

5. let $S = \begin{pmatrix} 1 & 1 & 1 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{pmatrix}$

Then, $A_2 \text{ Diagonalized} = S^{-1} A_2 S$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & -\sqrt{2} \\ 0 & -\sqrt{2} & 0 \end{pmatrix}$$

clearly $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is eigenvector with eigenvalue -3

now we solve $\left| \begin{pmatrix} 1 & -\sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix} - \lambda I \right| = 0$

$\Rightarrow 2$ is eigenvalue with $\frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \sqrt{2} \\ -1 \end{pmatrix}$ eigenvector

-1 is eigenvalue with $\frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ 1 \\ \sqrt{2} \end{pmatrix}$ eigenvector