AI – Aided KFs

Evaluation

Models

- Israel Group – Revach (ETHZ), Shlezinger (BGU), Eldar (Weizmann)
  - KalmanNet (IEEE TSP 2022)
  - RTSNet (IEEE TSP 2023)

- KTH Group
  - DANSE: Data-Driven Non-Linear State Estimation of Model-Free Process in Unsupervised Learning Setup
    IEEE TSP 2024

- North-Eastern Group
  - Hybrid Neural Network Augmented Physics-based Models for Nonlinear Filtering
    International Conference on Information Fusion (2022)

Lorentz Attractor

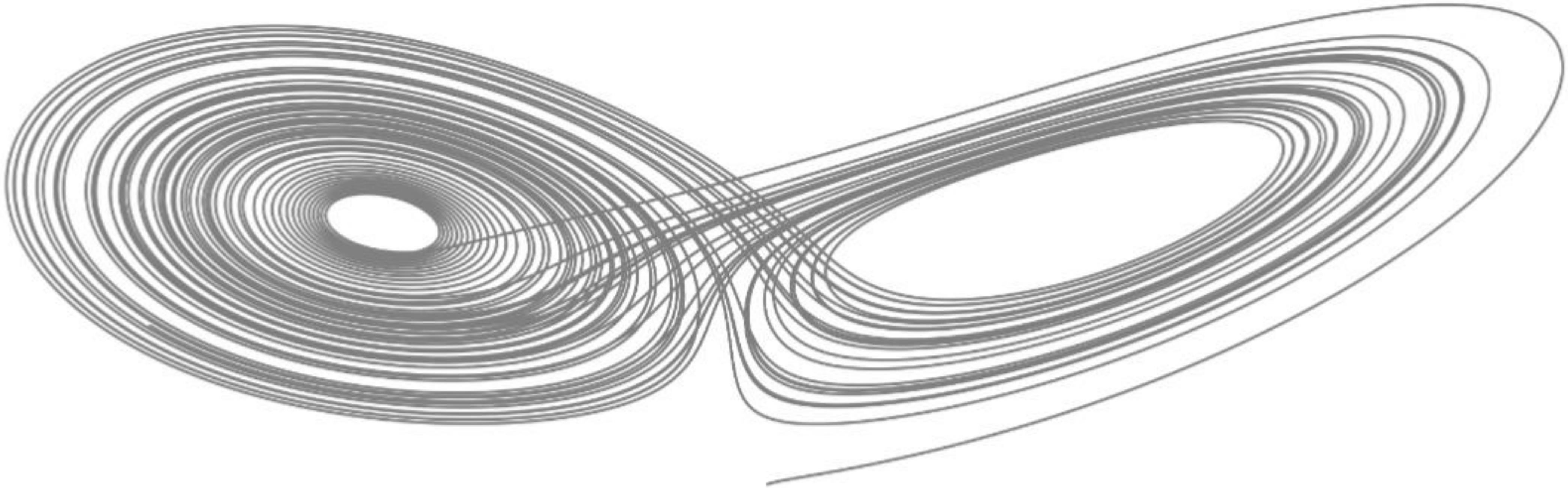

Figure 2.2: Lorenz attractor generated with $\Delta\tau = 10^{-5}$and $J = 5$.

Differential Equation

A general form of a deterministic differential equation is

$$\dot{\mathbf{x}}_\tau = \frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{x}_\tau = \mathbf{A} \cdot \mathbf{x}_\tau \tag{2.4}$$

Solution

The solution of (2.4), can be used to characterize a linear state evolution in a discrete-time interval $\Delta\tau$. More specifically, the extrapolation of $\mathbf{x}$ in from time τ to time $\tau + \Delta\tau$ is given by:

$$\mathbf{x}_{\tau+\Delta\tau} = \mathbf{F}_{\Delta\tau} \cdot \mathbf{x}_{\tau}, \quad \mathbf{F}_{\Delta\tau} = \exp\left(\mathbf{A} \cdot \Delta\tau\right). \tag{2.5}$$

Here, $\mathbf{F}_{\Delta\tau}$ is th evolution matrix that is given as an infinite *Taylor* expansion, namely

$$\exp\left(\mathbf{A} \cdot \Delta\tau\right) \triangleq \sum_{k=0}^{J} \mathbf{A} \cdot \frac{\Delta\tau^{k}}{k!}, \quad J \to \infty. \tag{2.6}$$

While there are cases, i.e., forms of A matrix, in which the evolution can be accurately described as a finite *Taylor* expansion, it does not hold in general. Therefore in these cases, one should use a finite approximation with sufficiently large J.

Lorentz System

The continuous-time state vector $\mathbf{x}_\tau$ captures the chaotic movement of a single particle through three-dimensional space.

Here, the continuous-time state vector $\mathbf{x}_\tau$ describes the chaotic movement of a single particle in a three-dimensional space. The dynamic forces governing the particle's chaotic motion are encapsulated by the system dynamics matrix

$$\mathbf{A}\left(\mathbf{x}_\tau\right) = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & -\mathrm{x}_{1,\tau} \\ 0 & \mathrm{x}_{1,\tau} & -\frac{8}{3} \end{pmatrix}. \tag{2.17}$$

This matrix's nonlinearity is underscored by its state dependence, particularly on $\mathrm{x}_{1,\tau}$, showcasing the system's complexity.

Sampling Mismatch – Testing Data

- 10 Trajectories are generated from the decimated data
- Observation noise $\mathrm{r}^2 = 0[\mathrm{dB}]$.
- Files:
  - [RTSNet_TSP/Simulations/Lorenz_Atractor/data/decimation at master · KalmanNet/RTSNet_TSP · GitHub]
  - [RTSNet_TSP/main_lor_decimation.py at master · KalmanNet/RTSNet_TSP · GitHub]

Results (RTSNet TSP)

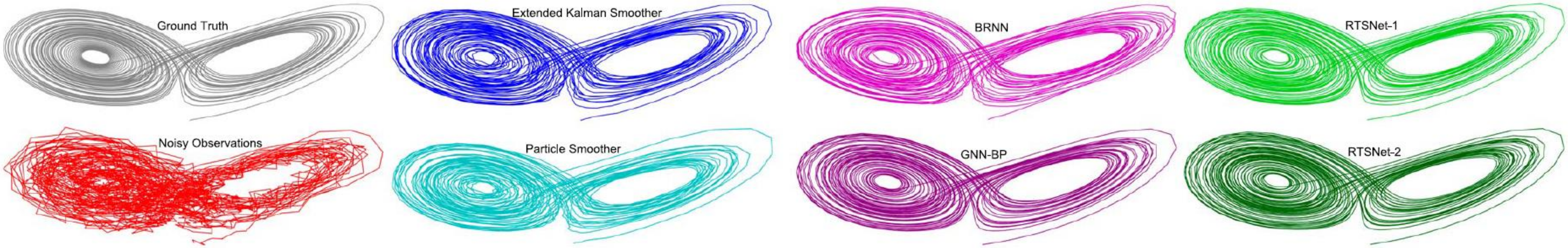

Fig. 8. Lorenz attractor with sampling mismatch (decimation), $T = 3000$.

TABLE VIII
MSE[dB] - LORENZ ATTRACTOR WITH SAMPLING MISMATCH

Noise	Extended KF (EKF)	PF	KalmanNet	EKS	PS	BRNN	GNN-BP	RTSNet-1	RTSNet-2
-0.024 $\pm$ 0.049	-6.316 $\pm$ 0.135	-5.333 $\pm$ 0.136	-11.106 $\pm$ 0.224	-10.075 $\pm$ 0.191	-7.222 $\pm$ 0.202	-2.342 $\pm$ 0.092	-16.479 $\pm$ 0.352	-15.436 $\pm$ 0.329	-16.803 $\pm$ 0.301