7- PSET4
MIT OCW
8.05 - 2013

6) Shankar 1.8.8 :

m^1, m^2, m^3, m^4 : $m^i m^j + m^j m^i = 2\delta^{ij} I$ $i,j = 1,2,3,4$

1) when $i=j$, $m^{2,i} = I$ $(m^{2,i} = m^i \cdot m^i)$

let λ be an eigenvalue of m^i with an eigenvector v $\Rightarrow$ $m^i v = \lambda v$ $\Rightarrow$ $m^{2,i} v = \lambda^2 v$

$\Rightarrow \lambda^2 v = I v$ $\Rightarrow$ $\lambda = \pm 1$

or this is directly deduced from eigenbasis too

2) $m^i m^j = -m^j m^i$; $i \neq j$

$\Rightarrow$ $m^i \cdot m^i m^j = -m^i m^j m^i$

$\Rightarrow$ $m^j = -m^i m^j m^i$ (since $m^i \cdot m^i = I$)

$\Rightarrow$ trace $(m^j) = -$ trace $(m^i m^j m^i)$

Given trace (ACB) = trace (CBA)

$\Rightarrow Tr(m^i m^j m^i) = Tr(m^j m^i \cdot m^i) = Tr(m^j)$

$\Rightarrow Tr(m^j) = -Tr(m^j) \Rightarrow \boxed{Tr(m^j) = 0}$

3) since $Tr(m^j) = 0$
and eigen values, $\lambda_i = \pm 1$

$\Rightarrow \sum \lambda_i = 0$ only when equal no. of $+1$ and -1 are there, but $\sum \lambda_i = Tr(m)$

$\Rightarrow$ They must be even - dimensional