"Exercise NO 15.2"

(17) Evalute The line integral with r. to s along The curve C.

$$\int_C \frac{1}{1+x}\, ds$$

$C;\ r(t) = ti\ \frac{2}{3} t^{3/2} j$

$(0 \le t \le 3)$

$\Rightarrow$ using

$$\int_C f(x,y,z)\, ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

$\Rightarrow$ $x = ti$ $\qquad \frac{dx}{dt} = 1$

$y = \frac{2}{3} t^{3/2} j$

$dy = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) t^{\frac{3}{2}-1} \Rightarrow t^{\frac{3-2}{2}} \Rightarrow t^{1/2}$

$\frac{dy}{dt} = t^{1/2}$

$$\Rightarrow \int_0^3 \frac{1}{1+t} \sqrt{(1)^2 + ((t)^{1/2})^2}\, dt$$

$$\Rightarrow \int_0^3 \frac{1}{1+t} \sqrt{1+t}\, dt$$

$$\int_0^3 (1+t)^{1/2} (1+t)^{-1}\, dt$$

$$\Rightarrow \int_0^3 (1+t)^{\frac{1}{2}-1}\, dt \Rightarrow \int_0^3 (1+t)^{-1/2}\, dt.$$

$$\int_0^3 (1+t)^{-1/2}\, dt \Rightarrow \left.\frac{(1+t)^{-1/2+1}}{-1/2+1}\right]_0^3$$

$$\Rightarrow \left.\frac{(1+t)^{1/2}}{1/2}\right]_0^3 \Rightarrow \left.2\sqrt{1+t}\right)_0^3$$

$$\Rightarrow 2\sqrt{1+3} \Rightarrow 2\sqrt{4} \Rightarrow 2(2) = 4$$

$\hookrightarrow$ Ans

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(22) $$\int_C \frac{e^{-z}}{x^2+y^2}\, ds$$

$C;\ r(t) = 2\cos t\, i + 2\sin t\, j + tk$

$(0 \le t \le 2\pi)$

Since

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

So,

$x = 2\cos t\, i \qquad \frac{dx}{dt} = -2\sin t$

$y = 2\sin t \qquad \frac{dy}{dt} = 2\cos t$

$z = t \qquad \frac{dz}{dt} = 1$

$$\Rightarrow \int_0^{2\pi} \frac{e^{-t}}{(2\cos t)^2 + (2\sin t)^2} \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (1)^2}$$

$$\int_0^{2\pi} \frac{e^{-t}}{4} \sqrt{5}\, dt$$

$$\Rightarrow \frac{\sqrt{5}}{4} \int_0^{2\pi} e^{-t}\, dt \Rightarrow -\frac{\sqrt{5}}{4} \left(e^{-t}\right)_0^{2\pi}$$

$$-\sqrt{5}/4 \left(e^{-2\pi} - e^{0}\right) \Rightarrow \frac{-\sqrt{5}}{4}\left(e^{-2\pi} - 1\right)$$

$$\Rightarrow \frac{\sqrt{5}}{4}\left(1 - e^{-2\pi}\right) \Rightarrow \text{Ans}$$

(Q3) Evalute The line integral along The curve C.

$$\int_C (x - 2y)\, dx + (x - y)\, dy$$

$C;\ x = 2\cos t \quad y = 4\sin t$

$(0 \le t \le \pi/4)$

Given

$x = 2\cos t \quad dx = -2\sin t$

$y = 4\sin t \quad dy = 4\cos t$

$0 \le t \le \pi/4$

$$\int_0^{\pi/4} (2\cos t - 2(4\sin t))(-2\sin t) + (2\cos t - 4\sin t)\, 4\cos t$$

$$\Rightarrow \int_0^{\pi/4} -4\cos t\sin t + 16\sin^2 t + 8\cos^2 t$$

$$+ 16\sin t\cos t$$

$$\Rightarrow \int_0^{\pi/4} -4\cos t\sin t + 16\sin^2 t + 8\cos^2 t - 16\sin t\cos t\,dt$$

$$\Rightarrow \int_0^{\pi/4} -20\cos t\sin t + 8(2\sin^2 t + \cos^2 t)\,dt$$

$$\Rightarrow -20\int_0^{\pi/4} \cos t\sin t + 16\sin^2 t + 8\cos^2 t\,dt$$

$\Rightarrow -20$

$u = \cos t$

$du = -\sin t\,dt \Rightarrow dt = \dfrac{du}{-\sin t}$

when $t \to \pi/4$. Then $u \to 1/\sqrt{2}$

" $t \to 0$ Then $u \to 1$

$$+20\left(\frac{u^2}{2}\right)_1^{1/\sqrt{2}} + \int_0^{\pi/4}\frac{16}{2}(1-\cos 2\theta) + \frac{8}{2}\int_0^{\pi}(1+\cos 2\theta)\,d\theta$$

$$\Rightarrow 10\left(\frac{1}{2}-1\right) + \frac{16}{2}\left(\theta - \frac{\sin 2\theta}{2}\right) + 4\left(\theta - \frac{\sin 2\theta}{2}\right).$$

$$\Rightarrow -\frac{10}{2} + 8\left(\frac{\pi}{4} - \frac{1}{2}\right) + 4\left(\pi/4 - \frac{1}{0}\right)$$

$$\Rightarrow -\frac{10}{2} + 8\left(\frac{\pi}{4} - \frac{1}{2}\right) + 4\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$-\frac{10}{2} + 12\left(\frac{\pi}{4} - \frac{1}{2}\right) \Rightarrow \text{Ans}$$

(25) $\int_c -y\,dx + x\,dy$

$C: y^2 = 3x$ from $(3,3)$ to $(0,0)$

Given $\int_c -y\,dx + x\,dy \rightarrow (A)$

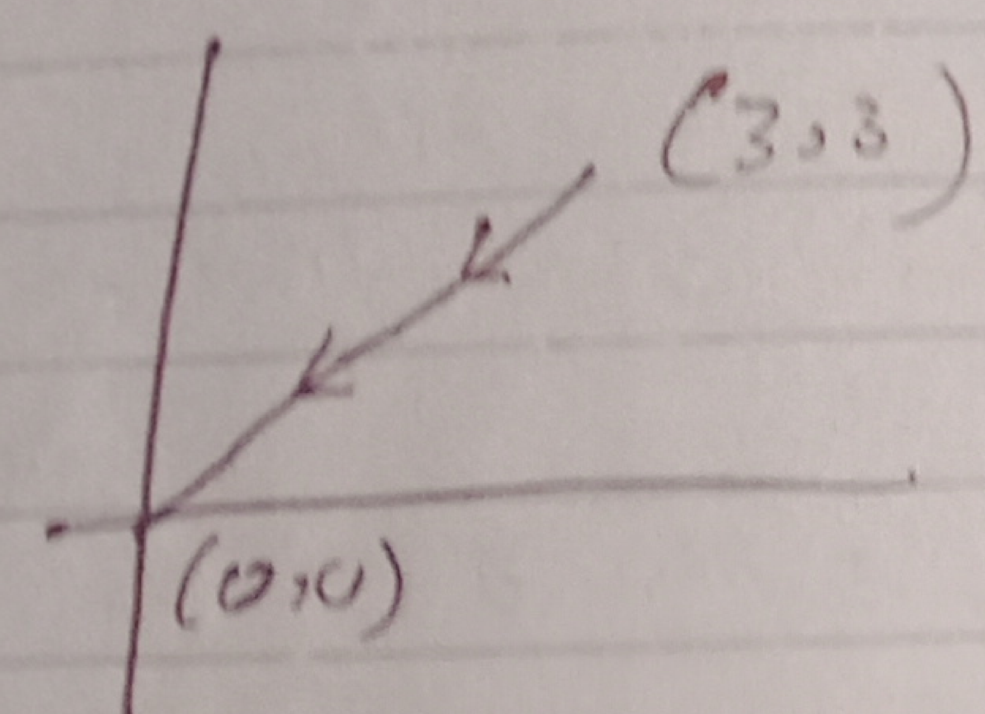
Sol,

let $t = y \Rightarrow dy = 1$

$t^2 = 3x$

$\Rightarrow x = \frac{t^2}{3} \quad dx = \frac{2t}{3}$

$\Rightarrow \int_3^0 -t\left(\frac{2t}{3}\right) + \frac{t^2}{3}(1)\,dt$



$\Rightarrow \int_3^0 \frac{-2t^2}{3} + \frac{t^2}{3}\,dt$

$\Rightarrow \int_3^0 \frac{-t^2}{3}\,dt$

$\Rightarrow +\frac{1}{3}\left(\frac{t^3}{3}\right)_0^3 \Rightarrow \frac{1}{9}(27 - 0)$

$\Rightarrow 27/9 \Rightarrow 3 \Rightarrow$ Ans

27) $\int_c (x^2+y^2)\,dx - x\,dy$

$C: x^2+y^2 = 1$ counter clock wise

from $(1,0)$ to $(0,1)$

(25) $\int_c -y\,dx + x\,dy$

C: $y^2 = 3x$ from $(3,3)$ to $(0,0)$

Given $\int_c -y\,dx + x\,dy \rightarrow (A)$

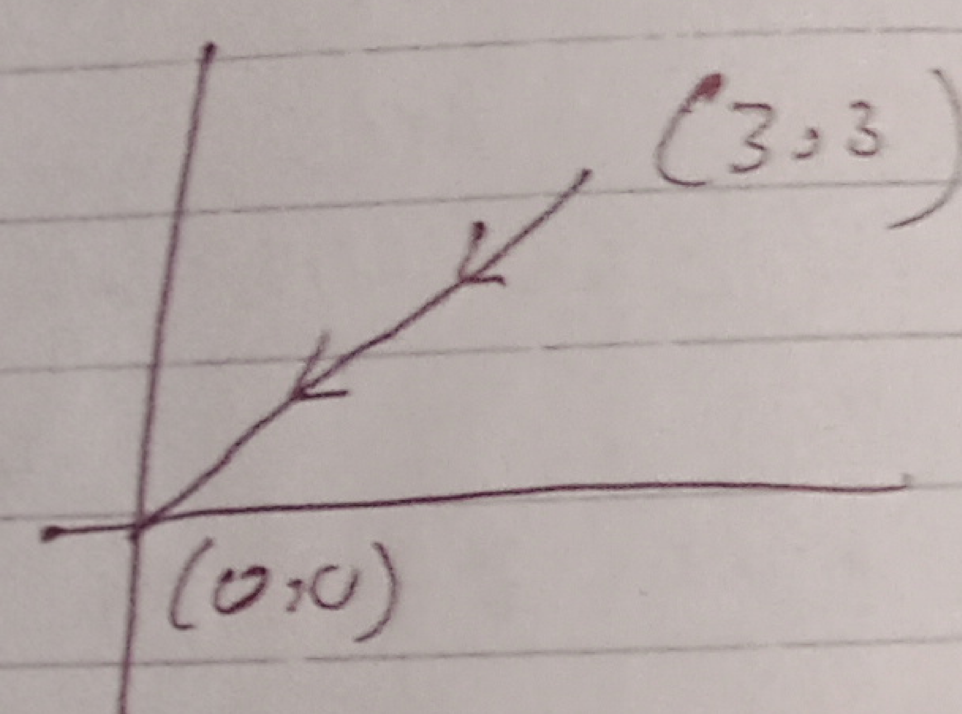
So;

let $t = y \Rightarrow dy = 1$

$t^2 = 3x$

$\Rightarrow x = \frac{t^2}{3}$ $\quad dx = \frac{2t}{3}$

$$\Rightarrow \int_3^0 -t\left(\frac{2t}{3}\right) + \frac{t^2}{3}(1)\,dt$$



$$\Rightarrow \int_3^0 \frac{-2t^2}{3} + \frac{t^2}{3}\,dt$$

$$\Rightarrow \int_3^0 \frac{-t^2}{3}\,dt$$

$$\Rightarrow +\frac{1}{3}\left(\frac{t^3}{3}\right)_0^3 \Rightarrow \frac{1}{9}(27-0)$$

$\Rightarrow 27/9 \Rightarrow 3 \Rightarrow$ Ans

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27) $\int_c (x^2+y^2)\,dx - x\,dy$

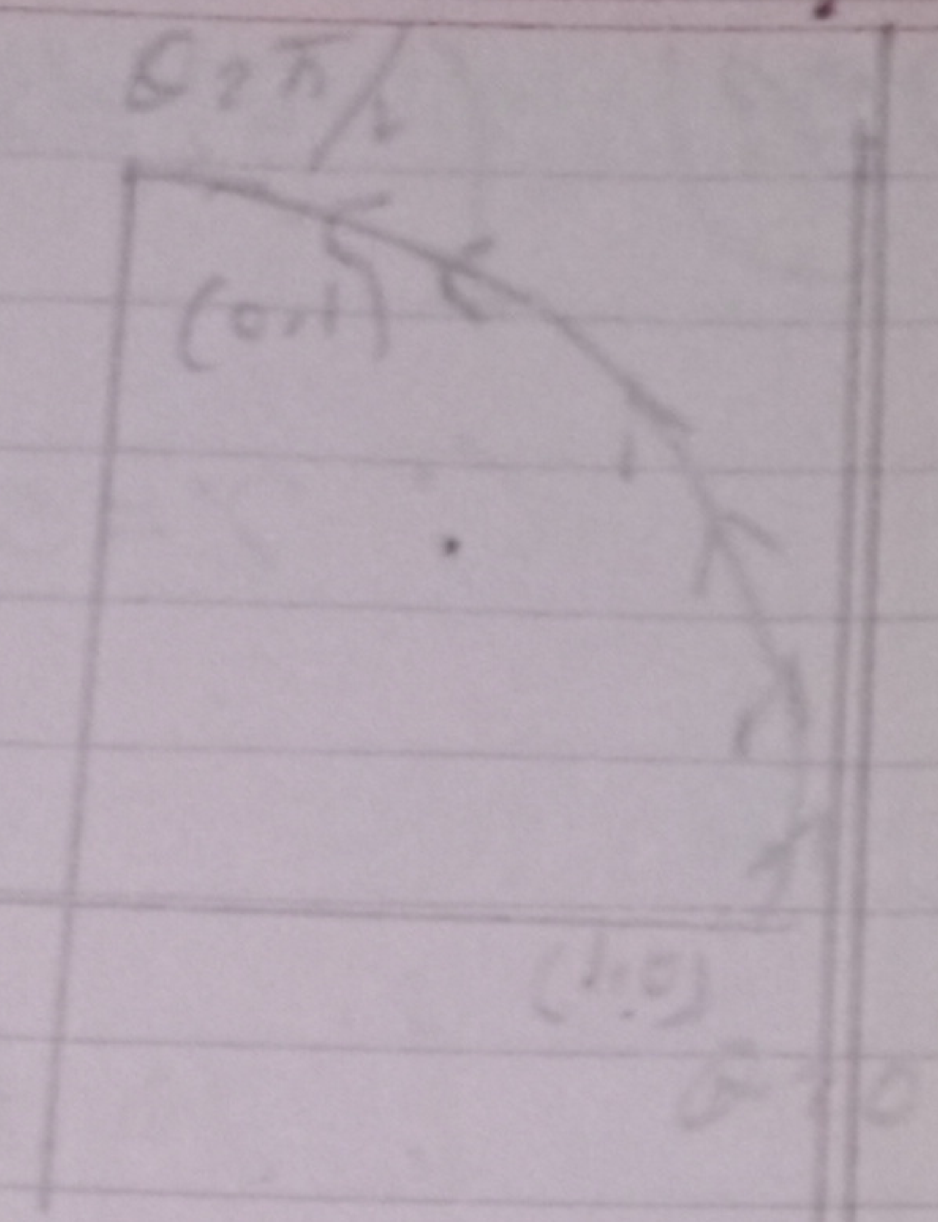
C: $x^2+y^2=1$ counter clock wise

from $(1,0)$ to $(0,1)$

$x^2+y^2=1$

So, $0 \leq \theta \leq \pi/2$

$0 \leq t \leq \pi/2$



$x = r\cos t \quad \because r=1 \; = \cos t$

$y = r\sin t \quad \because r=1 \; = \sin t$

$dx = -\sin t$

$dy = \cos t$

$$\Rightarrow \int_0^{\pi/2} (\cos t)^2 + (\sin t)^2(-\sin t) - \cos t(\cos t)$$

$$\int_0^{\pi/2} -\sin t - \cos^2 t \, dt$$

$$\Rightarrow -(-\cos t)\Big|_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2}\left(1 - \cos 2t\right) dt$$

$$\Rightarrow \cos t\Big|_0^{\pi/2} - \left(\frac{1}{2}\left(t - \frac{\sin 2t}{t}\right)\right)\Bigg|_0^{\pi/2}$$

$$\Rightarrow 0 - 1 - \left(\frac{1}{2}\left(\pi/2 - 0\right)\right)$$

$$\Rightarrow \left(-1 - \frac{\pi}{4}\right) \Rightarrow \left(\frac{-4-\pi}{4}\right)$$

So, $\left(-4 - \pi/4\right)$

$\longrightarrow$ Ans In parametrically

Q 29) $\int_c yz\,dx - xz\,dy + xy\,dz$

$C;\ x = e^{t} \quad y = e^{3t} \quad z = e^{-t}$

$0 \le t \le 1$

Sol

$\int_c yz\,dx - xz\,dy + xy\,dz \rightarrow (1)$

$x = e^{t} \Rightarrow dx = e^{t}$
$y = e^{3t} \Rightarrow dy = 3e^{3t}$
$z = e^{-t} \Rightarrow dz = -e^{-t}$

So, (1) becomes

$$\int_0^1 \left((e^{3t})(e^{-t})\right)e^{t} - 3\left((e^{t})(e^{-t})\right)e^{t} + (e^{t})(e^{3t})e^{-t}\,dt$$

$$\Rightarrow \int_0^1 e^{3t-t+t}\,dt - 3e^{3t-t+t}\,dt - e^{t+3t-t}\,dt$$

$$\Rightarrow \int_0^1 e^{3t}\,dt - 3e^{3t}\,dt - e^{3t}\,dt$$

$$\Rightarrow \int_0^1 (e^{3t} - 3e^{3t} - e^{3t})\,dt$$

$$\Rightarrow \int_0^1 (-2e^{3t} - e^{3t})\,dt \Rightarrow \int_0^1 -3e^{3t}$$

$$-\frac{3}{3}\left(e^{3t}\right]_0^1 \Rightarrow -1(e^3 - 1)$$

$$\Rightarrow (1 - e^3) \Rightarrow \text{Ans}$$

Evalate $\int_C F \cdot dr$ along the curve;

$F(x,y) = x^2 i + xy j$

$C: r(t) = 2\cos t\, i + 2\sin t\, j \quad (0 \le t \le \frac{\pi}{2})$

$\int_C F \cdot dr = \int_a^b F\, r(t) \cdot r'(t)\, dt$

$r(t) = 2\cos t\, i + 2\sin t\, j$

$r'(t) = -2\sin t\, i + 2\cos t\, j]\, dt$

$\int_0^{\pi} (2\cos t\, i + 2\sin t\, j)(-2\sin t\, i + 2\cos t\, j)\, dt$

$\Rightarrow \int_0^{\pi} (-4\cos t \sin t + 4\sin t \cos t)\, dt$

$\int_0^{\pi} 0\, dt \Rightarrow 0 \Rightarrow$ Ans

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(35) $\int_C (x^2+y^2)^{-3/2} (xi + yj)$

$C;\ e^t \sin t\, i + e^t \cos t\, j \quad (0 \le t \le 1)$

$\int_C (x^2+y^2)^{-3/2} (xi + yj) \to$ (A)

$r(t) = e^t \sin t\, i + e^t \cos t\, j$

$r'(t) = (e^t \cos t + \sin t e^t) i + (-e^t \sin t + \cos t e^t)$

S; (A) becomes

$$\int_0^1 \left[(e^t \sin t)^2 + (e^t \cos t)^2\right]^{-3/2} \left((e^t \sin t)i + (e^t \cos t)j\right) \cdot \left(e^t \cos t\, i + e^t \sin t\right) i - \left(e^t \sin t + \cos t\, e^t\right)$$

$$\int_0^1 e^{t^{2 \cdot -3/2}} (\sin t^2 + \cos t^2) \left(e^t \sin t\, i\right) i + (e^t \cos t) j$$

$$\cdot \left(e^t \cos t\, i + e^t \sin t\right) i - \left(e^t \sin t + \cos t\, e^t\right) j$$

$$\Rightarrow \int_0^1 (e^{-3t} e^t \sin t) i + (e^{-3t} e^t \cos t) j \cdot (e^t \cos t\, i + e^t \sin t) i$$

$$- (e^t \sin t + \cos t\, e^t) j$$

$$\Rightarrow \int_0^1 e^{-2t} e^t (\sin t \cos t) + e^{-2t} e^t \sin^2 t \, dt$$

$$- \left(e^{-2t} e^t \cos t \sin t + e^{-2t} e^t \cos^2 t \, dt\right)$$

$$\int_0^1 e^{-t} \left(\sin t \cdot \cos t \cancel{+ \sin^2 t} - \cos t \sin t \right.$$

$$\left. + \cos^2 t \right) dt$$

$$\int_0^1 + e^{-t} (+\sin^2 t + \cos^2 t)\, dt$$

$$\Rightarrow \int_0^1 e^{-t} \, dt$$

$$\Rightarrow \left.\frac{e^{-t}}{-1}\right]_0^1 \Rightarrow -(e^{-1} - 1)$$

$$\Rightarrow (1 - e^{-1})$$

$\Longrightarrow$ Ans

"Exercise No 15.3"

Examples:-

$$F(x,y) = 2xy^3 i + (1+3x^2y^2) j$$

ⓐ Find That f is Conservation Vector field
ⓑ Find $\phi = ?$
ⓒ Find ϕ first integrating $\partial\phi/\partial y$

ⓐ

$$f(x,y) = 2xy^3$$
$$g(x,y) = 1+3x^2y^2$$

$$\frac{\partial f}{\partial y} = 6xy^2 \qquad \frac{\partial g}{\partial x} = 6xy^2$$

Since $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

$f(x,y)$ is a Conservation Field.
So we find out a ϕ.

So, $\frac{\partial \phi}{\partial x} = 2xy^3$ and $\frac{\partial \phi}{\partial y} = 1+3x^2y^2$

we Find the ϕ,

so; $\partial \phi = 2xy^3 dx$

$$\phi = \int 2xy^3 \partial x$$

$$\phi = \frac{2x^2y^3}{2} + k(y)$$

$$\phi = x^2y^3 + k(y) \longrightarrow (A)$$

differentiate w.r.to y.

$$\frac{\partial \phi}{\partial y} = 3x^2y^2 + k'(y)$$

$$1+3x^2y^2 = 3x^2y^2 + k'(y)$$

$$\Rightarrow 1+\cancel{3x^2y^2}-\cancel{3x^2y^2} = k'(y)$$

$$\Rightarrow k'(y) = 1$$

integrate w.r.to y.

$$k(y) = y + k$$

so, (A) becomes

$$\phi = x^2y^3 + y + k \Rightarrow \text{Ans}$$

Exercise Question:-

Determine wether F is conservation vector field. and find potential function.

(1) $F(x,y) = xi + yj$

Solution

$f(x,y) = xi$ and $g(x,y) = y$.

$\frac{\partial f}{\partial y} = 0$ $\quad \frac{\partial g}{\partial x} = 0$

$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ So $F(x,y)$ is a conservation field. we find out the potential function.

$\frac{\partial \phi}{\partial x} = x$ & $\frac{\partial \phi}{\partial y} = y$

$\Rightarrow \partial \phi = x \, \partial x$

$\phi = \int x \, dx$

$\phi = \frac{x^2}{2} + k(y) \longrightarrow (A)$

diffrent w.r.to y

$\frac{\partial \phi}{\partial y} = 0 + k'(y)$

$y = k'(y)$

integrate w.r.to y.

(4)

$k(y) = y^2/2 + k$

Put in (A)

$\Rightarrow \quad \phi = \frac{x^2}{2} + \frac{y^2}{2} + k$

So the required potential function is above.

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(4) $F(x,y) = e^x \cos y\, i - e^x \sin y\, j$

$f = e^x \cos y \qquad g = -e^x \sin y$

$\Rightarrow \quad \frac{\partial f}{\partial y} = -e^x \sin y \qquad \frac{\partial g}{\partial x} = -e^x \cos y$

So it is clear to that from the above result $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

we find the potential function

$\frac{\partial \phi}{\partial x} = e^x \cos y \qquad \frac{\partial \phi}{\partial y} = -e^x \sin y$

$\Rightarrow \partial \phi = e^x \cos y\, \partial x \Rightarrow \phi = \int e^x \cos y\, \partial x$

$\phi = e^x \cos y + k(y) \rightarrow$ (A)

differentiate w.r.to y.

$\frac{\partial \phi}{\partial y} = -e^x \sin y + k'(y)$

$-e^x \sin y + e^x \sin y = k'(y)$

$\Rightarrow k'(y) = 0$

integrate w.r.t y.

$\Rightarrow k(y) = 0 + k$ so (A) becomes

$\Rightarrow \phi = e^x \cos y + k$

So The required Potential function functions.

(Q5) $F(x,y) = (\cos y + y\cos x)\hat{i} + (\sin x - x\sin y)\hat{j}$

$f = \cos y + y\cos x$

$g = \sin x - x\sin y$

$\frac{\partial f}{\partial y} = -\sin y + \cos x \rightarrow (i)$

$\partial g/\partial x = \cos x - \sin y \rightarrow (ii)$

from (i) & (ii)

$\partial g/\partial x = \frac{\partial f}{\partial y}$

So $F(x,y)$ is conservative field. and we find out the Potential function ϕ.

$\frac{\partial \phi}{\partial x} = \cos y + y\cos x$

$\frac{\partial \phi}{\partial y} = \sin x - x\sin y$

$\Rightarrow \partial\phi = \cos y + y\cos x \, dx$

$\phi = \int (\cos y + y\cos x)$

$\Rightarrow \phi = x\cos y + y\sin x + k(y)$ (A)

diffrete w.r.to y.

$\frac{\partial \phi}{\partial y} = -x\sin y + \sin x + k'(y)$

$\sin x - x\sin y = -x\sin y + \sin + k'(y)$

$k'(y) = 0$

integrate y,

$k(y) = 0 + k$ put in (A)

$\Rightarrow \phi = x\cos y + y\sin x + k$

So The required $f(x,y)$ is above.

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(6) $F(x,y) = x\ln y\, i + y\ln x\, j$

Sol $f = x\ln y \qquad g = y\ln x$

$\frac{\partial f}{\partial y} = x/y \qquad \frac{\partial g}{\partial x} = y/x$

$\frac{\partial f}{\partial y} \neq \partial g/\partial x$ So does not a conservation field.

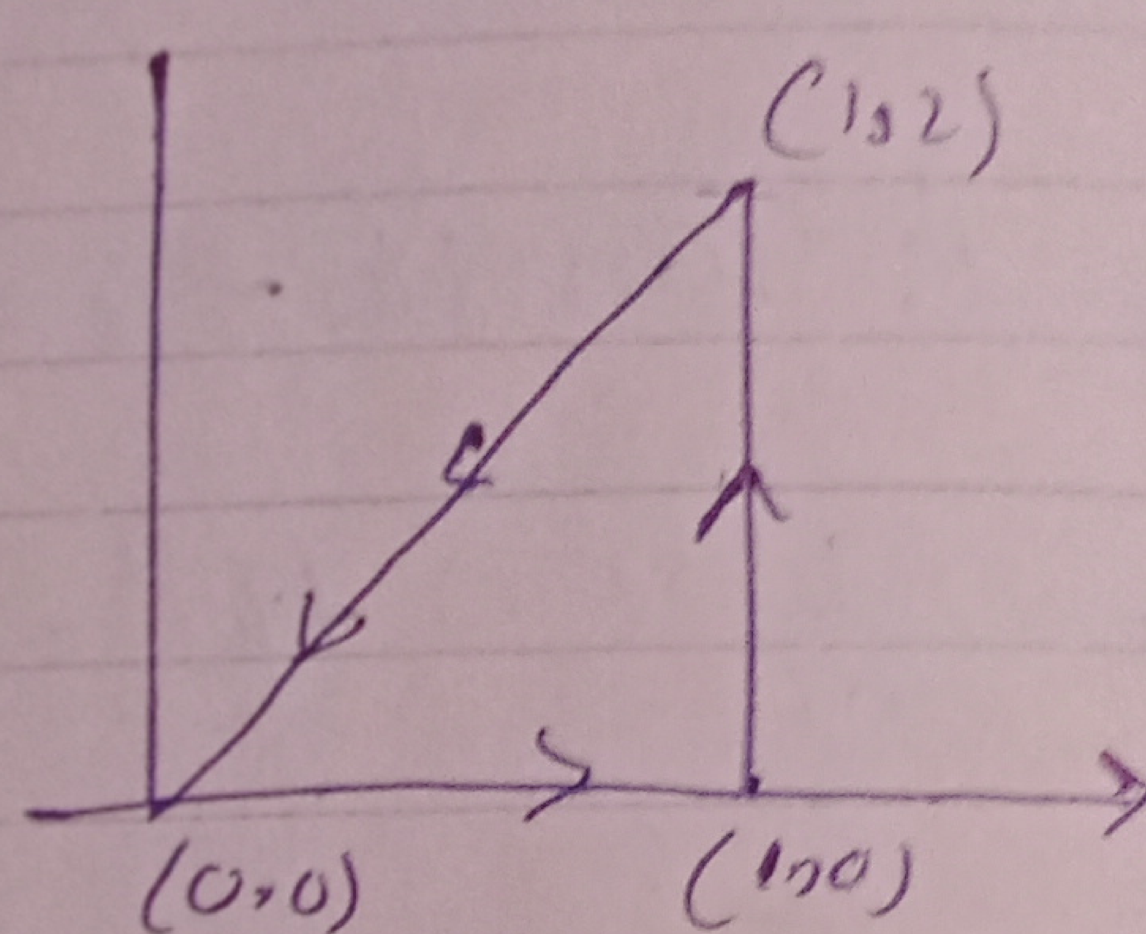
Exercise No 15.4

Use Green Theorm to evaluate?

$\int_c x^2y\,dx + x\,dy$

$f(x,y) = x^2y$ and $g(x,y) = x$

$\frac{\partial f}{\partial y} = x^2$ and $\frac{\partial g}{\partial x} = 1$



$\int_c x^2y\,dx + x\,dy =$

$\iint_R \left(\left(\frac{\partial g}{\partial x}\right) - \left(\frac{\partial f}{\partial y}\right) \right) dA$

$\Rightarrow \int_0^1 \int_0^{2x} (1 - x^2)\,dx\,dy$

$\Rightarrow \int_0^1 \left(x - \frac{x^3}{3} \right]_0^{2x} dy$

$\cancel{\Rightarrow \int_0^{2x} \left(1 - \frac{1}{3}\right) dy \Rightarrow \left(y - \frac{y}{3}\right]_0^{2x}}$

$\Rightarrow \int_0^1 \left(2x^2 - \frac{2x^4}{3}\right) dx$

$\Rightarrow \left(x^2 - \frac{x^4}{2}\right) = \frac{1}{2}$

Example (2)

Find The work done by Force Field.

$F(x,y) = (e^x - y^3)i + (\cos y + x^3)j$

particles moves ince circular path

$x^2 + y^2 = 1$

$w = \oint_c F \, dr.$

$\oint (e^x - e^y)\,dx + (\cos y + x^3)\,dy$

$\Rightarrow \frac{\partial g}{\partial x} = 3x^2$

$\frac{\partial f}{\partial y} = -3y^2$

$w = \oint_0^{2\pi} \int_0^1 3x^2 + 3y^2 \, dA$

$w = \int_0^{2\pi} \int_0^1 3(x^2 + y^2) \, r \, dr \, d\theta$

$\cancel{\int_0^{2\pi} \left[\frac{3r^3}{3}\right]_0^1} \Rightarrow \int_0^{2\pi} \int_0^1 3(r^2) r \, dr$

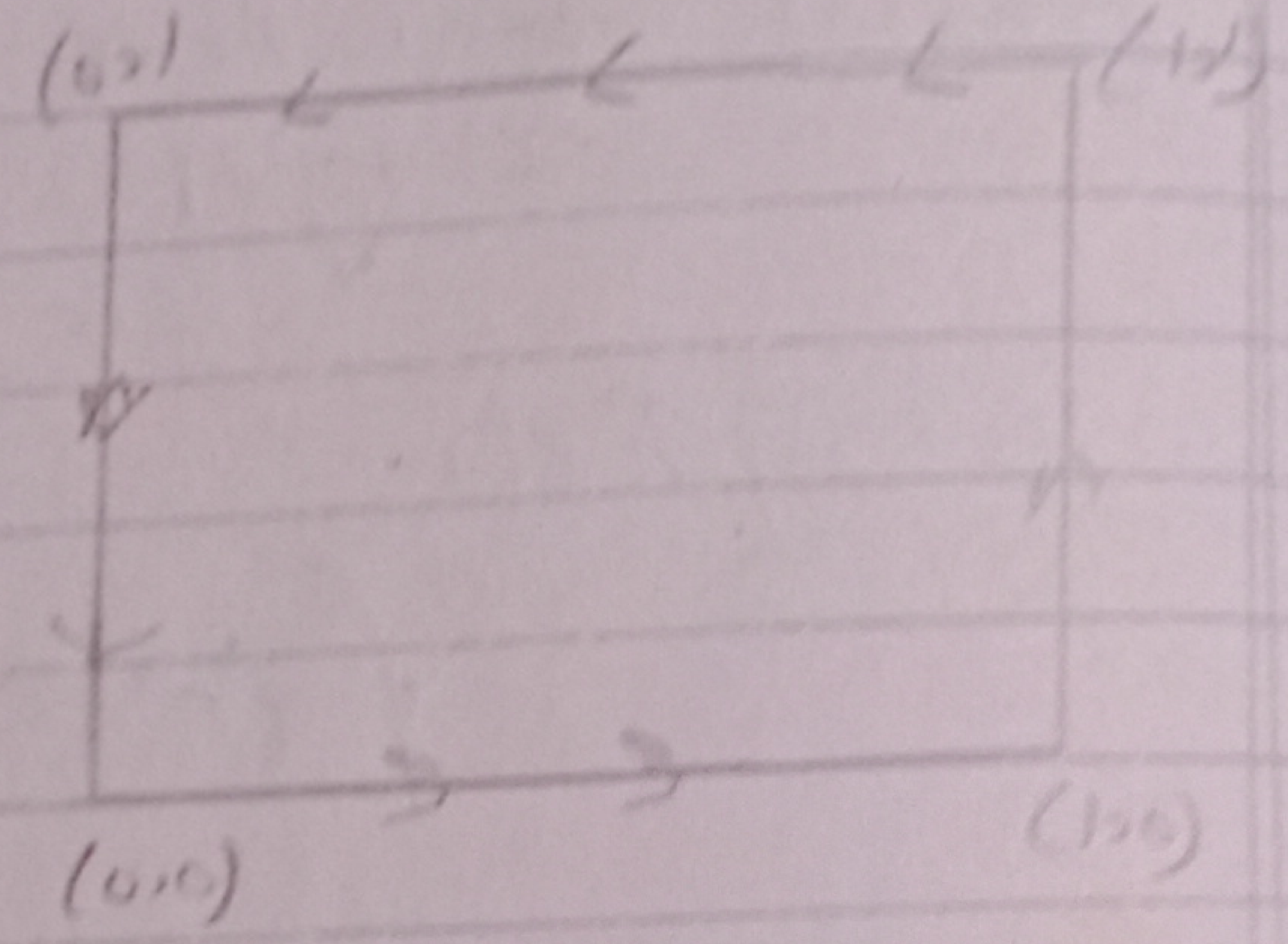
$\Rightarrow \int_0^{2\pi} \left(\left[\frac{3r^4}{4}\right]_0^1\right) \Rightarrow \frac{6\pi}{4} \Rightarrow \frac{3\pi}{2}$

$\Rightarrow$ Ans

Exercise Q (15.4)

Evalue using Geem Theorm

Q(1) $\oint_c y\,dx + x^2 dy$ where C, is the square with vertexes (0,0) (1,0) (1,1) (0,1) oriented counterclockwise.



From the figer it is clear to that

$0 \le x \le 1$

$0 \le y \le 1$

So

A/c To Green Theorm

$$\iint_D \left(\frac{\partial x}{\partial x} - \frac{\partial f}{\partial y} \right) dA \quad — (A)$$

$$\frac{\partial x}{\partial x} = \partial x$$

$$\frac{\partial f}{\partial y} = \partial y$$

(A) becomes

$$\Rightarrow \int_0^1\int_0^1 (\partial x - \partial y)\, dx\, dy$$

$$\Rightarrow \int_0^1 \left(\frac{\partial x^2 y}{\partial} - \partial y x \right)_0^1 dy$$

$$(d)\int_0 y - 2y \Rightarrow \left[\frac{y^2}{2} - \frac{2y^2}{2}\right]_0^1$$

$$(d) \Rightarrow \left(\frac{1}{2} - 1\right) \Rightarrow \left(\frac{1-2}{2}\right) \Rightarrow -1/2$$

$\hookrightarrow$ Ans

$$\Rightarrow \int_0^1 (1 - 2y)\,dy \Rightarrow \left(y - \frac{2y^2}{2}\right)_0^1$$

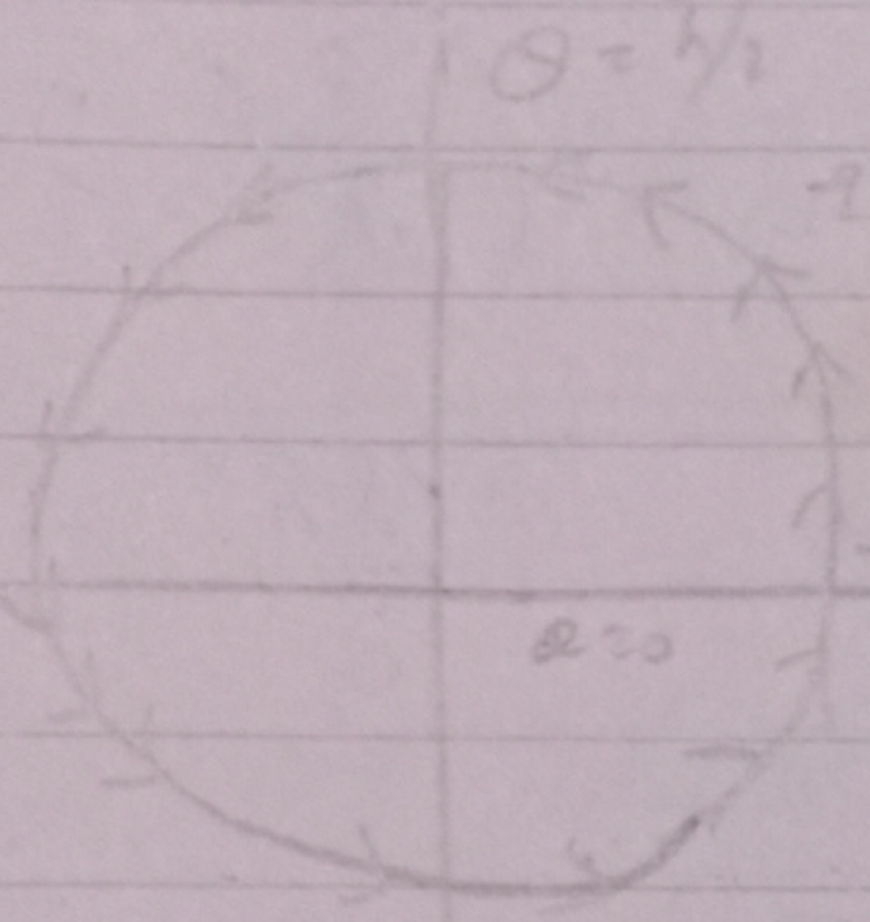
$$\Rightarrow (1-1) \Rightarrow 0 \Rightarrow \text{Ans}$$

Q4 $\oint_c (x^2 - y^2)\,dx + x\,dy$ where c is The circle $x^2 + y^2 = 9$

Given $x^2 + y^2 = 9$

$0 \leq r \leq 3$

$0 \leq \theta \leq 2\pi$



$f(x,y) = (x^2 - y^2)$

$g(x,y) = x$

$\frac{\partial f}{\partial y} = -2y$

$\frac{\partial g}{\partial x} = 1$

Acc to Green Theorm:

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$\Rightarrow$ So it is clea to Thate

$$\int_0^{2\pi}\int_0^{3} (1+2y)\, dA$$

$$\int_0^{2\pi}\int_0^{3} (1+2r\sin\theta)\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\int_0^{3} (r+2r^2\sin\theta)\, dr\right) d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\frac{r^2}{2}+\frac{2r^3}{3}\sin\theta\right)_0^{3} d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\frac{9}{2}+18\sin\theta\right) d\theta$$

$$\Rightarrow \left(\frac{9}{2}\theta-18\cos\theta\right)_0^{2\pi}$$

$$\Rightarrow \frac{18\pi}{2}-18(1-1)$$

$$\Rightarrow (9\pi) \Rightarrow \text{Ans.}$$

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(7) $(x^2-y)\,dx + x\,dy$ where C is the circle line $x^2+y^2=4$

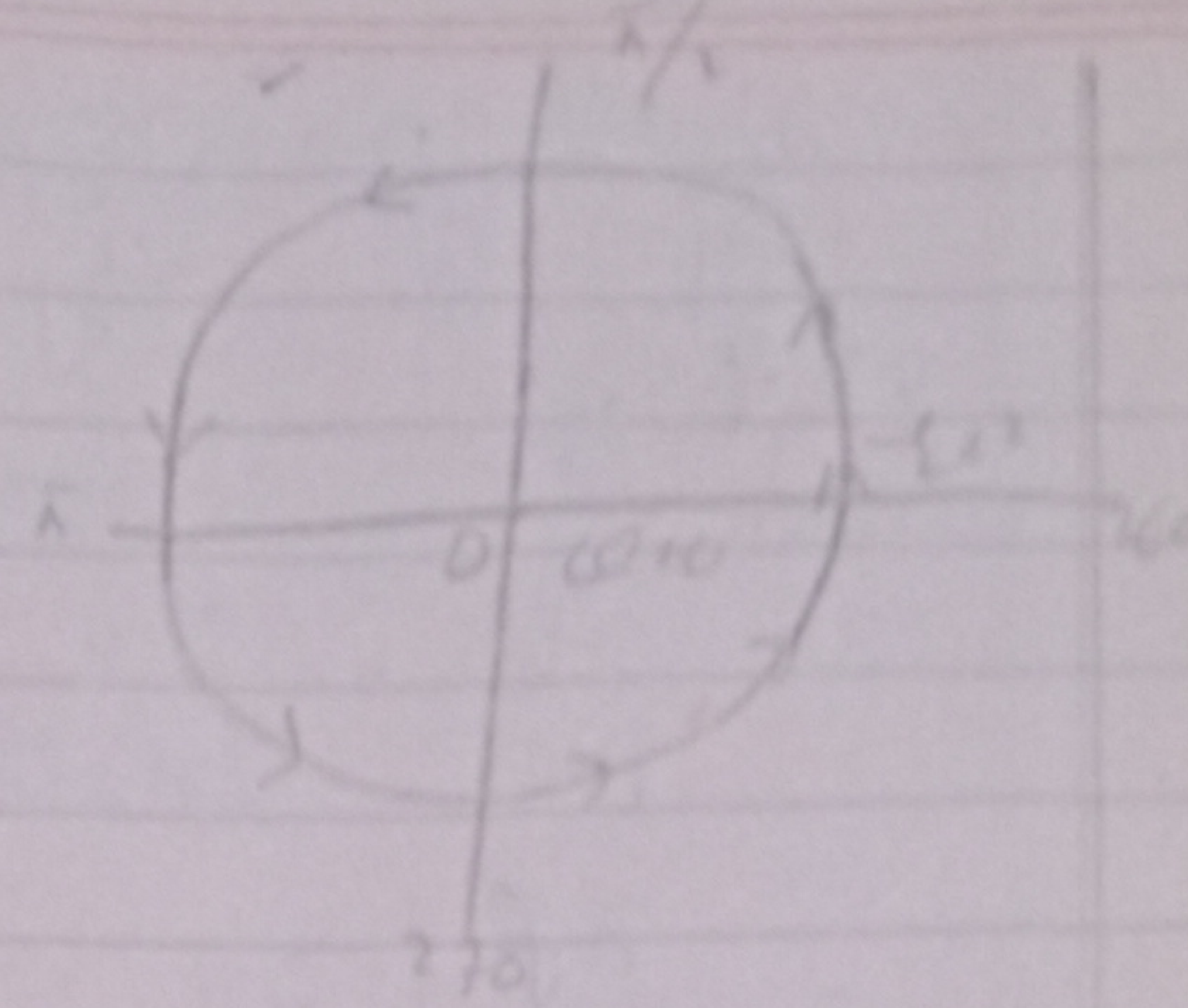
$$f(x,y) = (x^2-y)$$

$$g(x,y) = x$$

$$\frac{\partial f}{\partial y} = -1 \qquad \frac{\partial g}{\partial x} = 1$$

$x^2+y^2=4$

$0\le r\le 2$
$0\le\theta\le 2\pi$



$$\int_0^{2\pi}\int_0^2 (1+1)\, r\,dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\int_0^2 2r\,dr\right)d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(2r^2/2\right]_0^2 d\theta \Rightarrow \int_0^{2\pi}(4-0)\,d\theta$$

$$\Rightarrow \int_0^{2\pi} 4\theta \Rightarrow (8\pi) \Rightarrow \text{Ans}$$

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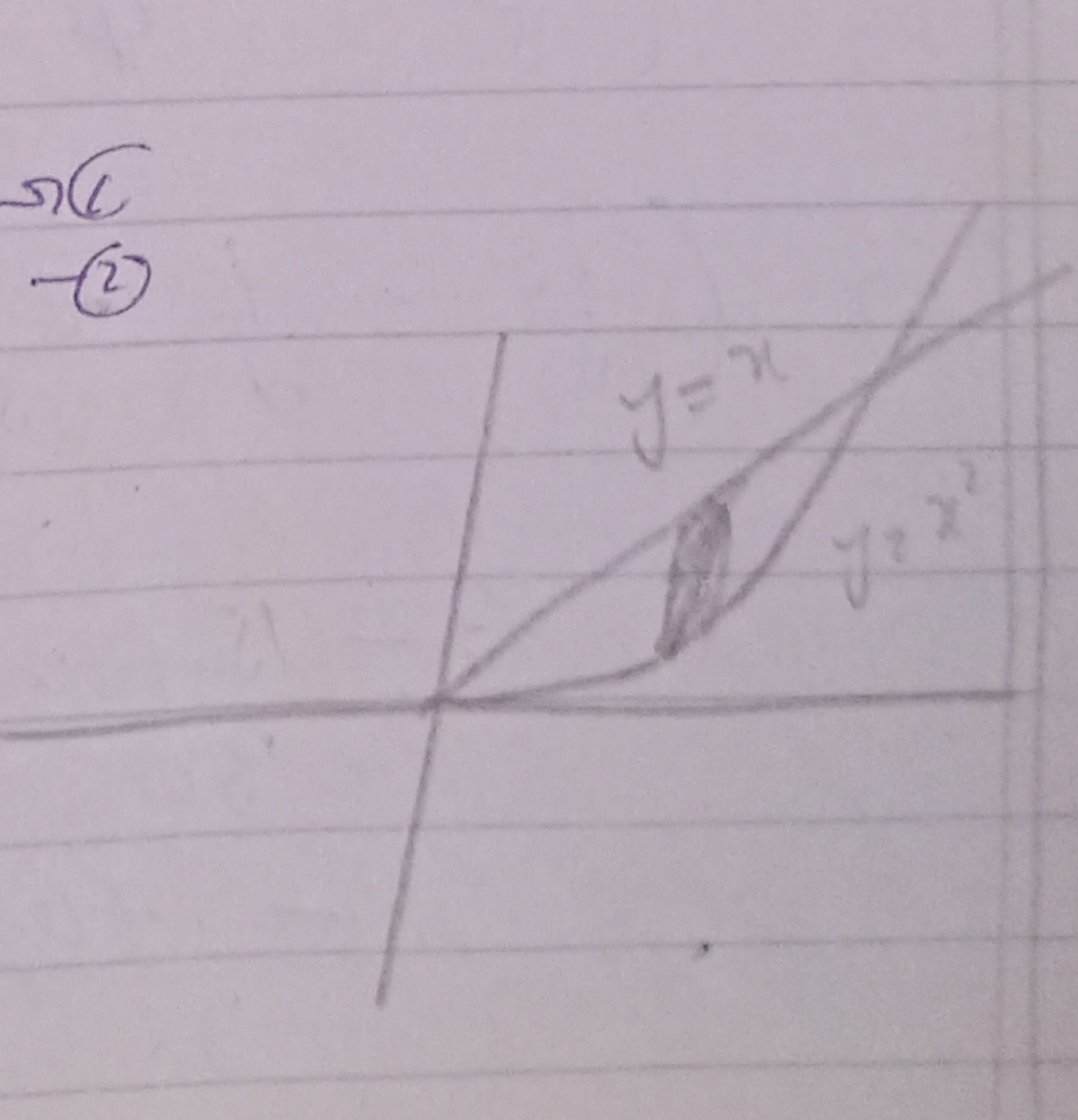
(8) $\oint_C (e^x+y^2)\,dx+(e^y+x^2)\,dy$ where c is The boundry region b/w $y=x^2$ & $y=x$

Given $y=x^2$ —(1)
& $y=x$ —(2)



$x\le y\le x^2$
$0\le x\le 1$

$\Rightarrow x^2-x=0$
$x(1-x)=0$
$x=1$ & $x=0$

$$\int_0^1\int_{x^2}^{x}\left(\frac{\partial g}{\partial x}-\frac{\partial f}{\partial y}\right) dA$$

$$\Rightarrow \frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\Rightarrow \int_0^1\int_{x^2}^{x}(2x-2y)\,dy\,dx$$

$$\Rightarrow \int_0^1 \left(2xy - \frac{\cancel{2}y^2}{\cancel{2}}\right)_{x^2}^{x} dx$$

$$\Rightarrow \int_0^1 2x(x-x^2) - (x^2-x^4)\,dx$$

$$\Rightarrow \int_0^1 2x^2-2x^3-x^2+x^4\,dx$$

$$\Rightarrow \int_0^1 x^4-2x^3+x^2\,dx$$

$$\Rightarrow \left(\frac{x^5}{5}-\frac{2x^4}{4}+\frac{x^3}{3}\right)_0^1$$

$$\Rightarrow \left(\frac{1}{5}-\frac{1}{2}+\frac{1}{3}\right)$$

$$\begin{array}{r|l} 2 & 5-2-3 \\ 3 & 5-1-3 \\ 5 & 5-1-1 \\ & 1-1 \end{array}$$

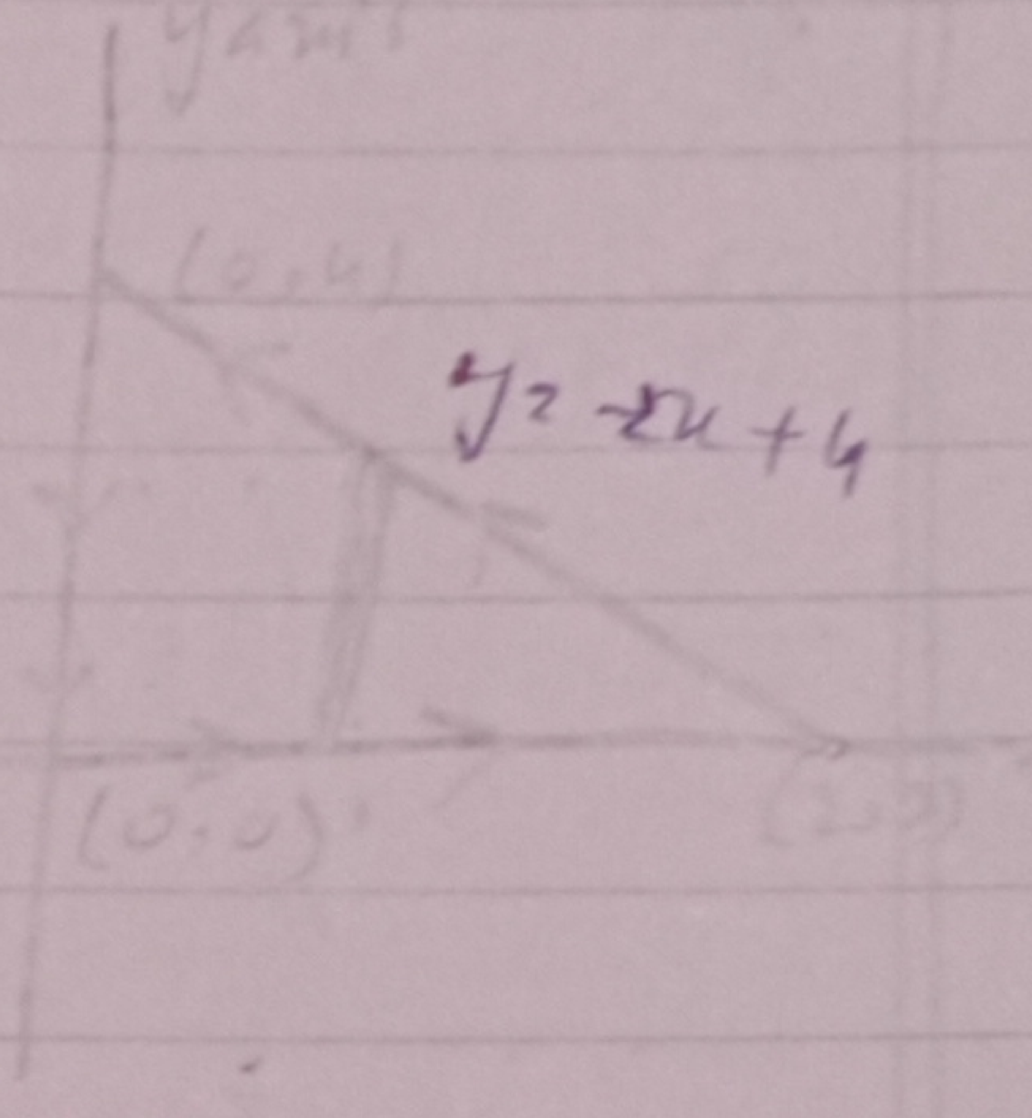
$$\frac{6-15+10}{30} \Rightarrow$$

$$\Rightarrow \frac{-9+10}{30} \Rightarrow \left(\frac{1}{30}\right) \Rightarrow \text{Ans}$$

(4) $\oint \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$ C is

The Triangulee region with verties $(0,0)$ $(2,0)$ and $(0,4)$

Given The Trigule vertues



It is clear to thele from a diagramm.

$0 \le x \le 2$

There two pom $(2,0)$ & $(0,4)$ a line segment using for the purpose of eq.

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-0}{4-0} = \frac{x-2}{0-2}$$

$$\Rightarrow -2y = 4x-8$$

$$\Rightarrow y = \frac{4x-8}{-2}$$

$$\Rightarrow y = -2x+4$$

So A/c Green theorm

$f(x,y) = \ln(1+y)$

$g(x,y) = \dfrac{-xy}{1+y}$

$\dfrac{\partial f}{\partial y} = \dfrac{1}{1+y}$

$\dfrac{\partial g}{\partial x} = \dfrac{-y}{1+y}$ So,

$$\Rightarrow \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} \left(\frac{-y}{1+y} - \frac{1}{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} \left(\frac{-y-1}{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} -1 \left({}^{y+1}/_{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \left(\int_0^{-2x+4} -1\, dy \right) dx$$

$$\Rightarrow \int_0^2 2x \cancel{+} 4 \, dx$$

$$\left(2x^2/x - 4x \right)_0^2$$

$\Rightarrow \quad 4 - 4(2) \quad \Rightarrow \quad 4 - 8$

$\Rightarrow \quad -4 \Rightarrow$ Ans

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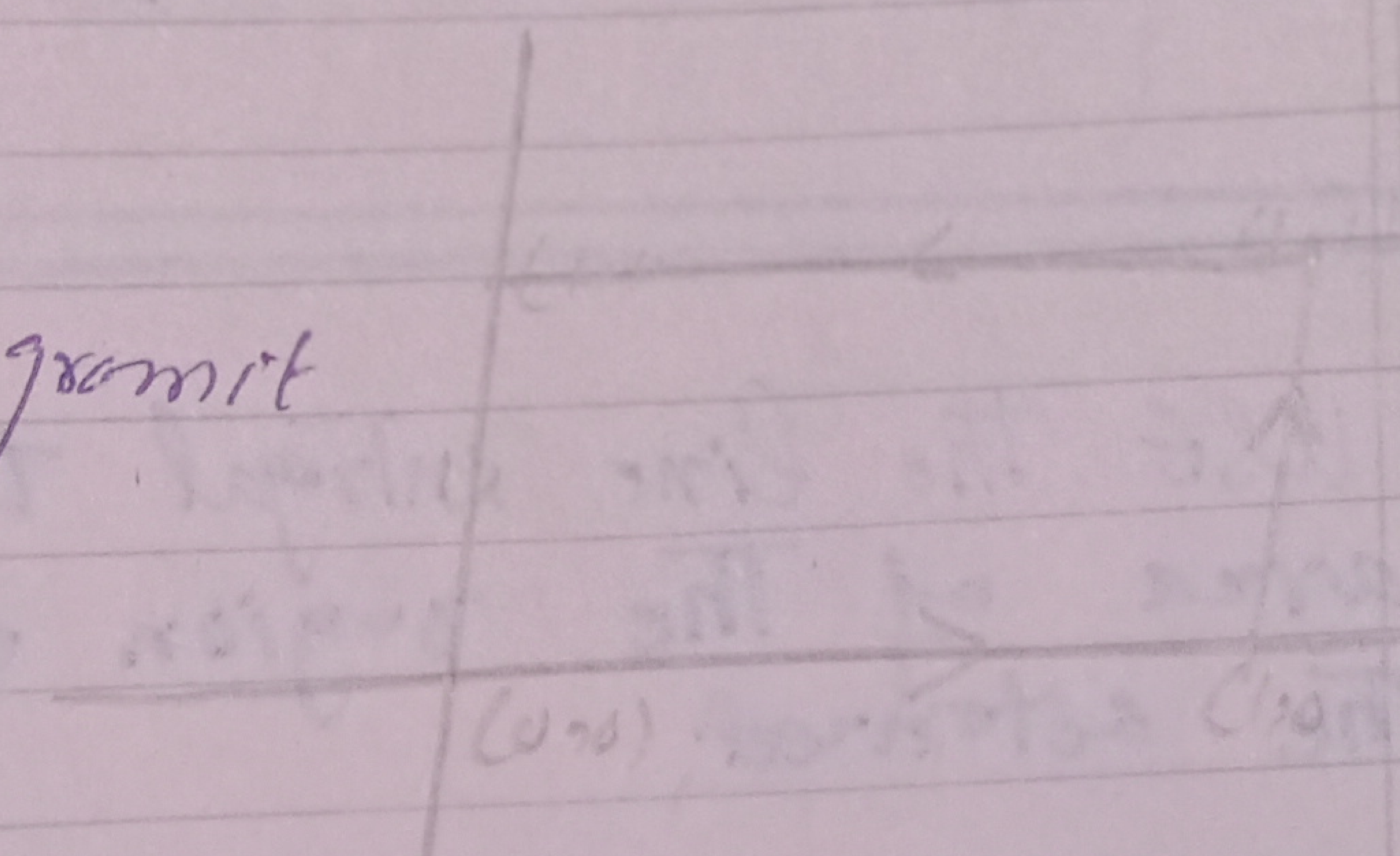
(11) $\oint \tan^{-1}y\,dx - \frac{y^2x}{1+y^2}\,dy$, C is the square with vertical $(0,0), (1,0)$ $(1,1)$ and $(0,1)$



From The diagramit is clear to Thate

$0 \leq x \leq 1$

$0 \leq y \leq 1$

So.

$f(x,y) = \tan^{-1}y$

$g(x,y) = -\frac{y^2x}{1+y^2}$

$\Rightarrow \quad \frac{\partial f}{\partial y} = \frac{1}{1+y^2}$

$\frac{\partial g}{\partial x} = -\frac{y^2}{1+y^2}$

$$\int_0^1\int_0^1 \left(\frac{-y^2}{1+y^2} - \frac{1}{1+y^2}\right) dx\,dy$$

$$\Rightarrow \int_0^1\int_0^1 \frac{\cancel{-y^2-1}}{\cancel{1+y^2}}\,dx\,dy$$

$$\Rightarrow \int_0^1 \left(\int_0^1 -1\,dx\right) dy$$

$$\Rightarrow \int_0^1 -1\,dy$$

$$\Rightarrow -1 \Rightarrow \text{Ans.}$$