Practise question (Tangent & Normal vector)

Ex 12.4

Question 5 to 11

Unit Binormal Vectors.

if C is The graph of vector valued function $r(t)$ in 3-space then we define The Bionormal vector to C at t to be

$$\boxed{B(t) = T(t) \times N(t)}$$

The cross product that $B(t)$ is orthogonal to both $T(t)$ and $N(t)$

Now,

$B(t)$ Can be expressed directly in term of $r(t)$ as.

$$\Rightarrow \boxed{B(t) = \frac{r'(t) \times r''(t)}{\| r'(t) \times r''(t) \|}}$$

(14) Find $B(t)$

$r(t) = 3\sin t\, i + 3\cos t\, j + 4t\, k$

As we know that

$$B(t) = \frac{r'(t) \times r''(t)}{\| r'(t) \times r''(t) \|}$$

$\Rightarrow \quad r(t) = 3\sin t\, i + 3\cos t\, j + 4t\, k.$

$\Rightarrow \quad r'(t) = 3\cos t\, i - 3\sin t\, j + 4k.$

$\Rightarrow \quad r''(t) = -3\sin t\, i - 3\cos t\, j + 0k.$

$\Rightarrow \quad r''(t) = -3\sin t\, i - 3\cos t\, j$

$\Rightarrow \quad \| r'(t) \times r''(t) \|$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 3\cos t & -3\sin t & 4 \\ -3\sin t & -3\cos t & 0 \end{vmatrix}$$

$$\Rightarrow \quad i \begin{vmatrix} -3\sin t & 4 \\ -3\cos t & 0 \end{vmatrix} - j \begin{vmatrix} 3\cos t & 4 \\ -3\sin t & 0 \end{vmatrix}$$

$$+ k \begin{vmatrix} 3\cos t & -3\sin t \\ -3\sin t & -3\cos t \end{vmatrix}$$

$\Rightarrow (0-(-12\cos t))i - j(0-(-12\sin t))$

$+k(-9\cos^2 t - 9\sin^2 t)$

$\Rightarrow 12\cos t\, i - j(12\sin t) - k9(\cos^2 t + \sin^2 t)$

$\Rightarrow = 12\cos t\, i - 12\sin t\, j - 9k$

$\|r'(t) \times r''(t)\| =$

$= \sqrt{(12\cos t)^2 + (-12\sin t)^2 + (-9)^2}$

$= \sqrt{144\cos^2 t + 144\sin^2 t + 81}$

$= \sqrt{144 + 81}$

$= \sqrt{225} \Rightarrow 15$

$\Rightarrow \|r'(t) \times r''(t)\| = 15$

$\Rightarrow B(t) = \dfrac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|}$

$\Rightarrow B(t) = \dfrac{12\cos t\, i - 12\sin t\, j - 9k}{15}$

$$B(t) = \frac{12\cos t\, i}{15} - \frac{12\sin t\, j}{15} - \frac{9k}{15}$$

$$\Rightarrow B(t) = \tfrac{4}{5}\cos t\, i - \tfrac{4}{5}\sin t\, j - \tfrac{3}{5}k$$

$\Rightarrow$ Ans.

(16) Find $B(t) = ?$

$$\Rightarrow r(t) = e^t \sin t\, i + e^t \cos t\, j + 3k$$

$$\Rightarrow B(t) = \frac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|}$$

$$\Rightarrow r(t) = e^t \sin t\, i + e^t \cos t\, j + 3k$$

$$\Rightarrow r'(t) = (e^t \cos t + \sin t\, e^t)\, i + (-e^t \sin t + \cos t\, e^t)\, j + 0k$$

$$\Rightarrow r''(t) = (-\cancel{e^t \sin t} + e^t \cos t + \cos t\, e^t + \cancel{e^t \sin t})\, i$$

$$+ (-\cancel{e^t \cos t} - e^t \sin t - \sin t\, e^t + \cancel{\cos t\, e^t})\, j$$

$$\Rightarrow r''(t) = 2e^t \cos t\, i - 2e^t \sin t\, j$$

$$\Rightarrow \|r'(t) \times r''(t)\|$$

$$\begin{vmatrix} i & j & k \\ e^t \cos t + \sin t\, e^t & -e^t \sin t + e^t \cos t & 0 \\ 2e^t \cos t & -2e^t \sin t & 0 \end{vmatrix}$$

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$i(0-0) - j(0-0)$

$+k(-2e^t \sin t(e^t \cos t + e^t \sin t) + 2e^t \cos t(e^t \cos t - e^t \sin t)$

$\Rightarrow k(-2e^{2t} \cancel{\sin t \cos t} - 2e^{2t} \sin^2 t + 2e^{2t} \cos^2 t + 2e^{2t} \cancel{\cos t \sin t})$

$\Rightarrow \quad k(-2e^{2t}(\underbrace{\sin^2 t + \cos^2 t}_{=1}))$.

$\Rightarrow \quad -2e^{2t} k$

$\Rightarrow \quad r'(t) \times r''(t) = -2e^{2t} k$.

$\Rightarrow \quad \|r'(t) \times r''(t)\| = \sqrt{(-2e^{2t})^2}$

$= \sqrt{4e^{4t}}$

$\Rightarrow \; = 2e^{2t}$

$\Rightarrow \quad \|r'(t) \times r''(t)\| = 2e^{2t}$

$B(t) = \left\| \dfrac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|} \right\|$

$\Rightarrow \quad B(t) = \dfrac{-2e^{2t} k}{2e^{2t}}$

$\Rightarrow \quad B(t) = \dfrac{-2\cancel{e^{2t}}\,k}{2\cancel{e^{2t}}}$

$\Rightarrow \quad B(t) = -k \Rightarrow$ Ans

↳ Binormal vector

(18) $r(t) = a\cos t\, i + a\sin t\, j + ct\, k$ find Binormal vector

$\Rightarrow \quad r(t) = a\cos t\, i + a\sin t\, j + ct\, k$

$\Rightarrow \quad r'(t) = -a\sin t\, i + a\cos t\, j + c\, k$

$\Rightarrow \quad r''(t) = -a\cos t\, i - a\sin t\, j$

$\Rightarrow \quad r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -a\sin t & a\cos t & c \\ -a\cos t & -a\sin t & 0 \end{vmatrix}$

$\Rightarrow \quad ac\sin t\, i + ac\cos t\, j + (a^2\sin^2 t + a^2\cos^2 t)k$

$\Rightarrow \quad \cancel{ac\sin t\, i + a^2\cos^2 t\, j + a^2 k}$

$\Rightarrow \quad ac\sin t\, i - ac\cos t\, j + a^2 k$

$$\|r'(t) \times r''(t)\| = \sqrt{(ac\sin t)^2 + (-ac\cos t)^2 + (a^2)^2}$$

$$\sqrt{a^2c^2\sin^2 t + a^2c^2\cos^2 t + a^4}$$

$$\Rightarrow \sqrt{a^2c^2 + a^4}$$

$$\Rightarrow \sqrt{a^2(c^2 + a^2)}$$

$$= a\sqrt{a^2 + c^2}$$

$$\Rightarrow B(t) = \frac{r'(t) \times r''(t)}{\cancel{a} \|r'(t) \times r''(t)\|}$$

$$\Rightarrow B(t) = \frac{ac\sin t\, i - ac\cos t\, j + a^2 k}{a\sqrt{a^2+c^2}}$$

$$\Rightarrow B(t) = \frac{\cancel{a}c\sin t\, i}{\cancel{a}\sqrt{a^2+c^2}} - \frac{\cancel{a}c\cos t\, j}{\cancel{a}\sqrt{a^2+c^2}} + \frac{\cancel{a^2}\, k}{\cancel{a}\sqrt{a^2+c^2}}$$

$$\Rightarrow B(t) = \frac{c\sin t\, i}{\sqrt{a^2+c^2}} - \frac{c\cos t\, j}{\sqrt{a^2+c^2}} + \frac{a k}{\sqrt{a^2+k^2}}$$

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Practice Question (Binormal vector)

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Question 15, 16, 17, 18