1 - PSET-4
MIT OCW - 2013
8.05

1. $A, B, C, q, p \rightarrow$ linear op.

a) $[A, BC] = [A,B]C + B[A,C]$

$$= ABC - BCA$$

$$= \underbrace{ABC - BAC} + \underbrace{BAC - BCA}$$

$$= [A,B]C + B[A,C]$$

QED

b) $[[A,B],C] + [[B,C],A] + [[C,A],B] =$

$$\cancel{ABC} - \cancel{BAC} - \cancel{CAB} + \cancel{CBA} +$$

$$\cancel{BCA} - \cancel{CBA} - \cancel{ABC} + \cancel{ACB} +$$

$$\cancel{CAB} - \cancel{ACB} - \cancel{BCA} + \cancel{BAC}$$

$$= 0$$

Thus, Jacobi identity established

c) $[q, p] = i\hbar$

$$[q^n, p] = q^n p - p q^n = \underbrace{q^n p - q^{n-1} p q} + \underbrace{q^{n-1} p q - p q^n}$$

$$= q^{n-1} i\hbar + [q^{n-1}, p] q$$

$$\Rightarrow [q^n, p] = q^{n-1} i\hbar + (q^{n-2} i\hbar + [q^{n-2}, p]q) q$$

$$= 2q^{n-1} i\hbar + [q^{n-2}, p] q^2$$

Doing this iteratively for $n-3$ more times:

$$= (2 + n - 3) q^{n-1} i\hbar + [q, p] q^{n-1}$$

$$\Rightarrow \boxed{[q^n, p] = n i\hbar q^{n-1}}$$

2 - PSET 4
8.05 - 2013
MIT OCW

d) Given $f(q) = \sum_{n=0}^{\infty} a(n) q^n$ and $[q,p] = i\hbar \Rightarrow [q^n, p] = i\hbar q^{n-1} n$

$$\Rightarrow [f(q), p] = \sum_{n=0}^{\infty} a[n] [q^n, p]$$

$$= i\hbar \sum_{n=0}^{\infty} n a(n) q^{n-1}$$

$$= i\hbar f'(q)$$

QED

e) $$[f(x), p]\psi = f \frac{\hbar}{i} \frac{\partial \psi}{\partial x} - \frac{\hbar}{i} \frac{\partial}{\partial x}(f\psi)$$

$$= i\hbar \frac{\partial f}{\partial x} \psi$$

This follows from d)