2. $\langle x|\hat{p}|\psi\rangle = \frac{\hbar}{i}\frac{d}{dx}\langle x|\psi\rangle$

a) $\langle x|\hat{p}^n|\psi\rangle = \left(\frac{\hbar}{i}\frac{d}{dx}\right)^n \psi(x)$

$\langle x|\hat{p}^n|\psi\rangle = \langle x|\hat{p}\,(\hat{p}^{n-1})|\psi\rangle$

$= \frac{\hbar}{i}\frac{d}{dx}\langle x|\hat{p}^{n-1}|\psi\rangle$

Thus, recursively,

$\langle x|\hat{p}^n|\psi\rangle = \left(\frac{\hbar}{i}\frac{d}{dx}\right)^n \langle x|\psi\rangle = \left(\frac{\hbar}{i}\frac{d}{dx}\right)^n \psi(x)$

b) Given that $\langle x|p\rangle = \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}$

$\langle p|\hat{x}|\psi\rangle = \int dx\,\langle p|x\rangle\langle x|\hat{x}\psi\rangle = \int dx\,(x\langle p|x\rangle)\langle x|\psi\rangle$

$x\langle p|x\rangle = i\hbar\frac{d}{dp}\langle p|x\rangle$

$\Rightarrow \langle p|\hat{x}|\psi\rangle = \int dx\left(i\hbar\frac{d}{dp}\langle p|x\rangle\right)\langle x|\psi\rangle$

$= i\hbar\frac{d}{dp}\int dx\,\langle p|x\rangle\langle x|\psi\rangle$

$= i\hbar\frac{d}{dp}\langle p|\psi\rangle$

c) $[\hat{x},\hat{p}]\langle p|\psi\rangle = \left(i\hbar\frac{d}{dp}(\hat{p}|\psi\rangle) - \hat{p}\,i\hbar\frac{d}{dp}|\psi\rangle\right)$

$= i\hbar\frac{d}{dp}(p\psi(p)) - p\,i\hbar\frac{d}{dp}(\psi(p))$

$= i\hbar\,\psi(p)$

Thus $[\hat{x},\hat{p}] = i\hbar$