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Pset-4 P5

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$|\psi\rangle = a|1\rangle - b|2\rangle + c|3\rangle$ and $|\phi\rangle = b|1\rangle + a|2\rangle$

a) $\langle\psi| = a^*\langle 1| - b^*\langle 2| + c^*\langle 3|$ and $\langle\phi| = b^*\langle 1| + a^*\langle 2|$

$\langle\psi|\phi\rangle = a^*b - b^*a$ and $\langle\phi|\psi\rangle = b^*a - a^*b$, clearly, $\langle\psi|\phi\rangle$=$\langle\phi|\psi\rangle^*$

b) $|\phi\rangle = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix}$ and $|\psi\rangle = \begin{pmatrix} a \\ -b \\ c \end{pmatrix}$, while $\langle\psi| = \begin{pmatrix} a^* & -b^* & c^* \end{pmatrix}$ and $\langle\phi| = \begin{pmatrix} b^* & a^* & 0 \end{pmatrix}$

c) $A = \begin{pmatrix} ba^* & -|b|^2 & bc^* \\ |a|^2 & -ab^* & ac^* \\ 0 & 0 & 0 \end{pmatrix}$

d) Q is Hermitian because all projection operators are Hermitian. Q can have a 0 eigenvalue because we are in a three-dimensional space and there is a third vector, $|v\rangle$ out there perpendicular to $|\psi\rangle$ and $|\phi\rangle$ and it's projection on $|\psi\rangle$ and $|\phi\rangle$ is 0,

$\Rightarrow Q|v\rangle = 0$