Derivative & intregrals of vector function:-

The derivative $r'(t)$ of a vector function is define in much the some way as for real-valued functions.

if $r(t) = \langle x(t), y(t), z(t) \rangle$

$r(t) = x(t)i + y(t)j + z(t)k$

where $x(t)$, $y(t)$ & $z(t)$ are diffrenable function

$\Rightarrow r'(t) = x'(t)i + y'(t)j + z'(t)k$

Example:-

let $r(t) = t^2 i + e^t j - (2\cos\pi t)k$

than

$r'(t) = \frac{d}{dt}(t^2)i + \frac{d}{dt}(e^t)j - \frac{d}{dt}(2\cos\pi t)k$

$\Rightarrow r'(t) = 2ti \; e^t j + 2\pi \sin\pi t\, k \Rightarrow$ Ans.

Example: Find $r'(t)$

$\Rightarrow r(t) = (\tan^{-1} t)i + (t\cos t)j - \sqrt{t}\,k$

$r'(t) = \frac{d}{dt}(\tan^{-1} t)i + \frac{d}{dt}(t\cos t)j - \frac{d}{dt}\sqrt{t}\,k.$

$$r'(t) = \frac{1}{1+t^2}\, i + (-t \sin t + \cos t (1))\, j - \frac{d}{dt}(t)^{1/2}\, k.$$

$$r'(t) = \frac{1}{1+t^2}\, i + (\cos t - t \sin t)\, j - \frac{1}{2\sqrt{t}}\, k$$

Rules of differentiation:

(a) $\frac{d}{dt}(C) = 0$

(b) $\frac{d}{dt}\, k \cdot r(t) = k \frac{d}{dt}[r(t)]$

(c) $\frac{d}{dt}\left[r_1(t) + r_2(t)\right] = \frac{d}{dt} r_1(t) + \frac{d}{dt} r_2(t)$

(d) $\frac{d}{dt}\left[r_1(t) - r_2(t)\right] = \frac{d}{dt} r_1(t) - \frac{d}{dt} r_2(t)$

(e) $\frac{d}{dt}\left[f(t)\, r(t)\right] = f(t) \frac{d}{dt}[r(t)] + \frac{d}{dt}[f(t)] . r(t)$

Derivative of Dot & Cross Products.

(i) $\frac{d}{dt}\left[r_1(t) \cdot r_2(t)\right] = r_1(t) \cdot \frac{dr_2}{dt} + \frac{dr_1}{dt} \cdot r_2(t)$

(2) $\frac{d}{dt}\left[r_1(t) \times r_2(t)\right] = r_1(t) \times \frac{dr_2}{dt} + \frac{dr_1}{dt} \times r_2(t)$

Rules of intregration:-

let $r(t)$, $r_1(t)$ and $r_2(t)$ be vector valued function in 2space or 3-space that are Continuous on the interval $a \le t \le b$ & let k be a scalar. then the following rule of intregation hold.

(a) $$\int_a^b k\, r(t)\, dt = k \int_a^b r(t)\, dt$$

(b) $$\int_a^b [r_1(t) + r_2(t)]\, dt = \int_a^b r_1(t)\, dt + \int_a^b r_2(t)\, dt$$

(c) $$\int_a^b [r_1(t) - r_2(t)]\, dt = \int_a^b r_1(t)\, dt - \int_a^b r_2(t)\, dt.$$

Question:-

$$\int \left(t^2 i - 2tj + \frac{1}{t}k\right) dt$$

$$\Rightarrow \int t^2 i\, dt - 2\int t j\, dt + \int \frac{1}{t} k\, dt$$

$$\Rightarrow \left(\frac{t^3}{3} + c_1\right) i - \left(\frac{\cancel{2}t^2}{\cancel{2}} + c_2\right) j + \ln(t) k$$

$$\Rightarrow \frac{t^3}{3} i + c_1 i - t^2 j + c_2 j + \ln(t) k + c_3 k$$

$$\Rightarrow \frac{t^3}{3} i - t^2 j + \ln(t) + c_1 i + c_2 j + c_3 k$$

$$\Downarrow$$

$$\Rightarrow \frac{t^3}{3} i - t^2 j + \ln(t) + C$$

(2) $\int (e^{-t}, e^{t}, 3t^2)\, dt$

$\Rightarrow \int e^{-t} i\, dt + \int e^{t} j\, dt + \int 3t^2 k\, dt$

$\Rightarrow -e^{-t} i + e^{t} j + \frac{3t^3}{3} k + C$

$\Rightarrow -e^{-t} i + e^{t} j + t^3 k + c \Rightarrow$ Ans.

(A) Evaluate the definite integral.

(1) $\int_0^{\pi/2} \langle \cos 2t, \sin 2t \rangle\, dt$

$\Rightarrow \int_0^{\pi/2} \cos 2t\, i\, dt + \int_0^{\pi/2} \sin 2t\, dt$

$\Rightarrow \left.\frac{\sin 2t\, i}{2}\right|_0^{\pi/2} + \left.\frac{\cos 2(t)}{2}\right|_0^{\pi/2}$

$\Rightarrow \frac{1}{2}\left[\sin 2(\pi/2) - \sin 2(0)\right] i - \frac{1}{2}\left[\cos 2(\pi/2) - \cos 2(0)\right] j$

$\Rightarrow \frac{1}{2}\left[\sin(\pi) - \sin(0)\right] i - \frac{1}{2}\left[\cos(\pi) - \cos(0)\right] j$

$\because \sin(\pi) = 0, \cos(\pi) = -1$

$\Rightarrow \frac{1}{2}\left[\cancel{0} - 0\right] i - \frac{1}{2}\left[-1 - 1\right] j$

$\Rightarrow -\frac{1}{2}[-2] = \frac{2}{2} = 1j$

$\Rightarrow \langle 0, j \rangle$

(2) $\int_0^1 (e^{2t} i + e^{-t} j + tk)\, dt$

$\Rightarrow \int e^{2t} i\, dt + \int_0^1 e^{-t} j + \int_0^1 tk.$

$\Rightarrow \frac{e^{2t}}{2}\Big|_0^1 i + \frac{e^{-t}}{-1}\Big|_0^1 + \frac{t^2}{2}\Big|_0^1$

$\Rightarrow \frac{1}{2}\left[e^{2} - e^{0}\right] i - \left[e^{-1} - e^{0}\right] j$

$+ \frac{1}{2}\left[(1)^2 - (0)^2\right] k$

$\Rightarrow \frac{1}{2}\left[e^{2} - 1\right] i - \left[e^{-1} - 1\right] j + \frac{1}{2} k$

$\Rightarrow \frac{1}{2}\left[e^{2} - 1\right] i - \left[\frac{1}{e} - 1\right] j + \frac{1}{2} k$

$\Rightarrow$ Ans

[Practise question Ex 14.2.

question 9, 10,
& 31 to 40]