S-P-7
8·05-2013

3. a) $x_H(t) = U^\dagger(t)\, x_S\, U(t)$

$$\frac{dx_H}{dt} = \frac{dU^\dagger(t)}{dt} x_S U(t) + U^\dagger(t) x_S \frac{d}{dt} U(t)$$

$$\Rightarrow \frac{dx_H}{dt} = \frac{i}{\hbar} H U^\dagger x_S U(t) + \frac{-i}{\hbar} U^\dagger(t) x_S H U(t)$$

$HU^\dagger = U^\dagger H$ and $HU = UH$

$$\Rightarrow \frac{dx_H}{dt} = \frac{i}{\hbar} U^\dagger [H, x_S] U$$

$$= \frac{i}{\hbar} U^\dagger \left[-\frac{i\hbar \hat{P}_S}{m} \right] U$$

$$= U^\dagger \frac{\hat{P}_S}{m} U = \frac{\hat{P}_H(t)}{m}$$

$$\Rightarrow \frac{dx_H}{dt} = \frac{\hat{P}_H(t)}{m} \Rightarrow \frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}$$

b) $\cancel{x_H}$ $P_H(t) = U^\dagger(t)\, P_S\, U(t)$

$$\Rightarrow \frac{dP_H(t)}{dt} = \frac{dU^\dagger(t)}{dt} P_S U(t) + U^\dagger(t) P_S U(t)$$

again :

$$\frac{dP_H(t)}{dt} = \frac{i}{\hbar} U^\dagger [H, P_S] U = \frac{i}{\hbar} U^\dagger [V(\hat{x}), P_S] U$$

as $[V(\hat{x}), P_S] = i\hbar \frac{\partial V_x}{\partial x}$

$$\Rightarrow \frac{dP_H(t)}{dt} = \cancel{\partial V(x_H(t))} - U^\dagger \frac{\partial V(\hat{x})}{\partial x} U$$

$$\Rightarrow \boxed{\frac{d\langle p\rangle}{dt} = -\left\langle \frac{\partial V(\hat{x})}{\partial x} \right\rangle} \Rightarrow \frac{d^2\langle \hat{x}\rangle}{dt^2} = -\frac{\langle V'(\hat{x})\rangle}{m}$$

To get newton's law in $\langle x \rangle$,

potential should be such that: $\boxed{\langle V'(x) \rangle = V'(\langle x \rangle)}$

some linear func^n?

b) $V = 0 \Rightarrow \frac{d^2 \langle \hat{x} \rangle}{dt^2} = 0$ and $\frac{d}{dt} \langle p \rangle = 0$

$\Rightarrow \langle \hat{x} \rangle = At + B$

$\frac{d \langle \hat{x} \rangle}{dt} = \frac{\langle \hat{p} \rangle}{m} \Rightarrow A = \frac{\langle \hat{p} \rangle}{m}$,

$$\Rightarrow \boxed{\langle \hat{x} \rangle = \frac{p_0 t}{m} + x_0}$$

c) clearly $[\hat{H}(t_1), \hat{H}(t_2)] \neq 0$

however, $\frac{\partial U(t, t_0)}{\partial t} = -\frac{i}{\hbar} H(t)\, U(t, t_0)$ is still true

$$\Rightarrow \frac{\partial U^\dagger(t, t_0)}{\partial t} = \frac{i}{\hbar} U^\dagger(t, t_0)\, H(t)$$ $\boxed{\text{Yay!}}$

this is all we needed.

d) $\langle \frac{d}{dx} V(\hat{x}) \rangle = q_0 E_0 \sin \omega t$

$$\Rightarrow \frac{d^2 \langle \hat{x} \rangle}{dt^2} = -\frac{q E_0 \sin \omega t}{m}$$

$$\Rightarrow \langle \hat{x} \rangle = \frac{q E_0 \sin \omega t}{m \omega^2}$$