{2} \omega_0 \gamma^3 \frac{y_{\max}}{1+\xi y_{\max}} , \qquad y_{\max} \sim 1 .$$

Of some interest is the radiation of weakly excited electrons (low-energy levels) in a strong magnetic field. It is peculiar for such a problem that the energy spectrum of an electron is discrete ('quantising' magnetic field). If electrons move perpendicularly to the field ($k_3 = 0$), the energy

$$E = mc^2 \left(1 + 2n \frac{H}{H_0} \right)^{1/2}$$

takes essentially discrete values for $n \sim 1, 2, \ldots$.

In this case, only numerical methods are applicable. It turns out that the probability of spontaneous transitions depends no longer on the orientation of the spin of an electron and the probability of a transition with a change in the spin orientation is the same as that with no change of polarisation.

The radiation powers of the σ and π components of the linear polarisation take the forms $W_\sigma = 0.742\,W$ and $W_\pi = 0.258\,W$, where

$$W = 0.453 \left(\frac{H_0}{H} \right)^2 W^{\mathrm{cl}}$$

(see Ref. [78]). Thus, the expression for the radiation power W differs from the classical formula by the invariant factor

$$f = \frac{1}{4} \frac{1}{H_0^2} F_{\mu\nu} F^{\mu\nu} = \left(\frac{H}{H_0} \right)^2 .$$

This result does not coincide with those obtained not only in the classical theory, but also in the ultraquantum case of an excited electron moving in a very strong field.

3.4 Quantum fluctuations of the trajectory of an electron

The quantum theory of synchrotron radiation deserves the credit for the discovery of a new physical phenomenon: that the radiation perturbs the trajectory of a particle because of its discrete nature. This influence manifests itself even for energies

$$E \geqslant E_{1/5} = mc^2 \left(\frac{mcR}{\hbar} \right)^{1/5}$$

(of several hundreds of MeV): quantum effects become an important factor which affects the dynamics of an electron. As a result, the recoil gives impetus to the excitation of radial degrees of freedom of an electron, and the trajectory of the particle suffers a quantum widening.

The quantum theory of an electron moving in synchrotron radiation conditions was first used to analyse the radial factor $I_{ss'}^2(x)$ [see Eqn (3.9)], entering the expressions for the radiation power and for the probability of quantum transitions, and accounting for the fluctuation nature of excitation of radial degrees of freedom of an electron which are characterised by the number s. The stochastic character of excitation is particularly striking under the assumption that the centre of the orbit of an electron coincides with the origin of coordinates at the initial instance and, therefore, $s = 0$. Then the radial factor

$$I_{0s'}^2(x) = \frac{\mathrm{e}^{-x} x^{s'}}{s'!} , \qquad x = \frac{\xi_1 y^2}{(1+\xi y)^2}$$

takes the form of the common Poisson distribution. In the classical limit ($\hbar \to 0$), the argument x vanishes. Therefore

$$\lim_{\hbar \to 0} I_{0s'}^2(x) = \delta_{0s'} ,$$

and in the general case

$$\lim_{\hbar \to 0} I_{ss'}^2(x) = \delta_{ss'} ,$$

†This result was also obtained by V I Ritus in Ref. [79] in the crossed field model ($H = E$, $\boldsymbol{H} \perp \boldsymbol{E}$). Although an electron behaves differently in magnetic and crossed fields, certain results on synchrotron radiation power coincide since the radiation is emitted over a very small portion of the trajectory. This is true, however, only for the quasiclassical motion of an electron (large quantum numbers) when the energy spectrum is continuous. The crossed field model is no longer acceptable for small quantum numbers ('quantising magnetic field').

i.e., the radial number does not vary in the classical limit. Then from the standpoint of the classical theory, radiation causes a shortening of the radius of the orbit of an electron which is compensated by the outer high-frequency electric field in a synchrotron and a storage ring. Thus, quantum fluctuations of the radius of the orbit of an electron cannot be considered as a 'quantum correction'. This is a new, proper quantum physical phenomenon (see Refs [32, 67, 25]).

The quantum number s which characterises the quadratic fluctuation of the radius of the orbit of an electron [Eqn (3.8)] can be found as follows. Using Eqn (3.9) to determine the probability of quantum transitions per unit time, we can write it in the form:

$$w_{ss'} = \frac{\sqrt{3}e^2}{2\pi\hbar R\sqrt{\varepsilon_0}} \int_y^\infty K_{5/3}(x)\,\mathrm{d}x I_{ss'}^2(y)\ , \tag{3.17}$$

where $\varepsilon_0 = 1 - \beta^2$. Then, considering

$$\sum_{s'} (s' - s)\, I_{ss'}^2(y) = y\ ,$$

we have

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{55}{48\sqrt{3}} \frac{e^2}{mcR^2} \left(\frac{E}{mc^2}\right)^6 , \tag{3.18}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t} \overline{\Delta r^2} = \frac{55}{48\sqrt{3}} \frac{e^2}{mc} \frac{\hbar}{mcR} \left(\frac{E}{mc^2}\right)^5 ; \tag{3.19}$$

the bar indicates the average value of the quantity. Thus, the stochastic process of radiation suggests a formula typical of the Brownian motion provided that the average energy of an electron is constant. This formula accounts for the action of random forces on the particle: $\overline{x^2} = 2Dt$, where D is the diffusion coefficient (see Ref. [80]). The quantum effect of the 'widening orbit' [Eqn (3.19)] predicted by the author together with Sokolov [32, 67] can be interpreted as a macroscopic manifestation of quantum fluctuations of radiation caused by discrete emission of photons and the recoil an electron experiences. In the radial direction, electrons move in accordance with quantum laws, whereas the motion along the circle is of a classical nature (a 'macroatom'). Such a pattern of motion is realisable only if the average energy of an electron is constant when the radiative energy losses of the particle are compensated by an outer source.

The situation is, however, different when the radiative losses of an electron are not compensated. This is the case, for example, in the astrophysical conditions or, especially, in the magnetosphere of a pulsar. It is interesting that an equilibrium is established as a result of two processes — the radiative shortening of the radius of the orbit of an electron and the growth of the quadratic fluctuation. Since

$$\overline{R^2} = \frac{2c\hbar}{e_0 H}\left(n + s + \frac{1}{2}\right) ,$$

it may be found that

$$\frac{\mathrm{d}}{\mathrm{d}t} R^2 = \frac{2c\hbar}{e_0 H}\left(\frac{\mathrm{d}n}{\mathrm{d}t} + \frac{\mathrm{d}s}{\mathrm{d}t}\right) , \tag{3.20}$$

where the derivative $\mathrm{d}s/\mathrm{d}t$ is specified by means of formula (3.18), and the principal quantum number n varies with energy:

$$\frac{\mathrm{d}}{\mathrm{d}t} n = -\frac{H_0}{H} \frac{EW}{(mc^2)^2} .$$

Then it follows from Eqn (3.20) that $\mathrm{d}R^2/\mathrm{d}t = 0$ provided that the radius is minimal:

$$R_{\min} = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \left(\frac{E}{mc^2}\right)^2 . \tag{3.21}$$

This expression is similar to the corresponding expression in the case of an electron moving in the focusing magnetic field of a storage ring when radiative damping forces (damping effect) act in parallel with the quantum widening of the trajectory of a particle and the amplitude of the oscillations of an electron is at a minimum (see Section 4).

3.5 Effect of radiative polarisation of electrons and positrons in storage rings

The effect is that the spins of particles are oriented in the same way under the influence of synchrotron radiation when they circulate in storage rings for a long time. This effect of radiative polarisation was predicted by the author [81] and strictly established together with Sokolov with the use of the exact solutions to the Dirac equations [82] (see also Refs [35, 83]).

The probability of quantum transitions accompanied by spin flips can be calculated with Eqn (3.11). The calculation shows that the probability still depends on the spin orientation and upon integration with respect to angles and to the spectrum:

$$w^{\uparrow\downarrow} = \frac{1}{2\tau}\left(1 + \zeta\,\frac{8\sqrt{3}}{15}\right) , \tag{3.22}$$

where the polarisation time τ has the form

$$\tau = \frac{8\sqrt{3}}{15} \frac{\hbar^2}{mce^2} \left(\frac{mc^2}{E}\right)^2 \left(\frac{H_0}{H}\right)^3 . \tag{3.23}$$

It follows that radiation induces electrons to transit predominantly in states with a spin oriented in the direction opposite to that of the magnetic field [81]. Positrons have the opposite spin orientation. States with a predominant spin orientation match the minimal potential energy of particles, the magnetic moment of which is $\boldsymbol{\mu} = -(e_0\hbar/2mc)\,\boldsymbol{\zeta}$ in the magnetic field $U = (-e/|e|)\,\boldsymbol{\mu}\cdot\boldsymbol{H}$.

Omitting the details of the kinetics of the polarisation process (see Refs [82, 35] and turning our attention to an ensemble of particles, we can characterise the polarisation of a beam of particles by the average value $\zeta(t) = \langle\zeta\rangle$, bearing in mind that the electron goes into the combined state as a result of interaction with the electromagnetic field. Then

$$\frac{\mathrm{d}}{\mathrm{d}t}\zeta(t) = \sum_{\zeta'} (\zeta' - \zeta)\, w^{\uparrow\downarrow} = -2\sum_{\zeta} \zeta w^{\uparrow\downarrow} ,$$

whence it follows that†

$$\zeta(t) = -\frac{8\sqrt{3}}{15}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right] . \tag{3.24}$$

The extreme degree of polarisation is $P(\infty) = 8\sqrt{3}/15 = 0.924$ (for $t \gg \tau$).

†This result was later obtained by Baier and Katkov [84] and by Schwinger [85].

The estimate for the polarisation time shows that the effect of radiative polarisation is accessible for observation in magnetic fields which are typical of accelerators only if the particles circulate in the magnetic field for a long time (about 1 hour). Storage rings provide a means for this possibility. Although there are effects depolarising the beam in an actual storage ring, the effect of radiative polarisation exists and provides a unique capability for creating polarisation beams of high-energy electrons and positrons.

The effect of radiative polarisation was experimentally observed in the USSR, France, Germany, USA, Japan, and Switzerland in storage rings with electrons of energy 1–50 GeV (see Ref. [83]).

3.6 Stimulated synchrotron radiation. Cyclotron resonance maser

In this section, we will study the induced quantum transitions of electrons under the action of the outer field of an electromagnetic wave by using methods of quantum theory (see Ref. [25]). The induced amplification and generation of electromagnetic waves are possible not only in atomic and molecular systems but also when electron beams move in an outer electromagnetic field. In contrast to induced quantum transitions in atomic and molecular systems, where electrons are in bound states, particles are 'quasifree' when they move in an outer electromagnetic field, and their energy spectrum is virtually quasicontinuous. So, for example, in the problem of an electron moving in an homogeneous magnetic field, only the motion in the plane perpendicular to the magnetic field (Landau levels) is quantised while the motion along the field is free. Quantum generators, whose 'working body' is an electron beam moving in an outer magnetic field (for example, in the field of an undulator), were called free-electron (not bound in an atom) lasers.

The detailed description of free-electron lasers goes beyond the scope of this review (see Refs [52, 100]). Here I shall only discuss the problem of how a cyclotron resonance maser (a CRM) can be realised. The solution can be obtained by means of the quantum theory of SR.

Let us consider the induced radiation of an electron moving in a constant and homogeneous magnetic field in the presence of an outer electromagnetic wave. Then the expression for the probability of induced transitions can be presented as the product $w_{nn'} = N(\boldsymbol{\omega})\, w_{nn'}^{\mathrm{SP}}$, where $w_{nn'}^{\mathrm{SP}}$ is the probability of spontaneous transitions and $N(\boldsymbol{\omega})$ is the number of photons in the volume L^3. The last quantity is related to the outer electromagnetic wave strength by the equation:

$$\frac{\mathcal{E}^2}{4\pi} = \frac{\hbar\omega N(\boldsymbol{\omega})}{L^3} .$$

If the outer wave propagates at an angle θ to the z axis, then the expression for the probability of induced transitions can be reduced to the form [35]

$$w_{nn'} = \frac{\pi e_0^2 \mathcal{E}^2 c^2}{\hbar^2 \omega^2} \left(|\bar{\alpha}_1|^2 + |\bar{\alpha}_2|^2 \cos^2\theta\right) g(\omega, \omega_{nn'}) , \quad (3.25)$$

where $\hbar\omega_{nn'} = E_n - E_{n'}$, and the function $g(\omega, \omega_{nn'})$ characterises the Lorentz width of the spectral line:

$$g(\omega, \omega_{nn'}) = \frac{1}{\pi} \frac{\tau}{1 + \tau^2(|\omega_{nn'}| - \omega)^2} . \quad (3.26)$$

Here τ is a characteristic finite lifetime of an electron in the excited state. It corresponds to the effective time in which the particle traverses the magnetic field. Note that the function $g(\omega, \omega_{nn'})$ is normalised to unity:

$$\int_{-\infty}^{\infty} g(\omega, \omega_{nn'})\, \mathrm{d}\omega = 1 .$$

If the lifetime of an electron in a state with the energy E_n is large enough, then $g(\omega, \omega_{nn'})$ goes into the delta function:

$$\lim_{\tau\to\infty} g(\omega, \omega_{nn'}) = \delta(\omega - \omega_{nn'}) .$$

Let us turn our attention to the energy spectrum of an electron moving in a homogeneous magnetic field. For simplicity I shall consider the motion of an electron in the orbital plane, and neglect the recoil along the field when a photon is emitted. This assumption makes it possible to describe the fundamental features of the maser effect in a simple form.

Under these conditions, the expression for the energy spectrum is

$$E_n = (m^2c^4 + 2ne_0\hbar cH)^{1/2} = mc^2 + \hbar n\Omega\left(1 - \frac{n\hbar\Omega}{2mc^2}\right) , \quad (3.28)$$

where $\Omega = e_0H/mc$ is the cyclotron frequency. In the expansion of the square root, relativistic corrections to the Landau energy levels are accurate to β^2. The discrete character of the energy spectrum manifests itself but in a weak form under conditions typical of an electron moving in an orbit of a macroscopic radius: the distance between adjacent energy levels ($\Delta n = 1$) is

$$\Delta E = mc^2 \frac{mc^2}{E} \frac{H}{H_0} ,$$

and the energy spectrum is virtually continuous.

In such problems the outer radiation cannot be chosen to be localised in frequency. Therefore, three adjacent levels, E_n and $E_{n\pm1}$, are to be considered. They correspond to absorption (E_{n+1}) and emission (E_{n-1}) of a photon. It is important that the coherent amplification is impossible in the case of an equidistant spectrum (nonrelativistic approximation) — photons of the outer magnetic field are only absorbed.

However, the relativistic corrections [Eqn (3.28)] change the situation: quantum transitions with emission $\omega_{n,\,n-1}$ and absorption $\omega_{n,\,n+1}$ of a photon have distinct resonance frequencies:

$$\omega_{n,\,n\mp1} = \Omega\left[1 - \frac{\hbar\Omega}{2mc^2}(2n \mp 1)\right] . \quad (3.29)$$

This offers new opportunities for amplifying an outer electromagnetic wave. Given, in particular, that $\omega_{n,\,n-1} \cong \Omega$ we have $|\omega_{n,\,n+1}| = \Omega - \Omega_1$, where $\Omega_1 = \hbar\Omega^2/mc^2$.

It suffices to take the elements of the Dirac matrices $|\bar{\alpha}_1|$ and $|\bar{\alpha}_2|$ in the dipole approximation:

$$|\bar{\alpha}_1|^2 = |\bar{\alpha}_2|^2 = \frac{\hbar\Omega}{2mc^2}\left[(n+1)\delta_{n',\,n+1} + n\delta_{n',\,n-1}\right] ,$$

Then the overall energy of the stimulated radiation and absorption is

$$W = \hbar(\omega_{n,\,n-1}\, w_{n,\,n-1} - |\omega_{n,\,n+1}|\, w_{n,\,n+1})$$
$$= -\frac{e_0^2\mathcal{E}^2\Omega^2\tau(1+\cos^2\theta)}{m\omega^2}\, \Phi(x) ,$$

where

$$\Phi(x) = \frac{1}{1+x^2}\left[1 + \frac{2\beta^2 \cdot Qx}{1+x^2}\right], \qquad Q = \Omega\tau\,, \tag{3.30}$$

and

$$x = 2\tau(\Omega - \omega)\,, \qquad \beta^2 = \frac{2n\hbar\Omega}{mc^2}\,.$$

Hence it follows that, if the levels are equidistant ($\beta = 0$), then the system always absorbs the energy from the electromagnetic wave since $W < 0$. Formula (3.30) was first obtained by Schneider [93]. When the resonance is violated ($\omega > \Omega$), the second term in the function $\Phi(x)$ can become negative and its absolute value can be greater than unity. Then the system of electrons in the magnetic field is a source of coherent radiation (Fig. 13). This idea provides the theoretical foundation for the cyclotron resonance maser [94, 95].

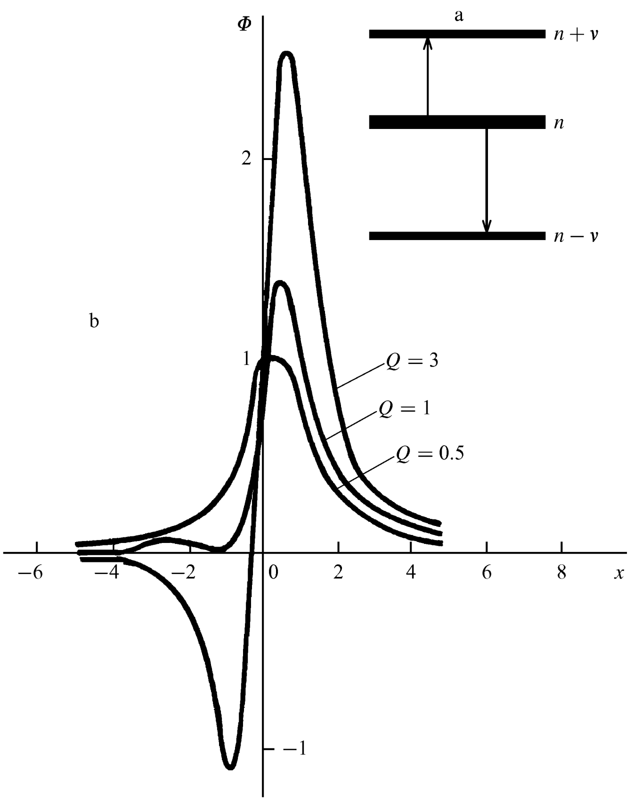


Figure 13. (a) Three-level system describing interaction of electrons with the radiation field at frequencies close to $\nu\omega_c$. (b) Absorption curves for electrons in a magnetic field $H = 10^4$, for $\theta = \pi/2$.

Note that the maser effect can also be obtained at the higher harmonics of the fundamental frequency for electrons of high energies in the relativistic case [96]:

$$W = 0.506\,\nu^{-4/3}\,\frac{e^2\mathcal{E}^2\tau c^2}{E}\left[1 - 0.726\,\nu^{4/3}\left(\frac{mc^2}{E}\right)^2\right]. \tag{3.31}$$

Hence it follows that radiation dominates over absorption for harmonics $\nu < \sqrt{\nu_{\max}}$, where $\nu_{\max} = (E/mc^2)^3$ is the index of harmonics corresponding to the maximum in the synchrotron radiation spectrum.

The problem of a cyclotron radiation maser described above can be solved by methods of the classical theory [97]. However the processes of induced radiation and absorption cannot be split within the framework of the classical theory: they must be considered as a whole. In this respect, the quantum theory has certain advantages (see Refs [25, 27]).

4. Influence of synchrotron radiation on the dynamics of electrons in cyclic accelerators and storage rings

The phenomenon of quantum fluctuations of the trajectory of an electron moving in a magnetic field is of fundamental importance in the analysis of the stability of motion of particles in a cyclic accelerator and, in particular, in a storage ring. The magnetic field in such apparatus not only induces an electron to move in a circle of a constant radius but also possesses focusing properties. The field which has these properties helps to return the particle to stationary orbits [24, 25, 86]. If the field has the form $H = \bar{H}(R/r)^q$ near a stationary orbit, then electrons subjected to random deviations will be engaged in betatron oscillations: the frequency of radial oscillations will be equal to $\omega_r = \omega_0\sqrt{1-q}$, and that of vertical oscillations will be $\omega_z = \omega_0\sqrt{q}$, where $\omega_0 = e_0cH/E = c/R$. Given an index of decrease of the field (q) less than unity (weak focusing), the orbit is stable in either direction. (The letter q is chosen to designate the index of decrease of the field since its common letter is used to designate the principal quantum number.) The case of strong focusing $|q| \gg 1$ is not considered here.

Synchrotron radiation is an important factor which determines the dynamics of particles in an accelerator and in a storage unit. It is sufficient to recall the fate of the betatron [7]. SR induces large radiative energy losses and the radius of the orbit shortens. This sets a physical limit on the induction method of acceleration, which is conditioned by the disturbance of the stability of motion of electrons.

In a synchrotron the radiative losses of the energy of a particle are compensated by an outer source — a periodic magnetic field which is tangential to the trajectory — and the radius of the orbit of an electron is constant since $\beta E = e_0HR$. The focusing properties of the magnetic field provide for stability of the motion of a particle in a circle of a stationary radius.

However, the radiative shortening of the radius of the orbit of an electron is not the only role of SR. The general analysis of the stability of motion with regard to SR leads to an interesting observation: an electron simultaneously experiences the action of classical radiative damping forces, which drastically decreases the amplitude of betatron oscillations (see the works of Kolomenskii, Lebedev, Orlov, and Tarasov [86]), and of quantum fluctuation forces, which result in quantum widening of the orbit [32, 67]. As a result, the amplitude of betatron oscillations assumes a steady value in the course of time. This value is a peculiar compromise of radiative damping of oscillations and their quantum excitations.

The strict solution to the problem of how quantum fluctuations of SR affect the motion of electrons in a homogeneous focusing magnetic field was obtained by Gutbrod [87] (see also Refs [88, 24, 25]). He showed, by the method of exact solutions to the Klein–Gordon equation (a spinless electron), that the square of the amplitude of radial oscillation,

$$a^2 = 2\overline{x^2} = 2\overline{(\bar{r}-R)^2} = \frac{2c\hbar s}{e_0 HR\sqrt{1-q}}$$

changes in time in accordance with the law

$$\frac{\mathrm{d}a^2}{\mathrm{d}t} = \frac{55}{24\sqrt{3}}\,\frac{e^2}{mc(1-q)^2}\,\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^5 - a^2\,\frac{qW}{(1-q)E}\,, \tag{4.1}$$

and, similarly, that the amplitude of vertical oscillations $b^2 = 2\overline{z^2}$ changes in accordance with the law

$$\frac{\mathrm{d}b^2}{\mathrm{d}t} = \frac{13}{24\sqrt{3}}\,\frac{e^2}{mcq}\,\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^3 - b^2\,\frac{W}{E}\,. \tag{4.2}$$

Here W is the synchrotron radiation power and q is the index of decrease of the magnetic field. Note that Eqn (4.1) goes into formula (3.19) in the case of an homogeneous magnetic field ($q = 0$).

It is seen from the time dependencies of the squares of amplitudes of radial and vertical betatron oscillations that, if the quantum excitation were absent, the classical radiative damping with the damping decrement

$$\Gamma_r = \frac{q}{1-q}\,\frac{W}{E}\,, \qquad \Gamma_z = \frac{W}{E}$$

would stop betatron oscillations and strongly compress the entire bunch of electrons. Then the radiation would be strongly coherent and the normal operation of an accelerator or a storage ring of electrons would be upset: a prolonged circulation of electrons in the magnetic field would be impossible in essence. This would set a physical limit on a synchrotron or a storage ring.

However, quantum fluctuations of the trajectory of a particle prevent the bunch from being compressed. As a result, the amplitude of oscillations takes a fixed value once the equilibrium of two processes — the classical radiative damping (the damping effect) and quantum excitation — is established:

$$a^2_{\mathrm{est}} = \frac{55}{16\sqrt{3}}\,\frac{1}{q(1-q)}\,\frac{R\hbar}{mc}\left(\frac{E}{mc^2}\right)^2\,, \tag{4.3}$$

$$b^2_{\mathrm{est}} = \frac{13}{16\sqrt{3}}\,\frac{1}{q}\,\frac{R\hbar}{mc}\,. \tag{4.4}$$

Note that quantum fluctuations affect radial betatron oscillations to a greater extent. As a result, the electron beam is compressed vertically and elongated radially.

Let us consider now how SR affects the dynamics of electrons in a storage ring, when particles move in it for a long time (tens of hours) and preserve their energy (their radiative losses are compensated by the outer electric field). We shall use the classical Dirac – Lorentz equation,

$$m\ddot{x}^{\mu} = \frac{e}{c}\,\dot{x}_{\nu}F^{\mu\nu} + F^{\mu}_{\mathrm{rad}}\,, \tag{4.5}$$

in which the radiative friction force in the relativistic case of interest to us has the form

$$F^{\mu}_{\mathrm{rad}} = -\frac{2e^2}{3c^5}\,\dot{x}^{\mu}\ddot{x}^{\nu}\ddot{x}_{\nu}\,, \tag{4.6}$$

where the dots indicate derivatives with respect to the intrinsic time. Then the Dirac – Lorentz notion takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{E\,v}{c^2}\right) = -\frac{e_0}{c}\,v\times\boldsymbol{H} - e_0\boldsymbol{E}_{\mathrm{comp}} - W\,\frac{v}{c^2}\,. \tag{4.7}$$

in its common three-dimensional notation. Here the electric field $\boldsymbol{E}_{\mathrm{comp}}$ compensates for energy losses and provides for a constant energy. Although Eqn (4.7) fully describes the motion of electrons in a storage ring, it does not account for the quantum fluctuations of radiation.

In particular, Eqn (4.7) is followed by the equation of betatron oscillations:

$$\ddot{x} + \Gamma_r\dot{x} + \omega_r^2 x = 0\,, \qquad x = r - R\,. \tag{4.8}$$

Thus, synchrotron radiation manifests itself as a factor which contributes to steadying the motion of a particle [86]. As noted above, the classical force of radiative friction [Eqn (4.6)] does not give a full description of the dynamics of an electron. The operation of a storage ring will be upset abruptly after a time $\tau \sim 1/\Gamma_r$, as a consequence of compression of the electron beam and very strong coherent radiation (this time is equal to 10^{-3} s for electrons of energy 1 GeV).

The reason is that expression (4.6) for the force of radiative friction does not account for the quantum nature of synchrotron radiation — the discrete character of photon emission. It is a pity that this most important factor has not been reflected in the review [89] dedicated, in particular, to equations of motion of particles with consideration for the radiative force of friction.

In view of the known complexity of the quantum theory, several authors (Sands [90], Kolomenskii and Lebedev [86], see also Ref. [88]) proposed models which add the classical expression for the radiative friction force to the quantum fluctuation force. So, for example, the expression for the fluctuation force F^{fl} responsible for quantum fluctuations can be chosen in the form [88, 25]

$$F^{\mathrm{fl}} = \frac{\hbar c v}{E(1-q)}\sum_i \dot{\delta}(t-t_i)\,, \tag{4.9}$$

and then the equation for betatron radial oscillations becomes inhomogeneous:

$$\ddot{x} + \Gamma_r\dot{x} + \omega_r^2 x = F^{\mathrm{fl}}\,. \tag{4.10}$$

Its right-hand side accounts for discrete properties of synchrotron radiation. Note that, unlike the classical formula for the force of radiative friction [Eqn (4.6)]†, the fluctuation force given by Eqn (4.9) is introduced here for the sake of the model only. The extent to which this model is true can be seen from comparison with the exact quantum calculation [86].

The general solution of Eqn (4.10) can be presented in the form (see Ref. [25])

$$\overline{(x-\bar{x})^2} = \frac{A}{\Gamma_r}\left[1 - \exp(-\Gamma_r t)\right]\,, \tag{4.11}$$

where the quantity

$$A = \frac{55}{48\sqrt{3}(1-q)^2}\,\frac{e^2}{mc}\,\frac{\hbar}{mcR}\left(\frac{E}{mc^2}\right)^5 \tag{4.12}$$

characterises the change in the quadratic fluctuation of radius due to the quantum widening of the trajectory; this formula is substituted into Eqn (3.19) for $q = 0$ (homogeneous field).

†The expression for radiative friction force [Eqn (4.6)] was obtained by Dirac [91] under the assumption that an electron experiences the action of the field equal to half the difference between the lagging and leading fields created by the particle in motion.

Of interest are two limiting cases. First I shall dwell on the case of small times: $t \ll 1/\Gamma_r$. Then

$$\overline{(x-\bar{x})^2} = At \ . \tag{4.13}$$

The particle moves in the classical trajectory and executes harmonic damping oscillations. Quantum fluctuations result in the quantum widening of this trajectory (Fig. 14). Moreover, the widening of the trajectory is similar to the law of Brownian motion for the time $t \ll 1/\Gamma_r$ ($\tau = 1/\Gamma_r \sim 10^{-3}$ sec):

$$\overline{(x-\bar{x})^2} = 2Dt .$$

It is important that quantum fluctuations responsible for widening trajectory cannot be eliminated in principle since they cannot be described by a continuous law.

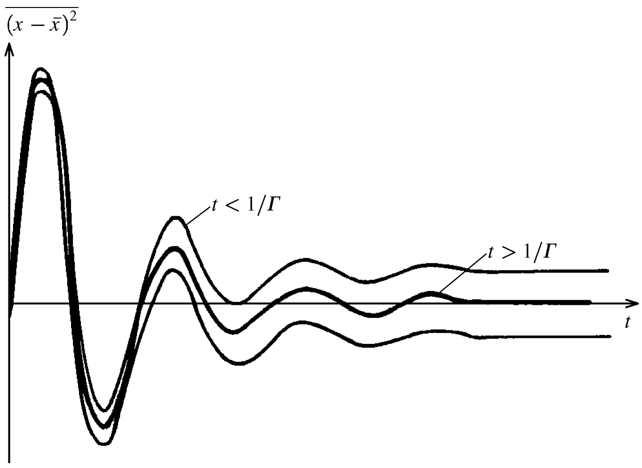


Figure 14. Quantum widening of the amplitude of damping betatron oscillations.

In another limiting case of $t > 1/\Gamma_r$, it follows from Eqn (4.11) that

$$\overline{(x-\bar{x})^2}\Big|_{t\to\infty} = \frac{A}{\Gamma_r} \ . \tag{4.14}$$

Moreover, the motion of the particle no longer depends on the initial amplitude of oscillations. The time $\tau = 1/\Gamma_r$ characterises the time for which the information about the initial state of the system is forgotten. As a result, the steady-state value of the quadratic fluctuation is established:

$$\overline{(x-\bar{x})^2} = \frac{55}{32\sqrt{3}}\,\frac{1}{q(1-q)}\,\frac{R\hbar}{mc}\left(\frac{E}{mc^2}\right)^2 . \tag{4.15}$$

This quantity is essential in determining the sizes of the cross-section of the beam (emittance) and is quantum in nature.

In the first experimental studies of the quantum properties of SR, quantum fluctuations were indirectly observed on the electron synchrotron in the California Institute of Technology [90]. There were problems in starting up the accelerator when attempts were made to achieve the rating energy of electrons of 1.2 GeV, because of quantum excitations of phase oscillations. The study of the dynamics of an electron beam was performed by means of the technique of rapid photography of synchrotron radiation in the FIAN on the synchrotrons in which the electron energies were 280 and 680 MeV [92]. The evolution of the luminous patch was presented in a series of photographs. The patches characterised the betatron oscillations of an electron and their changes during the course of a cycle of acceleration. The damping effect and the quantum widening of the trajectory were observed (Fig. 15). Using the same technique, Vorob'ev et al. performed studies in the Tomsk Polytechnical Institute on the synchrotron 'Sirius' for an electron energy of 1.5 GeV [42]. All the experiments supported the theoretical conclusions. It was also established that the focusing of a beam should be sharpened and that the field should have larger gradients (close focusing), in order to make the advance of the accelerating technique into the region of energies of more than $|q| \gg 1$ possible.

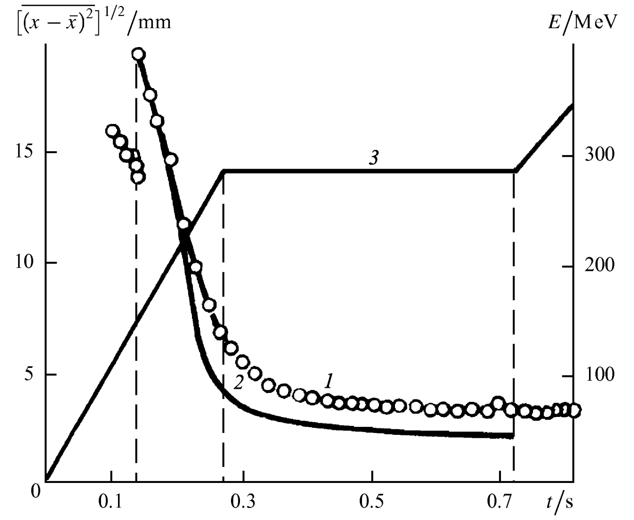


Figure 15. Time dependence of the mean square amplitude of radial betatron oscillations of electrons for the synchrotron 'FIAN-680 MeV': (1) experimental data; (2) theoretical data; (3) variations in the magnetic field.

5. Synchrotron radiation in physical experimentation

For a long time, synchrotron radiation was considered to be a nuisance in the operation of a cyclic accelerator: the reason was that it set a radiative 'ceiling' to the operation of a betatron [7]. Radiative losses of energy imposed a fundamental restriction on the inductive method of acceleration of electrons. Therefore, a new acceleration technique was adopted upon the discovery of automatic phase stabilisation (Wecksler, McMillan) — a synchrotron in which energy losses are compensated for. However, even early studies of the properties of SR [2, 16–18] attracted the attention of experimentalists and soon aroused an interest in SR as a new source of radiation.

In the 1960s the first laboratories of synchrotron radiation appeared with the object of finding an application for SR in the physical experiment. This special attention to the new source of radiation was due to its peculiar properties: a wide spectral range on the scale of electromagnetic waves from the infrared radiation to the x-ray radiation; sharp collimination because of which the brightness of radiation was very high; high power; and natural polarisation typical of this source. Of great importance was the fact that all properties of SR were fully described theoretically. This made it possible to calculate its characteristic with a high degree of precision†.

The 1980s were marked by a vigorous growth of a number of investigations in which SR was used, and by advances in new types of specialised sources of radiation. During these years scientific research centres of synchrotron radiation appeared. They were equipped with sources of SR, free-electron lasers (FELs), wigglers, systems of undulators, and auxiliary facilities built in the chamber of a storage ring. In Russia, centres of synchrotron radiation were established in the Institute of Nuclear Physics in Novosibirsk, in the P N Lebedev Physical Institute (FIAN), and in the I V Kurchatov Institute of Atomic Energy. Today scientific research using synchrotron radiation receives a wide acceptance all over the world: in the USA, FRG, Italy, Japan, United Kingdom, France, Switzerland and other countries‡.

The successful application of SR in the physical experiment had a pronounced effect on the progress of physics of atoms and molecules and also of physics of the solid body. It is impossible to cover all these issues in our review in any detail since they constitute a vast problem, which is in itself of interest. Fortunately there is no need for it since the issues related to applications of SR in the physical experiment are considered in detail in specific literature. So, for example, the review of Curdling was dedicated in particular to the application of SR in atomic spectroscopy (see Ref. [98]); Koch and Sonntag reviewed applications of SR in molecular spectroscopy; and Ling reviewed the spectroscopy of solid bodies (see Ref. [98]). Applications of SR in studies of the luminescence of crystals were considered in the monograph [25] and, finally, the review of Haensel is dedicated to applications of SR in studies of optical properties of alkali halide compounds (see Ref. [99] and also Refs [105 – 108]).

It should be stressed that synchrotron radiation possesses a number of benefits in comparison with other sources used in spectroscopy. Kulipanov and Shkrinskii cited a set of formulas handy for practical calculations and evaluations of SR as a source in the review [105] (see also Refs [25, 27]). So, for example, the spectral brightness of a source,

$$B_\lambda = \frac{N^{\mathrm{p}}\,\mathrm{d}^4}{\mathrm{d}t\,\mathrm{d}S\,\mathrm{d}\Omega\,\mathrm{d}\lambda/\lambda}\,, \tag{5.1}$$

is of significant importance for practical purposes. The brightness is the number of photons N^{p} which have been emitted for one second from a unit area of a source S into a unit solid angle Ω in a given spectral band $\mathrm{d}\lambda/\lambda$. It is a function of the size of the electron beam and of the angular spread of particles in the beam:

$$B_\lambda = \frac{N_\lambda}{\Delta x\,\Delta z\left[\psi_\lambda^2 + (\Delta\theta_z)^2\right]^{1/2}}\,. \tag{5.2}$$

Here N_λ is the spectral flux of photons, Δx and Δz are the effective sizes of the beam (horizontal and vertical), ψ_λ is the angular divergence of SR, and $\Delta\theta_z$ is the vertical angular spread of electrons in the beam.

The brightness of a source specifies the maximal attainable wavelength resolution and also the exposure time (biology, x-ray lithography). Therefore, one of the major aims in designing sources of SR is to attain a brightness as high as possible.

Note that SR is virtually a unique source of high-intensity radiation in the range 200 – 500 Å. In the short-wavelength range of the vacuum ultraviolet radiation and in the soft x-ray range, the power of the radiation emitted by electrons of energy of several GeV exceeds the power of radiation available from x-ray tubes by several orders of magnitude [25, 27].

Of special importance is the application of SR in experiments in the soft x-ray range of radiation, in which its power exceeds several times that of all other sources of x-ray radiation. It should be added that SR has an advantage over other sources since it allows for continuous adjustment of the wavelength of radiation, especially for application of long-wavelength x-ray radiation.

This peculiar feature of SR opened up a possibility for its application in biology, in the study of structures of biopolymers. The reduction of the exposure time, and the preservation of the object of investigation from being destroyed because of a much smaller radiative load make SR irreplaceable in studies of biological structures (see Ref. [25]).

The last few years have been marked by the successful application of SR in medicine, particularly in angiography by x-ray techniques. There is a possibility of obtaining more information when a smaller radiative load is applied to a patient. In 1986 Winick conducted angiographic inspection of a man in the Stanford Laboratory of SR (earlier such inspections were conducted on animals only) (see Refs [25, 103, 104].

SR has been applied in microlithograpy for obtaining elements of microschemes used in modern semiconductor devices. The unique properties of SR — sharp directivity, large power in the x-ray range — make it possible to improve the quality of elements of microschemes and to obtain new elements in microelectronics (see Refs [25, 109]).

I will now draw the reader's attention to new possibilities of experiments centered around the direct visual observation of 'electronic light' [25, 100]. This refers to the observation of an electron beam when it passes through an accelerating cycle or moves in a storage ring. As noted above, an outstanding success was the experimental examination of the dynamics of betatron oscillations of electrons in the presence of forces of radiative damping and quantum fluctuations [41, 101].

The basis for visual observation of dynamics of an electron beam is the high-speed photography of synchrotron radiation emitted from an electron beam, followed by the processing of photographs. High-speed photography of an electron beam for purposes of analysis of its dynamics was first performed by Pollock's group on the synchrotron 'General Electric-70 MeV' [2], and then this technique was developed by Ado [102] and also in a series of studies

†In 1968 a channel of vacuum ultraviolet synchrotron radiation was built in the synchrotron 'FIAN C-60' under the supervision of V V Mikhailin.

‡In the same period, a laboratory was established in the Physics Depart- ment of Moscow State University (under the supervision of the author and V V Mikhailin), which united theoreticians and experiment-alists for theoretical analysis and applications of synchrotron radiation. The laboratory collaborated with the Physical Institute of the Academy of Science, the Institute of Atomic Energy, the Institute of Nuclear Physics, Siberian Branch of the USSR

performed under the supervision of Korolev on the synchrotrons 'FIAN-280 MeV' and 'FIAN-680 MeV' [92]. The analysis of photographs obtained in the experiments mentioned above restored the full history of evolution of betatron oscillations under the action of forces of radiative damping and quantum fluctuations [41, 92]. I want to emphasise that the source of information about the motion of an electron was the particle itself through emitted electromagnetic waves — 'luminous electron' (for details, see Ref. [25]).

I shall describe in detail several new possibilities of applying SR in experiments, centred around the radiation of an electron having an oriented spin. Under the conditions typical of the magnetic field of accelerators and storage rings, the dynamic parameter

$$\chi = \frac{H}{H_0}\,\frac{p_\perp}{mc}$$

[see Eqn (3.1)] is still a small quantity and the quantum effects of synchrotron radiation manifest themselves as small contributions on the background of the classical formulas for radiation. Nevertheless the radiation of an electron beam with an oriented spin has interesting features. As mentioned earlier, there is an addition $\varDelta$ to the power of radiation in the short-wavelength range of the spectrum of SR. This addition is due to polarisation of the electron beam $|\zeta| = |\langle\zeta\rangle|$:

$$\varDelta = \frac{3}{2}\,|\zeta|\gamma y\,\frac{H}{H_0}\,,$$

where $\gamma = E/mc^2$, $y = \omega/\omega_{\mathrm{cr}} \gg 1$. In other words, the additional power of SR is proportional to the magnetic field strength, energy, and degree of polarisation of the electron beam for a fixed wavelength. The quantity $\varDelta$ can be considered as the jump of the radiation energy upon turning on the depolariser which destroys the spin orientation.

This method of observation of spin dependence of SR was for the first time applied in the Institute of Nuclear Physics, the Siberian Branch of the USSR Academy of Science in Novosibirsk [72]. The jump of the radiation energy $\varDelta$ was determined by comparing the powers of SR of the two clusters of particles — polarised and nonpolarised (one of the clusters was exposed to selective depolarisation). The results obtained showed a good agreement with the theory of the radiative polarisation effect for electrons and positrons in storage rings [81, 82].

Note that the observer's instrument has no direct influence on the particle in the above experiments (as was the case, for example, in the experiments of Stern and Gerlach). Here the source of information about the spin orientation is the electron itself: it emits SR which depends on the spin properties of the particle. The experiment on observation of spin dependence of SR presents new insights on the problem on measuring the spin of a free (not bound in an atom) electron.

As noted above, electrons and positrons moving in storage rings are polarised by synchrotron radiation. The interaction of opposing polarised beams of particles is of the utmost importance in experiments in the field of high energy physics. However, these issues are beyond the scope of our review (see Ref. [25]).

In conclusion I shall describe briefly ways in which SR sources can be improved. One of the crucial problems is how the brightness of a source can be increased. The term brightness is understood to refer to the number of photons which are emitted in one second from a unit area of an extended source into a unit solid angle. One way of increasing the brightness is to create a storage ring of small emittance — the emittance is a characteristic of a beam of particles, and is given by $\varepsilon = \pi\sigma\theta$, where σ is the Gaussian size of the beam in meters, and θ is the angle of the cone of radiation in radians. In advanced modern sources, the small emittance is achieved through the strong focusing of a beam of particles, combined with systems of permanent magnets of multiperiodic undulators (note that the angular size of the cone of radiation is the quantity $\delta\theta \sim 1/(\gamma\sqrt{N})$ for an undulator made up of N sections of magnets).

Further, it is clear that the brightness of a source depends primarily on the radiation power. Of interest in this context are coherent bunches of electrons clustered at distances less than the wavelength of their radiation. As noted in the early work [10], in this case coherence would make it possible to increase the radiation drastically since the bunch of electrons behaves as an effective charge $e_{\mathrm{eff}} = N_{\mathrm{e}}e$ (N_{e} is the number of electrons in the bunch).

The creation of such coherent bunches is a very complicated problem, even in the microwave range. So far as the possibility of clustering electrons in bunches of the size of the order of the optical wavelength was concerned, the difficulties seemed to be insurmountable.

At present, it is established that in the theory of free electron lasers, which considers the interaction of an electron beam in an undulator with an electromagnetic wave, there is a mechanism of self-modulation of an electron beam. Electrons are clustered in the longitudinal direction and form coherent bunches of length of the order of the optical wavelength.

The mechanism of self-modulation of an electron beam is similar to some extent to the clustering of electrons in a synchrotron under the combined action of the leading magnetic field of the accelerator and the vortical high-frequency field accelerating the particles (the self-modulation principle of Wecksler and McMillan). As a result of their action, the electron beam is divided into bunches the length of which is a function of parameters of the high-frequency electric field.

It is interesting that particles are clustered in the free-electron laser even if an outer electromagnetic wave is absent — the 'trigger' wave of spontaneous radiation plays its role. The self-amplification of spontaneous radiation is one of the important properties of the undulator. When electrons pass through a large-length undulator, there is no interference initially and the total radiation power is proportional to the number of particles $W^{\mathrm{total}} = N_{\mathrm{e}}W$. Then the clustering mechanism comes into play, and the radiation of a cluster of electrons becomes coherent. The spontaneous radiation amplifies itself and the radiation power is now proportional to N_{e}^2 because of the interference: $W^{\mathrm{total}} = N_{\mathrm{e}}^2 W$. This self-amplification phenomenon provides the grounds for a special 'strong' source of radiation — a large-length undulator.

The coherence problems of synchrotron radiation have recently attracted the attention not only of theoreticians but also of experimentalists [111 – 114].

6. Conclusions

I would like to say that synchrotron radiation has not only won recognition in physical experiments of the present time but will still be an experimental tool in the future.

In *The Thousand and One Nights*, there is a story about a boy named Aladdin and a magic lamp. The boy found a magic lamp and, when he rubbed it slightly with a pinch of sand, there appeared an enormous genie and said, ''I am at your disposal, I am your slave''. The electronic light burst out of the chamber of an accelerator in 1947 and since then synchrotron radiation, like the genie in Aladdin's magic lamp, has shown the way to knowledge in various fields of science.

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