Date______20___

Limits and Continuity:-

Let f be a function of two variable, and assume that f is defined at all ~~~~ point of some open disk centred at (x_0, y_0), except possibly at (x_0, y_0). We will write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

if given any $\epsilon > 0$ we can find number $\delta > 0$ s.t $f(x,y)$ satisfied

$$|f(x,y) - L| < \epsilon$$

whenever the distance b/w (x,y) & (x_0,y_0) satisfied

$$0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$

OR

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

$$\Rightarrow \quad f(x,y) \to L \quad \text{as } (x,y) \to (x_0,y_0)$$

Continuity of a Function:-

A function have two are more then two variable, we define a function $f(x,y,z)$ of three variable to be continuous at a point (x_0, y_0, z_0) if the limit of a function and the value of a function are the same at this point,

i.e

$$\lim_{(x,y,z)\to(x_0,y_0,z_0)} f(x,y,z) = f(x_0,y_0,z_0)$$

limiting value at point (x_0, y_0, z_0) — functional value at point (x_0, y_0, z_0)

Ex 13.2

(2) $\lim_{(x,y)\to(0,0)} \dfrac{4x-y}{\sin(y)-1}$

$\Rightarrow \dfrac{4(0)-0}{\sin(0)-1} \Rightarrow \dfrac{0-0}{0-1} = \dfrac{0}{-1} = 0$

(6) $\lim_{(x,y)\to(4,-2)} x\sqrt[3]{y^3+2x}$

$\Rightarrow \lim_{(x,y)\to(4,-2)} \left[x\,(y^3+2x)^{1/3}\right]$

$\Rightarrow 4\left((-2)^3+2(4)\right)^{1/3}$

$\Rightarrow 4\left[-8+8\right]^{1/3}$

$\Rightarrow 4[0]^{1/3} = 0$

(7) Show that limit does not exist by considering the limit as $(x,y)\to(0,0)$ along the coordinate axes.

Date ______ 20__

8(a) $\lim\limits_{(x,y)\to(0,0)} \left(\frac{x-y}{x^2+y^2}\right)$

First we find the limit of $f(x,y)$ along x axis, $y=0$.

$\Rightarrow \lim\limits_{x\to 0} \left(\frac{x-0}{x^2+0}\right) \Rightarrow \lim\limits_{x\to 0} \left(\frac{x}{x^2}\right)$

$\Rightarrow \lim\limits_{x\to 0} \left(\frac{1}{x}\right) \Rightarrow \frac{1}{0} = \infty$

$\Rightarrow$ along x axis the limit does not exist so the limit of $f(x,y)$ is also does not exist.

(b)

$\lim\limits_{(x,y)\to(0,0)} \left(\frac{\cos xy}{x^2+y^2}\right)$

along y axis $\Rightarrow x=0$

$\lim\limits_{y\to 0} \left(\frac{\cos((0)y)}{(0)^2+y^2}\right) \Rightarrow \lim\limits_{y\to 0} \left(\frac{\cos(0)}{y^2}\right)$

$\Rightarrow \lim\limits_{y\to 0} \left\{\frac{1}{y^2}\right) \Rightarrow \frac{1}{0} = \infty$

$\Rightarrow$ limit $f(x,y)$ does not exist.

Date______ 20__

(9–12) Evaluate the limit using the substitution $z = x^2 + y^2$ and observing that $z \to 0^+$ iff $(x, y) \to (0, 0)$

(10)

$$\lim_{(x,y)\to(0,0)} \left[\frac{1 - \cos(x^2+y^2)}{x^2+y^2}\right]$$

$\Rightarrow$ $\because z = x^2 + y^2$

$$\Rightarrow \lim_{z\to 0} \left[\frac{1-\cos(z)}{z}\right] \Rightarrow \boxed{\frac{1-1}{0} = \frac{0}{0}}$$

Since if we put the limit $z \to 0$ the we get Undeterminate form $\frac{0}{0}$,
so, we using l, Hopital rule.

$$\Rightarrow \lim_{z\to 0} \left[\frac{\frac{d}{dz}(1-\cos(z))}{\frac{d}{dz}(z)}\right] = \left(\frac{0-(-\sin z)}{(1)}\right)$$

$$\Rightarrow \lim_{z\to 0} \left(\frac{\sin z}{1}\right) \Rightarrow \sin(0) = 1$$

$\Rightarrow$ $0 \Rightarrow$ Ans

(12) $\lim\limits_{(x,y)\to(0,0)} \left[\dfrac{e^{-1/\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}\right]$

$\because \; z = x^2+y^2$

$\Rightarrow \lim\limits_{z\to 0} \dfrac{e^{-1/\sqrt{z}}}{\sqrt{z}} \Rightarrow$

$\Rightarrow \lim\limits_{z\to 0} \dfrac{1}{\sqrt{z}}\left(e^{-1/\sqrt{z}}\right)$ — eq (1)

let $t = 1/\sqrt{z}$

when $z \to 0 \Rightarrow t \to 0$?

$t = \dfrac{1}{\sqrt{0}} \Rightarrow t = 1/0 = \infty$

when $z \to 0 \Rightarrow t \to \infty$

(1) $\Rightarrow \lim\limits_{t\to\infty} t\,e^{-t}$

$\Rightarrow \lim\limits_{t\to\infty} \left(t/e^{t}\right) = \left(\frac{\infty}{\infty}\right)$ form

again using L-Hospital rule

$\Rightarrow \dfrac{d/dt\,(t)}{d/dt\,(e^{t})} \Rightarrow \dfrac{1}{e^{t}}$

$\Rightarrow \lim\limits_{t\to 0} = \dfrac{1}{e^{t}} = \dfrac{1}{e^{\infty}} = \dfrac{1}{\infty} = 0$

$\Rightarrow$ Ans.

13-22 determine weather the limit is exist if so find the value.

(14) $\lim_{(x,y)\to(0,0)} \left[\frac{x^4-16y^4}{x^2+4y^2}\right] = \frac{0}{0}$

(does not put limit direct)

$\Rightarrow$

$$= \frac{(x^2)^2 - (4y^2)^2}{x^2+4y^2}$$

$\because a^2-b^2 = (a-b)(a+b)$

$x^4 - 16y^4 = (x^2+4y^2)(x^2-4y^2)$

$$= \frac{\cancel{(x^2+4y^2)}(x^2-4y^2)}{\cancel{x^2+4y^2}}$$

$\lim_{\substack{x\to 0\\ y\to 0}} (x^2-4y^2) = (0)^2 - 4(0)^2 = 0$

(15) $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2}$

Take a path $\rightarrow$ at x, axis i.e. $\Rightarrow y=0$

$y=0$

~~$\lim_{x\to 0} \left(\frac{x(0)}{3(x)^2+y^2}\right)$~~ $\lim_{x\to 0}\left(\frac{x(0)}{3x^2+(0)^2}\right)$

$\Rightarrow \lim \left(\frac{0}{3x^2}\right) = 0$

Date_____ 20___

$\Rightarrow$ limit of $f(x,y)$ along x axis is 0'

along y axis $\Rightarrow$ $x=0$

$$\lim_{y\to 0}\left(\frac{(0)y}{3(0)^2+2y^2}\right) = \lim_{y\to 0}\left(\frac{0}{2y^2}\right) = 0$$

$\Rightarrow$ limit of $f(x,y)$ along y axis is 0

The line $y=x$

$$\lim_{x\to 0}\left(\frac{x(x)}{3x^2+2(x)^2}\right) = \frac{x^2}{3x^2+2x^2} = \frac{\cancel{x^2}}{5\cancel{x^2}}$$

$$\lim_{x\to 0}\left(\frac{1}{5}\right) = \frac{1}{5}$$

So The limit along $y=x$ is $1/5$

So, we can deside easly in diffrent way, Then we get a different limit point $\Rightarrow$

limit of $f(x,y)$ does not exist.

(21)

$$\lim_{(x,y,z)\to(0,0,0)} \frac{e^{\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}}$$

let

$$t = x^2+y^2+z^2$$

$\Rightarrow$ $(x,y,z) \to 0 \Rightarrow t \to 0$

$\Rightarrow$ $\lim_{t\to 0} \frac{e^{\sqrt{t}}}{\sqrt{t}}$ and let

$u = \sqrt{t}$

when $t \to 0$

$\Rightarrow u \to 0.$

$\Rightarrow$ $\lim_{u\to 0} \frac{e^{u}}{u} \Rightarrow \frac{e^{(0)}}{0} = \frac{1}{0} = \infty$

$\Rightarrow$ limit does not exist

Using Polar Coordinates

(23)

$$\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}\ \ln(x^2+y^2) \to ①$$

Since we know that in Polar Coordinate

$x = r\cos\theta$

& $y = r\sin\theta$

when $(x,y) \to 0 \Rightarrow r \to 0^+$ $\because r \geq 0$

So,

Date ______ 20__

$$\lim_{r\to 0^+} \sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}\, \ln\left((r\cos\theta)^2 + (r\sin\theta)^2\right)$$

$$\Rightarrow \lim_{r\to 0} r\sqrt{\cos^2\theta + \sin^2\theta}\, \ln\left(r^2(\cos^2\theta + \sin^2\theta)\right)$$

$\because \cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \lim_{r\to 0} r\ln(r^2)$$

$$\Rightarrow \lim_{r\to 0} 2r\ln(r)$$

$\left(2(0)\ln(0) \because \ln(0) = \infty \quad 0\cdot\infty \text{ Underminate} \right)$

Using L, Hopital rule

$$\lim_{r\to 0} \cancel{\tfrac{d}{dr}(2r)} \quad (2r\ln(r))$$

(First arrange)

$$\lim_{r\to 0} \left(\frac{2\ln(r)}{1/r}\right) \Rightarrow \left(\frac{2\,1/r}{-1/r^2}\right)$$

$$\Rightarrow \lim_{r\to 0} \left(\frac{-2r^2}{r}\right) \Rightarrow (-2r)$$

$$\lim_{r\to 0} (-2r) = -2(0) = 0$$

$\Rightarrow 0$ Ans

Date______20___

Practise Question

Complete exercise 13.2

QUALITY PAPER PRODUCTS