卒論　ノート

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概要

2023 年 A セメスターの卒論執筆に際して、勉強したことや考えたことのメモをここにまとめた。

1 Papers

1.1 Emergence of a resonance in machine learning

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勉強のためのキーワード：Resonance in nonlinear dynamical systems,

1.1.1 要旨

1. 問い：What is the physical or dynamical mechanism underlying the benet of noise and how do we nd the optimal level of noise?
2. この論文の意義：In this paper, we uncover a resonance phenomenon in which a certain amount of noise can signicantly enhance the short-term and long-term prediction accuracy and robust- ness for chaotic systems, where the optimal noise level can be found through a generalized scheme of hyperparameter
3. 難点：A challenging issue is to identify the underlying dynamical mechanism responsible for the emergence of a resonance in machine learning optimization.

1.1.2 状況設定

1.1.3 面白いと思ったところ

1.1.4 論文を受けての今後の研究方向

1.1.5 疑問点

1. Fig.4 と Fig.5 について，MG system における τ の値が 30 から 17 に変えると，σ にどのような影響があるか．なぜその影響が生まれるか.
2. なぜ，Fig.2 の（逆）ピークを与える σ 帯と Fig.6(c) の（逆）ピークを与える σ 帯が重なるのか.
3. III で，Machine learning における resonance が生まれる Physical reason を挙げているが，これは対象を正しく説明できているか．extraordinarily complicated な hidden layer の中身を解析することなく，physical

reason を与えることが，なにを説明しているのか/なにを説明していないのか.

1.1.6 関連する文献

1.1.7 用語まとめ

1.1.8 Abstract

1. stochastic/coherence resonance:
2. nonlinear dynamical system:
3. regularizer/regularization:
4. reservoir computing:
5. state variables/attractor:
6. hyperparameters:

1.1.9 I. Introduction

1. model-free/data-driven:
2. oscillatoin/Lyapnov times:
3. trajectory:
4. basin boundary:
5. robustness:
6. Baysian optimization:

1.1.10 II. Result

1. SURROGATEOPT function (MATLAB):
2. surrogate approximation function:
3. objective function:
4. global minimum:
5. sampling/updating:
6. radial basis function:
7. Mackey-Glass (MG) system:
8. spatiotemporal chaotic Kuramoto-Sivashinsky (KS) system:

1.1.11 A. Emergence of a resonance from short-term prediction

1. transient behavior:
2. z-score normalization:
3. periodic boundary condition:
4. Prediction horizon/stability:

1.1.12 B. Emergence of a resonance from long-term prediction

1. collapse:
2. wider/narrower resonance:

1.1.13 III. HEURISTIC REASON FOR THE OCCURRENCE OF A RESONANCE

1. time-scale match:
2. the mean first-passage time:
3. nonlinear activation:
4. linear reservoir computing:
5. noise-enhanced temporal regularity:
6. vector autoregressive process (VAR):

1.1.14 IV. DISCUSSION

1. magnitude:

1.1.15 Appendix A

1. recurrent neural network(RNN):
2. input/hidden/output layer:
3. linear regression:
4. adjacency matrix:
5. state vector:

6. dynamical state/evolution:
7. neuron:
8. leakage parameter α:
9. link probability p:
10. spectral radius:

2 Lectures

1. Reservoir Computing
   (a) Reservoir Computing for SDEs(Josef Teichmann:)
   (b) Reservoir Computing & Dynamical Systems - Second Sumposium on Machine Learning and Dynamical Systems(Josef Teichmann)
2. Machine Learning in general
   (a) Machine Learning in Finance(Josef Teichmann)

2.1 Josef Teichmann: Reservoir Computing for SDEs

Access from here.

1. We consider differential equations of the form

$$dY_t = \sum_i V_i(Y_t)\, du_t^i, Y_0 = y \in E$$

   to construction evolutions in state space E (could be a manifold of finite or infinite dimension) depending on local characteristics, initial value $y \in E$ and the control u.

2. Theorem (Universality) Let Evol be a smooth evolution operator on a convenient vector space E which satisfies (again the time derivative is taken with respect to the forward variable t) a controlled ordinary differential equation

$$d\mathrm{Evol}_{s,t}(x) = \sum_{i=1}^{d} V_i\left(\mathrm{Evol}_{s,t}(x)\right) du^i(t).$$

   Then for any smooth (test) function $f : E \to \mathbb{R}$ and for every $M \geq 0$ there is a time-homogenous linear $W = W(V_1, \ldots, V_d, f, M, x)$ from $\mathbb{A}_d^M$ to the real numbers $\mathbb{R}$ such that

$$f\left(\mathrm{Evol}_{s,t}(x)\right) = W\left(\pi_M\left(\mathrm{Sig}_{s,t}(1)\right)\right) + \mathcal{O}\left((t-s)^{M+1}\right)$$

   for $s \leq t$

3. Signature as universal dynamical system
   (a) This explains that any solution can be represented - up to a linear readout - by a universal reservoir, namely signature. Similar constructions can be done in regularity structures, too (branched rough

paths, etc).

(b) This is used in many instances of provable machine learning by, e.g., groups in Oxford (Harald Oberhauser, Terry Lyons, etc), and also ...

(c) ... at JP Morgan, in particular great recent work on 'Nonparametric pricing and hedging of exotic derivatives' by Terry Lyons, Sina Nejad and Imanol Perez Arribas.

(d) in contrast to reservoir computing: signature is high dimensional (i.e. infinite dimensional) and a precisely defined, non-random object.

(e) Can we approximate signature by a lower dimensional random object with similar properties?

3 TODOs

1. reservoirpy 関連
   (a) Understand and optimize ESN hyperparameters などの Tutorial ページを読む.
2. Reservoir Computing について学ぶ.
   (a) レクチャーノートなどを通じて、知識を準備する。
   (b) Github 環境を整備する。