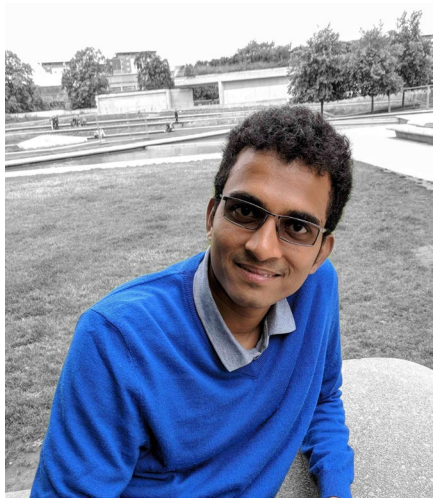
On Differentially Private Counting on Trees

Kewen Wu (UC Berkeley)

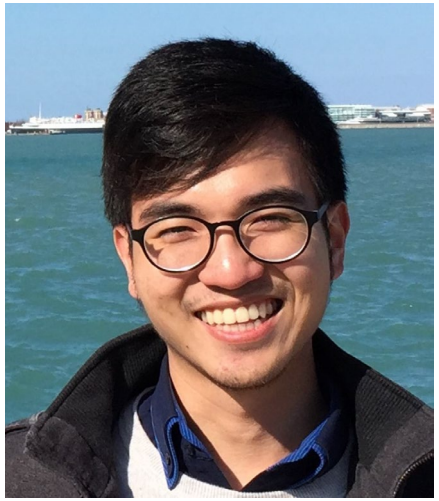
Badih Ghazi

Pritish Kamath



Ravi Kumar

Pasin Manurangsi



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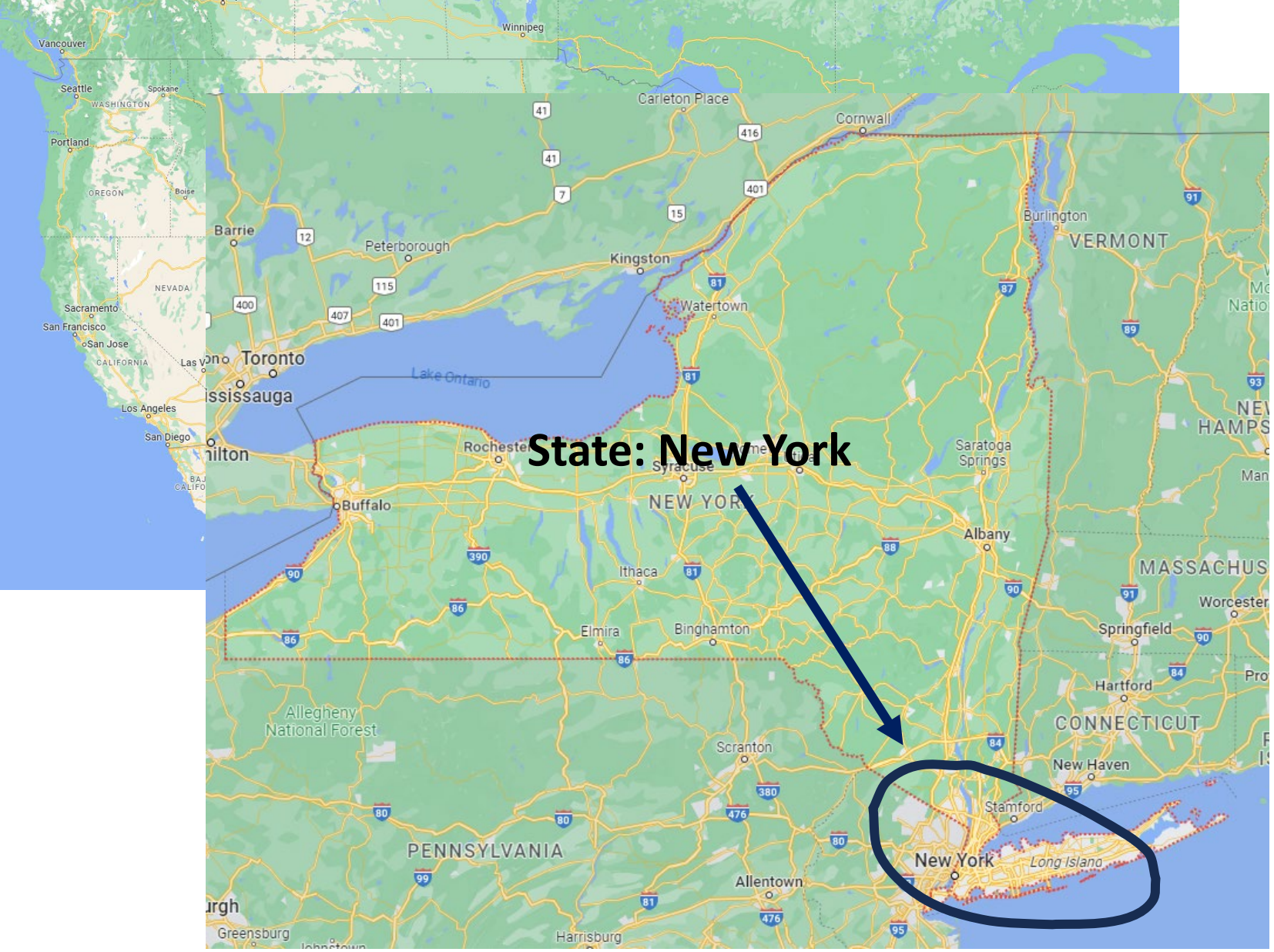
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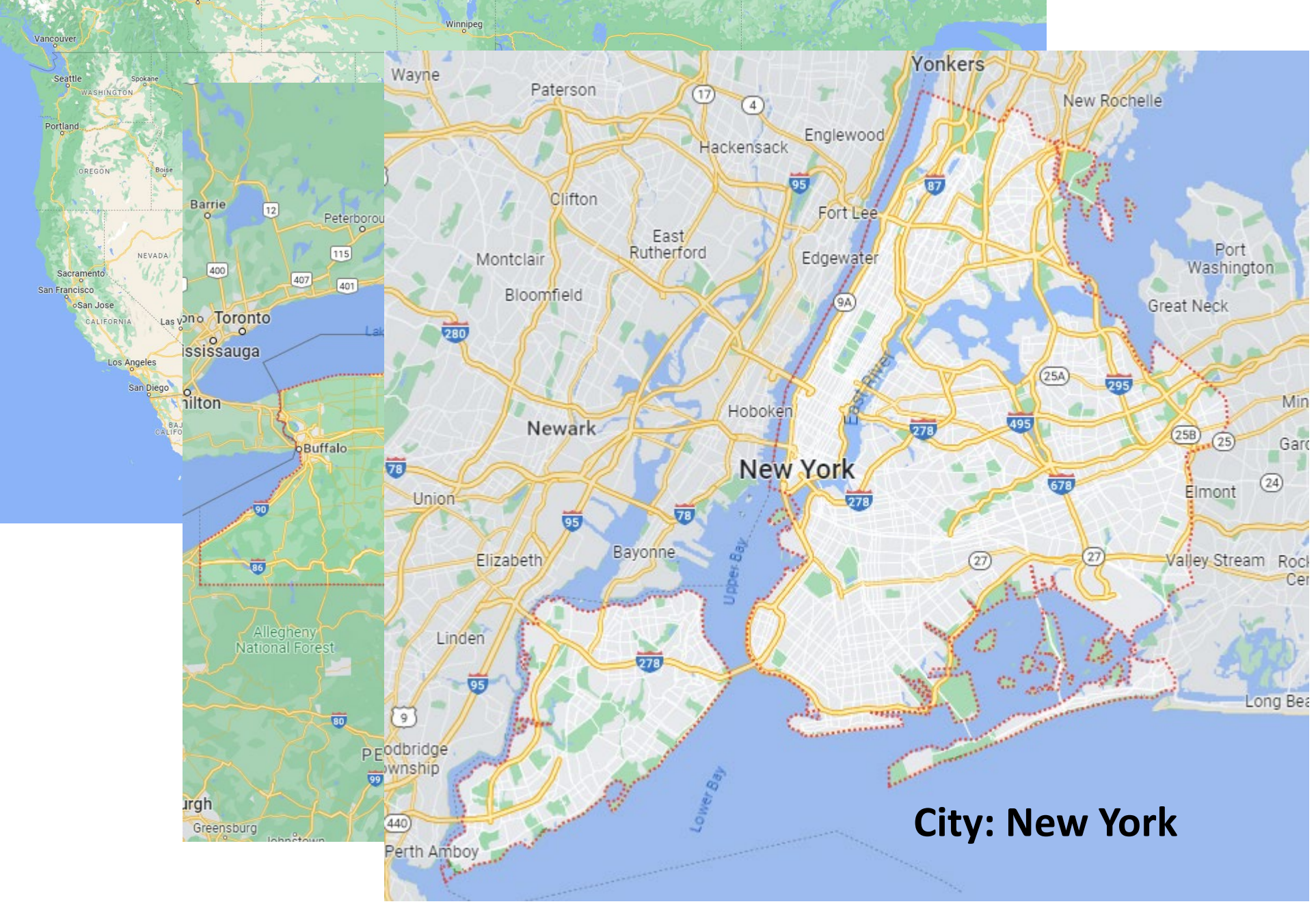

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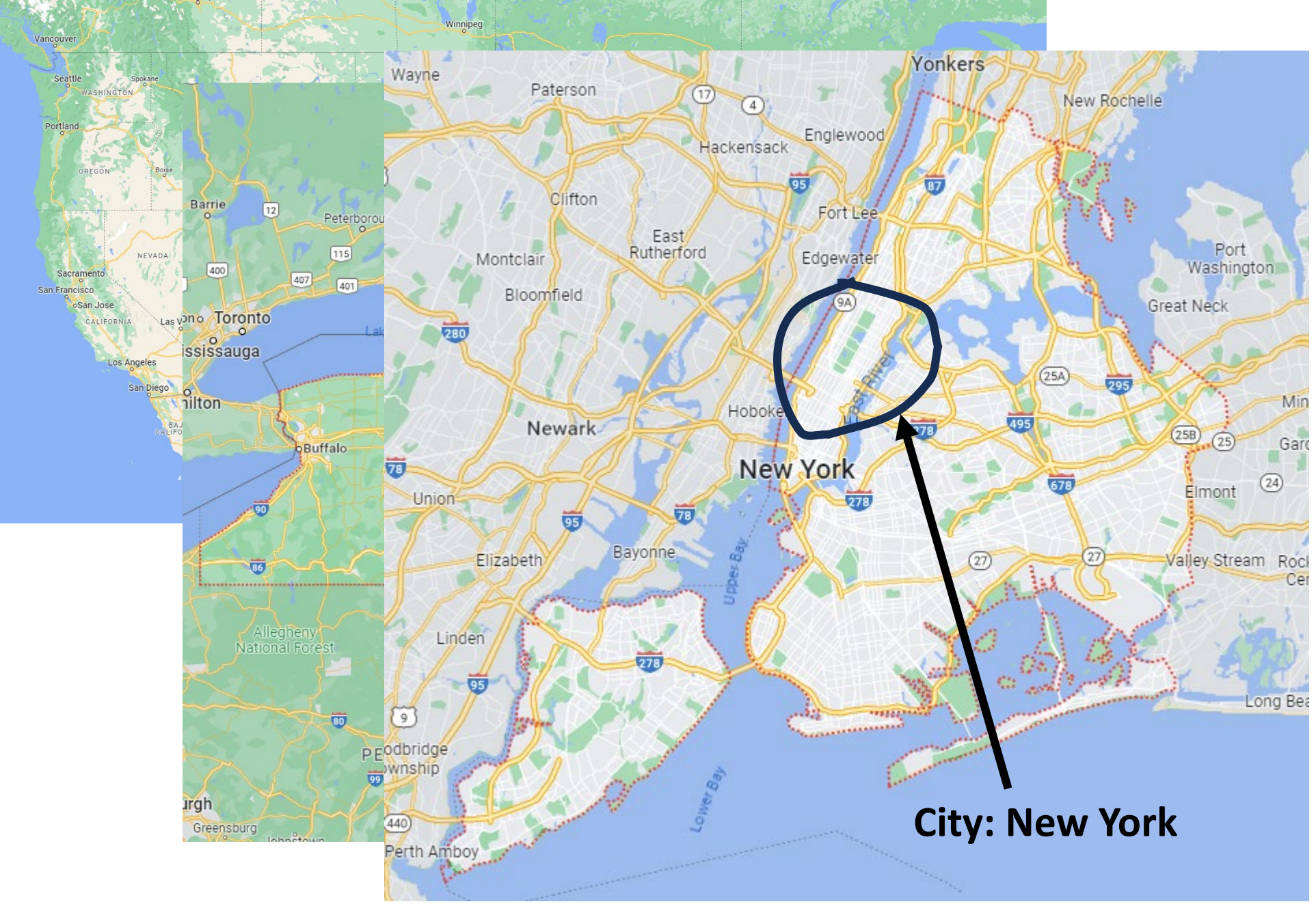
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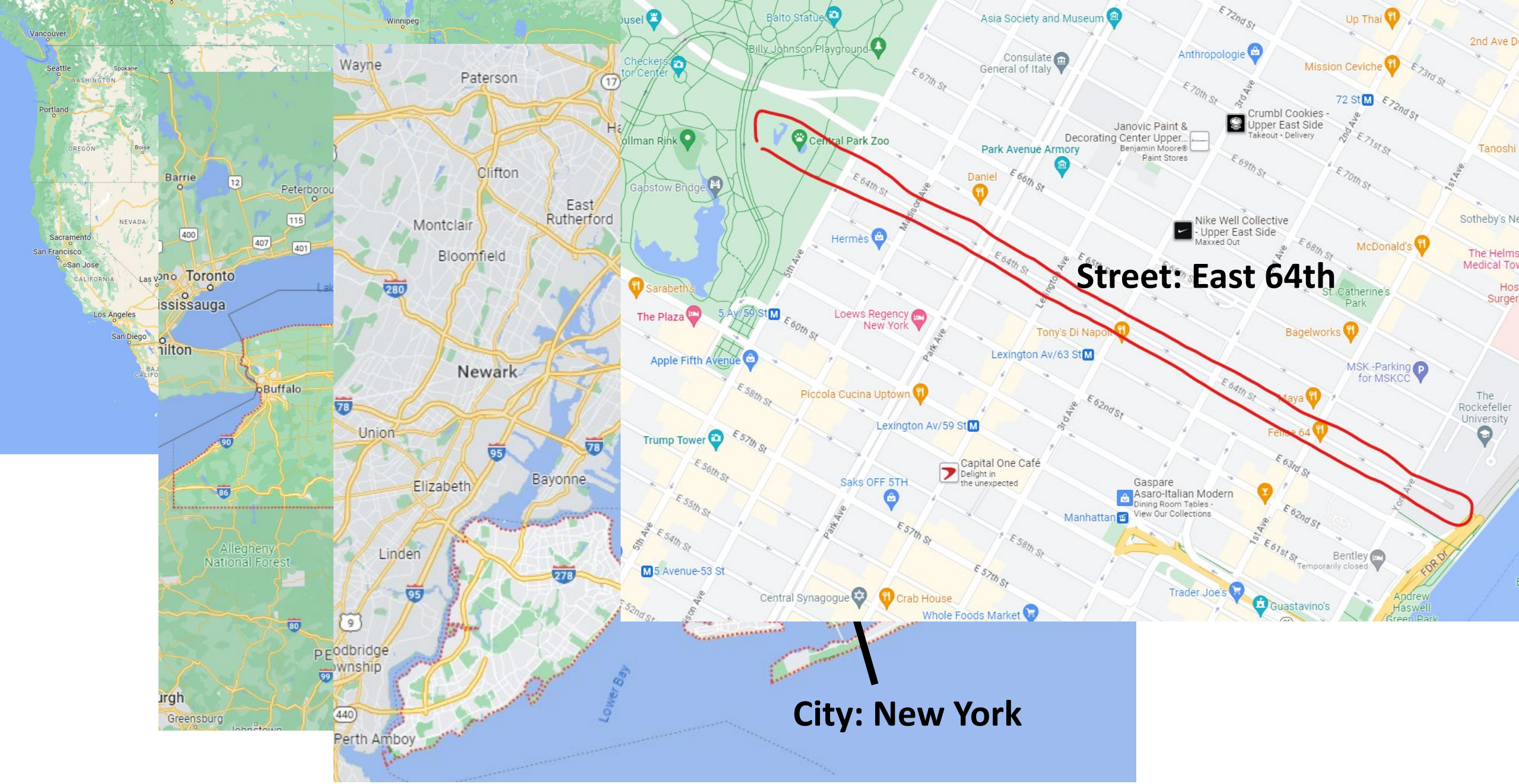
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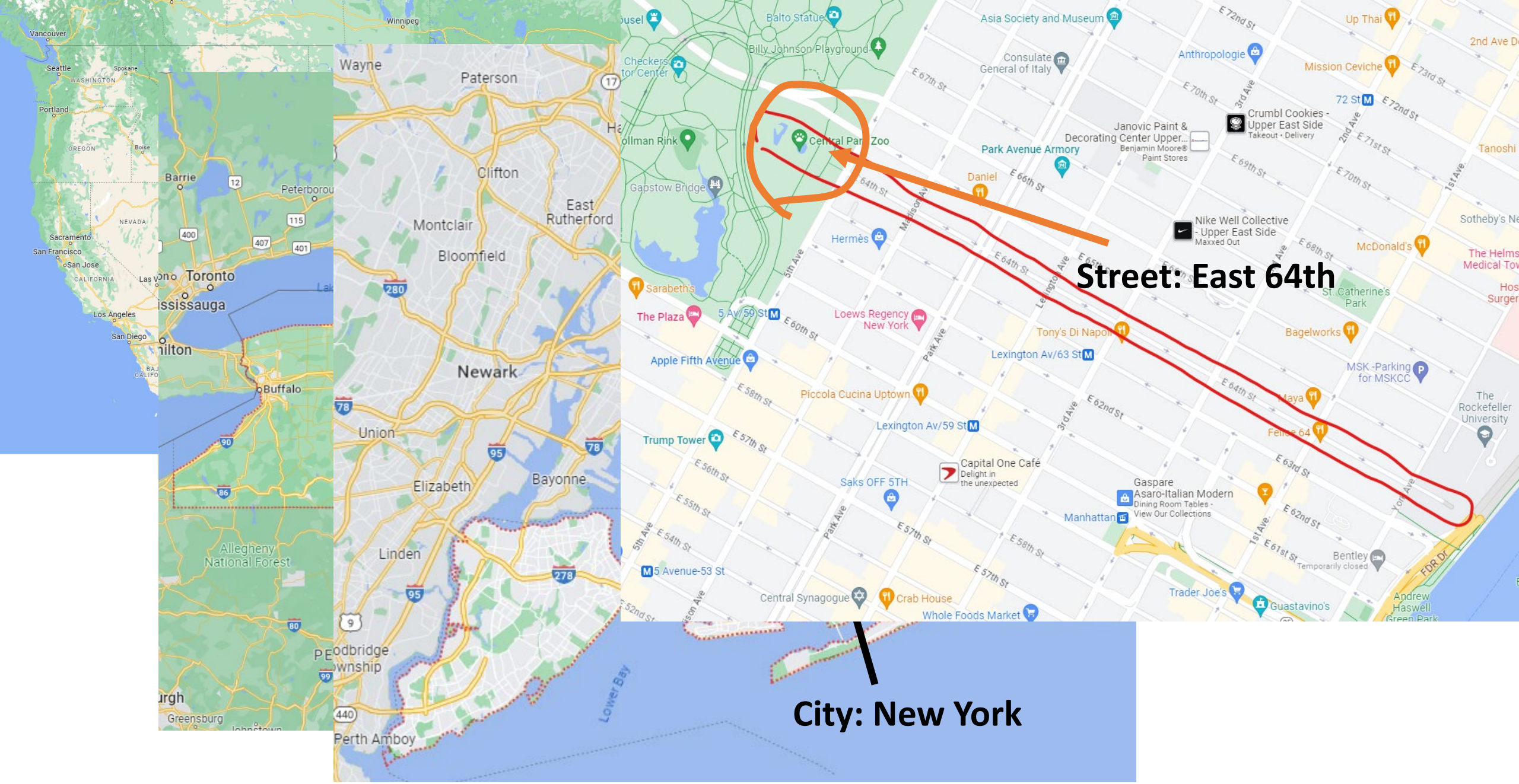
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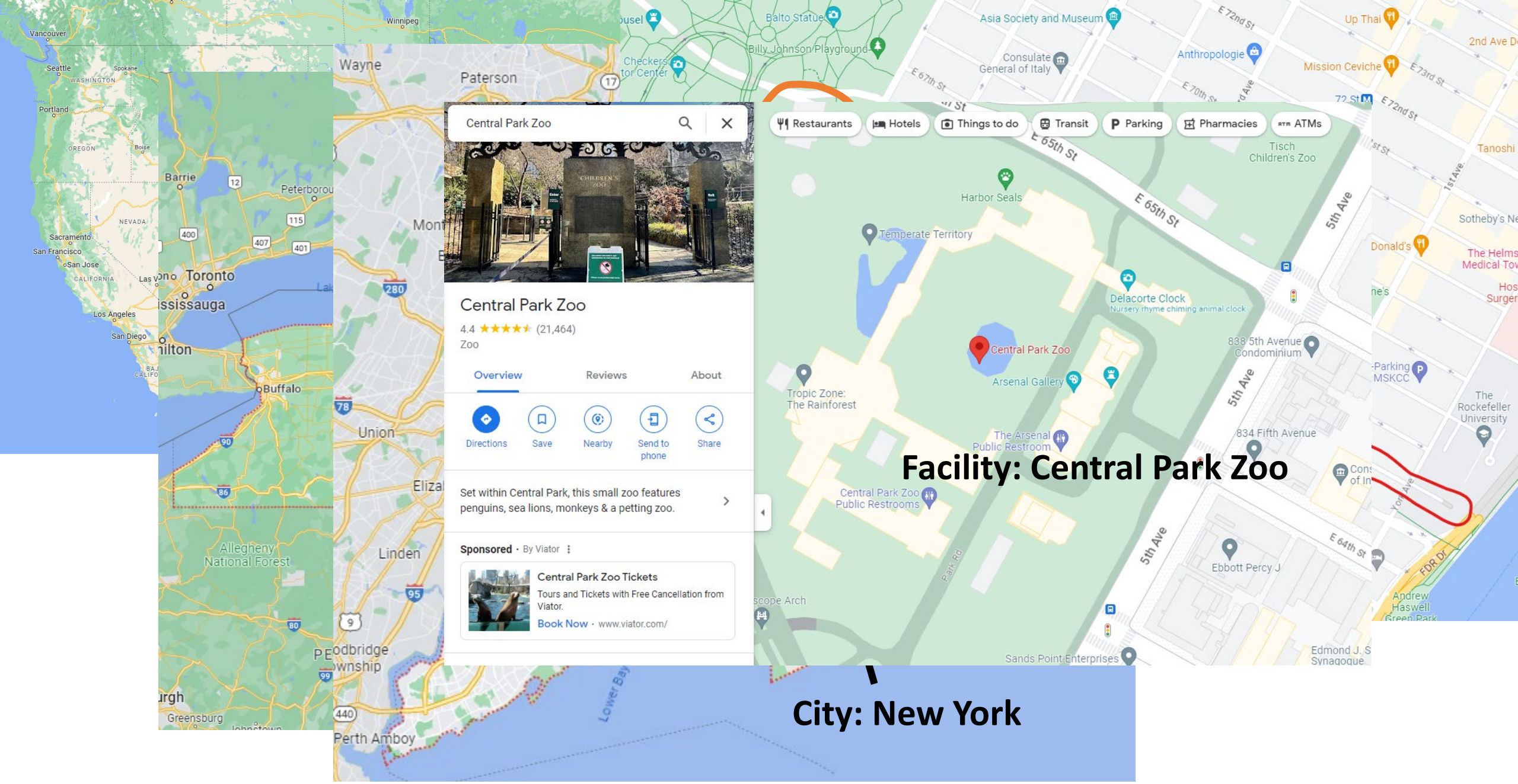
Central Park Zoo
Central Park Zoo
4.4 (21,464)
Zoo
Overview
Reviews
About
Directions
Save
Nearby
Send to phone
Share
Set within Central Park, this small zoo features penguins, sea lions, monkeys & a petting zoo.
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Things to do
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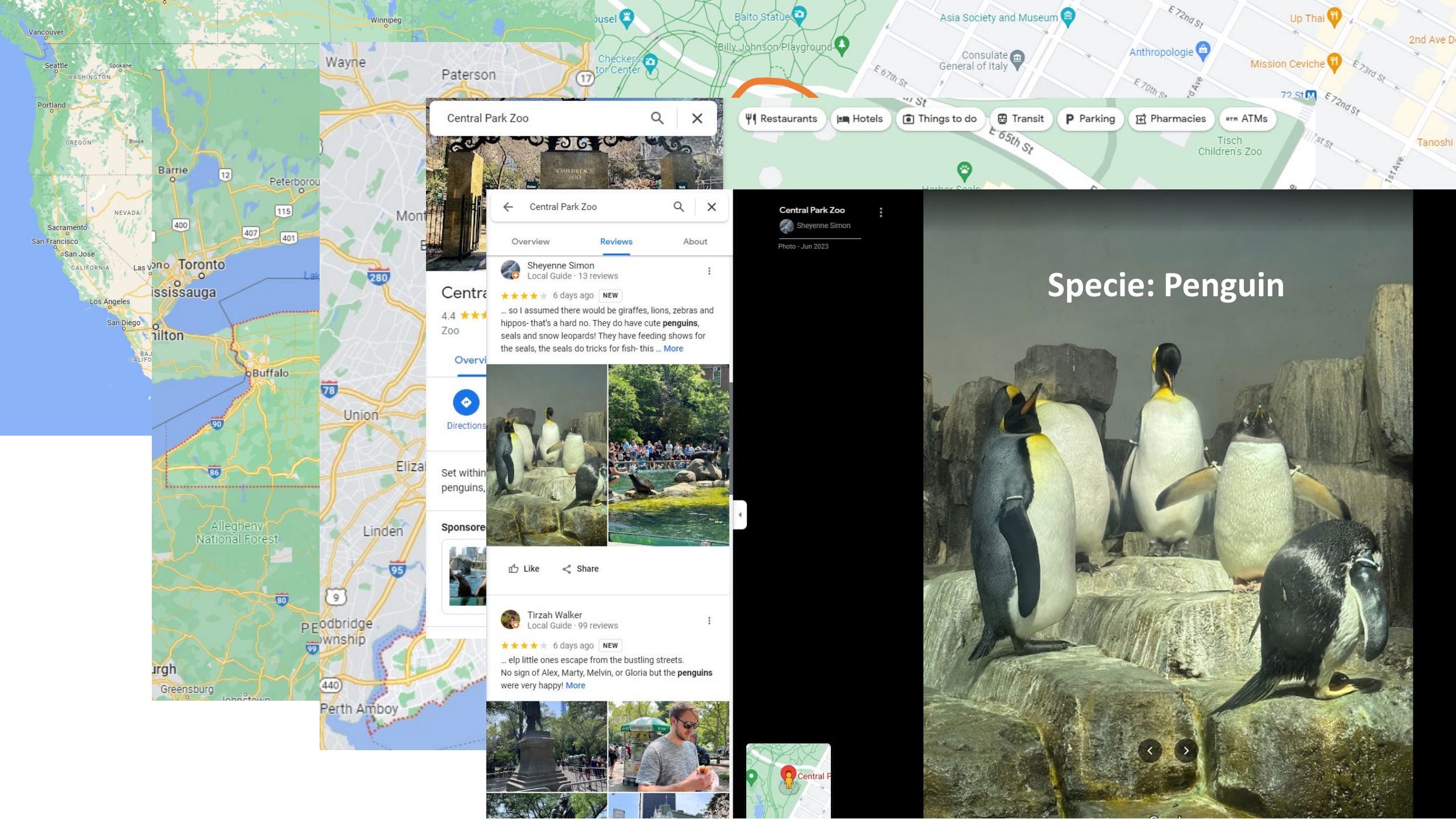
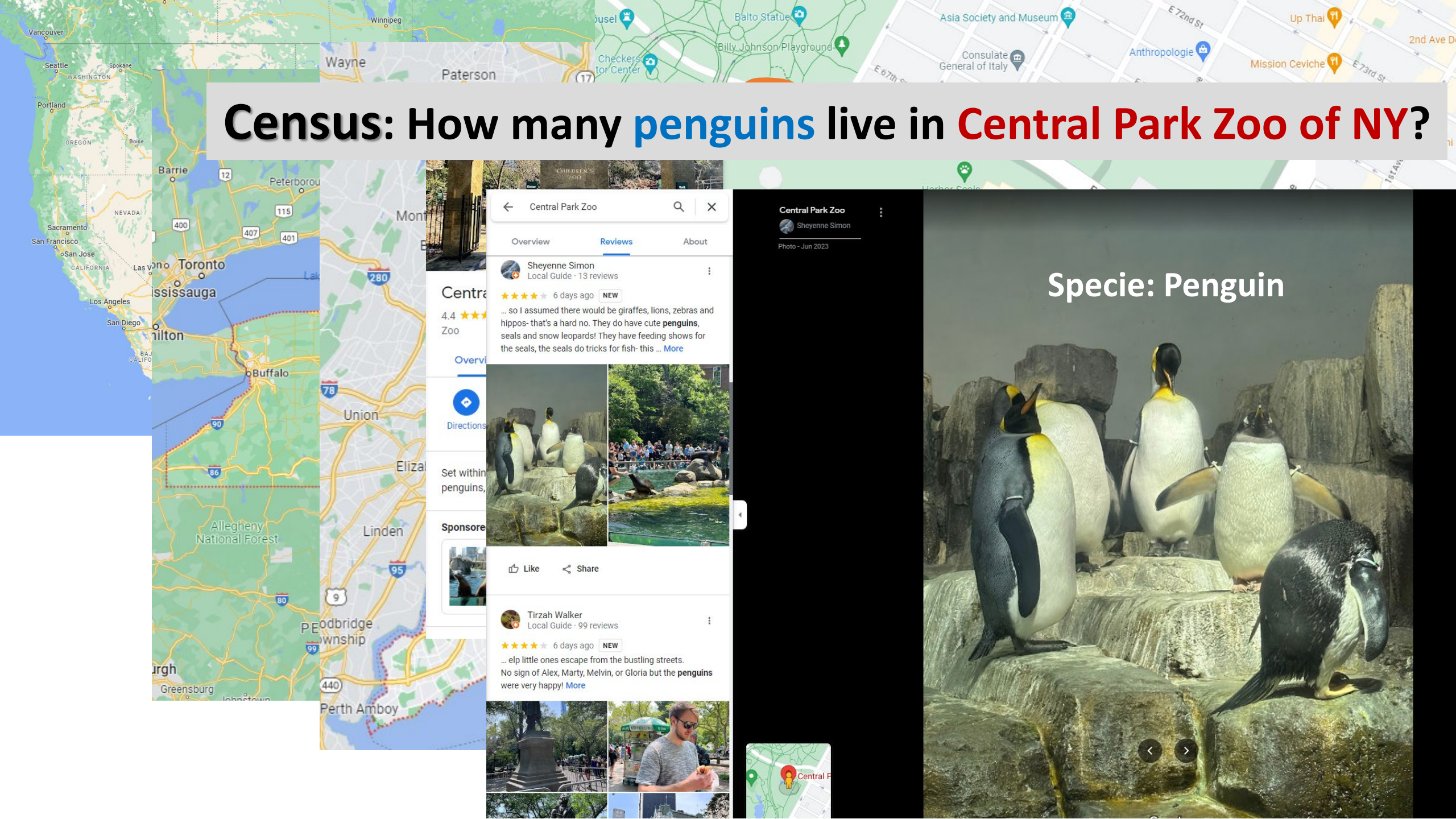
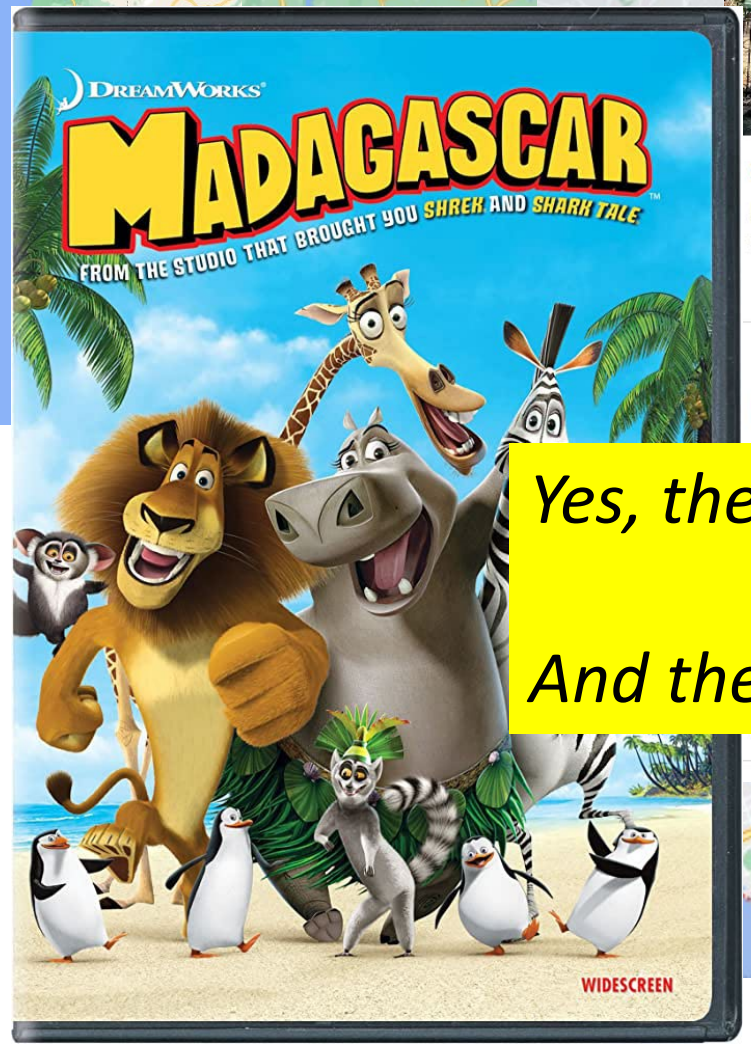
Nursery rhyme chiming animal clock
Central Park Zoo
Arsenal Gallery
Tropic Zone: The Rainforest
The Arsenal Public Restroom
Central Park Zoo Public Restrooms
Tisch Children's Zoo
838 5th Avenue Condominium
834 Fifth Avenue
Ebbott Percy J
Sands Point Enterprises
E 65th St
5th Ave
E 64th St
Facility: Central Park Zoo
City: New York

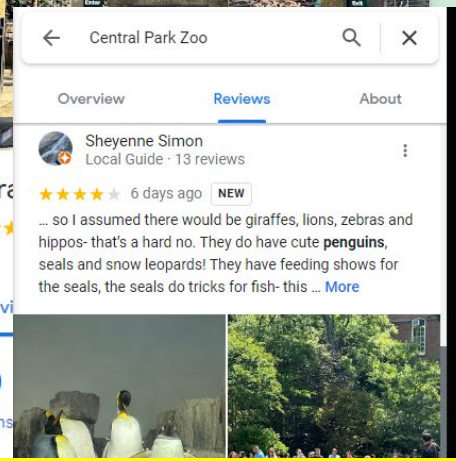

Central Park Zoo
Overview
Reviews
About
Sheyenne Simon
Local Guide · 13 reviews
6 days ago
NEW
... so I assumed there would be giraffes, lions, zebras and hippos- that's a hard no. They do have cute penguins, seals and snow leopards! They have feeding shows for the seals, the seals do tricks for fish- this ... More
Like
Share
Tirzah Walker
Local Guide · 99 reviews
6 days ago
NEW
... elp little ones escape from the bustling streets. No sign of Alex, Marty, Melvin, or Gloria but the penguins were very happy! More
Restaurants
Hotels
Things to do
Transit
Parking
Pharmacies
ATMs
Specie: Penguin

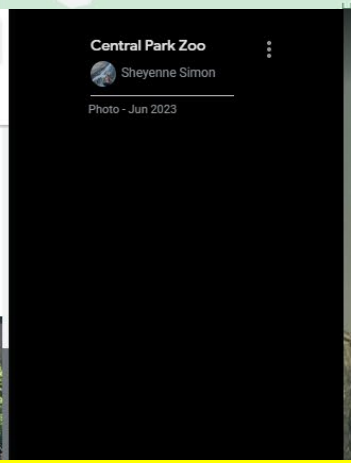
Census: How many penguins live in Central Park Zoo of NY?

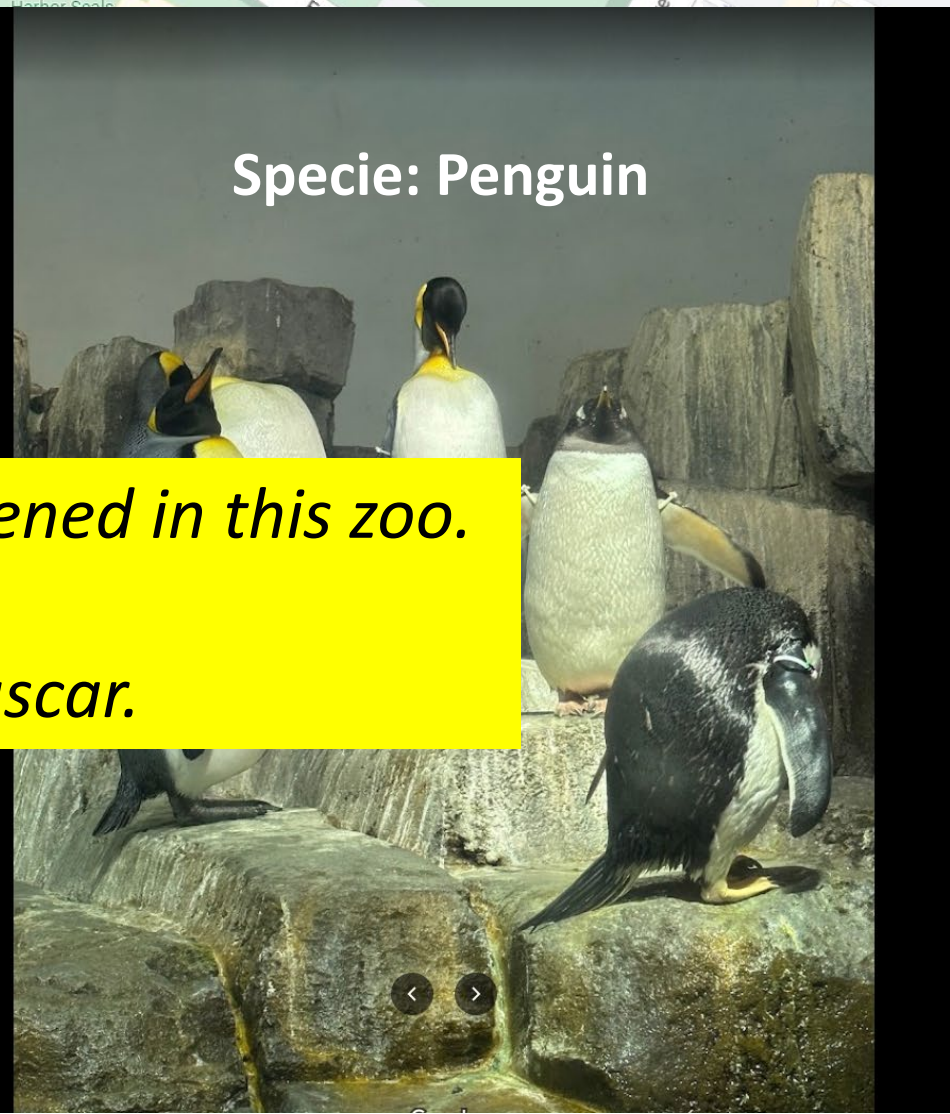


Census: How many penguins live in Central Park Zoo of NY?



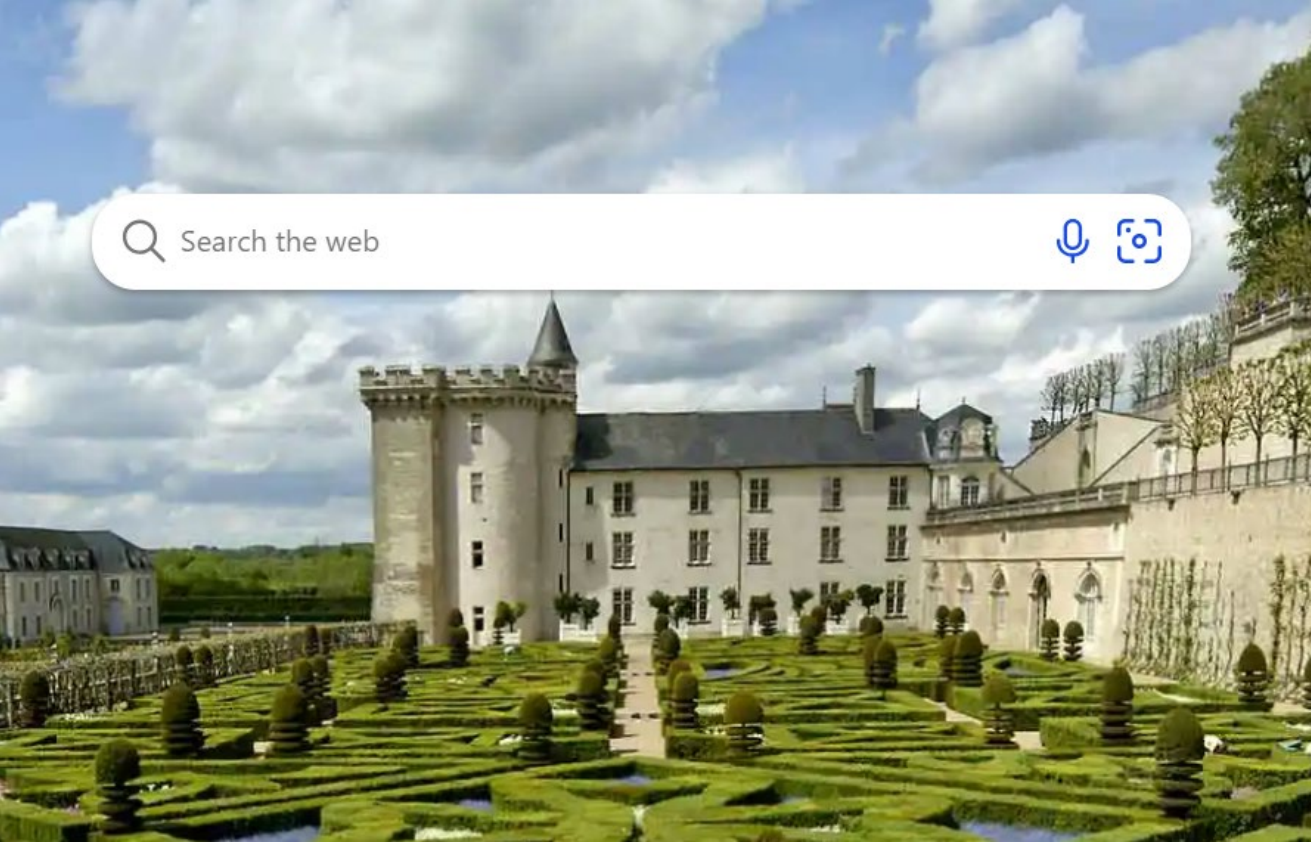






Yes, the story of Madagascar happened in this zoo.

And there is no penguin in Madagascar.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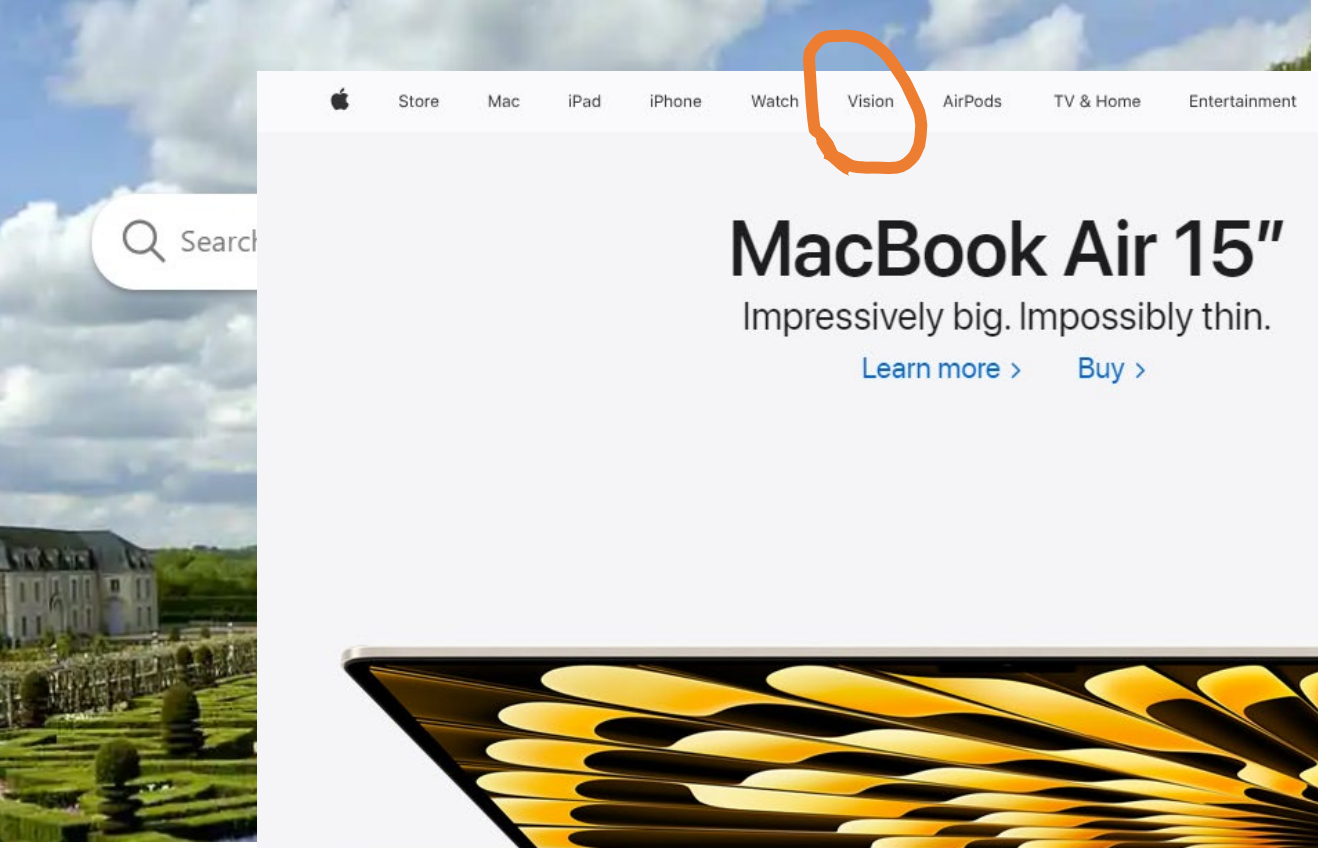

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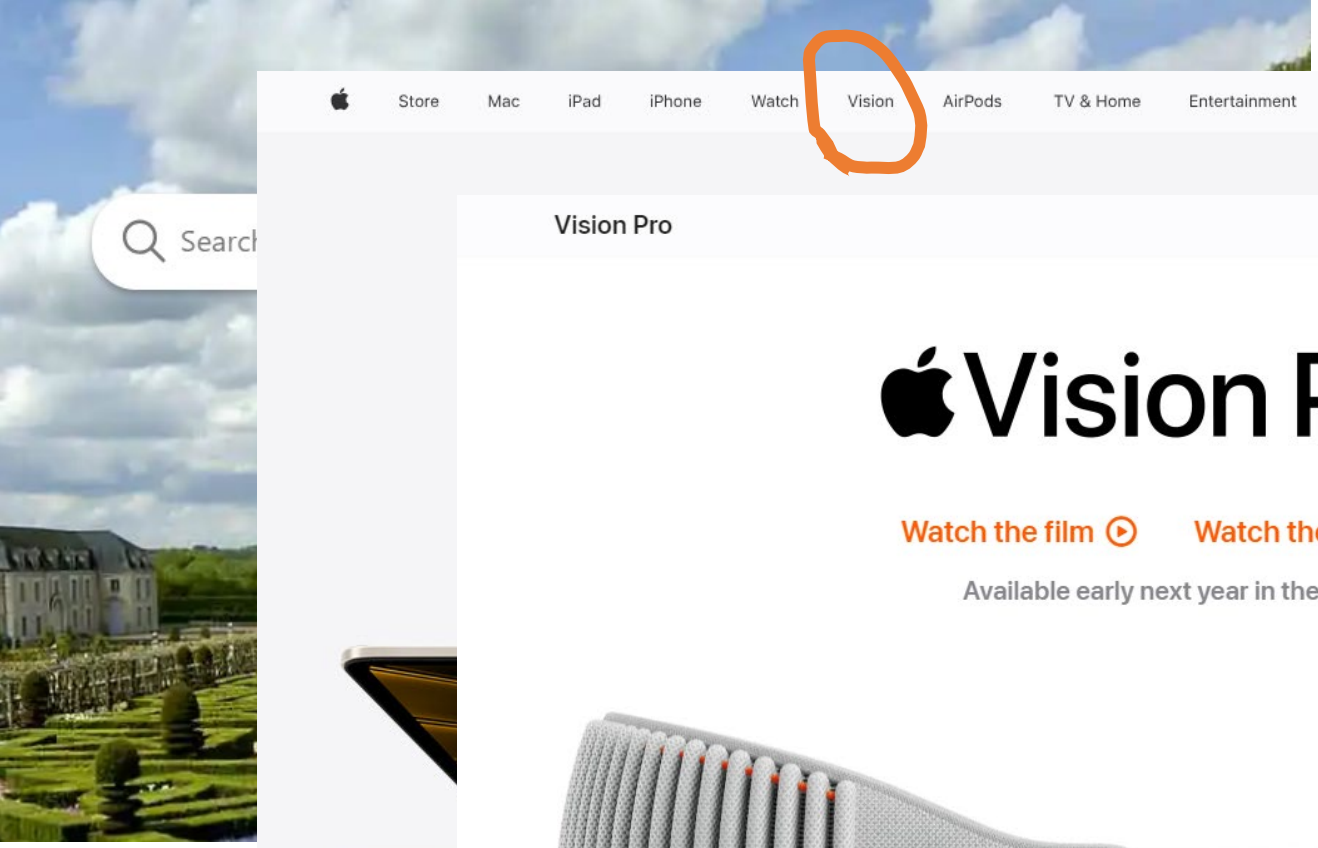

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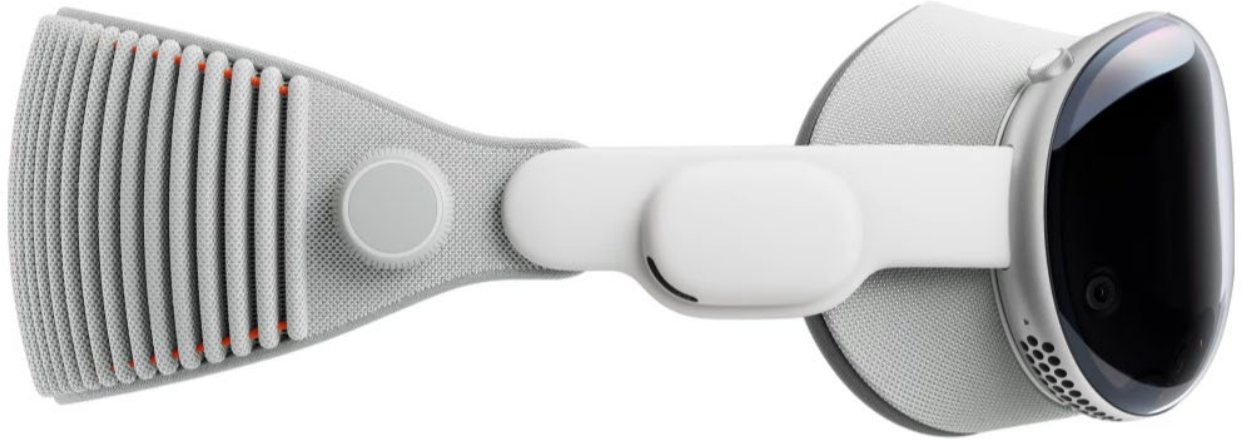

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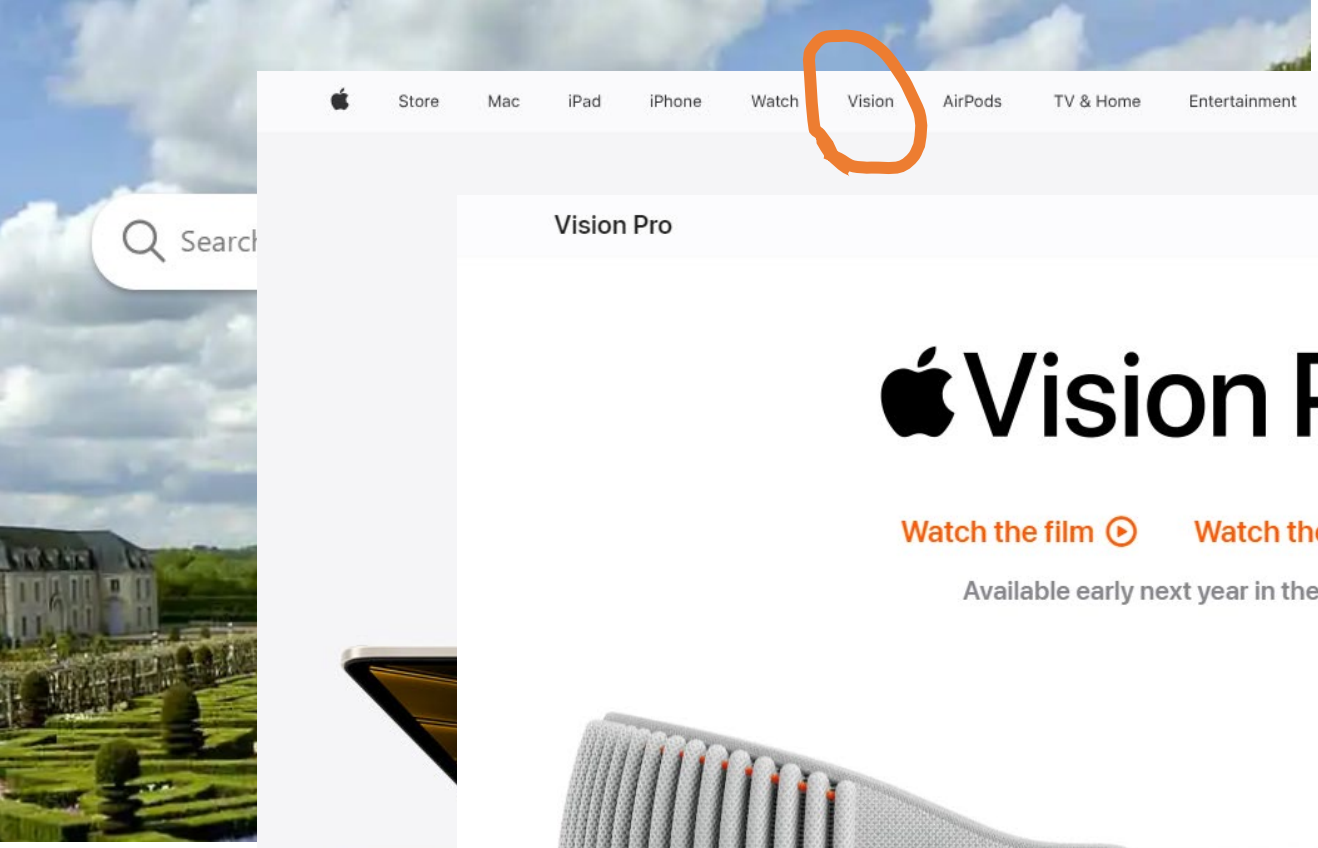

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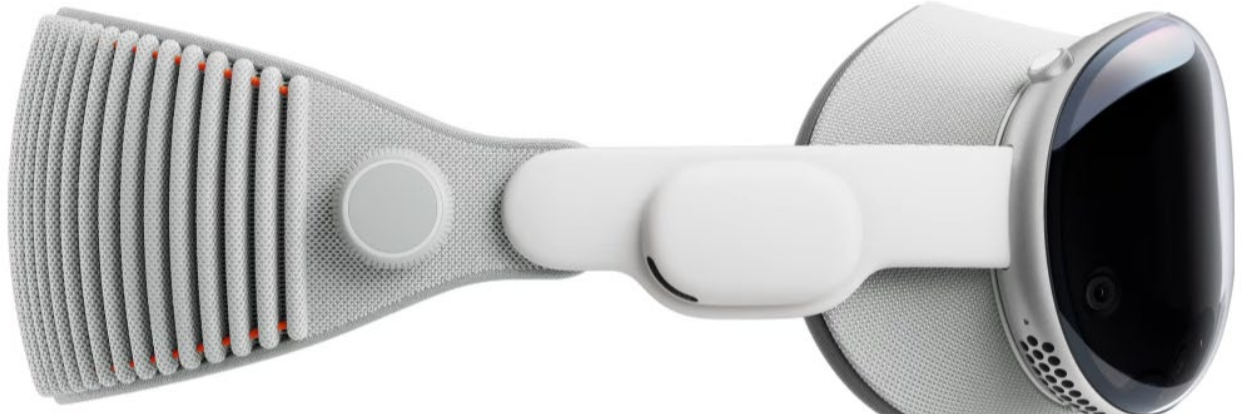

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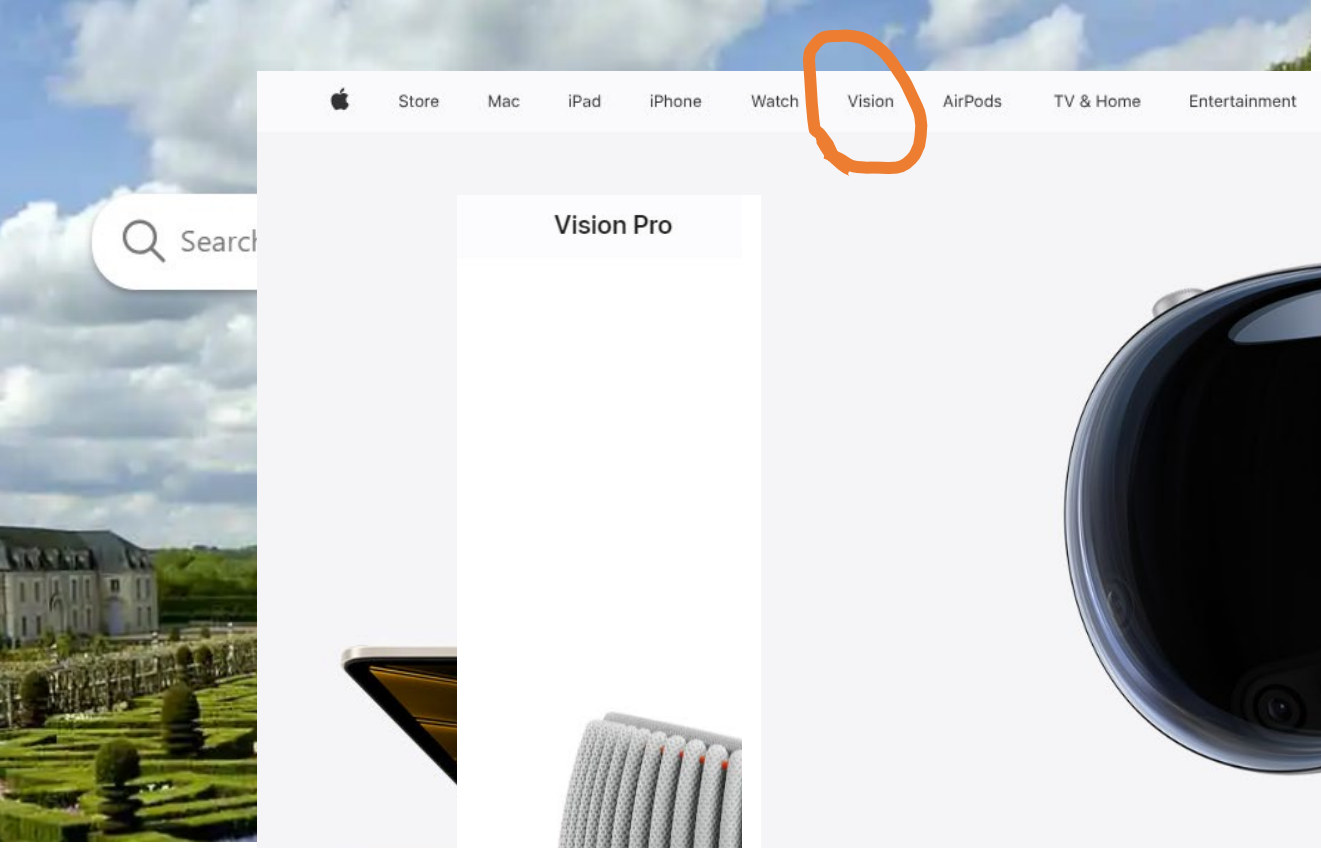

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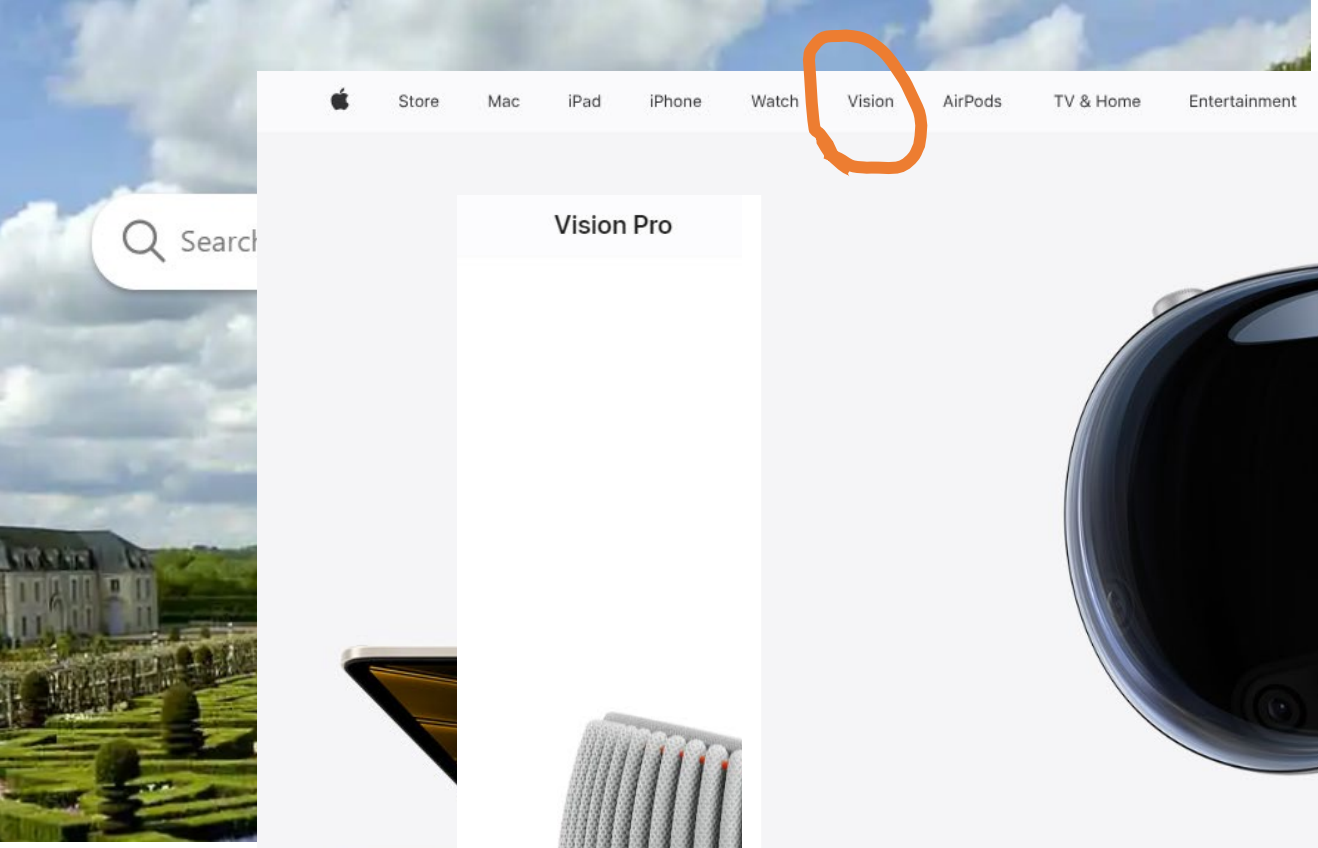
Vision Pro

Enclosure | Cameras and sensors | Audio Straps | Head Band | Displays | Light Seal | Digital Crown | Top button | Power

A singular piece of three-dimensionally formed laminated glass flows into an aluminum alloy frame that gently curves to wrap around your face.

+ Take a closer look at Vision Pro



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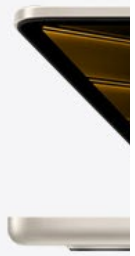
Vision Pro



Enclosure | Cameras and sensors | Audio Straps | Head Band | Displays | Light Seal | Digital Crown | Top button | Power

The external battery supports up to 2 hours of use, and all-day use when plugged in.[1]

Enclosure | Cameras and sensors | Audio Straps | Head Band | Displays | Light Seal | Digital Crown | Top button | Power

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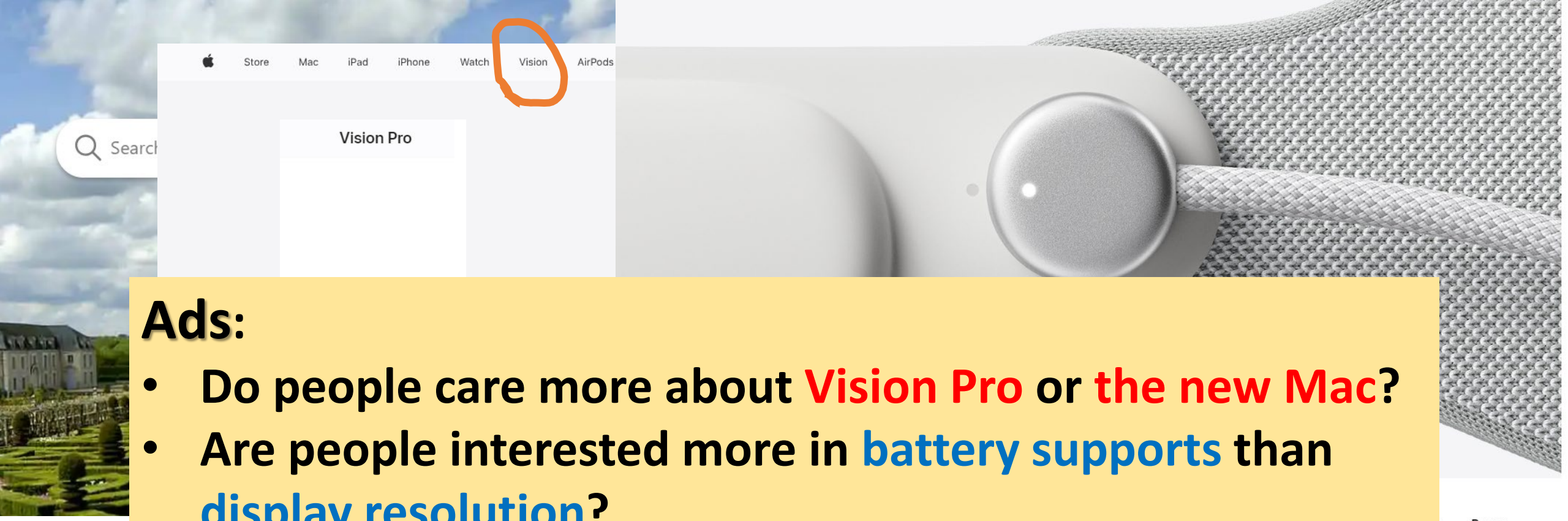

Ads:

- Do people care more about Vision Pro or the new Mac?
- Are people interested more in battery supports than display resolution?

Power

The external battery supports up to 2 hours of use, and all-day use when plugged in.[1]

Enclosure | Cameras and sensors | Audio Straps | Head Band | Displays | Light Seal | Digital Crown | Top button | Power

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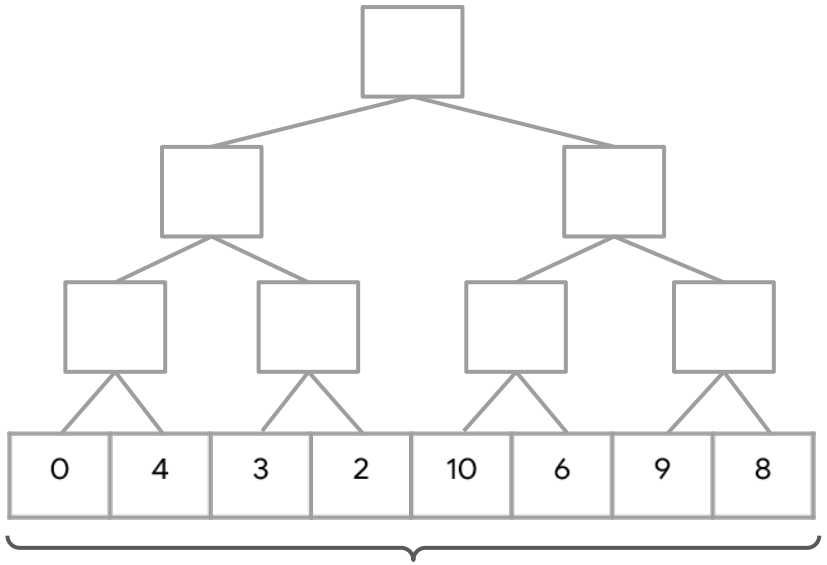
+ Take a closer look at Vision Pro

Counting on Tree

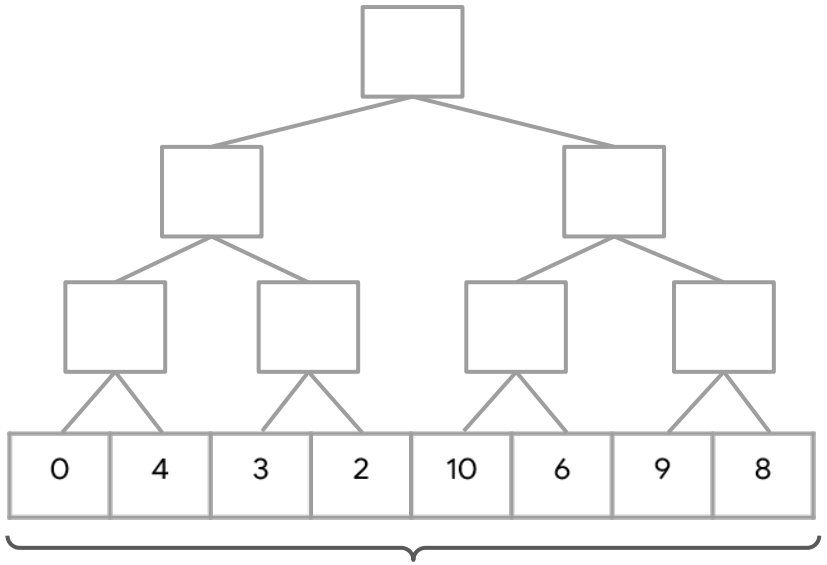


Input: length-N positive vector x

N = # Leaves

d = Tree depth

Counting on Tree



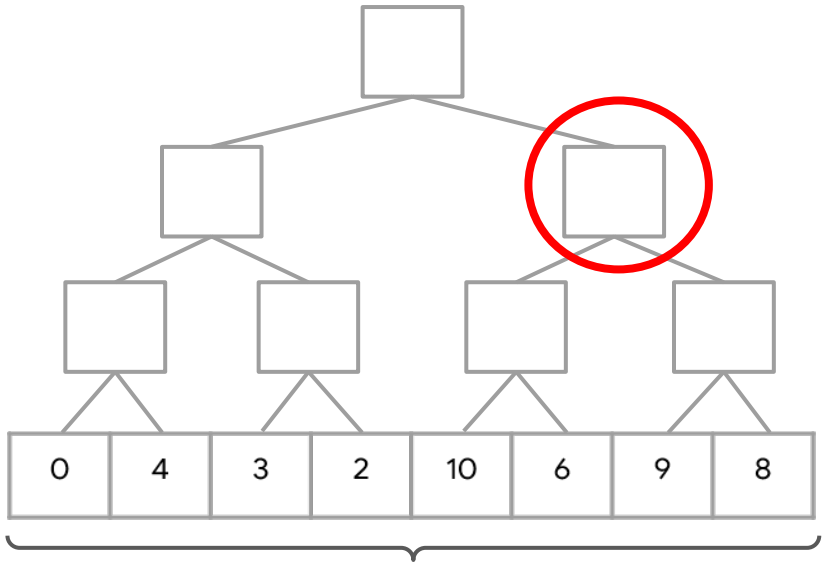
Input: length-N positive vector x

N = # Leaves

d = Tree depth

Node query v:
q_i = total of x_ℓ for all leaves ℓ under v_i

Counting on Tree



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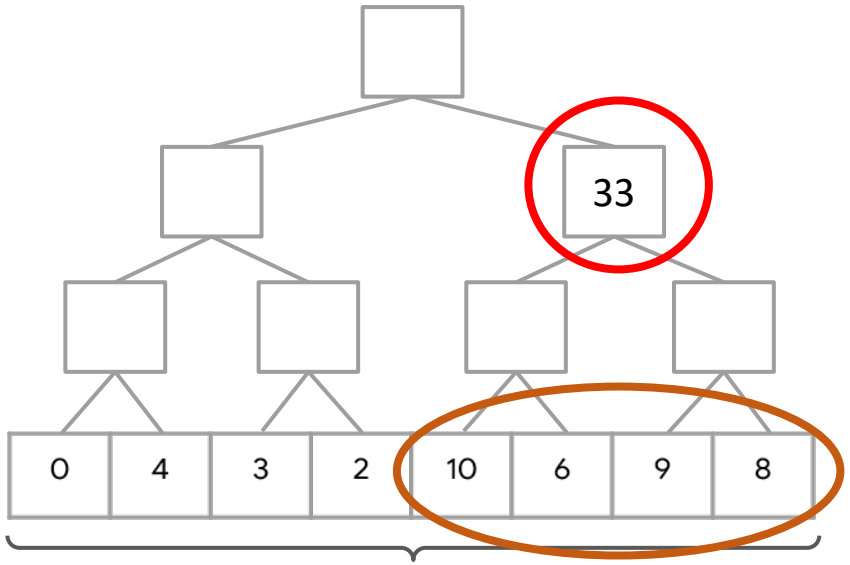

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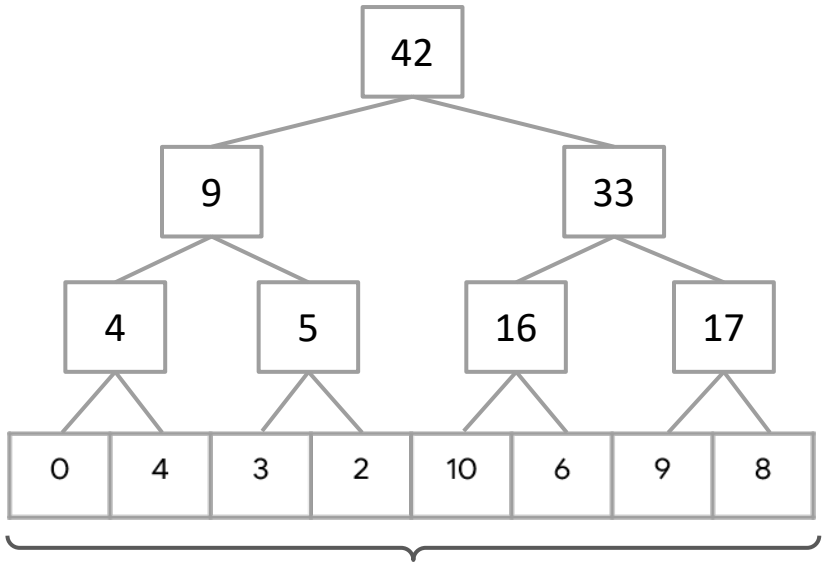
Counting on Tree



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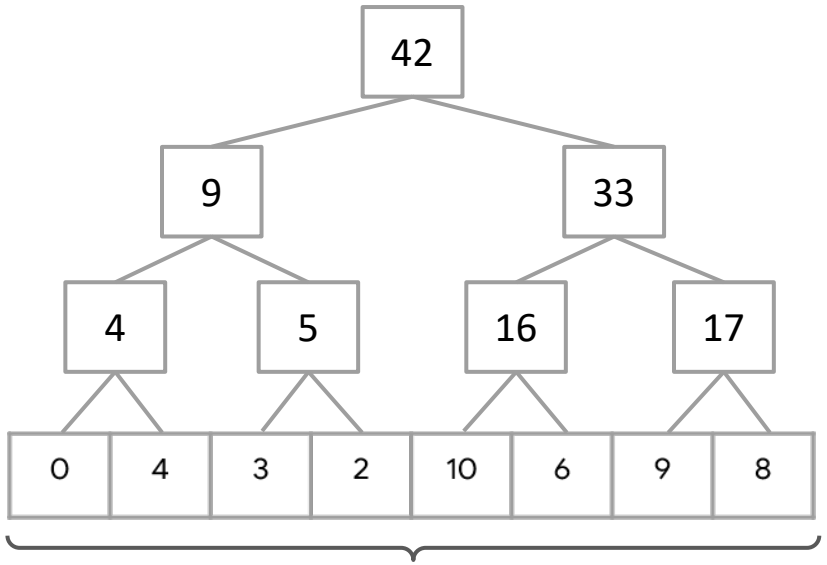

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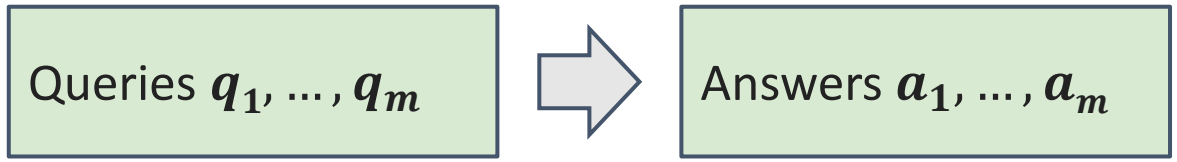
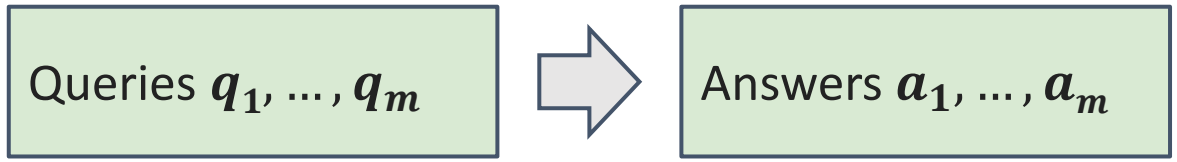
Captures:
- Census (Geographical hierarchy)
- Ads (Webpage hierarchy)
- ……

Accuracy



Accuracy

- How many people live in **US**?



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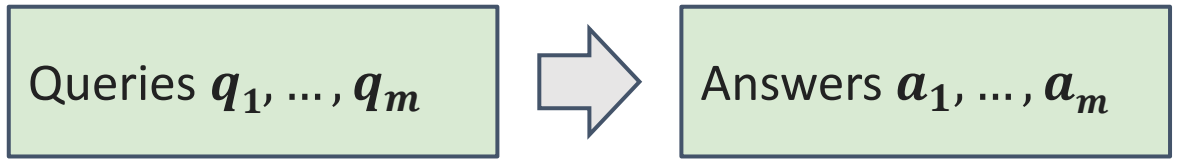

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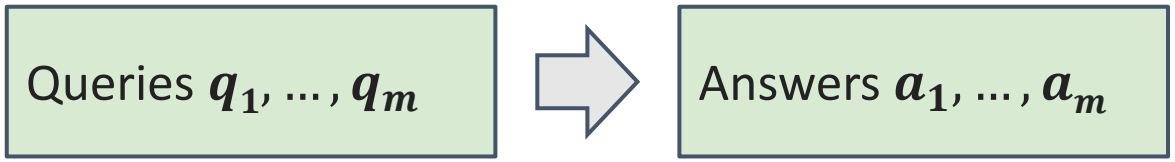

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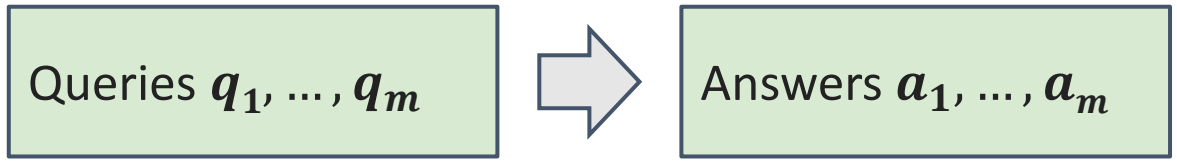

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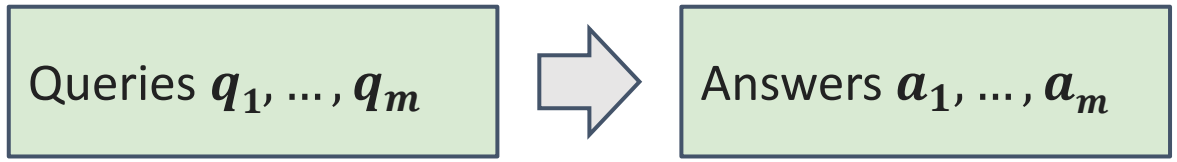

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Queries $q_1, \ldots, q_m$ → Answers $a_1, \ldots, a_m$

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- How many people li **ia, US**?
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Boundary Cases:

- $\alpha = 0$: $\mathrm{err}_\alpha = \mathrm{err}$
- $\alpha \geq 1$: $\mathrm{err}_\alpha = 0$ by $a_i \equiv 0$

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x, y are neighbors $\Leftrightarrow |x - y|_1 = 1$

$\delta = 0$: Pure-DP
$\delta \in (0, 1)$: Approximate-DP
$\delta \geq 1$: Trivial

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Fix $\epsilon, \delta, d, \alpha$. What is the smallest possible err_α?

Results

Error / Privacy	Pure-DP $\delta = 0$	Approximate-DP $\delta \in (0, 1)$
Additive only $\alpha = 0$	$O(d/\epsilon)$ [Laplace Mechanism]	$O\left(\sqrt{d \log(1/\delta)}/\epsilon\right)$ [Gaussian Mechanism]
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Simple mechanisms work best

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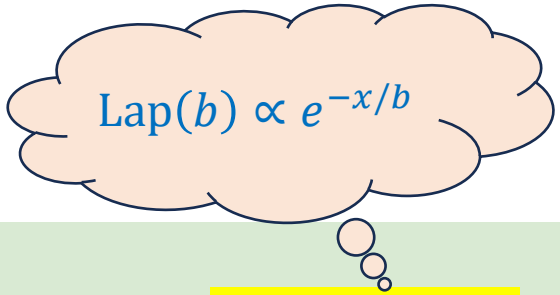

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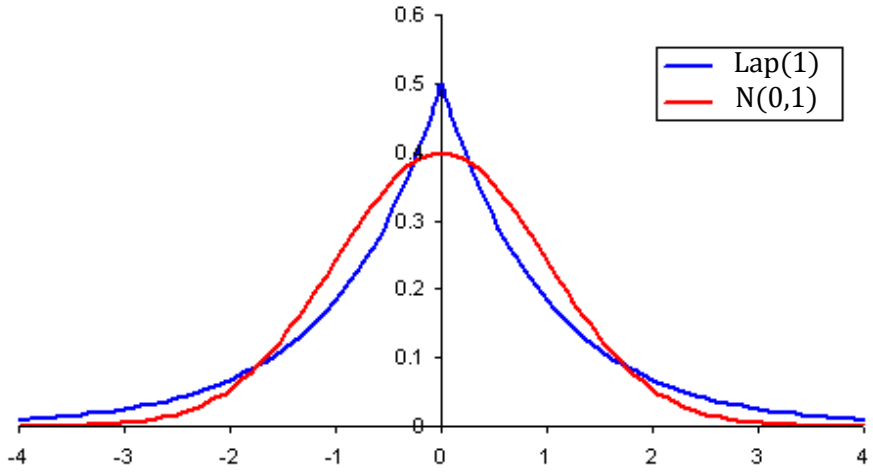

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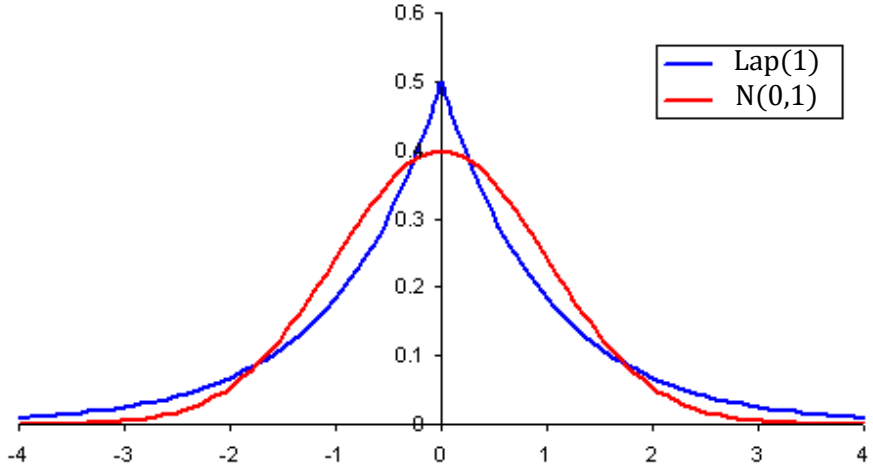

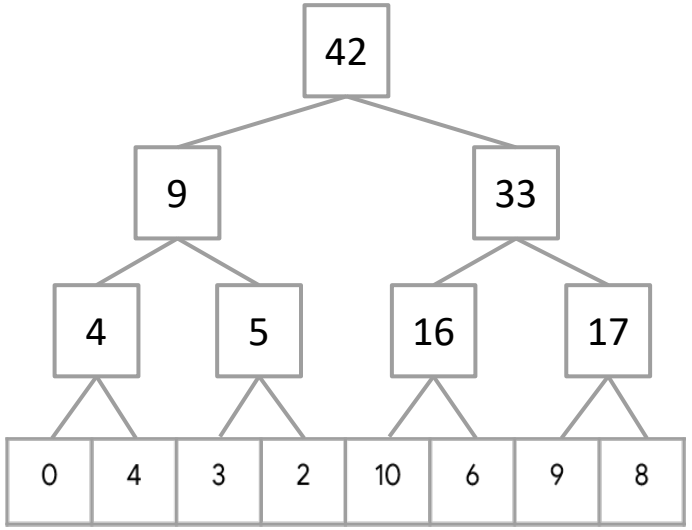

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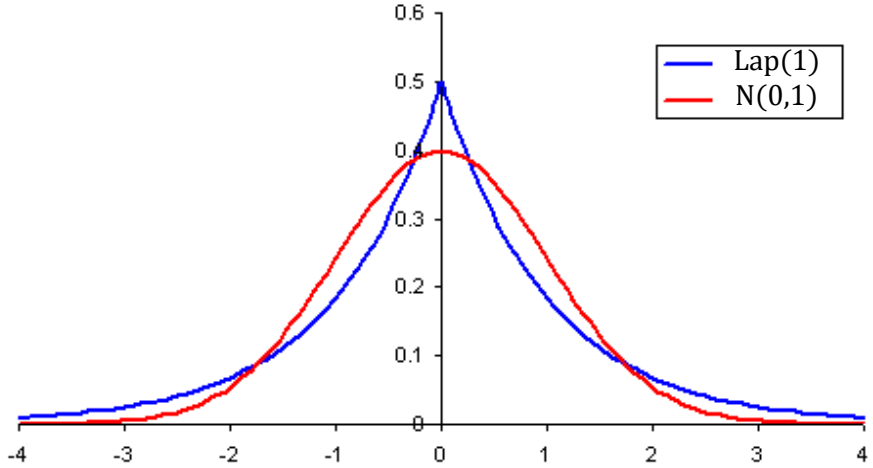

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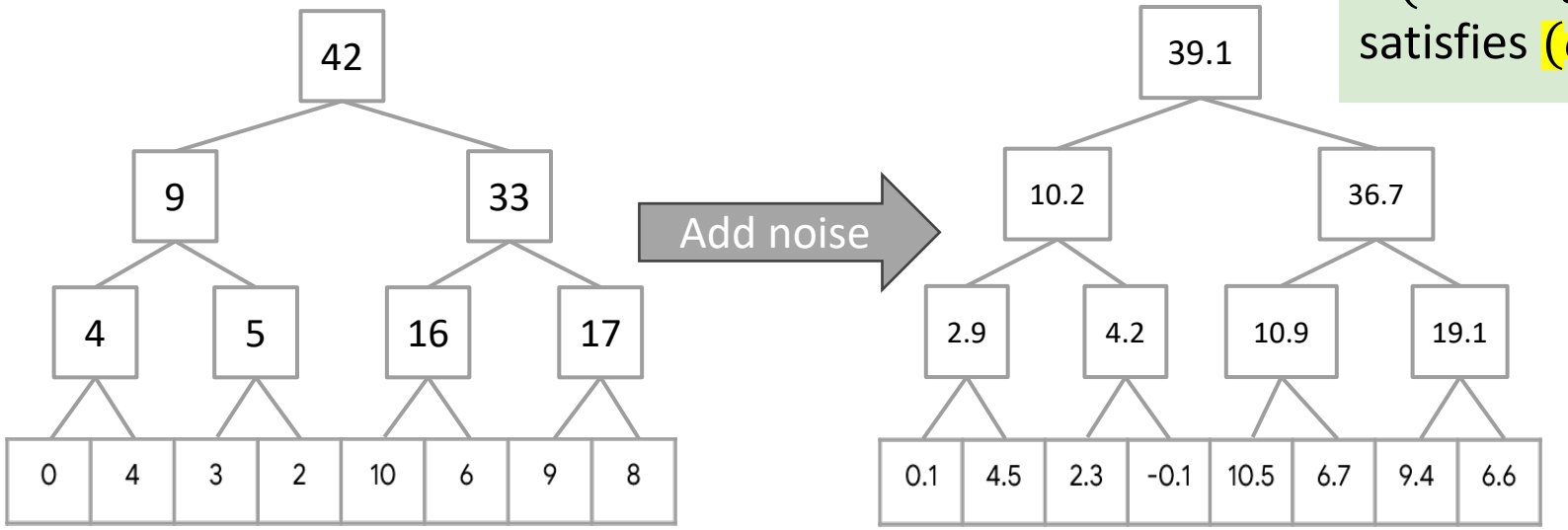

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Classification Problem

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- Given threshold $\boldsymbol{\tau}$
- Output for each node $\boldsymbol{v}$ whether
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Reduction to Classification

Example: $\tau = 30, \alpha = 0.2$
Above: ≥ 36
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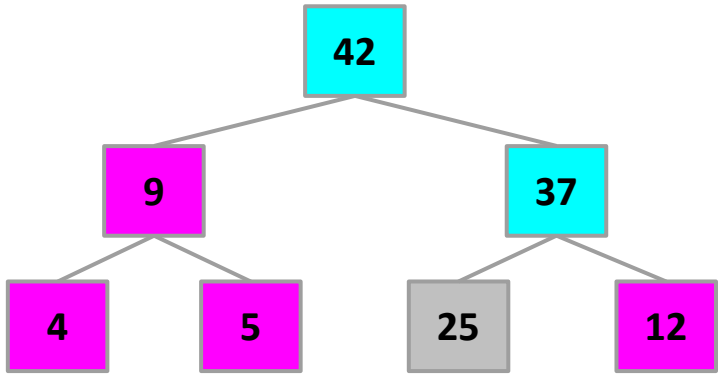

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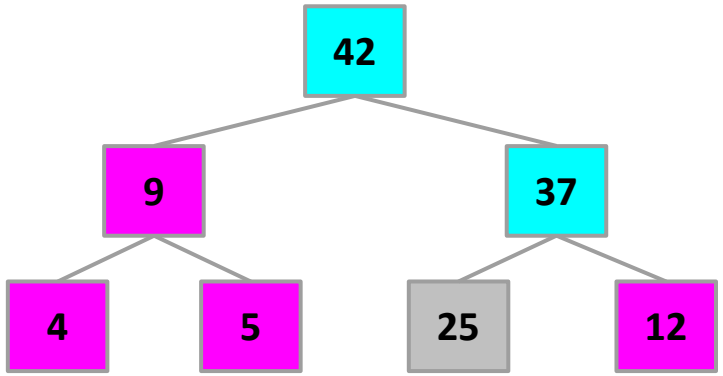

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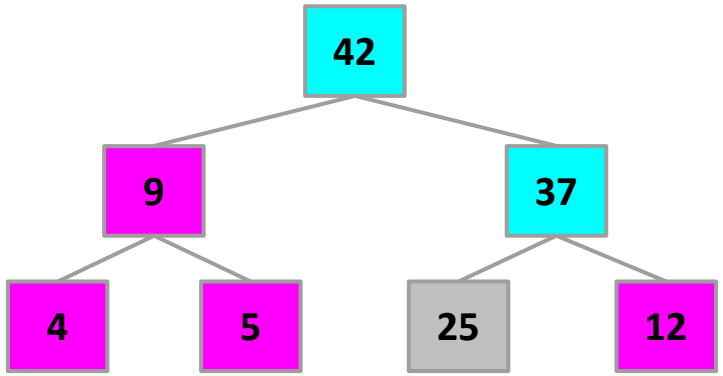

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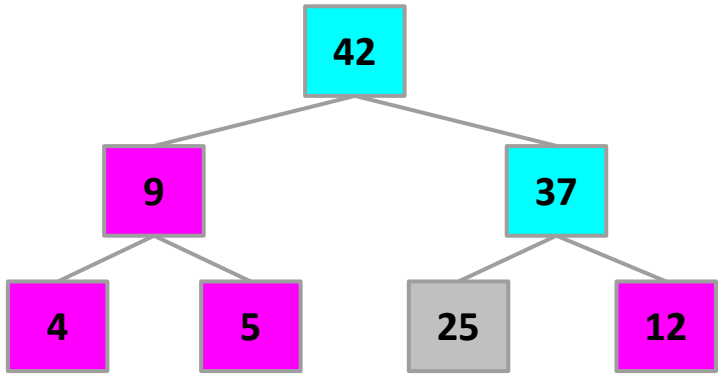

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If the classification algorithm is private, the reduction result is also private

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Reduction Example

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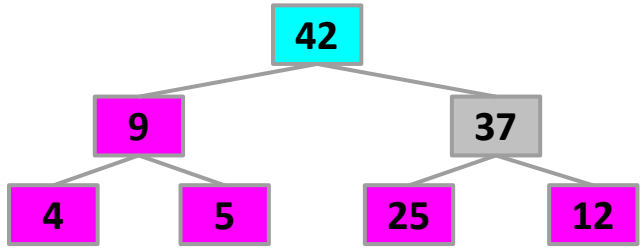

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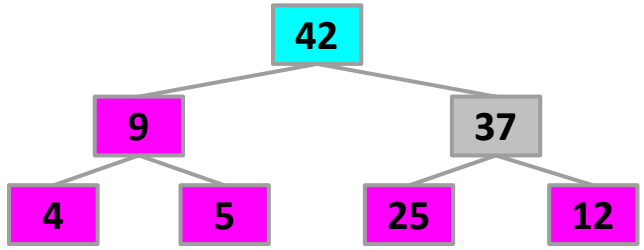

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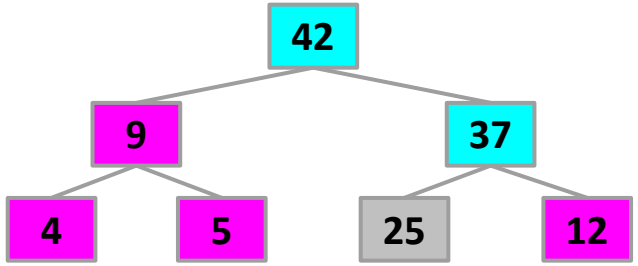

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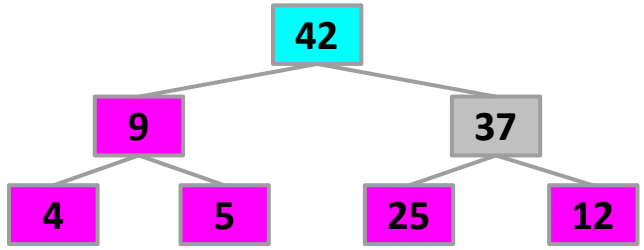
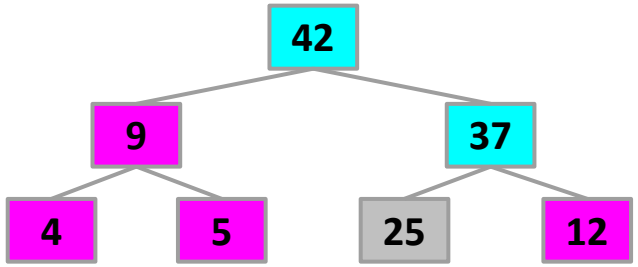


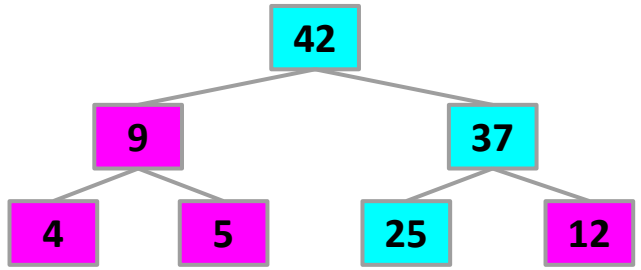
$\tau = 25, \alpha = 0.2$

Above: ≥ 30 Below: ≤ 20



$\tau = 15, \alpha = 0.2$

Above: ≥ 18 Below: ≤ 12



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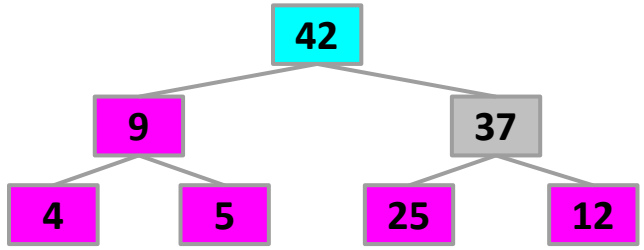

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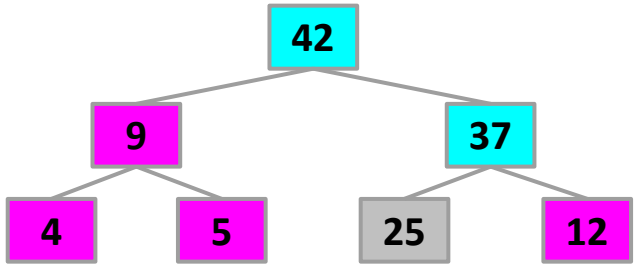

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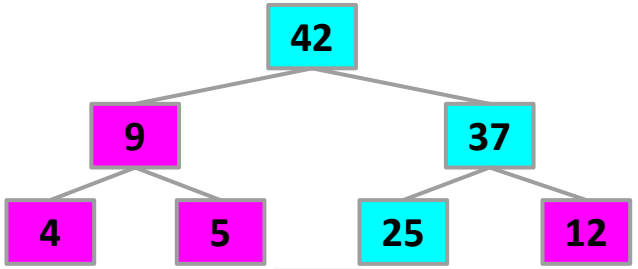
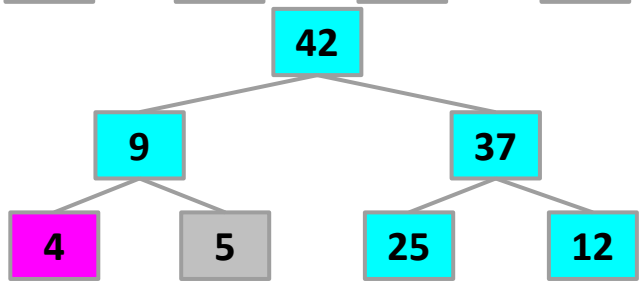


$\tau = 5, \alpha = 0.2$

Above: ≥ 6 Below: ≤ 4



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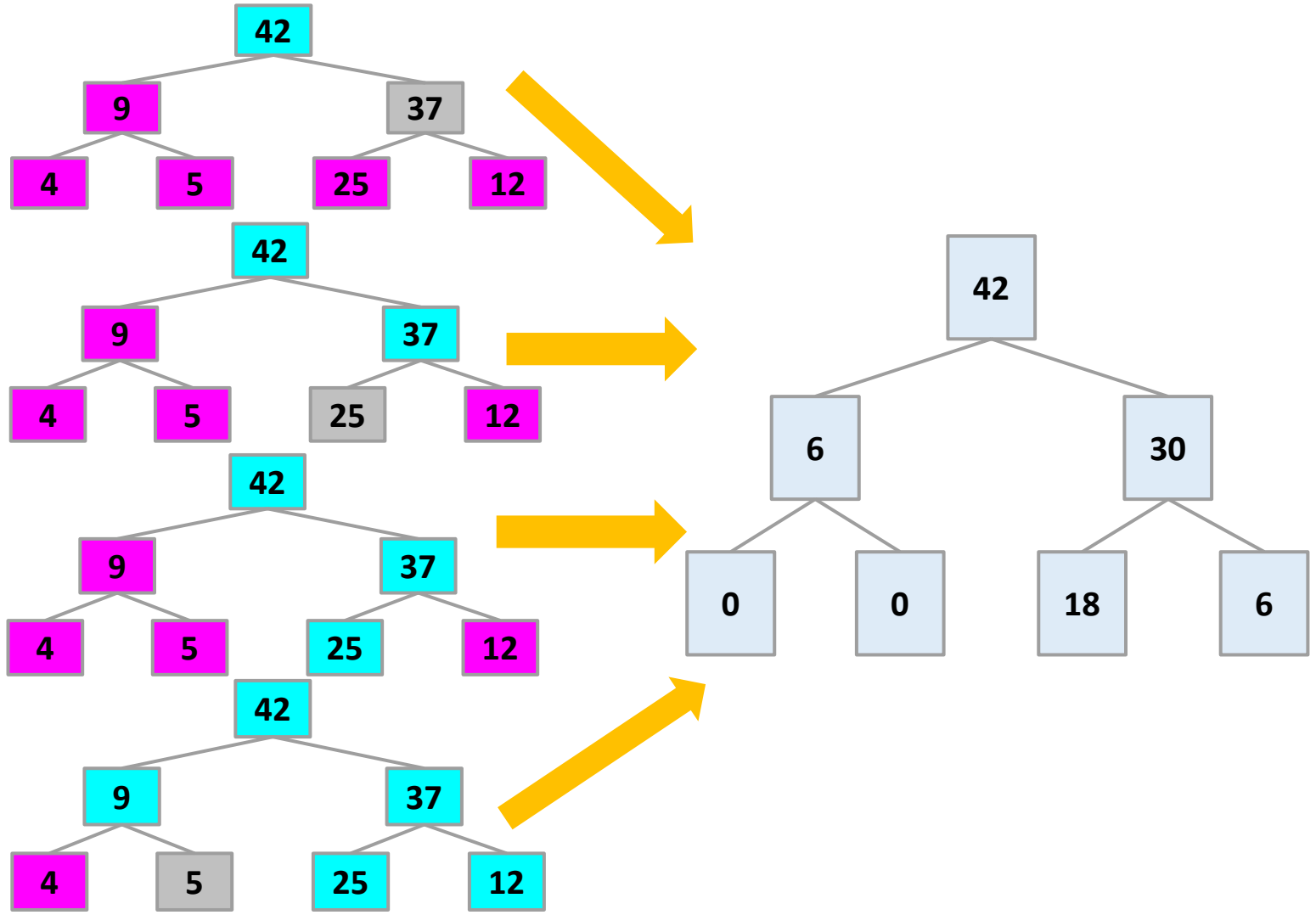


Classification Algorithm

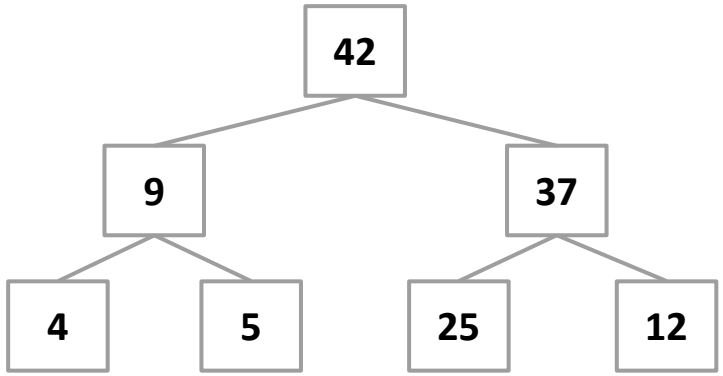
Classification Algorithm

Classification Problem

- Given threshold τ
- Promise that root count is $< 2\tau$
- Output for each node v whether
  - q_v is above $(1+\alpha)\tau$ or below $(1-\alpha)\tau$
  - If $q_v \in [(1-\alpha)\tau, (1+\alpha)\tau]$, any answer works

Classification Algorithm

Example: $\tau = 30$
Promise: root count is < 60



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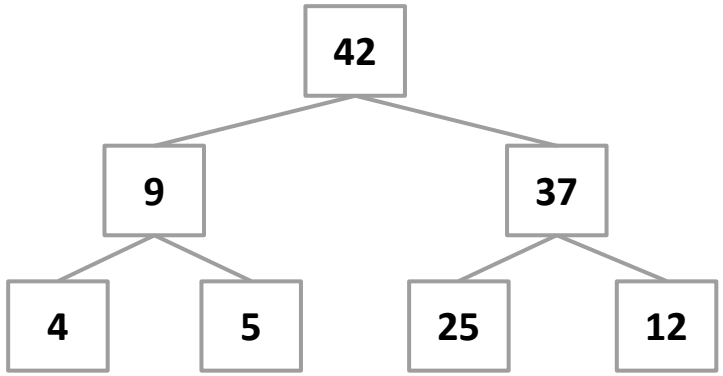

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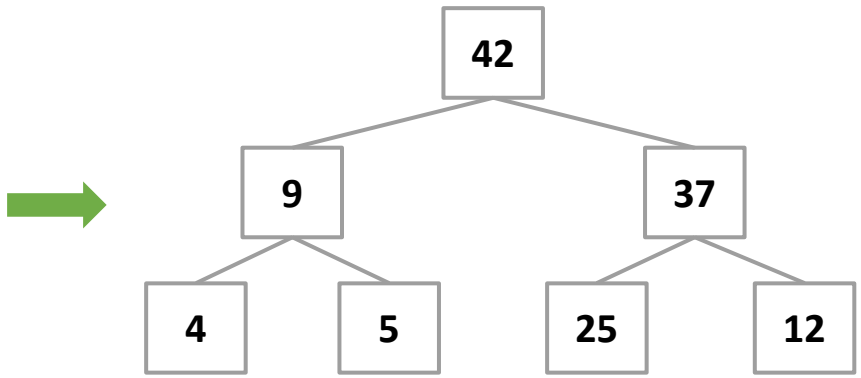

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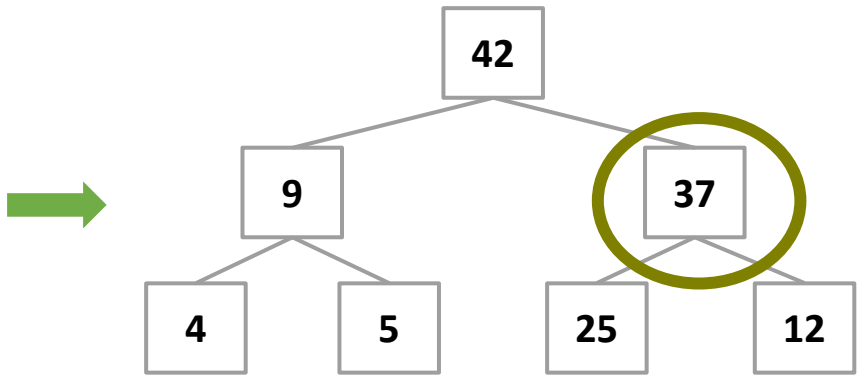

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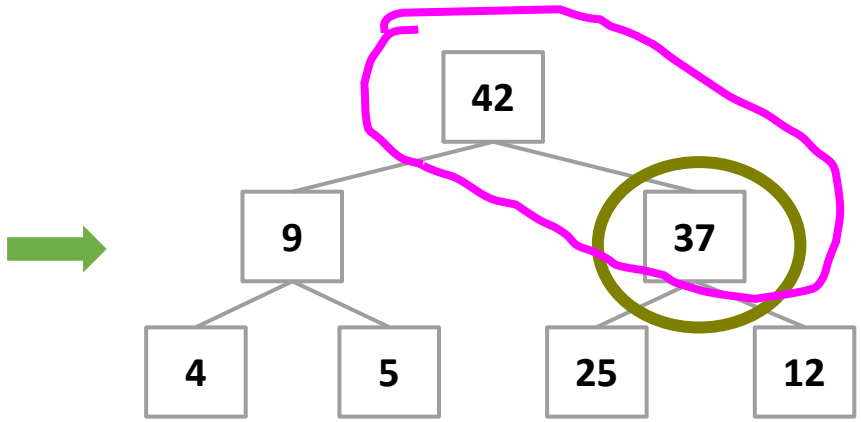

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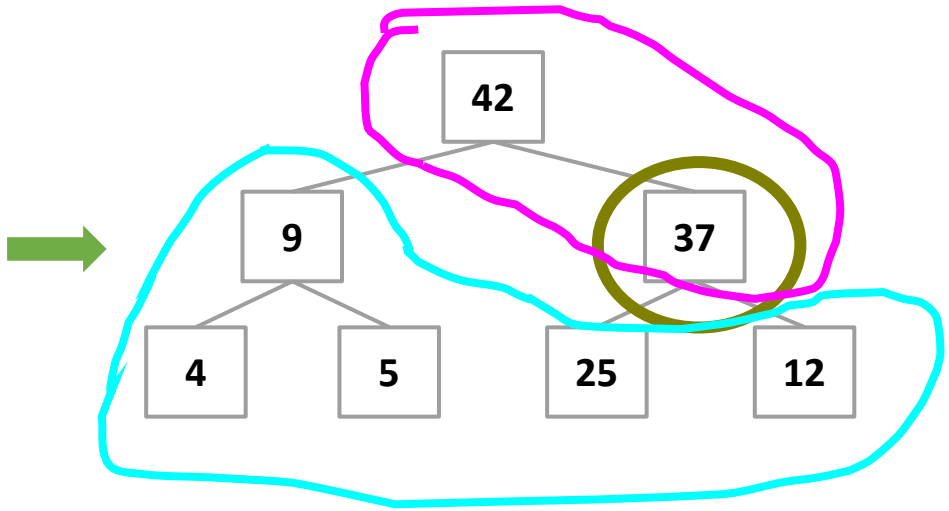

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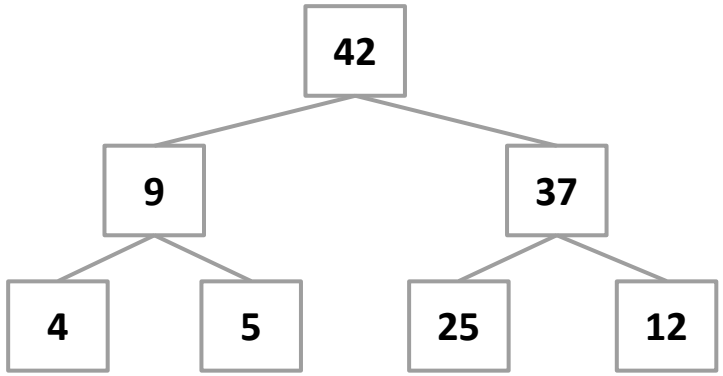

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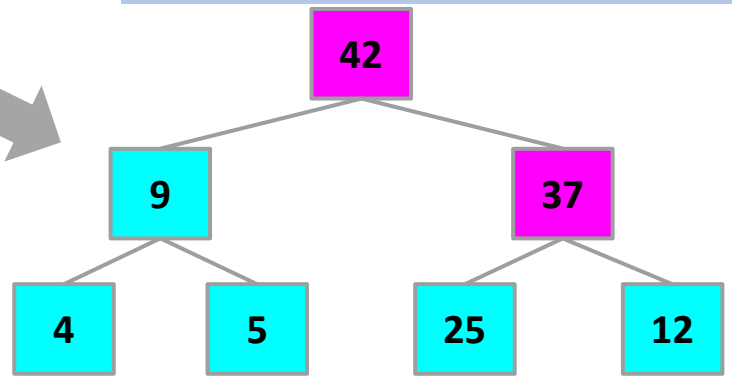

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Total Error: $O(\log(d/\delta)/\varepsilon)$

Results

Error / Privacy	Pure-DP $\delta = 0$	Approximate-DP $\delta \in (0, 1)$
Additive only $\alpha = 0$	$O(d/\epsilon)$ [Laplace Mechanism]	$O\left(\sqrt{d \log(1/\delta)}\,/\epsilon\right)$ [Gaussian Mechanism]
	$\Omega(d/\epsilon)$ [Our result]	$\Omega_{\epsilon,\delta}\left(\sqrt{d}\right)$ [Edmonds-Nikolov-Ullman'20]
Multiplicative & additive $\alpha \in (0, 1)$ is a constant	$\Omega(d/\epsilon)$ [Our result]	$O(\log(d/\delta)\,/\epsilon)$ [Our result]

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Assume for all i

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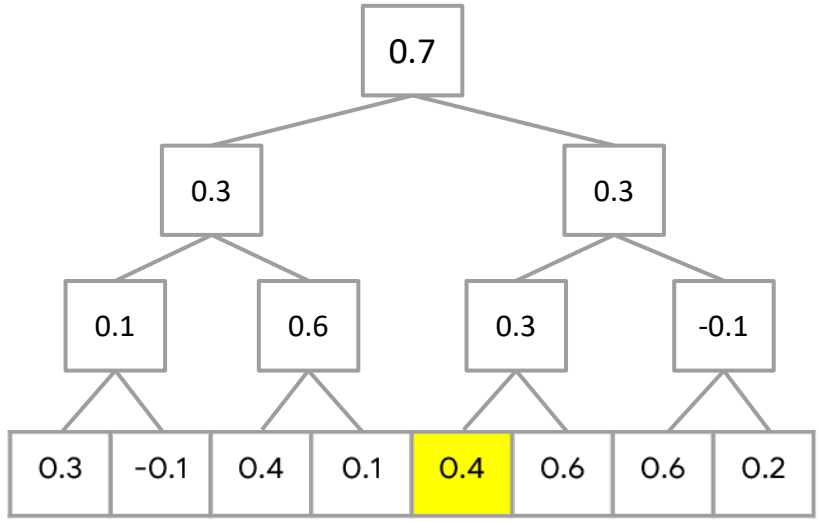

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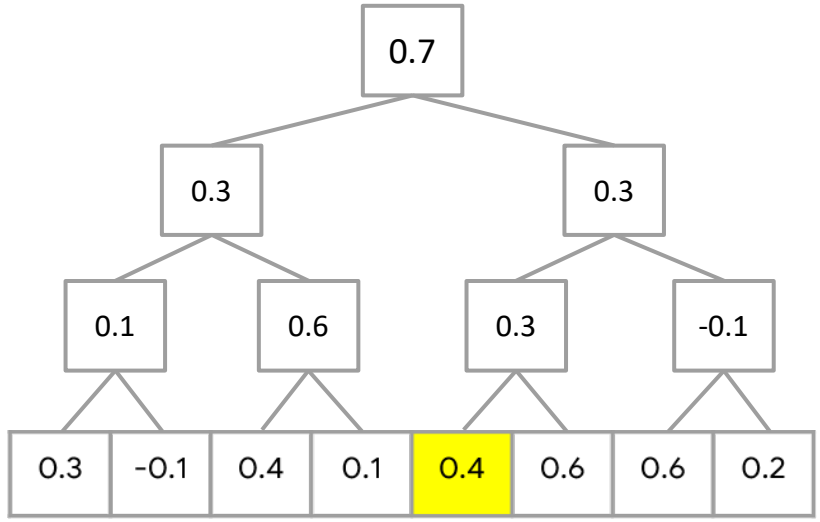

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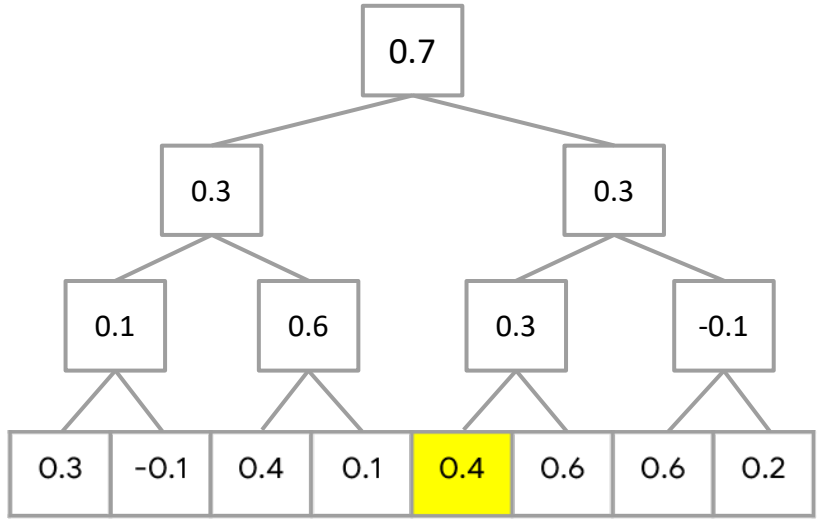
  - Satisfies $\boldsymbol{\Pr[i' = i] > \exp(\epsilon D)/m}$

$\boldsymbol{x_i}$: put $\boldsymbol{0.1d/\epsilon}$ at the $\boldsymbol{i}$-th leaf & **zero** elsewhere
$\boldsymbol{m = 2^d}$ and $\boldsymbol{D = 0.2d/\epsilon}$

$\boldsymbol{\Pr[i' = i] > \exp(0.2d)/2^d \approx 0.6^d}$

Output of $\boldsymbol{A(xi)}$



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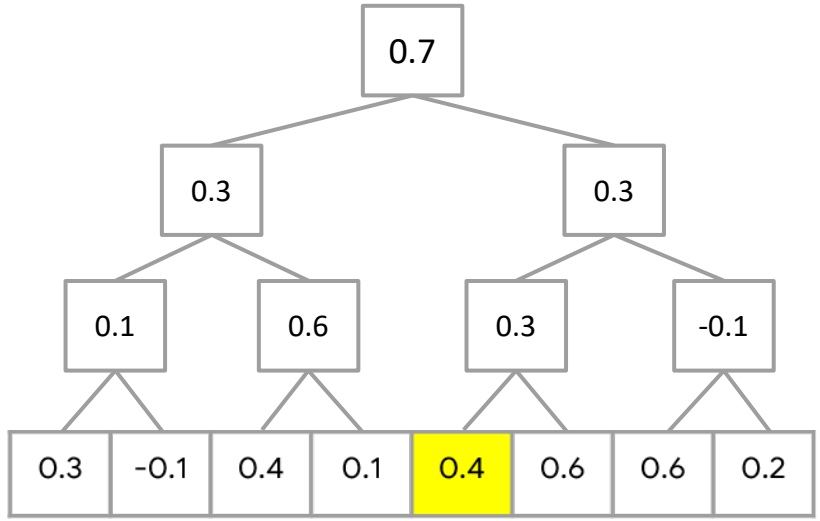

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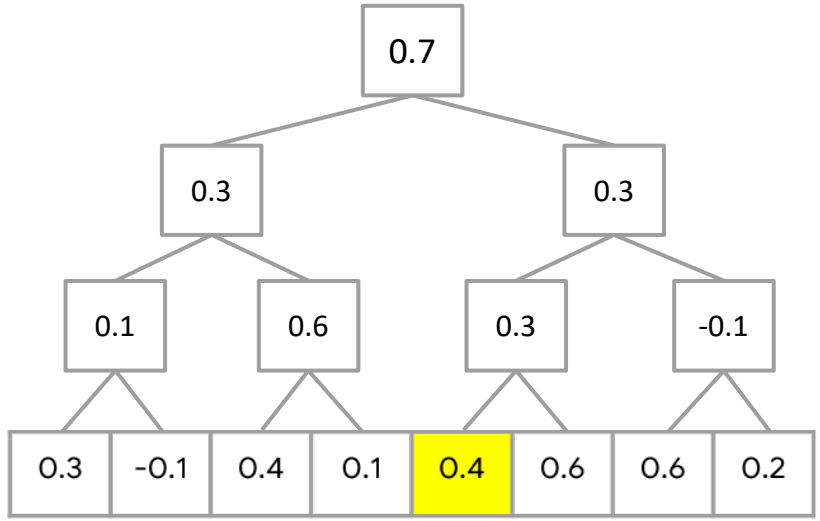

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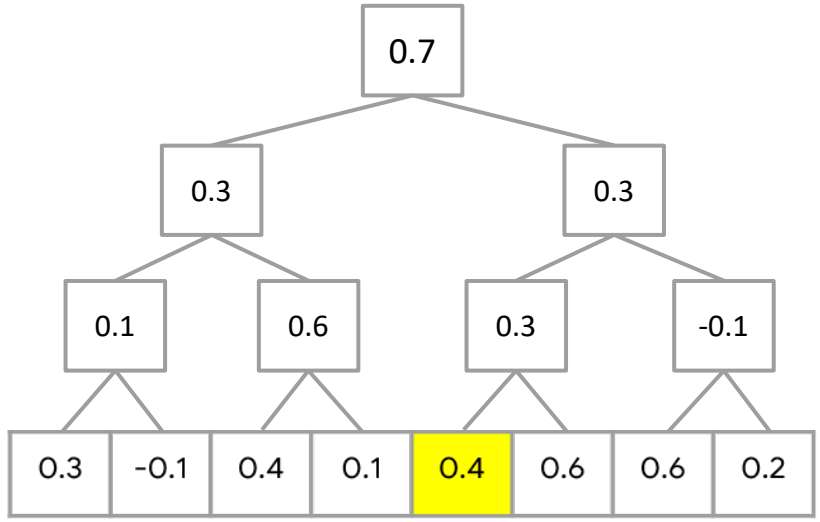

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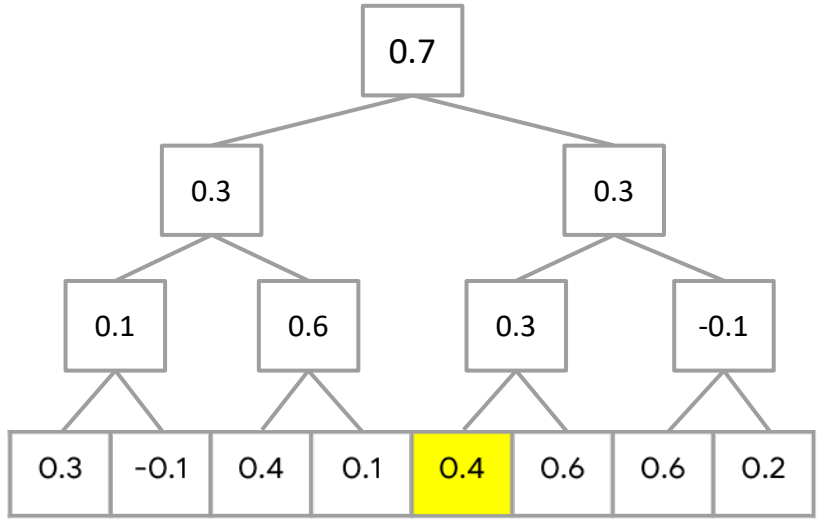

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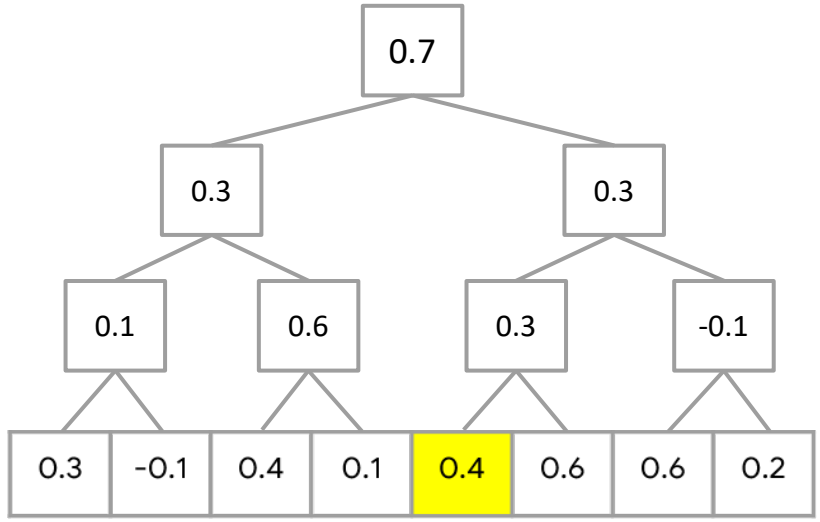

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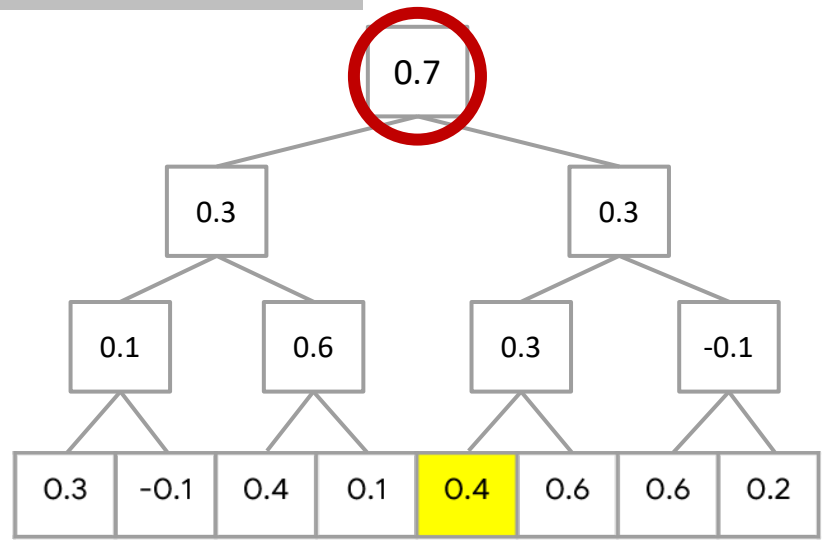

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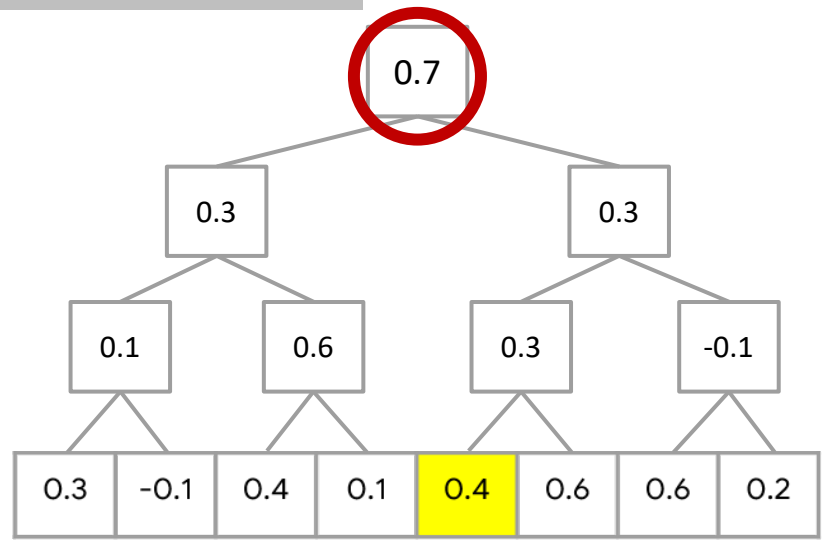

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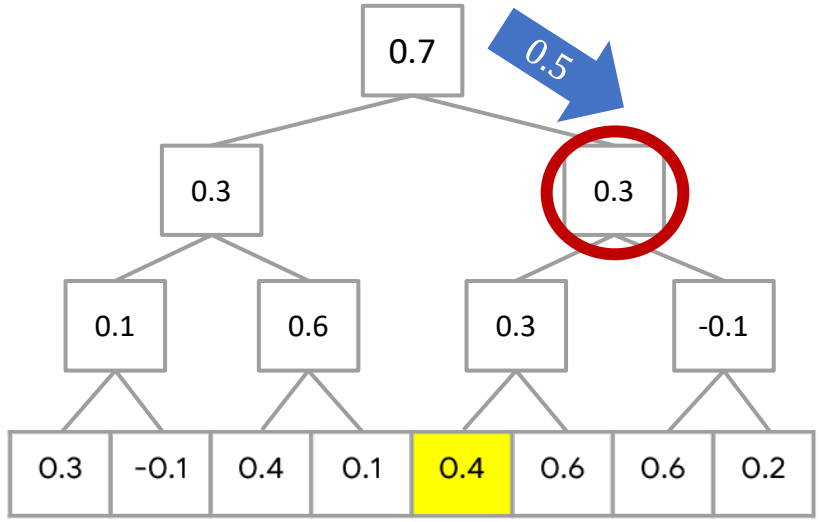

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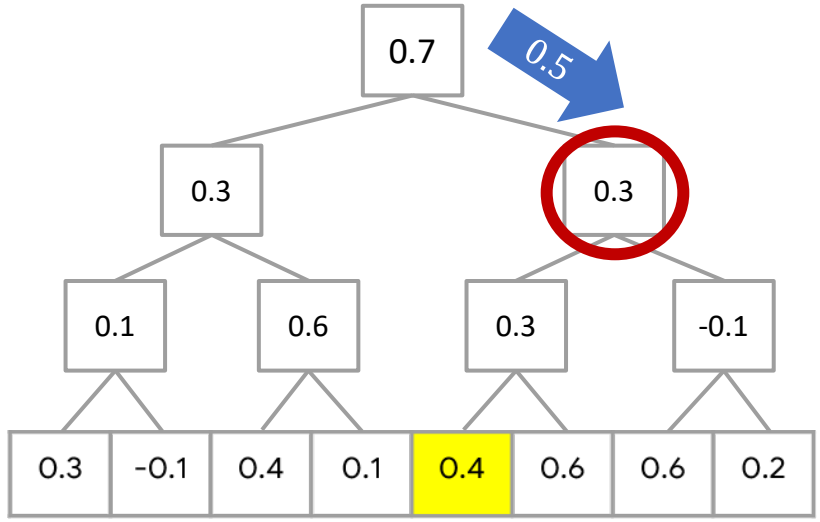

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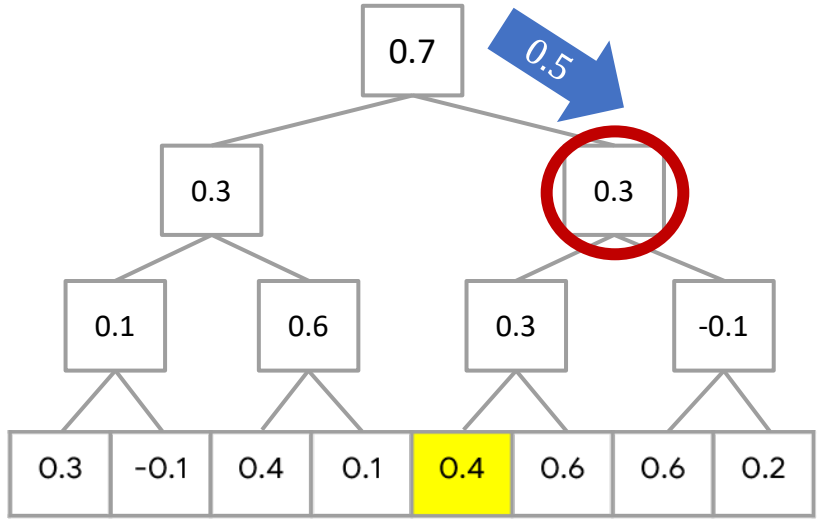

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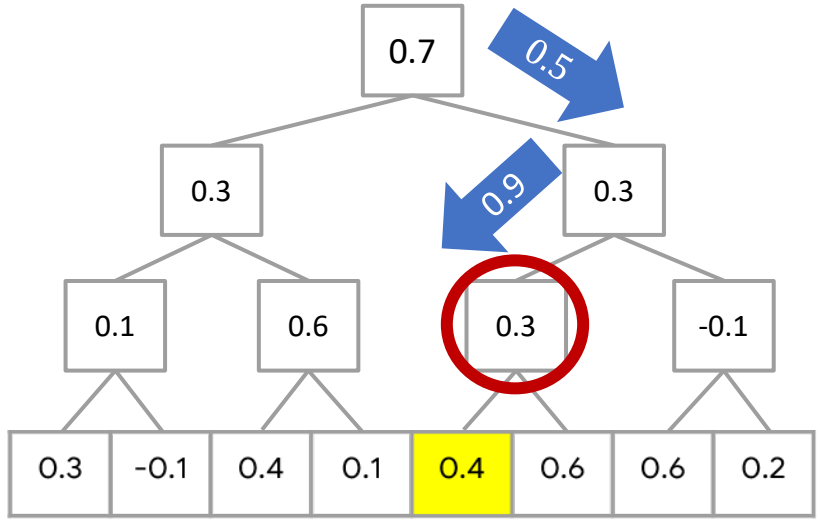

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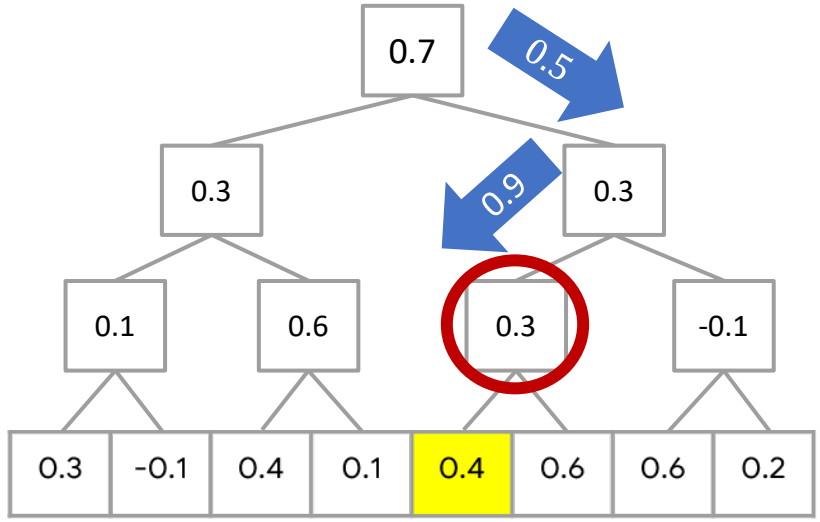

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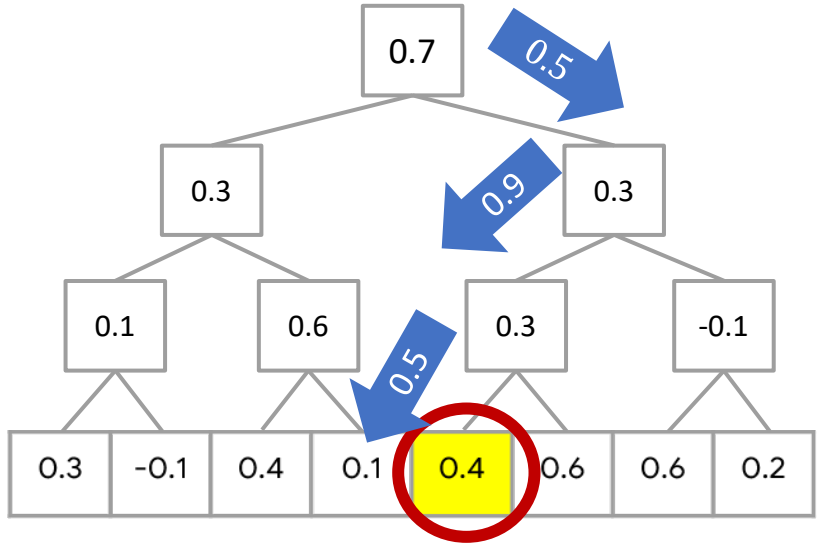

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In this example,

$$\mathbf{Pr}[i' = i] = 0.5 \cdot 0.9 \cdot 0.5 = 0.225$$

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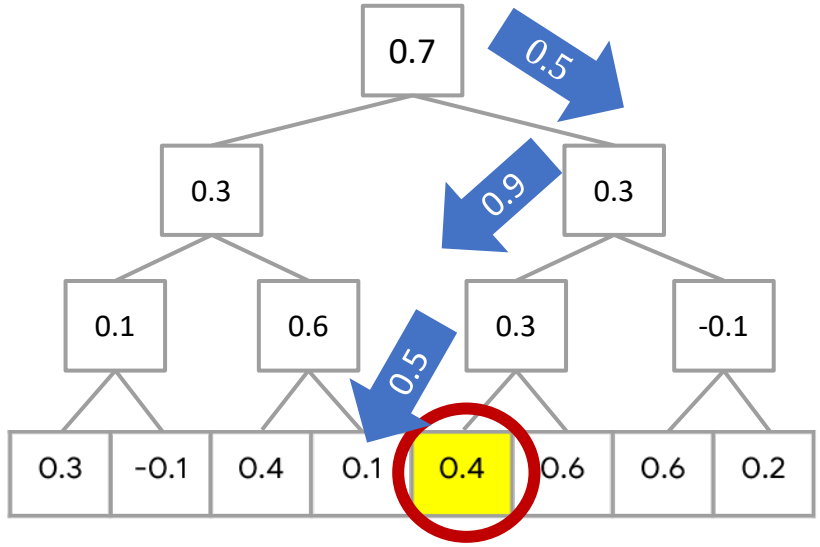

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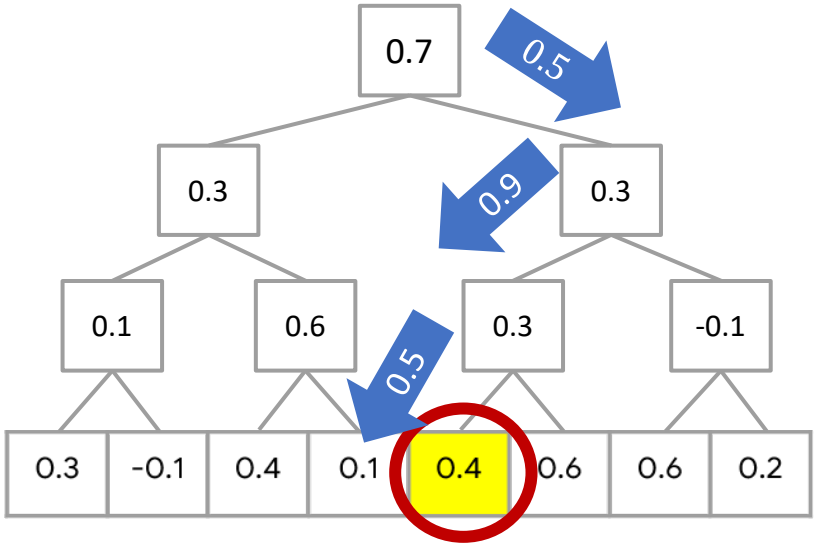

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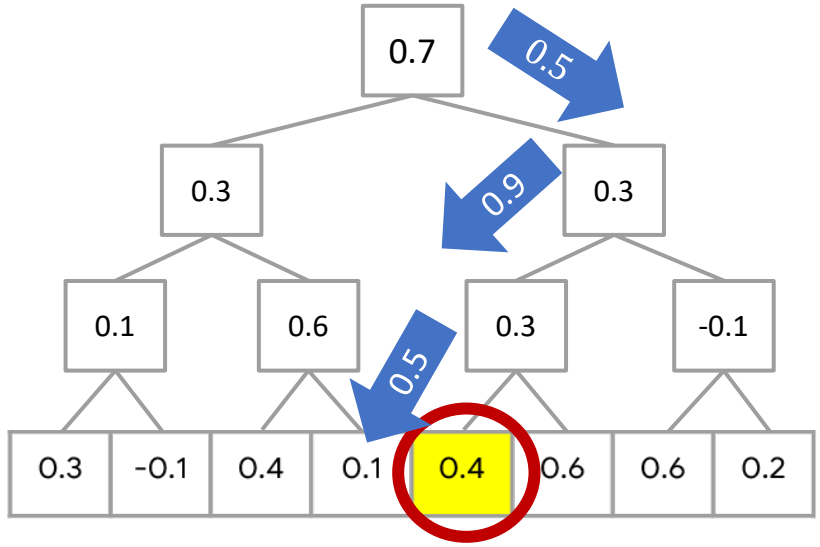

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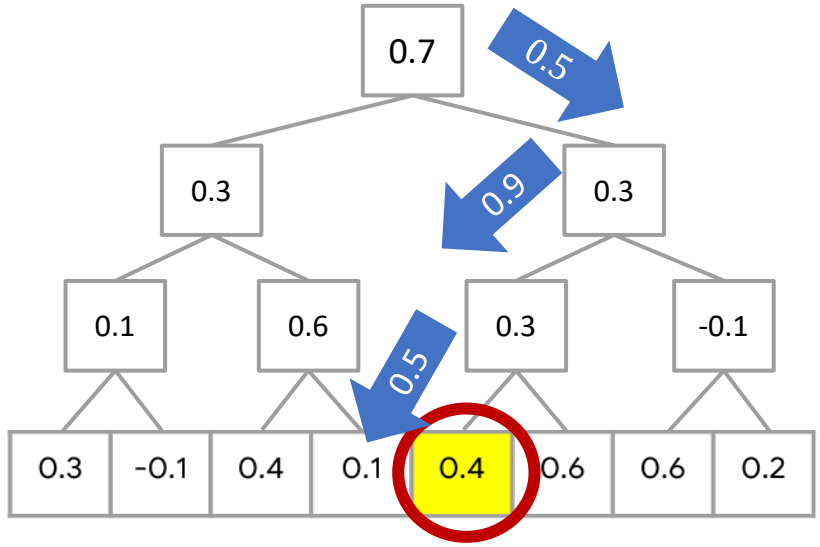

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Thank you!