3-PSet-9
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3. a) $\hat{H} = -\sqrt{\Delta^2 + \mu^2\varepsilon^2}\ \dfrac{(\Delta, 0, -\mu\varepsilon)}{\sqrt{\Delta^2+\mu^2\varepsilon^2}} \cdot \vec{\sigma}$

$\Rightarrow \vec{n} = \dfrac{(\Delta, 0, -\mu\varepsilon)}{\sqrt{\Delta^2+\mu^2\varepsilon^2}}$

$\Rightarrow \theta = \cos^{-1}\dfrac{-\mu\varepsilon}{\sqrt{\Delta^2+\mu^2\varepsilon^2}}$

$\phi = 0$

b) $\lambda = \pm\sqrt{\mu^2\varepsilon^2+\Delta^2}$

eigenstates: $\begin{pmatrix} -\dfrac{(\lambda+\mu\varepsilon)}{\Delta} \\ 1 \end{pmatrix}$

$\Rightarrow \lambda = \sqrt{\mu^2\varepsilon^2+\Delta^2} \Rightarrow \begin{pmatrix} -\operatorname{cosec}\theta + \cot\theta \\ 1 \end{pmatrix}$ is eigenket

$\lambda = -\sqrt{\mu^2\varepsilon^2+\Delta^2} \Rightarrow \begin{pmatrix} \operatorname{cosec}\theta + \cot\theta \\ 1 \end{pmatrix}$ is eigenket

Simplifying in terms of half-angles

$\lambda = \sqrt{\mu^2\varepsilon^2+\Delta^2} \Rightarrow \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$ is eigenket

$\lambda = -\sqrt{\mu^2\varepsilon^2+\Delta^2} \Rightarrow \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$ is eigenket

$\Rightarrow -\sin\theta/2\,|+\rangle + \cos\theta/2\,|-\rangle$

and $\cos\theta/2\,|+\rangle + \sin\theta/2\,|-\rangle$ are eigenkets

c) $\mu\varepsilon = 0 \Rightarrow \theta = \pi/2$

$\Rightarrow$ $|x;->$ has energy $\sqrt{\mu^2\varepsilon^2+\Delta^2} = \Delta$

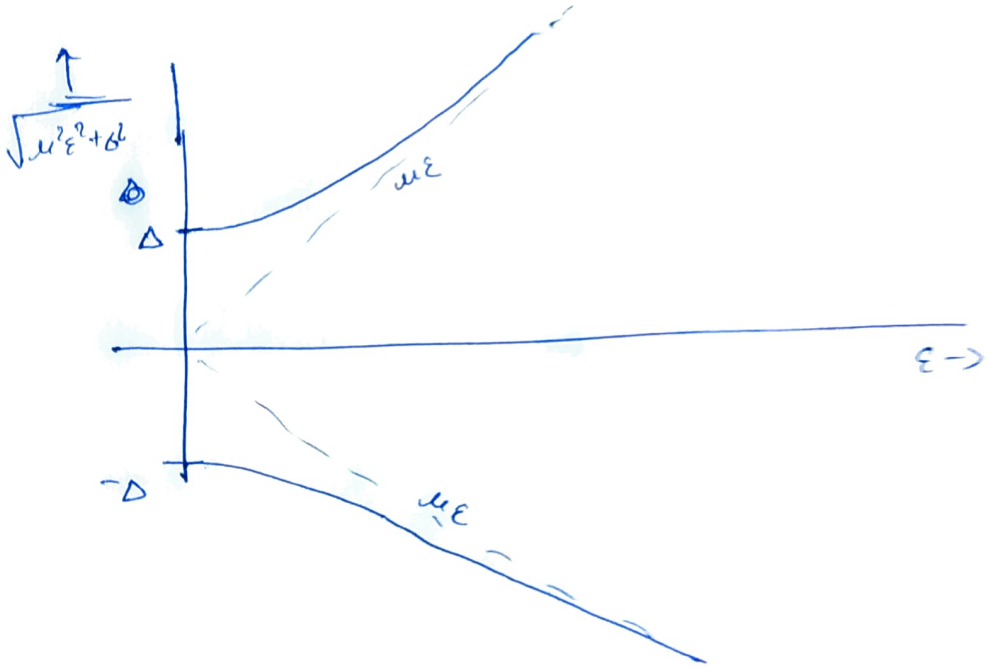
$|x;->$ has energy $-\sqrt{\mu^2\varepsilon^2+\Delta^2} = -\Delta$

$\Delta = 0 \Rightarrow \theta = \pi$

$|z;+>$ has energy $\sqrt{\mu^2\varepsilon^2+\Delta^2} = \mu\varepsilon$

$|z;->$ has energy $-\sqrt{\mu^2\varepsilon^2+\Delta^2} = -\mu\varepsilon$

d)



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c) $|\uparrow,0\rangle \cdot |\downarrow\rangle$

$$H = \omega_1 \cdot S = -\frac{2\Delta}{\hbar} \hat{S}_x$$

$$\Rightarrow \omega_L = -\frac{2\Delta}{\hbar}\hat{x}$$

$\Rightarrow |\downarrow\rangle$ will rotate in z-y plane anticlockwise with $\omega = 2\Delta/\hbar$

after $\mu\varepsilon >> \Delta$, precession happens very slowly along x-axis, very quickly along z-axis's

probability of finding $|\downarrow\rangle$ remains nearly constant after turning on $\mu\varepsilon >> \Delta$

experiencing only minor ripples.