Optics

Interference, Diffraction, Polarization

prepared by Shanku

December 27, 2024

Huygens Principal

According to Huygens, a light source in a *homogeneous isotropic medium* sends out light in every direction & these waves travel with equal velocity to carry energy with them to be transmitted in all directions.

1. Every point on a given wavefront may be regarded as the source of a new disturbance, called **secondary wavelets.**
2. The secondary wavelets(spherical) from each point spread out in all directions with the velocity of light.
3. The envelope of these wavelets in the forward direction at any instance constitutes the new wave front at the instance.

Wavefront- defined as a *surface* on which the phase of the disturbance is the same at any given instant of time. (two types = spherical /cylindrical)

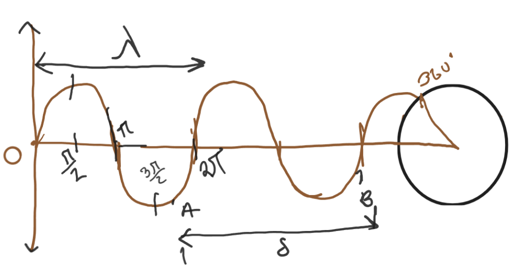
Huygens Principal can

- Refraction & reflection. Double Refraction in crystals.
- Explain polarization phenomena.

Huygens Principal cannot

- Geometrical shadow theory.
- Diffraction interference.

Path Difference vs Phase Difference



Suppose 2 points $A\, and\, B$

path difference $OB - OA = \delta$

let, Phase difference $= \theta$

we know if the difference between two waves equals to its wavelength(λ) then $\theta = 2\pi\,[360°]$

for path distance λ phase diff $= 2\pi$

for path distance δ phase diff $= \dfrac{2\pi}{\lambda}\delta$

so, $\boxed{\theta = 2\pi\delta/\lambda}$

Coherent Source: the phase diff between the interfering waves must be zero or constant.

Interference

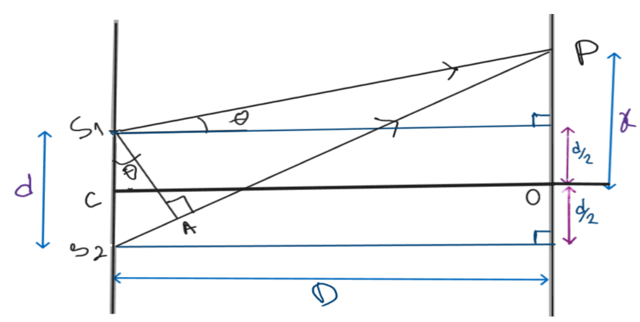
Superposition stats that.. the resultant disturbance of two or more waves acting on the same point simultaneously is the vector sum of the individual waves(disturbances).

Let, $y_1 = a\sin\omega t$, and $y_2 = a\sin(\omega t + \theta)$,
so, resultant will,

$$\begin{aligned} y =& y_1 + y_2 \\ =& a[\sin\omega t + \sin(\omega t + \theta)] \\ =& 2a\sin(\omega t + \frac{\theta}{2})\cos\frac{\theta}{2} \\ =& A\sin(\omega t + \frac{\theta}{2}) \end{aligned}$$

Interference of light - the phenomenon where *monochromatic* light waves coming from two or more *coherent sources* Superimpose, This leads to regions of constructive interference (brighter) and destructive interference (darker), creating an interference pattern.

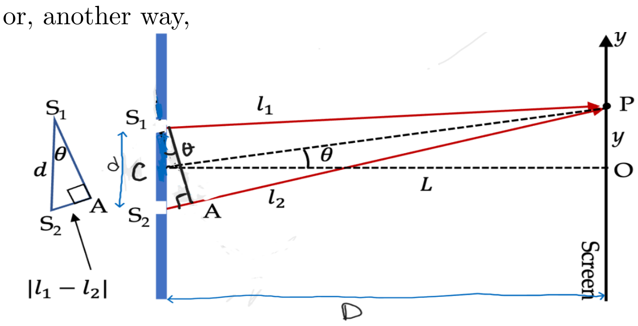
Young's Double slit Experiment



$$l_1^2 = D^2 + (x - d/2)^2$$
$$l_2^2 = D^2 + (x + d/2)^2$$
so, $(l_1 + l_2)(l_1 - l_2) = (x - d/2)^2 - (x + d/2)^2$
$\implies (l_1 - l_2)\,2D = 2xd$
$\implies l_1 - l_2 = path\,difference,$

$$\therefore\ \boxed{\delta = \mathbf{x_n d/D}}$$

or, another way,



$S_2A = \delta = S_1P - S_2P = l_1 - l_2, \quad (D = CP)$
and in $\triangle S_1AS_2, \quad \sin\theta = S_2A/S_1S_2$
or, $\quad S_2A = \delta = d\sin\theta$
$P\,\&\,O$ are very close when, $S_1P \approx S_2P \approx D$
as $[d << D]$ and y is very small
then, $\theta = \angle S_2S_1A = \angle PCO$

$$\therefore \quad \delta = d\sin\theta = \frac{dy}{PC} \approx \frac{dy}{D}\ [if,\ S_1S_2 = 2d \implies \frac{2dy}{D}]$$

now, *from principle of interference we know, resultant amplitude*

$$A = 2a\cos\left(\frac{\theta}{2}\right) = 2a\cos\left(\frac{2\pi\delta}{2\lambda}\right)$$

for bright fringe,

$A = max \implies 2a\cos(\theta/2) = 1$

so, $\theta = n\pi$ and $\delta = n\lambda$

for dark fringe,

$A = min \implies 2a\cos(\theta/2) = 0$

so, $\theta = (2n+1)\pi$ and $\delta = (2n+1)\lambda$

where $[n = 0, 1, 2, 3, 4 \ldots]$

Determination of fringe width:

spacing between nth & $(n+1)th$ fringe will be,

$$x_n - x_{n-1} = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$x_n - x_{n-1} = \boxed{\beta = \frac{\lambda D}{d}}$$

Conditions for sustained interference:

- must be coherent(same λ or θ), so we **can't use two real sources** they can't be coherent.(also $a_1 \neq a_2$)
- same frequency(f) or wavelength(λ). **so,** if we cover slits with different transparent color paper that's lead to $\lambda_a \neq \lambda_b$
- if Polarized, then must be in same state of Polarization
- medium matters cause in water, $\beta' = \frac{D\lambda'}{d} = \beta' = \frac{D}{d}\frac{\lambda}{\mu}$ so, $\beta' < \beta$,means width will reduced.

Conditions for good observation:

- d must be small, **but if** $d < \lambda$ then β is extremely large, lead to an **uniform illumination** got **no visible fringes**. $[\beta \propto (\lambda/d)]$
- D should be relatively large. $\beta \propto D$
- Background should be darker.

Conditions for good contrast:

- amplitudes should be equal or very nearly equal $I_m \propto (a_1 \pm a_2)^2$ so, $a_1 \approxeq a_2$
- the sources must be narrow
- sources must be monochromatic or very nearly so.

Conservation of energy in interference: In case of constructive interference, $I = I_{max}$ and bright fringes are formed in the screen. Whereas in case of destructive interference, $I = I_{min}$, dark fringes are formed.

This implies that in interference and diffraction pattern, the intensity of light is simply being redistributed **i.e. energy is only transferred from dark to bright fringe and no energy is created or destroyed in the process.**

$$I_{\text{total}} = I_1 + I_2$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

Diffraction

The phenomenon of bending of light waves around obstacles or (aperture of sizes comparable with the wavelengths of light) & resulting thereby in their spreading in *Geometrical shadow* of the object is known as Diffraction.

It is a fundamental phenomenon in wave physics that occurs with all types of waves, including sound waves, electromagnetic waves (such as visible light, X-rays, and radio waves), and even matter waves (such as electrons and neutrons).

Types of diffraction

1. **Fresnel type** the source and the screen *(i.e. the point of observation)* or both are at finite distance from the obstacle or the aperture.
2. **Fraunhofer type** both source and screen are *at* ∞ *distance* from each other.

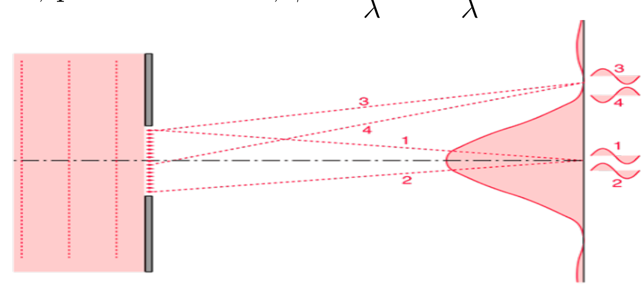
Analytical treatment

path difference, $\delta = l_1 - l_2$,

and in $\triangle S_1AS_2$, $\sin\theta = S_2A/S_1S_2$

or, $\delta = a\sin\theta \quad (a = d)$

so, phase difference, $\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}a\sin\theta$



for minima,

$a\sin\theta = n\lambda \quad [n = \pm1, \pm2 \ldots]$

so, $\delta = n\lambda \quad and, \quad \phi = n\pi$

for central maxima,

$\delta = 0, \quad so, a\sin\theta = 0$

for secondary maxima,

$a\sin\theta = (2n+1)\lambda/2 \quad [n = \pm1, \pm2 \ldots]$

so, $\delta = (2n+1)\lambda/2 \quad and, \phi = (2n+1)\pi/2$

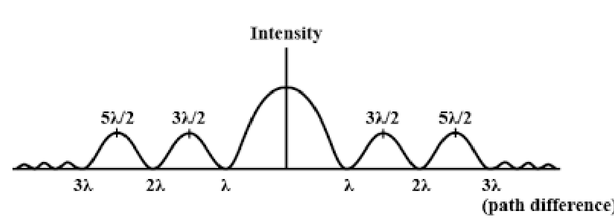
so, central fringe width,

$\sin\theta = n\lambda/d = x_n/D, \quad so, x = \frac{n\lambda D}{a}$

for, central fringe, $2x = \frac{2\lambda D}{a} \quad (n = 1)$

as θ is small so $\sin\theta = \theta$

for angular width $\theta = \lambda/a$



The key aspects of diffraction are:

- *fringewidth* $\propto \lambda$. so, for red light $(\lambda \uparrow)$ central width high$\uparrow$, same as for violet light $\downarrow$ width low $\downarrow$.
- if a $\downarrow$ then x $\uparrow$

- if monochromatic light used then we get a good contrast.

Interference vs Diffraction

1. **interaction occurs between:** In interference , it's two separate wave-fronts originating from two coherent source
In diffraction, it's among secondary wavelets originating from different points of the exposed part of the same wavefront.

2. **The widths:** fringes in interference may or may not be equal.
But in diffraction they will never equal.

3. **The regions of minimum intensity:** In interference they are perfectly dark.
But not so in case of diffraction.

4. **maxima:** are of uniform intensity in interference.
But the bright bands are of varying intensities in diffraction.

Polarization

Polarization is a property of transverse waves which specifies the geometrical orientation of the oscillations.

In a transverse wave, *the direction of the oscillation is perpendicular to the direction of motion of the wave.*

light is a transverse electromagnetic wave consisting of vibrating electric and magnetic field. ***the dominating electric vector is defined as light vector.***

Unpolarized light: light waves vibrating in all possible directions. *perpendicular to the direction of propagation.*

Polarized light: light waves that are oriented in a specific direction, meaning the light waves vibrate in a particular plane.

Plane of vibration: polarized light-waves vibrates in this plane.

Plane of polarization: plane perpendicular to the plane of vibration.

Isotropic medium: the physical properties of light are same in all direction.*[glass,water]*

Anisotropic substances: particularly crystals (except those with cubic symmetry) the physical properties are different in different direction.
[calcite, quarts, tourmaline]

O-Ray: obey laws of refraction.

E-Ray: extraordinary ray, doesn't obey laws.

Types of Polarization

1. **Linear polarization:** The electric field of the light waves oscillates in a single plane.
2. **Circular polarized light-** produced by *two plane polarized light* vibrating perpendicular to each-other; where, amplitude and frequency equal but phase differ by $\pi/2$.
3. **Elliptically polarized light-** produced if the amplitude and phase differ in previous case.

Production of plane polarized light

By– (1)reflection, (2)refraction, (3)double-refraction & (4)dichroism.

Brewster's Law

the *tangent of angle of polarization* equals the *refractive index* of refractive medium.

$$\boxed{\mu = \tan i_p}$$

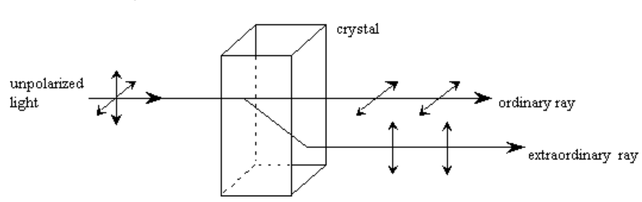
- angle of polarization **depends on** wave-length of light-wave.
- when light is incident at polarized angle, the *plane of vibration* being at right angles to the *plane of incidence.*

let take, incident angle i_p is angle of polarization & r is the angle of refraction.
from Brewster's law, $\tan i_p = \mu$
also from snail's law, $\mu = \sin i_p / \sin r$
so we get,

$$\tan i_p = \frac{\sin i_p}{\sin r} \implies \cos i_p = \sin r$$
$$\implies i_p = 90^\circ - r$$

so, the reflected beam is perpendicular to refractive beam ($i_p + r = \pi/2$).

Double Refraction



The optical phenomenon of splitting of a single incident ray of non-polarized light into two refracted E-rays and O-rays(vibrates in principal section) when passing through anisotropic substances known as Double Refraction.

Dichroism: Polaroids

The selective absorption Some doubly refractive crystals absorbs one of O/E ray strongly & allows other to pass by. (tourmaline)

When a film of *polyvinyl alcohol heated & stretched* 3-5 times it's original length, then it's molecules orients with long axis along the direction of stress. Then the film is *saturated with iodine* = **H-Polaroids.**

when the stretched polyvinyl alcohol heated with a catalyst(HCL) instead = **K-Polaroid.**