# Vector - valued function or Vector function:-

<u>def:-</u>

A vector-valued function or vector function, is simply a function whose domain is a set of real number and whose ~~range~~ ranges is a set of vector.

and denoted by $r(t)$

- $r(t)$ is represent $r$ is a function of $t$, where $t$ is a parameter & $t \in R$.

$\Rightarrow$ $r(t) = \langle f(t), g(t), h(t) \rangle = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$

Example:

$r(t) = (t^3, \sin t, e^t)$

or

$r(t) = t^3 i + \sin t\, j + e^t k.$

- $r(t)$ is vector valued function in $R^3$

$\Rightarrow$ $r(t) = \langle e^t, 1/t^2 \rangle$

$r(t) = e^t\hat{i} + 1/t^2\, j$

- $r(t)$ is ~~Valued~~ valued function in $R^2$.

$\Rightarrow$ $r = r(t) = \langle f(t), g(t), h(t) \rangle$

$= f(t)\, i + g(t)\, j + h(t)\, k$

the function $f(t), g(t), h(t)$

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are Called Component function or The Component of $r(t)$.

Example:- The Component function of.

$r(t) = \langle t, \sin t, e^t \rangle$

$= ti + \sin t j + e^t$

$\Rightarrow f(t) = t,$
$g(t) = \sin t, \quad h(t) = e^t$

(2) $r(t) = \langle t+1, t^3 - 9 \rangle.$

$\Rightarrow r(t) = (t+1)i + (t^3 - 9)j$

$\Rightarrow f(t) = t+1, \quad g(t) = t^3 - 9$

## Domain of Vector-Valued function:-

The Domain of The vector valued function $r(t)$ is the set of allowable values of for t. if $r(t)$ ~~is denoted in term of~~ defined in term of Component function and the domain is not specified explicity.

Then it will be Understood that The domain is the intersection of all natrul domain of The

Component function. This is called the natrul domain of $r(t)$.

Q find The natural domain of

$$r(t) = \langle \ln|t-1|, e^t, \sqrt{t} \rangle$$

$\Rightarrow$ $r(t) = \ln|t-1|\,i, \; e^t j, \; \sqrt{t}\,k$

$\Rightarrow$ $x(t) = \ln|t-1|,$
$y(t) = e^t,$
$z(t) = \sqrt{t}$

we need to we find the domain of $x(t)$, $y(t)$ and $z(t)$.

$\Rightarrow$ $x(t) = \ln|t-1|$
$x(t)$ is define all valur of $t$ except 1
note thate if $t=1$

$x(t) = \ln|1-1| = \ln|0| = \ln(0) =$ Undef.

So domain of $\ln|t-1|$

$\Rightarrow$ $(-\infty, 1) \cup (1, \infty)$

$y(t) = e^t \quad \forall\, t \in \mathbb{R}.$

Domin.

$\Rightarrow$ $(-\infty, \infty)$.

$z(t) = \sqrt{t}$

define onle $\mathbb{R}^+ \cup \{0\}$.

$[0, \infty)$

note that if we allow -ive value of t, then we get output is complex no.

i.e if $t = -1 \Rightarrow \sqrt{t} = \sqrt{-1} = i$

$\Rightarrow$ $i$ is not allow. or any complex number is not allow to become as a range element.

$\Rightarrow z(t) = \sqrt{t} \Rightarrow [0, \infty)$

So the natural domain of $r(t)$ is the intersection of three component of $r(t)$

$\Rightarrow x(t) \cap y(t) \cap z(t)$

$\Rightarrow (-\infty, 1) \cup (1, \infty) \cap (-\infty, \infty) \cap [0, \infty)$

So the intersection of these set is.

$[0, 1) \cup (1, \infty)$

So the natural domain $r(t)$ is

$[0, 1) \cup (1, \infty)$

(ii) Find the domain of $r(t)$

$\Rightarrow r(t) = \cos\pi t i - \ln t j + \sqrt{t-2} k,$

$\Rightarrow x(t) = \cos \pi t,$

$y(t) = -\ln t$

$z(t) = \sqrt{t-2}$

The domain of $x(t)$
$\cos \pi t$
$\Rightarrow$ $\cos \pi t$ is define all value of $t$

$\Rightarrow$ $(-\infty, \infty)$ or $\mathbb{R}$.

The domain of $y(t)$ $-\ln(t)$
So, we know that
$-\ln(t)$ is define all value of $t \in \mathbb{R}^+$.
$\Rightarrow$ $(0, \infty)$

$\Rightarrow$ $z(t) = \sqrt{t-2}$
$\Rightarrow$ $[2, \infty)$

$\sqrt{2-2} \geq 0$.
$\sqrt{1-2} \geq \sqrt{-1} \neq i$

$\Rightarrow$ $(-\infty, \infty) \cap (0, \infty) \cap [2, \infty)$

$\Rightarrow$ $[2, \infty) \Rightarrow$ Ans.

Practise quesstions.

Ex 12.1 (Find domain).
question 1, 2, 3, 4

## "GRAPH OF VECTOR-Valued functions."

if $r(t)$ is a vector-valued function in 2-space or 3-space, Then we define The graph of $r(t)$ to be parametric curve describe by The component function for $r(t)$.

Sketch the graph ~~and~~ of the following's

(a) $r(t) = \langle \cos t, \sin t \rangle \quad 0 \le t \le 2\pi$

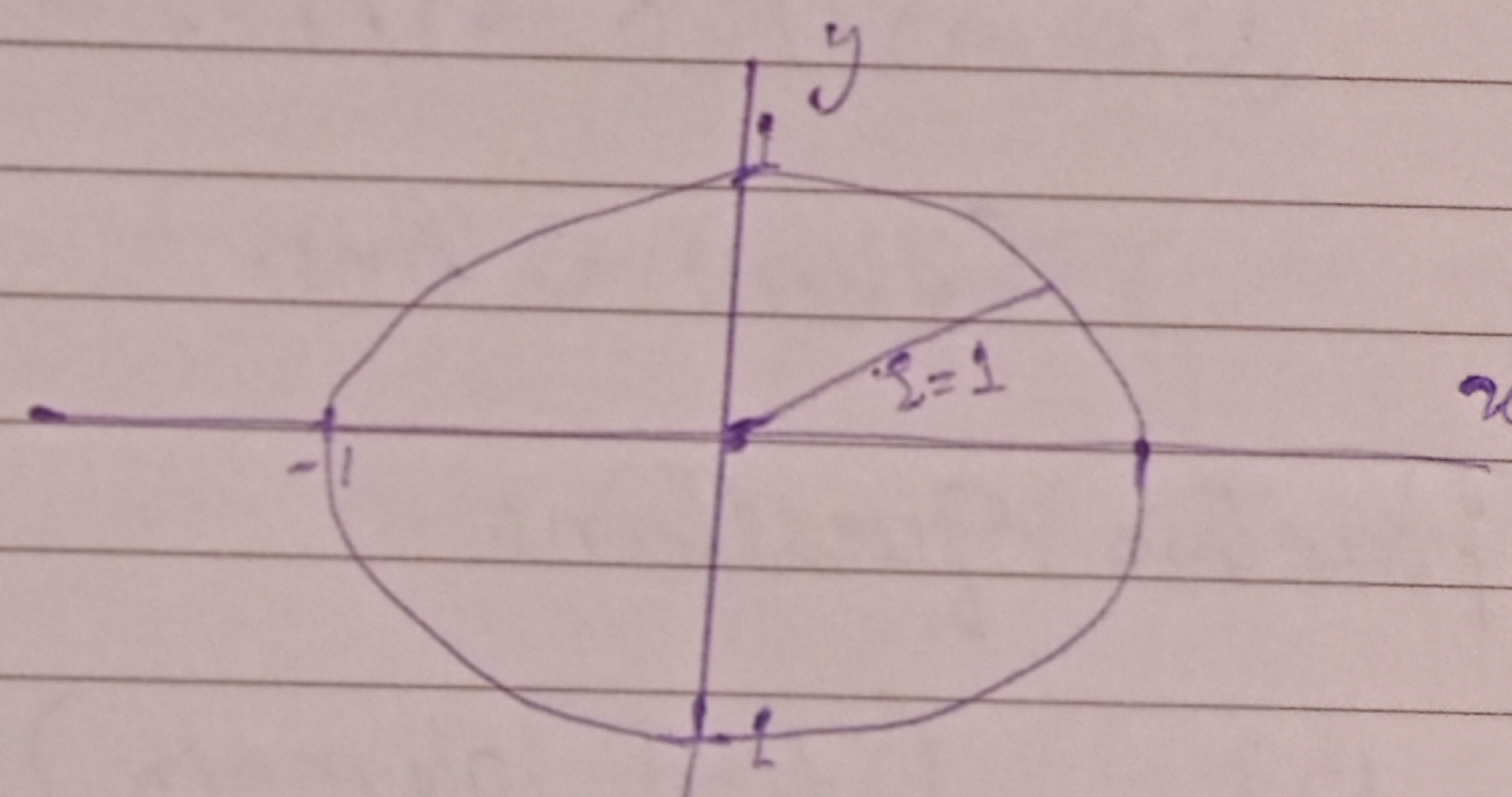
$\Rightarrow r(t) = \cos t\, i + \sin t\, j$

$r(t)$ is 2-space $t \in [0, 2\pi]$

$\Rightarrow \quad x(t) = \cos t, \quad \Rightarrow$

$y(t) = \sin t. \quad \Rightarrow \quad x^2 + y^2 = 1$

So the graph is circle of radius 1, centred at origin and oriented counter clock wise.




(b) Sketch the curve whose vector equation is

$r(t) = \cos t\, i + \sin t\, j + t k$

$\Rightarrow$ the parametric equation of this curve is:

$r(t) = \cos t\, i + \sin t\, j + t k.$

is

$x(t) = \cos t,$

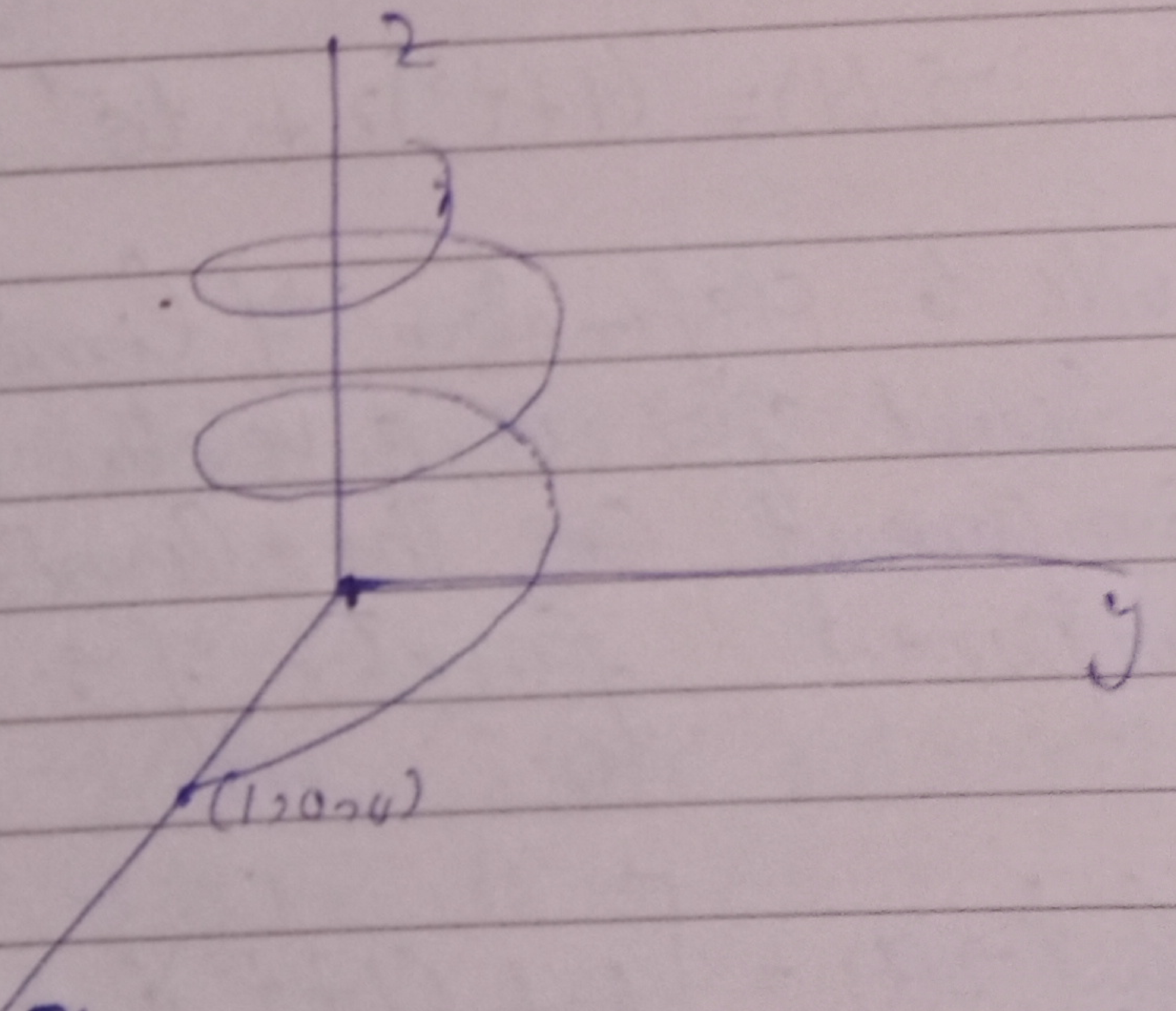
$y(t) = \sin t,$
$z(t) = t$

Since
$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

The curve must lies on the circular cylinder $x^2 + y^2 = 1$. the point $(x, y, z)$

lies directly above the point $(x, y, 0)$ which moves counter clockwise around the circle $x^2 + y^2 = 1$ in the $xy$ plane. Since $z = t$, the curve spirals upward around the cylinder as $t$ increase.

The curve is called a helix.




So,

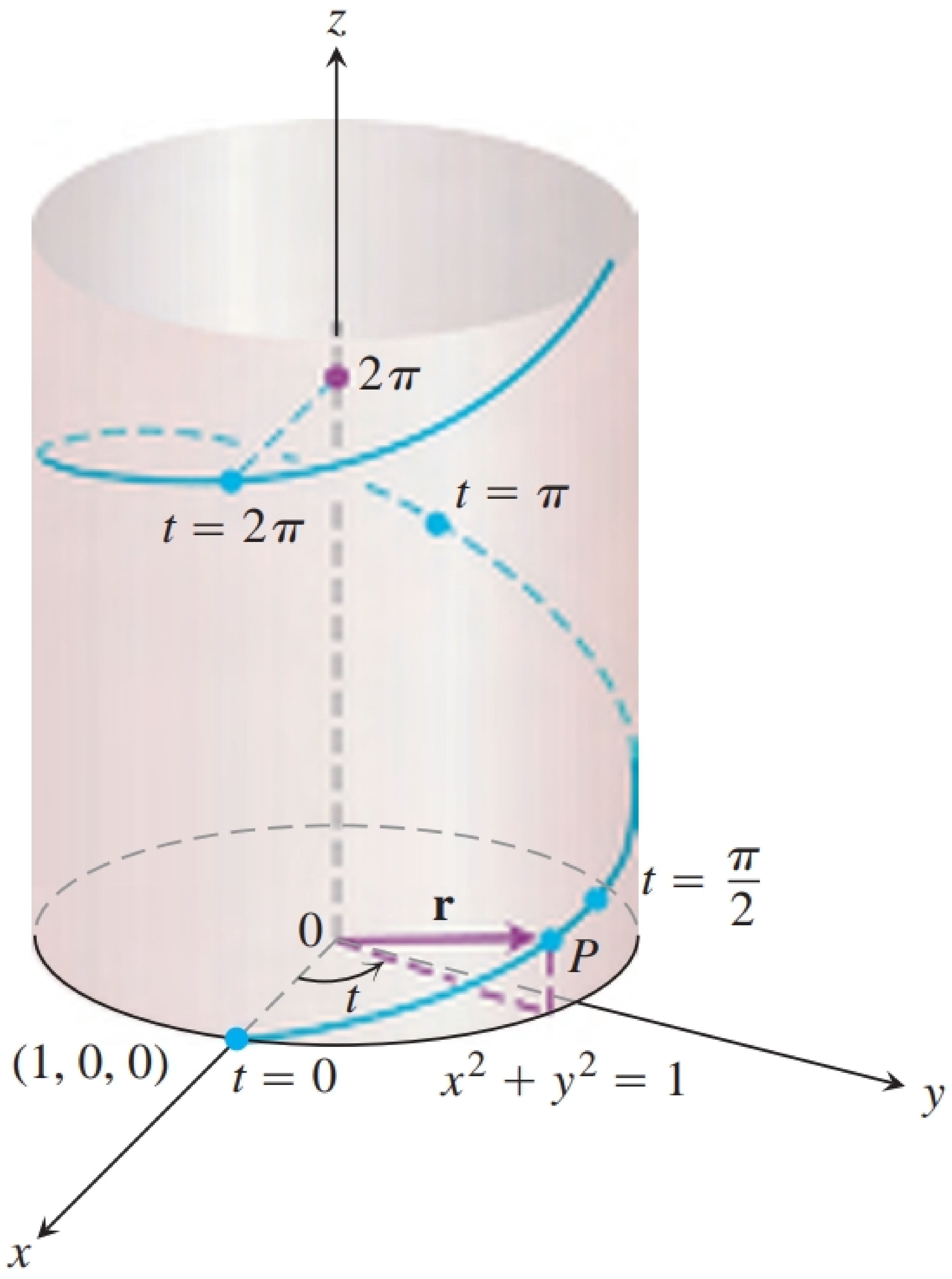



**FIGURE 13.3** The upper half of the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$ (Example 1).

# Limit of a Vector function:-

## & ~~Continuous~~ Continuity:-

The limit of a Vector function $r(t)$ is defined by taking the limit of its component function as fallows.

if $r(t) = \langle x(t), y(t), z(t) \rangle$
Then

$$\lim_{t\to a} r(t) = \left\langle \lim_{t\to a} x(t), \lim_{t\to a} y(t), \lim_{t\to a} z(t) \right\rangle$$

provided the limit of the Component function exist.

Q: Find $\lim_{t\to 0} r(t)$ where $r(t)$

$$r(t) = (1+t^3)i + te^{-t} + (t^3+3)k$$

A/c to defination of limit
The limit of $r$ is a vector whose Component are the limits of the Component function of $r$.

$$\lim_{t\to 0} r(t) = \left[\lim_{t\to 0} (1+t^3)i + \lim_{t\to 0} (te^{-t})j + \lim_{t\to 0} (t^3+3)k\right]$$

$$\lim_{t\to 0} r(t) = (1 + (0)^3)\, i + (0e^{-0})\, j + ((0)^3+3)\, k$$

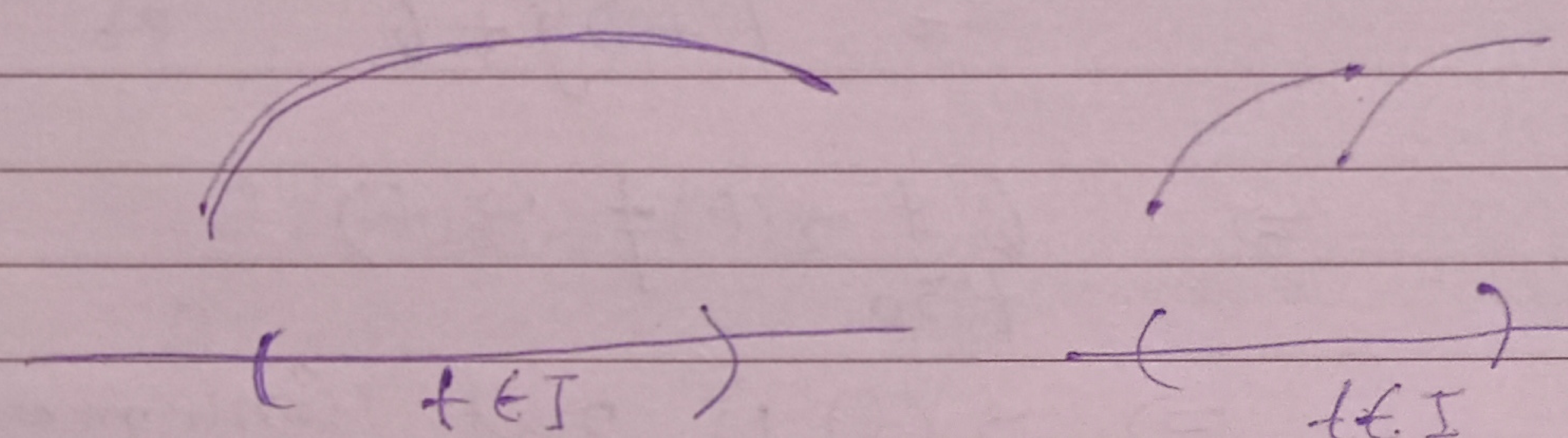
$$= 1i + 0j + 3k.$$

$\Rightarrow \lim_{t\to 0} \vec{r}(t) = i + 3k.$

A vector function $\vec{r}$ is continuous at a if

$$\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$$

$\vec{r}$ is continuous at a if and only if its components function $x, y, z$. are continuous at a.




$\Rightarrow$ $\vec{r}(t)$ is continuous

$\vec{r}(t)$ is not continuous.

Consider

$$\vec{r}(t) = (3+t)i + (\cos t)j + 3k.$$

$t = 0$

$\Rightarrow$

$\Rightarrow \vec{r}(0) = (3+0)i + (\cos(0)) + 3k$

$\Rightarrow \vec{r}(0) = 3i + j + 3k$

$\Rightarrow \boxed{\vec{r}(0) = 3i + j + 3k}$

$\lim_{t\to 0} = \lim_{t\to 0} \left((3+t)i + (\cos t)j + 3k\right)$

$= 3i + j + 3k$

$\Rightarrow \boxed{\lim_{t\to a} \vec{r}(t) = \vec{r}(a)}$ $\therefore \vec{r}(t)$ is Contin

Consider

$r(t) = \cos t\, i + 2e^t j + \frac{\sin t}{t} k$

at $t = 0$

$\Rightarrow r(0) = \cos(0) + 2e^0 j + \frac{\sin(0)}{0}$

$= i + 2j +$ doesn't exist

$\lim_{t \to 0} r(t) = \lim_{t \to 0} \cos t + \lim_{t \to 0} 2e^t + \lim_{t \to 0} \frac{\sin t}{t} k$

$\because \lim_{t \to 0} \frac{\sin t}{t} = 1$

$= 1 + 2j + k$

$\Rightarrow \lim_{t \to 0} r(t) \neq r(t)$

$\Rightarrow r(t)$ is not continuous.

Question:-

Find the limit

① $\lim_{t \to \infty} \left\langle \frac{t^2+1}{3t^2+2}, \frac{1}{t} \right\rangle$

$\Rightarrow \lim_{t \to \infty} \left( \frac{t^2+1}{3t^2+2} \right) i + \lim_{t \to \infty} \left( \frac{1}{t} \right)$

$= \lim_{t \to \infty} \frac{t^2}{t^2} \left[ \frac{1 + 1/t^2}{3 + 2/t^2} \right] i + \frac{1}{\infty}$

$\Rightarrow$

$\Rightarrow \left(\frac{1+1/\infty}{3+1/\infty}\right)i + \left(\frac{1}{\infty}\right)j$

$\Rightarrow \left(\frac{1+0}{3+0}\right)i + \left(\frac{1}{\infty}\right)j \quad \because 1/\infty = 0$

$\Rightarrow \frac{1}{3}i + 0j \Rightarrow \langle 1/3, 0 \rangle \Rightarrow$ Ans.

(ii) $\lim_{t \to 2} (ti - 3j + t^2k)$

$\Rightarrow \lim_{t \to 2} (t)i - \lim_{t \to 2} 3j + \lim_{t \to 2} t^2k$

$\Rightarrow 2i - 3j + (2)^2k$

$\Rightarrow 2i - 3j + 4k \Rightarrow \langle 2, -3, 4 \rangle$

Determine whether $r(t)$ is Continuous at $t = 0$

(a) $r(t) = 3\sin t\, i - 2tj$

so by defination of Continuity $\lim_{t \to a} r(a) = r(a)$

$\Rightarrow \lim_{t \to 0} r(0) = r(0) \quad \because a = 0$

$\Rightarrow r(0) = 3\sin(0)i - 2(0)j$

$r(0) = 0 - 0 = 0$

$\Rightarrow \lim_{t \to 0} (3\sin t)i - \lim (2t)j$

$\Rightarrow 3\sin(0)i - 2(0)j$

$\Rightarrow 0 - 0 = 0$

Hence $\lim_{t \to 0} r(0) = r(0)$ $r(t)$ is Continuous at $t = 0$

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(b) $r(t) = t^2 i + \frac{1}{t} j + t k.$

not that. limit $r(t)$ at $t=0$
does not exist
$\Rightarrow$ $r(t)$ is not continuous at $t=0$

$\because \lim_{t \to 0} (1/t)$ does not exist

{ Practise question Ex 14.2

question, $\underbrace{1, 2, 3, 4,}_{\Downarrow \text{ limit}}$ $\underbrace{5, 6}_{\Downarrow \text{ Continuity}}$. }