1-P7
8-03-2013

1. $\vec{B} = B_0 \cos(\omega t)\, \hat{e}_z$

$\Rightarrow H(t) = -(\vec{B} \cdot \hat{S}_z)\, r$

where r related spin to magnetic moment

since $[H(t), H(t')] = 0$

$\Rightarrow U(t, t_0) = \exp\left[-\frac{i}{\hbar} \int_{t_0}^{t} H(t')\, dt'\right]$

$\int_{t_0}^{t} H(t')\, dt' = -rB \frac{\sin \omega t}{\omega} \hat{S}_z \qquad (t_0 = 0)$

$\Rightarrow U(t, t_0) = \exp\left[\frac{i}{\hbar} \frac{rB}{\cancel{\omega}} \frac{\sin \omega t}{\omega} \hat{S}_z\right]$

From last Pset,

$\Rightarrow \boxed{U(t, t_0) = \cos\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right) \mathbb{1} + i \sin\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$

$|\psi, t\rangle = U(t, 0)\, |\psi, 0\rangle$ (Spin - component)

$\Rightarrow |\psi, t\rangle = \left[\cos\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right) + i \sin\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right)\right] |+\rangle$
$+ \left[\cos\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right) - i \sin\left(\frac{rB}{2} \frac{\sin \omega t}{\omega}\right)\right] |-\rangle$

let $\frac{rB}{\cancel{2}} \frac{\sin \omega t}{\omega} = \theta(t)$

$\Rightarrow |\psi, t\rangle = e^{i\theta(t)/2} |+\rangle + e^{-i\theta(t)/2} |-\rangle$

$= \frac{1}{\sqrt{2}} |+\rangle + \frac{e^{-i\theta(t)}}{\sqrt{2}} |-\rangle$ (since common phase is arbitrary) (and normalizing)

$\Rightarrow \boxed{\theta(t) = \frac{\pi}{2}}$

$$\boxed{\phi(t) = -\frac{\gamma B \sin \omega t}{\omega}}$$

c) Probability of find $S_x = -\frac{\hbar}{2}$

$$= |\langle x; -|\psi, t\rangle|^2$$

$$= \left(\frac{1}{\sqrt{2}}\langle +| - \frac{1}{\sqrt{2}}\langle -|\right)\left(\frac{e^{i\frac{\phi}{2}}}{\sqrt{2}}|+\rangle + \frac{e^{-i\frac{\phi}{2}}}{\sqrt{2}}|-\rangle\right)$$

$$= \left|\frac{1}{2}e^{i\phi/2} - \frac{1}{2}e^{-i\phi/2}\right|^2$$

$$= \sin^2\left(\frac{\gamma B}{2}\frac{\sin \omega t}{\omega}\right)$$

d) for full fillipng ; $\frac{\gamma B}{2}\frac{\sin \omega t}{\omega} = (2n+1)\frac{\pi}{2}$

or atleast $\frac{\gamma B}{2}\frac{|\sin \omega t|}{\omega} \geq \frac{\pi}{2}$

$$\Rightarrow \frac{\gamma B}{2}\frac{1}{\omega} \geq \frac{\pi}{2}$$

$$\Rightarrow \boxed{\frac{\gamma B}{\pi} \geq \omega}$$