4. $$\frac{d}{dt}\Omega_H(t) = \frac{d}{dt}U^\dagger(t)(\vec{r}\cdot\vec{p})U(t) + U^\dagger(t)(\vec{r}\cdot\vec{p})\frac{d}{dt}U(t)$$

$$= \frac{i}{\hbar}U^\dagger H(\vec{r}\cdot\vec{p})U(t) - U^\dagger(\vec{r}\cdot\vec{p})HU$$

$$= U^\dagger[H, \vec{r}\cdot\vec{p}]U$$

$$= \frac{i}{\hbar}[H_H, \vec{r}_H\cdot\vec{p}_H]$$

$$= \frac{i}{\hbar}\left(H_H(\vec{r}_H\cdot\vec{p}_H) - (\vec{r}_H\cdot\vec{p}_H)H_H\right)$$

$$= \frac{i}{\hbar}\left(H_H r p_r - r p_r H_H\right)$$

$$= \frac{i}{\hbar}\left[\frac{\hat{p}_r^2 r p_r}{2m} - \frac{r\hat{p}_r^3}{2m} + V r p_r - r p_r V\right]$$

$$= \frac{i}{\hbar}\left[-\frac{i\hbar\,\hat{p}_r^2}{m} + r i\hbar\frac{\partial V}{\partial r}\right]$$

$$= \frac{1}{\hbar}\frac{\hat{p}_r^2}{m} - \frac{r}{\hbar}\partial_r V = \frac{1}{\hbar m}\hat{p}_{rH}^2 - \frac{r_H}{\hbar}\partial_r V_H$$

b) for stationary state ; $U(t) = \mathbb{1}\cdot e^{-iEt/\hbar}$

$$\Rightarrow \frac{d}{dt}O_H(t) = \frac{dU^\dagger}{dt}OU + U^\dagger O\frac{dU}{dt}$$

$$= \frac{iE}{\hbar}\left(U^\dagger OU - U^\dagger OU\right)$$

$$\langle\psi,0|\frac{d}{dt}O_H(t)|\psi,0\rangle = \frac{iE}{\hbar}\langle\psi,t|O|\psi,t\rangle$$

But expectation of any operator is constant for a stationary state $\Rightarrow \langle\psi,0|\frac{d}{dt}O_H(t)|\psi,0\rangle = 0$

if O is TISO, $\psi$ is stationary

c) $O_H = \vec{r}\cdot\vec{p}$

$$\Rightarrow \langle \psi, 0 | \frac{d}{dt}(\vec{r}\cdot\vec{p}) | \psi, 0 \rangle = 0$$

$$\Rightarrow \langle \psi, 0 | \frac{\hat{p}_r^2}{m} - r\partial_r V_r | \psi, 0 \rangle = 0$$

$$\Rightarrow \cancel{2\langle T \rangle} \quad r\partial_r V_r = r\frac{\partial}{\partial r}\frac{c}{r^k} = \frac{-kc}{r^k} = -kV$$

$$\Rightarrow 2\langle T \rangle = -k\langle V \rangle$$

$$\Rightarrow \boxed{\langle T \rangle = -\frac{k}{2}\langle V \rangle}$$