PSET-6 P2

Shlok Vaibhav Singh

2020/05/19

2.

$$\cos^2\phi(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2; \qquad 0 \leq \phi \leq \pi/2 \tag{1}$$

Let Q be the projection operator on $|\Psi, 0\rangle \Rightarrow Q = |\Psi, 0\rangle \langle\Psi, 0|$

$$\Rightarrow \langle Q\rangle = \langle\Psi, t| Q |\Psi, t\rangle = \langle\Psi, t|\Psi, 0\rangle \langle\Psi, 0|\Psi, t\rangle \tag{2}$$

$$\Rightarrow \langle Q\rangle = |\langle\Psi, 0|\Psi, t\rangle|^2 = \cos^2\phi(t) \tag{3}$$

Since Q is a projection operator, $Q^2 = Q \Rightarrow \langle Q^2\rangle = \langle Q\rangle = \cos^2\phi(t)$. Since:

$$(\Delta Q)^2 = \langle Q^2\rangle - \langle Q\rangle^2 \tag{4}$$

$$\Rightarrow (\Delta Q)^2 = \cos^2\phi(t) - \cos^4\phi(t) \tag{5}$$

$$\Rightarrow (\Delta Q)^2 = \cos\phi(t)\sin\phi(t); \qquad 0 \leq \phi \leq \pi/2 \tag{6}$$

From 2,

$$\left|\frac{d\langle Q\rangle}{dt}\right| = 2\cos\phi(t)\sin\phi(t)\left|\frac{d\phi(t)}{dt}\right| \tag{7}$$

We know that:

$$\Delta H \Delta Q \geq \frac{\hbar}{2}\left|\frac{d\phi(t)}{dt}\right| \tag{8}$$

$$\boxed{\Rightarrow \frac{\Delta H}{\hbar} \geq \left|\frac{d\phi(t)}{dt}\right|} \tag{9}$$

If $\left|\frac{d\phi(t)}{dt}\right| = \frac{\Delta H}{\hbar}$, time taken to be self-orthogonal will be minimum, in this case, $\Delta t_{\perp}\left|\frac{d\phi(t)}{dt}\right| = \frac{\pi}{2} \Rightarrow \Delta t_{\perp} = \frac{h}{4\Delta H}$

$$\Rightarrow \boxed{\Delta H \Delta t_{\perp} \geq \frac{h}{4}} \tag{10}$$

b)

$$\frac{\Delta H}{\hbar} \geq \left|\frac{d\phi(t)}{dt}\right| \Rightarrow \frac{\Delta H t}{\hbar} \geq \left|\phi(t) - \phi(0)\right| \tag{11}$$

Since $\phi(0) = 0 \Rightarrow \frac{\Delta H t}{\hbar} \geq \left|\phi(t)\right|$

$$\Rightarrow \cos\frac{\Delta H t}{\hbar} \leq \cos\phi(t); \qquad t \leq \frac{\pi\hbar}{2\Delta H} \tag{12}$$

Using 1:

$$\boxed{|\langle\Psi(0)|\Psi(t)\rangle|^2 \geq \cos^2\left(\frac{\Delta H t}{\hbar}\right); \qquad t \leq \frac{\pi\hbar}{2\Delta H}} \tag{13}$$