A SIMPLE SOLUTION TO THE STRONG *CP* PROBLEM WITH A HARMLESS AXION

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Received 15 May 1981

We describe a simple generalization of the Peccei–Quinn mechanism which eliminates the strong *CP* problem at the cost of a very light, very weakly coupled axion. The mechanism requires no new fermions and is easily implemented in grand unified theories.

Since the discovery of instantons [1], the problem of strong *CP* violation has been a vexing one [2]. In a gauge theory like quantum chromodynamics (QCD), it is possible to add to the lagrangian a term

$$\mathcal{L}_\theta = (\theta g^2/32\pi^2)\, F\tilde{F}\,. \tag{1}$$

Such a term is a pure gradient, and one might argue that it can have no physical consequences. The presence of instantons, however, shows that such a term can have physical effects [2,3]. In particular, it will induce *CP* violation. QCD estimates [4], and the experimental limits on the neutron electric dipole moment [5] suggest $\theta < 10^{-8}$. It would be very surprising if this small value of θ was simply an accident.

Under certain circumstances, however, θ is an unobservable parameter and strong interactions automatically conserve *CP*. In particular, this is the case if the classical lagrangian possesses a U(1) symmetry which is explicitly broken by color anomalies. Several mechanisms for obtaining such a U(1) symmetry have been proposed. One, setting $M_u = 0$ [2], does not seem consistent with successful current algebra predictions. A second, proposed by Peccei and Quinn [6], requires that at least two scalar doublets be included in the Weinberg–Salam model. The chief difficulty with the Peccei–Quinn scheme is that it predicts a light pseudoscalar particle called the axion [7]. The properties of the axion can be determined with some confidence [7,8], and no such particle has been observed.

In this note we describe a simple generalization of the Peccei–Quinn scheme with a harmless axion. Other generalizations of the Peccei–Quinn scheme have been proposed; most seek to increase the mass of the axion [9] [‡1]. Kim [10], however, has proposed a mechanism for decreasing the mass of the axion while simultaneously decreasing the axion's couplings to ordinary matter. His model requires the existence of both additional quarks and additional scalars, all of which are neutral with respect to electroweak interactions.

In this paper, we discuss a model in which the axion is also very light and very weakly coupled to ordinary

[1] Research supported by the Department of Energy under Grant #DE-AC02-76ER02220.
[2] Research supported by the Department of Energy under Grant #DE-AC02-76ER03071.
[3] Research supported by the National Science Foundation under Grant #PHY80-19754.

[‡1] For models with heavy axions, see for example, ref. [9]. Note that in the model proposed by Dimopoulos, the Fermi constant comes out too small.

matter. Our model is quite economical. It requires the addition of just one scalar field to the Peccei–Quinn model. Moreover, this scheme is easily implemented in grand unified theories. Our model was suggested by a study of supersymmetric versions of technicolor [11], but it may be possible to implement the mechanism in other dynamical symmetry breaking schemes, such as extended technicolor models.

The model is identical to that of Peccei and Quinn, except for the addition of a complex scalar field, ϕ, which is a singlet under SU(2) × U(1). The model has two scalar doublets, ϕ_u and ϕ_d, with hypercharge −1 and +1, respectively. ϕ_u couples only to right-handed charge 2/3 quarks, ϕ_d only to right-handed charge −1/3 quarks and to right-handed charged leptons. Thus the Yukawa couplings have the structure

$$\mathcal{L}_Y = G_u(\bar{u}\bar{d})_L \phi_u u_R + G_d(\bar{u}\bar{d})_L \phi_d d_R + \text{h.c.}\,, \qquad (2)$$

and similarly for other quarks and leptons. The restriction of the couplings to this form is required by the symmetry we describe below.

We will demand that the lagrangian possesses a global symmetry at the classical level under which the scalar fields transform as

$$\phi_u \to \exp(i\alpha X_u)\phi_u\,, \quad \phi_d \to \exp(i\alpha X_d)\phi_d\,,$$
$$\phi \to \exp(i\alpha X_\phi)\phi\,, \qquad (3)$$

where

$$X_u + X_d = -2X_\phi = 1\,. \qquad (4)$$

The most general scalar potential consistent with this symmetry as well as the SU(2) × U(1) symmetry of electroweak interactions is

$$\begin{aligned} V(\phi, \phi_u, \phi_d) &= \lambda_u(|\phi_u|^2 - V_u^2)^2 + \lambda_d(|\phi_d|^2 - V_d^2)^2 \\ &+ \lambda(|\phi|^2 - V^2)^2 + (a|\phi_u|^2 + b|\phi_d|^2)|\phi|^2 \qquad (5) \\ &+ c(\phi_u^i \epsilon_{ij} \phi_d^j \phi^2 + \text{h.c.}) + d|\phi_u^i \epsilon_{ij} \phi_d^j|^2 + e|\phi_u^* \phi_d|^2\,. \end{aligned}$$

Here ϵ_{ij} is the completely antisymmetric symbol of SU(2).

For a finite range of values of the parameters in the potential, the desired symmetry breakdown occurs. In particular, we demand

$$\langle\phi_u\rangle = 2^{-1/2}\begin{pmatrix} f_u \\ 0 \end{pmatrix}, \quad \langle\phi_d\rangle = 2^{-1/2}\begin{pmatrix} 0 \\ f_d \end{pmatrix}, \qquad (6)$$

giving mass to the W and Z bosons and quarks and leptons. Note that, since only doublets are used, one has, at tree level, the important relation

$$M_W/M_Z = \cos\theta_W\,. \qquad (7)$$

Moreover, this structure insures the absence of dangerous strangeness-changing neutral currents.

We also require that the potential give ϕ a large vacuum expectation value,

$$\sqrt{2}\langle\phi\rangle = f_\phi \gg (f_u^2 + f_d^2)^{1/2} \equiv f\,. \qquad (8)$$

These expectation values, in addition to providing the breaking of weak isospin, also break the X-symmetry of eq. (1). This symmetry is anomalous, however, and the corresponding Goldstone boson gets a small mass due to instanton effects [more precisely, through those QCD effects which break the U(1) symmetry]. We can study the properties of this particle, which we refer to as the axion, using standard current algebraic techniques. In particular, the methods used by Bardeen and Tye [8] to study the axion of the Peccei–Quinn model allow us to determine the axion's mass, lifetime, and couplings.

Before instanton effects are considered (more generally, QCD effects which violate X-symmetry), the model possesses two conserved U(1) currents, the hypercharge current and the X-current. Writing the fields as

$$\begin{aligned} \phi_u &= 2^{-1/2}\begin{pmatrix} f_u + \eta^u + i\xi_1^u \\ \xi_2^u + i\xi_3^u \end{pmatrix}, \\ \phi &= 2^{-1/2}\begin{pmatrix} \xi_2^d + i\xi_3^d \\ f_d + \eta^d + i\xi_1^d \end{pmatrix}, \qquad (9) \\ \phi &= 2^{-1/2}(f_\phi + \eta^\phi + i\xi^\phi)\,, \end{aligned}$$

the field eaten by the Z boson is

$$\phi^Y = (f_u\xi_1^u - f_d\xi_1^d)/f\,. \qquad (10)$$

The X-current is given by

$$\begin{aligned} j_\mu^X &= X_\phi \phi^\dagger \overleftrightarrow{\partial}_\mu \phi + X_u \phi_u^\dagger \overleftrightarrow{\partial}_\mu \phi_u + X_d \phi_d^\dagger \overleftrightarrow{\partial}_\mu \phi_d \\ &+ X_u(\bar{u}\gamma_\mu\gamma_5 u + \bar{c}\gamma_\mu\gamma_5 c + \bar{t}\gamma_\mu\gamma_5 t + \ldots) \\ &+ X_d(\bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s + \bar{b}\gamma_\mu\gamma_5 b + \ldots) \qquad (11) \\ &+ X_d(\bar{e}\gamma_\mu\gamma_5 e + \bar{\mu}\gamma_\mu\gamma_5\mu + \bar{\tau}\gamma_\mu\gamma_5\tau + \ldots)\,. \end{aligned}$$

In order to isolate the axion and study its properties it is convenient to choose X_u and X_d so that j_μ^X does not couple to ϕ^Y, i.e.

$$\langle 0|j_\mu^X|\phi^Y\rangle = 0 . \tag{12}$$

This fixes

$$X_u = f_d^2/f^2 , \quad X_d = f_u^2/f^2 . \tag{13}$$

At the tree level, this current creates a massless particle, the axion. The axion field is easily determined to be

$$\begin{aligned} A = &[2f_u f_d(f_u \xi^d + f_d \xi^u) - f^2 f_\phi \xi^\phi] \\ &\times [f(f^2 f_\phi^2 + 4f_u^2 f_d^2)^{1/2}]^{-1} . \end{aligned} \tag{14}$$

Note that in the limit $f_\phi \gg f_u, f_d$,

$$A \simeq -\xi^\phi + (2f_u f_d/f_\phi f^2)(f_u \xi^\phi + f_d \xi^u) , \tag{15}$$

i.e. the axion is primarily composed of ϕ field. The axion decay constant, f_A, is defined by

$$\langle 0|j_\mu^X|A\rangle = f_A q_\mu . \tag{16}$$

Recalling that $X_\phi = -1/2$ [eq. (4)], and using eqs. (11), (13) and (14) one finds

$$f_A = (2f)^{-1}(4f_u^2 f_d^2 + f^2 f_\phi^2)^{1/2} . \tag{17}$$

In the limit that f_ϕ is very large

$$f_A = \tfrac{1}{2} f_\mu + O(f^2/f_\phi) . \tag{18}$$

The current j_μ^X is anomalous [12],

$$\partial^\mu j_\mu^X = N(g^2/32\pi^2) F_{\mu\nu}{}^a \tilde{F}_{\mu\nu}{}^a , \tag{19}$$

where N is the number of quark doublets and $F_{\mu\nu}{}^a$ is the color field strength tensor. This anomaly will lead to a mass for the axion. In order to calculate this mass, we follow Bardeen and Tye and define an anomaly-free, almost-conserved current. We do this by adding to the current j_μ^X a piece involving the light quarks (which we take to be u and d; including the strange quark would only change the results slightly). Specifically, we study the current

$$\tilde{J}_\mu^X = j_\mu^X - N\{(1+Z)^{-1}\bar{u}\gamma_\mu\gamma_5 u + [Z/(1+Z)]\,\bar{d}\gamma_\mu\gamma_5 d\} . \tag{20}$$

where Z will be defined below. This current is obviously anomaly-free, and its divergence is proportional to light quark masses. The axion mass may now be calculated in terms of the pion mass using Dashen's theorem [13] and the fact that A_μ^3 (the usual axial τ_3 current) is almost conserved. If we demand

$$\langle 0|[\tilde{Q}_X, [Q_3^5, \mathcal{H}]]|0\rangle = 0 , \tag{21}$$

where

$$\tilde{Q}_X = \int d^3x\, \tilde{J}_0^X(x) , \tag{22}$$

$$Q_3^5 = \int d^3x\, A_0^3(x) , \tag{23}$$

and $\mathcal{H}$ is the hamiltonian density, Dashen's formula gives

$$m_A^2 f_A^2 = \langle 0[\tilde{Q}^X, [\tilde{Q}^X, \mathcal{H}]]|0\rangle . \tag{24}$$

It is a straightforward matter to evaluate the commutators in these equations. One finds that eq. (19) implies (value of m_u/m_d from ref. [14])

$$Z \simeq m_u/m_d \simeq 0.56 . \tag{25}$$

The axion mass may finally be determined by computing

$$m_\pi^2 f_\pi^2 = \langle 0|[Q_5^3, [Q_5^3, \mathcal{H}]]|0\rangle , \tag{26}$$

and comparing with eq. (24).

One finds

$$m_A^2 = (f_\pi^2/f_A^2) m_\pi^2 N^2 Z(1+Z)^{-2} , \tag{27}$$

or

$$m_A \simeq [74\ \mathrm{KeV}\ (250\ \mathrm{GeV}/f_A)] \tag{28}$$

(where we have taken $N = 3$).

The axion lifetime can be computed through the two-photon anomaly, analogous to the calculation for the π^0. One obtains

$$\begin{aligned} \tau(A \to 2\gamma) &= \tau(\pi^0 \to 2\gamma)(m_\pi/m_a)^5 N^4 Z^3/(1+Z)^4 \\ &= 41\ \mathrm{s} \times (f_A/250\ \mathrm{GeV})^5 . \end{aligned} \tag{29}$$

A crucial feature of this axion is the strength of its couplings to ordinary matter. These are suppressed by a factor

$$r = f/f_A , \tag{30}$$

relative to those of the axion of the Peccei–Quinn model. Thus production of axions in any process (hadronic collisions, nuclear decays, etc.) is reduced by a factor r^2; the probability of subsequent detection of these axions is reduced by the same factor. In fact, if r is greater than about 10, no terrestrial experiment which is currently used to set limits on axions could have observed ours [15].

An axion of this type might have cosmological or astrophysical implications. If the axion decays after recombination time and before the present era, i.e. if

$$10^5\,\mathrm{yr} \lesssim \tau \lesssim 10^{10}\,\mathrm{yr}\,, \tag{31}$$

then axion decays might give an observable distortion of the cosmic microwave radiation background. A more stringent restriction arises from the existence of red giant stars [16]. Unless the axion is *extraordinarily* weakly coupled, it will be copiously produced in these stars (and for that matter, the sun), and rapidly carry off all of their thermal energy. The authors of ref. [16] find a limit

$$m_{\mathrm{A}} \lesssim 0.01\ \mathrm{eV}\,. \tag{32}$$

This corresponds to

$$\langle\phi\rangle > 10^9\ \mathrm{GeV}\,. \tag{33}$$

Such a huge vacuum expectation value might be expected in grand unified models [17]. For example, if one constructs an SU(5) model with two 5's of Higgs and a scalar singlet, one can implement our scheme for strong *CP* conservation explicitly. The natural scale for $\langle\phi\rangle$ is then 10^{15} GeV. In fact, our mechanism suggests that the *CP* problem may not be a problem of weak-interaction physics at all; rather it may be resolved only by physics at extremely short distances. The only requirement is the existence of an anomalous U(1) symmetry broken at a large energy scale.

References

[1] A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Yu. S. Tyupkin, Phys. Lett. 59B (1975) 85.
[2] G. 't Hooft, Phys. Rev. Lett. 37 (1976) 172.
[3] R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172; C.G. Callan, R.F. Dashen and D.J. Gross, Phys. Lett. 63B (1976) 334.
[4] V. Baluni, Phys. Rev. D19 (1979) 227; R. Crewther, P. DiVecchia, G. Veneziano and E. Witten, Phys. Lett. 88B (1979) 123.
[5] N.F. Ramsey, Phys. Rep. 43C (1978) 409.
[6] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791.
[7] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. lett. 40 (1978) 279.
[8] W.A. Bardeen and S.-H.H. Tye, Phys. Lett. 74B (1978) 229.
[9] S. Treiman and F. Wilczek, Phys. Lett. 74B (1978) 381; S.-H.H. Tye, Cornell preprint CLNS-81/479 (1981); S. Dimopoulos, Phys. Lett. 84B (1979) 435.
[10] J.E. Kim, Phys. Rev. Lett. 43 (1979) 103; see also: M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166 (1980) 493.
[11] M. Dine, W. Fischler and M. Srednicki, Princeton Univ. preprint.
[12] S. Adler, Phys. Rev. 177 (1969) 2426; J.S. Bell and R. Jackiw, Nuovo Cimento 60A (1960) 49; W.A. Bardeen, Phys. Rev. 184 (1969) 1848.
[13] R. Dashen, Phys. Rev. 183 (1969) 1245.
[14] S. Weinberg, Harvard Univ. preprint HUTP-77, A057.
[15] T.W. Donnelly, S.J. Freedman, R.S. Lytel, R.D. Peccei and M. Schwartz, Phys. Rev. D18 (1978) 1607.
[16] D.A. Dicus, E.W. Kolb, V.L. Teplitz and R.V. Wagoner, Phys. Rev. D18 (1978) 1829; Phys. Rev. D22 (1980) 839.
[17] H. Georgi and S. Glashow, Phys. Rev. Lett 32 (1974) 438.