5. $\hat{H} = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}$

a) we see that $\frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ $(|1\rangle)$ and $\frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$ $(|2\rangle)$

are normalized eigenstates with energy $\pm\Delta$

b) $|\Psi, 0\rangle = C_L|L\rangle + C_R|R\rangle$

$= \left(\frac{C_L + C_R}{2}\right)|1\rangle + \left(\frac{C_L - C_R}{2}\right)|2\rangle$

$|\Psi, t\rangle = e^{-iHt/\hbar}|\Psi, 0\rangle$

$= \frac{(C_L + C_R)}{2} e^{+i\Delta t/\hbar}|1\rangle + \frac{(C_L - C_R)}{2} e^{-i\Delta t/\hbar}|2\rangle$

$= [C_L \cos(\Delta t/\hbar) + C_R \sin(\Delta t/\hbar)]|L\rangle$

$+ [C_R \cos(\Delta t/\hbar) - C_L \sin(\Delta t/\hbar)]|R\rangle$

c) at $t = 0$, $C_L = 0$, $|C_R| = 1$

$\Rightarrow$ Probability of observing $|L\rangle$ state is

$\sin^2(\Delta t/\hbar)$

d) $\hat{H} = \begin{pmatrix} 0 & \Delta \\ 0 & 0 \end{pmatrix}$

$|\psi, 0\rangle = c_L|L\rangle + c_R|R\rangle$

$|\psi, t\rangle = e^{-iHt/\hbar}|\psi, 0\rangle$

$\cancel{e^{-iHt/\hbar} \frac{(c_L + c_R)}{2}|1\rangle + \frac{(c_L - c_R)}{2}|2\rangle}$

$e^{-iHt/\hbar}$ acting on $|L\rangle$ gives $0 + |L\rangle$

on $|R\rangle$, $e^{-iHt/\hbar} = 1 - \frac{iHt}{\hbar} - \frac{H^2t^2}{\hbar^2} + \dots$

$H|R\rangle = 2$ $\Rightarrow$ only first two terms contribute

$|\psi, t\rangle = c_R|R\rangle - c_R\frac{i\Delta t}{\hbar}|L\rangle + c_L|L\rangle$

clearly probability sum is not constant at 1