

6.

a) $R|+\rangle = |n;+\rangle$

\Rightarrow if $R = \begin{pmatrix} c & a \\ d & b \end{pmatrix}$, $c = \cos \theta/2$
 $d = \sin \theta/2 e^{i\phi}$

where $\vec{n} = \cos \theta \hat{z} + \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y}$

R is rotation operator $\Rightarrow R^\dagger R = I$

$\Rightarrow \begin{pmatrix} 1 & a \cos \theta/2 + e^{-i\phi} b \sin \theta/2 \\ \bar{a} \cos \theta/2 + \bar{b} \sin \theta/2 e^{i\phi} & |a|^2 + |b|^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow a \cos \frac{\theta}{2} + b \sin \frac{\theta}{2} = 0$

$|a|^2 + |b|^2 = 1$

solving these, $a = \sin \frac{\theta}{2} e^{i\Omega}$
 $b = -\cos \frac{\theta}{2} e^{i\phi} e^{i\Omega}$ $\Omega \in \mathbb{R}$

$\Rightarrow R|+\rangle$ is upto a phase $e^{i\Omega}$, equal to $|n;+\rangle$

b) $R \oplus R | \psi \rangle = \frac{1}{\sqrt{2}} (R|+\rangle \otimes R|-\rangle - R|-\rangle \otimes R|+\rangle)$

$= e^{i\Omega} \frac{1}{\sqrt{2}} \left(\left(\cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\phi} |-\rangle \right) \otimes \left(\sin \frac{\theta}{2} |+\rangle - \cos \frac{\theta}{2} e^{i\phi} |-\rangle \right) \right.$
 $\left. - \left(\sin \frac{\theta}{2} |+\rangle - \cos \frac{\theta}{2} e^{i\phi} |-\rangle \right) \otimes \left(\cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\phi} |-\rangle \right) \right)$

$= \frac{e^{i\Omega} e^{i\phi}}{\sqrt{2}} (|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle)$

Thus it is equal to $|\psi\rangle$ upto a phase