Simulating Random Variables

Inverse Transformation Method

EE 381 - Project 4, 5 Points

Date Due: 4-21-20

Introduction: There are a variety of ways of simulating random variables, (RV). In this project we will explore one of the common methods of simulation. This is the inverse transformation method. Further, we need a context in which to discuss this method. Consequently we will introduce a RV and apply the method to it.

Exponential RV, T: This RV can be used to model the reliability of an apparatus. If the apparatus has been in use for any number of hours it is as good as a new apparatus of the same kind in regards to the amount of time remaining until the item fails. The cumulative distribution function (CDF) and the probability density function (pdf) are:

$F_T(t) = 1 - e^{-\lambda t}$ for $t \geq 0$ and $f_T(t) = \lambda e^{-\lambda t}$ for $t \geq 0$.

The inverse transformation method: We will be using a linear congruential pseudorandom number generator to provide us with a random variable uniformly distributed between zero and one. This pseudorandom number generator is provided in computer languages. We will characterize this as: The random variable U such that U is uniform on the interval $[0,1)$ or equivalently $f_U(u) = 1$ for $0 \leq u < 1$.

Then the method is based in the argument: For the CDF function F if we define the RV T by $T = F^{-1}(U)$ then the RV T has CDF F.

The application of the inverse transformation method to the exponential distribution. (In doing Monte Carlo studies it is sometimes necessary to generate a series of exponential RV's.) Let U be a uniform RV on the interval $[0,1)$. Find a transformation such that it possess an exponential distribution with mean $1/\lambda$.

The CDF $F_T(t)$ is strictly increasing on the interval $[0, \infty)$. Let $0 < u < 1$ and observe that there is a unique value of t such that $F_T(t) = u$. Thus $F_T^{-1}(u)$ for $0 < u < 1$ is well defined. In this case $F_T(t) = 1 - e^{-\lambda t} = u$ if and only if $t = -\frac{1}{\lambda}\ln(1-u) = F_T^{-1}(u)$. So, consequently given a list of random numbers that are uniformly distributed a list of random numbers that are exponentially distributed can be determined using the derived transformation.

Deliverables: Write a program in Python that simulates an exponential RV using the inverse transformation method. In addition to the code the plot(s) of the input and output are wanted.

References

Mathematical Statistic with Applications, 5th Ed. By Wackerly, Mendenhall, and Scheaffer 1995

Introduction to Probability Models, 5th Ed. By S. Ross 1993