

$$7 \quad a) \quad |v-u|^2 = |u-v|^2 = |u - P_U v - (v - P_U v)|^2$$

$$= |u - P_U v|^2 + |v - P_U v|^2$$

$$+ \langle u - P_U v | v - P_U v \rangle + \langle v - P_U v | u - P_U v \rangle$$

$$\text{now, } |u - P_U v\rangle \in U$$

$$|v - P_U v\rangle \in V - U$$

$$\Rightarrow \langle u - P_U v | v - P_U v \rangle = 0$$

$$\Rightarrow |v-u|^2 = |u - P_U v|^2 + |v - P_U v|^2$$

if  $v \notin U$ ,  $v - P_U v$  is not null

$$\Rightarrow |v - P_U v|^2 \geq 0$$

$$\Rightarrow |v-u|^2 \geq |u - P_U v|^2$$

$$\text{If } u \neq P_U v, |u - P_U v|^2 > 0$$

$$\Rightarrow |v-u|^2 > |v - P_U v|^2$$

$$\text{when } u = P_U v$$

$$|v-u|^2 = |v - P_U v|^2$$

$$\Rightarrow |v-u|^2 \geq |v - P_U v|^2 \quad \forall u \in U, v \in (V - U)$$

b)

c)

d)

} P. 7. nb Mathematica Notebook