

$$1. |n; +\rangle = \cos\frac{\theta}{2} |+\rangle + \sin\frac{\theta}{2} e^{i\phi} |-\rangle$$

$$\langle S_x \rangle_{\hat{x}} = \frac{\hbar}{2} (\cos\theta/2 \quad \sin\theta/2 e^{-i\phi}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 e^{i\phi} \end{pmatrix}$$

$$= \frac{\hbar}{2} \sin\theta \cos\phi \hat{x}$$

$$\text{similarly } \langle S_y \rangle_{\hat{y}} = \frac{\hbar}{2} \sin\theta \sin\phi \hat{y}$$

$$\langle S_z \rangle_{\hat{z}} = \frac{\hbar}{2} \cos\theta \hat{z}$$

$$\Rightarrow \boxed{\langle S \rangle_n = \frac{\hbar}{2} \vec{n}}$$

$$\langle S \cdot n' \rangle_n = \langle S \rangle_n \cdot n' = \vec{n} \cdot \vec{n}'$$

$$\Rightarrow \boxed{\langle S \cdot n' \rangle_n = \vec{n} \cdot \vec{n}'}$$