Assignment 1

AI1110: Probability and Random Variables
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12.13.6.3: Question. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer: $\frac{20}{21}$.

Solution: Let us conside a random variable X which depicts whether a person is a male of female.

$$X = \begin{cases} 1 & \text{a person being a male} \\ 0 & \text{a person being a female} \end{cases} \tag{1}$$

As given the question that there are equal number of males and females,

$$\Pr(X = i) = \begin{cases} \frac{1}{2} & i = 0 \\ \frac{1}{2} & i = 1 \end{cases} \tag{2}$$

And let E be the event where a person has grey hair. Given,

$$\Pr(E|X = i) = \begin{cases} \frac{5}{100} & i = 0 \\ \frac{0.25}{100} & i = 1 \end{cases} \tag{3}$$

Now ,the probability that the selected person is male given that he's grey haired is $\Pr(X = 0|E)$ which is equal to

$$\Pr(X = 0|E) = \frac{\Pr(E, (X = 0))}{\Pr(E)} \tag{4}$$

$$= \frac{\Pr(E|X = 0)\Pr(X = 0)}{\Pr(E)} \tag{5}$$

$$= \frac{\Pr(E|X = 0)\Pr(X = 0)}{\Pr(E|X = 0)\Pr(X = 0) + \Pr(E|X = 1)\Pr(X = 1)} \tag{6}$$

Substituing eqs.(2) and(3) in(6),

$$\Pr(X = 0|E) = \frac{\frac{5}{100}\frac{1}{2}}{\frac{5}{100}\frac{1}{2} + \frac{0.25}{100} * \frac{1}{2}} \tag{7}$$

$$= \frac{5}{5.25} \tag{8}$$

$$= \frac{20}{21} \tag{9}$$

Therefore, the required probability is $\frac{20}{21}$.