DOUBLE intregation:-

Ex 14.1

Q1:- $\int_0^1\int_0^2 (x+3)\,dy\,dx$

using

$$\int_c^d\int_a^b f(x,y)\,dx\,dy = \int_c^d\left[\int_a^b f(x,y)\,dx\right]dy.$$

$$\int_a^b\int_c^d f(x,y)\,dy\,dx = \int_a^b\left[\int_c^d f(x,y)\,dy\right]dx$$

$$\Rightarrow \int_0^1\left[\int_0^2 (x+3)\,dy\right]dx$$

$$\Rightarrow \int_0^1\Big[\,y(x+3)\,\Big]_0^2\,dx \Rightarrow \int_0^1\Big[\,2(x+3)\,\Big]dx.$$

$$\Rightarrow \frac{2x^2}{2} + 6x\Big|_0^1 \Rightarrow 1+6(1) = 1+6 = \cancel{\text{Ans}}.$$

$$\Rightarrow 1+6 = 7 \Rightarrow \text{Ans.}$$

---

(5) $\int_0^{\ln 3}\int_0^{\ln(x)} e^{x+y}\,dy\,dx$

Date _____ 20__

$$\Rightarrow \int_0^{\ln(3)} \left[\int_0^{\ln(2)} e^{x+y}\, dy \right] dx$$

$$\Rightarrow \int_0^{\ln(3)} \left[\int_0^{\ln(2)} e^{x} \cdot e^{y} \cdot dy \right] dx$$

$$\Rightarrow \int_0^{\ln(3)} \left[e^{x} e^{y} \right]_0^{\ln(2)} dx$$

$$\Rightarrow \int_0^{\ln(3)} e^{x} \left(e^{\ln(2)} - e^{0} \right) dx$$

$$\Rightarrow \int_0^{\ln(3)} e^{x} (2-1)\, dx \Rightarrow \int_0^{\ln(3)} e^{x}\, dx$$

$$e^{\ln(3)} - e^{0} \Rightarrow 3-1 = 2 \Rightarrow \text{Ans}$$

Exercise 14.2

Practise work / Question

Q 1 to 16

with (complet) example
all

Chapter 14 "MULTIPLE INTEGRALS"

"Exercise No 14.1"

Ex (2):-

Evalate

$$\int_1^3 \int_2^4 (40-2xy)\,dy\,dx$$

Slocution

$$\Rightarrow \int_1^3 \left[\int_2^4 (40-2xy)\,dy \right] dx$$

$$\Rightarrow \int_1^3 \left[40y - \frac{2xy^2}{2} \right]_2^4 dx$$

$$\Rightarrow \int_1^3 \left[40(4-2) - x(16-4) \right] dx$$

$$\Rightarrow = \int_1^3 (80 - 12x)\,dx$$

$$\Rightarrow = \int_1^3 80\,dx - 12x\,dx \Rightarrow \left[80x - \frac{12x^2}{2} \right]_1^3$$

$$\Rightarrow 80(3-1) - \frac{12}{2}(9-1)$$

$$\Rightarrow 80(2) - 6(8)$$

$$\Rightarrow 160 - 48 \Rightarrow (42) \Rightarrow \text{Ans}$$

Evalute $\int_2^4 \int_1^3 (40 - 2xy)\,dx\,dy$

Sl $\int_2^4 \left[40x - \frac{2x^2}{2} y\right]_1^3 dy$

$\Rightarrow \int_2^4 \left(40(3-2) - (9-1)y\right) dy$

$\Rightarrow \int_2^4 80 - 8y \, dy$

$\Rightarrow 80y - \frac{8y^2}{2}\Big]_2^4$

$\Rightarrow 80(4-2) - 4(16-4)$

$\Rightarrow 80(2) - 4(12 \Rightarrow 160 - 48$

$\Rightarrow 112 \Rightarrow$ Ans

Ex (4):-

Evalute:- $\iint_R y^2 x \, dA$

$-3 \le x \le 2$

$0 \le y \le 1$

Sl:-

$$\Rightarrow \int_{-3}^{2}\int_{0}^{1} y^2 x \, dy\, dx$$

$$\Rightarrow \int_{-3}^{2}\left[\int_{0}^{1} y^2 x \, dy\right] dx$$

$$\Rightarrow \int_{-3}^{2}\left(\frac{y^3 x}{3}\right)_{0}^{1} dx$$

$$\Rightarrow \int_{-3}^{2}\left(\frac{x}{3}\left((1)^3-0\right)\right) dx$$

$$\Rightarrow \int_{-3}^{2} \frac{x}{3}\, dx \Rightarrow \left.\frac{x^2}{6}\right]_{0-3}^{2}$$

$$\Rightarrow \frac{1}{6}(4-9) \Rightarrow -5/6$$

$\longrightarrow$ Ans

(Q 13) $\iint_R 4xy^3 \, dA$

$R = \{(x,y) : -1 \le x \le 1$
$-2 \le y \le 2$

Sol $\Rightarrow \int_{-1}^{1}\int_{-2}^{2} 4xy^3 \, dy\, dx$

$$\Rightarrow \int_{-1}^{1}\left[\frac{4xy^4}{4}\right]_{-2}^{2} dx$$

$$\Rightarrow \int_{-1}^{1}\left(xy^4\right)_{-2}^{2} dx$$

$$\int_{-1}^{1} x(16-16)\,dx$$

$$\Rightarrow 0 \Rightarrow \text{Ans}$$

Q (15) $\iint_R x\sqrt{1-x^2}\,dA$

(14)

$0 \le x \le 1$
$2 \le y \le 3$

$$\Rightarrow \int_0^1\int_2^3 x\sqrt{1-x^2}\,dy\,dx$$

$$\Rightarrow \int_2^3\int_0^1 x\sqrt{1-x^2}\,dx\,dy$$

let $u = 1-x^2$
$du = -2x\,dx$
$dx = \frac{du}{-2x}$

when $x \to 1$, $u \to 0$
" $x \to 0$, $u \to 1$

So, $\int_2^3 \left[\frac{-1}{2}\int_1^0 \cancel{x}(u)^{1/2}\frac{du}{\cancel{x}}\right]dy$

$$\Rightarrow \int_2^3 \left(\frac{1}{2}\frac{(u)^{3/2}}{3/2}\right)_0^1 dy$$

$$\Rightarrow \left(\frac{1}{3}\right)(1-0)^{3/1}\, dy$$

$$\Rightarrow \int_2^3 \frac{1}{3}\, dy \Rightarrow \frac{1}{3}(3-2)$$

$$\Rightarrow 1/3 \Rightarrow \text{Ans}$$

---

(14)

$$\frac{xy}{\sqrt{x^2+y^2+1}}\, dA$$

$0 \le x \le 1$

$0 \le y \le 1$

Solution:

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\, dy\,dx. \quad \text{let } u = x^2+y^2+1$$

$$\frac{du}{dx} = 2x$$

$x \to 0$

$u \to y^2+1$

$x \to 1$

$u \to y^2+2$

$$\frac{du}{2x} = dx$$

$$\int_0^1\int_{y^2+1}^{y^2+2} \frac{xy}{2\sqrt{u}}\, \frac{du}{2x}\, dy$$

$$\frac{1}{2}\int_0^1\int_{y^2+1}^{y^2+2} \frac{y}{\sqrt{u}}\, du\, dy$$

$$\frac{1}{2}\int_0^1 y\left(\frac{u^{1/2}}{1/2}\right)_{y^2+1}^{y^2+2} dy$$

Date _____ 20__

Ex 14.2 :-

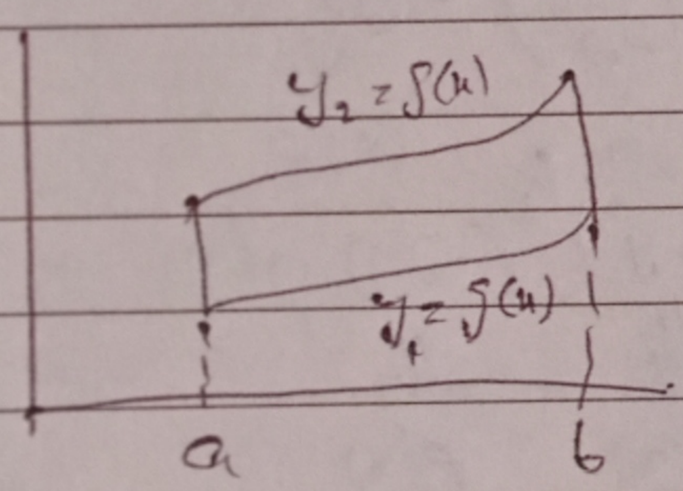
Double integral ~~with~~ over non rectangular Region :-

$$\Rightarrow \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right] dx$$

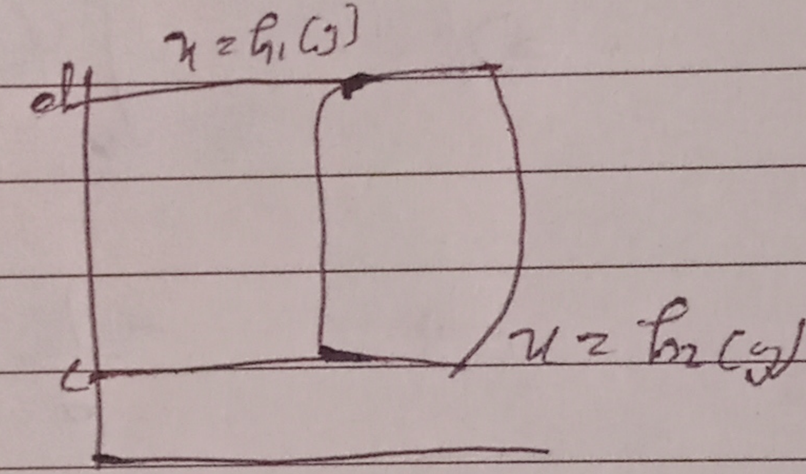
& $$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right] dy.$$

(& read Theorum (4.2.1 Page 1009)
i.e region Type 1 & type II

i.e



Type I



Evaluate the integral.

(3) $$\int_0^3 \int_0^{\sqrt{9-y^2}} y\,dx\,dy$$

(Sin)
$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right] dy$$

QUALITY PAPER PRODUCTS

$\because h_1(y) = 0$

$h_2(y) = \sqrt{9-y^2}$

Date _____ 20__

$$\Rightarrow \int_0^3 \left[\int_0^{\sqrt{9-y^2}} y \, dx \right] dy$$

$$\Rightarrow \int_0^3 \left[xy \right]_0^{\sqrt{9-y^2}} dy \Rightarrow \int_0^3 \left[y\sqrt{9-y^2} - 0 \right] dy.$$

$$\Rightarrow \int_0^3 \left(\sqrt{9-y^2}\right) y \, dy \Rightarrow \int_0^3 (9-y^2)\, y \, dy$$

$$\int [f(x)]^n f'(x) = \frac{(f(x))^{n+1}}{n+1}$$

(or Substitution method)

$$\Rightarrow \int_0^3 (9-y^2)^{1/2} y \left(\frac{-2}{-2}\right) dy$$

$$\Rightarrow -\frac{1}{2} \int_0^3 \underbrace{(9-y^2)^{1/2}}_{f(x)} \underbrace{(-2y)}_{f'(x)} dy$$

$$\Rightarrow \frac{-\frac{1}{2}\left[9-y^2\right]^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Rightarrow \frac{-\frac{1}{2}\left[9-y^2\right]^{3/2}}{3/2}\Bigg|_0^3$$

$$\Rightarrow \frac{\frac{(9-y^2)^{3/2}}{-\cancel{2}}}{3/\cancel{2}}\Bigg|_0^3 \Rightarrow -\frac{(9-y^2)}{3}\Bigg|_0^3$$

$$\Rightarrow \frac{9-(3)^2}{3} - \frac{9-(0)^2}{3} \Rightarrow \cancel{\frac{9}{3}}$$

$$\Rightarrow \frac{(9)^{3/2}}{3} \Rightarrow -\frac{(3)^{\cancel{2}\times 3/\cancel{2}}}{3} = \frac{27}{3} = 9$$

(7) $$\int_0^1 \int_0^x y\sqrt{x^2 - y^2}\, dy\, dx$$

$\Rightarrow$ using

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\, dy \right] dx$$

$$\Rightarrow \int_0^1 \left[\int_0^x \left(\sqrt{x^2 - y^2}\right) y\, dy \right] dx$$

$$\Rightarrow \int_0^1 \left[\int_0^x \underbrace{\left(\sqrt{x^2 - y^2}\right)}_{f(x)} \underbrace{\left(\frac{-2y}{-2}\right)}_{f'(x)} dy \right] dx \qquad f'(x) = \frac{f(x)^{n+1}}{n+1}$$

$$\Rightarrow \frac{-1}{2} \int_0^1 \left[\int_0^x (x^2 - y^2)^{1/2} (-2y)\, dy \right] dx$$

$$\Rightarrow \frac{-1}{2} \int_0^1 \left[\frac{(x^2 - y^2)^{3/2}}{3/2} \right]_0^x \Rightarrow \frac{-1}{2} \int_0^1 \left[\cancel{\frac{(x^2 - (x)^2)^{3/2}}{3/2}} - \frac{(x^2 - (0)^2)^{3/2}}{3/2} \right]$$

$$\Rightarrow \frac{-1}{2} \int_0^1 \left[-\frac{2(x^2)^{3/2}}{3} \right] dx \Rightarrow \cancel{\int_0^1 \frac{x^3}{3}\, dx}$$

$$\cancel{\frac{x^3}{9} \Big|_0^1 \Rightarrow \frac{1}{9} - 0 \Rightarrow \frac{1}{9}}$$

$$\Rightarrow \frac{-1}{\cancel{2}} \int_0^1 -\frac{\cancel{2}(x^2)^{3/2}}{3}\, dx \Rightarrow \int_0^1 \frac{(x^2)^{3/2}}{3}\, dx$$

Date _____ 20 ___

$$\cancel{\frac{1}{3}\left[\frac{(x)^{3/2+1}}{3/2+1}\right]_0^1 \Rightarrow \frac{(x)^{5/2}}{\frac{3}{5/2}}\Big|_0^1}$$

$$\Rightarrow \cancel{\frac{2(x)^{5/2}}{15}\Big|_0^1 \Rightarrow}$$

$$\Rightarrow \int_0^1 \frac{x^3}{3}\,dx$$

$$\Rightarrow \frac{x^4}{12}\Big|_0^1 \Rightarrow \frac{1}{12} \Rightarrow \text{Ans}$$

Practise Question Ex 14.2.

all example

Ex Question 1 to 8,

15, 18, 19, 21, 25, 26

Exercise No 14.2

Evalute

(1) $$\int_0^1 \int_{-x}^{x^2} y^2 x \, dy \, dx$$

Sol:-

$$\Rightarrow \int_0^1 \left[\int_{-x}^{x^2} y^2 x \, dy \right] dx$$

$$\Rightarrow \int_0^1 \left(\frac{y^3}{3} x \right)_{-x}^{x^2} dx$$

$$\Rightarrow \int_0^1 \left(\frac{x}{3} \left(x^6 + x^4 \right) \right. dx$$

$$\Rightarrow \int_0^1 \frac{x^6}{3} + \frac{x^4}{3} \, dx$$

$$\Rightarrow \left. \frac{x^7}{21} + \frac{x^5}{15} \right]_0^1 \Rightarrow \frac{1}{21}(1-0) + \frac{1}{15}(1-0)$$

$$\Rightarrow \frac{1}{21} + \frac{1}{15}$$

$$\Rightarrow \frac{13}{120} \Rightarrow \text{Ans}$$

Evalute $$\int_0^{\pi/3} \int_0^{\cos y} x \sin y \, dx \, dy$$

Sol:-

$$\int_0^{\pi/3}\left[\int_0^{\cos y} x\sin y\,dx\right]dy$$

$$\Rightarrow \int_0^{\pi/3}\left[\frac{x^2}{2}\sin y\right]_0^{\cos y}dy$$

$$\Rightarrow \int_0^{\pi/3}\frac{1}{2}(\cos^2 y\sin y)\,dy$$

let $u = \cos y$

$du = -\sin y\,dy$

$\frac{du}{-\sin y} = dy$

when $y \to \pi/3$; $u \to 1/2$

" $y \to 0$; $u \to 1$

$$-\frac{1}{2}\int_1^{1/2}(u)^2\frac{du\sin y}{\sin y} \Rightarrow \frac{1}{2}\int_{1/2}^{1}\left(\frac{u^3}{3}\right)$$

$$\Rightarrow \frac{1}{6}(u)^3\Big]_{1/2}^{1} \Rightarrow \frac{1}{6}(1-(1/2)^3)$$

$$\Rightarrow \frac{1}{6}\left(1-\frac{1}{8}\right)$$

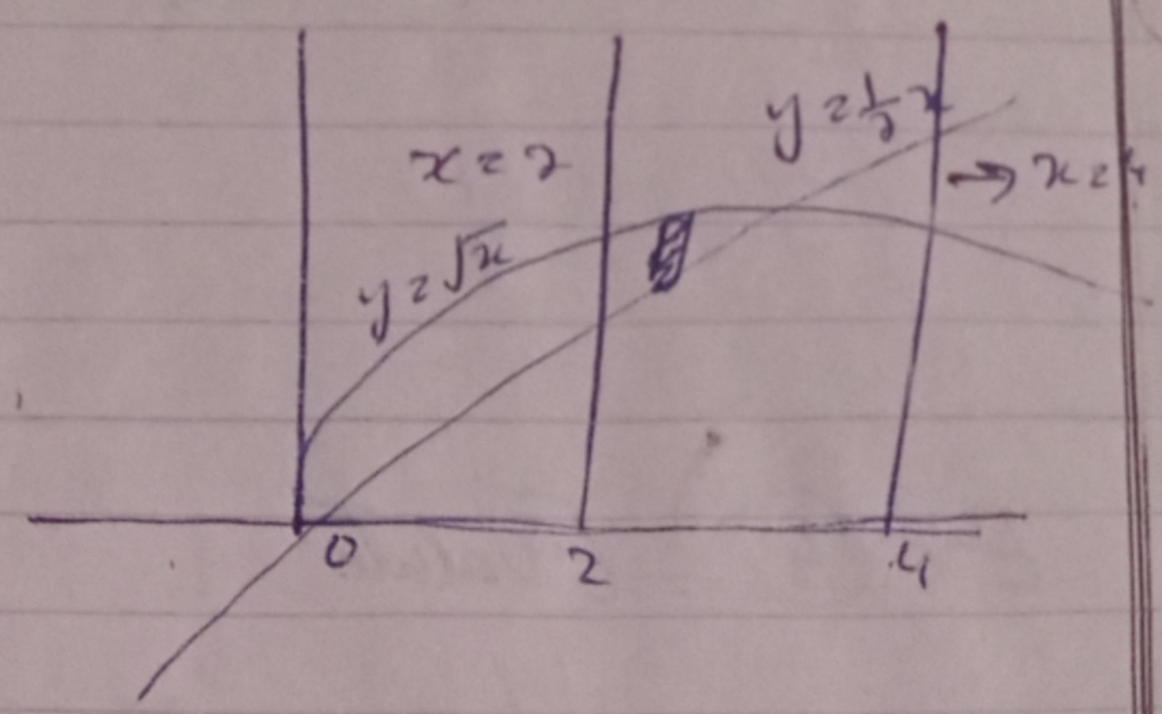
$$\Rightarrow \frac{1}{6}\left(\frac{8-1}{8}\right) \Rightarrow (7/48) \Rightarrow \text{Ans}$$

Evalute $\iint_R xy\,dA$

$y = \frac{1}{2}x$, $y = \sqrt{x}$

and $x = 2$ $x = 4$

Sol



$$E_1 \qquad \begin{array}{c} 2 \le x \le 4 \\ \sqrt{x} \le y \le \frac{1}{2}x \end{array}$$

$$\Rightarrow \int_2^4 \int_{x/2}^{\sqrt{x}} xy \, dy \, dx$$

$$\Rightarrow \int_2^4 \left[x \frac{y^2}{2} \right]_{x/2}^{\sqrt{x}} dx$$

$$\Rightarrow \int_2^4 \frac{x}{2}\left(x - \frac{x^2}{4}\right) dx$$

$$\Rightarrow \quad \Rightarrow \int_2^4 \frac{x}{2}\left(\frac{4x - x^2}{4}\right) dx$$

$$\Rightarrow \int_2^4 \frac{4x^2 - x^3}{8} \, dx$$

$$\int_2^4 \frac{x^2}{2} - \frac{x^3}{8} \, dx$$

$$\Rightarrow \frac{x^3}{6} - \frac{x^4}{32} \Big]_2^4$$

$$\Rightarrow \frac{1}{6}(64 - 8) - \frac{1}{32}(256 - 16)$$

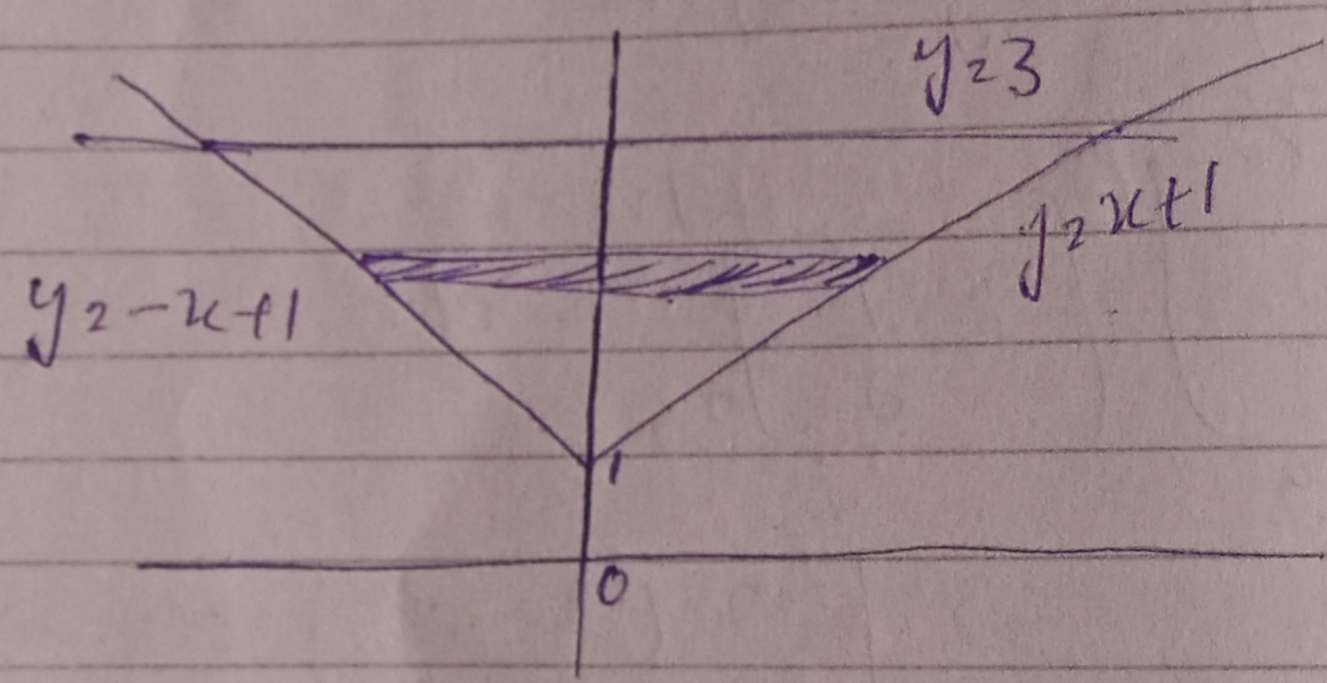
$56/6 - \frac{240}{32} \Rightarrow 11/6$ ← Ans

$20/3 - \frac{15}{2}$

Ex (4) Evalute

$$\iint_R (2x - y^2)\, dA$$

lines $y = -x+1$, $y = x+1$ & $y=3$



$1 \le y \le 3$

$y-1 \le x \le 1-y$

$y = -x+1$

$-x+1 = y \Rightarrow -x = y-1$

$x = -y+1$

$x = 1-y$

& $y = x+1 \Rightarrow x = y-1$

$$\int_1^3 \int_{1-y}^{y-1} (2x - y^2)\, dx\, dy$$

$$\Rightarrow \left(\frac{2x^2}{2} - xy^2\right)_{1-y}^{y-1} dy$$

$$\Rightarrow \int_1^3 \left((y-1)^2 - (1-y)^2 - \cancel{x(y}-(y-1)-(1-y)y^2\right) dy$$

$$\Rightarrow \int_1^3 \left(y^2+1-2y\right) - (1+y^2-2y) - \left((y-1)-(1+y)y^2\right) dy$$

23

$$\Rightarrow y^2+1-2y-1-y^2+2y-(y-1-1+y)\,y^2\, dy$$

$$\Rightarrow \left(\cancel{y^2}+\cancel{1}-\cancel{2y}-\cancel{1}-\cancel{y^2}+\cancel{2y} - y^3+y^2+y^2+y^3\right) dy$$

$$\Rightarrow \int_1^3 \left(+2y^2-2y^3\right) dy$$

$$\Rightarrow \frac{2y^3}{3} - \frac{\cancel{2}y^4}{\cancel{4}}\Big]_1^3$$

$$\Rightarrow \frac{2}{3}(27-1) - \frac{1}{2}(81-1)$$

$$\Rightarrow \frac{52}{3} - \frac{80}{2} \Rightarrow \frac{\quad}{6}$$

$$\Rightarrow \frac{104-240}{6} \Rightarrow \frac{-136}{6}$$

$$\Rightarrow \left(-\frac{68}{3}\right)$$

$\square \longrightarrow$ Ans

Find the volume of the tetrahedron bounded by the coordinate planes $z = 4 - 4x - 2y$

Given $z = 4 - 4x - 2y$

in xy plane $z = 0$

$$-4x - 2y = -4$$

$$-2y = -4 + 4x$$

$$\Rightarrow y = \frac{-4 + 4x}{-2}$$

$$y = 2 - 2x$$

$$0 \le y \le 2 - 2x$$

z is allready zero and xy plane $y = 0$,

$$4 - 4x = 0$$

$$-4x = -4$$

$$x = -4/-4$$

$$x = 1$$

$$0 \le x \le 1$$

$$\int_0^1 \int_0^{2-2x} (4 - 4x - 2y)\, dy\, dx$$

$$\Rightarrow \int_0^1 \left(4y - 4xy - 2y^2/2\right)_0^{2-2x} dx$$

$$\Rightarrow 4(2 - 2x) - 4x(2 - 2x) - (2 - 2x)^2$$

$$\Rightarrow 8 - 8x - 8x + 8x^2 - (4 + 4x^2 - 8x)$$

$8 - 8x - \cancel{8x} + 8x^2 - 4 - 4x^2 + \cancel{8x}$

$$\int_0^1 (4 - 8x + 4x^2)\,dx$$

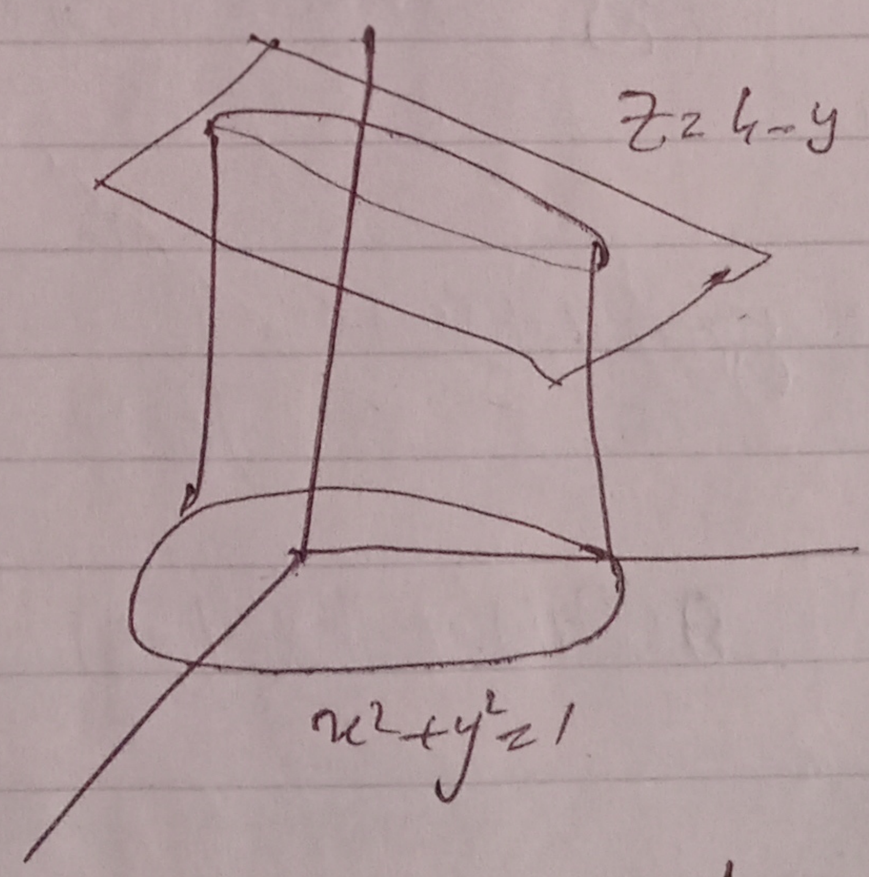
$$\Rightarrow \left. 4x - \frac{8x^2}{2} + \frac{4x^3}{3} \right]_0^1$$

$$\Rightarrow 4 - 4 + \frac{4}{3}$$

$$\Rightarrow 0 + 4/3 \Rightarrow 4/3$$

$\hookrightarrow$ Ans

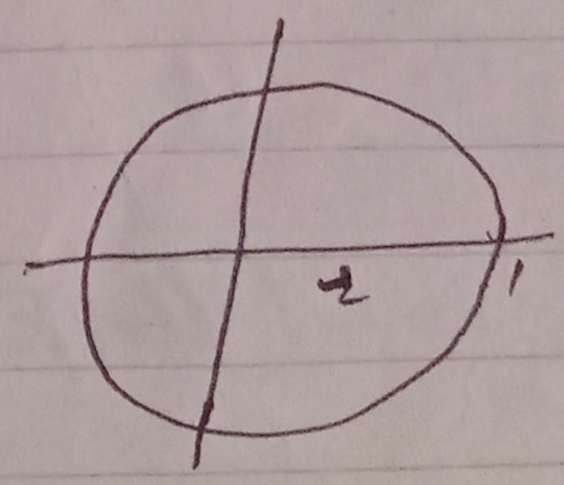
Q Find The volume of solid cylinder bounded by $x^2+y^2=4$ and The planes $y+z=4$ & $z=0$



$\Rightarrow x^2+y^2=1$

$r = \pm 1$

$0 \le r \le 1$



$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{2} (4 - 2\sin\theta)\, r\, dr\, d\theta$$

using polar coordinates (r, θ)

$\because y = 2\sin\theta$

$$\int_0^{2\pi} \left[(4r - r^2 \sin\theta)\, dr \right] d\theta$$

$$\int_0^{2\pi} \left(\frac{4r^2}{2} - \frac{r^3}{3} \sin\theta \right)_0^2 d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(2r^2 - r^3/3 \sin\theta \right)_0^2 d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(2(4) - 8/3 \sin\theta \right) d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(8 - \frac{8\sin\theta}{3} \right) d\theta$$

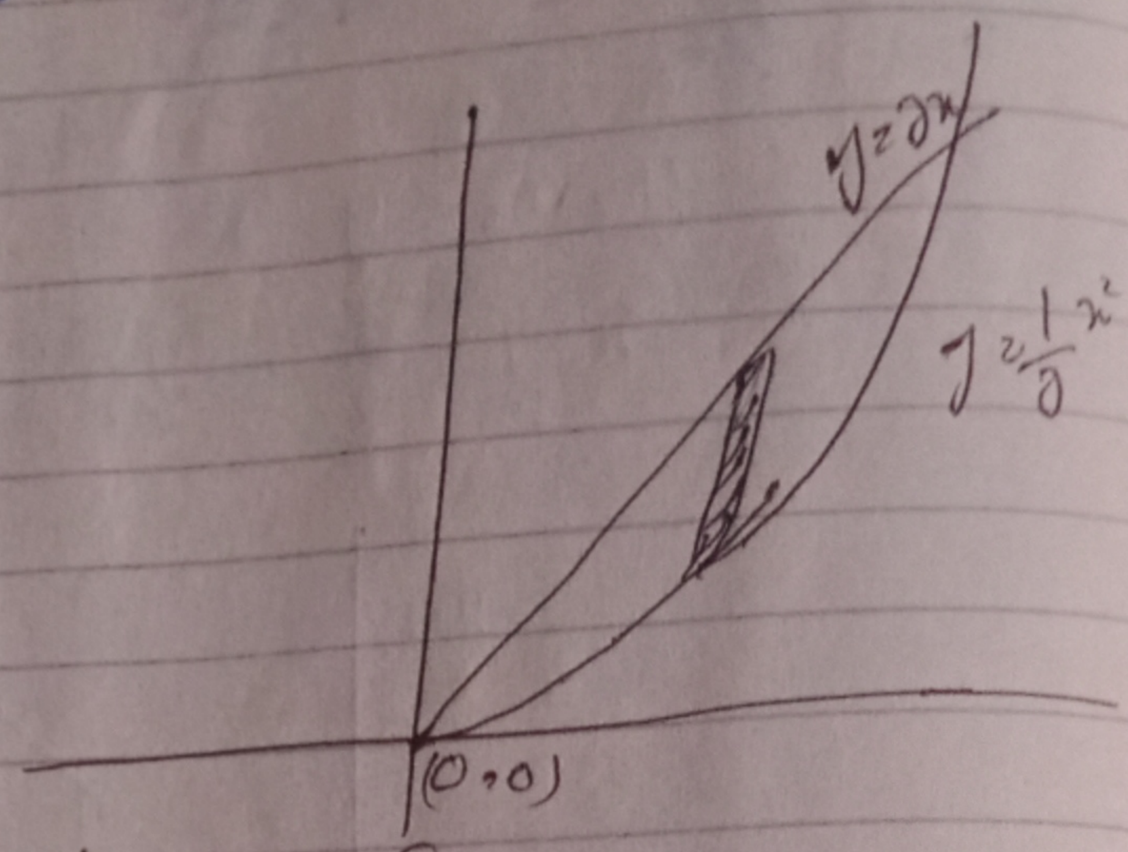
$$\Rightarrow 8\theta + \frac{8\cos\theta}{3} \Big]_0^{2\pi}$$

$$\Rightarrow 8(2\pi) + \frac{1}{3}(1-1)$$

$$\Rightarrow 16\pi + 0$$

$$\Rightarrow (16\pi) \Rightarrow \text{Ans}$$

$y = \frac{1}{2}x^2$ and line $y = 2x$



$y = \frac{1}{2}x^2 \rightarrow (i)$

$y = 2x \rightarrow (ii)$

Comparing (i) & (ii), we get

$\Rightarrow \frac{1}{2}x^2 = 2x$

$\Rightarrow x^2 = 4x \Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x-4) = 0 \quad x = 0, \; x = 4$

& $\quad 0 \le x \le 4$

$2x \le y \le \frac{x^2}{2}$

From figure

$$\int_0^4 \int_{x^2/2}^{2x} dy\, dx$$

$$\Rightarrow \int_0^4 \left(2x - \frac{x^2}{2}\right) dx$$

$$\Rightarrow \int_0^4 \frac{4x - x^2}{2}\, dx$$

$$\int_0 \left(2x - \frac{x^2}{2}\right) dx$$

$$\Rightarrow \frac{2x^2}{2} - \frac{x^3}{6}\Big]_0^4$$

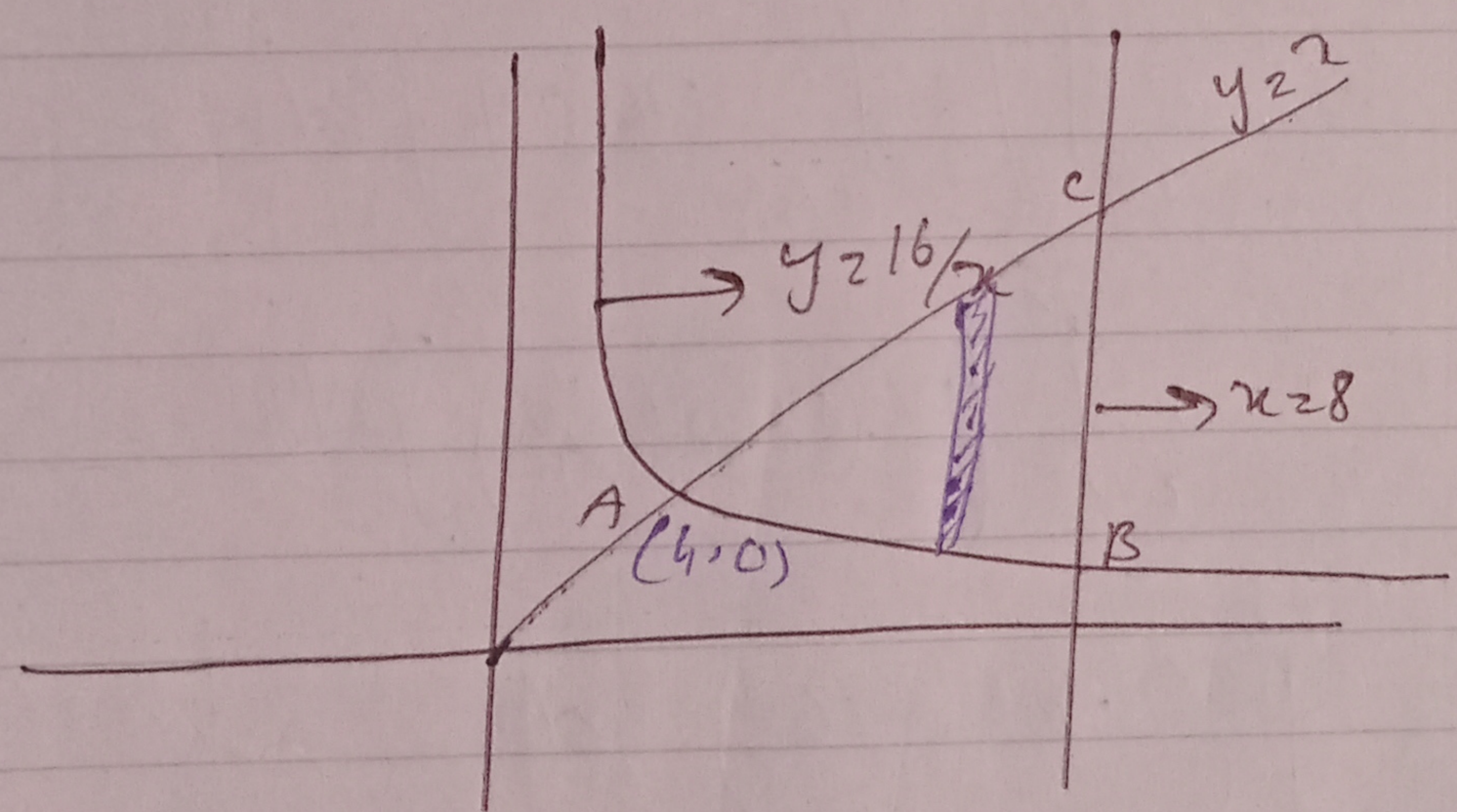
$$\Rightarrow 16 - \frac{64}{6} \Rightarrow \frac{96-64}{6}$$

$$\Rightarrow 32/6 \Rightarrow (16/3) \rightarrow \text{Ans}$$

---

(15)

Q $\iint_{R^2} dA$ R is The Region bounded bounded by $y = 16/x$ & $y = x$ and $x = 8$



$y = 16/x \rightarrow (1)$

$y = x \rightarrow (2)$

$y = 8 \rightarrow (3)$

Comprig (1) & (2), we get

$x = 16/x$

$\Rightarrow x^2 = 16 \quad \Rightarrow x^2 = \pm 4$

$x = 4,$

b/c Region is bounded in first quad.

From the figer. It is clear to that

$$4 \le x \le 8$$

$$x \le y \le 16/x$$

From the figer, in triangle A, B, C are bounded are we calculated.

So,

$$\iint_R x^2 \, dA = \int_4^8 \int_{16/x}^{x} x \, dy \, dx$$

$$\Rightarrow \int_4^8 \left[\int_{16/x}^{x} x \, dy \right] dx$$

$$\Rightarrow \int_4^8 \left[x^2 \left(x - 16/x \right) \right] dx$$

$$\Rightarrow \int_4^8 \left(x^3 - 16 \right) dx$$

$$\Rightarrow \left. \frac{x^4}{4} - \frac{16x}{2} \right]_4^8$$

$$\cancel{\frac{1}{3}(512 - 64) - 16(8-4)}$$

$$\Rightarrow \cancel{\frac{448}{3} - 64}$$

Q18

$\frac{1}{4}(4096-256) - 8(64-16)$

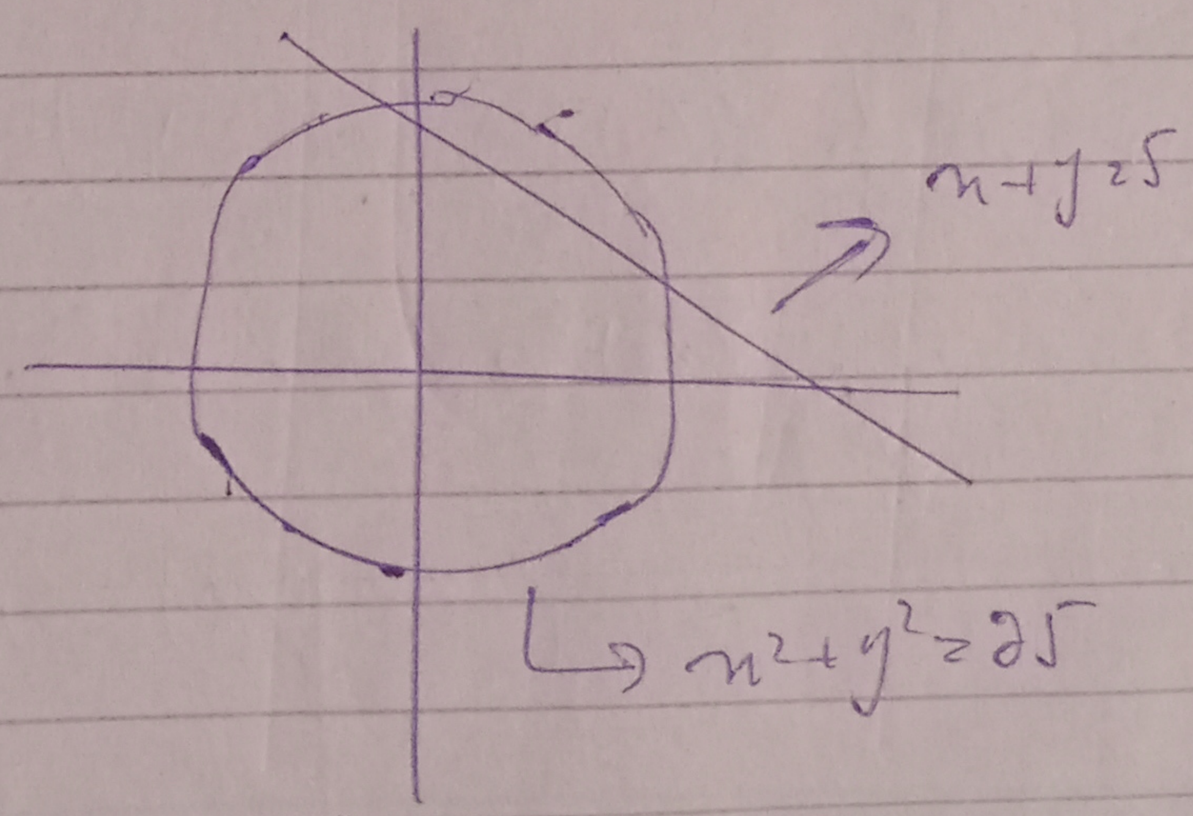
$\Rightarrow \frac{1}{4}(3840) - 384$

$\Rightarrow 960 - 384$

$\Rightarrow 576$

$\hookrightarrow$ Ans

Q18 $\iint_R y\,dA$; R is the region in the first quadrant enclosed b/w circle $x^2+y^2=25$ and lines $x+y=5$



$x^2+y^2 = 25 \rightarrow (1)$

$x+y = 5 \rightarrow (2)$

$y = 5-x$

$5-x \le y \le \sqrt{25-x^2}$

$0 \le x \le 5$

$$\int_0^5 \int_{5-x}^{\sqrt{25-x^2}} y\,dy\,dx$$

$$\Rightarrow \quad y^2/2 \Big]_{5-x}^{\sqrt{25-x^2}} dx$$

$$\Rightarrow \quad \frac{1}{2}(25-x^2) - (25+x^2-10x)$$

$$\int_0^5 \frac{1}{2}(\cancel{25}-\cancel{x^2}-\cancel{25}-\cancel{x^2}+10x)$$

$$\int_0^5 \frac{1}{2}(-2x^2+10x)\,dy$$

$$\Rightarrow \frac{1}{2}\left(-\frac{2x^3}{3}+\frac{10x^2}{2}\right)_0^5$$

$$\Rightarrow \frac{1}{2}\left(\frac{-2}{3}(625-0)+\frac{5(25)}{2}\right)$$

$$\Rightarrow \frac{-1}{3}(625)+\frac{625}{0})$$

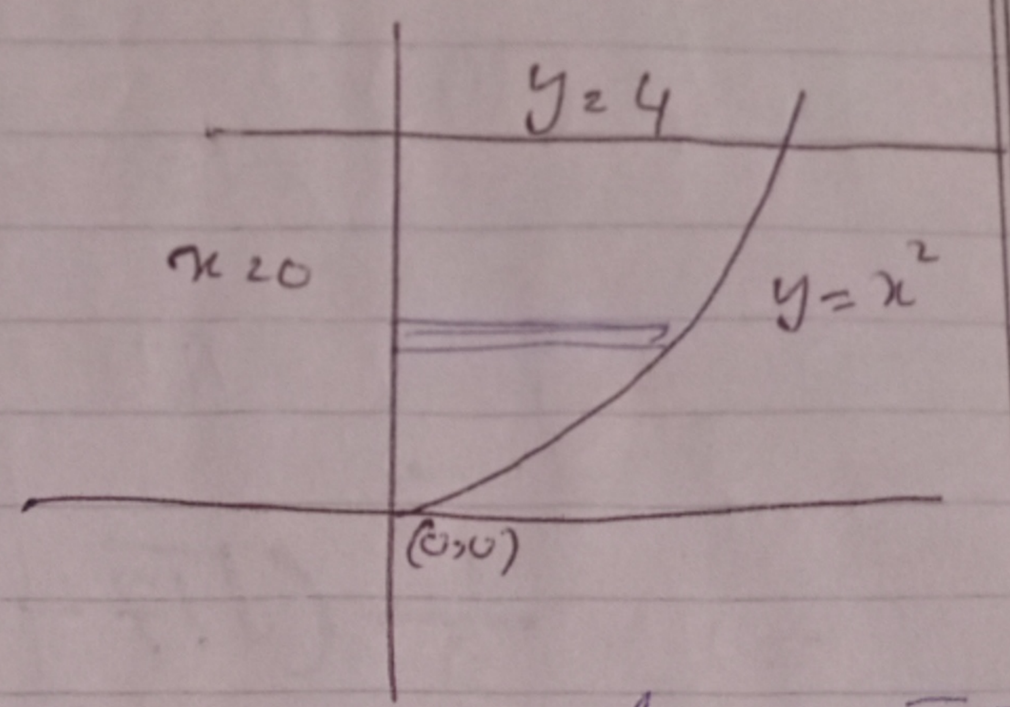
$$\Rightarrow -1250+1875/6$$

$$\Rightarrow (625/6) \Rightarrow \text{Ans}$$

---

(19) Evalute $\iint_R x(1+y)^{-1/2}\,dA$

R is the region inclosed in the first quadrant by $y=x^2$, $y=4$



From The figer it is clear to That

$$0 \le y \le 4$$
$$0 \le x \le \sqrt{y}$$

&

$$\int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-1/2}\, dx\, dy$$

$$\Rightarrow \int_0^4 \left[\frac{x^2}{2}(1+y^2)^{-1/2}\right]_0^{\sqrt{y}} dy$$

$$\Rightarrow \int_0^4 \left(\frac{y}{2}(1+y^2)^{-1/2}\right) dy$$

let $u = 1+y^2$

$du = 2y\,dy$

$\frac{du}{2y} = dy$

when $y \to 4$, $u \to 17$

$y \to 0$, $u \to 1$

$$\Rightarrow \frac{1}{4}\int_1^{17} \cancel{y}(u)^{-1/2}\, du/\cancel{y}$$

$$\Rightarrow \frac{1}{4}\left(\frac{(u)^{-1/2+1}}{-1/2+1}\right)_1^{17}$$

$$\left(\frac{1}{4}\left(u\right)^{1/2}\Big/ 1/2\right)_1^{17}$$

$\Rightarrow \quad \Rightarrow \frac{1}{2}\left(\sqrt{17}-1\right)$

$\Rightarrow \quad \frac{1}{2}\left(\sqrt{17}-1\right) \Longrightarrow$ Ans

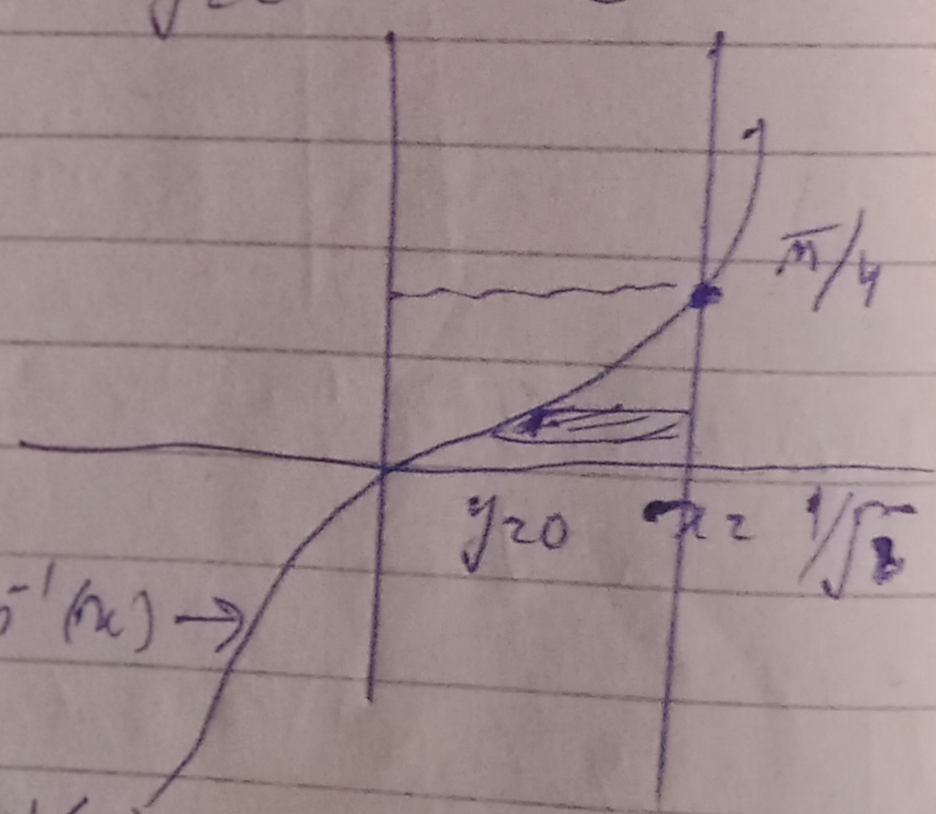
---

Q2) Q $\iint_R x\,dA$; R is the region enclosed by $y = \sin^{-1}x$, $x = 1/\sqrt{2}$ and $y = 0$

Given $y = \sin^{-1}x \rightarrow (1)$

$x = 1/\sqrt{2} \rightarrow (2)$

$y = 0 \rightarrow (3)$



$y = \sin^{-1}(x)$

$y = \sin^{-1}(1/\sqrt{2}) \Rightarrow \quad \sqrt{2} = \frac{\pi}{4}$

$\Rightarrow \quad 0 \le y \le \pi/4 \quad \Rightarrow \quad y = \frac{x}{\sin}$

$\sin^{-1} y \le x \le 1/\sqrt{2} \qquad x = \sin y$

$$\int_0^{\pi/4} \int_{\sin y}^{1/\sqrt{2}} (x)\, dx\, dy$$

$$\Rightarrow \int_0^{\pi/4} \left(\frac{x^2}{2}\right)_0^{\sin y} dy \Rightarrow$$

$$\Rightarrow \int_0^{\pi/4} \left(x^2/2\right]_{\sin y}^{1/\sqrt{2}} dy$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{2}\left(\frac{1}{2} - \sin^2 y\right) dy$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{2}\left(\frac{1 - 2\sin^2 y}{2}\right) dy$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{4}\left(1 - 2\sin^2 y\right) dy$$

$$\Rightarrow \int_0^{\pi/4} \frac{1}{4}\left(1 - \cancel{2}\left(\frac{1 - \cos 2y}{\cancel{2}}\right)\right) dy$$

$$\int_0^{\pi/4} \frac{1}{4}\left(\cancel{1} - \cancel{1} + \cos 2y\right) dy$$

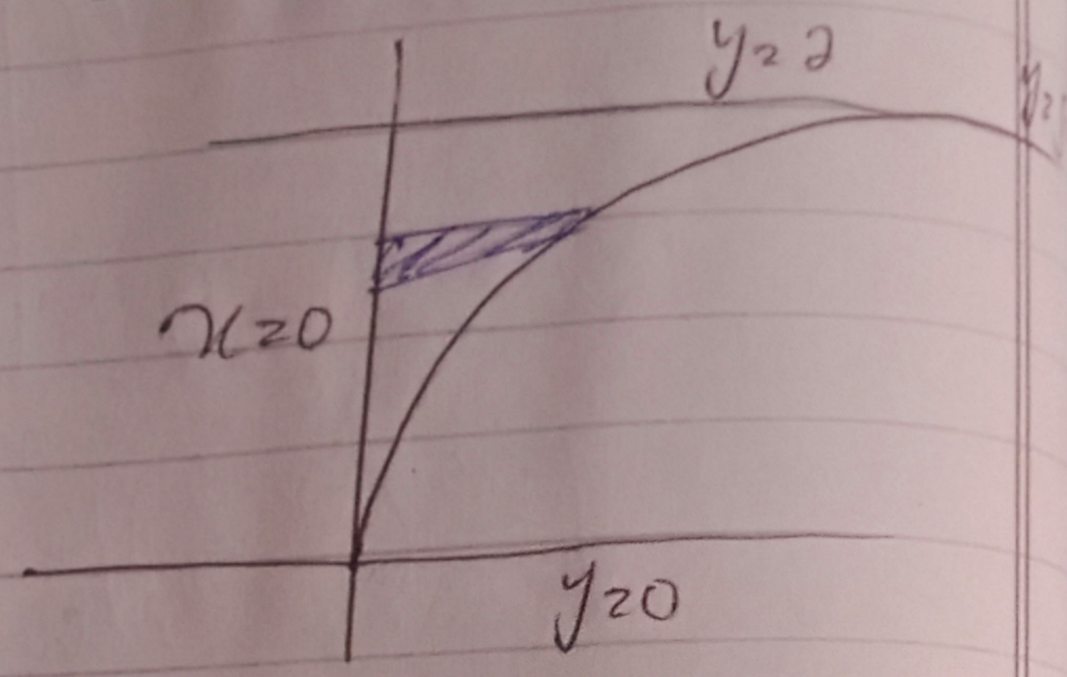
$$\frac{1}{4}\int_0^{\pi/4} (\cos 2y)\, dy \Rightarrow \frac{1}{4}\left[\frac{\sin 2y}{2}\right]_0^{\pi/4}$$

$$\frac{1}{8}\left(\sin(\pi/2) - \sin(0)\right)$$

$$\frac{1}{8}(1) = 1/8 \text{ Ans}$$

(25) Evaluate $\iint_R \sin(y^3)\,dA$

$y=\sqrt{x}$ $y=2$ and $x=0$



$y=\sqrt{x}$
$x=y^2$

$0 \le y \le 2$
$y^2 \le x \le 0$

$$\int_0^2 \int_0^{y^2} \sin(y^3)\,dA$$

$$\Rightarrow \int_0^2 \int_0^{y^2} \sin(y^3)\,dx\,dy$$

$$\Rightarrow \int_0^2 \left[\int_0^{y^2} \sin(y^3)\,dx\right] dy$$

$$\Rightarrow \int_0^2 \left(\sin(y^3)\,y^2\,dy \right)$$

$\Rightarrow$ $u = y^3$

$du = 3y^2\,dy$

$\frac{du}{3y^2} = dy$

when $y \to 2$; $u \to 8$

" $y \to 0$; $u \to 0$

14.2

$$\int_0^8 \sin(u)^{3y^2} du / 3y^2$$

$$\Rightarrow \frac{1}{3}\int_0^8 \sin(u)^{3} du$$

$$\Rightarrow \frac{1}{3}\left(-\cos u\right)_0^8 \Rightarrow \frac{-1}{3}\left(\cos 8 + 1\right)$$

$$\Rightarrow \left(\frac{1-\cos 8}{3}\right) \hookrightarrow \text{Ans}$$

---

(43) :-

$4x^2+y^2=9$

by The plane $z=0$ $z=y+3$

Given $4x^2+y^2=9$

at xy & y axis