7 - Pset 9
MIT OCW - 8.05
2013

4. a) let $\hat{H} = \begin{pmatrix} E_0 & \Delta & 0 \\ \Delta & E_c & \Delta \\ 0 & \Delta & E_0 \end{pmatrix} \begin{pmatrix} |R\rangle \\ |C\rangle \\ |L\rangle \end{pmatrix}$

Eigenvalues: $\lambda_1 = E_0$

$\lambda_2 = \frac{1}{2}\left(E_0 + E_c - \sqrt{(E_0 - E_c)^2 + 8\Delta^2}\right)$

$\lambda_3 = \frac{1}{2}\left(E_0 + E_c + \sqrt{(E_0 - E_c)^2 + 8\Delta^2}\right)$

b) $E_c = E_0$ Then

$\lambda_1 = E_0$, $|1\rangle = \frac{1}{\sqrt{2}}(-1, 0, 1)^T$

$\lambda_2 = E_0 - \sqrt{2}\Delta$, $|2\rangle = \frac{1}{2}(1, -\sqrt{2}, 1)^T$

$\lambda_3 = E_0 + \sqrt{2}\Delta$, $|3\rangle = \frac{1}{2}(1, \sqrt{2}, 1)^T$

c) $|\psi, 0\rangle = \frac{1}{2}(|2\rangle + |3\rangle + \sqrt{2}|1\rangle) = |L\rangle$

$\Rightarrow |\psi, t\rangle = e^{-iHt/\hbar}|\psi, 0\rangle$

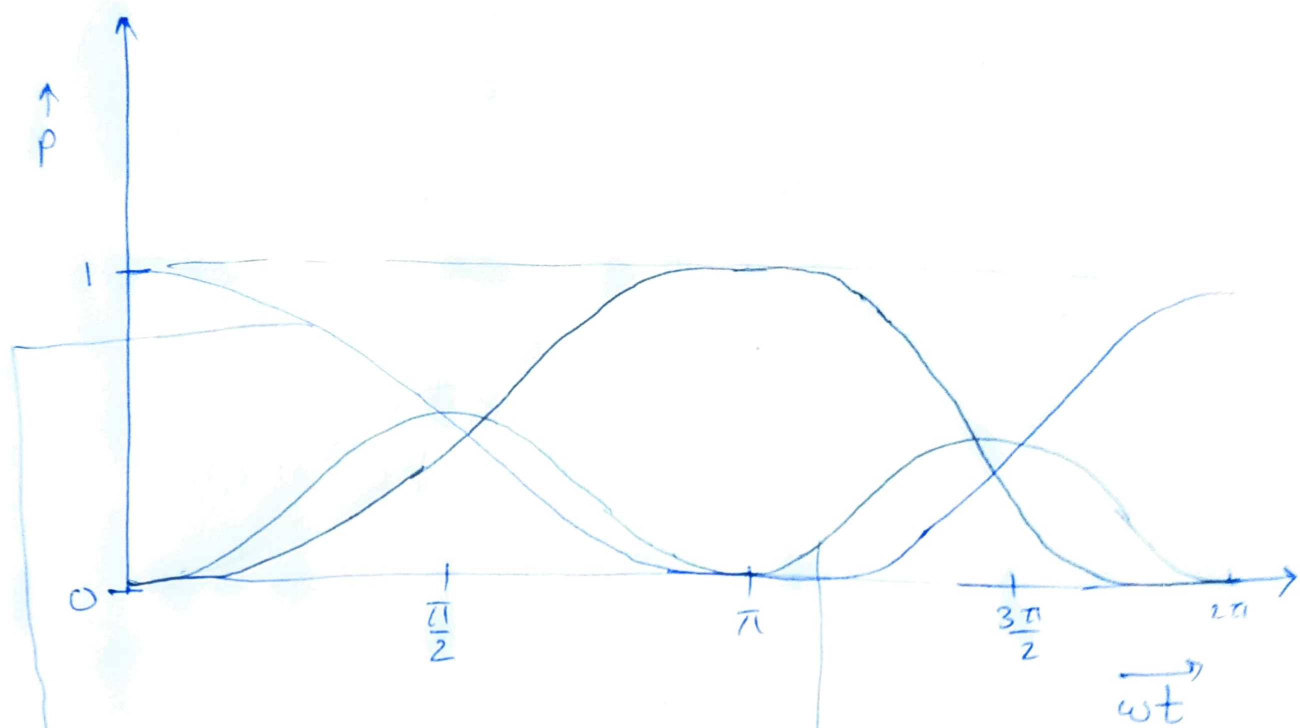
$= \frac{1}{4} e^{-iE_2t/\hbar}(|R\rangle - \sqrt{2}|C\rangle + |L\rangle)$

$+ \frac{1}{4} e^{-iE_3t/\hbar}(|R\rangle + \sqrt{2}|C\rangle + |L\rangle)$

$+ \frac{1}{2} e^{-iE_1t/\hbar}(-|R\rangle + |L\rangle)$

removing the common phase $e^{-iE_0t/\hbar}$:
and rearranging:

$|\psi, t\rangle = \left[\frac{\cos(\omega t) + 1}{2}\right]|L\rangle + \left[\frac{\cos(\omega t) - 1}{2}\right]|R\rangle - \frac{\sin(\omega t)}{\sqrt{2}}|C\rangle$; $\omega = \frac{\sqrt{2}\Delta}{\hbar}$



$$P(|L\rangle) = \cos^4(\omega t/2)$$

$$P(|R\rangle) = \sin^4(\omega t/2)$$

$$P(|e\rangle) = \sin^2(\omega t)/2$$