# Using Wavelets to Analyse Heart Beat Patterns in an ECG and Compression of ECG Data

Group 1 | Application Assignment | EE 338 Autumn 2020

This is the application assignment for the course EE 338 Digital Signal Processing taken by Prof.Vikram Gadre for Autumn 2020. In this project we have demonstrated the use of the wavelet transform to analyse Heart Beat Patterns in an ECG and compress the ECG data for smaller storage requirement.

Group Members

Group 1

- Amol Girish Shah | 18D070005
- Mantri Krishna Sri Ipsit | 180070032
- Shlok Vaibhav Singh | 18D070064

# Table of Contents


I Part One

1 Abstract . . . . . 7

1.1 Application of DSP concepts and contribution to Aatmanirbhar Bharat 8

2 Getting started . . . . . 9

2.1 Short introduction to ECG Waves 9

2.2 Overview of wavelets 9

2.3 Selecting wavelet for ECG dilineation 11

2.4 Filter bank design for ECG dilineation 11

II Part Two

3 QRS Complex detection . . . . . 16

3.1 R peak detection 16

3.2 QRS Dilineation 18

3.3 Results 19

4 T Wave Detection and Delineation . . . . . 20

4.1 Overview 20

4.2 Multiresolution Analysis 20

4.3 T Wave Detection 21

**4.4 T Wave Delineation** 21

**4.5 Results** 21

5 P Wave Detection and Delineation . . . . . 23

**5.1 Overview** 23

**5.2 Multiresolution Analysis** 23

**5.3 P Wave Detection** 23

**5.4 P Wave Delineation** 24

**5.5 Results** 24

III Part Three

6 Compression of ECG . . . . . 28

**6.1 Overview** 28

**6.2 Implementation** 29

6.2.1 Periodic Segmentation . . . . . 29

6.2.2 Period Normalization and Matrix Conversion . . . . . 29

6.2.3 SVD Transformation for Signal Decomposition . . . . . 30

6.2.4 Data compression . . . . . 31

6.2.5 Decompression . . . . . 31

**6.3 Results** 32

IV Appendix

A Source Code . . . . . 36

B About Databases . . . . . 37

Bibliography . . . . . 38

**References** 38

# Part One


1 Abstract . . . . . 7

1.1 Application of DSP concepts and contribution to Aatmanirbhar Bharat

2 Getting started . . . . . 9

2.1 Short introduction to ECG Waves

2.2 Overview of wavelets

2.3 Selecting wavelet for ECG dilineation

2.4 Filter bank design for ECG dilineation

# 1. Abstract

The analysis of the ECG is a prominent problem in medical science, Most of the useful information in the ECG is found in the intervals and amplitudes defined by its significant points as shown here:

The development of accurate and robust methods for automatic ECG delineation is a subject of major importance, especially for the analysis of long recordings. As a matter of fact, QRS detection is necessary to determine the heart rate, and as reference for beat alignment. ECG wave delineation provides fundamental features (amplitudes and intervals) to be used in subsequent automatic analysis. [2].

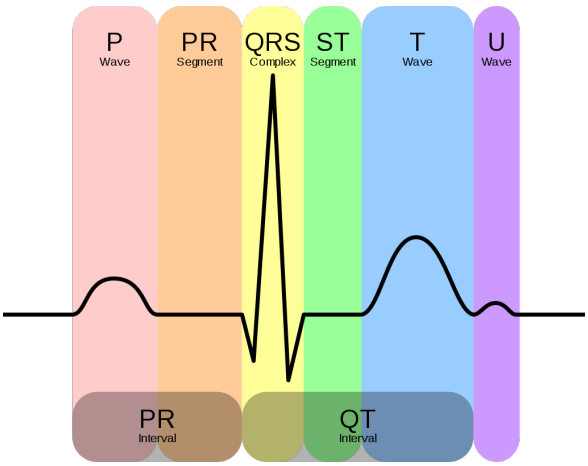



Figure 1.1: ECG intervals

We use wavelets and filter-banks to perform the required dilineation as proposed in [2] and [1] to determine the onset and end of P, T and QRS intervals and determination of R-peak. We also demonstrate the compression of ECG waves for efficient storage as suggested in [3].

In this assignment we have mainly explored and implemented three research papers - [1] “Detection of ECG characteristic points using wavelet transforms”

[2] “A Wavelet-Based ECG Delineator: Evaluation on Standard Databases”.

[3] “ECG Data Compression Using Truncated Singular Value Decomposition”.

We have mostly stayed on track of our abstract submitted on 23/10/2020

## 1.1 Application of DSP concepts and contribution to Aatmanirbhar Bharat

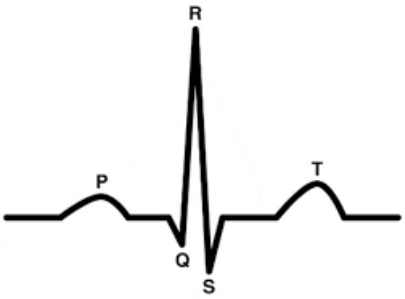
In this project we explored various Digital Signal Processing concepts like **Wavelet Transform, filter bank design, resampling, upsampling, downsampling, discrete transformations, compression of discrete quasi-periodic ECG signal, reconstruction of signals, etc** Thus we covered many parts of DSP and applied the knowledge gained throughput the semester to bear this project to fruition. We feel that this application assignment was a grand celebration of various concepts we learned in addition to us gaining valuable experience in actually implementing various research papers ourselves.

We specifically chosen this topic since we observed that there is no major Indian manufacturer who produces ECG detection equipment and most equipment is from foreign companies whose costs are thus much higher than if it were made in India. Nowadays we are seeing that heart problems are on the rise and thus to make the cardiac tests affordable for the common man is it essential to get reliable and cheaper equipment. **With this application assignment we wish to contribute towards the formation of ECG equipment which is made in India**. For this purpose, the ECG machine needs to have functions like automatic QRS complex detectors, PT wave detector, heart rate detector which we have implemented in this assignment. Many-a-times patients need to be monitored at home or in the hospital for 24 hours or over a span of many days in which the ECG data collected requires large amount of memory. With the compression technique that we implemented and is described here, smaller and more affordable heart monitoring devices can be made.

# 2. Getting started

## 2.1 Short introduction to ECG Waves

An electrocardiogram(ECG) is a tracing of the electrical activity that is taking place within the heart. Under normal circumstances, an electrical impulse will travel in various sections the heart. This electrical impulse causes the four chambers of the heart to contract and relax in a coordinated fashion. Studying these electrical impulses allows us to understand how the heart is functioning. Many heart diseases are detected by analysing the shape and structure of the ECG. We observe the following import components of the ECG waves - QRS complex, P and T waves which are used by cardiologists to determine any regularities in the functioning of the heart.




## 2.2 Overview of wavelets

Fourier Transform is a well-known transform studied in undergraduate years. However there are certain demerits of this, the first one being that the basis functions, i.e. $\exp(j\omega t)$ are not at all localized in space while highly localized in frequency domain. Second is the inability of transform to handle discontintuities in data, it is well known that around discontinuity, a large number of fourier coefficients are needed for synthesis, this problem is solved in JPEG standard by use of DCT.

A different approach to analysis was first proposed by Alfred Haar, namely the use of haar transform, where we use piecewise constant basis functions for analysis and synthesis. Further research in last 40

years has matured the field of wavelets, the most celebrated wavelets being those of Daubechies family of wavelets.

The wavelet approach is also a multi-resolution analysis (MRA) since the basis functions extract signal contents at various levels of details (by level, we mean frequency componenets). The wavelet based MRA can be obtained by designing a filter bank (a collection of related filters) as follows:

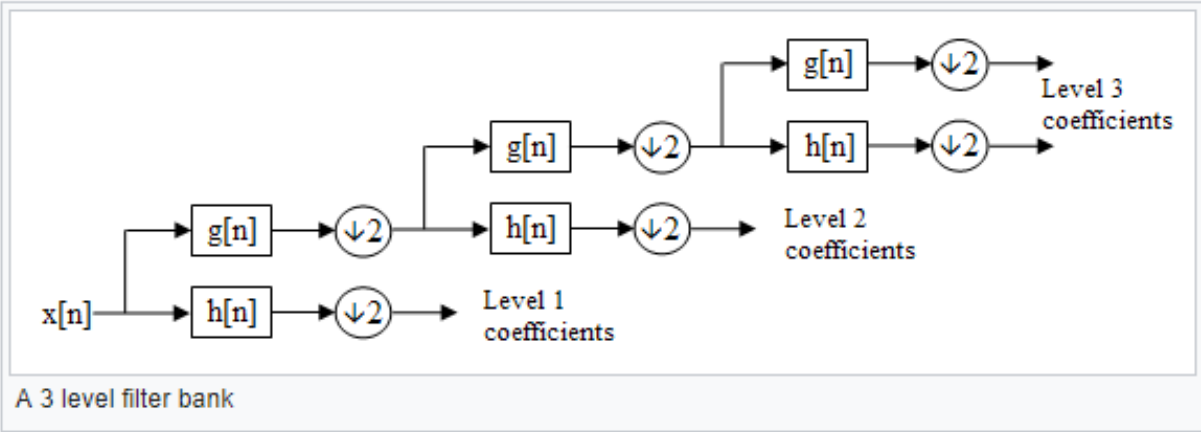



Figure 2.1: Analysis Filter bank

There is also a synthesis filter bank to reconstruct the analyzed signal from the coefficients of various levels, however, in our ECG dilineation, reconstruction is not needed so we will focus only on the analysis filter bank. Formally, The wavelet transform (WT) is a decomposition of the signal as a combination of a set of basis functions, obtained by means of dilation (a) and translation (b) of a single prototype wavelet, $\psi(t)$ [2]. If the time domain signal is $x(t)$, we define WT as:

$$W_a x(b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi(\frac{t-b}{a}) dt, \quad a > 0 \tag{2.1}$$

As we increase dilation by increasing a, the WT gives information about lower and lower frequency components.

As shown here, a and b are continuouos however in practice they are not, it is sufficient to choose a as a discrete set of form : $\{2^n : n \in Z\}$ leading us to the filter bank in figure 2.1 if we choose n as positive integer only. A typical procedure for making this filter bank is to choose a scaling function, by expressing this scaling function as a linear combination of its dilates (scaling function squeezed in time by a factor of 2) and translates (squeezed scaling function shifted in time).

## 2.3 Selecting wavelet for ECG dilineation

In our assignment, inspired by [2], we have chosen the quadratic spline wavelet as scaling function, $\psi(t)$, the fourier transform of this function is :

$$\Psi(\Omega) = j\Omega\left(\frac{\sin(\frac{\Omega}{4})}{\frac{\Omega}{4}}\right)^4 \tag{2.2}$$

It is called quadratic spline since in time-domain, the basis functions are piecewise quadratic functions. As explained in [2] and [1], the WT corresponding to this prototype has a maxima whenever $x(t)$ has a large slope and it is zero whenveer $x(t)$ is a maxima, making this WT a good tool for finding maxima and max slopes of $x(t)$.

## 2.4 Filter bank design for ECG dilineation

Finally, we proceed to filter bank design. From [2], the FIR impulse responses for the first level filters, H(z) and G(z) are

$$h[n] = \frac{1}{8}[\delta[n+2]+3\delta[n+1]+3\delta[n]+\delta[n-1]] \tag{2.3}$$

$$g[n] = 2[\delta[n+1]-\delta[n]] \tag{2.4}$$

To obtain the equivalent filter impulse repsonse for higher levels without decimation, we use algorithm a trous as shown above , essentially, removing downsampling. Thus our filter bank is a collection $Q_k(e^{j\omega}) =:$

$$\begin{cases} G(e^{j\omega}) & k=1 \\ G(e^{j2^{k-1}\omega})\Pi_{l=0}^{k-2}H(e^{j2^l\omega}) & k\geq 2 \end{cases} \tag{2.5}$$

We now generate this collection of filters using Matlab and show the obtained frequency domain repsonses at Sampling frequency of 250Hz:

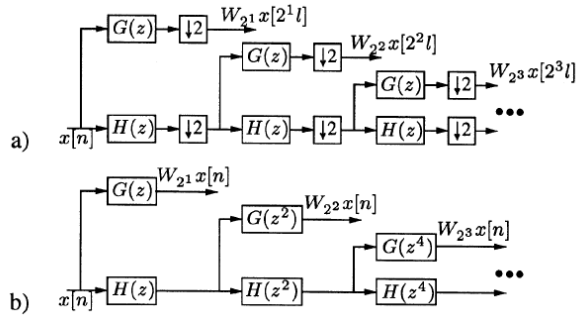



Figure 2.2: Filter bank using a) Mallat's algorithm b) Algortihm a trous

The databases can contain ecg signals that have been sampled at frequencies other than 250Hz, in this case it is imperative that we transfer these filter characteristics to other frequecies (higher than 250Hz), use a resampling technique described in oppenheim section 6.4 in chapter titled on sampling of continuous time signal. Suppose we have to obtain response ar 360Hz, then first we upsample the filter repsonse by 3, then we apply a Low-pass filter on reponses with cutoff at $\pi/3$, giving us the filter repsonses at 750Hz, then we downsample the resulting impulse response by a factor of 2, thus getting the transferred characteristics at 375Hz, quite close to 360Hz. For designing the lowpsass filter, we directly used the Kaiser window based approach directly built-in in Matlab.

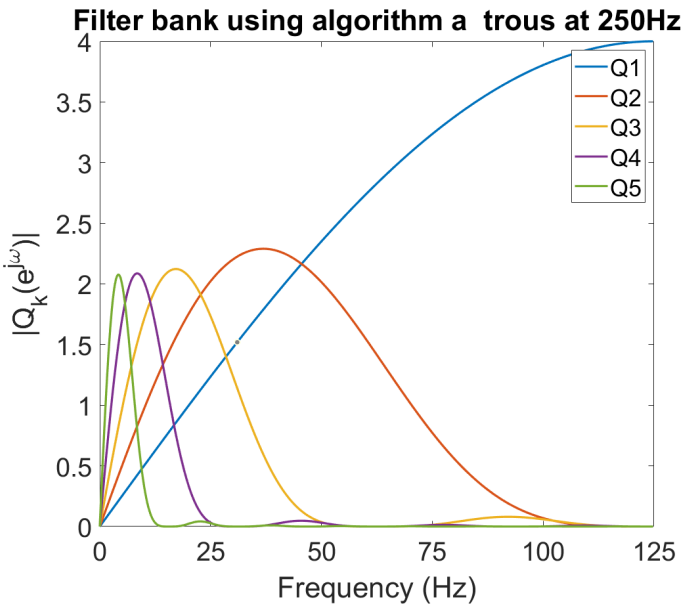
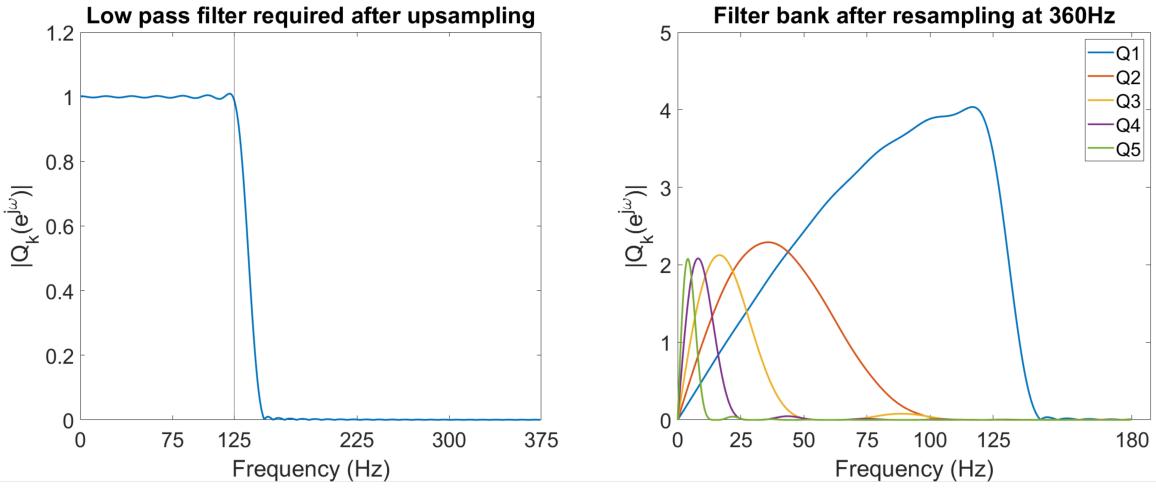



Figure 2.3: Responses of DWT filters at scales k=1,2,3,4,5 at sampling of 250Hz

The low pass filter used and the new filter reponses are shown below:




a) Low pass filter required after upsampling

b) Filter bank at 360Hz

Figure 2.4:

We can see the repsonses before 125Hz have been transferred, only that they have been slightly downscaled to 360Hz from 375Hz, however we do not expect it to have much impact on analysis

Now we demonstrate via an example a representative filter-bank output. Shown below is the output of first four filter banks, we see that as claimed, the high frequency compenents decrease as we go down while lower frequency content increases.

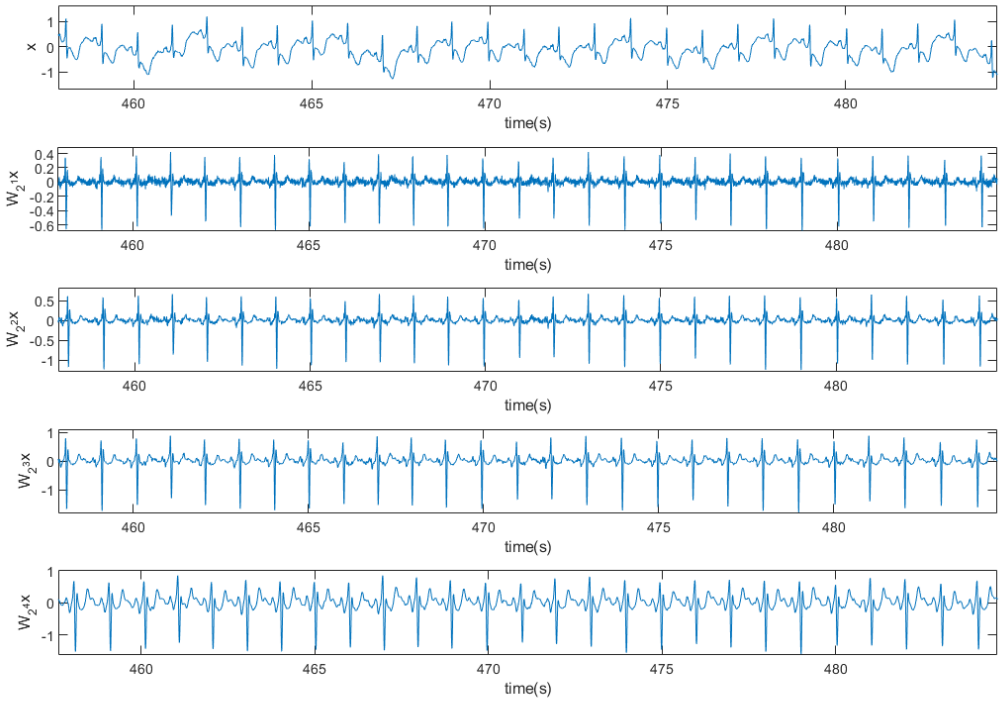



Figure 2.5: A sample signal and the output of first 4 filter banks
Notice as we take successive outputs, the high frequency variations decrease while the effect of low frequency components becomes significant.

# Part Two


**3 QRS Complex detection . . . . . 16**

3.1 R peak detection

3.2 QRS Dilineation

3.3 Results

**4 T Wave Detection and Delineation . . . . 20**

4.1 Overview

4.2 Multiresolution Analysis

4.3 T Wave Detection

4.4 T Wave Delineation

4.5 Results

**5 P Wave Detection and Delineation . . . 23**

5.1 Overview

5.2 Multiresolution Analysis

5.3 P Wave Detection

5.4 P Wave Delineation

5.5 Results

# 3. QRS Complex detection

Detection of R-peaks and QRS complex onset and end is the first and most important step towards dilineation of ECG wave. Essentially, first an R peak is found and then the search for other segments is carried around the detected R waves.

## 3.1 R peak detection

The algorithm for detection of R-peaks is described in detail in [1]. We briefly describe our approach: The logic behind starting the detection from fourth filter output is that it was observed that this output is much more immune to noise than the upper outputs, we cannot also go to fifth or higher scales since the effect of baseline wandering becomes significant there. Since we are rejecting noisy artifactual peaks in fourth scale, the only major source of error can be missing for peaks, i.e. when there is a peak, but it is smaller is amplitude and rejected due to thresholding. We thus expect our predictivity to be very high and sensitivity to be high.

**Algorithm 1** Pseudo-code for R-peaks detection

1: Define thresholds for each filter output, $e_1, e_2, e_3, e_4$
2: Determine the set of samples, $\{n_4\}$ for which $|W_{2^4}x[n_4]| > e_4$
3: Define a neighbourhood of surrounding 11 samples around each $n \in \{n_4\}$
4: Choose the largest sample in $W_{2^3}x$ above threshold $e_3$ in neighbourhood and push it in set $\{n_3\}$, if no such sample, then push a zero to $\{n_3\}, \{n_2\}$ and $\{n_1\}$
5: Repeat step 3 and 4 for second and first filter outputs
6: Assign a peak to all non-zero $n \in \{n1\}$ ▷ Threshold values were used as given in [2] computed over 2000 nearest samples

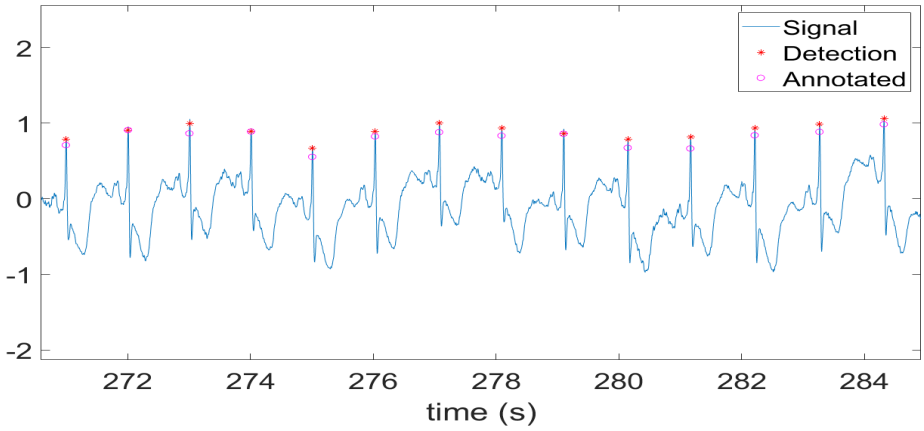



Figure 3.1: R-peak detection

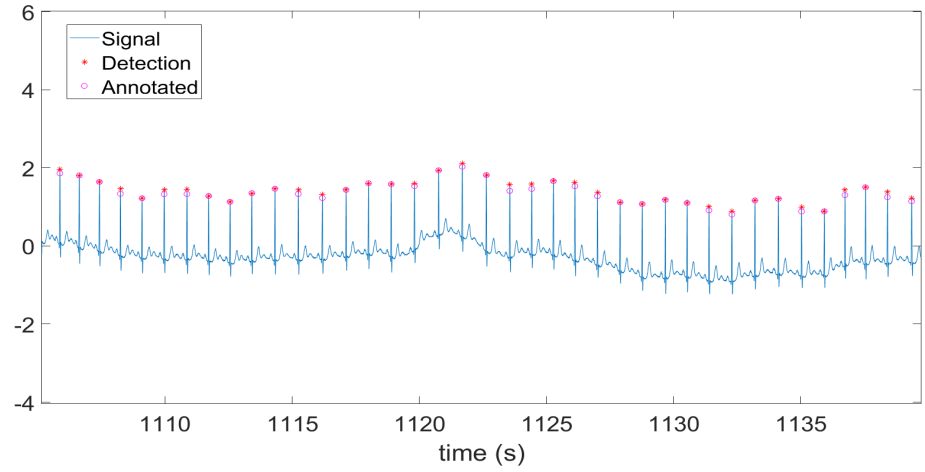



Figure 3.2: R-peak detection in presence of baseline wandering

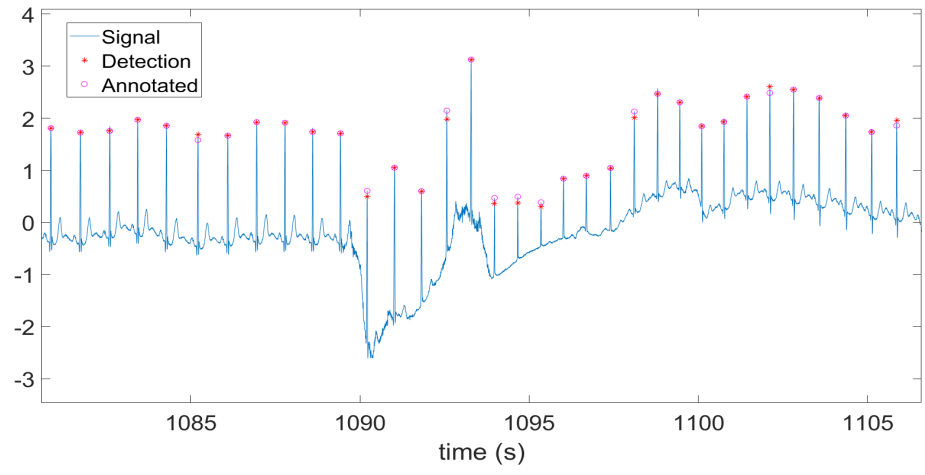



Figure 3.3: R-peak detection in presence of motion artifact

## 3.2 QRS Dilineation

After R peak detection has been done, we move to Q onset and S end detection. The algorithm we have followed is the same as prescribed in [2]. We first find two maxima of $|W_{2^2}x|$ on both sides of the R peaks marked by our detector from previous stage.

Let these peaks be $n_1, n_2, n_3, n_4 : n_1 < n_2 <$ Location of R $< n_3 < n_4$, then we look before $n_1$ and look after $n_4$ to find the first sample below certain thresholds, these samples are then assigned as Q-onset and S-end. Thus the QRS detection is compelete. **caution**: In the following plots, the onset and end provided along databases were for QRS, T and P waves, thus it is not possible to plot only annotations belonging to QRS onset and end, the annotations nearest to the R peak should be considered as the onset or end

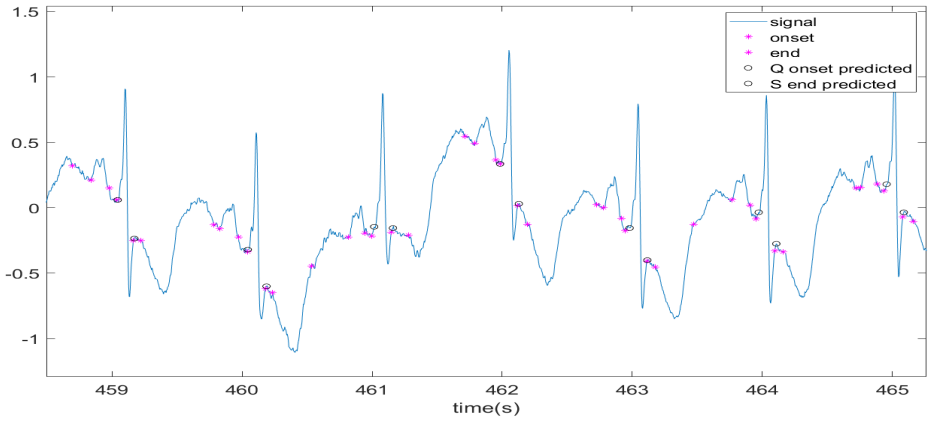



Figure 3.4: QRS dilineation

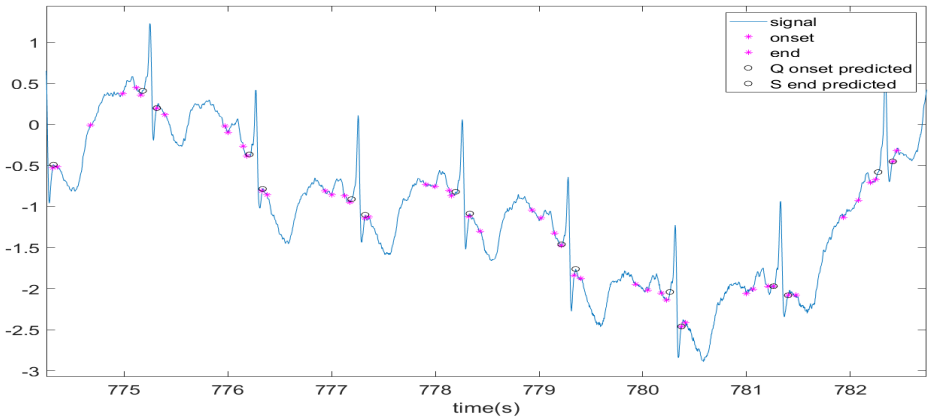



Figure 3.5: QRS dilineation in presence of artifacts

## 3.3 Results

Upon tesing on some of the random signals from qtdb and mitdb that we tested, the over 98% of annotated R-peaks were detected while false positives were less than 5% of total detections observed. In particular on qtdb/sel102, the detection of ground truth was only 90%, this is the least rate that was observed in our testing. This database can be used to study further improvements in code.
(We tolerated a separation of upto 4 samples or 16ms)

For QRS dilineation, the results were unsatisfactory, for sizeable amount of data tested, the algorithm was not able to detect locations with satisfactory. As a result of this, **in further dilineation and detection of T and P waves, the annotations provided along with database have been used for locating QRS boundaries.**

# 4. T Wave Detection and Delineation

## 4.1 Overview

The T wave on an electrocardiogram (ECG) typically represents ventricular repolarization. This phase of the cardiac cycle is rapid and the general morphology of the T wave consists of a low amplitude broad hump following the QRS complex. Sometimes the T wave is uniphasic and sometimes it is biphasic. In the latter case, we happen to observe two local maxima instead of one. T wave is usually found within an interval of 420 ms after the end of the QRS complex in a cardiac cycle.

## 4.2 Multiresolution Analysis

It can be observed that the T wave has its major component in higher scales like the 4th or 5th. This is expected because the T wave varies rapidly. As we know that in the case of *algorithme à trous*, the zero-crossings of the WT correspond to the local maxima or minima of the smoothed signal at different scales, and the absolute values of the wavelet transform are associated with maximum slopes in the filtered signal. We will not use the last scale because Baseline wander affects only scales higher than $2^4$

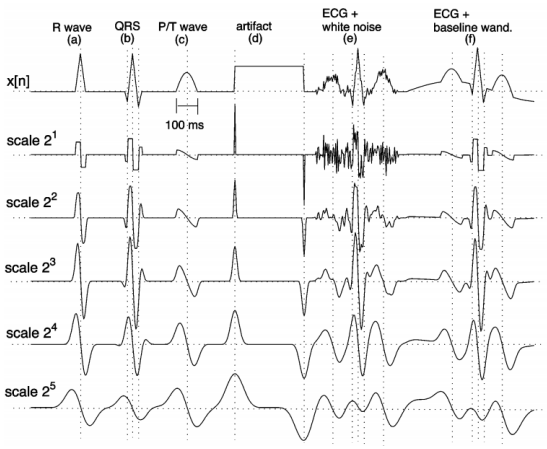



Figure 4.1: WT at the first five scales of ECG-like simulated waves (Taken from [2], Fig. 5)

## 4.3 T Wave Detection

For this, we first define a search window depending on the end position of QRS wave and R-R interval. As we know the window length in time is 420ms, and $f_s$ is the sampling frequency, our search window will be $0.420\,f_s$ samples in length.

Within this window, we look for local maxima of $\left|W_{2^4}x[n]\right|$. If atleast two of them exceed the threshold $\varepsilon_T$, a T wave is considered to be present. In this case, the local maxima of WT with amplitude greater than $\gamma_T$ are considered as significant slopes of the wave, and the zero crossings between them as the wave peaks. Let us denote the locations of these maxima as $n_{first}$ and $n_{last}$

$$\varepsilon_T = 0.25\,\mathrm{RMS}\left(W_{2^4}x[n]\right)$$

$$\gamma_T = 0.125\,\max\left(\left|W_{2^4}x[n]\right|\right) \quad n \in sw_T$$

We try to detect two peaks, to account for uniphasic and biphasic nature. But the latter case is slightly rare.

## 4.4 T Wave Delineation

To detect the onset and end of the T wave we make use of a search window on either side of the maxima. It is believed that the duration of T wave is roughly around 100-150 ms. So we used a window of 80ms duration on either side of maxima detected to identify the onset and the end. This will be $0.08\,f_s$ samples in length.

The candidates to onset and end are determined by applying two criteria:

i. searching for the sample where $\left|W_{2^4}x[n]\right|$ is below a threshold ($\xi_{Ton}$ or $\xi_{Tend}$)

ii. searching for a local minimum of $\left|W_{2^4}x[n]\right|$ before $n_{first}$ or after $n_{last}$

$$\xi_{Ton} = 0.25\,W_{2^4}x[n_{first}]$$

$$\xi_{Tend} = 0.4\,W_{2^4}x[n_{last}]$$

This part is very similar to what we did while detecting the QRS onset and ends.

## 4.5 Results

We tested our algorithm on some sample ECG signals taken from the QT Database.

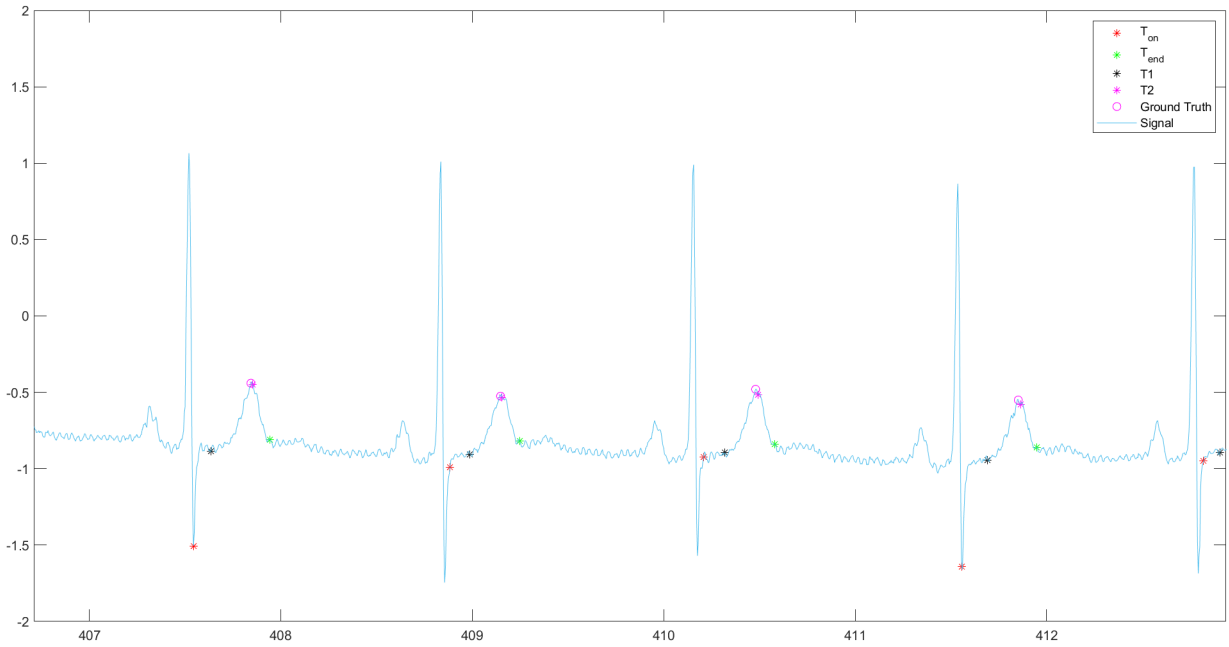



Figure 4.2: Sample Predictions vs Ground Truths

As we can see, there is a very good agreement between our predictions and the ground truths. We had tested on few other signals from the QT database and found that the detection gave a peak predictibility of more than **95%** and reaching to 99% in some cases. We considered a sensitivity of 5 samples.

# 5. P Wave Detection and Delineation

## 5.1 Overview

The P wave on an electrocardiogram (ECG) typically represents electrical depolarization of the atria of the heart. It is typically a small positive deflection from the isoelectric baseline that occurs just before the QRS complex. Sometimes the P wave is uniphasic and sometimes it is biphasic. Inthe latter case, we happen to observe two local maxima instead of one. P wave is usually found within aninterval of 200 ms before the start of the QRS complex in a cardiac cycle.

## 5.2 Multiresolution Analysis

The characteristics of P wave are very similar to those of T wave. This wave too has its major component in higher scales. As we know that in the case of *algorithme à trous*, the zero-crossings of the WT correspond to the local maxima or minima of the smoothed signal at different scales, and the absolute values of the wavelet transform are associated with maximum slopes in the filtered signal. We will not use the last scale because Baseline wander affects only scales higher than $2^4$.

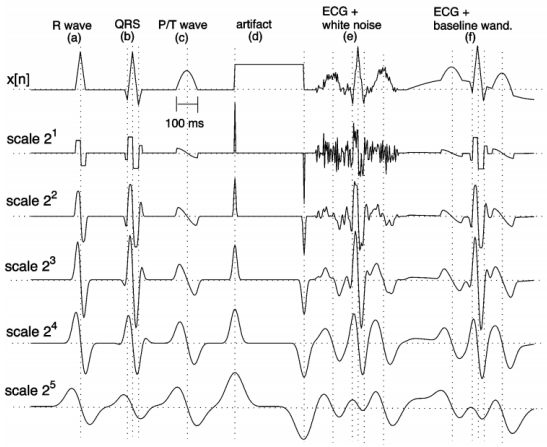



Figure 5.1: WT at the first five scales of ECG-like simulated waves (Taken from [2], Fig. 5)

## 5.3 P Wave Detection

For this, we first define a search window depending on the start of the QRS wave and the R-R interval. As we know the window length in time is 200ms, and $f_s$ is the sampling frequency, our search window

will be $0.2\,f_s$ samples in length.

Within this window, we look for local maxima of $\left|W_{2^4}x[n]\right|$. If atleast two of them exceed the threshold $\varepsilon_P$, a P wave is considered to be present. In this case, the local maxima of the WT with amplitude greater than $\gamma_P$ are considered as significant slopes of the wave, and the zero crossings between them as the wave peaks. Let us denote the locations of these maxima as $n_{first}$ and $n_{last}$.

$$\varepsilon_P = 0.02\,\mathrm{RMS}\left(W_{2^4}x[n]\right)$$

$$\gamma_P = 0.125\max\left(\left|W_{2^4}x[n]\right|\right), \quad n \in sw_P$$

Here again, we try to detect two peaks to account for uniphasic and biphasic nature.

## 5.4 P Wave Delineation

To detect the onset and end of the P wave we make use of a search window on either side of the maxima. It is believed that the duration of the P wave is roughly around 80ms. So we used a window of 40ms duration on either side of maxima to identify the onset and the end. This will be $0.04\,f_s$ samples in length.

The candidates to onset and end are determined by applying two criteria:

i. searching for the sample where $\left|W_{2^4}x[n]\right|$ is below a threshold ($\xi_{Pon}$ or $\xi_{Pend}$)

ii. searching for a local minimum of $\left|W_{2^4}x[n]\right|$ before $n_{first}$ or after $n_{last}$

$$\xi_{Pon} = 0.5\,W_{2^4}x[n_{first}]$$

$$\xi_{Pend} = 0.9\,W_{2^4}x[n_{last}]$$

## 5.5 Results

We tested our algorithm on some sample ECG signals taken from the QT Database.

As we can see, there is a very good agreement between our predictions and the ground truths. We had tested on few other signals from the QT database and found that the detection gave a peak predictibility of more than **95%** and reaching to 99% in some cases. We considered a sensitivity of 5 samples.

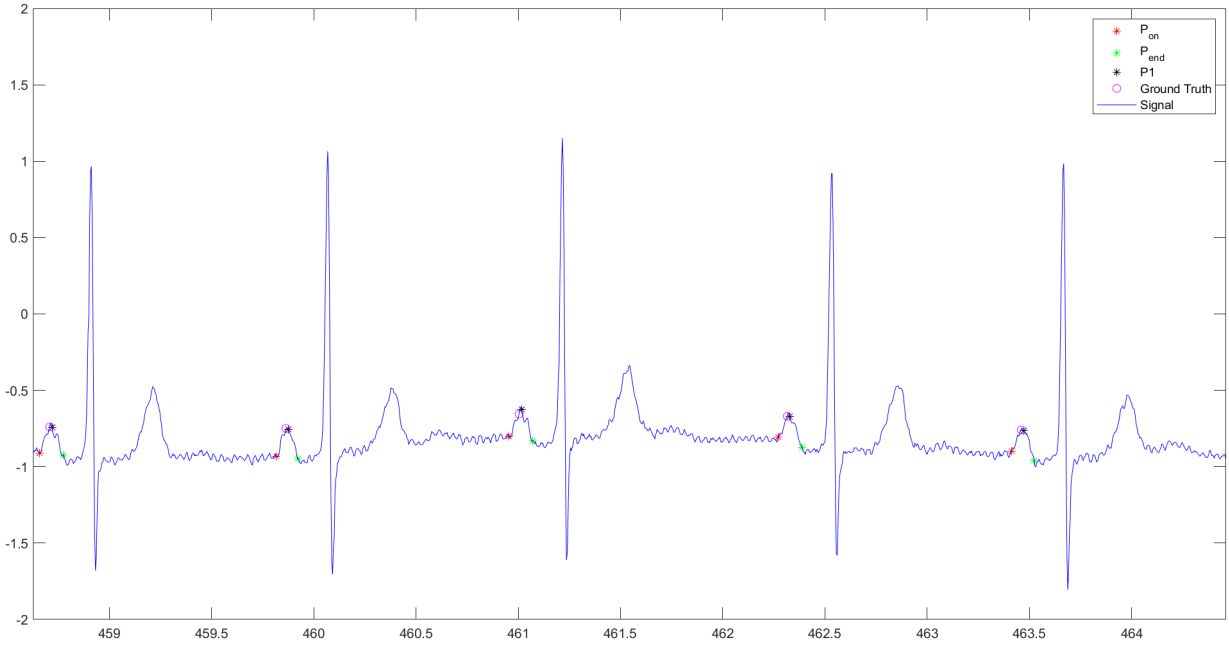



Figure 5.2: Sample Predictions vs Ground Truths

# Part Three

**6 Compression of ECG . . . . . 28**

6.1 Overview

6.2 Implementation

6.3 Results

# 6. Compression of ECG

In this section we explore using Truncated Singular Value Decomposition for ECG Data Compression

## 6.1 Overview

The method of truncated singular value decomposition (SVD) is proposed for electrocardiogram (ECG) data compression in [3]. The ECG signal is quasi-periodic and this property is exploited to extract the main features.

The method of singular value decomposition (SVD) was proposed in 1970 and has been applied in a wide range of areas, such as image compression, texture processing, and feature extraction. Hence, a truncated SVD was proposed to control noise artifacts in the electroencephalography (EEG) and magnetoencaphalography (MEG) of a reconstructed image. The signal decomposition capability of SVD is used to extract the significant feature components of the ECG by decomposing the ECG into a set of basic patterns with associated scaling factors. The signal informations are mostly concentrated within a certain number of singular values with related singular vectors due to the strong interbeat correlation among ECG cycles. Therefore, only the relevant parts of the singular triplets need to be retained as the compressed data for retrieving the original signals. The insignificant overhead can be truncated to eliminate the redundancy of ECG data compression. The Massachusetts Institute of Technology– Beth Israel Hospital (MIT -BIH) arrhythmia database was applied to evaluate the compression performance and recoverability in the retrieved ECG signals. Combining the specific interbeat correlation of ECGs, the algorithm described in [3] has been implemented that provides an efficient performance with higher compression ratios (CRs) and a relatively low reconstructed error.

## 6.2 Implementation

The block diagram below showcases the main steps used for ECG Compression:

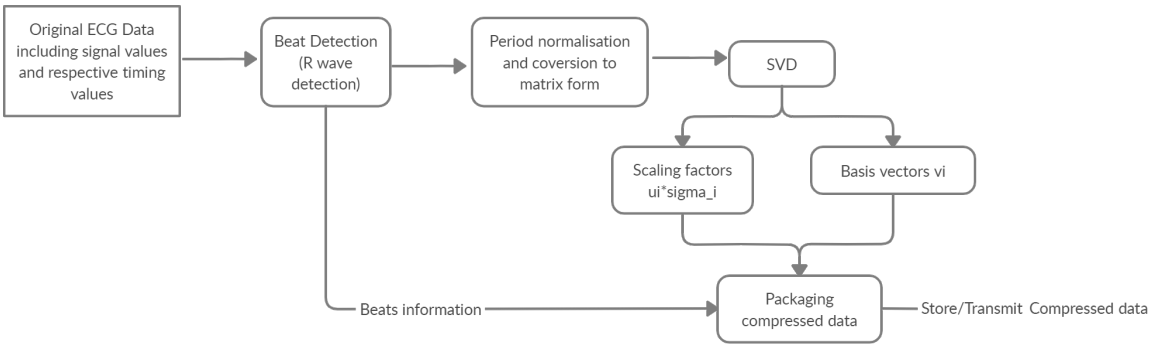



Figure 6.1: SVD flowchart

We have coded the complete implementation ourselves on MATLAB.

### 6.2.1 Periodic Segmentation

The first step of the algorithm is to separate each period of the ECG using QRS detection. By picking up the R-wave position among heartbeats, we define a segment from one -wave to the next for delineating ECG segments and store the R–to-R interval as the beat information for period transformation.

### 6.2.2 Period Normalization and Matrix Conversion

The segmented ECG cycles are normalized to the same periodic length using the transformation using which one of the ECG segments $y_i = [y_i(1), y_i(2), \ldots, y_i(n^*)]$ can be converted into a segment $x_i = [x_i(1), x_i(2), \ldots, x_i(n)]$ that holds the same signal morphology, but in different data length (i.e., $n^* \neq n$ ) using the following equation:

$$x_i(j) = y_i(j^*) + (y_i(j^*+1) - y_i(j^*))(r_j - j^*)$$

where $r_j = (j-1)(n^*-1)/(n-1)+1$ and $j^*$ is the integral part of $r_j$ Therefore, the various lengths of the ECG segments will be compressed or extended into a set of ECG segments with the same periodic length . The mean beat period (MBP) is chosen as the normalized length and is also retained as the beat information. Now, the normalized set of ECG segments can be rearranged into an m x n matrix A filled with consecutive ECG cycles as the consecutive rows, m where is the number of consecutive ECG cycles and n is the length of the normalized period. Note that the MIT-BIH dataset was sampled at 360Hz i.e 360

samples per second.

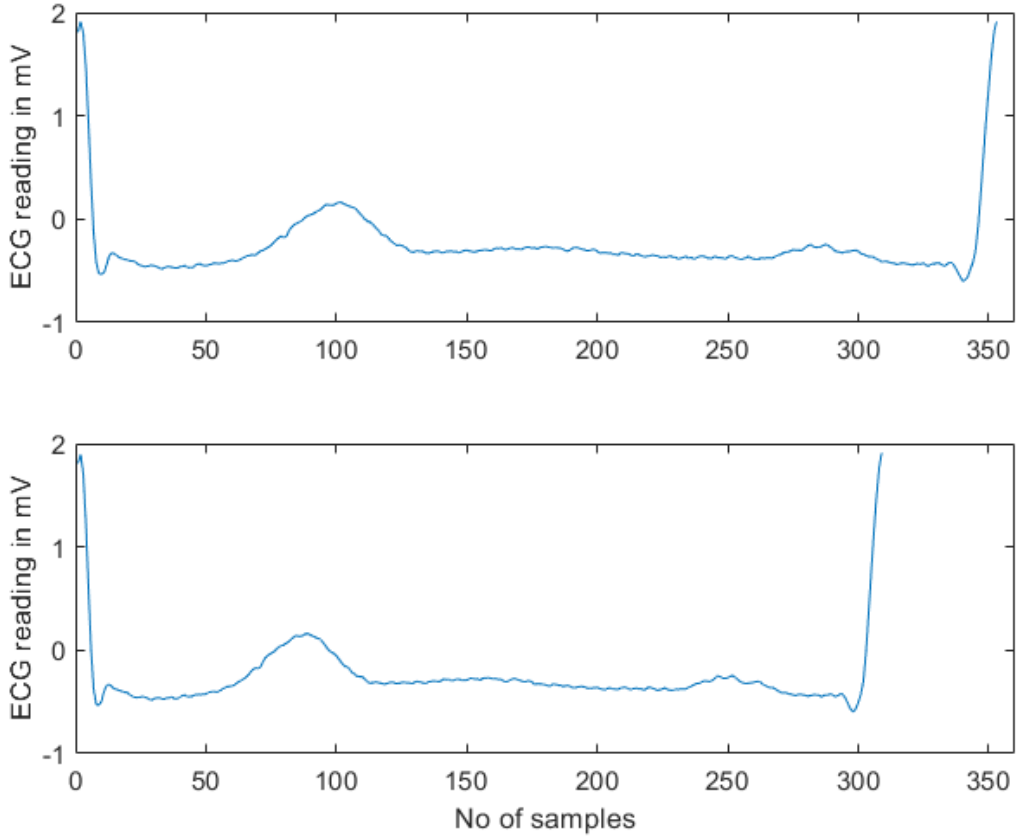



Figure 6.2: Comparison of unnormalised and normalized period waveforms(unnormalised period is 352 and normalized is 309)

The above figure shows how the shape of the waveform has been retained but the period scaled to the normalised period. The illustration is for the extreme case having the maximum interval. Similarly,while reconstruction, we can reconstruct the first waveform using the same transformation from the second one.

### 6.2.3 SVD Transformation for Signal Decomposition

Now, the represented ECG matrix is then decomposed using

$$A = USV^T$$

where $U \in R^{m\times m}, V \in R^{n\times n}$ are the left and right singular vectors, respectively and both U and V are orthogonal matrices. The $m \times n$ matrix $S = [\text{diag}\{\sigma_1, \ldots, \sigma_R\} : 0]$ or its transpose depend on whether $m < n$ or $m > n$ and $\sigma_1, \ldots, \sigma_R$ are the singular values of matrix $A$ with $\sigma_1 \geq \cdots \geq \sigma_R \geq 0$ and $R := \min(mn)$. The set $\{u_i, \sigma_i, v_i\}$ is called the ith singular triplet. We note that *any* matrix A has a valid svd transformation,

for more information any standard linear algebra textbook/website can be referred.

Now we select the first q singular triplets corresponding to the highest singular values. These are the constituents which determine or affect the shape of the wave the most. Since the ECG signal is quasi-periodic the first singular value is very high and the later ones are much lower and almost insignificant after the third.

### 6.2.4 Data compression

Let us consider that we have 'p' cycles the heartbeat with average no of samples per period 'd'. Thus the original uncompressed data consists of about p*d datapoints. Now we store/transmit only the first q singular triplets where q « p ( normally q is in range 1-20 and there are on average 4000 cycles per hour of ECG data). Now the compressed data has p*q (extracted U vectors) + q (extracted singular values) + d*q (extracted V vectors) + p (period information for the cycles) datapoints which are much less than the original datapoints due to small value of q.

Thus say we have 1000 cycles of heartbeats in consideration with average 309 samples/period and we take q = 3. Then the compression ratio(CR) is

$$CR = \frac{1000 * 309 + 1000}{3 * (1000 + 309 + 1) + 1000} = 62.67$$

Thus we acheive a compression of the signal data of about 62 times! Note that we are not compressing the timing data(the time at which the sample occured) and it is transmitted as it is but we are compressing the signal data i.e what is the amplitude of the ECG signal at various samples.

### 6.2.5 Decompression

To retrieve ECG waveforms, the compressed data must include the set of truncated singular triplets and the associated beat information(periods of R-R cycles).

Usually, we can assume that top $q$ of the singular values are predominant. Therefore, the prime information of the matrix $A$ is contained in

$$\hat{A} = \sum_{j=1}^{q} u_j S_j v_j^T$$

Having gotten $\hat{A}$ we can now use the beats information and the transformation described in 6.2.2 to reconstruct the original intervals of the original signal. We linearize the matrix $\hat{A}$ to get the reconstructed signal.

## 6.3 Results

We now see the results: The plot in red is the reconstructed and the plot in blue is the original signal. We see that they overlap very nicely that the reconstructed and original signal is nearly indistinguishable. Here q = 3 and the CR for the signal data is around 62 as calculated earlier.

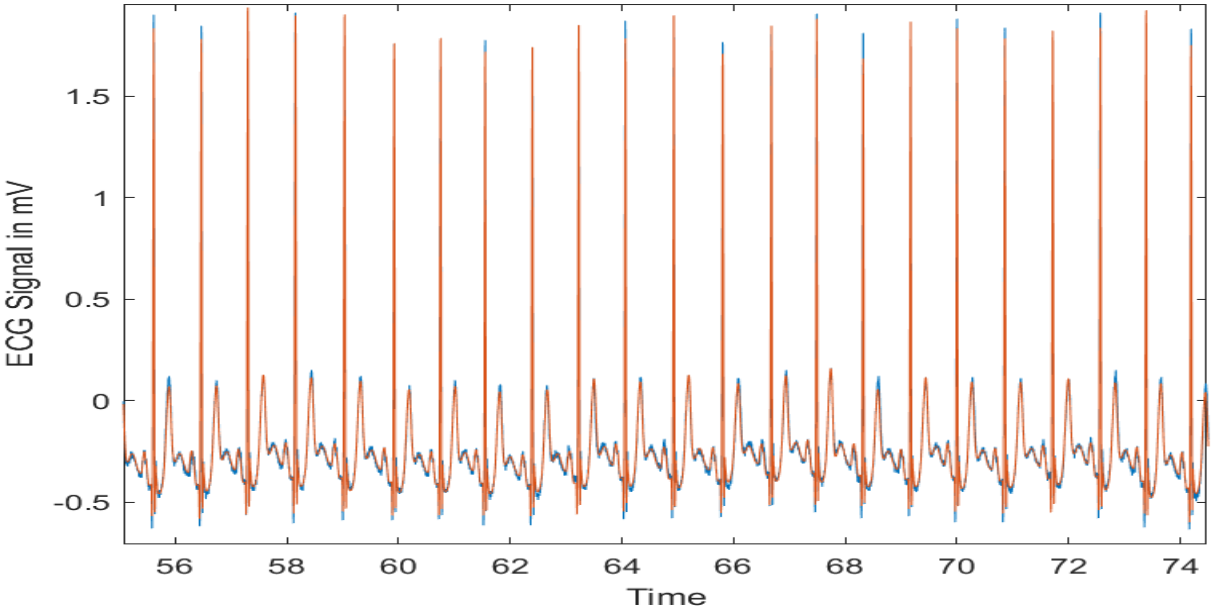



Figure 6.3: Comparison of original and reconstructed signal

We now zoom to 2 cycles and observe the reconstruction. **We see that the reconstructed signal is very similar to the original signal, almost indistinguishable.**

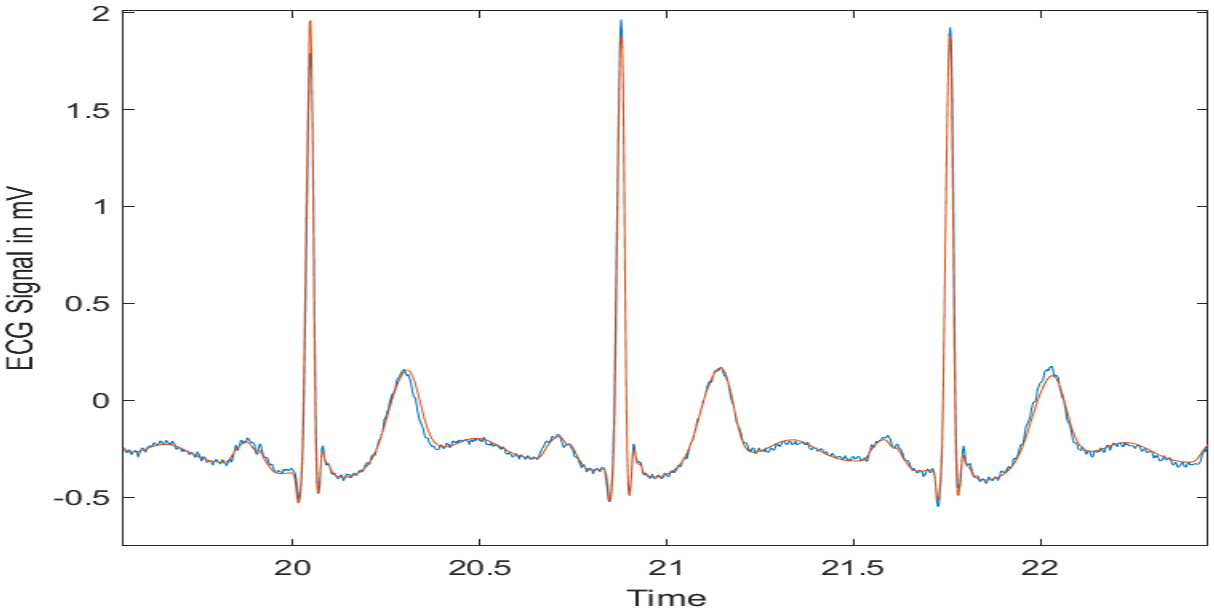



Figure 6.4: Comparison of original and reconstructed signal(zoomed)

The percent root mean square difference (PRD) is included to evaluate the error between the original and reconstructed waveforms, which is computed by the expression

$$\mathrm{PRD}(\%) = \sqrt{\frac{\sum_{i=1}^{L}[x_o(i) - x_r(i)]^2}{\sum_{i=1}^{L} x_\alpha^2(i)}} \times 100$$

where $x_o$ denotes the original data.

For the above case with 1000 cycles and q = 3, the PRM comes out to be 7.7% which is well within accepted ranges.

**Thus we see that our implementation has achieved high compression of the signal wave with minimal reconstruction error**

# IV

# Appendix

A Source Code . . . . . 36

B About Databases . . . . . 37

Bibliography . . . . . 38

References

# A. Source Code

The GitHub link for all the codes we created is : Link

All the implementations have been done by us and no code has been adapted from anywhere.

Shlok also created a tutorial for how to work with Physionet databases since he faced problems with that, so the tutorial is for helping people who might be later

# B. About Databases

The MIT-BIH Arrhythmia Database contains 48 half-hour excerpts of two-channel ambulatory ECG recordings, obtained from 47 subjects studied by the BIH Arrhythmia Laboratory between 1975 and 1979.The recordings were digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range. Two or more cardiologists independently annotated each record; disagreements were resolved to obtain the computer-readable reference annotations for each beat (approximately 110,000 annotations in all) included with the database.

This database was used for detecting R peak and SVD

The QTDB contains over 100 fifteen-minute two-lead ECG recordings (many excerpted from other databases), with onset, peak, and end markers for P, QRS, T, and (where present) U waves of from 30 to 50 selected beats in each recording.

This datbase has been used for QRS complex and P,T wave detection.

# Bibliography

## Reference papers


[1] Cuiwei Li, Chongxun Zheng, and Changfeng Tai. “Detection of ECG characteristic points using wavelet transforms”. In: *IEEE Transactions on Biomedical Engineering* 42.1 (Jan. 1995), pages 21–28 (cited on pages 7, 11, 16).

[2] J.P. Martinez et al. “A Wavelet-Based ECG Delineator: Evaluation on Standard Databases”. In: *IEEE Transactions on Biomedical Engineering* 51.4 (Apr. 2004), pages 570–581 (cited on pages 7, 10, 11, 16, 18, 20, 23).

[3] Jyh-Jong Wei et al. “ECG Data Compression Using Truncated Singular Value Decomposition”. In: *IEEE TRANSACTIONS ON INFORMATION TECHNOLOGY IN BIOMEDICINE* 5.4 (Dec. 2001), pages 290–299 (cited on pages 7, 28).