4. $$E_x(z,t) = \sqrt{\frac{2}{\varepsilon_0 V}}\,\omega\, q(t) \sin kz$$

$$cB_y(z,t) = \sqrt{\frac{2}{\varepsilon_0 V}}\, p(t) \cos kz$$

$$\nabla \cdot E = 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial x} = 0 \qquad \text{satisfied}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\sqrt{\frac{\cancel{2}}{\cancel{\varepsilon_0} V}}\,\omega\, q(t)\, k\, \cancel{\cos kz} = -\frac{1}{c}\sqrt{\frac{\cancel{2}}{\varepsilon_0 V}}\,\dot{p}(t)\,\cancel{\cos kz}$$

$$\Rightarrow \quad \boxed{q(t) = -\frac{\dot{p}(t)}{\omega^2}}$$

$$\nabla \cdot B = 0 \qquad \text{satisfied clearly}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}$$

$$-\frac{\partial B_y}{\partial z} = \mu_0 \varepsilon_0 \sqrt{\frac{2}{\varepsilon_0 V}}\,\omega\, \dot{q}(t) \sin kz$$

$$\frac{1}{\cancel{c}}\sqrt{\frac{\cancel{2}}{\cancel{\varepsilon_0} V}}\, k\, p(t)\, \cancel{\sin kz} = \frac{1}{c\cancel{c}}\sqrt{\frac{\cancel{2}}{\cancel{\varepsilon_0} V}}\,\omega\, \dot{q}(t)\,\cancel{\sin kz}$$

$$\boxed{p(t) = \dot{q}(t)}$$

$$\hat{q}(t) = \sqrt{\frac{\hbar}{2\omega}} \left(\hat{a} e^{-i\omega t} + \hat{a}^{+} e^{i\omega t}\right)$$

$$\hat{p}(t) = \frac{1}{i}\sqrt{\frac{\omega\hbar}{2}} \left(\hat{a} e^{-i\omega t} - \hat{a}^{+} e^{i\omega t}\right)$$

$$\dot{\hat{q}}(t) = i\omega\sqrt{\frac{\hbar}{2\omega}} \left(-\hat{a} e^{-i\omega t} + \hat{a}^{+} e^{i\omega t}\right)$$

$$\Rightarrow \dot{\hat{q}}(t) = \hat{p}(t)$$

$$\dot{\hat{p}}(t) = \frac{1}{i}\sqrt{\frac{\omega\hbar}{2}} \left(-i\omega\hat{a} e^{-i\omega t} - i\omega \hat{a}^{+} e^{i\omega t}\right)$$

$$\dot{\hat{p}}(t) = -\omega^{2} q(t)$$

Thus two set of conditions agree