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40. $\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{x})$

$H|a\rangle = E_a|a\rangle$

a) $[[\hat{x},\hat{H}],\hat{x}] = 2\hat{x}\hat{H}\hat{x} - \hat{H}\hat{x}^2 - \hat{x}^2\hat{H}$

$\hat{H}$ contains $V(\hat{x})$ which will commute with $\hat{x}$

$\Rightarrow \quad 2\hat{x}\frac{\hat{p}^2}{2m}\hat{x} - \frac{\hat{p}^2}{2m}\hat{x}^2 - \frac{\hat{x}^2\hat{p}^2}{2m}$

$[\hat{x},\hat{p}^2]\frac{\hat{x}}{2m} - \frac{\hat{x}}{2m}[\hat{x},\hat{p}^2]$

$= \frac{i\hbar}{m}[\hat{p}\hat{x} - \hat{x}\hat{p}] = \frac{\hbar^2}{m}$

$\Rightarrow (2\hat{x}\hat{H}\hat{x} - \hat{H}\hat{x}^2 - \hat{x}^2\hat{H})|a\rangle = \frac{\hbar^2}{m}|a\rangle$

$\Rightarrow \langle a|2\hat{x}\hat{H}\hat{x} - \hat{H}\hat{x}^2 - \hat{x}^2\hat{H}|a\rangle = \frac{\hbar^2}{m}$ (Assuming $\langle a|a\rangle = 1$)

~~$\Rightarrow 2\langle a|\hat{x}|a'\rangle\langle a'|\hat{H}\hat{x}|a\rangle - \langle a|\hat{H}|a'\rangle\langle a'|\hat{x}^2|a\rangle - \langle a|\hat{x}^2|a'\rangle$~~

The first term is $\sum_{a'} 2\langle a|\hat{x}|a'\rangle\langle a'|\hat{H}|\hat{x}|a\rangle$

since By completeness $\sum_{a'}|a'\rangle\langle a'| = I$

now $\langle a'|\hat{H} = E_{a'}\langle a'|$

$\Rightarrow$ first term is $\sum_{a'} 2\langle a|\hat{x}|a'\rangle\langle a'|\hat{x}|a\rangle E_{a'}$

since $\hat{x}$ is hermitian $\Rightarrow \langle a|\hat{x}|a'\rangle = \langle a'|\hat{x}|a\rangle^*$

$\Rightarrow$ first term is : $\sum_{a'} 2E_{a'}|\langle a|\hat{x}|a'\rangle|^2$

second term is : $-\langle a|\hat{H}|\hat{x}^2|a\rangle = -E_a\langle a|\hat{x}^2|a\rangle$

$= -E_a\sum_{a'}\langle a|\hat{x}|a'\rangle\langle a'|\hat{x}|a\rangle = -E_a\sum_{a'}|\langle a|\hat{x}|a'\rangle|^2$

third term is also same

thus $\sum_{a'}|\langle a|\hat{x}|a'\rangle|^2(E_{a'} - E_a) = \frac{\hbar^2}{2m}$

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b) Measurement of spin along z-axis gives $+\frac{\hbar}{2}$ with prob. $\frac{1}{2}$

$$\Rightarrow \langle H \rangle = \frac{1}{2} \cdot \frac{-\gamma B \hbar}{2} + \frac{1}{2} \frac{\gamma B \hbar}{2} = 0$$

$$\langle H^2 \rangle = \frac{1}{2} \frac{\gamma^2 B^2 \hbar}{4} + \frac{1}{2} \frac{\gamma^2 B^2 \hbar^2}{4} = \frac{\gamma^2 B^2 \hbar^2}{4}$$

(same value is obtained by $\langle \psi(0) | H | \psi(0) \rangle$)

$$\Rightarrow \Delta H = \frac{\gamma B h}{4\pi}$$

while $\cos^2(\theta(t)) = |\langle \psi(0) | \psi(t) \rangle|^2$

$$= \left[\cos\left(\frac{\gamma B \hbar t}{2\hbar}\right)\right]^2, \text{ for orthogonality, } \frac{\gamma B \cancel{\hbar} t}{2\cancel{\hbar}} = \frac{\pi}{2}$$

$$\Rightarrow \Delta t_\perp = \frac{\pi}{\gamma B}$$

$$\Rightarrow \boxed{\Delta H \Delta t_\perp = \frac{h}{4}}$$

Inequality is saturated thus

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c) Virial's Theoram:

$$[\hat{x}\hat{p}, \hat{H}] = \hat{x}\hat{p}\hat{H} - \hat{H}\hat{x}\hat{p}$$

$$\langle a| \hat{x}\hat{p}\hat{H} - \hat{H}\hat{x}\hat{p} |a\rangle = E\langle a| \hat{x}\hat{p} - \hat{x}\hat{p} |a\rangle = 0$$

$$\Rightarrow \hat{x}\hat{p}\hat{H} - \hat{H}\hat{x}\hat{p} = \frac{\hat{x}\hat{p}^3}{2m} + \hat{x}\hat{p}V(\hat{x}) - \frac{\hat{p}^2\hat{x}\hat{p}}{2m} - \cancel{\hat{x}\hat{p}}\, V(\hat{x})\hat{x}\hat{p}$$

$$= \frac{i\hbar\hat{p}^2}{m} + \hat{x}\hat{p}V(\hat{x}) - V(\hat{x})\hat{x}\hat{p}$$

$= \cancel{\frac{i\hbar \hat{p}^2}{m}}$ in x-basis, $[\hat{p}, V(\hat{x})]$ is

$= \cancel{i\hbar} = \cancel{p}\ \hat{p}V(\hat{x})f - V(\hat{x})\hat{p}f$

$$= -i\hbar \frac{\partial}{\partial x} V(x) f$$

$$\Rightarrow \hat{x}\hat{p}V(\hat{x}) - V(\hat{x})\hat{x}\hat{p} = -i\hbar\, \hat{x}\frac{\partial V(\hat{x})}{\partial x}$$

$$\Rightarrow \langle a|\frac{\hat{p}^2}{m}|a\rangle - \langle a|\hat{x}\frac{\partial V(\hat{x})}{\partial x}|a\rangle = 0$$

~~ex~~

or $$\boxed{\langle a|\frac{\hat{p}^2}{2m}|a\rangle = \frac{1}{2}\langle a|\hat{x}\,\partial_{\hat{x}} V(\hat{x})|a\rangle}$$

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