chapter 13 Date______20___

Partial Derivative:-

Example ①

let $f(x,y) = \sqrt{y+1} + \ln(x^2-y)$

Find $f(e,0)$ and sketch the natrul domain.

Solu:-

$\Rightarrow f(x,y) = \sqrt{y+1} + \ln(x^2-y)$

at $x=e$, $y=0$

$\Rightarrow f(e,0) = \sqrt{0+1} + \ln(e^2-0)$

$\because \ln(a^n) = n\ln(a)$

$\Rightarrow f(e,0) = \sqrt{0+1} + 2\ln(e)$

$\Rightarrow f(e,0) = \sqrt{1} + 2\ln(e)$

$\Rightarrow f(e,0) = 1+2 \Rightarrow 3.$

Now,

to find the domain of f,

note that $\sqrt{y+1}$ is defined only.

when $y+1 \geq 0$

$\Rightarrow y \geq -1$ → ①

Now,

Similarly $\ln(x^2-y)$ is define.

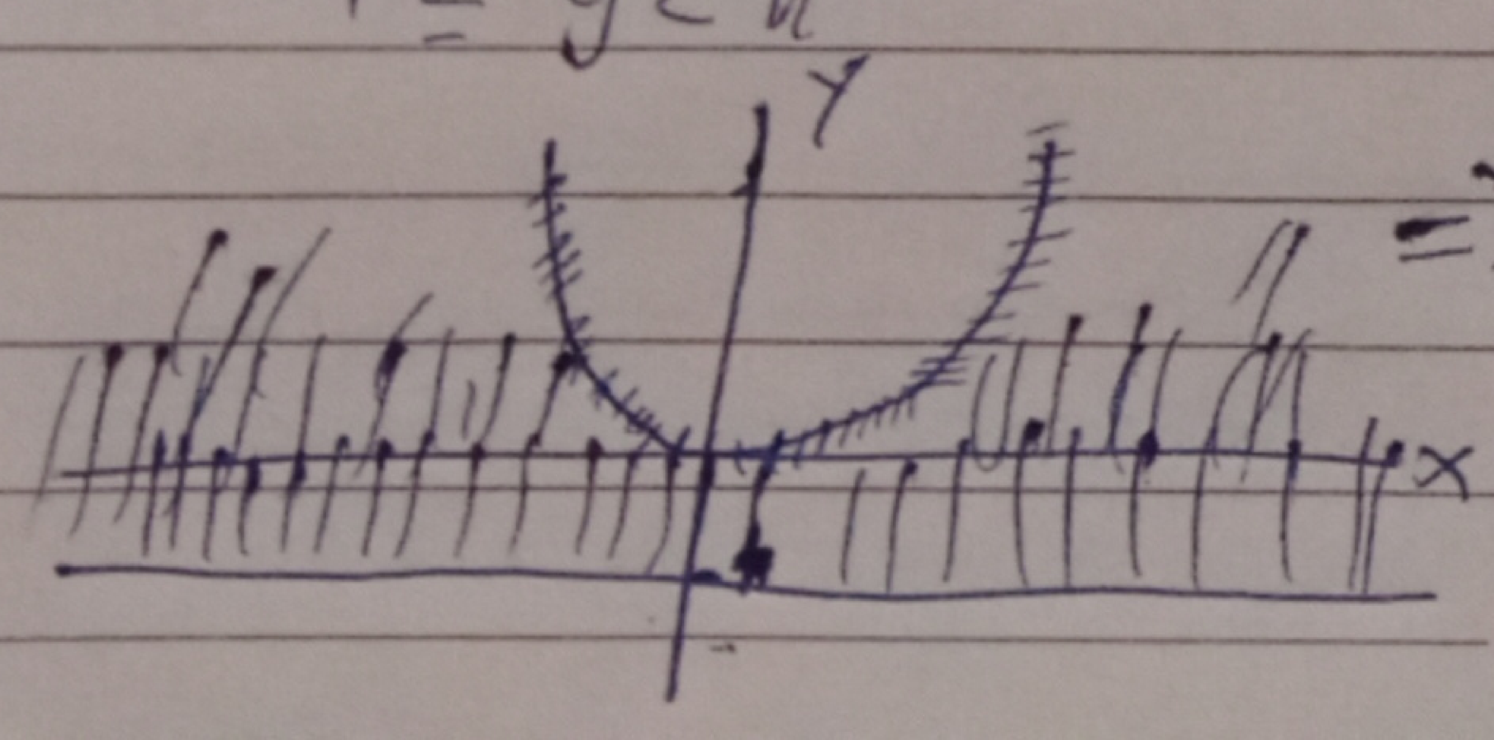
when $x^2-y > 0$

$\Rightarrow x^2 > y \Rightarrow y < x^2$ → ②

So the natural domain is from ① & ②

$-1 \leq y < x^2$

Geometrically



$\Rightarrow$ domain

and Range is

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Ex 2:

let $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$ find $f(0, \frac{1}{2}, -\frac{1}{2})$ and the natural domain of f.

$\Rightarrow$ $f(x,y,z) = f(0, \frac{1}{2}, -\frac{1}{2}) = \sqrt{1-(0)^2-(-\frac{1}{2})^2-(\frac{1}{2})^2}$

$= 1 - \frac{1}{4} + \frac{1}{4} \Rightarrow 1 - \frac{2}{4} \Rightarrow 1 - \frac{1}{2}$

$= \frac{2-1}{2} = \frac{1}{2}$

$\Rightarrow$ $f(0, \frac{1}{2}, -\frac{1}{2}) = \frac{1}{2}$

Now,

we find the domain

Since $f(x,y,z)$ is square root function

$\Rightarrow$ & $f(x,y,z)$ is define when

$\Rightarrow$ $f(x,y,z) \geq 0$

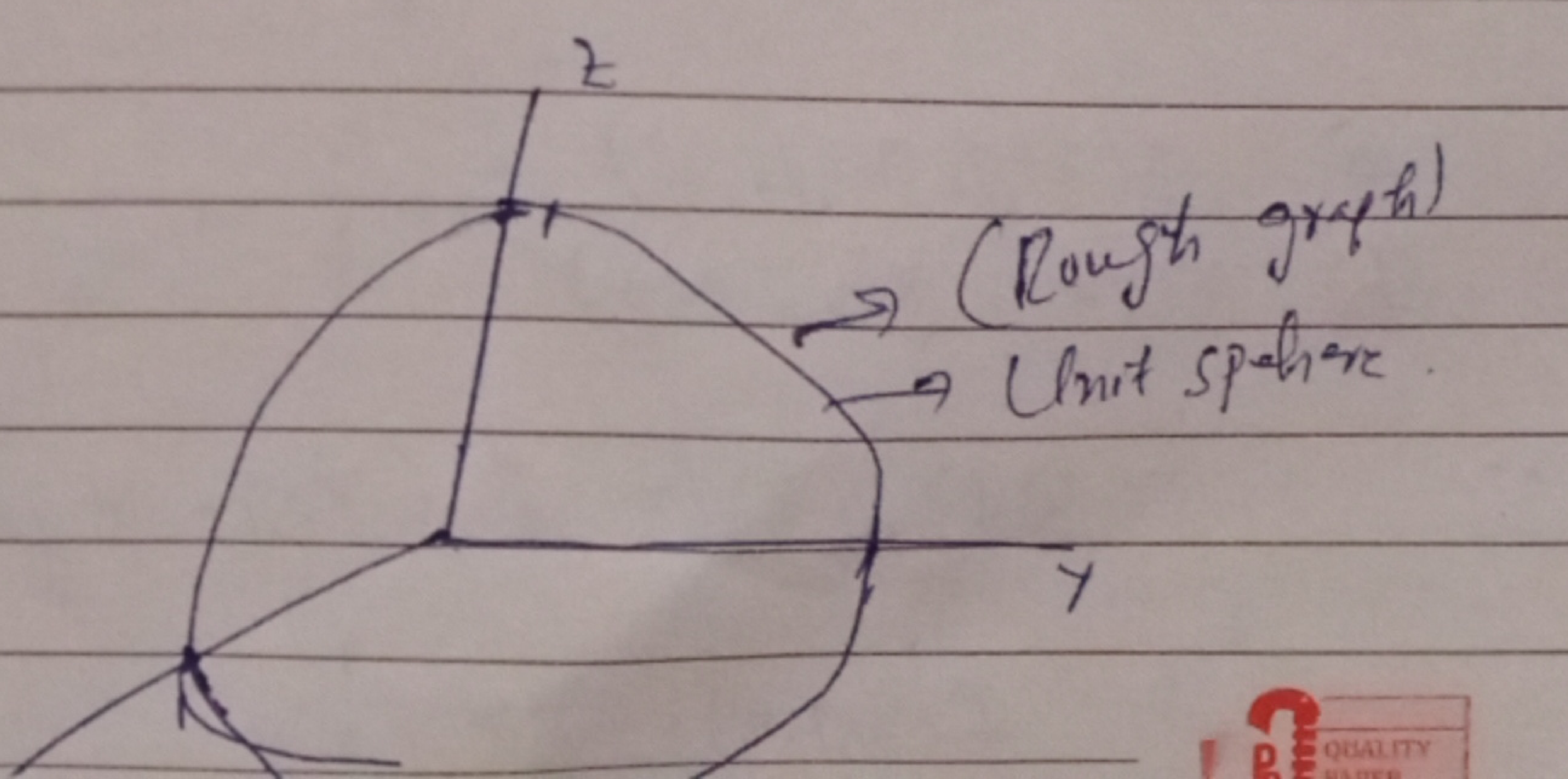
$\Rightarrow$ $1-x^2-y^2-z^2 \geq 0$

$\Rightarrow$ $-x^2-y^2-z^2 \geq -1$

$\Rightarrow$ $x^2+y^2+z^2 \leq 1$ $\rightarrow$ (1)

(1) show that the domain of $f(x,y,z)$ is the set of point of R^3 lies on or inside the unit sphere

Geometrically



Ex 13.1

(Q2):- let $(x,y) = x + \sqrt[3]{xy}$. Find

ⓐ $f(t, t^2)$. ⓑ $f(x, x^2)$ ⓒ $f(2y^2, 4y)$

ⓐ $f(x,y) = x + \sqrt[3]{xy}$.

$\Rightarrow f(t, t^2) = t + \sqrt[3]{(t)(t^2)}$

$= t + \sqrt[3]{t^3}$

$= t + (t^3)^{1/3} \Rightarrow t + t = 2t$

ⓑ $f(x, x^2) = x + \sqrt[3]{(x)(x^2)} = x + \sqrt[3]{x^3} = 2x$

ⓒ $f(x,y) = f(2y^2, 4y)$

$= 2y^2 + \sqrt[3]{(2y^2)(4y)}$

$= 2y^2 + (8y^3)^{1/3}$

$= 2y^2 + ((2y)^3)^{1/3} = 2y^2 + 2y$.

$= 2y(y+1) \Rightarrow$ Ans

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(18)

let $f(x,y,z) = zxy + x$ Find

ⓐ $f(x+y, x-y, x^2)$

ⓑ $f(xy, y/x, xz)$

ⓐ $f(x,y,z) = f(x+y, x-y, x^2)$

and $f(x,y,z) = zxy + x$.

$\Rightarrow$

$= \cancel{(x^2 y)}$

$= (x^2)(x+y)(x-y) + x+y$

$= (x^2)(x^2-y^2) + x+y.$

$= x^4 - x^2y^2 + x + y.$

$\Rightarrow$ Ans.

(b) $f(x,y,z) = (xy,\ y/x,\ xz)$

$= (x.z)(xy)(y/x) + xy$

$= \dfrac{x^2y^2z}{x} + xy$

$= xy^2z + xy \Rightarrow$ Ans.

23-26

Sketch The domain of f,

(23) $\ln(1-x^2-y^2)$

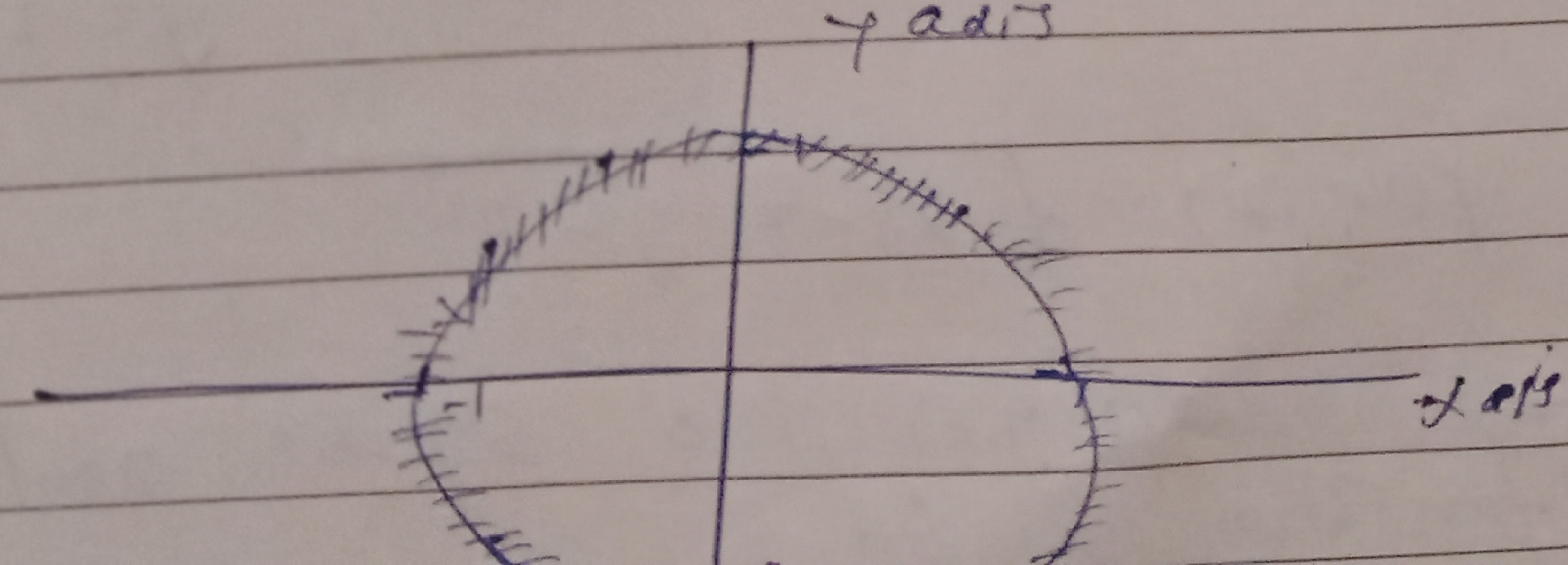
$\because$ ln is defin $\forall$ value of x & y. $\Rightarrow$

$1-x^2-y^2 > 0$

$\Rightarrow -x^2-y^2 > -1$

$\Rightarrow x^2+y^2 < 1 \rightarrow$ (A)

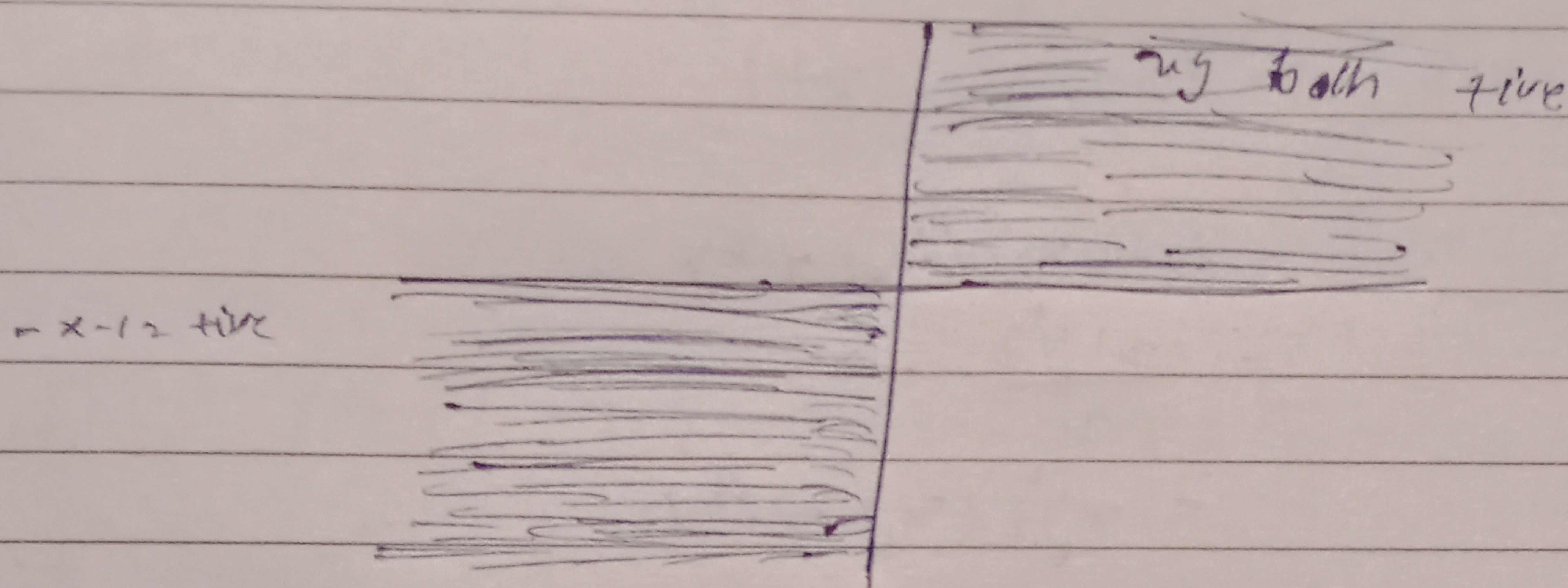
equation (A) Show that x,y lies in xy plane & we get a Circle (not encluded boundary) or Unit disk.



(26) $f(x,y) = \ln(xy)$

⇒ ∵ $\ln$ is define a +ive value

⇒ $xy > 0$ ⇒ $\ln(xy)$



domain is 1st & 3rd qudrent

(27) (a) $f(x,y) = xe^{-\sqrt{y+2}}$

Since $f(x,y)$ is define all value of x & y ~~except $\sqrt{y+2} \geq 0$~~
~~except $\sqrt{y+2} = 0$~~

⇒ $y+2 < 0$ ⇒ $y \geq -2$

⇒ All point in 2-space above on the line $y = -2$

(b) $f(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$

⇒ $f(x,y,z)$ is a square root function so

$f(x,y,z)$ is define when

all value value of $xyz > 0$

$$\Rightarrow 25 - x^2 - y^2 - z^2 \geq 0$$

$$\Rightarrow -x^2 - y^2 - z^2 \geq -25$$

$$\Rightarrow x^2 + y^2 + z^2 \leq 25$$

So The $f(x, y, z)$ is define all value of x, y & z lies on & inside the sphere having radius is 5.

(28) (c)

$$f(x, y, z) = \frac{xyz}{x+y+z}$$

domain all value of x, y, z except

$$\Rightarrow x + y + z \neq 0.$$

Practise Question :-

(1) 1, 2, 3, 4, 5, 17, 18, 19, 20, 23 - 26, 27 - 28.