## Sinusoidal Time Embedding Explanation and Implementation

## 1 Mathematical Formulation

Given a scalar timestep  $t \in \mathbb{R}$  and an embedding dimension  $d \in \mathbb{N}$  (where d is even), the sinusoidal time embedding is computed using alternating sine and cosine functions applied at exponentially scaled frequencies. The output is a vector  $\mathbf{e}_t \in \mathbb{R}^d$  defined as:

$$e_t[2i] = \sin\left(\frac{t}{10000^{2i/d}}\right)$$
 
$$e_t[2i+1] = \cos\left(\frac{t}{10000^{2i/d}}\right)$$

for  $i = 0, 1, \dots, \frac{d}{2} - 1$ .

This allows to encode continuous, smooth representations of time.

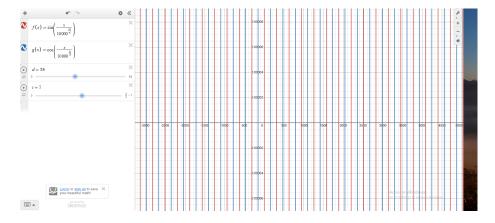


Figure 1: Cool desmos graph of the above functions.

## 2 Explanation

- The embedding uses a fixed, non-learned set of sinusoidal functions (compared to other methods where we use machine learning and fully-connected layers to learn the representation of time embeddings).
- Even indices (0, 2, 4, ...) use the sine function.

• Odd indices (1, 3, 5, ...) use the cosine function.

## 3 PyTorch Implementation

```
# My version
   def get_time_embeddings(timesteps: torch.Tensor, embedding_dim: int) -> torch.Tensor:
2
       assert embedding_dim % 2 == 0, "embedding_dim must be even"
3
        # Create the frequency spectrum
       half_dim = embedding_dim // 2
6
       exponent = -math.log(10000.0) / (half_dim - 1)
       freq = torch.exp(torch.arange(half_dim, dtype=torch.float32) * exponent)
        # Expand timesteps for broadcasting
10
       timesteps = timesteps.float().unsqueeze(1) # (N, 1)
11
        args = timesteps * freq.unsqueeze(0)
                                                    # (N, half_dim)
12
13
        # Concatenate sin and cos
14
       embedding = torch.cat([torch.sin(args), torch.cos(args)], dim=1) # (N, embedding_dim)
15
       return embedding
```