Assignment 1

AI1110: Probability and Random Variables
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12.13.6.3: Question. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer: $\frac{20}{21}$.

Solution: Let us conside a random variable X which depicts whether a person is a male of female.

$X = 0$, a person being a male
$X = 1$, a person being a female

As given the question that there are equal number of males and females,

$$\Pr(X = 0) = \frac{1}{2}$$
$$\Pr(X = 1) = \frac{1}{2}$$

And let E be the event where a person has grey hair. Given,

$$\Pr(E|X = 0) = \frac{5}{100}$$
$$\Pr(E|X = 1) = \frac{0.25}{100}$$

Now ,the probability that the selected person is male given that he's grey haired is $\Pr(X = 0|E)$ which is equal to

$$\Pr(X = 0|E) = \frac{\Pr(E + (X = 0))}{\Pr(E)}$$
$$\Rightarrow \frac{\Pr(E|X = 0)\Pr(X = 0)}{\Pr(E)}$$
$$\Rightarrow \frac{\Pr(E|X = 0)\Pr(X = 0)}{\Pr(E|X = 0)\Pr(X = 0) + \Pr(E|X = 1)\Pr(X = 1)}$$

Substitutuing all the values,

$$\Pr(X = 0|E) = \frac{\frac{5}{100}\frac{1}{2}}{\frac{5}{100}\frac{1}{2} + \frac{0.25}{100} * \frac{1}{2}}$$
$$= \frac{5}{5.25}$$
$$= \frac{20}{21}$$

Therefore, the required probability is $\frac{20}{21}$.