

# The Portable Document Format and pdfTeX

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*April, 2025*

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# 1 Introduction

The Portable Document Format (PDF) is a file format developed by the Adobe corporation in 1992 to render and display documents. It is a rich file format capable of displaying a diverse variety of documents, and its immense popularity has lead to its use around the world. Still today it is the leading format for displaying documents in a cross-platform manner. It has undergone various updates and standardizations, keeping it modern and usable.

T<sub>E</sub>X is a program and language for typesetting (generally academic) documents. Historically, it compiled to a dvi (device independent) file format, but a more modern T<sub>E</sub>X engine called pdfT<sub>E</sub>X was developed to compile to PDF. T<sub>E</sub>X includes a powerful macro-based programming layer, as well as a versatile typesetting engine. We assume basic knowledge of plain-T<sub>E</sub>X for this article (if you don't know what plain-T<sub>E</sub>X is, try reading the T<sub>E</sub>Xbook by D. Knuth before this article).

Despite the multitude of literature on PDF as well as T<sub>E</sub>X, there exists little literature on pdfT<sub>E</sub>X. Specifically, there does not exist much literature on how to utilize pdfT<sub>E</sub>X-primitives to create PDF graphics. The pdfT<sub>E</sub>X manual lists primitives, but does not give explanations on how to use them, and instead assumes intimate familiarity with PDF. In this article we will both explain PDF as well as how to utilize pdfT<sub>E</sub>X primitives to create PDFs.

We give thanks to the resources which were invaluable for this article:

- The pdfT<sub>E</sub>X manual by Hàn Thê Thành and team;
- The PDF reference by Adobe;
- Petr Olšák's article on pdfT<sub>E</sub>X primitives;
- The TikZ and PGF manual, as well as PGF source code.
- The paper <https://dl.acm.org/doi/10.1145/964965.808606> by Porter and Duff.

## Structure of the article

This article will be split largely into two parts: an introduction to PDF, and an explanation of pdfT<sub>E</sub>X primitives. The first part will not cover the entirety of the PDF, and we will leave certain subjects for the second part as well.

# I. THE PORTABLE DOCUMENT FORMAT

The Portable Document Format (henceforth, PDF) is a powerful format for displaying documents. In this section we will discuss certain features of the PDF, focusing on its structure. In the following section we will discuss how to utilize pdfTeX primitives to alter the output PDF.

# 1 Datatypes and Functions

## 1.1 Datatypes

PDF supports the following datatypes:

- booleans;
- integers;
- real numbers;
- strings;
- names;
- arrays;
- dictionaries;
- streams;
- the null object.

### 1.1.1 Booleans

Booleans are determined by the keywords **true** and **false**. Integers are written with or without a sign, and a decimal point turns a number into a real number.

### 1.1.2 Strings

There are two ways to write a string: enclosing characters in parentheses, or, as hexadecimal data wrapped in single angle brackets (<...>). So for example, (hello world) represents the string “hello world”. So does <68656C6C6F776F726C64>. To add special characters (line feed, unbalanced parentheses, etc) you can add a backslash before (as one would normally).

### 1.1.3 Names

A name begins with a forward slash /, and may contain any characters except for whitespace and delimiter characters (brackets, parentheses and friends, forward slash, or a percent sign). You can add any non-null character to a name (including special characters) by preceding its hexadecimal code with #. So for example, the following are names:

`/name, /name*with_special&characters, /#28parentheses#29`

### 1.1.4 Arrays

Arrays are one-dimensional array objects, written in square brackets. They are delimited by whitespace, and can include any other object as a member. For example, the following is an array:

`[1 1.2 true (S. Lurp) /A_Name [true false]]`

As displayed in the above example, an array can contain an array as well, thus allowing multi-dimensional arrays.

### 1.1.5 Dictionaries

A dictionary is a mapping between name objects and instances of any datatype. A dictionary is enclosed in double angle brackets, like so:

```
1 <<
2   /Type /A_Dictionary
3   /Subtype /Example
4   /IntItem 12
5   /NumItem 1.2
6   /StringItem (hello world)
7   /ArrItem [12 1.2 (hello world)]
8   /DictItem <<
9     /Item1 true
10    /Item2 false
```

```
11 >>
12 >>
```

Dictionaries are of extreme importance in PDFs. They will show up a lot later.

### 1.1.6 Streams

A stream is similar to a string, except for a few key differences:

- (1) a string must be read in its entirety, while a stream may be read incrementally;
- (2) a string has a maximum limit based on implementation, while a stream has no such limit (which is why larger data like images or pages are stored in streams).

A stream is structured as follows:

```
1 << dictionary >>
2 stream
3 ...
4 endstream
```

A stream must be an *indirect object*. The stream's dictionary which precedes the `stream...endstream` must have a `/Length` field which is equal to the byte length of the stream's content (...).

### 1.1.7 Null

The null object is an object referencable by the keyword `null`. It is not equal to any other object. An indirect object which references a non-existent object is equivalent to the null object. And giving a dictionary entry the value `null` is equivalent to omitting the entry.

### 1.1.8 Indirect Objects

Any object can be labeled as an *indirect object*. This allows other objects to reference it via a unique *object identifier*. The object identifier has two parts:

- (1) a unique positive integer called the *object number*;
- (2) a (not necessarily unique) non-negative integer called the *generation number*. For our purposes, these are always zero.

An indirect object is declared using the `obj` postfix operator. Preceding it is the object identifier. The object's value itself is written between the `obj` and `endobj` keywords. For example:

```
1 12 0 obj
2   [1 2 3]
3 endobj
```

Creates an array indirect object of value `[1 2 3]`. Its object number is 12, and its generation number is 0. To reference this object, simply use the *indirect reference* `12 0 R`.

For example, to create an indirect stream object, you could do:

```
1 7 0 obj
2   << /Length 8 0 R >>
3   stream
4     BT
5       /F1 12 Tf
6       72 712 Td
7       (A stream with an indirect length) Tj
8     ET
9   endstream
10 endobj
11
12 8 0 obj
13   77
14 endobj
```

This defines object `7 0` to be a stream object containing the above contents. Its length is an indirect reference to object `8 0`, whose value is 77.

## 1.2 Trees

A tree is a composite datatype made up of other datatypes. Its purpose is similar to a dictionary: it maps keys to values, but by different means. They vary in some ways:

- (1) the keys in a tree are either strings (name trees) or numbers (number trees);
- (2) the keys are ordered;
- (3) the values associated with the keys may be of any type (including `null`);
- (4) a tree can represent an arbitrarily large map, and can be read in parts, unlike a dictionary.

There are two types of trees: name trees and number trees. The difference between them is the datatype of their keys: name trees use strings as keys while number trees use numbers.

Every tree is constructed from nodes, which are dictionary objects. There are three kinds of nodes: a root node, intermediate nodes, and leaf nodes. The meaning of these are self-explanatory.

### 1.2.1 Number trees

A number tree node is a dictionary with the following fields:

Key	Type	Value
<b>Kids</b>	array	(Root and intermediate nodes only; present in root only if <b>Names</b> isn't) an array of indirect references to the children of this node (either intermediate or leaf nodes).
<b>Names</b>	array	(Root and leaf nodes; present in root only if <b>Kids</b> isn't) an array of the form <div style="text-align: center;"> <math>[key_1 \ value_1 \ \dots \ key_n \ value_n]</math> </div> <p>where each <math>key_i</math> is a string, and the <math>value_i</math> is the associated value. The keys are sorted by lexical value as explained below.</p>
<b>Limits</b>	array	(Intermediate and leaf nodes only) an array of two strings, specifying the lexically least and greatest keys included in the <b>Names</b> array of a leaf node, or in the case of an intermediate node, the <b>Names</b> of the children nodes.

For example, we may have the following tree which describes the grades of students, say:

```

1 1 0 obj      % root
2  <<
3     /Kids
4     [ 2 0 R
5       3 0 R ]
6  >>
7 endobj
8
9 2 0 obj      % intermediate
10 <<
11    /Limits [(Andrew) (Gordon)]
12    /Kids
13    [ 4 0 R
14      5 0 R
15      6 0 R ]
16  >>
17 endobj
18
19 3 0 obj      % intermediate
20 <<
21    /Limits [(Howard) (Zack)]
22    /Kids
23    [ 7 0 R
24      8 0 R ]
25  >>
26 endobj
27
28 4 0 obj      % leaf
29 <<
30    /Limits [(Andrew) (Avery)]
31    /Names
32    [ (Andrew) 100
33      (Avery) 80 ]
34  >>
35 endobj

```

```

36
37 5 0 obj      % leaf
38 <<
39   /Limits [(Bob) (Dylan)]
40   /Names
41   [ (Bob) 90
42     (Chris) 100
43     (Drew) 100
44     (Dylan) 60 ]
45 >>
46 endobj
47
48 6 0 obj      % leaf
49 <<
50   /Limits [(Fred) (Gordon)]
51   /Names
52   [ (Fred) 50
53     (Gordon) 85 ]
54 >>
55 endobj
56
57 7 0 obj      % leaf
58 <<
59   /Limits [(Howard) (Howard)]
60   /Names
61   [ (Howard) 10 ]
62 >>
63 endobj
64
65 7 0 obj      % leaf
66 <<
67   /Limits [(Zack) (Zack)]
68   /Names
69   [ (Zack) 70 ]
70 >>
71 endobj

```

### 1.2.2 Number trees

A number tree is similar to a name tree except that its keys are integers instead of strings, and are sorted in ascending numerical order. And instead of the key-value array being named **Names**, it is named **Nums**.

## 1.3 Functions

An important quote from the PDF reference:

*“PDF is not a programming language, and a PDF file is not a program.”*

Despite this, PDF provides the ability to define certain kinds of functions. Though of course their use is limited and restricted.

All PDF functions are pure functions  $\mathbb{R}^m \rightarrow \mathbb{R}^n$  (pure meaning they have no side-effects). Importantly, their inputs and outputs must be numbers, not just any PDF datatype. PDF functions must have a domain defined in their definition. If a function has a domain of, say,  $[-1, 1]$  and is called with input 6, the input will be clipped to the domain; so the function will be called with input 1. Similarly some functions may define a range, and the output may be similarly clipped.

A function may be either a dictionary or a stream. A *function dictionary* refers either directly to the function (if it is a dictionary) or to the stream dictionary (if it is a stream). The dictionary must provide a **FunctionType** entry, which is one of 0, 2, 3, 4. For a function of type  $\mathbb{R}^m \rightarrow \mathbb{R}^n$ , the function dictionary may have the following fields (in addition to fields specific to the function type).

Key	Type	Value
<b>FunctionType</b>	integer	(Required) the function type.
<b>Domain</b>	array	(Required) an array of $2m$ numbers. For $0 \leq i \leq m - 1$ , <b>Domain</b> <sub><math>2i</math></sub> must be less than or equal to <b>Domain</b> <sub><math>2i+1</math></sub> . The domain of the function is

$$\prod_{0 \leq i \leq m-1} [\mathbf{Domain}_{2i}, \mathbf{Domain}_{2i+1}]$$



<b>Range</b>	array	(Required for type 0 and 4 functions, otherwise required) an array of $2n$ numbers. Similar to the domain, for every $0 \leq i \leq n-1$ , $\mathbf{Range}_{2i} \leq \mathbf{Range}_{2i+1}$ . The range (codomain) of the function is
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$$\prod_{0 \leq i \leq n-1} [\mathbf{Range}_{2i}, \mathbf{Range}_{2i+1}]$$

### 1.3.1 Type 0 (Sampled) Functions

Type 0 functions use a sequence of sampled values (which are contained in a stream) to approximate a function whose domain and range are both bounded. In addition to the fields already listed, the function dictionary of a type 0 function may include the following fields as well:

Key	Type	Value
<b>Size</b>	array	(Required) an array of $m$ positive integers which specifies the number of samples in each input dimension.
<b>BitsPerSample</b>	integer	(Required) the number of bits used to represent each sample. Valid values are 1, 2, 4, 8, 16, 24, 32.
<b>Order</b>	integer	(Optional) the order of interpolation between samples. Valid values are 1 and 3, specifying linear and cubic spline interpolation, respectively. The default value is 1.
<b>Encode</b>	array	(Optional) an array of $2m$ numbers specifying a linear mapping of input values into the domain of the function's sample table. The default value is $[0 (\mathbf{Size}_0 - 1) 0 (\mathbf{Size}_1 - 1) \dots]$ .
<b>Decode</b>	array	(Optional) an array of $2n$ numbers specifying a linear mapping of sample values into the range appropriate for the function's output values.

The dictionary may include other fields common to stream objects. Given an input dimension of  $m$ , we must have  $\prod_{0 \leq i < m} \mathbf{Size}_i$  values in the stream. In order, these give the multi-dimensional array

$$g(0, \dots, 0), g(1, \dots, 0), \dots, g(\mathbf{Size}_0 - 1, 0, \dots, 0), g(\mathbf{Size}_0 - 1, 1, \dots, 0), \dots, g(\mathbf{Size}_0 - 1, \dots, \mathbf{Size}_{m-1} - 1)$$

We now describe how to use  $g$  to compute  $f$ , the type 0 function.

To explain how the function is calculated, we first define the following function:

$$\text{Interpolate}(x; x_0, x_1, y_0, y_1) = y_0 + \left( (x - x_0) \cdot \frac{y_1 - y_0}{x_1 - x_0} \right)$$

this simply projects  $x$  onto the line between  $(x_0, y_0)$  and  $(x_1, y_1)$ .

When a sampled function is called with input values  $(x_0, \dots, x_{m-1})$ , the following steps are taken in order to compute the result:

- (1) Each  $x_i$  is clipped to the domain:

$$x'_i = \min(\max(x_i, \mathbf{Domain}_{2i}), \mathbf{Domain}_{2i+1})$$

- (2) The input value is then encoded:

$$e_i = \text{Interpolate}(x'_i; \mathbf{Domain}_{2i}, \mathbf{Domain}_{2i+1}, \mathbf{Encode}_{2i}, \mathbf{Encode}_{2i+1})$$

That is, given an input  $x$ , we project it onto the line whose endpoints are  $(\mathbf{Domain}_{2i}, \mathbf{Encode}_{2i})$  and  $(\mathbf{Domain}_{2i+1}, \mathbf{Encode}_{2i+1})$ . The effect is that the lower end of the domain is mapped to  $\mathbf{Encode}_{2i}$  and the higher end is mapped to  $\mathbf{Encode}_{2i+1}$ .

- (3) Then the input value is clipped

$$e'_i = \min(\max(e_i, 0), \mathbf{Size}_i - 1)$$

This gives us a real matrix  $(e'_m, \dots, e'_{m-1})$  which is the encoding of the input vector.

(4) We can then use interpolation (of order **Order**) to compute  $g(e') = r$ .

(5) We interpolate  $r$  in order to decode it:

$$r'_j = \text{Interpolate}(r_j; 0, 2^{\text{BitsPerSample}} - 1, \text{Decode}_{2j}, \text{Decode}_{2j+1})$$

(6) And then we clip the output to the range:

$$y_j = \min(\max(r'_j, \text{Range}_{2j}), \text{Range}_{2j+1})$$

This gives us the output:  $f(x) = y$ .

So for example, suppose we have a sampled function  $\mathbb{R}^2 \rightarrow \mathbb{R}$  whose domain is  $[-1, 1]^2$ . The sampling contains 21 columns and 31 rows. So we must encode the input to  $[0, 20] \times [0, 30]$ , and then decode to  $[-1, 1]$ . So the code will be

```
1 14 0 obj
2  <<
3    /FunctionType 0
4    /Domain [-1.0 1.0 -1.0 1.0]
5    /Size [21 31]
6    /Encode [0 20 0 30]
7    /BitsPerSample 4
8    /Range [-1.0 1.0]
9    /Decode [-1.0 1.0]
10   /Length ...
11  >>
12 stream
13 ... 651 sampled values ...
14 endstream
15 endobj
```

### 1.3.2 Type 2 (Exponential Interpolation) Functions

Type 2 functions are functions  $\mathbb{R} \rightarrow \mathbb{R}^n$ . Exponential interpolation is given by the following parameters  $c_0, c_1 \in \mathbb{R}^n$  and  $N \in \mathbb{R}$ . The value of the function at  $x$  is

$$f(x; c_0, c_1, n) = c_0 + (c_1 - c_0)x^N$$

The interpretation of the parameters is as follows:  $c_0, c_1$  are the values of  $f$  at  $x = 0$  and  $x = 1$  respectively.  $N$  is the interpolation exponent, which dictates how the curve behaves. When  $N = 1$  this is simply linear interpolation.

A type 2 function dictionary may include the following additional fields:

Key	Type	Value
<b>C0</b>	array	(Optional) an array of $n$ numbers, defining the parameter $c_0$ . Its default value is $[0.0]$ .
<b>C1</b>	array	(Optional) an array of $n$ numbers, defining the parameter $c_1$ . Its default value is $[1.0]$ .
<b>N</b>	number	(Required) the interpolation exponent.

The values of **Domain** must constrain  $x$  such that if  $N$  is not an integer,  $x \geq 0$ . And if  $N$  is negative  $x \neq 0$ .

### 1.3.3 Type 3 (Stitching) Functions

Type 3 functions define a stitching of  $k$  one-input functions together. Suppose you're given a sequence of functions  $f_0, \dots, f_{k-1}: \mathbb{R} \rightarrow \mathbb{R}^n$  with domains  $[d_{00}, d_{01}], \dots, [d_{k-1,0}, d_{k-1,1}]$ . Now we define another vector of *bounds*, **Bounds** =  $[b_0, \dots, b_{k-2}]$ , so that if  $b_{i-1} \leq x < b_i$  we want to input  $x$  into  $f_i$  ( $b_{-1}$  and  $b_{k-1}$  are the endpoints of the domain specified for  $f$ ). But  $[b_i, b_{i+1}]$  may not align with the domain of  $f_i$  ( $[d_{i0}, d_{i1}]$ ), and we don't necessarily want it to anyway. So we now we want to linearly interpolate  $[b_i, b_{i+1}]$  into  $[d_{i0}, d_{i1}]$ . We do so by specifying two points  $e_{i0}, e_{i1} \in [d_{i0}, d_{i1}]$  which will be the new endpoints.

Explicitly, we have

(1) a new domain  $[d_0, d_1]$ ;

- (2)  $k$  one-input functions **Functions** =  $[f_0, \dots, f_{k-1}]$  each with a domain  $[d_{i0}, d_{i1}]$ ;
- (3) a vector of bounds **Bounds** =  $[b_0, \dots, b_{k-2}]$  (and define  $b_{-1} = d_0$  and  $b_{k-1} = d_1$ );
- (4) a vector of encoding values **Encode** =  $[e_{00}, e_{01}, \dots, e_{k-1,0}, e_{k-1,1}]$ ;

and given an input value  $x$ , if  $b_{i-1} \leq x < b_i$  (the right inequality is weak if  $i = k - 1$ ), then we compute  $x' = \text{Interpolate}(x; b_{i-1}, b_i, e_{i0}, e_{i1})$ . Then we output  $f_i(x')$ .

Of course, the ranges of each  $f_i$  must be compatible with the range specified for  $f$  (if specified).

Key	Type	Value
<b>Functions</b>	array	(Required) the array of $k$ one-input functions to be stitched together. Each function must have the same output dimensionality $n$ .
<b>Bounds</b>	array	(Required) the array of $k-1$ numbers determining the bounds for which to map into each function.
<b>Encode</b>	array	(Required) the array of $2k$ numbers which determines the mapping of bounds into domains of each function.

Notice that if we have a function  $f: \mathbb{R} \rightarrow \mathbb{R}^n$ , and we want to compute  $g(x) = f(1-x)$ . We can compute this by defining  $g$  as a stitching function, where the **Encode** array is  $[1 \ 0]$ .

### 1.3.4 Type 4 (PostScript Calculator) Functions

A type 4 function utilizes a small subset of PostScript code to compute values. The following PostScript operators can be used in type 4 functions:

- Arithmetic operators: **abs**, **cvi**, **floor**, **mod**, **sin**, **add**, **cvr**, **idiv**, **mul**, **sqrt**, **atan**, **div**, **ln**, **neg**, **sub**, **ceiling**, **exp**, **log**, **round**, **truncate**, **cos**
- Boolean and bitwise operators: **and**, **false**, **le**, **not**, **true**, **bitshift**, **ge**, **lt**, **or**, **xor**, **eq**, **gt**, **ne**
- Conditional operators: **if**, **ifelse**
- Stack operators: **copy**, **exch**, **pop**, **dup**, **index**, **roll**

The operand syntax for type 4 functions follows PDF conventions rather than PostScript ones. The entire code defining the function must be wrapped in curly braces  $\{\dots\}$ . Braces are also used to delimit expressions executed conditionally in **if** and **ifelse** operators.

The **Range** field is required for PostScript functions.

I may later update this document to provide information on how to use PostScript operators (specifically in PDFs).

## 2 File Structure

### 2.1 File structure

A “canonical” PDF file (pdfTeX only creates canonical PDF files. PDF files can be updated to create non-canonical PDFs, but we will not deal with those) has the following structure:

- (1) a one-line *header* which specifies the PDF version of the file;
- (2) a *body* containing all the objects which make up the file;
- (3) a *cross-reference stream* containing information regarding the indirect objects in the file;
- (4) a *trailer* which specifies the location of the cross-reference stream.

For example, if we were to compile the following TeX file:

```

1 \pdfcompresslevel=0 % don't compress the PDF
2 \nopagenumbers
3 Hello world!
4 \bye
hello-world.tex

```

we'd get the following PDF:

```

1  %PDF-1.5
2  %
3  3 0 obj
4  <<
5  /Length 66
6  >>
7  stream
8  BT
9  /F1 9.9626 Tf 91.925 759.927 Td [(Hello)-333(w)27(orld!)]TJ
10 ET
11
12 endstream
13 endobj
14 8 0 obj
15 <<
16 /Length1 1472
17 /Length2 9273
18 /Length3 0
19 /Length 10745
20 >>
21 stream
22 %!PS-AdobeFont-1.0: CMR10 003.002
23 %%Title: CMR10
24 %Version: 003.002
25 ...
26 endstream
27 endobj
28 11 0 obj
29 <<
30 /Producer (pdfTeX-1.40.26)
31 ...
32 >>
33 endobj
34 5 0 obj
35 <<
36 /Type /ObjStm
37 /N 7
38 /First 43
39 /Length 1150
40 >>
41 stream
42 2 0 1 106 7 168 9 649 4 878 6 1010 10 1062
43 % 2 0 obj
44 <<
45 /Type /Page
46 /Contents 3 0 R
47 /Resources 1 0 R
48 /MediaBox [0 0 595.276 841.89]
49 /Parent 6 0 R
50 >>
51 % 1 0 obj
52 <<
53 /Font << /F1 4 0 R >>
54 /ProcSet [ /PDF /Text ]
55 >>
56 % 7 0 obj
57 [ ... ]
58 % 9 0 obj
59 <<

```

```

60 /Type /FontDescriptor
61 /FontName /IEQIOR+CMR10
62 /Flags 4
63 /FontBBox [-40 -250 1009 750]
64 /Ascent 694
65 /CapHeight 683
66 /Descent -194
67 /ItalicAngle 0
68 /StemV 69
69 /XHeight 431
70 /CharSet (</H/d/e/exclam/l/o/r/w>)
71 /FontFile 8 0 R
72 >>
73 % 4 0 obj
74 <<
75 /Type /Font
76 /Subtype /Type1
77 /BaseFont /IEQIOR+CMR10
78 /FontDescriptor 9 0 R
79 /FirstChar 33
80 /LastChar 119
81 /Widths 7 0 R
82 >>
83 % 6 0 obj
84 <<
85 /Type /Pages
86 /Count 1
87 /Kids [2 0 R]
88 >>
89 % 10 0 obj
90 <<
91 /Type /Catalog
92 /Pages 6 0 R
93 >>
94
95 endstream
96 endobj
97 12 0 obj
98 <<
99 /Type /XRef
100 /Index [0 13]
101 /Size 13
102 /W [1 2 1]
103 /Root 10 0 R
104 /Info 11 0 R
105 /ID [<804F5DFAA39FA28E85E2211C48BC665E> <804F5DFAA39FA28E85E2211C48BC665E>]
106 /Length 52
107 >>
108 stream
109 ...
110 endstream
111 endobj
112 startxref
113 12479
114 %%EOF

```

I have truncated the PDF to save space and increase readability, but we still end up with over a hundred lines. Oh well.

Let's go over this file:

```
1 %PDF-1.5
```

this is the header, it specifies that this file conforms to PDF version 1.5.

Lines 3 through 96 comprise the body of the PDF file.

Lines 97 through 111 comprise the cross-reference table. This contains information on how to randomly-access any indirect object within the file. This information (in PDF-1.5) is stored in the cross-reference stream (lines 108–110). The cross-reference stream is preceded by its stream dictionary, which may have the following fields:

Key	Type	Value
<b>Type</b>	name	(Required) the type of PDF object that the dictionary describes. Must be <b>XRef</b> .
<b>Size</b>	integer	(Required; must not be indirect) the highest object number used in the file, plus one.
<b>Root</b>	dictionary	(Required; must be indirect) an indirect reference to the catalog of the PDF (see below).
<b>Info</b>	dictionary	(Optional; must be indirect) an indirect reference to the document's information dictionary.
<b>ID</b>	array	(Optional) an array of two byte-strings (hexadecimal strings) which uniquely identifies the file.
<b>Index</b>	array	(Optional) an array of two non-negative integers. The first is the first object number in the file; the second is the number of objects. The default value is then [0 <b>Size</b> ].
<b>W</b>	array	(Required) an array of integers corresponding to the size of the fields in a single cross-reference entry. We won't develop this further.

The cross reference stream in this file is

hello-world.pdf

```

97 12 0 obj
98 <<
99 /Type /XRef
100 /Index [0 13]
101 /Size 13
102 /W [1 2 1]
103 /Root 10 0 R
104 /Info 11 0 R
105 /ID [<804F5DFAA39FA28E85E2211C48BC665E> <804F5DFAA39FA28E85E2211C48BC665E>]
106 /Length 52
107 >>
108 stream
109 ...
110 endstream
111 endobj

```

and the trailer, which just specifies the byte offset of the cross reference stream, is

hello-world.pdf

```

112 startxref
113 12479
114 %%EOF

```

## 2.2 Document structure

### 2.2.1 The document catalog

The **Root** field in the file's cross-reference stream is an indirect reference to the document's *catalog*. The

document catalog stores information for objects in the body of the file which define the document's content, outline, etc. It also contains information on how the document should be displayed.

We see on line 103 that the **Root** of the cross-reference stream is object 10 0. This is the document catalog:

hello-world.pdf

```
89 % 10 0 obj
90 <<
91 /Type /Catalog
92 /Pages 6 0 R
93 >>
```

We see that the document catalog is a dictionary, and here it only has two entries. But in general it can have more:

Key	Type	Value
<b>Type</b>	name	(Required) the type of PDF object the dictionary describes. Must be <b>Catalog</b> .
<b>Pages</b>	dictionary	(Required; must be indirect) the <i>page tree node</i> (see below) that is the root of the document's <i>page tree</i> (see below).
<b>PageLabels</b>	number tree	(Optional) a number tree defining the page labeling for the document. The keys in the tree are page indicies, and the values are <i>page label dictionaries</i> (see below).
<b>Dests</b>	dictionary	(Optional) A dictionary of names and corresponding destinations (see below when we discuss hyperlinks).
<b>URI</b>	dictionary	(Optional) A dictionary containing information for URI actions (see below when we discuss hyperlinks).

## 2.2.2 The page tree

The pages of a document are accessed through a structure known as the *page tree*. This defines the structure of the pages in a document. A page tree has two types of nodes: intermediate nodes and leaf nodes. The most simple structure would be a single root intermediate node which contains as children all the pages in the document as leaf nodes. This is not efficient, so instead the tree is kept balanced generally.

A page tree node (intermediate node) is a dictionary with the following fields:

Key	Type	Value
<b>Type</b>	name	(Required) the type of PDF object that this dictionary describes. Must be <b>Pages</b> .
<b>Parent</b>	dictionary	(Required except in root; must be indirect) a reference to the parent of the tree node.
<b>Kids</b>	array	(Required) an array of indirect references to the immediate children of the node.
<b>Count</b>	integer	(Required) the number of leaf nodes (page objects) that are descendants of this node.

Note that the page tree does not necessarily reflect the logical structure of the document (chapters, sections, etc.).

A page object is a leaf in the page tree. It is a dictionary with the fields listed below. Some of the fields (those which are listed as such) may be inherited by ancestor nodes in the page tree.

Key	Type	Value
<b>Type</b>	name	(Required) the type of PDF object that this dictionary describes. Must be <b>Page</b> .
<b>Parent</b>	dictionary	(Required; must be indirect) a reference to the parent of the page object in the page tree.

<b>Resources</b>	dictionary	(Required; inheritable) a dictionary containing any resources required by the page. Omitting this indicates that it should be inherited.
<b>MediaBox</b>	array	(Required; inheritable) an array of four numbers defining the boundaries of the page.
<b>Contents</b>	stream	(Optional) a <i>content stream</i> (see below) which contains the contents of the page.
<b>Group</b>	dictionary	(Optional) a <i>group attributes dictionary</i> specifying the attributes of the page's page group for use in transparency (see below).
<b>Annots</b>	array	(Optional) an array of <i>annotation dictionaries</i> representing annotations associated with the page (see below).

More fields exist, but we ignore them for the sake of brevity. In order to inherit an attribute, place it in a page tree node, and all page objects which are its descendants will inherit the attribute.

We see in our document's catalog that the page tree root is object 6 0:

```
hello-world.pdf
83 % 6 0 obj
84 <<
85 /Type /Pages
86 /Count 1
87 /Kids [2 0 R]
88 >>
```

We see that there is a single page, whose object is 2 0:

```
hello-world.pdf
43 % 2 0 obj
44 <<
45 /Type /Page
46 /Contents 3 0 R
47 /Resources 1 0 R
48 /MediaBox [0 0 595.276 841.89]
49 /Parent 6 0 R
50 >>
```

The contents of the page are in object 3 0. This is a content stream, which is a stream with operators telling the renderer how to display the page:

```
hello-world.pdf
3 3 0 obj
4 <<
5 /Length 66
6 >>
7 stream
8 BT
9 /F1 9.9626 Tf 91.925 759.927 Td [(Hello)-333(w)27(orld!)]TJ
10 ET
11
12 endstream
13 endobj
```

and the resources of the page are in object 1 0:

```
hello-world.pdf
51 % 1 0 obj
52 <<
53 /Font << /F1 4 0 R >>
```



```

54 /ProcSet [ /PDF /Text ]
55 >>

```

The resources dictionary defines `/F1` to be a font which is a reference to object 4 0, which is a dictionary defining how to access the font.

### 2.2.3 Resource Dictionaries

Operands supplied to operators in content streams may only be direct objects. This places a heavy restriction that must be somehow overcome. For this reason, a content stream has a *resource dictionary*, defined by the **Resources** entry associated with it, in one of the following ways:

- for a content stream that is the value of a page's **Contents** entry, the resource dictionary is named by the page's **Resources** entry.
- for other content streams, the stream dictionary's **Resources** specifies the resource dictionary.
- A form XObject (see below) may omit the **Resources** entry, in which case the resources are looked up in the **Resources** entry of the page it is used in.

The entries in the **Resources** dictionary are as follows (all fields are optional). All entries are explained in more depth in their respective sections below.

Key	Type	Value
<b>ExtGState</b>	dictionary	A dictionary that maps resource names to graphics state parameter dictionaries.
<b>ColorSpace</b>	dictionary	A dictionary that each resource names to either the name of a device-dependent color space or an array describing a color space (see below).
<b>Pattern</b>	dictionary	A dictionary that maps resource names to pattern objects (see below).
<b>Shading</b>	dictionary	A dictionary that maps resource names to shading dictionaries (see below).
<b>XObject</b>	dictionary	A dictionary that maps resource names to XObjects. (see below).
<b>Font</b>	dictionary	A dictionary that maps resource names to font dictionaries.
<b>ProcSet</b>	array	An array of predefined procedure set names.
<b>Properties</b>	dictionary	A dictionary that maps resource names to property list dictionaries for marked content.

Each field is explained in more depth later in this article.

## 3 Graphics

PDF provides support for drawing and including graphics. In this section we will cover the code for doing so, and later on we will discuss how to interact with this code through pdfTeX.

PDF inherits the postfix syntax from PostScript. This inheritance is entirely syntactical, as PDF does not support the concept of an argument stack or other features PostScript provides.

PDF defines the following graphics objects for use within content streams:

- *path objects* are arbitrary shapes made up of straight lines, rectangles, and cubic Bézier curves. A path object ends with painting operators which indicate whether the path is opened or closed, stroked, filled, etc.
- *text objects* consist of one or more character strings that identify sequences of glyphs to be painted. It can also be stroked, filled, or used as a clipping boundary.
- *external objects* (XObjects) are objects defined outside of the content stream, but can be referenced from within the content stream through use of the stream's **Resources**. There are different kinds of XObjects:
  - *image XObjects* define a rectangular array of color samples to be painted;

- *form XObjects* define an entire content stream to be treated as a single graphics object;
- *reference XObjects* are a type of form XObject used to import content from one PDF into another;
- *group XObjects* are a type of form XObject used to group graphical elements together (e.g. for use in the transparency model, which uses *transparency group XObjects*).
- *inline image objects* use a special syntax to express data for a small image directly within the content stream.
- *shading objects* describes a geometric shape whose color is an arbitrary function of position within the shape.

PDF 1.3 and early use an opaque imaging model, meaning that every object is painted in its entirety and at every point, only the object at the top has an effect on the color painted. PDF 1.4 and later use a transparent imaging model, meaning that objects may be specified to have a certain amount of transparency, so that objects underneath it may also affect the color painted. By default objects are painted as opaque.

### 3.1 Coordinate systems

Positions in the document are determined in terms of coordinates on a plane. A coordinate space is determined by the following properties relative to the current page:

- the location of the origin;
- the orientation of the  $x$  and  $y$  axes;
- the lengths of the units along each axis.

There are several coordinate spaces defined by the PDF, which will be described in this section. Transformations between coordinate spaces are done via affine transformations (transformations of the form  $x \mapsto Ax + b$ , where  $A$  is a matrix and  $b$  a vector).

The coordinate space which is native to a specific device is called its *device space*.

#### 3.1.1 User space

To avoid the issues arising from using device-dependent coordinate spaces, PDF defines a device-independent coordinate system that remains the same relative to the current page no matter the medium in which it is displayed or printed. This is called the *user space* coordinate system.

The **CropBox** entry in a page dictionary specifies the rectangle of user space corresponding to the visible area on the output medium. The length of a unit along both the  $x$  and  $y$  axes is set by **UserUnit** (in PDF-1.6). If the entry is not supplied (or supported), the default value is 1/72th of an inch. **CropBox** defines the rectangular region for which the page is to be displayed in the infinite plane that is the user space.

The transformation from user to device space is defined by the CTM (current transformation matrix). This is stored in the PDF graphics state (to be discussed below). A PDF content stream can modify user space by using the **cm** operator (coordinate transformation operator).

#### 3.1.2 Other coordinate spaces

In addition to device and user space, PDF utilizes a variety of other coordinate systems:

- The coordinates of text are defined in *text space*. The translation from text space to user space is defined by a *text matrix* as well as several text-related parameters in the graphics state (see below).
- All sampled images are defined in *image space*. The transformation from image to user space is predefined and cannot be changed. All images are one-unit by one-unit in user space. To paint them, the CTM must be temporarily changed.
- A form XObject as a self-contained content stream is defined in a *form space*. When painted in another content stream, its space is transformed into user space using the *form matrix* which is defined in the form XObject.
- A pattern (which is content invoked repeatedly to tile an area) is defined in a space called *pattern space*. The transformation from pattern space to user space is defined in a *pattern matrix* contained in the pattern.

#### 3.1.3 Transformation matrices

A transformation, as discussed previously, is an affine transformation of the form  $x \mapsto Ax + b$  where  $A$  is a  $2 \times 2$  matrix and  $b$  a vector of size 2. Suppose we want to transform  $(x, y)$  to  $(x', y')$  via such an affine transformation, then

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

or, we can write this as

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & e \\ b & d & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

The reason we add the final line of the matrix is to make it square. So if the current CTM is  $M$  and we'd like to transform it by the affine transformation described by matrix  $A$ , then we must change CTM to be  $M \cdot A$ . Thus if we are given an input coordinate  $X$ , then we first apply  $A$  and then  $M$ .

In the PDF reference, they take the convention of multiplying by row vectors on the left. So the affine transformation is represented by the tranpose of the matrix provided here. Nevertheless, the results are the same.

The affine transformation represented by

$$\begin{pmatrix} a & b & e \\ b & d & f \\ 0 & 0 & 1 \end{pmatrix}$$

is represented in PDF code by  $[a\ b\ c\ d\ e\ f]$ .

### 3.2 Graphics State

When rendering a PDF, an internal state must be held by the application which determines the current state to be used when rendering graphics. The graphics state is initialized at the beginning of each page according to the table below.

Parameter	Type	Value
CTM	array	The current transformation matrix, which maps user space to device space. The CTM can be modified by use of the <b>cm</b> operator.
clipping path	(internal)	The current <i>clipping path</i> , which defines the boundary against which all output is to be cropped. The initial value is the boundary of the entire imageable portion of the page.
color space	name or array	The current <i>color space</i> which determines how color values are to be interpreted (e.g. RGB). There are two separate color spaces: one for stroking and one for non-stroking operations. The initial value is <b>DeviceGray</b> .
color	(various)	The current color to be used when painting. The type and interpretation of the color depends on the current color space. There are two color parameters: one for stroking and one for non-stroking operations. The initial value is black.
text state	(various)	A set of nine graphics state parameters that affect the painting of text (see below).
line width	number	The thickness, in user space units, of paths to be stroked. Initial value: 1.0.
line cap	integer	A code specifying the shape of endpoints for any stroked open paths (see below). Initial value: 0 (square caps).
line join	integer	A code specifying the shape of joints between connected segments of a stroked path. Initial value: 0 (mitered joints).
miter limit	number	The maximum length of mitered line joins for stroked paths (the length of the spikes when lines join at sharp angles). Initial value: 10.0.
dash pattern	array and number	A description of the dash pattern to be used when paths are stroked (see below). Initial value: a solid line.
blend mode	name or array	The current <i>blend mode</i> to be used in the transparent imaging model (see below). This parameter is reset to its initial value at the beginning of execution of a transparency group XObject. Initial value: <b>Normal</b> .




soft mask	dictionary or name	A <i>soft-mask</i> dictionary (see below), specifying the mask shape or opacity values to be used in the transparent imaging model, or <b>None</b> . This parameter is reset to its initial value at the beginning of execution of a transparency group XObject. Initial value: <b>None</b> .
alpha constant	number	The constant shape or constant opacity value to be used in the transparent imaging model. There are two alpha constant parameters: one for stroking and another for non-stroking operations. This parameter is reset to its initial value at the beginning of execution of a transparency group XObject. Initial value: 1.0.
alpha source	boolean	A flag specifying whether the current soft mask and alpha constant parameters are to be interpreted as shape values ( <b>true</b> ) or opacity values ( <b>false</b> ). Initial value: <b>false</b> .

Some parameters are set with specific operators, while others are set by including a particular entry in a graphics state parameter dictionary. Some can be set either way. For example, the line width can be set using the **w** operator, or with the **LW** entry.




The *graphics state stack* allows for local changes to the graphics state, so you can change it without affecting things outside the current scope. The stack is a stack (LIFO — last in first out) data structure. The **q** operator pushes a copy of the graphics state onto the stack, while **Q** pops from the stack. Occurrences of **q** and **Q** must be balanced within a content stream.

### 3.2.1 Line caps

We demonstrate in the table below the three styles of line caps. These are the ends of open subpaths (and dashes) when they are stroked.

Style	Appearance	Description
0		<i>Butt cap</i> : the stroke is squared off at the endpoint of the path. There is no projection beyond the end of the path.
1		<i>Round cap</i> : the stroke is rounded off with semicircular ends on both sides of the stroke. The diameter of the capp is equal to the line width.
2		<i>Projecting square cap</i> : the stroke continues beyond the endpoint of the path for a distance equal to half the line width and is squared off.






### 3.2.2 Line Joins

Style	Appearance	Description
0		<i>Miter join</i> : the outer edges of the strokes for the segments are extended until they meet at an angle. If the angles meet at too sharp an angle (as defined by the miter limit parameter, though we won't go into this), a bevel join is used instead.
1		<i>Round join</i> : an arc of a circle with diameter equal to the line width is drawn around the point where the two segments meet.
2		<i>Bevel join</i> : the two segments are finished with butt caps.

### 3.2.3 Line dash pattern

The *line dash pattern* controls the pattern of dashes and gaps used by stroked paths. It is specified by a *dash array* and a *dash phase*. The dash array's elements are numbers that specify the lengths of alternating dashes and gaps (they must all be nonnegative and not all zero). The dash phase specifies the distance into

the dash pattern at which to start the dash. When dashing begins, the elements of the dash array are cyclicly summed up, and when the sum equals the dash phase, the stroking of the phase begins.

Dash	Appearance	Description
<code>[] 0</code>		No dash
<code>[3] 0</code>		3 units on, 3 units off,...
<code>[2] 1</code>		1 on, 2 off, 2 on,...
<code>[2 1] 0</code>		2 on, 1 off, 2 on,...
<code>[3 5] 6</code>		2 off, 3 on, 5 off,...

### 3.2.4 Graphics state operators

We list the operators which can be used to alter the graphics state. Like all PDF operators, they are postfix.

Operands	Operator	Description
—	<b>q</b>	Push the current graphics state onto the state stack.
—	<b>Q</b>	Pop from the state stack into the current graphics state.
$a\ b\ c\ d\ e\ f$	<b>cm</b>	Modify the current transformation matrix (CTM) by multiplying with the matrix $\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ e & f & 1 \end{pmatrix}.$
<i>lineWidth</i>	<b>w</b>	Sets the line width.
<i>lineCap</i>	<b>J</b>	Sets the line cap style according to the provided code.
<i>lineJoin</i>	<b>j</b>	Sets the line join style according to the provided code.
<i>miterLimit</i>	<b>M</b>	Sets the miter limit.
<i>dashArray dashPhase</i>	<b>d</b>	Set the line dash pattern in the graphics state.
<i>dictName</i>	<b>gs</b>	Set the specified parameters in the graphics state. <i>dictName</i> is the name of a graphics state dictionary in the <b>ExtGState</b> subdictionary of the current <b>Resources</b> dictionary.

As written, you can use the **gs** to alter the graphics state. This is by providing the name of a graphics state parameter dictionary which is provided in the **ExtGState** field of the current **Resources** dictionary. Graphics state parameter dictionaries may have the following fields (all fields are optional):

Key	Type	Value
<b>Type</b>	name	The type of PDF object that this dictionary describes; must be <b>ExtGState</b> .
<b>LW</b>	number	The line width.
<b>LC</b>	integer	The line cap code.
<b>LJ</b>	integer	The line join code.
<b>ML</b>	number	The miter limit.
<b>D</b>	array	The line dash pattern, expressed as an array of the form <code>[dashArray dashPhase]</code> where <i>dashArray</i> is itself an array.
<b>Font</b>	array	An array of the form <code>[font size]</code> , where <i>font</i> is an indirect reference to a font dictionary and <i>size</i> is a number expressed in text space units for the font size. These can also be altered by the <b>Tf</b> operator (see below).

<b>BM</b>	name or array	The current blend mode to be used in the transparent imaging model (see below).
<b>SMask</b>	dictionary or name	The current soft mask, which specifies the mask shape or mask opacity values to be used in the transparent imaging model.
<b>CA</b>	number	The current stroking alpha constant, specifying the shape or constant opacity value to be used for stroking operations in the transparent imaging model.
<b>ca</b>	number	Same as <b>CA</b> but for non-stroking operations.
<b>AIS</b>	boolean	The alpha source flag (“alpha is shape”), which specifies whether the current soft mask and alpha constant are to be interpreted as shape values ( <b>true</b> ) or opacity values ( <b>false</b> ).
<b>TK</b>	boolean	The text knockout flag, which determines the behavior of overlapping glyphs within a text object in the transparent imaging model.

For example, we may have the single-page PDF:

```

1 10 0 obj % the page object
2 <<
3   /Type /Page
4   /Parent 5 0 R
5   /Resources 20 0 R
6   /Contents 40 0 R
7 >>
8 endobj
9
10 20 0 obj % resource dictionary for the page
11 <<
12   /ProcSet [/PDF /Text]
13   /Font << /F1 25 0 R >>
14   /ExtGState <<
15     /GS1 30 0 R
16     /GS2 35 0 R
17   >>
18 >>
19 endobj
20
21 30 0 obj % first graphics state
22 <<
23   /Type /ExtGState
24   /LW .3
25   /D [[3] 0]
26 >>
27 endobj
28
29 35 0 obj % second graphics state
30 <<
31   /Type /ExtGState
32   /LC 1
33   /LJ 1
34   /LW 1
35 >>
36 endobj
37
38 40 0 obj % contents of the page
39 <<
40   /Length ...
41 >>
42 stream
43 q
44 /GS1 gs
45 0 0 m 10 0 l S
46 /GS2 gs
47 0 -5 m 10 -5 l S
48 Q
49 endstream
50 endobj

```

We have two graphics states, **GS1** and **GS2**, which both alter the graphics state to change how lines are drawn. We now discuss the code for drawing paths.

### 3.3 Path construction and painting

Paths define the boundaries of areas within the PDF. They can be stroked to draw the boundaries of shapes, or filled, or added to the current clipping path in order to crop material on the page.

A path is made up of one or more disconnected subpaths, each of which is a sequence of connected segments, which are lines or curves. Two segments are connected only if they are defined consecutively: if you define a line from (0,0) to (10,0), then a line to (10,10), the two lines are connected. But if between those two lines you draw another line, they are not connected. This is not entirely true: a subpath may be closed, that is the first and last segments may be connected by the use of a special operator.

There are three types of path operators:

- (1) *path construction operators* which define the geometry of the path;
- (2) *path painting operators* which define how the path should be painted and/or stroked;
- (3) *clipping path operators* which say if a path should be added to the current clipping area.

#### 3.3.1 Path construction operators

Operands	Operator	Description
$x\ y$	<b>m</b>	Begin a new subpath by moving the current point to coordinate $(x, y)$ in user space.
$x\ y$	<b>l</b>	Append a line from the current point to $(x, y)$ in the current subpath. The new current point is $(x, y)$ .
$x_1\ y_1\ x_2\ y_2\ x_3\ y_3$	<b>c</b>	Append a cubic Bézier curve to the current subpath. The curve extends from the current point to $(x_3, y_3)$ using $(x_1, y_1)$ and $(x_2, y_2)$ as control points.
$x_2\ y_2\ x_3\ y_3$	<b>v</b>	Append a cubic Bézier curve to the current subpath. The curve extends from the current point to $(x_3, y_3)$ using the current point and $(x_2, y_2)$ as control points.
$x_1\ y_1\ x_3\ y_3$	<b>y</b>	Append a cubic Bézier curve to the current subpath. The curve extends from the current point to $(x_3, y_3)$ using $(x_1, y_1)$ and $(x_3, y_3)$ as control points.
—	<b>h</b>	CLOSE the current subpath by appending a straight line segment from the current point to the starting point of the subpath. This terminates the current subpath.
$x\ y\ width\ height$	<b>re</b>	Append a rectangle to the current path as a complete subpath.

Doing

```
1 x y width height re
```

is equivalent to

```
1 x y m
2 (x + width) y l
3 (x + width) (y + height) l
4 x (y + height) l
5 h
```

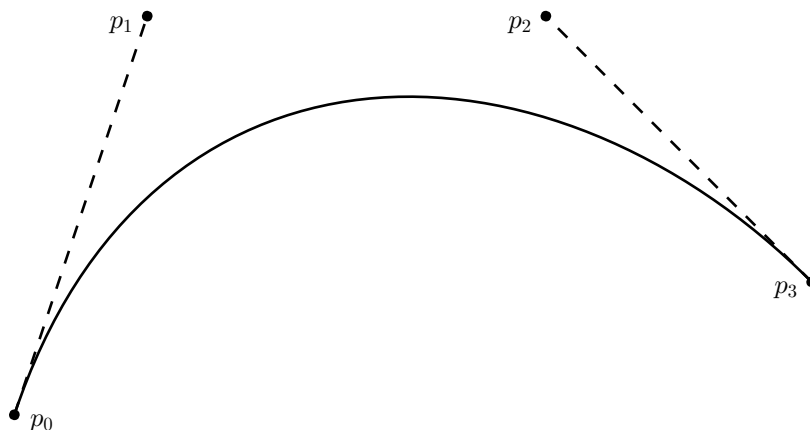
#### Cubic Bézier curves

Curved path segments are defined using *cubic Bézier curves*. Bézier curves are useful since they can be scaled, and subpaths are also Bézier curves, which makes them very useful in computer graphics.

A cubic Bézier curve is defined by four points: beginning and end points  $p_0, p_3$ , as well as two control points  $p_1, p_2$ . Given these points, the cubic Bézier curve is parameterized by

$$\mathbf{B}(t) = (1-t)^3 p_0 + 3t(1-t)^2 p_1 + 3t^2(1-t) p_2 + t^3 p_3$$

Note that  $\mathbf{B}(0) = p_0$  and  $\mathbf{B}(1) = p_3$ , so this defines a curve between  $p_0$  and  $p_3$ .  $\mathbf{B}$  generally does not intersect  $p_1, p_2$ , but it always lies within the convex hull of the four points. Another big reason for the use of Bézier curves is that changing the control points can *intuitively* alter the curvature of the curve.



The above figure demonstrates a useful property of Bézier curves: the lines  $p_0p_1$  and  $p_2p_3$  are tangent to the curve! This helps explain how they can intuitively alter the curvature of the curve.

### 3.3.2 Path painting operators

We now describe the operators which can be used for painting (stroking and filling) paths:

Operands	Operator	Description
—	<b>S</b>	Stroke the path.
—	<b>s</b>	Close and stroke the path. Equivalent to <b>h S</b> .
—	<b>f</b>	Fill the path, using the nonzero winding number rule to determine the region to fill (see below). All open subpaths are implicitly closed.
—	<b>f*</b>	Same as <b>f</b> , except using the even-odd rule.
—	<b>B</b>	Fill then stroke the path, using the nonzero winding number rule.
—	<b>B*</b>	Fill and then stroke the path, using the even-odd rule.
—	<b>b</b>	Close, fill, then stroke the path, using the nonzero winding number rule. (The closing affects only the stroking.)
—	<b>b*</b>	Same as <b>b</b> except using the even-odd rule.
—	<b>n</b>	End the path without stroking or filling it.

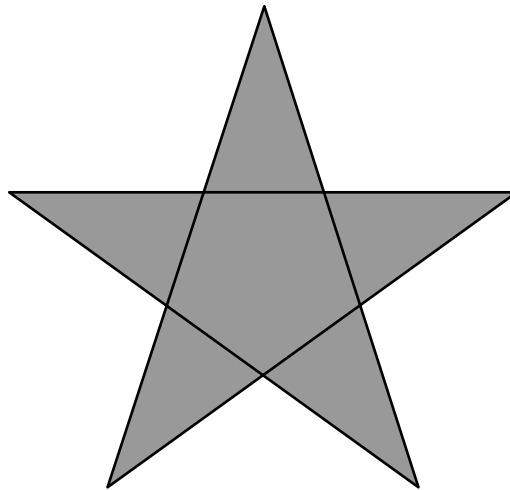
#### The nonzero winding number rule

The *nonzero winding number rule* determines whether a given point is inside the path by drawing a ray from the given point to infinity in every direction. For each ray, we count how many times the ray intersects the path. When it intersects the path as it is going left-to-right, it adds one; and when it intersects the path as it is going right-to-left, it subtracts one. If the result is zero (for any ray), the point is outside the path; otherwise it is inside.

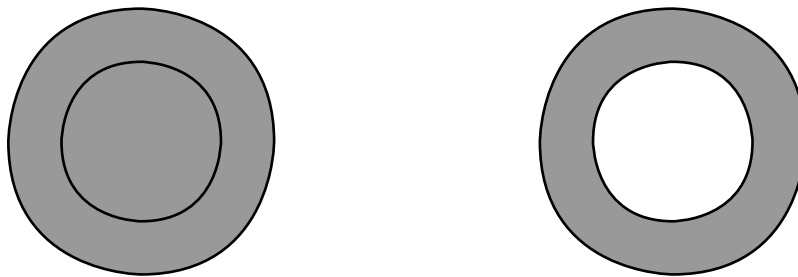
Instead of doing this for every ray, an arbitrary ray is chosen.

For convex shapes, this defines the inside and outside as you'd expect. For more complicated boundaries, such as those which intersect themselves, we can get weirder results. For example the five-point star:





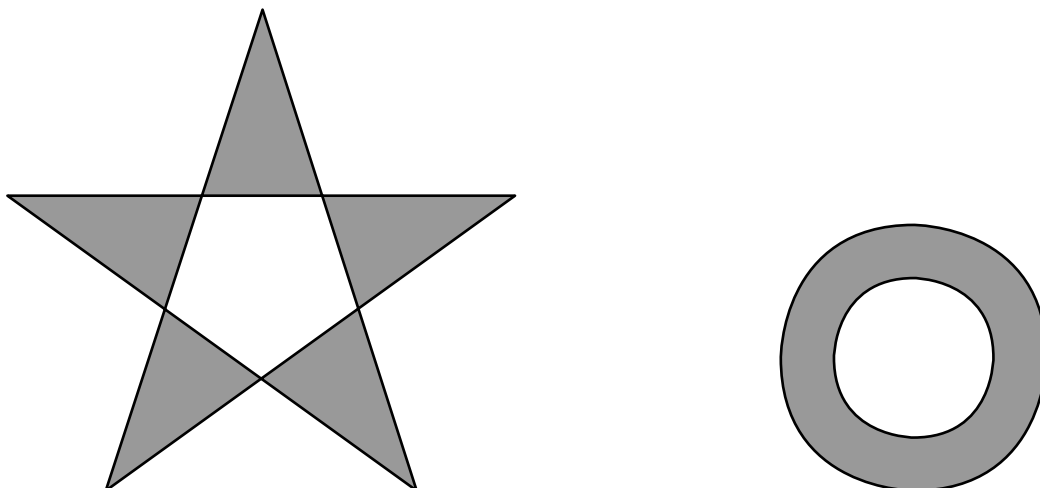
And if we draw a donut shape, the direction we choose to draw it matters. For example, if both circles are drawn in the same direction we get the left, but if they are drawn in opposite directions, we get the right:



### The even-odd rule

The even-odd rule is similar to the nonzero winding number rule. Similar to the nonzero winding number rule, a ray is drawn in any direction from the point, and the number of intersections is counted (without any subtractions). If the number is odd, the point is inside; and if it is even, outside.

For convex shapes, this produces the same result as the nonzero winding number rule. But for more complicated shapes, it produces different shapes:



### 3.3.3 Clipping path operators

The graphics state contains a *current clipping path* which defines a boundary of a shape through which painting operators are cropped. Anything outside the boundary is cropped from the page. The initial current clipping path includes the entire page. After constructing a path and before painting it, a clipping operator (**W** or **W\***) may be added. The clipping path is altered to be the intersection of the path with the current clipping path, and this is done only after the path is painted.

Operands	Operator	Description
----------	----------	-------------

—	<b>W</b>	Modify the current clipping path by intersecting it with the current path, using the nonzero winding number rule to determine what areas lay inside and outside the path.
—	<b>W*</b>	Like <b>W</b> , except it uses the even-odd rule.

In order to alter the clipping path without painting a path, one can use the **n** operator, which just terminates the path without painting it. So for example, one could do:

```
1 \pdfliteral{q}\rlap{\pdfliteral{0 0 100 4 re W n}hello}\pdfliteral{Q}
```

(\pdfliteral injects PDF code into the document, we'll discuss this later.) This will write “hello”, but only the coordinates which fall inside the rectangle of height 4. This results in:

```
neuo
```

It's important to push the color state before setting the clipping path and pop after you've done your clipping. Otherwise all subsequent material will also be cropped.

### 3.4 Color Spaces

PDF provides the ability to input color in multiple ways. For example, it supports both RGB and CMYK color spaces. The current color space dictates the colors used for stroking and non-stroking operations (there exists separate color spaces for each).

To provide a color, first a color space must be given. We will discuss only the *device color spaces* now, and later we will discuss some *special color spaces*. There are three device color spaces: **DeviceGray**, **DeviceRGB**, and **DeviceCMYK**.

**DeviceGray** uses only shades of gray, which range between 0.0 and 1.0. **DeviceRGB** uses combinations of red, green, and blue whose values range between 0.0 and 1.0 to produce color. Similarly **DeviceCMYK** uses combination of cyan, magenta, yellow, and black whose values range between 0.0 and 1.0. Importantly, RGB is an additive color system (it mixes light), and CMYK is subtractive (it mixes material; adding colors makes it darker).

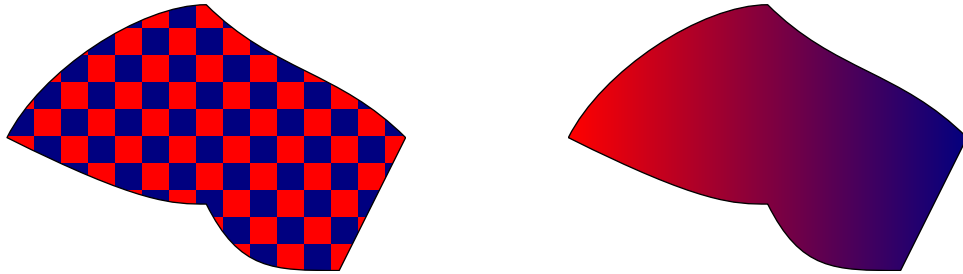
We can use the **CS** and **cs** operators to change the stroking and non-stroking color spaces respectively. Their only input is the name of a color space (**DeviceGray**, **DeviceRGB**, **DeviceCMYK**). Then to set the actual color value, one uses **SC** or **sc** to set the color. Their input is a stream of numbers which corresponds to a color value. For example, in the **DeviceRGB** color space the stream must have 3 numbers, while in the **DeviceCMYK** color space it must have 4.

To summarize:

Operands	Operator	Description
<i>name</i>	<b>CS</b>	Sets the current color space for stroking operations. <i>name</i> must be a name object ( <b>DeviceGray</b> , <b>DeviceRGB</b> , or <b>DeviceCMYK</b> ).
<i>name</i>	<b>cs</b>	Sets the current color space for non-stroking operations. <i>name</i> must be a name object.
$c_1 \dots c_n$	<b>SC</b>	Set the color to be used for stroking operations. Each value $c_i$ must be a number between 0.0 and 1.0. The number of operators $n$ depends on the current color space: 1 for <b>DeviceGray</b> , 3 for <b>DeviceRGB</b> , 4 for <b>DeviceCMYK</b> .
$c_1 \dots c_n$	<b>sc</b>	Same as <b>SC</b> but for non-stroking operations.
<i>gray</i>	<b>G</b>	The same as doing /DeviceGray <b>CS</b> followed by <i>gray</i> <b>SC</b> .
<i>gray</i>	<b>g</b>	The same as doing /DeviceGray <b>cs</b> followed by <i>gray</i> <b>sc</b> .
<i>r g b</i>	<b>RG</b>	The same as doing /DeviceRGB <b>CS</b> followed by <i>r g b</i> <b>SC</b> .
<i>r g b</i>	<b>rg</b>	The same as doing /DeviceRGB <b>cs</b> followed by <i>r g b</i> <b>sc</b> .
<i>c m y k</i>	<b>K</b>	The same as doing /DeviceCMYK <b>CS</b> followed by <i>c m y k</i> <b>SC</b> .
<i>c m y k</i>	<b>k</b>	The same as doing /DeviceCMYK <b>cs</b> followed by <i>c m y k</i> <b>sc</b> .

## 4 Patterns and Shadings

This section should probably belong to the “Graphics” section; but it’s a big enough of a concept that I (and hopefully you) will let letting it have its own section slide. A pattern is a repeating graphical figure or smooth gradient used to fill regions of space. Take for example the two patterns below:



There are two types of patterns:

- (1) *Tiling patterns* which repeat a small graphical figure (a *pattern cell*) over fixed horizontal and vertical intervals to fill the area to be painted. (The above left example is one of a tiling pattern.)
- (2) *Shading patterns* form a gradient that smoothly transitions between colors and fills the area to be painted. (The above right example is one of a shading pattern.)

Every pattern has a *pattern matrix*, which is a transformation matrix which maps the pattern’s internal coordinate system to the default coordinate system of the pattern’s parent content stream (the content stream from which the pattern was called). The multiplication of the pattern matrix with the transformation matrix of the parent content stream establishes the *pattern coordinate space*. Importantly this is the multiplication of the transformation matrix of the parent stream, and not the CTM. Thus after a pattern is defined in the resources of a content stream, its pattern coordinate space is not affected by changes to the CTM.

### 4.1 Tiling patterns

A tiling pattern takes a small graphical figure, called the *pattern cell*, and repetitively paints it horizontally and vertically. The pattern cell is a content stream which may contain anything a content stream can contain (paths, images, XObjects, etc.). When painting using a tiling pattern, the pattern cell is tiled as many times as needed to fill the region specified.

A pattern cell is defined by a content stream, and thus has a stream dictionary. Along with fields common to all stream dictionaries, a pattern dictionary may have the following fields:

Key	Type	Value
<b>Type</b>	name	(Optional) the type of PDF object the dictionary describes; must be <b>Pattern</b> .
<b>PatternType</b>	integer	(Required) a code identifying what kind of pattern this dictionary describes. Must be 1 for a tiling pattern.
<b>PaintType</b>	integer	(Required) a code that determines how the color of the pattern cell is to be specified: <ol style="list-style-type: none"><li>1 <i>Colored tiling pattern</i>: the pattern’s content stream specifies the colors to be used to paint the pattern cell. When the pattern cell’s content stream begins execution, the current color is the one that was in effect in the parent content stream.</li><li>2 <i>Uncolored tiling pattern</i>: the pattern’s content stream does not specify any color information. Instead, the entire pattern cell is painted a separately specified color each time the pattern is used.</li></ol>

<b>TilingType</b>	integer	(Required) a code that controls adjustments to the spacing of tiles relative to the device pixel grid: <ol style="list-style-type: none"> <li>1 <i>Constant spacing</i>: pattern cells are spaced consistently, i.e. by a multiple of a device pixel. To achieve this, the application may distort the pattern cell slightly by making small adjustments to <b>XStep</b>, <b>YStep</b>, and the transformation matrix. This distortion does not exceed one device pixel.</li> <li>2 <i>No distortion</i>: the pattern cell is not distorted, but the spacing between the pattern cells may vary by as much as one device pixel.</li> <li>3 <i>Constant spacing and faster tiling</i>: pattern cells are spaced consistently as with <i>constant spacing</i>, but additional distortion is permitted to enable faster tiling.</li> </ol>
<b>BBox</b>	array	(Required) an array of four numbers specifying the bottom left and top right corners of the bounding box of the pattern cell. That is, <b>BBox</b> is of the form $[x_0 \ y_0 \ x_1 \ y_1]$ where $(x_0, y_0)$ is the bottom left corner of the pattern cell (relative to its own coordinates), and $(x_1, y_1)$ is the top right corner.
<b>XStep</b>	number	(Required) the desired horizontal spacing between pattern cells, measured in the pattern coordinate system.
<b>YStep</b>	number	(Required) the desired vertical spacing between pattern cells, measured in the pattern coordinate system.
<b>Resources</b>	dictionary	(Required) a resource dictionary containing all the named resources used by the pattern's content stream.
<b>Matrix</b>	array	(Optional) an array of six numbers representing the pattern matrix.

We use the following operators to paint with patterns:

Operands	Operator	Description
$c_1 \dots c_n \ name$	<b>SCN</b>	Sets the current pattern to be stroked, if the current color space is <b>Pattern</b> , or if <i>name</i> refers to a shading pattern. <i>name</i> must refer to a pattern defined in the <b>Pattern</b> subdictionary of the current resource dictionary. If this is an uncolored tiling pattern ( <b>PatternType</b> of 1 and <b>PaintType</b> of 2), $c_1, \dots, c_n$ are parameters in the specified underlying color space (see below) which set the color to use for the tiling.
$c_1 \dots c_n \ name$	<b>scn</b>	Same as <b>SCN</b> but for non-stroking operations.

For example, to draw the colored tiling pattern at the beginning of the section, the following PDF code was produced:

First we have the pattern object, which defines the pattern cell and necessary data for tiling:

```

1 1 0 obj
2  <<
3     /Type /Pattern      % pattern dictionary
4     /PatternType 1      % tiling pattern
5     /PaintType 1       % colored tiling pattern
6     /TilingType 1       % constant spacing
7     /BBox [0 0 20 20]  % bounding box
8     /XStep 20           % xstep and ystep are precisely
9     /YStep 20           % the size of the pattern cell
10    /Resources << >>   % no resources
11    /Length 84
12  >>
13 stream
14 q
15 1 0 0 rg               % set color to red

```

```

16 0 0 10 10 re 10 10 10 10 re f % draw the two red rectangles
17 0 0 .5 rg % set color to blue
18 10 0 10 10 re 0 10 10 10 re f % draw the two blue rectangles
19 Q
20 endstream
21 endobj

```

Then we have the content stream (which here is an XObject) in which the pattern is painted.

```

1 2 0 obj
2 <<
3 /Type /XObject % fields for the XObject...
4 /Subtype /Form
5 /BBox [0 0 149.44 99.626]
6 /FormType 1
7 /Matrix [1 0 0 1 0 0]
8 /Resources 9 0 R % resources are in object 9 0
9 /Length 193
10 >>
11 stream
12 q
13 .996264 0 0 .996264 0 0 cm % this will be explained later
14 .5 0 0 .5 0 0 cm % scale down by half
15 1 j 1 J % rounded lines
16 /Pattern cs % set current non-stroking color space to be a pattern
17 /P scn % set the current non-stroking pattern
18 0 100 m % draw the region to paint...
19 25 150 100 200 150 200 c
20 200 150 250 150 300 100 c
21 275 50 250 0 y
22 200 0 175 0 150 50 c
23 125 50 100 50 0 100 c
24 h
25 B % fill
26 Q
27 endstream
28 endobj

```

The resources for this XObject are in object 9 0. This object must define the name /P. Indeed it does:

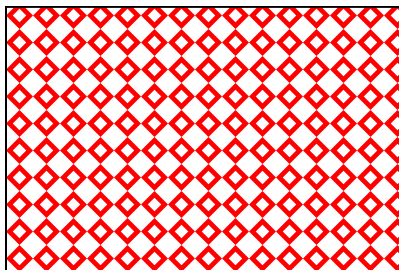
```

1 % 9 0 obj
2 <<
3 /Pattern << /P 1 0 R >>
4 /ProcSet [ /PDF ]
5 >>

```

So /P is the pattern defined in object 1 0.

For an uncolored tiling pattern, we must also specify what color we want to paint it with. So when we call **CS** or **cs**, we need to tell PDF that we're switching to a **Pattern** color space, as well as the underlying color space (e.g. **DeviceRGB**). Then when we call **SCN** or **scn**, we need to tell PDF what color we want to paint the pattern with. For example, suppose we have the following uncolored pattern:



The pattern cell here is simply a hollowed-out diamond. The PDF code for the pattern cell is as follows:

```

1 1 0 obj
2 <<
3 /Type /Pattern % pattern cell
4 /PatternType 1 % tiling pattern
5 /PaintType 2 % uncolored tiling pattern
6 /TilingType 1 % constant spacing

```

```

7      /BBox [0 0 10 10]      % bounding box
8      /XStep 10              % xstep and ystep are precisely
9      /YStep 10              % the size of the pattern cell
10     /Resources << >>      % no resources
11     /Length 66
12 >>
13 stream
14 5 10 m
15 0 5 1 5 0 1 10 5 1 h      % draw outer diamond
16 5 7.5 m
17 2.5 5 1 5 2.5 1 7.5 5 1 h % draw inner diamond
18 f*                        % fill using even-odd
19                          % (so that the diamond is hollowed out)
20 endstream
21 endobj

```

The code which draws the pattern is as follows:

```

1 2 0 obj
2 <<
3   /Type /XObject
4   /Subtype /Form
5   /BBox [0 0 149.44 99.626]
6   /FormType 1
7   /Matrix [1 0 0 1 0 0]
8   /Resources 9 0 R
9   /Length 78
10 >>
11 stream
12 q
13 .996264 0 0 .996264 0 0 cm % to be explained later
14 1 j 1 J                  % rounded lines
15 /CSP cs                  % set color space to /CSP
16 1 0 0 /P scn             % set pattern to /P with color 1 0 0
17 0 0 150 100 re           % draw rectangle
18 B                        % and fill
19 Q
20 endstream
21 endobj

```

We see that here we set the (non-stroking) color space to a name **CSP**. This name is defined in the resource dictionary, in object 9 0:

```

1 % 9 0 obj
2 <<
3   /ColorSpace << /CSP [ /Pattern /DeviceRGB ] >>
4   /Pattern << /P 1 0 R >>
5   /ProcSet [ /PDF ]
6 >>

```

Here **CSP** is defined to be a **ColorSpace**. But instead of being defined as a name like **DeviceRGB**, it is defined to be an array of two names: **Pattern** and **DeviceRGB**. This is because since **P** is an uncolored tiling pattern, we must specify the color to color it with. In object 2 0, we specified the color code 1 0 0, but in order to interpret this color code we need to know what the underlying color space is. That's why the second color space in the array is **DeviceRGB**, telling us to set the underlying color space to be **DeviceRGB**.

## 4.2 Shading Patterns

A shading pattern is one which defines a gradient between colors. It is a pattern for all intents and purposes, and painting it is done in the same way as a normal pattern. Unlike a tiling pattern, a shading pattern is defined by a dictionary instead of a content stream. Fields in the pattern dictionary for a shading pattern are as follows:

Key	Type	Value
<b>Type</b>	name	(Optional) the type of PDF object the dictionary describes; must be <b>Pattern</b> .
<b>PatternType</b>	integer	(Required) the code defining the type of pattern this dictionary describes. Must be 2 for a shading pattern.
<b>Shading</b>	dictionary or stream	(Required) a shading object (see below) defining the shading pattern gradient.

<b>Matrix</b>	array	(Optional) An array of six numbers defining the pattern matrix.
<b>ExtGState</b>	dictionary	(Optional) a graphics state parameter dictionary to be put into effect when the shading is painted.

The most important field is **Shading**, which defines the shading object; the object which controls the look of the gradient. A shading object may be a content stream or a dictionary, and in either case we refer to the dictionary (either the object itself or the content stream's dictionary) as the shading dictionary. The shading dictionary may have the following fields, which are common to all types of shadings (but different types of shadings will have additional fields):

Key	Type	Value
<b>ShadingType</b>	integer	(Required) the shading type (discussed below): <ul style="list-style-type: none"> <li>1 function-based shading;</li> <li>2 axial shading;</li> <li>3 radial shading;</li> <li>4 free-form Gourand-shaded triangle mesh;</li> <li>5 lattice-form Gourand-shaded triangle mesh;</li> <li>6 coons patch mesg;</li> <li>7 tensor-product patch mesh.</li> </ul>
<b>ColorSpace</b>	name	(Required) the color space in which the color values are to be interpreted. This cannot be a <b>Pattern</b> space.
<b>Background</b>	array	(Optional) an array containing a color value (appropriate to <b>ColorSpace</b> ) determining a background color value. If present, this is used to color any region outside of the region defined by the shading object.
<b>BBox</b>	array	(Optional) an array of the form $[x_0\ y_0\ x_1\ y_1]$ which defines the bottom-left and top-right coordinates of the bounding box to clip the shading to.

We will focus on shading types 1 through 3, as 4 through 7 are much more complicated. If you're interested, consult the PDF reference.

#### 4.2.1 Type 1 (Function-Based) Shadings

A function-based shading is one where the color at every point on the gradient is defined by a specified function. This is the most general form of shading. If a shading object is specified to be of type 1, it may have the following fields:

Key	Type	Value
<b>Domain</b>	array	(Optional) an array of four numbers $[x_0\ x_1\ y_0\ y_1]$ specifying the rectangular domain of coordinates over which the color function(s) are defined. Default is $[0\ 1\ 0\ 1]$ .
<b>Matrix</b>	array	(Optional) an array of six numbers specifying a transformation mapping the coordinate space specified by <b>Domain</b> to the shading's target coordinate space. For example, to map the default <b>Domain</b> to a one-inch square starting at coordinate (100,100), the <b>Matrix</b> would be $[72\ 0\ 0\ 72\ 100\ 100]$ (since the default value of <b>UserUnit</b> is 1/72th of an inch).
<b>Function</b>	function	A 2-in, $n$ -out function; or an array of $n$ 2-in 1-out functions (where $n$ is the number of color values required by the <b>ColorSpace</b> specified in the shading dictionary). Each function's domain must be a superset of <b>Domain</b> .

For example, suppose we want to shade using the following function, which simply computes the average of its two inputs:

```

1 1 0 obj
2 <<
3   /FunctionType 4
4   /Domain [0 1 0 1]
5   /Range [0 1]
6   /Length 13
7 >>
8 stream
9 {
10    add
11    2 div
12 }
13 endstream
14 endobj

```

We create a shading object which uses this function:

```

1 % 2 0 obj
2 <<
3   /ShadingType 1           % functional shading
4   /ColorSpace /DeviceGray  % use grayscale coloring
5   /Domain [0 1 0 1]       % domain is [0,1] x [0,1]
6   /Matrix [100 0 0 100 0 0] % scale gradient by 100
7   /Function 1 0 R         % use above function
8 >>

```

and a shading pattern object which sets this as its **Shading** attribute

```

1 % 4 0 obj
2 <<
3   /Type /Pattern % pattern object
4   /PatternType 2 % shading pattern
5   /Shading 2 0 R % shading object is above dictionary
6 >>

```

The object in which the pattern is drawn is

```

1 5 0 obj
2 <<
3   /Type /XObject
4   /Subtype /Form
5   /BBox [0 0 99.626 99.626]
6   /FormType 1
7   /Matrix [1 0 0 1 0 0]
8   /Resources 11 0 R
9   /Length 42
10 >>
11 stream
12 q
13 /P scn           % set current pattern
14 0 0 100 100 re  % draw rectangle
15 B               % fill
16 Q
17 endstream
18 endobj

```

Notice that we don't need to change the color space, as the color space is determined by the shading pattern (**P** is defined in the resources of this object, object 11 0 as a reference to object 4 0). The resulting pattern is:



#### 4.2.2 Type 2 (Axial) Shadings

Axial shadings are shadings which vary along a linear axis between two endpoints, and extends indefinitely



perpendicular to that axis. The shading may also extend indefinitely beyond either or both endpoints by continuing the boundary colors. The following table lists the fields specific to type 2 shadings:

Key	Type	Value
<b>Coords</b>	array	(Required) an array of the form $[x_0 \ y_0 \ x_1 \ y_1]$ which specifies the start and endpoints of the axis, expressed in the shading's target coordinate space.
<b>Domain</b>	array	(Optional) an array of two numbers $[t_0 \ t_1]$ which specifies the limiting values of the parameteric value $t$ (i.e. $t_0 \leq t \leq t_1$ ). Default value is $[0 \ 1]$ .
<b>Function</b>	function	(Required) a 1-in $n$ -out function, or an array of $n$ 1-in 1-out functions (where $n$ is the number of color values required by the <b>Color Space</b> ). These functions are used to determine the color along the axis. Each function's domain must be a superset of <b>Domain</b> .
<b>Extend</b>	array	(Optional) an array of two boolean values which specifies whether to extend the shading beyond the start and end points of the axis, respectively. Default value is $[\text{false} \ \text{false}]$ .

Suppose we want to create a linear gradient between red and blue. We can use the following linear interpolation function

```

1 % 1 0 obj
2 <<
3   /FunctionType 2 % exponential interpolation function
4   /Domain [0 1] % domain is [0,1]
5   /C0 [1 0 0] % interpolate between red ...
6   /C1 [0 0 .5] % ... and blue
7   /N 1 % using linear interpolation
8 >>

```

Now we define the shading object

```

1 % 3 0 obj
2 <<
3   /ShadingType 2 % axial shading
4   /ColorSpace /DeviceRGB % use RGB color sapce
5   /Coords [0 0 100 0] % axis between origin and (100,0)
6   /Domain [0 1] % domain of function is [0 1]
7   /Function 1 0 R % function is above linear interpolation function
8   /Extend [false false] % do not extend
9 >>

```

and the pattern object

```

1 % 4 0 obj
2 <<
3   /Type /Pattern % pattern object
4   /PatternType 2 % shading pattern
5   /Shading 3 0 R % using the above shading object
6 >>

```

And the actual content stream where the gradient is drawn is

```

1 5 0 obj
2 <<
3   /Type /XObject
4   /Subtype /Form
5   /BBox [0 0 99.626 99.626]
6   /FormType 1
7   /Matrix [1 0 0 1 0 0]
8   /Resources 11 0 R
9   /Length 94
10 >>
11 stream
12 q
13 /P scn

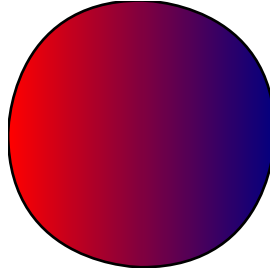
```

```

14 50 100 m
15 100 100 100 50 v 100 0 50 0 v 0 0 0 50 v 0 100 50 100 v % draw circle
16 h
17 B % fill
18 Q
19 endstream
20 endobj

```

Again, **P** is defined in the **Resources** of the content stream as a reference to the pattern object defined. The resulting pattern is



### 4.2.3 Type 3 (Radial) Shadings

Radial shadings define a gradient that varies between two circles. This can be used to depict three-dimensional spheres and cones. Radial shading dictionaries may contain the following fields:

Key	Type	Value
<b>Coords</b>	array	(Required) an array of the form $[x_0 \ y_0 \ r_0 \ x_1 \ y_1 \ r_1]$ , which specifies the centers and radii of the starting and ending circles. $r_0$ and $r_1$ must be at least zero. If one is zero, it is treated as a point, and if both are, nothing is painted.
<b>Domain</b>	array	(Optional) an array of two numbers $[t_0 \ t_1]$ specifying the limiting values of the parametric value $t$ . This variable varies linearly between $t_0$ and $t_1$ as it goes between the two circles. $t$ is the input to the function specified by <b>Function</b> .
<b>Function</b>	function	(Required) a 1-in $n$ -out, or an array of $n$ 1-in 1-out functions, (where $n$ is the number of values in the <b>ColorSpace</b> ) which specify the color to be drawn, called with parametric value $t$ . Each function's domain must be a superset of <b>Domain</b> .
<b>Extend</b>	array	(Optional) an array of two boolean values which specifies whether or not to extend the shading beyond the starting and ending circles. Default value $[false \ false]$ .

For example, if we want to shade a circle with an inner color of red and an outer color of blue, we first define the interpolation function which linearly interpolates between red and blue (same as our previous example). Then we define a shading object:

```

1 % 3 0 obj
2 <<
3   /ShadingType 3 % radial shading
4   /ColorSpace /DeviceRGB % rgb color space
5   /Coords [75 75 0 50 50 50] % first circle is a point at (75,75),
6   % second is the circle of radius 50 at (50,50)
7   /Domain [0 1] % domain of linear interpolation function
8   /Function 1 0 R % use linear interpolation function
9   /Extend [false false] % do not extend
10 >>

```

The object which draws the pattern is

```

1 4 0 obj
2 <<
3   /Type /XObject
4   /Subtype /Form
5   /BBox [0 0 99.626 99.626]

```

```

6      /FormType 1
7      /Matrix [1 0 0 1 0 0]
8      /Resources 10 0 R
9      /Length 7
10   >>
11   stream
12   /Sh sh
13   endstream
14   endobj

```

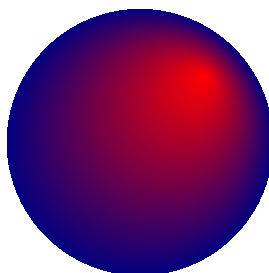
Notice here we just use the **sh** operator, which we explain below. Let's look at the resources dictionary:

```

1  % 10 0 obj
2  <<
3      /Shading << /Sh 3 0 R >>
4      /ProcSet [ /PDF ]
5  >>

```

So we define **Sh** to be a **Shading** object, referencing the shading object defined in object 3 0. The resulting pattern is



#### 4.2.4 The shading operator

If you have a shading pattern *name* and you'd like to draw it as-is (without using it to fill a path), you can use the shading operator **sh**. *name* must be specified in the **Shading** subdictionary of the resource dictionary of the current content stream. This operator should only be applied to bounded or geometrically defined shadings.

## 5 External Objects

An external object (XObject) is a graphics object whose content is defined by a self-contained content stream, separate from the content stream in which it is used. There are three types of XObjects, but we won't discuss the third (PostScript XObjects) as their use is discouraged. The other two types of XObjects are

- (1) *image XObjects* which represent images;
- (2) *form XObjects* which are self-contained descriptions of an arbitrary sequence of graphics objects. Two subtypes of form XObjects are *group XObjects* and *reference XObjects*.

If an XObject is named by *name* in the **XObject** subdictionary of the **Resource** dictionary of a content stream, it can be painted using the **Do** operator. For example, we may define the following form XObject:

```

1  1 0 obj
2  <<
3      /Type /XObject
4      /Subtype /Form
5      /BBox [0 0 10 1]
6      /FormType 1
7      /Matrix [1 0 0 1 0 0]
8      /Resources 7 0 R
9      /Length ...
10 >>
11 stream
12 1 w
13 1 0 0 1 0 .5 cm
14 0 0 m 10 0 l S
15 endstream
16 endobj

```

which simply paints a line of length 10. Now, in another content stream we can paint this XObject using the **Do** operator:

```

1 4 0 obj
2 <<
3   /Length 34
4 >>
5 stream
6 q
7 1 0 0 1 72 765.905 cm
8 /Fm1 Do
9 Q
10 endstream
11 endobj

```

We see that **Do** references the name **Fm1**, which is defined in the page’s **Resource** dictionary to be a reference to object 1 0.

```

1 % 2 0 obj
2 <<
3   /XObject << /Fm1 1 0 R >>
4   /ProcSet [ /PDF ]
5 >>

```

## 5.1 Images

There are two ways to insert images into a PDF. One is using image XObjects, and another is using inline images where the content of the image is represented inline directly in a content stream. Image XObjects will be discussed later, with their respective pdfTeX primitives.

An inline image is declared in a content stream in the following way:

```

1 BI % begin inline image object
2 ... key-value pairs ...
3 ID % begin image data
4 ... image data ...
5 EI % end inline image object

```

That is, the **BI** operator begins the inline image dictionary (except its just a stream of key-value pairs, not wrapped in <<...>>), **ID** ends it and begins the image data (which specifies how to color each pixel), and **EI** ends the inline image object. The fields that can be placed in between **BI** and **ID** are

Key	Type	Value
<b>BitsPerComponent</b>	integer	(Required) the number of bits used to represent each color component (a single value in a color value). Valid values are 1, 2, 4, 8, 16.
<b>Width</b>	integer	(Required) the width of the image, in samples.
<b>Height</b>	integer	(Required) the height of the image, in samples.
<b>ColorSpace</b>	name	(Required) the color space that the image is defined in.
<b>Filter</b>	array	(Required) an array of filters to be applied to the image data to decode it.

We can use the following abbreviations for the fields:

- **BitsPerComponent** — **BPC**
- **Width** — **W**
- **Height** — **H**
- **ColorSpace** — **CS**
- **Filter** — **F**

So if **CS** requires  $n$  values per color (e.g. 3 for RGB), **W** is  $w$ , and **H** is  $h$ , the image data should have  $n \cdot w \cdot h$  samples. Each sample must have a size of **BPC** after being decoded by the filter. The only filter we will discuss is **AHx** (or **ASCIHexDecode**), which decodes ASCII hex codes. That is, it takes two ASCII characters (which are either 0–9 or  $a-z$  or  $A-Z$ ), and computes the hex value. Whitespace is ignored.

For example, we may have the inline image:

```

1  q
2  .996264 0 0 .996264 0 0 cm
3  100 0 0 100 0 0 cm
4  BI
5      /BPC 8
6      /CS /DeviceRGB
7      /F [ /AHx ]
8      /W 9
9      /H 9
10 ID
11  ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000
12  ff0000 ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ff0000
13  ff0000 ffffffff 00ff00 00ff00 ffffffff 00ff00 00ff00 ffffffff ff0000
14  ff0000 ffffffff 00ff00 00ff00 ffffffff 00ff00 00ff00 ffffffff ff0000
15  ff0000 ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ff0000
16  ff0000 ffffffff 0000ff ffffffff ffffffff ffffffff 0000ff ffffffff ff0000
17  ff0000 ffffffff ffffffff 0000ff 0000ff 0000ff ffffffff ffffffff ff0000
18  ff0000 ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ffffffff ff0000
19  ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000 ff0000
20 EI
21 Q

```

Which draws the following:

Recall that an image is by default one unit by one unit, so we must scale it to get the desired size.

## 5.2 Form XObjects

A form XObject is a self-contained content stream which can be painted within other content streams. The graphics defined in a form XObject are defined relative to its *form coordinate space*, which can then be scaled to user space using its **Matrix** field.

Form XObjects may have the following fields in their stream dictionary, as well as fields common to all content streams:

Key	Type	Value
<b>Type</b>	name	(Optional) the type of PDF object the dictionary describes; must be <b>XObject</b> .
<b>Subtype</b>	name	(Required) the type of XObject the dictionary describes; must be <b>Form</b> .
<b>BBox</b>	array	(Required) a bounding box which defines the boundary of the XObject in form space. The XObject is clipped to this boundary.
<b>Matrix</b>	array	(Optional) a matrix representing the transformation from form space to user space.
<b>Resources</b>	dictionary	(Required) a dictionary specifying the resources required by the form XObject.
<b>Group</b>	dictionary	(Optional) a <i>group attributes dictionary</i> indicating that the contents of the form XObject are to be treated as a group and specifying the attributes of that group (see below).

### 5.2.1 Group XObjects

A *group XObject* is a type of form XObject that can be used to group graphical elements together for various purposes. It is declared by adding the **Group** field to a form XObject's dictionary. This is a dictionary called the *group attributes dictionary*, which may have the following fields:

Key	Type	Value
<b>Type</b>	name	(Optional) the type of PDF object that this dictionary describes; must be <b>Group</b> .
<b>S</b>	name	(Required) the <i>group subtype</i> , which identifies the kind of group the dictionary describes. The only available group subtype is <b>Transparency</b> . Transparency group XObjects are discussed later.

## 6 Transparency

### 6.1 The idea

#### 6.1.1 Compositing semi-transparent images

In a normal, opaque, image model, every pixel is given a color in some color space like RGB. When two objects are placed on top of one another, the bottom object has no effect on the color of the pixels which overlap with the top object. In a transparent image model, objects may be given an opacity value (called an  $\alpha$ -value) which determines how transparent the object is. Thus, if a semi-transparent object (an object with an  $\alpha$ -value less than 1) is placed on top of another object, the bottom object's color will affect the pixels that overlap.

We use the following notation: the underlying (opaque) color space is denoted by  $\mathcal{C}$ . Colors in the color space  $\mathcal{C}$  are denoted with an uppercase  $C$  and some subscript to uniquely distinguish them. A color in the transparent color space whose underlying space is  $\mathcal{C}$  is denoted with a lowercase  $c$ ; it is made up of an opaque color  $C \in \mathcal{C}$  and an  $\alpha$ -value  $\alpha \in [0, 1]$  where  $\alpha = 0$  means the color is totally transparent, and an  $\alpha$ -value of 1 means the color is totally opaque.

How should we interpret the  $\alpha$  value of a color (the  $\alpha$ -component of an image is also called the  $\alpha$  channel)? The idea that we will use is that given any pixel (or area of an image), if it is colored with color  $C$  and  $\alpha$ -value  $\alpha$ , then the color  $C$  is uniformly spread out over the pixel to take up an  $\alpha$  fraction of the space. For example, if we want to color a pixel with RGB value  $(1, 0, 0)$  and  $\alpha = 0.5$ , then half the pixel will be colored red  $(1, 0, 0)$ , so the resulting color will be  $(0.5, 0, 0)$ .

In order to store a transparent color  $(C, \alpha)$ , it is often useful to instead store the tuple  $(c = \alpha C, \alpha)$ . This is since most procedures around transparent colors use  $\alpha C$  instead of  $\alpha$ ; and multiplying every component by  $\alpha$  in every pixel is time-consuming.

Now suppose we'd like to blend two pixels  $A$  and  $B$ . The first has color  $(C_A, \alpha_A)$  and the second has color  $(C_B, \alpha_B)$ . We split the pixel into four regions: the region with neither  $A$  nor  $B$  (which doesn't contribute to the pixel color, so we ignore it); the region with  $A$  and not  $B$  (denoted  $A - B$ ); the region with  $B$  and not  $A$  (denoted  $B - A$ ); and the region with both  $A$  and  $B$  (denoted  $A \cap B$ ). Each of these regions takes up a certain amount of area in the pixel, and thus has an associated  $\alpha$ -value. We can compute the  $\alpha$ -values using the assumption that colors are spread uniformly. So if a region has an  $\alpha$ -value of  $\alpha_1$ , and another has an  $\alpha$ -value of  $\alpha_2$ , their overlap has an  $\alpha$ -value of  $\alpha_1 \cdot \alpha_2$  (due to uniformity). Thus

- region  $A - B$  has an  $\alpha$ -value of  $\alpha_A(1 - \alpha_B)$  (since the region outside  $B$ ,  $B^c$ , has an  $\alpha$ -value of  $1 - \alpha_B$ );
- similarly region  $B - A$  has an  $\alpha$ -value of  $\alpha_B(1 - \alpha_A)$ ;
- region  $A \cap B$  has an  $\alpha$ -value of  $\alpha_A \cdot \alpha_B$ .

Now we have to determine which color each region has. Obviously  $A - B$  and  $B - A$  should have colors  $C_A$  and  $C_B$  respectively. But what about  $A \cap B$ ? For this reason we must use a *blending function*  $\mathfrak{B}: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ , which takes two opaque colors and outputs how they should be blended. We leave this as a user-defined variable. So  $A \cap B$  will have a color of  $\mathfrak{B}(C_A, C_B)$ .

Thus the transparent color of the resulting pixel  $A \cup B$  as a whole will be

$$\alpha_{A-B}C_{A-B} + \alpha_{B-A}C_{B-A} + \alpha_{A \cap B}C_{A \cap B} = \alpha_A(1 - \alpha_B)C_A + \alpha_B(1 - \alpha_A)C_B + \alpha_A\alpha_B\mathfrak{B}(C_A, C_B)$$

That is,

$$c_{A \cup B} = \alpha_A(1 - \alpha_B)C_A + \alpha_B(1 - \alpha_A)C_B + \alpha_A\alpha_B\mathfrak{B}(C_A, C_B)$$

This is called the *basic color compositing formula*. Recall that  $c_{A \cup B} = \alpha_{A \cup B} \cdot C_{A \cup B}$ , so now we can ask ourselves: what is the value of  $\alpha_{A \cup B}$  and the value of the underlying opaque color  $C_{A \cup B}$ ?

Well we can write the region  $A \cup B$  as a disjoint union  $(A - B) \sqcup B$ , thus

$$\alpha_{A \cup B} = \alpha_{A-B} + \alpha_B = \alpha_A(1 - \alpha_B) + \alpha_B = \alpha_A + \alpha_B - \alpha_A\alpha_B$$

Then, we get the following formula for  $C_{A \cup B}$ :

$$C_{A \cup B} = \left(1 - \frac{\alpha_B}{\alpha_{A \cup B}}\right) \cdot C_A + \frac{\alpha_B}{\alpha_{A \cup B}} \cdot [(1 - \alpha_A) \cdot C_B + \alpha_A \cdot \mathfrak{B}(C_A, C_B)]$$

The benefit of this formula, over the naive one obtained by simply dividing by  $\alpha_{A \cup B}$ , is that only one division needs to be performed.

Note that  $A \cup B$  is not the best choice of notation for this region: it is not commutative. That is,  $C_{A \cup B}$  does not necessarily equal  $C_{B \cup A}$ . This is because the blend function may not be commutative. Instead, we will use the notation  $B/A$ , or  $B$  over  $A$ .

To summarize, the  $\alpha$ -premultiplied color of  $B$  over  $A$  is

$$c_{B/A} = \alpha_A(1 - \alpha_B)C_A + \alpha_B(1 - \alpha_A)C_B + \alpha_A\alpha_B\mathfrak{B}(C_A, C_B)$$

and the color of  $B$  over  $A$  is

$$C_{B/A} = \left(1 - \frac{\alpha_B}{\alpha_{B/A}}\right) \cdot C_A + \frac{\alpha_B}{\alpha_{B/A}} \cdot [(1 - \alpha_A) \cdot C_B + \alpha_A \cdot \mathfrak{B}(C_A, C_B)]$$

We will define this to be  $C_{B/A} = \text{BASICCOMPOSITE}(C_A, \alpha_A, C_B, \alpha_B, \mathfrak{B})$ . and the  $\alpha$ -value of  $B$  over  $A$  is

$$\alpha_{B/A} = \alpha_{A-B} + \alpha_B = \alpha_A(1 - \alpha_B) + \alpha_B = \alpha_A + \alpha_B - \alpha_A\alpha_B$$

It is useful to define the union function  $\cup: [0, 1]^2 \rightarrow [0, 1]$  by  $x \cup y = 1 - (1 - x)(1 - y) = x + y - xy$ . Thus,  $\alpha_{B/A} = \alpha_A \cup \alpha_B$ .

This gives us the theoretical background for composing two semi-transparent images together. We now discuss different types of blend functions:

### 6.1.2 Blend functions

A blend function is a function  $\mathfrak{B}: \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ . Generally color spaces  $\mathcal{C}$  are sets of tuples, these are the only kind of color space we will consider. In fact, we will only consider color spaces of the form  $[0, 1]^n$  for some  $n$ .

A blend function  $\mathfrak{B}$  is called *separable* there exists a function  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  (which we will also denote  $\mathfrak{B}$ ), such that

$$\mathfrak{B}((c_b^1, \dots, c_b^n), (c_s^1, \dots, c_s^n)) = (\mathfrak{B}(c_b^1, c_s^1), \dots, \mathfrak{B}(c_b^n, c_s^n))$$

That is, the blend of two colors (the backdrop  $C_b$  and the new source  $C_s$ ) is the blend of each component.

The separable blend functions supported by PDF are as follows:

Name	Result
<b>Normal</b>	Selects the source color, ignoring background: $\mathfrak{B}(c_b, c_s) = c_s$
<b>Multiply</b>	Multiplies the backdrop with the source: $\mathfrak{B}(c_b, c_s) = c_b \cdot c_s$  This can also be thought of as taking the “intersection” of the colors. The result is always at most as light as the darkest color.
<b>Screen</b>	Takes the complement of the multiplication of the complements: $\mathfrak{B}(c_b, c_s) = 1 - [(1 - c_b) \cdot (1 - c_s)] = c_b + c_s + c_b c_s$  This can be thought of as taking the “union” of the colors. The result is always at most as dark as the lightest color.
<b>HardLight</b>	Multiplies or screens the colors (intersection or union), depending on the source color value. The effect is similar to shining a harsh spotlight on the backdrop:

$$\mathfrak{B}(c_b, c_s) = \begin{cases} \text{Multiply}(c_b, 2c_s) & c_s \leq 0.5 \\ \text{Screen}(c_b, 2c_s - 1) & c_s > 0.5 \end{cases}$$

**Overlay** Multiplies or screens the colors (intersection or union), depending on the backdrop color value.

$$\mathfrak{B}(c_b, c_s) = \text{HardLight}(c_s, c_b)$$

**Darken** Selects the darker color

$$\mathfrak{B}(c_b, c_s) = \min(c_b, c_s)$$

**Lighten** Selects the lighter color

$$\mathfrak{B}(c_b, c_s) = \max(c_b, c_s)$$

**ColorDodge** Brightens the backdrop color to reflect the source color:

$$\mathfrak{B}(c_b, c_s) = \begin{cases} \min(1, c_b/(1 - c_s)) & c_s < 1 \\ 1 & c_s = 1 \end{cases}$$

**ColorBurn** Darkens the backdrop color to reflect the source color:

$$\mathfrak{B}(c_b, c_s) = \begin{cases} 1 - \min(1, (1 - c_b)/c_s) & c_s > 0 \\ 0 & c_s = 0 \end{cases}$$

**SoftLight** Darkens or lightens the colors, depending on source color value. The effect is similar to shining a diffused spotlight on the backdrop:

$$\mathfrak{B}(c_b, c_s) = \begin{cases} c_b - (1 - 2c_s)c_b(1 - c_b) & c_s \leq 0.5 \\ c_b + (2c_s - 1)(D(c_b) - c_b) & c_s > 0.5 \end{cases}$$

where

$$D(x) = \begin{cases} x((16x - 12)x + 4) & x \leq 0.25 \\ \sqrt{x} & x > 0.25 \end{cases}$$

**Difference** Subtracts the darker from the lighter color

$$\mathfrak{B}(c_b, c_s) = |c_b - c_s|$$

**Exclusion** Produces an effect similar to **Difference** but with less contrast.

$$\mathfrak{B}(c_b, c_s) = c_b + c_s - 2c_b c_s$$

PDF also supports non-separable blend functions. These blend functions follow essentially the same principal:

- (1) convert both colors from the blending space to an intermediate HSL (hue-saturation-luminosity) representation;
- (2) create a new color from some combination of the HSL values of the colors;
- (3) transform the color back to the blending color space.

The non-separable functions use the following auxiliary functions (we assume the blending space is RGB; for a color  $C$ , we denote it by  $(C_r, C_g, C_b)$ ; we also let  $C_{min}$  be the minimum of the color components,  $C_{mid}$  the middle, and  $C_{max}$  the maximum):

1. **function** LUM( $C$ )
2.     **return**  $0.3C_r + 0.59C_g + 0.11C_b$
3. **end function**
4. **function** SETLUM( $C, \ell$ )
5.      $\Delta = \ell - \text{Lum}(C)$



```

6.   return ClipColor( $C + (\Delta, \Delta, \Delta)$ )
7. end function

8. function CLIPCOLOR( $C$ )
9.    $\ell = \text{Lum}(C)$ 
10.  if ( $C_{\min} < 0$ )
11.     $C_r = \ell + (C_r - \ell) \cdot \ell / (\ell - C_{\min})$ 
12.     $C_g = \ell + (C_g - \ell) \cdot \ell / (\ell - C_{\min})$ 
13.     $C_b = \ell + (C_b - \ell) \cdot \ell / (\ell - C_{\min})$ 
14.  end if
15.  if ( $C_{\max} > 1$ )
16.     $C_r = \ell + (C_r - \ell) \cdot (1 - \ell) / (C_{\max} - \ell)$ 
17.     $C_g = \ell + (C_g - \ell) \cdot (1 - \ell) / (C_{\max} - \ell)$ 
18.     $C_b = \ell + (C_b - \ell) \cdot (1 - \ell) / (C_{\max} - \ell)$ 
19.  end if
20.  return  $C$ 
21. end function

22. function SAT( $C$ )
23.  return  $C_{\max} - C_{\min}$ 
24. end function

25. function SETSAT( $C, s$ )
26.  if ( $C_{\max} > C_{\min}$ )
27.     $C_{\text{mid}} = (C_{\text{mid}} - C_{\min}) \cdot s / (C_{\max} - C_{\min})$ 
28.     $C_{\max} = s$ 
29.  else
30.     $C_{\text{mid}} = 0$ 
31.     $C_{\max} = 0$ 
32.  end if
33.   $C_{\min} = 0$ 
34.  return  $C$ 
35. end function

```

The non-separable blend functions supported by PDF are:

Name	Result
<b>Hue</b>	Creates a color with the hue of the source and saturation and luminosity of the backdrop:  $\mathfrak{B}(C_b, C_s) = \text{SETLUM}(\text{SETSAT}(C_s, \text{SAT}(C_b)), \text{LUM}(C_b))$
<b>Saturation</b>	Creates a color with the saturation of the source color and the hue and luminosity of the backdrop.  $\mathfrak{B}(C_b, C_s) = \text{SETLUM}(\text{SETSAT}(C_b, \text{SAT}(C_s)), \text{LUM}(C_b))$
<b>Color</b>	Creates a color with the luminosity of the backdrop and hue and saturation of the source:  $\mathfrak{B}(C_b, C_s) = \text{SETLUM}(C_s, \text{LUM}(C_b))$
<b>Luminosity</b>	Creates a color with the luminosity of the source and hue and saturation of the backdrop:  $\mathfrak{B}(C_b, C_s) = \text{SETLUM}(C_b, \text{LUM}(C_s))$

### 6.1.3 $\alpha$ -values and shape and opacity

The  $\alpha$ -value of an object is actually dictated by two other parameters: *shape* and *opacity*. These are

denoted by  $f$  and  $q$  respectively, and they range between 0 and 1. The  $\alpha$ -value of a pixel is actually the product of its shape and opacity:  $\alpha = f \cdot q$ . When the shape of an object at a pixel is 0, its opacity is undefined there. We adopt the convention that  $0/0 = 0$ .

Shape and opacity can be derived from multiple different sources:

- Objects can provide an *object shape* and *object opacity*. Elementary objects such as strokes, fills, and text have an intrinsic shape. The value of this shape is 1 for points inside the object and 0 for those outside. The shape of a group object is the union of the shapes of the objects it contains. We denote object shape by  $f_j$ .

Elementary objects have an object opacity, denoted  $q_j$ , of 1 everywhere.

- A *soft mask* is a source of shape and opacity independent of other objects. It can be used to alter the shape and opacity of another object, for example if a soft mask specifies a gradient which slowly goes from opaque to transparent, applying this to text will have the effect of making the text fade out. The shape of a soft mask is denoted  $f_m$ , and its opacity is  $q_m$ .
- The shape and opacity can be altered by a scalar constant. The shape constant is denoted  $f_k$  and the opacity constant is denoted  $q_k$ .

The shape and opacity of a shape at a point are called the *source shape* and *source opacity*, which are defined to be

$$f_s = f_j \cdot f_m \cdot f_k, \quad q_s = q_j \cdot q_m \cdot q_k$$

Then the *source- $\alpha$ -value* is defined to be  $f_s \cdot q_s$ .

Compositing two objects together must form a result shape and opacity. If the source shape and opacity are  $f_s, q_s$  respectively, and the backdrop shape and opacity are  $f_r, q_r$  then we know the following:

$$\alpha_r = \alpha_s \cup \alpha_b, \quad f_r = f_s \cup f_r$$

thus we get that  $q_r = \alpha_r / f_r = (\alpha_s \cup \alpha_b) / (f_s \cup f_r)$ .

#### 6.1.4 Transparency groups

In a PDF, objects form what is called a *transparency stack*, which determines how the objects are layered in the transparent imaging model. Suppose the transparency stack looks like  $E_1, \dots, E_n$ , then we begin with some *initial backdrop*, then composite element  $E_1$  onto it, then  $E_2$  on that, and so on.

Each element in the transparency stack may itself be a group of elements, called a *transparency group*; and the elements in a transparency group may be transparency groups as well, and so on. So the transparency stack is more like a tree structure. We can view the transparency stack as a whole as a single transparency group.

Each element in a transparency group is an object specifying the element's color, shape, opacity, and blend mode.

For a given transparency group  $G$ , we consider three backdrops:

- (1) the *group backdrop* is the result of compositing all elements up to, but not including, the first element in the group (for non-knockout groups, discussed later).
- (2) the *initial backdrop* is a backdrop that is selected for compositing the group's first element against. This is either the group backdrop (for non-isolated groups) or a transparent backdrop (for isolated groups, see below).
- (3) the *immediate backdrop* of an element in the group  $E_i$  is the result of compositing all elements in the group up to but not including it.

To composite a group  $G$ , we start with a backdrop which specifies a color  $C_0$  and an  $\alpha$ -value  $\alpha_0$ . Then the function  $\text{COMPOSITE}(C_0, \alpha_0, G)$  returns a triple  $(C, f, \alpha)$  which is the group's color, the group's shape, and the group's  $\alpha$ -value. Once the group is composited, it is treated like a new single item on the outer transparency stack.

Suppose a non-isolated non-knockout group  $G$  is composed of  $E_1, \dots, E_n$ . We define  $\text{COMPOSITE}(C_0, \alpha_0, G)$  recursively as follows:

1. **function**  $\text{COMPOSITE}(C_0, \alpha_0, G)$
2.  $f_{g_0} = \alpha_{g_0} = 0$
3. **for** ( $i = 1, \dots, n$ )
  - ▷  $C_{s_i}$  is the source color of element  $i$ ,  $f_{j_i}$  is the object shape of element  $i$ ,  
and  $\alpha_{j_i} = f_{j_i} \cdot q_{j_i}$  is the object  $\alpha$ -value of element  $i$ .
4. **if** ( $E_i$  is a group)
5.  $(C_{s_i}, f_{j_i}, \alpha_{j_i}) = \text{COMPOSITE}(C_{i-1}, \alpha_{i-1}, E_i)$
6. **else if** ( $E_i$  is an object)

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7.       $C_{s_i}$  is the intrinsic color of the object
8.       $f_{j_i}$  is the intrinsic shape of the object
9.       $\alpha_{j_i}$  is the intrinsic  $\alpha$ -value of the object
10.    end if
11.     $\triangleright f_{s_i}$  is the source shape of element  $i$ .
      It is obtained from the product of the object shape with the mask shape and shape constant.
12.     $f_{s_i} = f_{j_i} \cdot f_{m_i} \cdot f_{k_i}$ 
13.     $\triangleright \alpha_{s_i}$  is the source  $\alpha$ -value of element  $i$ .
      It is obtained from the product of the object  $\alpha$ , with the mask  $\alpha$  and  $\alpha$  constant.
14.     $\alpha_{s_i} = \alpha_{j_i} \cdot (f_{m_i} \cdot q_{m_i}) \cdot (f_{k_i} \cdot q_{k_i})$ 
15.     $\triangleright f_{g_i}$  is the accumulated shape of group elements  $E_1, \dots, E_i$  excluding the backdrop.
16.     $f_{g_i} = f_{g_{i-1}} \cup f_{s_i}$ 
17.     $\triangleright \alpha_{g_i}$  is the accumulated source  $\alpha$  of group elements  $E_1, \dots, E_i$  excluding the backdrop.
18.     $\alpha_{g_i} = \alpha_{g_{i-1}} \cup \alpha_{s_i}$ 
19.     $\triangleright \alpha_i$  is the accumulated  $\alpha$  after compositing  $E_i$  onto the rest of the group, including the backdrop.
20.     $\alpha_i = \alpha_0 \cup \alpha_{g_i}$ 
21.     $\triangleright C_i$  is the accumulated color after compositing  $E_i$ .
22.     $C_i = \left(1 - \frac{\alpha_{s_i}}{\alpha_i}\right) \cdot C_{i-1} + \frac{\alpha_{s_i}}{\alpha_i} \cdot [(1 - \alpha_{i-1}) \cdot C_{s_i} + \alpha_{i-1} \cdot \mathfrak{B}_i(C_{i-1}, C_{s_i})]$ 
23.  end for
24.   $C = C_n + (C_n - C_0) \cdot \left(\frac{\alpha_0}{\alpha_{g_n}} - \alpha_0\right)$ 
25.   $f = f_{g_n}$ 
26.   $\alpha = \alpha_{g_n}$ 
27.  return  $(C, f, \alpha)$ 
28. end function

```

Let's take a look at each variable.  $\alpha_{s_i}$  is defined as you'd expect: it's precisely  $\alpha_{j_i} \cdot \alpha_{m_i} \cdot \alpha_{k_i}$ . If we look at the definition of  $\alpha_i$ , we see that it's equal to

$$\alpha_i = \alpha_0 \cup \alpha_{g_i} = \alpha_0 \cup \alpha_{s_i} \cup \alpha_{g_{i-1}} = \dots = \alpha_0 \cup \bigcup_{j \leq i} \alpha_{s_j}$$

Thus, we have that

$$\alpha_i = \alpha_{i-1} \cup \alpha_{s_i}$$

This means that line 16 is another way of writing

$$C_i = \text{BASICCOMPOSITE}(C_{i-1}, \alpha_{i-1}, C_{s_i}, \alpha_{s_i}, \mathfrak{B}_i)$$

What's the rationale behind the  $\alpha_i$  variable? Well,  $\alpha_i$  is the  $\alpha$  of the group composited with its backdrop. We don't want to store this in  $\alpha_{g_i}$ , as it shouldn't affect later computations of compositing elements with the immediate backdrop. Importantly, every element of a group is composited onto a backdrop including the initial backdrop. This is done because most blend modes are dependent on the backdrop and source colors. This is also precisely the difference between non-isolated and isolated transparency groups.

The formulas at the end (after the loop) essentially remove the contribution of the initial backdrop from the computation. This is since when the group is composited with the initial backdrop, it may be composited with a different blend mode or mask. This is more clear by viewing it as follows: first we define the *backdrop fraction* which measures the relative contribution of the backdrop color to the overall color:

$$\phi_b = \frac{(1 - \alpha_{g_n})\alpha_0}{\alpha_0 \cup \alpha_{g_n}}$$

Indeed,  $(1 - \alpha_{g_n})\alpha_0$  measures the area covered by the backdrop and not the group. And  $\alpha_0 \cup \alpha_{g_n}$  is the total area covered. So  $\phi_b$  measures the ratio between the area covered by just the backdrop and the total area. Then we get

$$C = \frac{C_n - \phi_b \cdot C_0}{1 - \phi_b}$$

Notice that  $C_n - \phi_b \cdot C_0$  has the effect of removing the backdrop color from  $C_n$ , since  $\phi_b$  is the amount that  $C_0$  contributed. And

$$\frac{1}{1 - \phi_b} = \frac{\alpha_0 \cup \alpha_{g_n}}{\alpha_{g_n}}$$

so we are looking at the inverse ratio between the amount of area the group takes and the total area. So multiplying  $C_n - \phi_b \cdot C_0$  by this scales back up the color to take the entirety of the area.

### Isolated groups

An isolated group differs from a non-isolated groups in that each element is composited with a transparent backdrop as opposed to the group's backdrop. The effect that this has is that the group itself is transparent *relative to itself*, but not to its backdrop.

The only change to the above algorithm is that  $\alpha_0$  is set to zero.

### Knockout groups

In a knockout group, each element is composited with the group's initial backdrop rather than the stack of preceding elements in the group. If elements have a binary shape (1 inside, 0 outside), this has the effect of overlaying (or knocking-out) the effect of elements beneath it. This is similar to the opaque imaging model, except that "topmost object wins" applies to both color and opacity.

We alter the algorithm above as follows:

- $\alpha_{g_i}$  is instead defined to be  $(1 - f_{s_i})\alpha_{g_{i-1}} + \alpha_{s_i}$
- we also define

$$C_t = (f_{s_i} - \alpha_{s_i})\alpha_b C_b + \alpha_{s_i} \cdot [(1 - \alpha_b)C_{s_i} + \alpha_b \mathfrak{B}_i(C_b, C_{s_i})]$$

and then define

$$C_i = \frac{(1 - f_{s_i})\alpha_{i-1}C_{i-1} + C_t}{\alpha_i}$$

### 6.1.5 Soft masks

As previously discussed, shape and opacity values can be altered by soft masks, which are just other sources of shape and opacity values. These are generalizations of clipping paths, as a soft mask which specifies binary values does the same as clipping.

A mask can be defined by creating a transparency group and painting objects into it, which defines the color, shape, and opacity in the usual way. Then the resulting group can be used to derive the mask in one of two ways, discussed below.

A soft mask specifies a single value, called the *mask value* at every point. The mask value either specifies shape or opacity according to a flag (alpha is shape) in the graphics state.

#### Deriving a soft mask from group alpha

The first method involves first computing the color, shape, and  $\alpha$ -value of a transparency group  $G$  using the normal formula

$$(C, f, \alpha) = \text{COMPOSITE}(C_0, \alpha_0, G)$$

where  $C_0, \alpha_0$  are arbitrary and don't contribute to the result. Since  $C$  isn't used, we don't need to compute it.

The user then specifies a *transfer function*  $[0, 1] \rightarrow [0, 1]$  which maps  $\alpha$  to a mask value.

#### Deriving a soft mask from group luminosity

The second method of deriving a soft mask from a transparency group utilizes the luminosity of the group. The group is composited with a fully opaque backdrop of some color, then the mask value is defined to be the luminosity of the resulting color,  $C$ . The color  $C$  is created by first compositing  $G$  with a fully opaque background  $C_0$ , this gives

$$(C_g, f_g, \alpha_g) = \text{COMPOSITE}(C_0, 1, G)$$

then  $C$  is the weighted average between  $C_0$  and  $C_g$ :

$$C = (1 - \alpha_g) \cdot C_0 + \alpha_g \cdot C_g$$

The color is then converted to its luminosity as follows:

- if the underlying color space is **DeviceGray**, the luminosity is the same as the value;
- if the underlying color space is **DeviceRGB**, the luminosity is

$$0.3 \cdot C_r + 0.59 \cdot C_g + 0.11 \cdot C_b$$

- if the underlying color space is **DeviceCMYK**, the luminosity is

$$0.3 \cdot (1 - C_c)(1 - C_k) + 0.59 \cdot (1 - C_m)(1 - C_k) + 0.11 \cdot (1 - C_y)(1 - C_k)$$

Then the value computed is passed to a transfer function  $[0, 1] \rightarrow [0, 1]$  to compute the mask value.

## 6.2 Specifying transparency in PDF

Single graphic objects are treated as elementary graphics objects in a transparency stack. Even if they overlap themselves (like a path which intersects itself), this does not affect its transparency. An object's source color  $C_s$  is the same as its color in the opaque imaging model. Its backdrop color  $C_b$  is the result of previous painting operations.

The current blend mode is specified in the current graphics state under the **BM** entry. The **BM** entry is either a name corresponding to one of the names given above, or an array of names. In the latter case, the first name that device recognizes is used (or **Normal** if it recognizes none of them). This way more blending modes can be implemented in the future.

The current soft mask can be specified in the current graphics state under the **SMask** entry. The value is a *soft mask dictionary*, defined below:

### 6.2.1 Soft mask dictionaries

A soft mask dictionary may have the following fields:

Key	Type	Value
<b>Type</b>	name	(Optional) the type of PDF object this dictionary describes; must be <b>Mask</b> .
<b>S</b>	name	(Required) a subtype specifying the method to be used to derive the mask values from the transparency group specified by <b>G</b> . This is either <ul style="list-style-type: none"> <li>• <b>Alpha</b>: compute the group's <math>\alpha</math>-value and pass it to the transfer function specified by <b>TR</b> to derive the mask value.</li> <li>• <b>Luminosity</b>: derive the mask value from the group's computed luminosity.</li> </ul>
<b>G</b>	stream	(Required) a transparency group XObject (see below) to be used as the source of the $\alpha$ or luminosity values for the mask. If <b>S</b> is <b>Luminosity</b> , the transparency group XObject must contain a <b>CS</b> entry defining the color space in which the compositing is to be performed.
<b>BC</b>	array	(Optional) an array of component values specifying the backdrop to be used as the initial backdrop when compositing <b>G</b> . This is only used if <b>S</b> is <b>Luminosity</b> . The array must have $n$ values, where $n$ is the number of components in the XObject specified by <b>G</b> 's <b>CS</b> entry. Default value: whatever the code is for black in <b>CS</b> .
<b>TR</b>	function or name	(Optional) a 1-in 1-out function specifying the transfer function to be used in deriving the mask values from $\alpha$ or luminosity values. The name <b>Identity</b> may be used in place of a function to represent the identity function. Default value: <b>Identity</b> .

### 6.2.2 Transparency group XObjects

A transparency group is represented in PDF as a special subtype of group XObject, called a *transparency group XObject*. A group XObject is in turn a subtype of a form XObject, distinguished by the presence of a **Group** entry in its form dictionary, whose value is a *group attributes dictionary*. For a transparency group, this may have the following fields:

Key	Type	Value
<b>S</b>	name	(Required) the <i>group subtype</i> , specifying what kind of group this dictionary describes. Must be <b>Transparency</b> .

<b>CS</b>	name or array	(Required) the group color space, which is used for the following: <ul style="list-style-type: none"> <li>(1) as the color space into which colors are converted when painted into the group;</li> <li>(2) as the blending color space in which objects are composited within the group;</li> <li>(3) as the color space of the group as a whole when it in turn is painted as an object onto its backdrop.</li> </ul>
<b>I</b>	boolean	(Optional) a flag specifying if this transparency group is isolated. Default value: <b>false</b> .
<b>K</b>	boolean	(Optional) a flag specifying if this transparency group is knock-out. Default value: <b>false</b>

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## 6.3 An example

Suppose we want to create the following effect of fading in the word “hello”:

The idea is as follows: we create a soft mask containing the word “hello”, and then in another XObject we draw the gradient and use the soft mask as a mask. The code to do this is as follows:

```

1 11 0 obj                                % XObject which draws the gradient and sets the soft mask
2 <<
3 /Type /XObject
4 /Subtype /Form
5 /BBox [0 0 20.479 6.918]
6 /FormType 1
7 /Matrix [1 0 0 1 0 0]
8 /Resources 16 0 R
9 /Length 48
10 >>
11 stream
12 q
13 .996264 0 0 .996264 0 0 cm
14 /GS1 gs      % set soft mask
15 /Sh sh       % draw gradient
16 Q
17 endstream
18 endobj
19
20 % 16 0 obj                                resources of above XObject
21 <<
22 /Shading << /Sh 4 0 R >>
23 /ExtGState << /GS1 10 0 R >>
24 /ProcSet [ /PDF ]
25 >>
26
27 % 4 0 obj                                pattern object
28 <<
29 /Type /Pattern
30 /PatternType 2
31 /Shading 3 0 R
32 >>
33
34 % 3 0 obj                                shading object
35 <<
36 /ShadingType 2
37 /ColorSpace /DeviceGray
38 /Coords [0 6.94444 0 0]    % 6.94 is height of "hello"
39 /Domain [0 1]
40 /Function 1 0 R
41 /Extend [false false]
42 >>
43
44 % 1 0 obj                                linear interpolation between white and black
45 <<
46 /FunctionType 2
47 /Domain [0 1]
48 /CO [1]

```

```

49 /C1 [0]
50 /N 1 >>
51
52 % 10 0 obj          graphics state object
53 <<
54   /Type /ExtGState
55   /SMask 9 0 R
56 >>
57
58 % 9 0 obj           mask object
59 <<
60   /Type /Mask
61   /S /Luminosity
62   /G 6 0 R
63 >>
64
65 6 0 obj             % transparency groupXObject containing "hello"
66 <<
67   /Type /XObject
68   /Subtype /Form
69   /Group 5 0 R
70   /BBox [0 0 20.479 6.918]
71   /FormType 1
72   /Matrix [1 0 0 1 0 0]
73   /Resources 7 0 R
74   /Length 71
75 >>
76 stream
77 1 g 1 G
78 1 0 0 1 0 6.918 cm
79 BT
80 /F1 9.9626 Tf 0 -6.918 Td [(hello)]TJ
81 ET
82 endstream
83 endobj
84
85 % 5 0 obj           group attributes dictionary
86 <<
87   /Type /Group
88   /S /Transparency
89   /CS /DeviceGray
90 >>

```