6. $H = \begin{pmatrix} 0 & -\Delta \\ -\Delta & 0 \end{pmatrix} = -\Delta\sigma_1 \qquad \Delta > 0$

$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

let $H = a\sigma_3 - \Delta\sigma_1 \quad (H^2 = a^2+\Delta^2)$

$\Rightarrow U(t) = e^{-iHt/\hbar} = 1 - i\frac{(a\sigma_3 - \Delta\sigma_1)}{\hbar}t - \frac{(a^2+\Delta^2)t^2}{2\hbar^2} + \dots$

$= \cos\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right) - \frac{ia}{\sqrt{a^2+\Delta^2}}\sin\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right)\sigma_3 + \frac{i\Delta\sigma_1}{\sqrt{a^2+\Delta^2}}\sin\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right)$

$|\psi, t\rangle = U(t,0)\,|\psi, 0\rangle = U(t,0)\,|1\rangle$

$\Rightarrow |\psi, t\rangle = \left[\cos\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right) - \frac{ia}{\sqrt{a^2+\Delta^2}}\sin\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right)\right]|1\rangle$

$+ \frac{i\Delta}{\sqrt{a^2+\Delta^2}}\sin\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right)|2\rangle$

$P_{min} = \min_{t\in R}\left[\cos^2\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right) + \frac{a^2}{a^2+\Delta^2}\sin^2\left(\frac{\sqrt{a^2+\Delta^2}\,t}{\hbar}\right)\right]$

$\Rightarrow p_{min} = \frac{a^2}{a^2+\Delta^2} \Rightarrow q = \sqrt{\frac{1}{1-p_{min}^2}}\,\Delta p_{min}$

$\Rightarrow H' = \begin{pmatrix} \Delta p_{min}/\sqrt{1-p_{min}^2} & -\Delta \\ -\Delta & -\frac{\Delta p_{min}}{\sqrt{1-p_{min}^2}} \end{pmatrix}$