5. It was pointed is problem 3 that Newton's law in $\langle x \rangle$ is observed if $\langle \frac{\partial V(\hat{x})}{\partial \hat{x}} \rangle = \partial_x V(\langle \hat{x} \rangle)$

since $V = g\hat{x}$, , $\frac{\partial V}{\partial \hat{x}} = g\mathbb{1}$

$\Rightarrow \langle \frac{\partial V(\hat{x})}{\partial \hat{x}} \rangle = g = \frac{\partial}{\partial x} V(\langle \hat{x} \rangle)$

$\Rightarrow$ a) Newton's laws obeyed by $\hat{x}_H(t)$ and $\hat{p}_H(t)$

$\frac{d}{dt} \hat{p}_H(t) = -g$

$\Rightarrow \hat{p}_H(t) = -gt\mathbb{1} + \hat{p}_H(0)$

$\frac{d}{dt} \hat{x}_H(t) = \frac{\hat{p}_H(t)}{m} = -\frac{g}{m} t\mathbb{1} + \frac{\hat{p}_H(0)}{m}$

$\Rightarrow \hat{x}_H(t) = -\frac{gt^2}{2m}\mathbb{1} + \frac{\hat{p}_H(0)t}{m} + \hat{x}_H(0)$

b) $\Rightarrow \boxed{\hat{x}_H(t) = -\frac{gt^2}{2m}\mathbb{1} + \frac{\hat{p}t}{m} + \hat{x}}$

b) $\langle \psi | \hat{x}_H(t) | \psi \rangle = -\frac{gt^2}{2m} \langle \psi | \psi \rangle + \frac{t}{m} \langle \psi | \hat{p} \psi \rangle + \langle \psi | \hat{x} \psi \rangle$

$= -\frac{gt^2}{2m}$

c) $\hat{x}_H^2(t) = \frac{g^2t^4}{4m^2} - \frac{gt^3\hat{p}}{2m^2} - \frac{gt^2\hat{x}}{2m} - \frac{gt^3\hat{p}}{2m^2} + \frac{\hat{p}^2t^2}{m^2} + \frac{\hat{x}\hat{p}t}{m}$

$- \frac{gt^2\hat{x}}{2m} + \frac{\hat{p}\hat{x}t}{m} + \hat{x}^2$

$\langle \psi | \hat{x}_H^2(t) | \psi \rangle = \frac{g^2t^4}{4m^2} + \frac{t^2}{m^2} \langle \psi | \hat{p}^2 | \psi \rangle + \langle \psi | \hat{x}^2 | \psi \rangle + \frac{t}{m} \langle \psi | \hat{p}\hat{x} + \hat{x}\hat{p} | \psi \rangle$

$= \frac{g^2t^4}{4m^2} + \frac{t^2}{m^2} \times \frac{\hbar^2 N^2 \sqrt{\pi}}{2\Delta} + (\Delta x(0))^2 + 0$

$\Rightarrow (\Delta x(t))^2 = (\Delta x(0))^2 + \frac{\hbar^2 N^2 \sqrt{\pi}\, t^2}{2m^2\Delta} \left(+ \frac{g^2t^4}{4m^2} - \frac{g^2t^4}{4m^2} \right)$

$\Rightarrow$ spreading doesn't depend on g

as $N^2 \sqrt{\pi}\, \Delta = 1$

$$\Rightarrow \quad (\Delta x(t))^2 = (\Delta x(0))^2 + \frac{\hbar^2 t^2}{2m^2\Delta^2}$$

6. $$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 - F\hat{x}$$

Clearly $[\hat{y}, \hat{p}] = i\hbar$

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \left(\hat{x}^2 - 2P\hat{x} \cdot \frac{F}{m\omega^2} + \frac{F^2}{m^2\omega^4}\right) - \frac{F^2}{2m\omega^2}$$

$$\Rightarrow H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{y}^2 - \frac{F^2}{2m\omega^2}$$

clearly $H = a_y^\dagger a_y + \left(\frac{1}{2}\hbar\omega - \frac{F^2}{2m\omega^2}\right)$

a) $\Rightarrow$ ground state energy is $-\frac{F^2}{2m\omega^2} + \frac{1}{2}\hbar\omega$

$\langle \hat{y} \rangle = 0 \quad \Rightarrow \quad \boxed{\langle \hat{x} \rangle = \frac{F}{m\omega^2}}$

b) Since we want to translate solution to $x_0 = \frac{F}{m\omega^2}$

$$|0'\rangle = \exp\left(-\frac{i\hat{p}x_0}{\hbar}\right)|0\rangle = \exp\left(\frac{x_0}{\sqrt{2}\, d_0} a^\dagger\right)|0\rangle \quad , \; d_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$\Rightarrow \boxed{\alpha = \frac{F}{m\omega^2} \frac{\sqrt{m\omega}}{\sqrt{2\hbar}}}$$