1-PSEFS
MZZ-OCCO
8-05-2015

1. $T_x = \exp\left(-\frac{i\hat{p}\hat{x}}{\hbar}\right)$ $\tilde{T}_p = \exp\left(\frac{ip\hat{x}}{\hbar}\right)$

a) $T_x^{\dagger}\hat{x}T_x = \exp\left(\frac{i\hat{p}\hat{x}}{\hbar}\right)\hat{x}\exp\left(-\frac{i\hat{p}\hat{x}}{\hbar}\right)$

By Hadomard's lemma,

$$T_x^{\dagger}\hat{x}T_x = \hat{x} + \left[\frac{i\hat{p}x}{\hbar}, \hat{x}\right]$$

$$= \hat{x} + x$$

$$\tilde{T}_p^{\dagger}\hat{p}\tilde{T}_p = \exp\left(-\frac{ip\hat{x}}{\hbar}\right)\hat{p}\exp\left(\frac{ip\hat{x}}{\hbar}\right)$$

$$= \hat{p} + \left[-\frac{ip\hat{x}}{\hbar}, \hat{p}\right]$$

$$= \hat{p} + p$$

b) $[T_x, \tilde{T}_p] = T_x\tilde{T}_p - \tilde{T}_pT_x$

$$= \exp\left(-\frac{i\hat{p}x}{\hbar}\right)\exp\left(\frac{ip\hat{x}}{\hbar}\right) - \exp\left(\frac{ip\hat{x}}{\hbar}\right)\exp\left(-\frac{i\hat{p}x}{\hbar}\right)$$

$$= \exp\left(\frac{ip\hat{x}}{\hbar}\right)\exp\left(-\frac{i\hat{p}x}{\hbar}\right)\left[\exp(-ixp) - 1\right]$$

$\Rightarrow$ commute whenever $xp = 2n\pi, n \in \mathbb{Z}$