6. Trivial, Just solve integral

7. No such problem in my copy of griffiths

8. By variational principle :

$$E_g \le \frac{\int_{-\infty}^{\infty} \psi \hat{H} \psi \, dx}{\int_{-\infty}^{\infty} \psi \psi \, dx}$$

let $\psi = e^{-ax^2/2}$

$$\Rightarrow E_g \le \frac{a\hbar^2}{4m} + \frac{k}{4a}$$

The bound is minimal when $a^2 = km/\hbar^2$

$$\Rightarrow E_g \le \frac{\hbar}{2}\sqrt{\frac{k}{m}} \qquad \text{let } \sqrt{\frac{k}{m}} = \omega_0$$

$$\Rightarrow \boxed{E_g \le \frac{\hbar\omega_0}{2}} \qquad — (1)$$

now, let ψ_{gs} be ground state function

$$\Rightarrow \langle \hat{H} \rangle = \frac{\langle \hat{p}^2 \rangle}{2m} + \frac{1}{2}k\langle \hat{x}^2 \rangle$$

$\langle \hat{p} \rangle = \langle \hat{x} \rangle = 0$ due to symmetry of ψ_{gs}

$$\Rightarrow \langle \hat{H} \rangle = \frac{(\Delta P)^2}{2m} + \frac{1}{2}k(\Delta x)^2$$

$$(\Delta x)^2 (\Delta P)^2 \ge \frac{\hbar^2}{4} \quad \Rightarrow \langle \hat{H} \rangle \ge \frac{(\Delta P)^2}{2m} + \frac{1}{2}\frac{k\hbar^2}{4}\frac{1}{(\Delta P)^2}$$

$\frac{(\Delta P)^2}{2m} + \frac{1}{2}\frac{k\hbar^2}{4}\frac{1}{(\Delta P)^2}$ is min at $(\Delta P)^2 = \frac{\hbar}{2}\sqrt{km}$

$$\Rightarrow \boxed{\langle \hat{H} \rangle \ge \frac{\hbar\omega_0}{2}} \; -(2) \Rightarrow E_{gs} = \frac{\hbar\omega_0}{2} \; ((1) \text{ and } (2))$$