S-PSET-4
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MIT-OCW

3. We already know from P2 that $F(t) = e^{tA} B e^{-tA}$

then $\frac{dF(t)}{dt} = e^{tA} [A, B] e^{-tA}$

then $\frac{d^2F(t)}{dt^2} = e^{tA} [A, [A, B]] e^{-tA}$

since we can replace B by $[A, B]$

thus $\frac{d^nF(t)}{dt^n} = e^{tA} \underbrace{[A, [A, [, \dots, [A, [A, B]]] \dots]}_{n\text{- } A\text{s}}$

$\Rightarrow e^{-tA} B e^{tA} = B + t e^{tA} [A, B] e^{-tA} + \frac{1}{2!} e^{tA} [A, [A, B]] e^{-tA}$
$+ \frac{1}{3!} e^{tA} [A, [A, [A, B]]] e^{-tA} + \dots$

Put $t = 1$ to recover (1)

Given $\operatorname{ad} A(X) = [A, X]$

$\Rightarrow \operatorname{ad} A(\operatorname{ad} A) = [A, [A, x]]$

Clearly $e^{A} B e^{-A} = e^{\operatorname{ad} A}(B)$