# Assignment 1

**AI1110**: Probability and Random Variables
Indian Institute of Techonology Hyderabad

### S Bhavya Shloka
### CS22BTECH11056

**12.13.6.3: Question**. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

**Answer:** $\frac{20}{21}$.

**Solution**: Let us conside a random variable X which depicts whether a person is a male of female.

$$X = \begin{cases} 1 & \text{a person being a male} \\ 0 & \text{a person being a female} \end{cases} \quad (1)$$

As given the question that there are equal number of males and females,

$$\Pr(X = i) = \begin{cases} \frac{1}{2} & i = 0 \\ \frac{1}{2} & i = 1 \end{cases} \quad (2)$$

And let E be the event where a person has grey hair. Given,

$$\Pr(E|X = i) = \begin{cases} \frac{5}{100} & i = 0 \\ \frac{0.25}{100} & i = 1 \end{cases} \quad (3)$$

Now ,the probability that the selected person is male given that he's grey haired is $\Pr(X = 0|E)$ which is equal to

$$\Pr(X = 0|E) = \frac{\Pr(E, X = 0)}{\Pr(E)} \quad (4)$$

$$= \frac{\Pr(E|X = 0) \times \Pr(X = 0)}{\Pr(E)} \quad (5)$$

$$= \frac{\Pr(E|X = 0) \times \Pr(X = 0)}{\sum_{i=0}^{1} \Pr(E|X = i) \times \Pr(X = i)} \quad (6)$$

Substituing eqs.(2) and(3) in(6),

$$\Pr(X = 0|E) = \frac{\frac{5}{100} \times \frac{1}{2}}{\frac{5}{100} \times \frac{1}{2} + \frac{0.25}{100} \times \frac{1}{2}} \quad (7)$$

$$= \frac{5}{5.25} \quad (8)$$

$$= \frac{20}{21} \quad (9)$$

Therefore, the required probability is $\frac{20}{21}$.