PSET-3 P2

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$$|\mathbf{n};+\rangle = \cos\frac{\theta}{2}\,|+\rangle + \sin\frac{\theta}{2}e^{i\phi}\,|-\rangle \tag{1}$$

$$|\mathbf{n}';+\rangle = \cos\frac{\theta'}{2}\,|+\rangle + \sin\frac{\theta'}{2}e^{i\phi'}\,|-\rangle \tag{2}$$

$$\Rightarrow \langle\mathbf{n};+|\mathbf{n}';+\rangle = \cos\frac{\theta}{2}\cos\frac{\theta'}{2}\,\langle+|+\rangle + \sin\frac{\theta}{2}\sin\frac{\theta'}{2}e^{-i\phi}e^{i\phi'}\,\langle-|-\rangle \tag{3}$$

$$\Rightarrow |\langle\mathbf{n};+|\mathbf{n}';+\rangle\,|^2 = \cos^2\frac{\theta}{2}\cos^2\frac{\theta'}{2} + \sin^2\frac{\theta}{2}\sin^2\frac{\theta'}{2} + \cos\frac{\theta}{2}\cos\frac{\theta'}{2}\sin\frac{\theta}{2}\sin\frac{\theta'}{2}(e^{i(\phi-\phi')} + e^{-i(\phi-\phi')}) \tag{4}$$

$$\Rightarrow |\langle\mathbf{n};+|\mathbf{n}';+\rangle\,|^2 = \cos^2\frac{\theta}{2}\cos^2\frac{\theta'}{2} + \sin^2\frac{\theta}{2}\sin^2\frac{\theta'}{2} + \frac{\sin\theta\sin\theta'\cos(\phi-\phi')}{2} \tag{5}$$

If γ be angle between vectors $\mathbf{n}$ and $\mathbf{n}'$, then :

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi') \tag{6}$$

$$\Rightarrow \cos^2\frac{\gamma}{2} = \frac{\cos\theta\cos\theta'+1}{2} + \frac{\sin\theta\sin\theta'\cos(\phi-\phi')}{2} \tag{7}$$

Writing 1 as $(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2})(\cos^2\frac{\theta'}{2}+\sin^2\frac{\theta'}{2})$ and expanding cosines in terms of half angles, we see that $|\langle\mathbf{n};+|\mathbf{n}';+\rangle\,|^2 = \cos^2\frac{\gamma}{2}$ thus.