3-P-6
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2. $\cos^2(\phi(t)) = |\langle\psi(0)|\psi(t)\rangle|^2$; $0 \le \phi(t) \le \pi/2$

Q: Projector on $|\psi(0)\rangle$:= $Q = |\psi(0)\rangle\langle\psi(0)|$

$\langle Q\rangle = \langle\psi(t)|Q|\psi(t)\rangle = \langle\psi(t)|\psi(0)\rangle\langle\psi(0)|\psi(t)\rangle$

$= |\langle\psi(0)|\psi(t)\rangle|^2 = \cos^2(\phi(t))$

since Q is a projection operator $\Rightarrow Q^2 = Q$

$\Rightarrow \langle Q^2\rangle = \langle Q\rangle = \cos^2(\phi(t))$

$\Rightarrow (\Delta Q)^2 = \langle Q^2\rangle - \langle Q\rangle^2 = \cos^2(\phi(t)) - \cos^4(\phi(t))$

$\Rightarrow \Delta Q = |\cos(\phi(t))\sin(\phi(t))| = \cos(\phi(t))\sin(\phi(t))$

(since $0 \le \phi(t) \le \pi/2$)

$\left|\frac{d\langle Q\rangle}{dt}\right| = 2[\sin\phi(t)]\cos[\phi(t)]$

$\Rightarrow \Delta H\,\Delta Q \ge \frac{\hbar}{2}\left|\frac{d\langle Q\rangle}{dt}\right| \Rightarrow \boxed{\frac{\Delta H}{\hbar} \ge \left|\frac{d\phi}{dt}\right|}$

if $\left|\frac{d\phi}{dt}\right| = \frac{\Delta H}{\hbar}$, time taken to be self-orthogonal

is minimum, $\Delta t_\perp = \frac{\pi\hbar}{2\Delta H} = \frac{h}{4\Delta H}$

$\Rightarrow \boxed{\Delta H\,\Delta t_\perp \ge \frac{h}{4}}$

b) $\left|\frac{d\phi}{dt}\right| \le \frac{\Delta H}{\hbar} \Rightarrow t\left|\frac{d\phi}{dt}\right|_{max} \le \frac{\Delta H t}{\hbar}$ ~~$\Rightarrow \cos t$~~

since this implies $\phi(t) \le \Delta H t/\hbar \Rightarrow \cos[\phi(t)] \ge \cos\left(\frac{\Delta H t}{\hbar}\right)$

$\Rightarrow |\langle\psi(0)|\psi(t)\rangle|^2 \ge \cos^2\left(\frac{\Delta H t}{\hbar}\right)$

since this inequality holds for $\phi(t) \le \frac{\pi}{2}$

$\Rightarrow$ for $t \le \frac{\pi\hbar}{2\Delta H}$ this holds