3-Pset-1
MIT OCW
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2. This proof is from Eisberg-Resnick.

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V(x) - E)\psi(x)$$

so $\frac{d^2\psi}{dx^2}$ will have same sign as ψ

when $E = V_{min}$, ψ will be a straight line ($\because \frac{d^2\psi}{dx^2} = 0$)

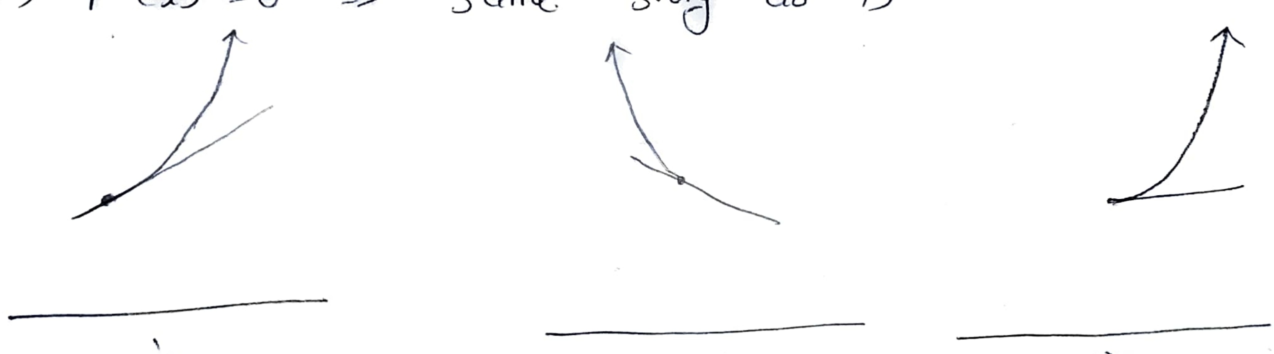
WLOG, let at a point $\psi(x) > 0$

we have 3 possibilities for $\psi'(x)$: >0, $=0$, <0

i) $\psi'(x) > 0$ $\Rightarrow$ $\psi(x)$ increases moving along $+x$ axis since $\frac{d^2\psi}{dx^2} > 0$, this portion is convex and $\psi(x) \to \infty$ as $x \to \infty$, clearly such wavefunc^n is non-normalizable.

ii) $\psi'(x) < 0$ $\Rightarrow$ same story as i) but this time moving along $-x$ axis, $\psi(x) \to \infty$ as $x \to -\infty$

iii) $\psi'(x) = 0$ $\Rightarrow$ same story as i)



i) ii) iii)

Note : if $V(x) = V_{min} = E$ at x, we will have a segment of line and sol^n will diverge from where line ends.