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3-Pset-5
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3. a) $\langle u, Su\rangle = (u_1 \ u_2)\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{pmatrix}\begin{pmatrix} u_1 \\ u_2\end{pmatrix}$

$= S_{11}u_1^2 + S_{22}u_2^2 + (S_{12}+S_{21})(u_1u_2)$

$\langle u, Su\rangle = 0 \ \forall u \Rightarrow S_{11} = S_{22} = S_{12}+S_{21} = 0$

$\Rightarrow S_{12} = -S_{21}$ is possible s.t. $S_{12} \neq 0$

$\langle v, Tv\rangle = (v_1^* \ v_2^*)\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22}\end{pmatrix}\begin{pmatrix} v_1 \\ v_2\end{pmatrix}$

$= |v_1|^2 T_{11} + |v_2|^2 T_{22} + T_{12}v_1^* v_2 + T_{21} v_1 v_2^*$

$\langle v, Tv\rangle = 0 \Rightarrow T_{11} = T_{22} = T_{12} = T_{21} = 0$

since $v_1^* v_2 \neq v_1 v_2^*$ as identity

b) for general size matrix $(n \times n)$,

$\langle u, Su\rangle = \sum_{i,j} u_i u_j S_{ij}$

$\Rightarrow \langle u, Su\rangle = 0 \ \forall u \Rightarrow S_{ii} = 0 \ \forall i \in \{0, \dots, n\}$
$S_{ij} + S_{ji} = 0 \ \forall i \neq j$

$\langle v, Tv\rangle = \sum_{i,j} v_i^* v_j T_{ij}$

$\langle v, Tv\rangle = 0 \ \forall v \Rightarrow T_{ij} \ \forall i,j$

since each $v_i^\dagger v_j$ can be made different arbitrarily many times

c) Suppose an arbitrary vector be such that $\langle u, Su \rangle = 0$ $\forall u$, then $S_{ij} + S_{ji} = 0$, $S_{ii} = 0$

If we reduce it to upper triangular form,

$S_{ij} = 0 \quad \forall \; i > j \Rightarrow S_{ji} = 0 \quad \forall \; i < j$

$\Rightarrow$ S is a null vector

i.e. a contradiction

$\Rightarrow$ we can't reduce any arbitrary matrix $\in \mathcal{L}(\mathbb{R}^n)$ into upper triangular form