6. $H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 - F\hat{x}$

Clearly $[\hat{y}, \hat{p}] = i\hbar$

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \left(\hat{x}^2 - 2P\hat{x} \cdot \frac{F}{m\omega^2} + \frac{F^2}{m^2\omega^4} \right) - \frac{F^2}{2m\omega^2}$$

$$\Rightarrow H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{y}^2 - \frac{F^2}{2m\omega^2}$$

clearly $H = a_y^\dagger a_y + \left(\frac{1}{2}\hbar\omega - \frac{F^2}{2m\omega^2} \right)$

a) $\Rightarrow$ ground state energy is $-\frac{F^2}{2m\omega^2} + \frac{1}{2}\hbar\omega$

$\langle \hat{y} \rangle = 0 \quad \Rightarrow \quad \boxed{\langle \hat{x} \rangle = \frac{F}{m\omega^2}}$

b) Since we want to translate solution to $x_0 = \frac{F}{m\omega^2}$

$|0'\rangle = \exp\left(-\frac{i\hat{p}x_0}{\hbar} \right) |0\rangle = \exp\left(\frac{x_0}{\sqrt{2}\, d_0} a^\dagger \right) |0\rangle$, $d_0 = \sqrt{\frac{\hbar}{m\omega}}$

$\Rightarrow \boxed{\alpha = \frac{F \sqrt{m\omega}}{m\omega^2 \sqrt{2\hbar}}}$