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Parial derivative and Continuty:-

=> In Contrast to the case of function of a Single variable. The existence of Partial derivative for a multi-variable function does not guarantee the Continuity of the function.

This fact Show in the following example.

Ex (1):

let

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(a) Show Thate $f_x(x,y)$ and $f_y(x,y)$ exist at all Point (x,y)

(b) explain f is not Continous at $(0,0)$

Ex (2):-

let

$$f(x,y) = \begin{cases} 0 & xy \neq 0 \\ 1 & xy = 0 \end{cases}$$

(a) Find the limit $f(x,y)$ $(0,0)$ along the line $y=x$

(b) Prove Thate f is not Continuous at the origin i.e $(0,0)$

(c) Show thate both Partial derivative

$\partial f/\partial x$ and $\partial f/\partial y$ exist at the origin.

(Do it)

Theorem :- (Alexis clairauts)

let f be a function of Two variable if f_{xy} and f_{yx} are Continuous on some open disk Then $f_{xy} = f_{yx}$ on the disk.

Ex :-
(ex q 87) $\quad f(x,y) = e^{x} \cos y$

$\Rightarrow \dfrac{\partial f}{\partial x} = e^{x}\cos y \Rightarrow \dfrac{\partial^2 f}{\partial x \partial y} = -e^{x}\sin y \rightarrow (1)$

$\Rightarrow \dfrac{\partial f}{\partial y} = -e^{x}\sin y \Rightarrow \dfrac{\partial^2 f}{\partial y \partial x} = -e^{x}\sin y \rightarrow (2)$

$\Rightarrow f_{xy} = f_{yx}$

$\Rightarrow$ ~~$f(x,y) = e^{x}\cos y$ is~~

So from (1) & (2) f_{xy} & f_{yx} are Continuous.

$\Rightarrow f_{xy} = f_{yx}$

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Some important Partial differential Equations:

① WAVE EQUATION:- (One dimensional wave eq)

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u^2}{\partial x^2}$$

where C is +ive Constant that depends upon the physical characteristics of the string.

② Laplace's Equation:-

$$\frac{\partial z^2}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

③ heat equation:-

$$\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2} \quad (c>0, \text{ constant})$$

④ Cauchy - Reimann equation (CR equation):-

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$$

(Q 102):-

Show that the function satisfies the heat equations.

$$\Rightarrow z = e^{-t} \sin(x/c)$$

we know that the heal equation is

$$\Rightarrow \quad \frac{\partial z}{\partial t} = c^2 \frac{\partial z}{\partial x^2} \quad (c>0) \rightarrow (1)$$

$$z = e^{-t} \sin(x/c)$$

$$\Rightarrow$$

$$\partial z/\partial t = -e^{-t} \sin(x/c)$$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = \frac{1}{c} e^{-t} \cos(x/c)$$

$$\Rightarrow \quad \frac{\partial^2 z}{\partial^2 x} = \frac{-1}{c^2} e^{-t} \sin(x/c)$$

$$\Rightarrow (1) \Rightarrow \quad \frac{\partial z}{\partial t} = c^2 \frac{\partial z}{\partial x^2}$$

$$\Rightarrow \quad \cancel{-e^{-t} \sin(x/c)} = -\cancel{c^2}\left(\cancel{\frac{e^{-t}}{c^2} \sin(x/c)}\right)$$

$$\Rightarrow \quad 1 = 1$$

hence $z = e^{-t} \sin(x/c)$ is satisfied as a heat Equation.

Q (1.4):-

(a) $\quad u = x^2 - y^2 \qquad v = 2xy$

Show that $u(x,y)$ and $v(x,y)$ is satisfied The CR equation

Solution:-

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$\Rightarrow$ we Prove that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$\Rightarrow$ $u = x^2 - y^2$ $\qquad$ $v = 2xy$

$\Rightarrow$ $\frac{\partial u}{\partial x} = 2x$ $\qquad$ $\frac{\partial v}{\partial x} = 2y$

$\frac{\partial u}{\partial y} = -2y$ $\qquad$ $\frac{\partial v}{\partial y} = 2x$

$\Rightarrow$ $2x = 2x$ and $-2y = -2y$

$1 = 1$ and $1 = 1$

$\Rightarrow$ $u(x,y)$ and $v(x,y)$ is satisfied the CR equation.

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Differentiability :-

def: A function f of two variable is said to be differentiable at (x_0, y_0) provided $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ both exist and

$$\lim_{(\Delta x, \Delta y) \to (0,0)} \frac{\Delta f - f_x(x_0, y_0)\Delta x - f_y(x_0, y_0)\Delta y}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} = 0$$

$\Rightarrow$ $f(x,y) = x^2 + y^2$ is differentiable at $(0,0)$

$\Rightarrow$ " Δf =
The symbol Δf, called the increment of $f(x,y)$, denotes the change in the value of $f(x,y)$ the results when (x,y) varies from some initial position (x_0, y_0) to some new position $(x_0 + \Delta x, y_0 + \Delta y)$ "

Thus

$$\Delta f = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$$

$\Rightarrow$ we show that $f(x,y)$ is diff at origin

$\Rightarrow$ $\Delta f = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0 y_0)$

$\Delta f = (0 + \Delta x)^2, (0 + \Delta y)^2 - f(0,0)$

$= (\Delta x)^2 + (\Delta y)^2 - 0 = \Delta x^2 + \Delta y^2$

$\Rightarrow \Delta f = \Delta x^2 + \Delta y^2$

$f_x(x_0, y_0) = \frac{\partial}{\partial x}(x^2+y^2) = 2x\Big|_{(0,0)} = 0$

$f_y(x_0, y_0) = \frac{\partial}{\partial y}(x^2+y^2) = 2y\Big|_{(0,0)} = 0$

So using the def of diff.

$$\Rightarrow \lim_{(\Delta x, \Delta y)\to(0,0)} \frac{\Delta f - f_x(x_0,y_0)\,\Delta x - f_y(x_0,y_0)\,\Delta y}{\sqrt{\Delta x^2 + \Delta y^2 + \cancel{\Delta z}^2}} = 0$$

$$\Rightarrow \lim_{(\Delta x, \Delta y)\to(0,0)} \left[\frac{(\Delta x^2 + \Delta y^2) - (0)\Delta x - 0(\Delta y)}{\sqrt{\Delta x^2 + \Delta y^2}}\right] = 0$$

$$\Rightarrow \lim_{(\Delta x, \Delta y)\to(0,0)} \left[\frac{\Delta x^2 + \Delta y^2}{\sqrt{\Delta x^2 + \Delta y^2}}\right] = 0$$

$$\Rightarrow \quad '' \quad \left[(\Delta x^2 + \Delta y^2)^{1-1/2}\right] = 0$$

$$\lim_{(\Delta x, \Delta y)\to(0,0)} \left[\sqrt{(\Delta x^2 + \Delta y^2)}\right] = 0$$

$\Rightarrow \sqrt{(0+0)} = 0 \Rightarrow 0 = 0$

There for $f(x,y)$ is differentiable.

⇒ **if function f of Three Variables :-**

def:-

A function f of Three variables is said to be differentiable at (x_0, y_0, z_0) provided $f_x(x_0, y_0, z_0)$, $f_y(x_0, y_0, z_0)$ and $f_z(x_0, y_0, z_0)$ exist and

$$\lim_{(\Delta x, \Delta y, \Delta z) \to (0,0,0)} \frac{\Delta f - f_x(x_0,y_0,z_0)\,\Delta x - f_y(x_0,y_0,z_0)\,\Delta y - f_z(x_0,y_0,z_0)\,\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}} = 0$$

(most importat Resut)

Theorem:-

if a function is differentiable at a point, Then it is Contineous at thate point.

Theorem:-

if all first-order partial derivative of f exist and are Contineous at a point, Then f is differentiable at the point.

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Consider $f(x,y,z) = x + yz$

Since $f_x(x,y,z) = 1$

$f_y(x,y,z) = z$

$f_z(x,y,z) = y$

are defined and continuous everywhere

$\Rightarrow$ $f(x,y,z)$ is diffrable.

Differentials :- (total derivative)

if $z = f(x,y)$ is differentiable at a point (x_0, y_0)

$$\Rightarrow \quad dz = f_x(x_0,y_0)\,dx + f_y(x_0,y_0)\,dy.$$

Similarly if a function $w = f(x,y,z)$ is three variable

$$\Rightarrow \quad dw = f_x(x_0,y_0)\,dx + f_y(x_0,y_0)\,dy + f_z(x_0,y_0)\,dz$$

or

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy.$$

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Ex 13.4

Q 9–20

Compute the differential dz or dw of the function.

(9) $z = 7x - 2y$

$\Rightarrow$ total derivative or differential of $f(x, y)$ is given by

$\Rightarrow dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \longrightarrow (A)$

$\Rightarrow \frac{\partial f}{\partial x} = 7$

$\frac{\partial f}{\partial y} = -2$

$\Rightarrow (A) \quad dz = 7\,dx - 2y \Rightarrow$ Ans.

(20) $w = \sqrt{x} + \sqrt{y} + \sqrt{z}$

$\Rightarrow$ differet is given by

$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz$

$\Rightarrow \frac{\partial w}{\partial x} = \frac{1}{2}(x)^{\frac{1}{2}-1} \Rightarrow \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}}$

$$\Rightarrow \frac{\partial w}{\partial y} = \frac{1}{2\sqrt{y}}.$$

$$\Rightarrow \frac{\partial w}{\partial z} = \frac{1}{2\sqrt{z}}$$

$$\Rightarrow dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$dw = \frac{1}{2\sqrt{x}}dx + \frac{1}{2\sqrt{y}}dy + \frac{1}{2\sqrt{z}}dz$$

$\Rightarrow$ Ans

Practise question

Ex 13.4 Q 9 — 20

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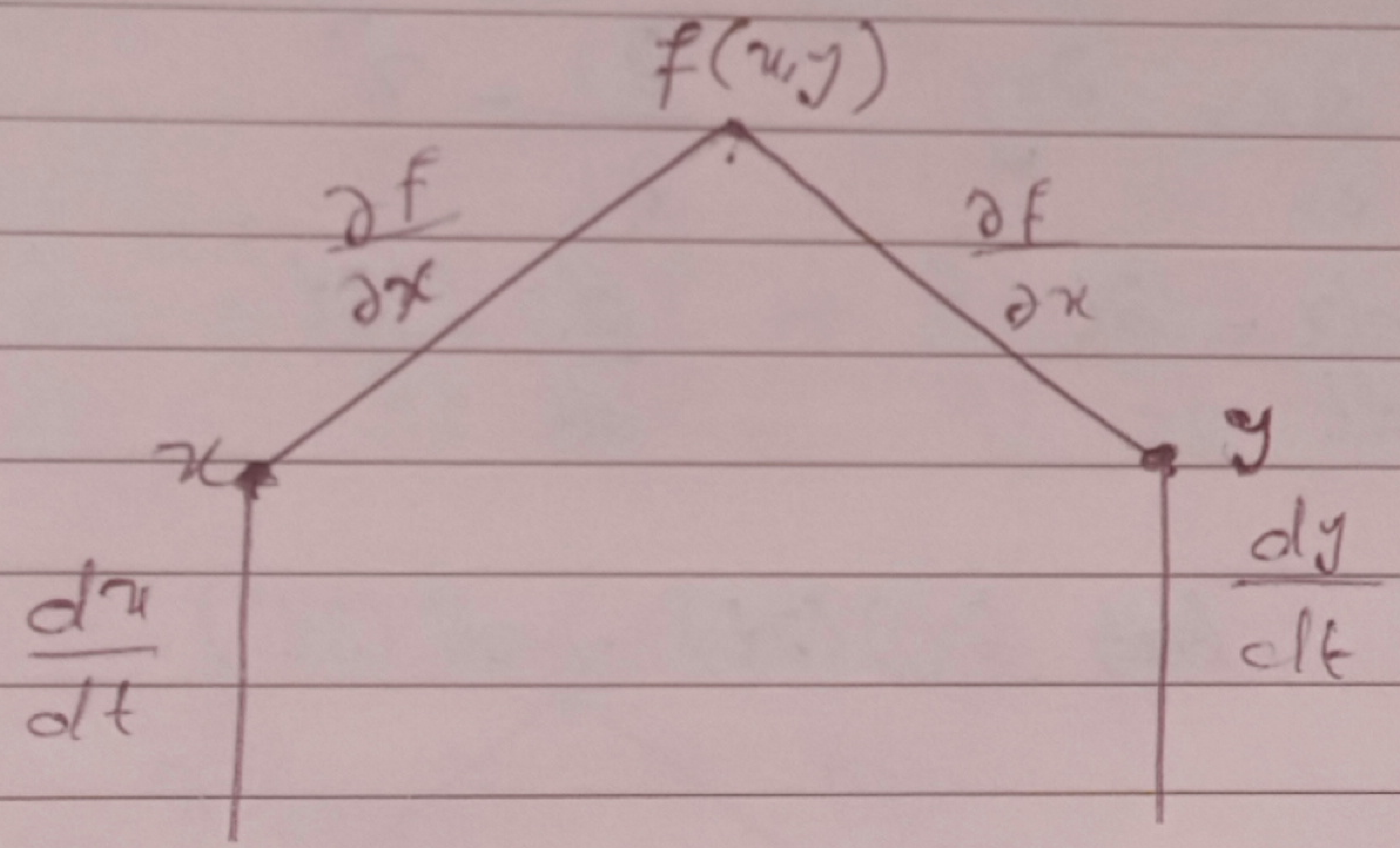
CHAIN RULE:-

Theorem:- (Chain Rules for Derivatives)

if $x = x(t)$ and $y = y(t)$ are diff at t. and $z = f(x,y)$ is differentiable at The Point $(x,y) = (x(t), y(t))$ Then $z = f(x(t), y(t))$ is diff at point t and

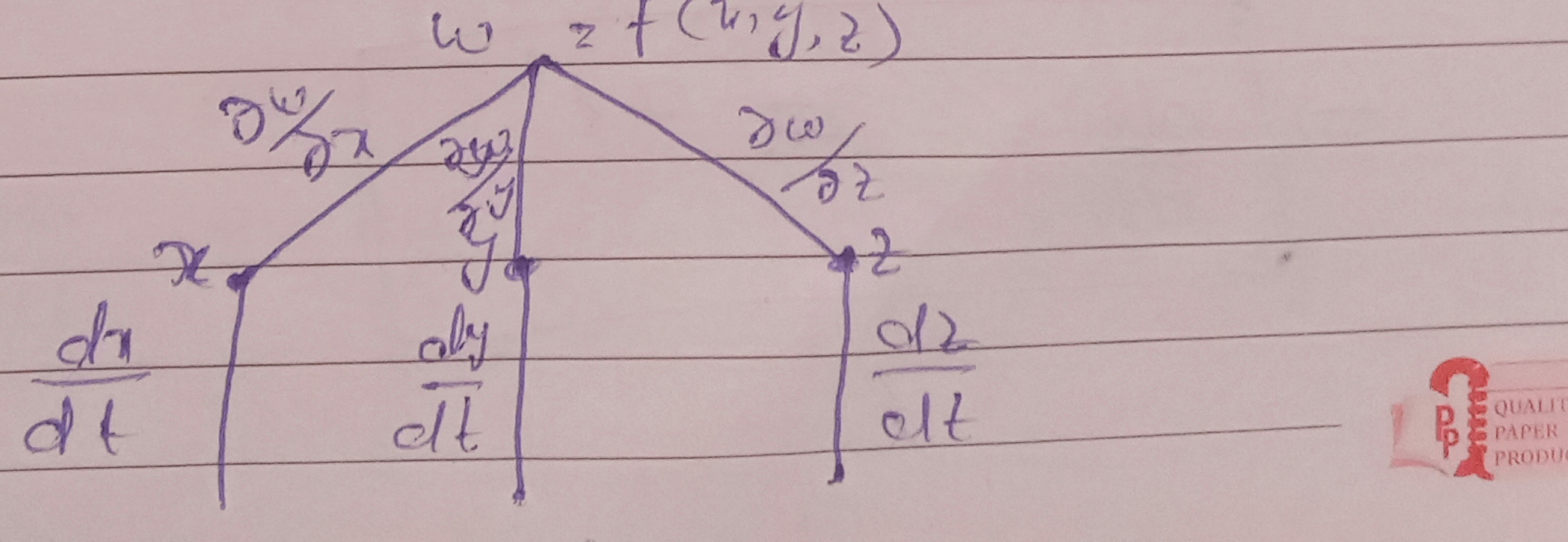
$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$



$$\Rightarrow \frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial x}\frac{dy}{dt}$$

if $w = f(x,y,z)$

$$\Rightarrow \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$



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Example:

$$z = x^2 y \qquad x = t^2, \quad y = t^3$$

$\Rightarrow$ Since z is depend upon x & y.

$\Rightarrow$ x is depend t. and y is depend upon t.

$$\Rightarrow \frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \longrightarrow (1)$$

$$\Rightarrow \frac{\partial z}{\partial x} = 2xy \qquad \frac{\partial z}{\partial y} = x^2$$

$$\Rightarrow \frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2$$

$$\Rightarrow \frac{dz}{dt} = (2xy)(2t) + x^2(3t^2)$$

$$\Rightarrow \frac{dz}{dt} = 4xyt + 3x^2t^2$$

$$\because x = t^2, \quad y = t^3$$

$$\frac{dz}{dt} = 4(t^2)(t^3)t + 3(t^2)^2 t^2$$

$$\Rightarrow \frac{dz}{dt} = 4t^6 + 3t^6 \Rightarrow 7t^6$$

$$\Rightarrow \boxed{\frac{dz}{dt} = 7t^6}$$

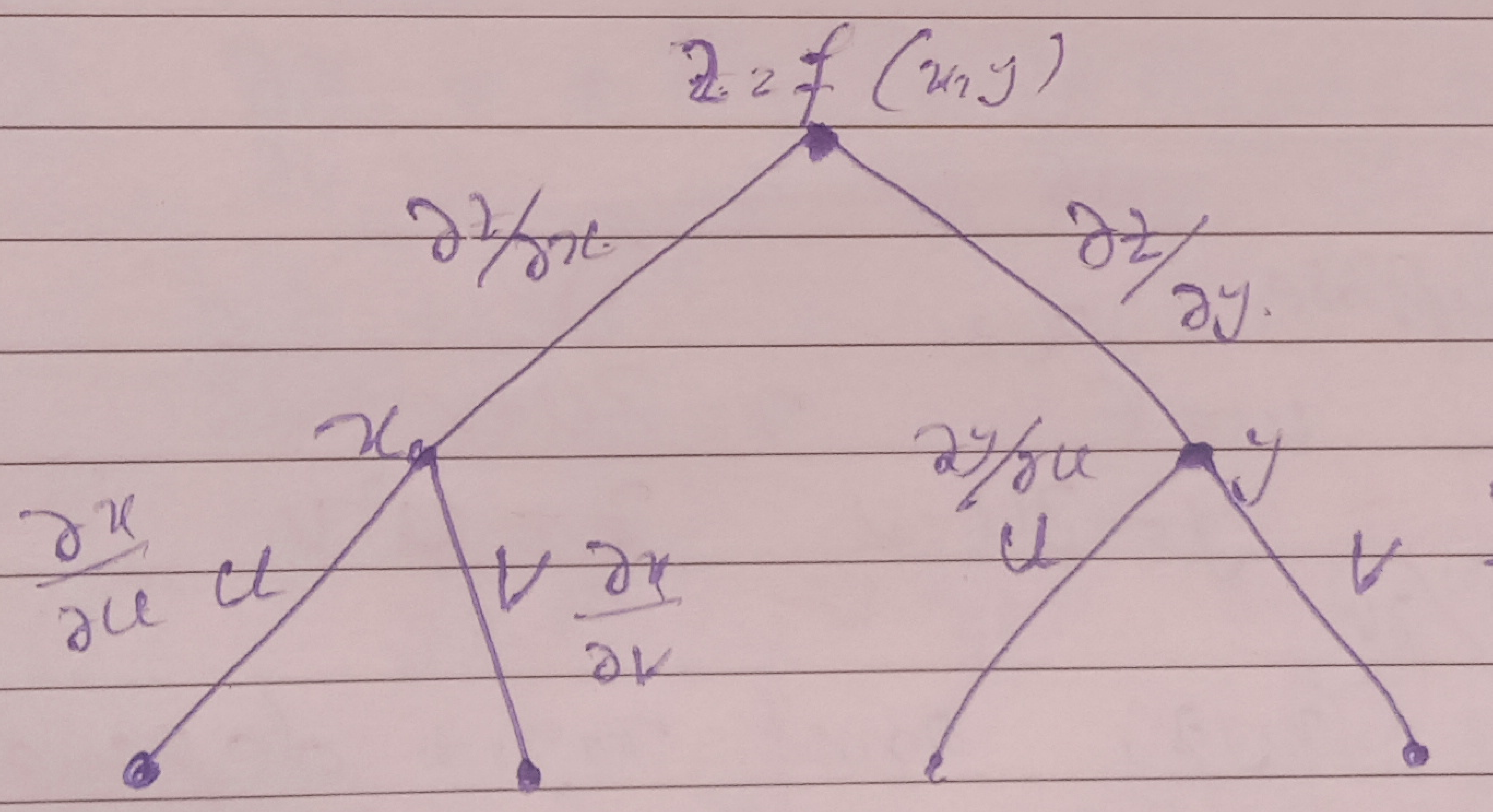
Theorem :-

if $x = x(u,v)$ and $y = y(u,v)$ have First order Partial derivative at a point (u,v) and if $z = f(x,y)$ is diff at the point $(x,y) = (x(u,v), y(u,v))$ Then $z = f(x(u,v), y(u,v))$ has First-order Partial derivative at the Point (u,v) given by

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

and

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$



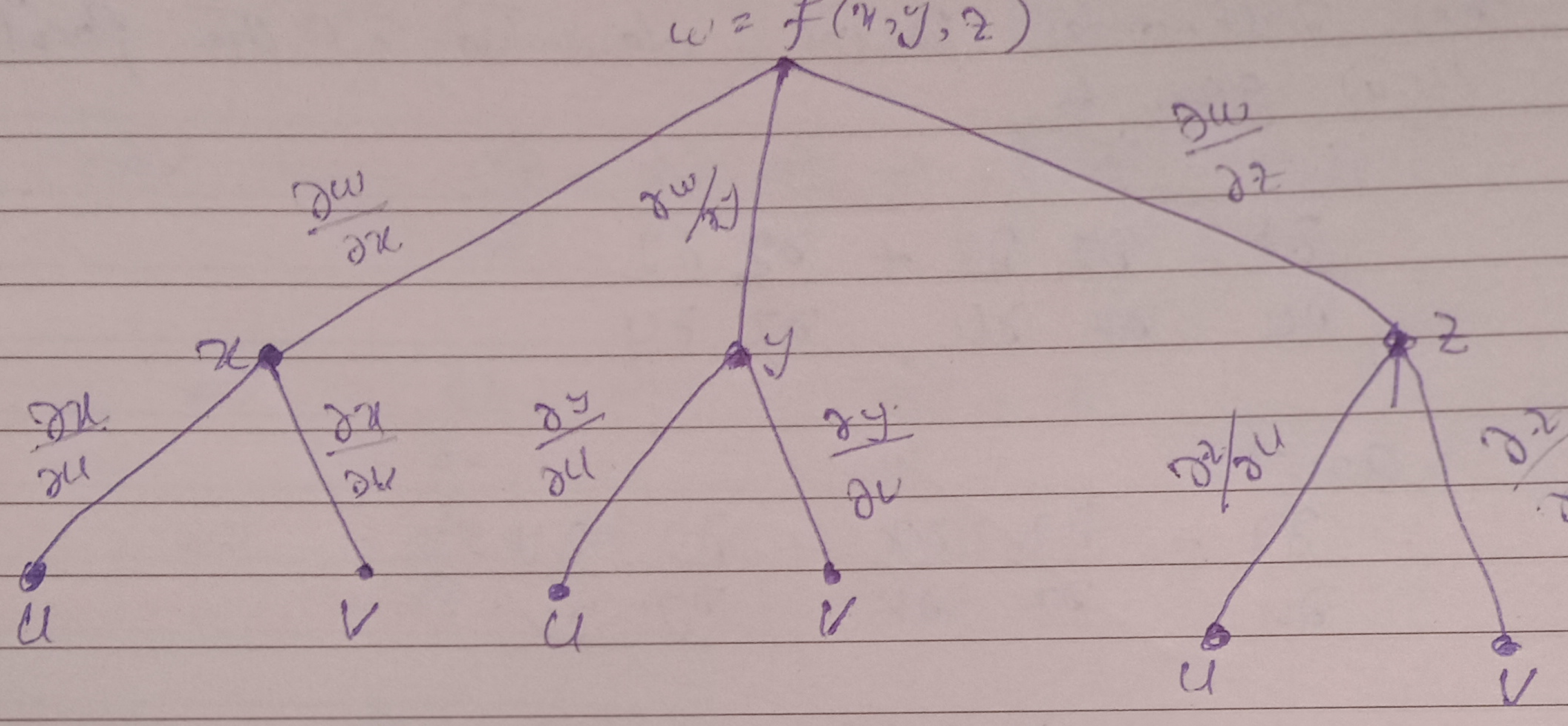
in Case of Three variable

$\Rightarrow \quad w = f(x, y, z)$

$$\Rightarrow \frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

and

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}.$$



Example Suppose

$$w = e^{xyz} \qquad x = 3u + v$$

$$y = 3u - v \qquad z = u^2 v$$

Find $\partial w/\partial u$ & $\partial w/\partial v$

Since $w = f(x, y, z)$ and x, y, z depend upon u & v. $\Rightarrow (x, y, z) = (x(u,v), y(u,v), z(u,v))$

$$\Rightarrow \frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}. \rightarrow (A)$$

$$\Rightarrow \frac{\partial w}{\partial x} = yz e^{xyz}$$

$$\frac{\partial w}{\partial y} = xz e^{xyz}$$

$$\Rightarrow \frac{\partial w}{\partial z} = xy\,e^{xyz}$$

$$\frac{\partial x}{\partial u} = 3 \qquad \frac{\partial y}{\partial u} = 3$$

$$\frac{\partial z}{\partial u} = 2uv.$$

$$\Rightarrow (A)\quad \frac{\partial w}{\partial u} = yz\,e^{xyz}(3) + xz\,e^{xyz}(3) + xy\,e^{xyz}(2uv)$$

$$\Rightarrow \frac{\partial w}{\partial u} = 3yz\,e^{xyz} + 3xz\,e^{xyz} + (2uv)xy\,e^{xyz}$$

$$\Rightarrow \frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} \longrightarrow (B)$$

$$\Rightarrow \frac{\partial x}{\partial v} = +1 \quad , \quad \frac{\partial y}{\partial v} = -1$$

$$\frac{\partial z}{\partial v} = u^2$$

$$\Rightarrow (B) = yz\,e^{xyz}(1) + xz\,e^{xyz}(-1) + xy\,e^{xyz}(u^2)$$

$$\Rightarrow yz\,e^{xyz} - xz\,e^{xyz} + u^2xy\,e^{xyz}$$

$$= e^{xyz}\left[yz - xz + u^2xy\right] \Rightarrow \text{Ans.}$$

(Read example 5 and example 6
page no 954)

Implicit differentiation :-

Theorem :-

if The equation $f(x,y) = c$ defines y implicity as a differentiable function of x, and if $\partial f/\partial y \neq 0$. Then

$$\Rightarrow \frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

Example $x^3 + y^2 x - 3 = 0$

$$\Rightarrow x^3 + y^2 x = 3$$

$$\Downarrow \quad \Downarrow$$

$$f(x,y) = c$$

$$\Rightarrow \frac{\partial f}{\partial x} = \cancel{3x^2 + y^2 + 2y\frac{dy}{dx}} \; 3x^2 + y^2$$

$$\frac{\partial f}{\partial y} = 2yx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(3x^2 + y^2)}{2yx} \Rightarrow \text{Ans.}$$

(old method i.e one variable)

$$x^3 + y^2 x - 3 = 0$$

$$\Rightarrow$$

$$3x^2 + y^2(1) + 2y\frac{dy}{dx}x - 0 = 0$$

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$$\Rightarrow \quad 2y\frac{dy}{dx}x = -\frac{(3x^2+y^2)}{1}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{(3x^2+y^2)}{2xy} \Rightarrow \text{(Ans}$$

Theorem:-

if the ~~function~~ equation $f(x,y,z)=c$ define z implicity as a differentiable function of x and y, and if

$$\frac{\partial f}{\partial z} \neq 0. \quad \text{Then}$$

$$\frac{\partial z}{\partial x} = -\frac{\partial f/\partial x}{\partial f/\partial z} \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{\partial f/\partial y}{\partial f/\partial z}.$$

Ex 13.5

Q 1–6. Use an appropriate form of the chain rule to find dz/dt.

(2) $z = \ln(2x^2 + y)$

$x = \sqrt{t}$, $y = t^{2/3}$

Since $z = f(x,y)$ and y, x depend upon t,

$\Rightarrow \dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$

$\dfrac{\partial z}{\partial x} = \dfrac{\partial}{\partial x}\left(\ln(2x^2+y)\right)$

$= \dfrac{1}{2x^2+y}(4x) \Rightarrow \dfrac{4x}{2x^2+y}$

$\dfrac{\partial z}{\partial y} = \dfrac{1}{2x^2+y}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{1}{2\sqrt{t}}$

$\dfrac{dy}{dt} = \dfrac{2}{3}(t)^{2/3-1}$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{2}{3}(t)^{-1/3}$

$\Rightarrow dy/dt = \dfrac{2}{3\sqrt[3]{t}}$

$\Rightarrow \dfrac{dz}{dt} = \left(\dfrac{4x}{2x^2+y}\right)\dfrac{1}{2\sqrt{t}} + \left(\dfrac{1}{2x^2+y}\right)\left(\dfrac{2}{3t^{1/3}}\right)$

~~$\Rightarrow$ Ans.~~

Since $x = \sqrt{t}$, $y = t^{2/3}$

$\dfrac{dy}{dt} = \dfrac{2}{3}(t)^{-1/3}$

$$\frac{dz}{dt} = \left(\frac{4(\sqrt{t})}{2(\sqrt{t})^2 + t^{2/3}}\right)\frac{1}{2\sqrt{t}} + \frac{1}{2(\sqrt{t})^2 + y^{2/3}}\left(\frac{2}{3t^{1/3}}\right)$$

$$\cancel{= \frac{1}{2t+t^{2/3}} + \left(\frac{1}{2t+t^{2/3}}\right)\left(\frac{2}{3t^{1/3}}\right)}$$

$$\cancel{= \frac{1}{2t+t^{2/3}} + \frac{2}{6t^{4/3}+3t}}$$

$$\cancel{= \frac{1}{2t+t^{2/3}} + \frac{2}{2(t^{4/3}+t)}}$$

$$\frac{dz}{dt} = \left(\frac{4\sqrt{t}}{2(\sqrt{t})^2 + t^{2/3}}\right)\frac{1}{2\sqrt{t}} + \left(\frac{1}{2(\sqrt{t})^2 + t^{2/3}}\right)\left(\frac{2t^{-1/3}}{3}\right)$$

$$\frac{dz}{dt} = \frac{2}{2t+t^{2/3}} + \frac{2t^{-1/3}}{3(2t+t^{2/3})}$$

$$\frac{dz}{dt} = \frac{6 + 2t^{-1/3}}{3(2t+t^{2/3})} \Rightarrow \text{Ans.}$$

(7) Find $\frac{dw}{dt}$

$$w = 5x^2y^3z^4 \qquad x = t^2 \quad y = t^3 \quad z = t^5$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$w = 5x^2y^3z^4 \quad x = t^2, \; y = t^3, \; z = t^5$

$$\Rightarrow \frac{\partial w}{\partial z} = 20x^2y^3z^3 \qquad \frac{dx}{dt} = 2t$$

$$\frac{\partial w}{\partial y} = 15x^2y^2z^4 \qquad \frac{dy}{dt} = 3t^2$$

$$\frac{\partial w}{\partial x} = 10xy^3z^4 \qquad \frac{dz}{dt} = 5t^4$$

$$\Rightarrow \frac{dw}{dt} = (10xy^3z^4)\,2t + (15x^2y^2z^4)\,3t^2 + (20x^2y^3z^3)(5t^4)$$

$$\Rightarrow \frac{dw}{dt} = (20t^2 \cdot t^9 t^{20})t + (45t^4 t^6 t^{20})\,t^2 + (20t^4 t^9 t^{15})(5t^4)$$

$$\Rightarrow \frac{dw}{dt} = 20t^{32} + 45t^{32} + 100t^{32}$$

$$\Rightarrow \frac{dw}{dt} = 165t^{32} \Rightarrow \text{Ans.}$$

17-22 Find $\partial z/\partial u$ and $\partial z/\partial v$

(17) $z = 8x^2y - 2x + 3y$

$x = uv, \quad y = u - v$

First we find $\partial z/\partial u$.

$\Rightarrow \dfrac{\partial z}{\partial u} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial u} \longrightarrow (A)$

$\Rightarrow \quad z = 8x^2y - 2x + 3y$

$x = uv$

$y = u - v.$

$\Rightarrow \dfrac{\partial z}{\partial x} = 16xy - 2$

$\dfrac{\partial x}{\partial u} = v$

$\Rightarrow \dfrac{\partial z}{\partial y} = 8x^2 + 3$

$\dfrac{\partial y}{\partial u} = 1$

$\Rightarrow (A) \quad \dfrac{\partial z}{\partial u} = (16xy - 2)v + (8x^2+3)1.$

$\Rightarrow \dfrac{\partial z}{\partial u} = (16xy - 2)v + (8x^2+3) \Rightarrow$ Ans

$\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v} \longrightarrow (B)$

$\Rightarrow \dfrac{\partial z}{\partial x} = 16xy - 2$

$\dfrac{\partial x}{\partial v} = u$

$\dfrac{\partial z}{\partial y} = 8x^2 + 3$

$\dfrac{\partial y}{\partial v} = -1$

$\Rightarrow (B) \quad \dfrac{\partial z}{\partial v} = (16xy - 2)u + (8x^2+3) - 1$

$\Rightarrow \dfrac{\partial z}{\partial v} = (16xy - 2)u - (8x^2+3) \Rightarrow$ Ans

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Practice question

ex 13.5

1 - 6 and 7 - 10 and 17 - 22