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6. a) $u_h = A v_h \Rightarrow u_h = A_{hj} v_j$

simillarly $v_j = B_{ji} u_i$

$\Rightarrow u_k = A_{kj} B_{ji} u_i$

let $C_{ki} = A_{kj} B_{ji} \Rightarrow C = AB$

C is mapping coefficients of $\{u\}$ to $\{u\}$

$\Rightarrow C = I$

$\Rightarrow C_{ki} = \delta_{ik} \Rightarrow \boxed{A_{kj} B_{ji} = \delta_{ik}}$

Thus A and B are inverses of each other

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b) $T z_v = y_v$ where z_v and y_v are written in v-basis

or $T(\{v\}) z_v = y_v$

$z_u = A z_v \qquad y_u = A y_v$

$\Rightarrow T(\{v\}) A^{-1} z_u = A^{-1} y_u$

$\Rightarrow (A T(\{v\}) A^{-1}) z_u = y_u$

$\Rightarrow \boxed{T(\{u\}) = A T(\{v\}) A^{-1}}$

c) $Tr(T\{u\}) = Tr(AT(\{v\}) A^{-1}) = Tr(AA^{-1} T\{v\})$

$\Rightarrow Tr(T\{u\}) = Tr(T\{v\})$ (Since $Tr(ABC) = Tr(ACB)$)

Thus, trace is basis independent

d) $|T(\{u\})| = |A T(\{v\}) A^{-1}|$

$= |A| |T(\{v\})| |A|^{-1}$

$= |T(\{v\})|$ (since $|A| |A|^{-1} = 1$)

Thus, determinant is basis independent