A Review Note on Digital twins of nonlinear dynamical systems

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1 Overview

1.1 INTRODUCTION

- Digital twins: through mathematical modeling or data.
- Reservoir Computers: self-dynamical evolution with memory / no back-propagation.
- **novelty**: ”Introducing a control mechanism into the RC structure with an exoge- nous control signal acting directly onto the RC network distinguishes our work from existing ones in the litera- ture of RC as applied to nonlinear dynamical systems.”
- **Purpose**: ”Of particular interest is whether the collapse of the tar- get chaotic system can be anticipated from the digital twin. The purpose of this paper is to demonstrate that the digital twin so created can accurately produce the bifurcation diagram of the target system and faithfully mimic its dynamical evolution from a statistical point of view. The digital twin can then be used to monitor the present and future “health” of the system. More impor- tantly, with proper training from observational data the twin can reliably anticipate system collapses, providing early warnings of potentially catastrophic failures of the system”
- **Targets**:
  1. extrapolation of the dynamical evolution of the target system into certain “uncharted territories” in the param- eter space
  2. long-term continual forecasting of nonlin- ear dynamical systems subject to non-stationary external driving with sparse state updates
  3. inference of hidden variables in the system and accurate prediction of their dynamical evolution into the future
  4. adaptation to external driving of different waveform
  5. extrapo-lation of the global bifurcation behaviors of network sys-tems to some different sizes.

1.2 METHODS

- Digital twins: a recurrent RC neural network with a control mechanism, which requires two types of input signals: the observational time series for training and the con-trol signal $f(t)$ that remains in both the training and self-evolving phase.
  - During the train-ing, the hidden recurrent layer is driven by both the in- put signal $u(t)$ and the control signal $f(t)$.
- Reservoir updating equations:

$$\begin{aligned}\text{training phase: } \mathbf{r}(t+\Delta t) &= (1-\alpha)\mathbf{r}(t) + \alpha \tanh\left[\mathcal{W}_r \mathbf{r}(t) + \mathcal{W}_{\text{in}}\, \mathbf{u}(t) + \mathcal{W}_c f(t)\right] \\ \text{self-evolving phase: } \mathbf{r}(t+\Delta t) &= (1-\alpha)\mathbf{r}(t) + \alpha \tanh\left[\mathcal{W}_r \mathbf{r}(t) + \mathcal{W}_{\text{in}}\, \mathcal{W}_{\text{out}}\, \mathbf{r}'(t) + \mathcal{W}_c f(t)\right]\end{aligned}$$

- **"sense, learn, andmingle"**: During the training, several trials of data are typically used un-der different driving signals so that the digital twin can the responses of the target sys-tem to gain the ability to extrapolate a response to a new driving signal that has never been encountered before. We input these trials of training data, i.e., a few pairs of $\mathbf{u}(t)$ and the associated $f(t)$, through the matrices $\mathcal{W}_{\text{in}}$ and $\mathcal{W}_c$ sequentially.
- **validation/testing**: the validation of the RC net-works are done with the same driving signals $f(t)$ as in the training data. We test driving signals $f(t)$ that are different from those generating the training data (e.g., with different amplitude, frequency, or waveform).
- **warming − up** During the warming-up process to initialize the RC networks prior to making the predic-tions, we feed randomly chosen short segments of the training time series to feed into the RC network. That is, no data from the target system under the testing driving signals $f(t)$ are required for making the predic- tions.

1.3 RESULTS

1.4 DISCUSSION

1.5 APPENDIX