5. a) $x = \beta u \Rightarrow \frac{\partial}{\partial x} = \frac{\partial}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial}{\partial u}\frac{1}{\beta}$

$\Rightarrow \frac{\partial^2}{\partial x^2} = \frac{1}{\beta^2}\frac{\partial^2}{\partial u^2}$

$\Rightarrow$ ~~So~~ TISE: $-\frac{\hbar^2}{2m\beta^2}\frac{\partial^2\psi}{\partial u^2} + \alpha\beta^4\psi = E\psi$

let a constant r be multiplied

$\Rightarrow -\frac{\hbar^2}{2m}\frac{r}{\beta^2}\frac{\partial^2\psi}{\partial u^2} + \alpha\beta^4 r \psi u^4 = rE\psi$

we have $\frac{\hbar^2 r}{m\beta^2} = 1 \quad \alpha\beta^4 r = 1$

$\Rightarrow \frac{m\alpha}{\hbar^2}\beta^6 = 1 \Rightarrow \beta = \left(\frac{\hbar^2}{m\alpha}\right)^{1/6}$

$\Rightarrow r = \frac{1}{\alpha}\left(\frac{m\alpha}{\hbar^2}\right)^{2/3}$

$\Rightarrow rE = e \Rightarrow \boxed{E = \alpha\left(\frac{\hbar^2}{m\alpha}\right)^{2/3} e}$

(Check that $\alpha\left(\frac{\hbar^2}{m\alpha}\right)^{2/3}$ has dimensions of energy

b) see wolfram notebook, wagging tail method from Griffiths Ch-2 footnotes is useful

c) we shall choose $\frac{\sqrt{2}}{\pi^{1/4}}\left(\frac{m\alpha}{\hbar^2}\right)^{1/4} x e^{-(m\alpha/\hbar^2)^{1/3}x^2/2}$

since $\left(\frac{m\alpha}{\hbar^2}\right)^{1/6}$ has dimens^n of length and $\psi(x)$ must have dimension of length$^{-1/2}$

$\frac{\sqrt{2}}{\pi^{1/4}}$ accounts for normalization,

However, we will solve problem in u-coordinate

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$$\langle H \rangle = \frac{\int u e^{-u^2/2} \hat{H}(u e^{-u^2/2}) du}{\int u^2 e^{-u^2/2} \cdot e^{-u^2/2} du}$$

$$= \frac{\int u e^{-u^2/2} \left(\frac{3u}{2} e^{-u^2/2} - \frac{u^3}{2} e^{-u^2/2} \right) + u^6 e^{-u^2} du}{\int u^2 e^{-u^2} du}$$

$$= \frac{\frac{2 \cdot 3}{2 \cdot 2} \frac{\sqrt{\pi}}{2} - \frac{3\sqrt{\pi}}{8} + \frac{15\sqrt{\pi}}{8}}{\frac{\sqrt{\pi}}{2}} = \frac{\frac{18\sqrt{\pi}}{\cancel{8}4}}{\frac{\sqrt{\pi}}{\cancel{2}}} = \frac{9}{2} = 4.5$$

Using wolfram, we find $\varepsilon_2 = 2.3936$

now $\psi_1(x)$ is even, our trial funcⁿ was odd

so $|c_1| = 0$ so automatically, 4.5 is an upper-bound for ε_2 indeed (Griffiths prob 7.5)