3-P-7
8·05-2013

2. $H = -\gamma B S_z$

$$\Rightarrow U(t,t_0) = U(t,0) = \left(\cos\theta\, \mathbb{1} + i\sin\theta \frac{S_z}{\hbar/2}\right)$$

$$\theta = \frac{\gamma B t}{2}$$

$$\hat{S}_z(t) = U(0,t)\, \hat{S}_z\, U(t,0)$$

$$= \left(\cos\theta\, \mathbb{1} - i\sin\theta \frac{2S_z}{\hbar}\right) S_z \left(\cos\theta\, \mathbb{1} + \frac{2i\sin\theta\, S_z}{\hbar}\right)$$

$$= S_z$$

$$\hat{S}_x(t) = U^{\dagger}(t,0)\, \hat{S}_x\, U(t,0)$$

$$= \left(\cos\theta\, \mathbb{1} - i\sin\theta \frac{S_z}{\hbar/2}\right) S_x \left(\cos\theta\, \mathbb{1} + i\sin\theta \frac{S_z}{\hbar/2}\right)$$

$$= \cos^2\theta\, S_x + \sin^2\theta(-S_x)$$

$$\frac{i\cos\theta\sin\theta}{\hbar/2}(S_x S_z - S_z S_x)$$

$$= \cos 2\theta\, S_x + \frac{\sin 2\theta}{\hbar}[S_x ; S_z]$$

$$\Rightarrow \boxed{\hat{S}_x(t) = \cos(\gamma B t)\, S_x}$$

$$\hat{S}_y(t) = U^{\dagger}(t,0)\, \hat{S}_y\, U(t,0)$$

$$= \left(\cos\theta\, \mathbb{1} - i\sin\theta \frac{S_z}{\hbar/2}\right) \hat{S}_y \left(\cos\theta\, \mathbb{1} + i\sin\theta \frac{S_z}{\hbar/2}\right)$$

$$= \cos^2\theta\, \hat{S}_y + \sin^2\theta \frac{\hat{S}_z \hat{S}_y \hat{S}_z}{\hbar^2/4}$$

now $[\hat{S}_z , \hat{S}_x] = i\hbar \hat{S}_y$

$$\Rightarrow \hat{S}_x(t) = \cos 2\theta \, \hat{S}_x \, \cancel{\mp} \, i \cdot i \sin 2\theta \, \hat{S}_y$$

$$\Rightarrow \boxed{\hat{S}_x(t) = \cos(\gamma B t)\, \hat{S}_x + \cancel{\phi} \sin(\gamma B t)\, \hat{S}_y}$$

$$\hat{S}_y(t) = \left(\cos\theta \, \mathbb{1} - i \sin\theta \, \frac{S_z}{\hbar/2}\right) \hat{S}_y \left(\cos\theta \, \mathbb{1} + i \sin\theta \, \frac{S_z}{\hbar/2}\right)$$

$$= \cos^2\theta \, \hat{S}_y - \sin^2\theta \, \hat{S}_y + \frac{i \sin 2\theta}{\hbar} [\hat{S}_y , \hat{S}_z]$$

$$= \cos 2\theta \, \hat{S}_y - \sin 2\theta \, \hat{S}_{\cancel{y}x}$$

$$\Rightarrow \boxed{\hat{S}_y(t) = \cos(\gamma B t)\, \hat{S}_x - \sin(\gamma B t)\, \hat{S}_y}$$