

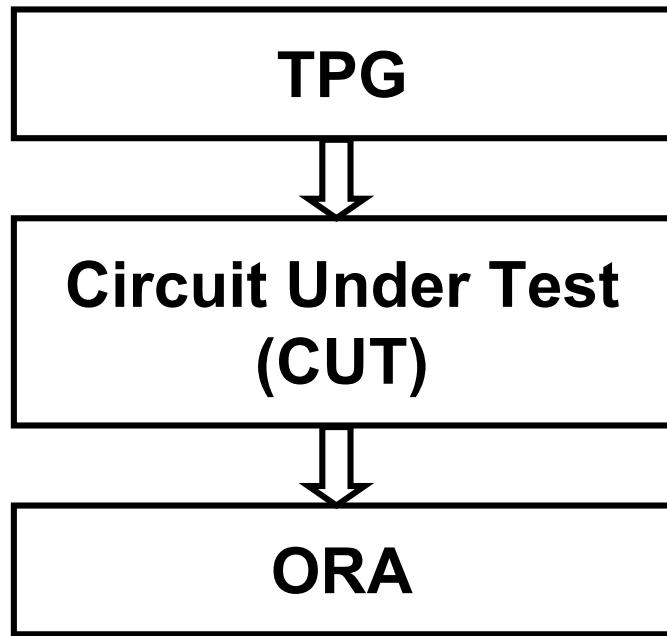
# **Built-in Self-Test (BIST)**

- **Introduction**
- **Test Pattern Generation**
- **Test Response Analysis**
- **BIST Architectures**
- **Scan-Based BIST**

# **Built-in Self-Test (BIST)**

- **Capability of a circuit to test itself**
- **On-line:**
  - **Concurrent : simultaneous with normal operation**
  - **Nonconcurrent : idle during normal operation**
- **Off-line:**
  - **Functional : diagnostic S/W or F/W**
  - **Structural : LFSR-based**
- **We deal primarily with structural off-line testing here.**

# Basic Architecture of BIST



- **TPG:** Test pattern generator
- **ORA** Output response analyzer

# Glossary of BIST

- **TPG - Test pattern generator**
- **PRPG - pseudorandom pattern generator  
(Pseudorandom number generator)**
- **SRSG – Shift register sequence generator (a single output PRPG)**
- **ORA – Output response analyzer**
- **SISR – Single-input signature register**
- **MISR – Multiple-input signature register**
- **BILBO – Built-in logic block observer**

# Built-in Self Testing

- Test pattern generation
  - Exhaustive
  - Pseudoexhaustive
  - Pseudorandom
- Test response compression
  - One's count
  - Transition count
  - Parity checking
  - Syndrome checking
  - Signature analysis

# **Test Pattern Generation for BIST**

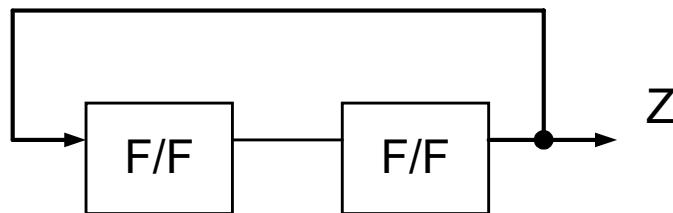
- **Exhaustive testing**
- **Pseudorandom testing**
  - **Weighted and Adaptive TG**
- **Pseudoexhaustive testing**
  - **Syndrome driver counter**
  - **Constant-weight counter**
  - **Combined LFSR and shift register**
  - **Combined LFSR and XOR**
  - **Cyclic LFSR**

# Exhaustive Testing

- **Apply all  $2^n$  input vectors, where n = # inputs to CUT**
- **Impractical for large n**
- **Detect all detectable faults that does not cause sequential behavior**
- **In general not applicable to sequential circuits**
- **Can use a counter or a LFSR for pattern generator**

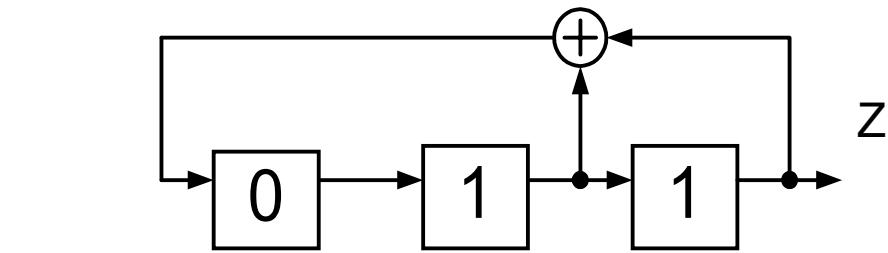
# Linear Feedback Shift Register (LFSR)

- Example



S1	1	0	
S2	0	1	
S1	1	0	
:			

$Z = 0\ 1\ 0\ 1\ \dots$   
2 states

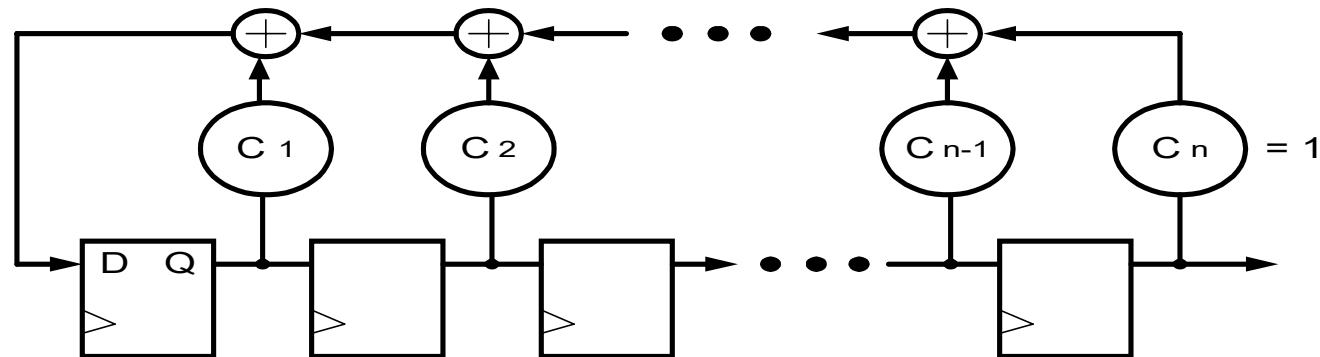


S1	0	0	1
S2	1	0	0
S3	0	1	0
S4	1	0	1
S5	1	1	0
S6	1	1	1
S7 = S0	0	1	1
:			

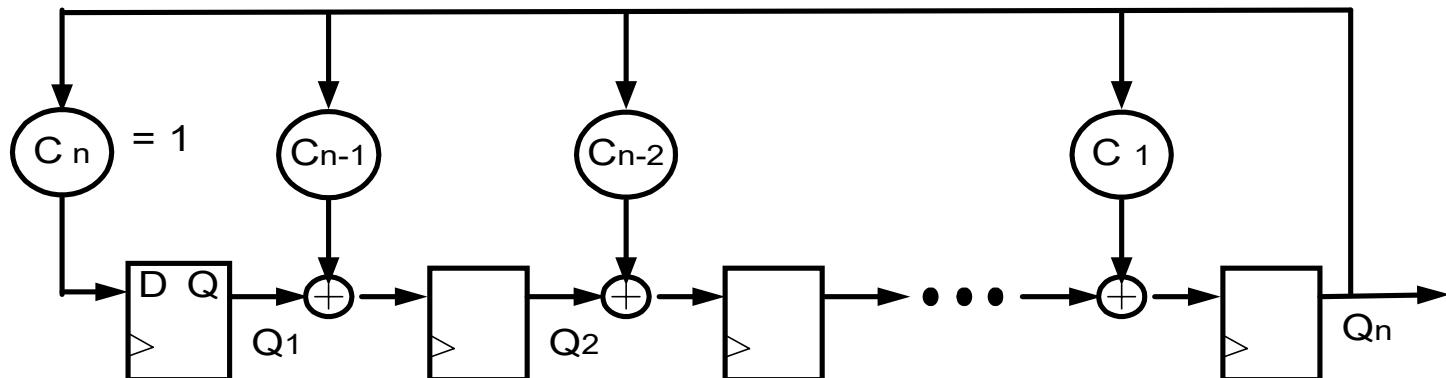
$Z = 1\ 1\ 0\ 0\ 1\ 0\ 1\ \dots$   
7 states

# Two Types of LFSRs

- Type 1: External type



- Type 2: Internal type



# Mathematical Operations over GF(2)

- Multiplication ( $\bullet$ )

$\bullet$	0	1
0	0	0
1	0	1

- Addition (  $\oplus$  or simply + )

$\oplus$	0	1
0	0	1
1	1	0

**Example:**

$$C_1 = 0, \quad C_2 = 1, \quad C_3 = 1$$

$$a_{-1} = 0, \quad a_{-2} = 1, \quad a_{-3} = 1,$$

**if  
then**

$$a_0 = a_{-1} \bullet C_1 + a_{-2} \bullet C_2 + a_{-3} \bullet C_3$$

$$a_0 = 0 + 1 + 1 = 0$$

# **Analysis of LFSR using Polynomial Representation**

- A sequence of binary numbers can be represented using a generation function (polynomial)
- The behavior of an LFSR is determined by its initial “seed” and its feedback coefficients, both can be represented by polynomials.

# Characteristic Polynomials

- $a_0, a_1, \dots, a_m, \dots$  : **sequence of binary numbers.**
- **Generating function :**  $G(x) = a_0 + a_1x + a_1x^2 + \dots + a_mx^m + \dots = \sum_{m=0}^{\infty} a_mx^m$
- Let  $\{a_m\} = a_0, a_1, \dots, a_m, \dots$   
**be output sequence of an LFSR of type1**

$$\Rightarrow a_m = \sum_{i=1}^n C_i * a_{m-i}$$

- Let initial state be  $a_{-1}, a_{-2}, \dots, a_{-n}$

$$\begin{aligned}
 \rightarrow G(x) &= \sum_{m=0}^{\infty} a_m * x^m = \sum_{m=0}^{\infty} \sum_{i=1}^n C_i * a_{m-i} * x^m \\
 &= \sum_{i=1}^n C_i * x^i \sum_{m=0}^{\infty} C_i * a_{m-i} * x^{m-i} && G(x) \\
 &= \sum_{i=1}^n C_i * x^i [a_{-i} * x^{-i} + \dots + a_{-1} * x^{-1} + \sum_{m=0}^{\infty} a_m * x^m] \\
 &\rightarrow G(x) = \frac{\sum_{i=1}^n C_i * x^i (a_{-i} * x^{-i} + \dots + a_{-1} * x^{-1})}{1 + \sum_{i=1}^n C_i * x^i}
 \end{aligned}$$

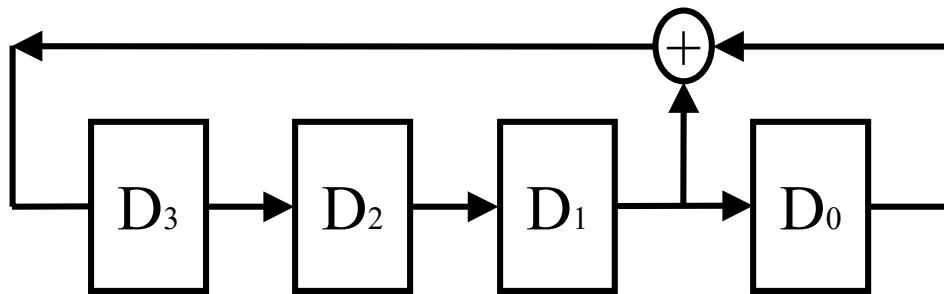
$\therefore G(x)$  depends on initial state and feedback coefficients

# Denominator

$$P(x) = 1 + c_1 * x + c_2 * x^2 + \dots + c_n * x^n$$

is called the characteristic polynomial of the LFSR

Example:



$$P(x) = 1 + x^3 + x^4$$

# LFSR Theory

- **Definition:** If period  $p$  of sequence generated by an LFSR is  $2^n - 1$ , then it is a maximum length sequence
- **Definition:** The characteristic polynomial associated with a maximum length sequence is a primitive polynomial
- **Theorem:** # of primitive polynomials for an  $n$ -stage LFSR is given by

$$\lambda_2(n) = \phi(2^n - 1) / n$$

where

$$\phi(n) = n * \prod_{P|n} \left(1 - \frac{1}{P}\right)$$

# Primitive Polynomial

- # primitive polynomials of degree n
- Some primitive polynomials

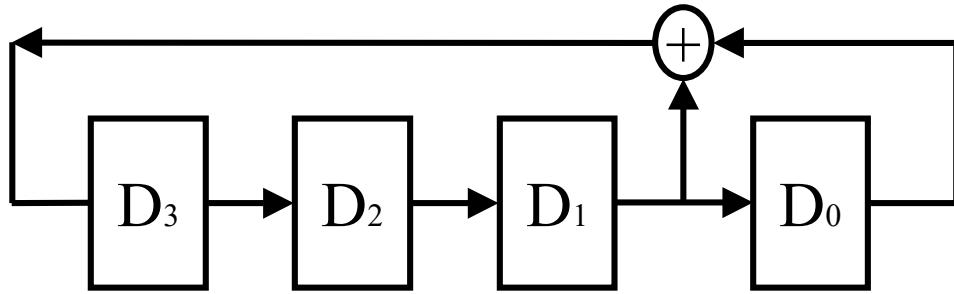
N	$\lambda_2(n)$
1	1
2	1
4	2
8	16
16	2048
32	67108864

1: 0	13: 4 3 1 0	25: 3 0
2: 1 0	14: 12 11 1 0	26: 8 7 1 0
3: 1 0	15: 1 0	27: 8 7 1 0
4: 1 0	16: 5 3 2 0	28: 3 0
5: 2 0	17: 3 0	29: 2 0
6: 1 0	18: 7 0	30: 16 15 1 0
7: 1 0	19: 6 5 1 0	31: 3 0
8: 6 5 1 0	20: 3 0	32: 28 27 1 0
9: 4 0	21: 2 0	33: 13 0
10: 3 0	22: 1 0	34: 15 14 1 0
11: 2 0	23: 5 0	35: 2 0
12: 7 4 3 0	24: 4 3 1 0	36: 11 0

# Primitive Polynomial (Cont.)

- Characteristic of maximum-length sequence:
  - Pseudorandom though deterministic and periodic
  - $\# \text{ 1's} = \# \text{ 0's} + 1$
- ⇒ Can be used as a (pseudo)-random or exhaustive number generator.

# LFSR Example



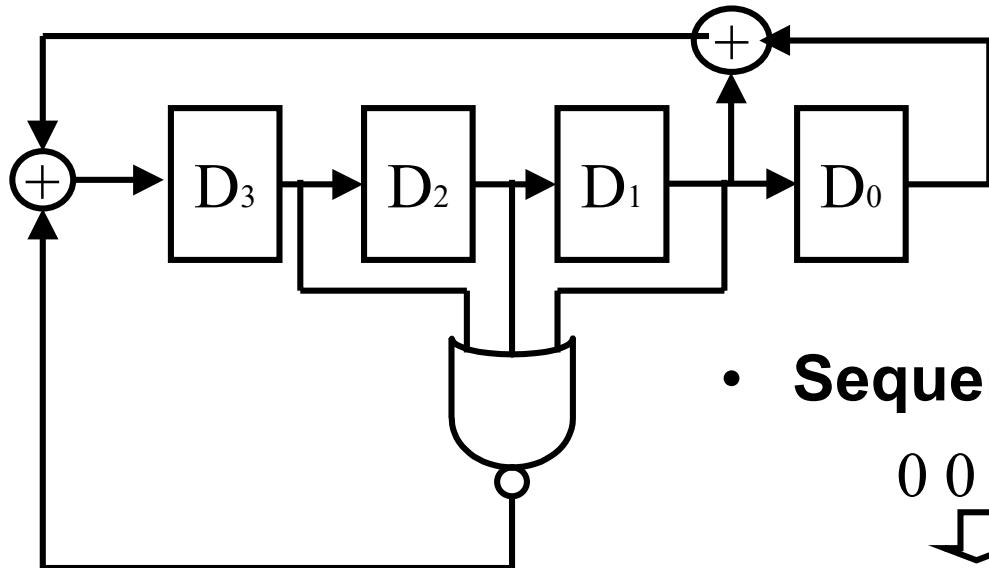
$$P(x) = 1 + x^3 + x^4$$

D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
1	0	0	0
0	0	0	1
0	0	1	1
0	1	1	1
1	1	1	1
1	1	1	0
1	1	0	1
1	0	1	0
0	1	0	1
1	0	1	1
0	1	1	0
1	1	0	0
1	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0 (repeat)

- $2^4 - 1 = 15$  “near” complete patterns are generated

# LFSR Example (Cont.)

- To generate  $2^n$  patterns using LFSR



- Sequence becomes:

0 0 0 1



0 0 0 0



1 0 0 0



⋮

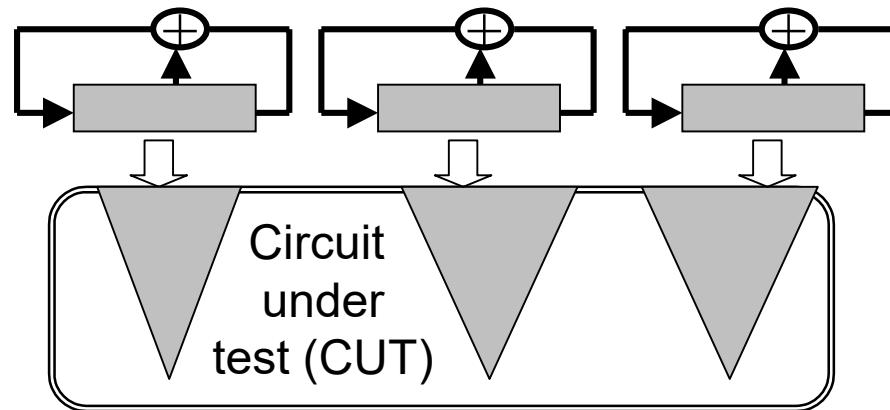
0 0 0 1

0 0 0 0

# What to Do if $2^n$ is too Large?

- Using “pseudorandom”  
e.g. generate  $2^{32}$  pattern only

- Partitioning



- Using pseudo-exhaustive

# Constant Weight Patterns (for pseudoexhaustive testing)

- $T$ , set of binary  $n$ -tuples, exhaustively covers all  $k$ -subspaces if for all subsets of  $k$  bits positions, each of the  $2^k$  binary patterns appears at least once in  $T$ , where
- Example:

$$T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} k \leq n \\ n = 3 \\ |T| = 4 \\ k = 2 \end{array}$$

⇒  $T$  can be a pseudoexhaustive test for a  $(n,w)$ -CUT if  $k \geq n$

# Constant Weight Patterns (Cont.)

- A binary  $n$ -tuple has weight  $k$  if it contains exactly  $k$  1's

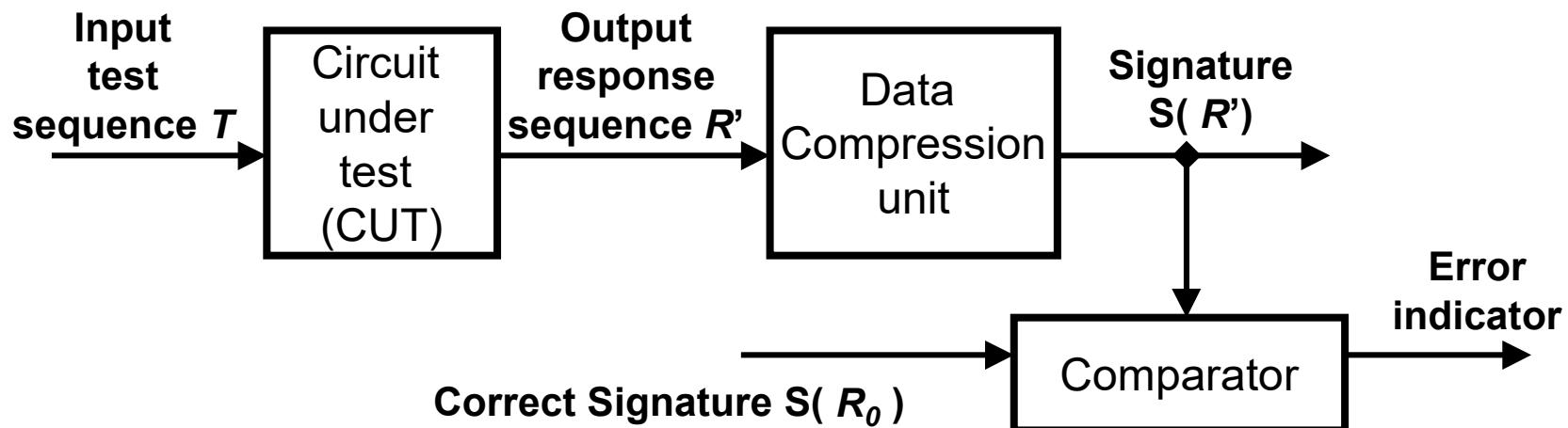
→ There are  $C_k^n$  binary  $n$ -tuples having weight  $k$ .

- Theorem: Given  $n$  and  $k$ ,  $T$  exhaustively covers all binary  $k$ -subspaces if it contains all binary  $n$ -tuples of weight(s)  $w$  such that  $w \equiv c \pmod{n-k+1}$  for some integer  $c$ , where

$$\begin{cases} 0 \leq c \leq n - k, \\ 0 \leq w \leq n \end{cases}$$

# Compression Techniques

- Bit-by-bit comparison is inefficient
- Compress information into “signature”
  - ➡ Response compacting or compressing



# **Compression Techniques to be Discussed**

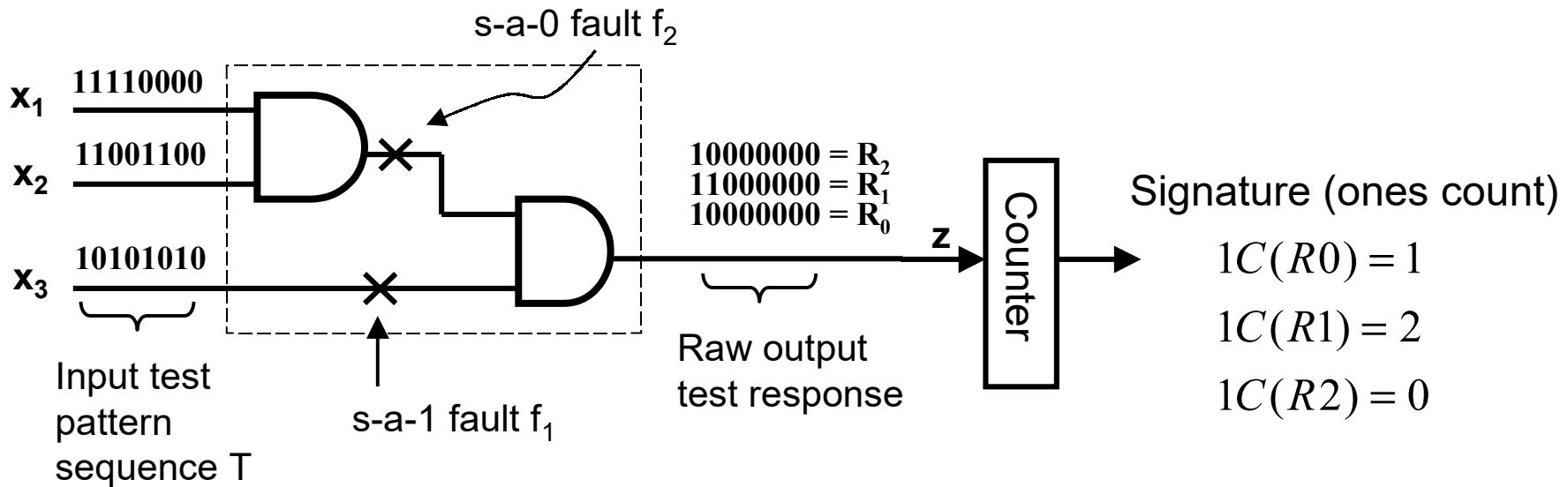
- **Ones Count**
- **Transition Count**
- **Parity Checking**
- **Syndrome Checking**
- **Signature Analysis**

# Practical Compression Techniques

- Easy to implement, part of BIST logic
- Small performance degradation
- High degree of compaction
- No or small aliasing errors
- Problems:
  1. Aliasing error:  
**signature (faulty ckt) = signature (fault-free ckt)**
  2. Reference (good) signature calculation:
    - ⇒ Simulation
    - ⇒ Golden unit
    - ⇒ Fault tolerant

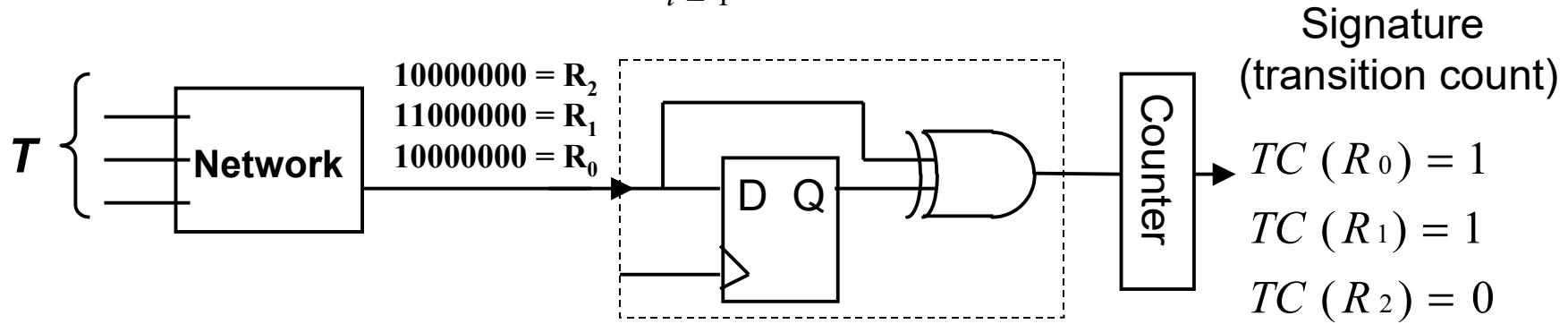
# Ones-count Compression

- **C: single-output circuit**
- **R: output response**  $R = r_1r_2\dots r_m$
- **1C(R) = # ones in**  $R = \sum_i r_i$



# Transition-count Compression

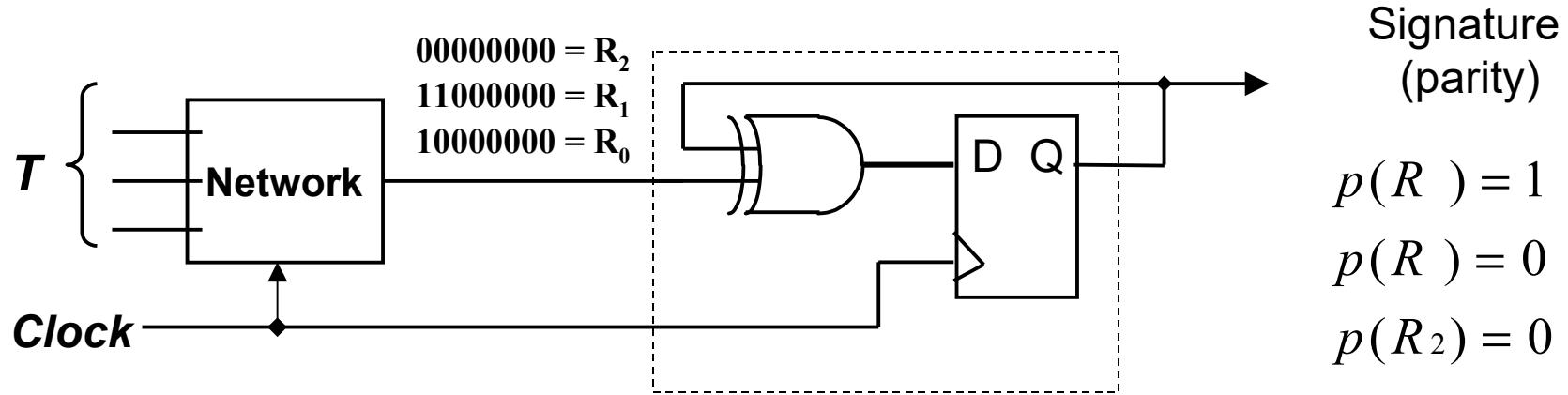
$$TC(R) = \sum_{i=1}^m r_i \oplus r_{i+1}$$



- $TC(|, ) = \frac{2C_r^{m-1} - 1}{2^m - 1}$

- Does not guarantee the detection of single-bit errors
- Prob. (single-bit error masked)  $= \frac{m-2}{2^m}$
- Prob. (masking error)  $\rightarrow (\pi m)^{-\frac{1}{2}}$

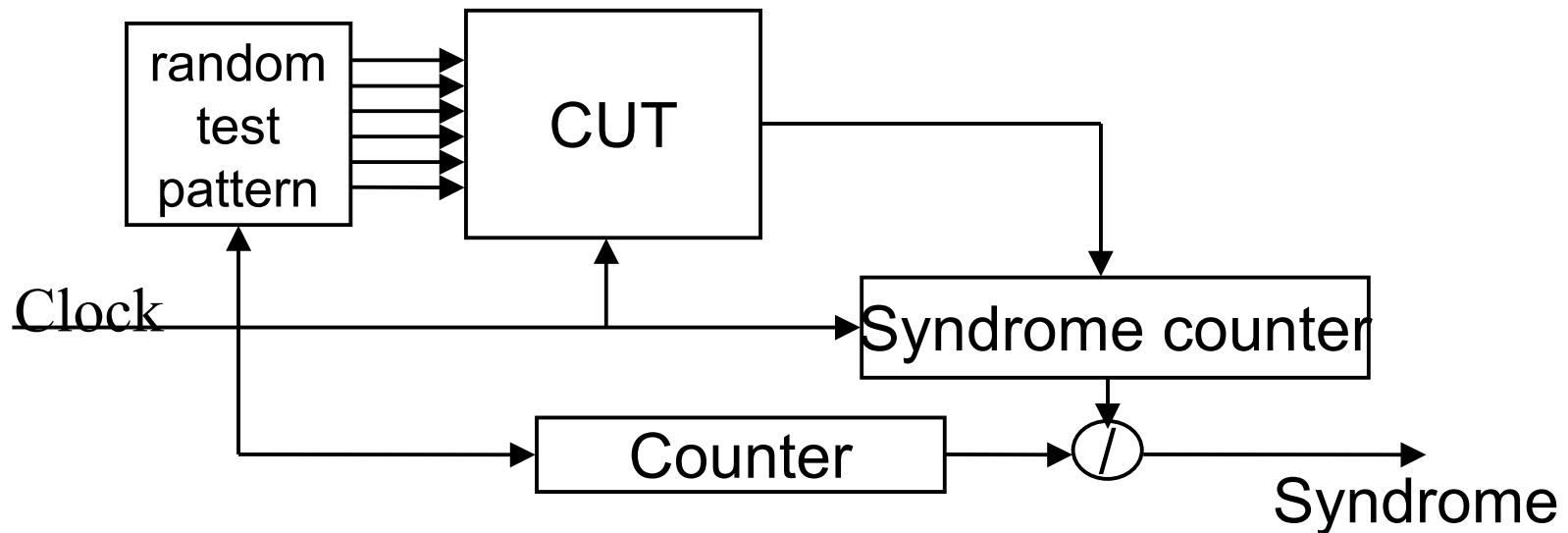
# Parity-check Compression



- Detect all single bit errors
- All errors consisting of odd number of bit errors are detected
- All errors consisting of even number of bit errors are masked
- Prob. (error masked)  $\approx \frac{1}{2}$

# Syndrome Checking

- Apply random patterns.
- Count the probability of 1.
- The property is similar to that of ones count.



# Signature Analysis

- Based on linear feedback shift register (LFSR)
- A linear ckt. composed of
  - ⇒ unit delay or D FFs
  - ⇒ Modulo-2 scalar multipliers or adders

+/-	0	1
0	0	1
1	1	0

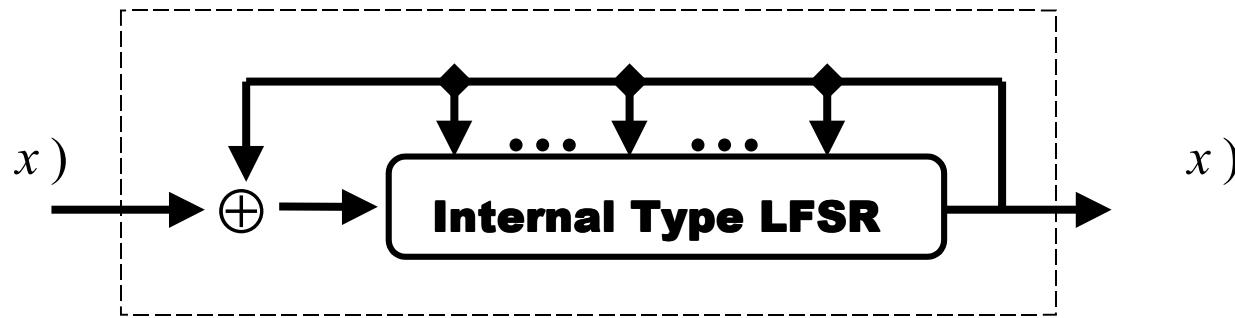
*	0	1
0	0	0
1	0	1

- Linear ckt.:  
**Response of linear combination of inputs**  
= **Linear combination of responses of individual inputs**

$$I_1 + I_2) = (I_1) + (I_2)$$

# Use LFSR as Signature Analyzer

- Single-input LFSR



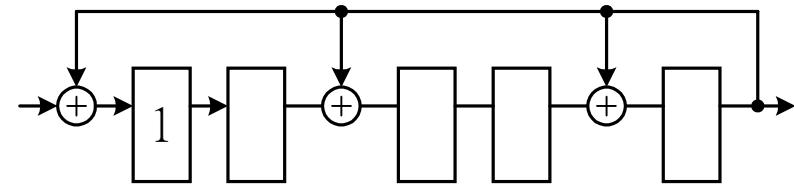
- Initial state  $x = 0$
- Final state  $x)$   
 $\Rightarrow \frac{x)}{x)} = x) + \frac{x)}{x)} \text{ or } x) = x) - x) + x)$   
 $x)$  : The remainder, or the signature

# Signature Analyzer (SA)

**Input sequence:**

1 1 1 1 0 1 0 1 (8 bits)

$$G(x) = x^7 + x^6 + x^5 + x^4 + x^2 + 1$$



$$P(x) = 1 + x^2 + x^4 + x^5$$

Time	Input stream	Register contents	Output stream
------	--------------	-------------------	---------------

0	1 0 1 0 1 1 1	1 2 3 4 5 0 0 0 0 0	Initial state
1	1 0 1 0 1 1 1	1 0 0 0 0	
:	:	:	
5	1 0 1	0 1 1 1 1	
6	1 0	0 0 0 1 0	1
7	1	0 0 0 0 1	0 1
8	Remainder	→ 0 0 1 0 1	1 0 1
			Quotient
	Remainder	$R(x) = x^7 + x^4$	$Q(x) = 1 + x^2$

# Signature Analyzer (SA) (Cont.)

$$x \quad x + x + x^2 + 1$$

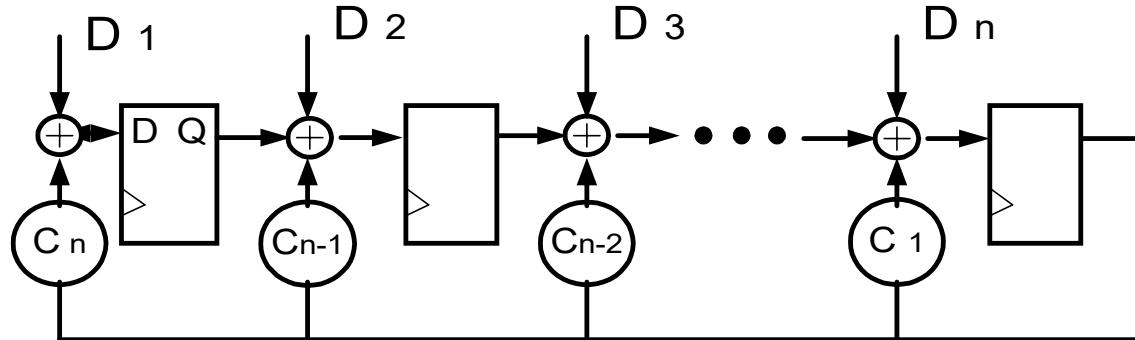
$$x \quad x^2 + 1$$

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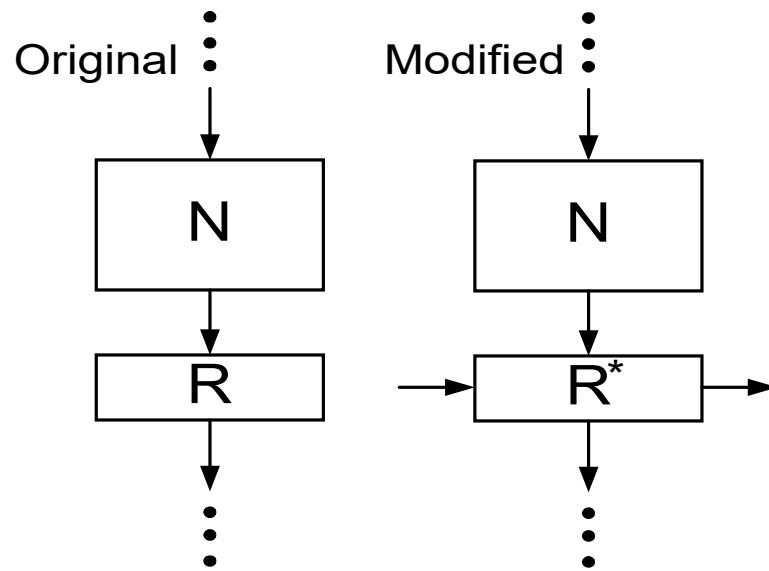
$$= \quad x + x + x^5 + 1$$

$$x) \quad x) + x) = x + x + x + x + x^2 + 1 = x)$$

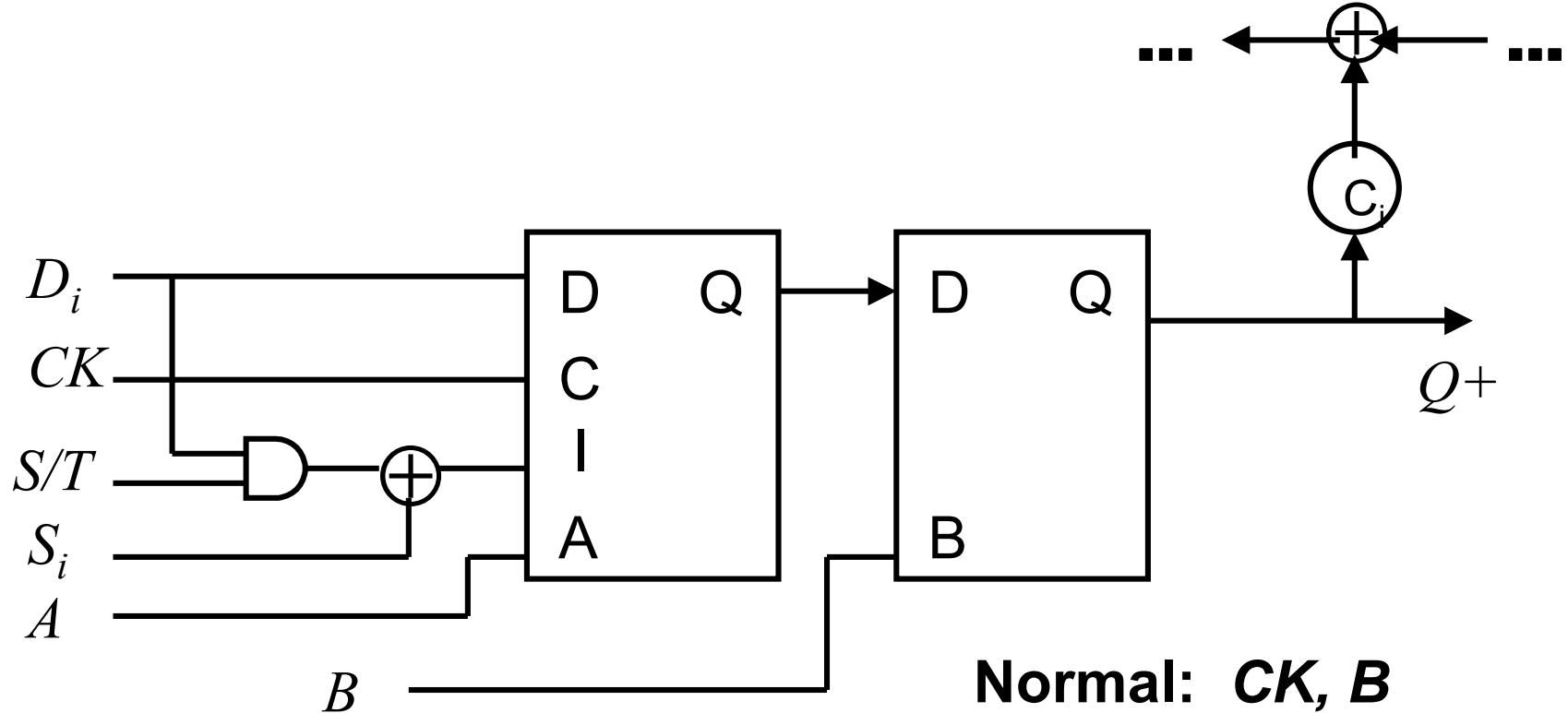
# Multiple-input Signature Register (MISR)



- **Implementation:**



# Storage Cell of a SA



**Normal:**  $CK, B$   
**Shift:**  $S/T=0, A, B$   
**MISR:**  $S/T=1, A, B$

# Performance of Signature Analyzer

- For a test bit stream of length  $m$  :  
**# possible response =  $2^m$ , of which only one is correct**
- The number of bit stream producing a specific signature is

$$\frac{2}{2^n} = 2^{-n}$$

# Performance of Signature Analyzer (Cont.)

- Among these stream , only one is correct.

$$SA(\quad | \quad, n) = \frac{2^{m-n} - 1}{2^m - 1} \cong 2^{-n}$$

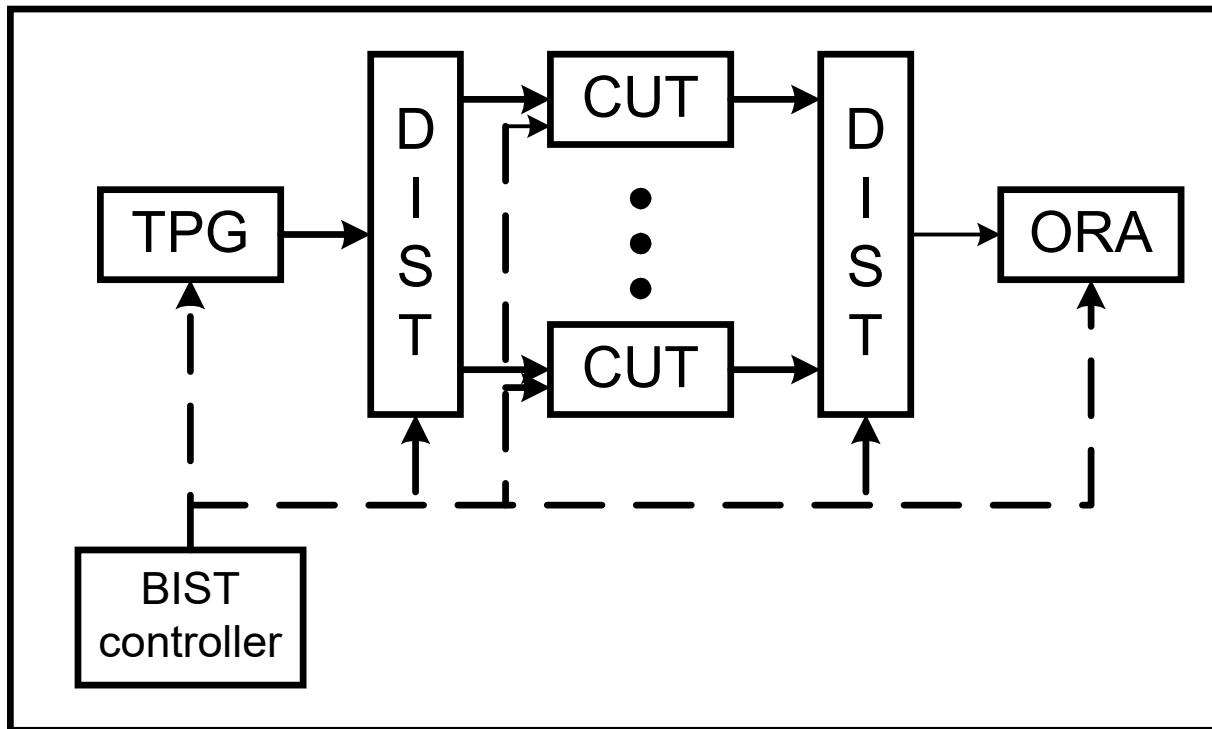
- If  $n=16$ , then  $1 - 2^{-16}$  ) 1 % = 99.99%  
of erroneous response are detected.  
*(Note that this is not % of faults !)*

# **Generic Off-line BIST Architecture**

- **Categories of architectures**
  - Centralized or Distributed
  - Embedded or Separate BIST elements
- **Key elements in BIST architecture**
  - Circuit under test (CUTs)
  - Test pattern generators (TPGs)
  - Output-response analyzers (ORAs)
  - Distribution system for data transmission between TPGs, CUTs and ORAs
  - BIST controllers

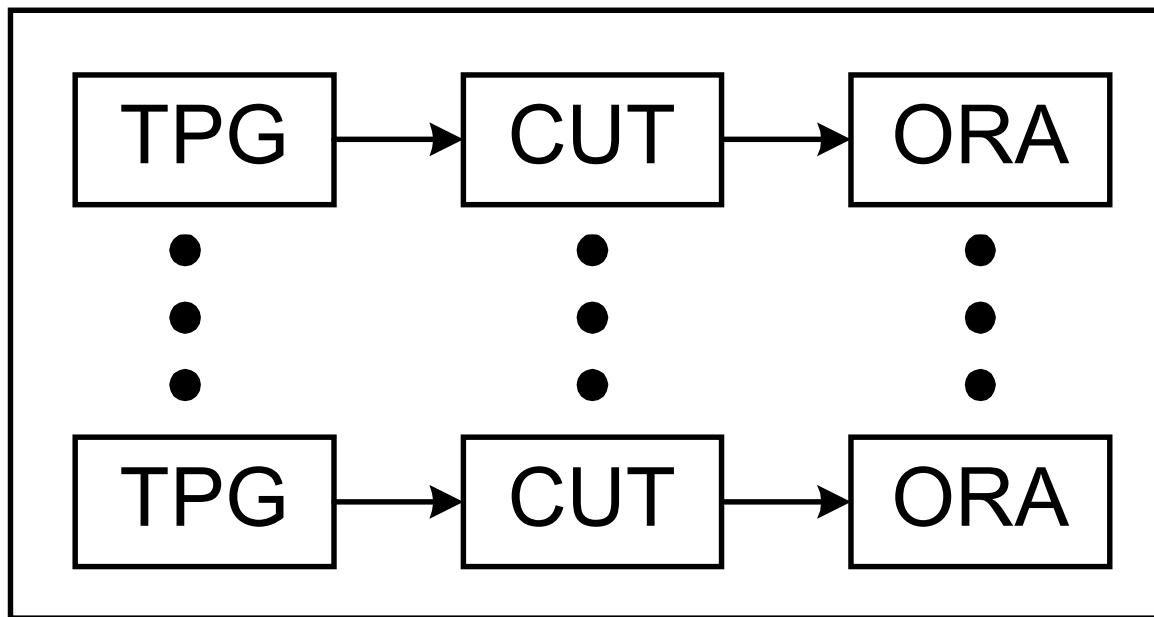
# Centralized/ Separate BIST

Chip, board, or system



# Distributed / Separate BIST

Chip, board, or system



# Distributed / Embedded BIST

Chip, board, or system

