9-PSET9
M270Cω-8.05
2013

5. a) $|\psi(t)\rangle = \exp\left(\frac{i\omega t \hat{S}_z}{\hbar}\right)\exp\left(\frac{i\gamma B_R \cdot S t}{\hbar}\right)|+\rangle$

$\omega_0 = \gamma B_0 = B_R = B_1 \hat{x}$

now $\exp\left(\frac{i\gamma B_R \cdot S t}{\hbar}\right) = \exp\left(\frac{i\gamma B_R \sigma_1 t}{2}\right)$

$= \cos\left(\frac{B_1 \gamma t}{2}\right)\mathbb{1} + i\sin\left(\frac{B_1 \gamma t}{2}\right)\sigma_1$

$\Rightarrow \exp\left(\frac{i\gamma B_R \cdot \vec{S} t}{\hbar}\right)|+\rangle$

$= \cos\left(\frac{B_1 \gamma t}{2}\right)|+\rangle + i\sin\left(\frac{B_1 \gamma t}{2}\right)|-\rangle$

$\exp\left(\frac{i\omega t \hat{S}_z}{\hbar}\right)\left[\cos\left(\frac{B_1 \gamma t}{2}\right)|+\rangle + i\sin\left(\frac{B_1 \gamma t}{2}\right)|-\rangle\right)$

$= e^{i\omega t/2}\cos\left(\frac{B_1 \gamma t}{2}\right)|+\rangle + ie^{-i\omega t/2}\sin\left(\frac{B_1 \gamma t}{2}\right)|-\rangle$

$\Rightarrow |\psi(t)\rangle = \cos\frac{\theta(t)}{2}|+\rangle + \sin\frac{\theta(t)}{2}e^{i\phi(t)}|-\rangle$

$\theta(t) = B_1 \gamma t$

$\phi(t) = -\omega t$

b) $\phi = -\theta(\omega/B_1\gamma)$

when spin lies for first time in xy plane,

$\theta = \pi/2 \Rightarrow \boxed{\phi = -\frac{\pi}{2}\frac{\omega}{B_1\gamma}}$