S-P-7
8·05-2013

3. a) $x_H(t) = U^{\dagger}(t)\, x_S\, U(t)$

$$\frac{dx_H}{dt} = \frac{dU^{\dagger}(t)}{dt} x_S U(t) + U^{\dagger}(t) x_S \frac{d}{dt} U(t)$$

$$\Rightarrow \frac{dx_H}{dt} = \frac{i}{\hbar} H U^{\dagger} x_S U(t) + \frac{-i}{\hbar} U^{\dagger}(t) x_S H U(t)$$

$HU^{\dagger} = U^{\dagger}H$ and $HU = UH$

$$\Rightarrow \frac{dx_H}{dt} = \frac{i}{\hbar} U^{\dagger} [H, x_S] U$$

$$= \frac{i}{\hbar} U^{\dagger} \left[-\frac{i\hbar \hat{P}_S}{m}\right] U$$

$$= U^{\dagger} \frac{\hat{P}_S}{m} U = \frac{\hat{P}_H(t)}{m}$$

$$\Rightarrow \frac{dx_H}{dt} = \frac{\hat{P}_H(t)}{m} \Rightarrow \frac{d}{dt}\langle x\rangle = \frac{\langle p\rangle}{m}$$

b) ~~xq~~ $P_H(t) = U^{\dagger}(t)\, P_S\, U(t)$

$$\Rightarrow \frac{dP_H(t)}{dt} = \frac{dU^{\dagger}(t)}{dt} P_S U(t) + U^{\dagger}(t) P_S U(t)$$

again :

$$\frac{dp_H(t)}{dt} = \frac{i}{\hbar} U^{\dagger} [H, P_S] U = \frac{i}{\hbar} U^{\dagger} [V(\hat{x}), P_S] U$$

as $[V(\hat{x}), P_S] = i\hbar \frac{\partial V_x}{\partial x}$

$$\Rightarrow \frac{dp_H(t)}{dt} = \cancel{\partial V(x_H(t))} - U^{\dagger} \frac{\partial V(\hat{x})}{\partial x} U$$

$$\Rightarrow \boxed{\frac{d\langle p\rangle}{dt} = -\left\langle \frac{\partial V(\hat{x})}{\partial x} \right\rangle} \Rightarrow \frac{d^2}{dt^2}\langle \hat{x}\rangle = -\frac{\langle V'(\hat{x})\rangle}{m}$$

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To get newton's law in $\langle x \rangle$,

potential should be such that: $\boxed{\langle V'(x) \rangle = V'(\langle x \rangle)}$

some linear func^n !

b) $V = 0 \Rightarrow \frac{d^2 \langle \hat{x} \rangle}{dt^2} = 0$ and $\frac{d}{dt} \langle p \rangle = 0$

$\Rightarrow \langle \hat{x} \rangle = At + B$

$\frac{d \langle \hat{x} \rangle}{dt} = \frac{\langle \hat{p} \rangle}{m} \Rightarrow A = \frac{\langle \hat{p} \rangle}{m}$,

$\Rightarrow \boxed{\langle \hat{x} \rangle = \frac{P_0 t}{m} + x_0}$

c) clearly $[\hat{H}(t_1), \hat{H}(t_2)] \neq 0$

however, $\frac{\partial U(t, t_0)}{\partial t} = -\frac{i}{\hbar} H(t)\, U(t, t_0)$ is still true

$\Rightarrow \frac{\partial U^\dagger(t, t_0)}{\partial t} = \frac{i}{\hbar} U^\dagger(t, t_0)\, H(t)$ $\boxed{\text{Yay!}}$

this is all we needed.

$\cancel{\langle V(\hat{x}) \rangle = qE_0 \langle \hat{x} \rangle \sin\omega t}$

$\Rightarrow \cancel{\frac{d \langle \hat{p} \rangle}{dt} = -qE_0 \frac{d}{dx} \langle \hat{x} \rangle \sin\omega t}$

$\Rightarrow$ or $\cancel{\frac{d^2 \langle x \rangle}{dt^2} = -\frac{d \langle V(\hat{x}) \rangle}{dx\, m} - \frac{qE_0 \sin\omega t}{m}}$

$\Rightarrow \cancel{\langle \hat{x} \rangle}$

$\langle \frac{d}{dx} V(\hat{x}) \rangle = qE_0 \sin\omega t$

$\Rightarrow \frac{d^2 \langle \hat{x} \rangle}{dt^2} = -\frac{qE_0 \sin\omega t}{m}$ $\Rightarrow \boxed{\langle \hat{x} \rangle = \frac{qE_0 \sin\omega t}{m\omega^2}}$

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