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Partial Derivatives :-

def:-

if $z = f(x,y)$ and (x_0, y_0) is a point in the domain (xy plane) of f, Then The partial derivative of f w.r. to x at (x_0, y_0) is the derivative at x_0 of The function thate result when $y = y_0$ is held fixed and x is allowed to vary. The partial derivative is denoted by $f_x(x_0, y_0)$ and is given by

$$f_x(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

Similarly

The Partial derivative of f with respect to y at (x_0, y_0) is the derivative at y_0 of the function thate result $x = x_0$ is held fixed and y is allowed to vary. This partial derivative is denoted by $f_y(x_0, y_0)$ and is given by

$$f_y(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right] = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

(read book page no 927.928)

► **Example 1** Find $f_x(1,3)$ and $f_y(1,3)$ for the function $f(x,y)=2x^3y^2+2y+4x$.

Solution. Since

$$f_x(x,3)=\frac{d}{dx}[f(x,3)]=\frac{d}{dx}[18x^3+4x+6]=54x^2+4$$

we have $f_x(1,3)=54+4=58$. Also, since

$$f_y(1,y)=\frac{d}{dy}[f(1,y)]=\frac{d}{dy}[2y^2+2y+4]=4y+2$$

we have $f_y(1,3)=4(3)+2=14$. ◄

THE PARTIAL DERIVATIVE FUNCTIONS

Formulas (1) and (2) define the partial derivatives of a function at a specific point (x_0, y_0). However, often it will be desirable to omit the subscripts and think of the partial derivatives as functions of the variables x and y. These functions are

$$f_x(x,y)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}\qquad f_y(x,y)=\lim_{\Delta y\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$$

The following example gives an alternative way of performing the computations in Example 1.

► **Example 2** Find $f_x(x,y)$ and $f_y(x,y)$ for $f(x,y)=2x^3y^2+2y+4x$, and use those partial derivatives to compute $f_x(1,3)$ and $f_y(1,3)$.

Solution. Keeping y fixed and differentiating with respect to x yields

$$f_x(x,y)=\frac{d}{dx}[2x^3y^2+2y+4x]=6x^2y^2+4$$

and keeping x fixed and differentiating with respect to y yields

$$f_y(x,y)=\frac{d}{dy}[2x^3y^2+2y+4x]=4x^3y+2$$

Thus,

$$f_x(1,3)=6(1^2)(3^2)+4=58 \quad\text{and}\quad f_y(1,3)=4(1^3)3+2=14$$

which agree with the results in Example 1. ◄

...OLOGY MASTERY

...er algebra systems have spe- ...mmands for calculating partial ...ves. If you have a CAS, use it ... the partial derivatives $f_x(x,y)$ ... in Example 2.

PARTIAL DERIVATIVE NOTATION

If $z=f(x,y)$, then the partial derivatives f_x and f_y are also denoted by the symbols

$$\frac{\partial f}{\partial x},\ \frac{\partial z}{\partial x} \quad\text{and}\quad \frac{\partial f}{\partial y},\ \frac{\partial z}{\partial y}$$

...tion ∂ is called a partial deriva- ... It is derived from the Cyrillic ...

Some typical notations for the partial derivatives of $z=f(x,y)$ at a point (x_0,y_0) are

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0,\,y=y_0},\quad \left.\frac{\partial z}{\partial x}\right|_{(x_0,y_0)},\quad \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)},\quad \frac{\partial f}{\partial x}(x_0,y_0),\quad \frac{\partial z}{\partial x}(x_0,y_0)$$

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notation:-

if $z = f(x,y)$ then the Partial derivative f_x and f_y are also denoted by the symbol.

$$\Rightarrow \quad \frac{\partial f}{\partial x}, \frac{\partial z}{\partial x} \quad \text{and} \quad \frac{\partial f}{\partial y}, \frac{\partial z}{\partial y}$$

Some Typical notation for the Partial derivative is.

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0,\, y=y_0} \qquad \left.\frac{\partial z}{\partial x}\right|_{(x_0,y_0)} \qquad \frac{\partial f}{\partial x}(x_0,y_0)$$

$$\frac{\partial z}{\partial x}(x_0,y_0)$$

"Ex 13.3"

① let $f(x,y) = 3x^3y^2$ Find

ⓐ $f_x(x,y)$
ⓑ $f_y(x,y)$
ⓒ $f_x(1,y)$
ⓓ $f_x(x,1)$

$$\Rightarrow \quad f(x,y) = 3x^3y^2$$

ⓐ

$$\Rightarrow \quad \frac{\partial f}{\partial x} = 9x^2y^2$$

$$\Rightarrow \quad ⓑ \quad \frac{\partial f}{\partial y} = 6x^3y$$

$$ⓒ \quad \left.\frac{\partial f}{\partial x}\right|_{x=1,\, y=y} = 9(1)^2y^2 = 9y^2$$

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(d) $\frac{\partial f}{\partial x}_{(x=x,\, y=1)} = 9x^2(1)^2 = 9x^2$

Q 3-10 :-

(4) $f(x,y) = 10x^2y^4 - 6xy^2 + 10x^2$

Find

$f_x(x,y)$, $f_y(x,y)$

$\Rightarrow$ Solutions $f_x(x,y) = \frac{\partial f}{\partial x}\left[10x^2y^4 - 6xy^2 + 10x^2\right]$

$= 20xy^4 - 6y^2 + 20x$

$\Rightarrow$ $f_y(x,y) = \frac{\partial f}{\partial y}\left[10x^2y^4 - 6xy^2 + 10x^2\right]$

$= 40x^2y^3 - 12xy$

(9) $z = \sin(5x^3y + 7xy^2)$,

$\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

$\Rightarrow$ $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\sin(5x^3y + 7xy^2)\right]$

$= \cos(5x^3y + 7xy^2)\ \partial/_{\partial x}(5x^3y + 7xy^2)$

$= \cos(5x^3y + 7xy^2)\ (14xy + 15x^2y^2)$

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$\Rightarrow \quad = (15x^2y + 14xy)\cos(5x^3y + 7x^2y^2)$

$\Rightarrow \dfrac{\partial f}{\partial y} = \dfrac{\partial}{\partial y}\left[\sin(5x^3y + 7xy^2)\right]$

$\Rightarrow \quad = \cos(5x^3y + 7xy^2)\, \partial/\partial y\, (5x^3y + 7xy^2)$

$= (5x^3 + 14xy)\cos(5x^3y + 7xy^2)$

$\Rightarrow$ Ans

(12) let $f(x,y) = xe^{-y} + 5y$

(a) Find the slope of a surface $z = f(x,y)$ in the x direction at the point $(3,0)$

(b) Find the slope of a surface $z = f(x,y)$ in the y direction at the point $(4,2)$

(a) we know that slope of surface $z = f(x,y)$ in the x direction is the derivative of a function in the x direction or w.r.t to x, i.e

$\dfrac{\partial f}{\partial x}\Big|_{(3,0)}$

$\Rightarrow f(x,y) = xe^{-y} + 5y$

slope along x direction

$\Rightarrow \dfrac{\partial f}{\partial x} = \dfrac{\partial}{\partial x}\left[xe^{-y} + 5y\right]$

$= e^{-y}\Big|$ at point $(3,0)$

$$\left.\frac{\partial f}{\partial x}\right|_{(3,0)} = \left.e^{-y}\right|_{(3,0)} = e^{-0} = 1.$$

So The slope along x direction at point (3,0) is 1.

(b)

$$f(x,y) = xe^{-y} + 5y.$$

$$\Rightarrow \quad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[xe^{-y} + 5y\right]$$

$$= -xe^{-y} + 5$$

at Point (3,0)

$$\Rightarrow \quad \left.\frac{\partial f}{\partial y}\right|_{(3,0)} = -3e^{-(0)} + 5$$

$$= -3 + 5 = 2$$

$\Rightarrow$ slope $= 2$

(2b)

$$z = \cos(x^5 y^4)$$

Find $\partial z/\partial x$, $\partial z/\partial y$

$\Rightarrow$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\cos(x^5y^4)\right] \Rightarrow -\sin(x^5y^4)\,\partial/\partial x\,(x^5y^4)$$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = -(5x^4y^4)\cos(x^5y^4)$$

$\Rightarrow$ Ans

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$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left[\cos(x^5 y^4)\right]$$

$$= -\sin(x^5y^4)\ \partial/\partial y\ (x^5 y^4)$$

$$= -(4x^5y^4)\ \sin(x^5y^4) \Rightarrow Ans$$

(38) Evaluate the indicated Partial derivative:-

$$f(x,y) = x^2 y e^{xy} \qquad \left.\frac{\partial f}{\partial x}\right|_{(1,1)} \qquad \left.\frac{\partial f}{\partial y}\right|_{(1,1)}$$

$\Rightarrow$ Solution = $$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[x^2 y\ e^{xy}\right]$$

$$= x^2y\ e^{xy}(y) + y e^{xy}(2x)$$

$$= x^2y^2e^{xy} + 2xye^{xy}.$$

at Point (1,1)

$$\Rightarrow \frac{\partial f}{\partial x} = \left. x^2y^2e^{xy} + 2xye^{xy}\right|_{(1,1)}$$

$$= (1)^2(1)^2 e^{(1)(1)} + 2(1)(1)\ e^{(1)(1)}$$

$$= e + 2e \Rightarrow 3e$$

$\Rightarrow$ Similarly $$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\underset{u}{x^2y}\ \underset{v}{e^{xy}}\right]$$

$$= x^2y\ e^{xy}(x) + x^2 e^{xy}$$

$$\Rightarrow \quad \frac{\partial f}{\partial y} = x^3 y e^{xy} + x^2 e^{xy}$$

at Point $(1,1)$

$$\Rightarrow \quad \left.\frac{\partial f}{\partial y}\right|_{(1,1)} = \left(x^3 y e^{xy} + x^2 e^{xy}\right)_{(1,1)}$$

$$= (1)^3 e^{(1)(1)} + (1)^2 e^{(1)(1)}$$

$$= e + e \Rightarrow 2e$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,1)} = 2e$$

69-72:

Calculate $\partial z/\partial x$ and $\partial z/\partial y$ using implicit differentiation. leave your ans in Term of x, y and z.

(69) $\quad (x^2+y^2+z^2)^{3/2} = 1$

Solution:

$\Rightarrow$ First we find $\partial z/\partial x$

$$\Rightarrow \quad \frac{\partial}{\partial x}(x^2+y^2+z^2)^{3/2} = \frac{\partial}{\partial x}(1)$$

$$\Rightarrow \quad \cancel{\frac{\partial z}{\partial x} = \left(2x + 0 + 2z\frac{\partial z}{\partial x}\right)}$$

$$\frac{\partial W}{\partial v}(25, 10) \approx \frac{W(25, 10-5) - 15}{-5} = \frac{W(25, 5) - 15}{-5} = \frac{19 - 15}{-5} = -\frac{4}{5}\ \frac{°F}{mi/h}$$

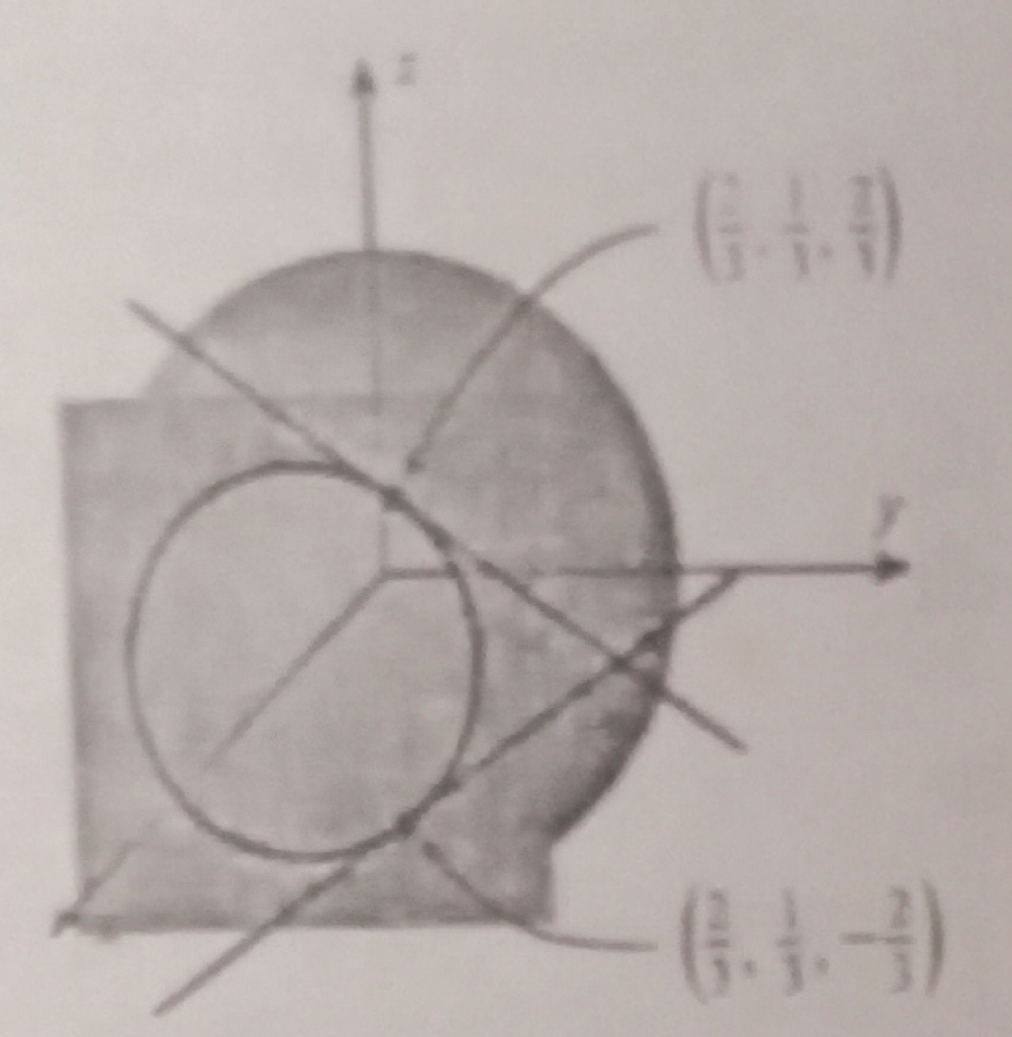
We will take the average, $-\frac{3}{5} = -0.6°F/(mi/h)$, of these two approximations as our estimate of $(\partial W/\partial v)(25, 10)$. This is close to the value

$$\frac{\partial W}{\partial v}(25, 10) = (-4.01)10^{-0.84} \approx -0.58\ \frac{°F}{mi/h}$$

found in Example 4. ◂

IMPLICIT PARTIAL DIFFERENTIATION

▸ **Example 7** Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y-direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ (Figure 13.3.2).



▲ Figure 13.3.2

Solution. The point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ lies on the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$, and the point $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ lies on the lower hemisphere $z = -\sqrt{1 - x^2 - y^2}$. We could find the slopes by differentiating each expression for z separately with respect to y and then evaluating the derivatives at $x = \frac{2}{3}$ and $y = \frac{1}{3}$. However, it is more efficient to differentiate the given equation

$$x^2 + y^2 + z^2 = 1$$

implicitly with respect to y, since this will give us both slopes with one differentiation. To perform the implicit differentiation, we view z as a function of x and y and differentiate both sides with respect to y, taking x to be fixed. The computations are as follows:

$$\frac{\partial}{\partial y}[x^2 + y^2 + z^2] = \frac{\partial}{\partial y}[1]$$

$$0 + 2y + 2z\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{y}{z}$$

Check the results in Example 7 by differentiating the functions

$$z = \sqrt{1 - x^2 - y^2}$$

and

$$z = -\sqrt{1 - x^2 - y^2}$$

directly.

Substituting the y- and z-coordinates of the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ in this expression, we find that the slope at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ is $-\frac{1}{2}$ and the slope at $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is $\frac{1}{2}$. ◂

▸ **Example 8** Suppose that $D = \sqrt{x^2 + y^2}$ is the length of the diagonal of a rectangle whose sides have lengths x and y that are allowed to vary. Find a formula for the rate of change of D with respect to x if x varies with y held constant, and use this formula to find the rate of change of D with respect to x at the point where $x = 3$ and $y = 4$.

Solution. Differentiating both sides of the equation $D^2 = x^2 + y^2$ with respect to x yields

$$2D\frac{\partial D}{\partial x} = 2x \quad \text{and thus} \quad D\frac{\partial D}{\partial x} = x$$

Since $D = 5$ when $x = 3$ and $y = 4$, it follows that

$$5\left.\frac{\partial D}{\partial x}\right|_{x=3,\,y=4} = 3 \quad \text{or} \quad \left.\frac{\partial D}{\partial x}\right|_{x=3,\,y=4} = \frac{3}{5}$$

Thus, D is increasing at a rate of $\frac{3}{5}$ unit per unit increase in x at the point $(3, 4)$. ◂

PARTIAL DERIVATIVES AND CONTINUITY

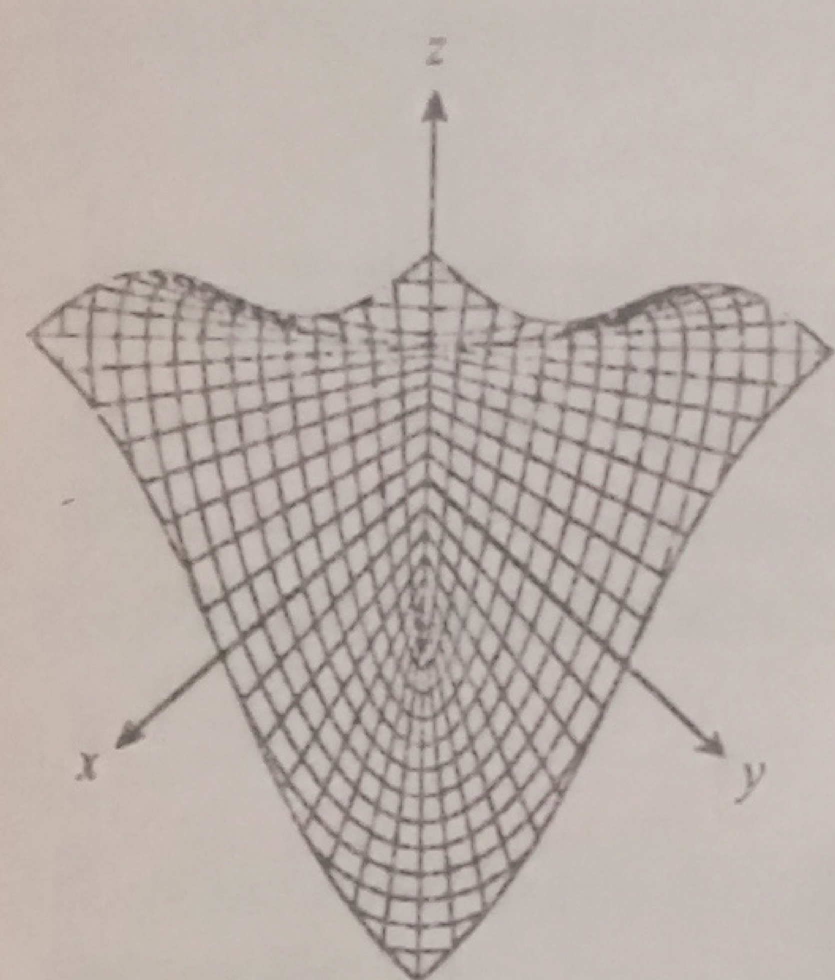
In contrast to the case of functions of a single variable, the existence of partial derivatives for a multivariable function does not guarantee the continuity of the function. This fact is shown in the following example.

▶ Example 9 Let

$$f(x, y) = \begin{cases} -\dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} \tag{3}$$

(a) Show that $f_x(x, y)$ and $f_y(x, y)$ exist at all points (x, y).

(b) Explain why f is not continuous at $(0, 0)$.



▲ Figure 13.3.3

Solution (a). Figure 13.3.3 shows the graph of f. Note that f is similar to the function considered in Example 1 of Section 13.2, except that here we have assigned f a value of 0 at $(0, 0)$. Except at this point, the partial derivatives of f are

$$f_x(x, y) = -\frac{(x^2+y^2)y - xy(2x)}{(x^2+y^2)^2} = \frac{x^2y - y^3}{(x^2+y^2)^2} \tag{4}$$

$$f_y(x, y) = -\frac{(x^2+y^2)x - xy(2y)}{(x^2+y^2)^2} = \frac{xy^2 - x^3}{(x^2+y^2)^2} \tag{5}$$

It is not evident from Formula (3) whether f has partial derivatives at $(0, 0)$, and if so, what the values of those derivatives are. To answer that question we will have to use the definitions of the partial derivatives (Definition 13.3.1). Applying Formulas (1) and (2) to (3) we obtain

$$f_x(0, 0) = \lim_{\Delta x \to 0} \frac{f(\Delta x, 0) - f(0, 0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{0 - 0}{\Delta x} = 0$$

$$f_y(0, 0) = \lim_{\Delta y \to 0} \frac{f(0, \Delta y) - f(0, 0)}{\Delta y} = \lim_{\Delta y \to 0} \frac{0 - 0}{\Delta y} = 0$$

This shows that f has partial derivatives at $(0, 0)$ and the values of both partial derivatives are 0 at that point.

Solution (b). We saw in Example 3 of Section 13.2 that

$$\lim_{(x,y)\to(0,0)} -\frac{xy}{x^2+y^2}$$

does not exist. Thus, f is not continuous at $(0, 0)$. ◀

We will study the relationship between the continuity of a function and the properties of its partial derivatives in the next section.

PARTIAL DERIVATIVES OF FUNCTIONS WITH MORE THAN TWO VARIABLES

For a function $f(x, y, z)$ of three variables, there are three ***partial derivatives***:

$$f_x(x, y, z), \quad f_y(x, y, z), \quad f_z(x, y, z)$$

The partial derivative f_x is calculated by holding y and z constant and differentiating with respect to x. For f_y the variables x and z are held constant, and for f_z the variables x and y are held constant. If a dependent variable

$$w = f(x, y, z)$$

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$$\Rightarrow \quad \frac{\partial f}{\partial y} = x^3 y e^{xy} + x^2 e^{xy}$$

at Point $(1,1)$

$$\Rightarrow \quad \frac{\partial f}{\partial y}\Big|_{(1,1)} = \left(x^3 y e^{xy} + x^2 e^{xy}\right)_{(1,1)}$$

$$= (1)^3 e^{(1)(1)} + (1)^2 e^{(1)(1)}$$

$$= e + e \Rightarrow 2e$$

$$\frac{\partial f}{\partial y}\Big|_{(1,1)} = 2e$$

69-72 :-

Calculate $\partial z/\partial x$ and $\partial z/\partial y$ using implicit differentiation. leave your ans in Term of x, y and z.

(69) $\quad (x^2+y^2+z^2)^{3/2} = 1$

Solution:

$\Rightarrow$ First we find $\partial z/\partial x$

$$\Rightarrow \quad = \frac{\partial}{\partial x}\left(x^2+y^2+z^2\right)^{3/2} = \frac{\partial}{\partial x}(1)$$

$$\Rightarrow \quad \cancel{\frac{\partial z}{\partial x} = \left(2x + 0 + 2z \frac{\partial z}{\partial x}\right)}$$

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$$\Rightarrow \frac{3}{2}(x^2+y^2+z^2)^{\frac{3}{2}-1}\,\frac{\partial}{\partial x}(x^2+y^2+z^2) = \frac{\partial}{\partial x}(1)$$

$$\Rightarrow \frac{3}{2}(x^2+y^2+z^2)^{1/2}\left(2x + 2z\frac{\partial z}{\partial x}\right) = 0$$

$$\Rightarrow 2x + 2z\frac{\partial z}{\partial x} = \frac{0}{\frac{3}{2}(x^2+y^2+z^2)^{1/2}}$$

$$\Rightarrow 2z\frac{\partial z}{\partial x} = -2x$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{-2x}{2z}$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial x} = \frac{-x}{z}}$$

(b) $\frac{\partial z}{\partial y} = ?$

$$\Rightarrow \frac{3}{2}(x^2+y^2+z^2)^{\frac{3}{2}-1}\,\frac{\partial}{\partial y}(x^2+y^2+z^2) = \frac{\partial}{\partial y}(1)$$

$$\Rightarrow \frac{3}{2}(x^2+y^2+z^2)^{1/2}\left(0 + 2y + 2z\frac{\partial z}{\partial y}\right) = 0$$

$$\Rightarrow 2y + 2z\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow 2z\frac{\partial z}{\partial y} = -2y \Rightarrow = \frac{-y}{z}$$

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(7.2)

$$e^{xy}\sinh z - z^2x + 1 = 0$$

$\frac{\partial z}{\partial x} = ?$ (Using implicity diff)

$$\Rightarrow \frac{\partial}{\partial x}\left(e^{xy}\sinh z - z^2x + 1\right) = \frac{\partial}{\partial x}(0)$$

$$\Rightarrow ye^{xy}\sinh z + e^{xy}\cosh(z)\frac{\partial z}{\partial x} - \left[\partial z\frac{\partial z}{\partial x}x + z^2\right] = 0$$

$$\Rightarrow ye^{xy}\sinh z + e^{xy}\cosh(z)\frac{\partial z}{\partial x} - \partial zx\frac{\partial z}{\partial x} - z^2 = 0$$

$$\Rightarrow \frac{\partial z}{\partial x}\left(e^{xy}\cosh z - \partial zx\right) = z^2 - ye^{xy}\sinh z$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{z^2 - ye^{xy}\sinh z}{e^{xy}\cosh z - \partial zx}$$

(5) $\frac{\partial z}{\partial y} = ?$

$$\frac{\partial}{\partial y}\left[e^{xy}\sinh z - z^2x + 1\right] = \frac{\partial}{\partial y}(0)$$

$$\Rightarrow xe^{xy}\sinh z + e^{xy}\cosh z\frac{\partial z}{\partial y} - \partial zx\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{\partial z}{\partial y}\left[e^{xy}\cosh z - 2zx\right] = -xe^{xy}\sinh z$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{-xe^{xy}\sinh z}{e^{xy}\cosh z - 2zx} \Rightarrow \text{Ans}$$

---

73. $(x^2+y^2+z^2+w^2)^{3/2} = 1$

$\Rightarrow \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z}$ (using implicity diff)

$$\Rightarrow \frac{\partial w}{\partial x} = ?$$

$$\Rightarrow \frac{\partial}{\partial x}\left(x^2+y^2+z^2+w^2\right)^{3/2} = \frac{\partial}{\partial x}(1)$$

$$\frac{3}{2}(x^2+y^2+w^2)^{3/2-1}\frac{\partial}{\partial x}(-)$$

$$\Rightarrow (\ )^{1/2}\left(2x+0+0\ 2w\frac{\partial w}{\partial x}\right) = 0$$

$$\Rightarrow \frac{3}{2}\left(x^2+y^2+z^2+w^2\right)^{1/2}\left(2x+2w\frac{\partial w}{\partial x}\right) = 0$$

$$\Rightarrow 2w\frac{\partial w}{\partial x} = -2x$$

$$\Rightarrow \boxed{\frac{\partial w}{\partial x} = -x/w}$$

$\frac{\partial w}{\partial y} = ?$

$$\frac{\partial}{\partial y}\left(x^2+y^2+z^2+w^2\right)^{3/2} = 4$$

$$\frac{3}{2}\left(x^2+y^2+z^2+w^2\right)^{3/2-1} \frac{\partial}{\partial y}\left(x^2+y^2+z^2+w^2\right)$$

$$\Rightarrow (\;)^{1/2}\left(0+2y+0+\frac{\partial w\,\partial w}{\partial y}\right) = 0$$

$$\Rightarrow \frac{3}{2}\left(x^2+y^2+z^2+w^2\right)^{1/2}\left(\cancel{0}\,2y+0+\frac{\partial w\,\partial w}{\partial y}\right) = 0$$

$$\Rightarrow \frac{\partial w\,\partial w}{\partial y} = -2y$$

$$\frac{\partial w}{\partial y} = \frac{-2y}{\partial w} \Rightarrow -y/w$$

$\frac{\partial w}{\partial z} = ?$

$$\frac{\partial}{\partial z}\left(x^2+y^2+z^2+w^2\right)^{3/2}$$

$$\Rightarrow \frac{3}{2}\left(x^2+y^2+z^2+w^2\right)^{3/2-1}\frac{\partial}{\partial z}\left(x^2+y^2+z^2+w^2\right)$$

$$\Rightarrow \frac{3}{2}\left(x^2+y^2+z^2+w^2\right)^{1/2} \cancel{\left(\frac{\partial z\,\partial z}{\partial x}\right)} = $$

$$\left(0+0+2z+\frac{\partial w\,\partial w}{\partial z}\right) = 0$$

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$$\Rightarrow \frac{\partial w \, \partial w}{\partial z} = -\partial z$$

$$\Rightarrow \frac{\partial w}{\partial z} = \frac{-\partial z}{\partial w} = \frac{-z}{w}$$

$$\Rightarrow \boxed{\frac{\partial w}{\partial z} = \frac{-z}{w}}$$

(82):-

let $f(x,y) = 4x^2 - 2y + 7x^4y^5$

Find ⓐ f_{xx} ⓑ f_{yy} ⓒ f_{xy} ⓓ f_{yx}

$\Rightarrow$ ⓐ $f_{xx} = \frac{\partial}{\partial x^2} \Rightarrow$

So, $f_x = \frac{\partial}{\partial x}\left(4x^2 - 2y + 7x^4y^5\right)$

$\Rightarrow f_x = 8x + 28x^3y^5$

again p. diff w.r.t x

$\Rightarrow f_{xx} = \frac{\partial}{\partial x}\left(8x + 28x^3y^5\right)$

$f_{xx} = 8 + 84x^2y^5 \Rightarrow$ Ans

ⓑ f_{yy}:

$\Rightarrow f_y = \frac{\partial}{\partial y}\left(4x^2 - 2y + 7x^4y^5\right)$

$\Rightarrow f_y = -2 + 35x^4y^4$

again P. diff w.r.t y.

$\Rightarrow$

$\Rightarrow f_{yy} = \dfrac{\partial}{\partial y}\left(-2 + 35x^4y^4\right)$

$f_{yy} = \left(0 + 140x^4y^3\right)$

$f_{yy} = 140x^4y^3 \Rightarrow$ Ans.

Imp

(101):- Show That The function is Laplace equation satisfied:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial y^2} = 0 \longrightarrow (B)$$

(a) $z = x^2 - y^2 + 2xy$

$\Rightarrow$ First we find $\partial z/\partial x$

$\Rightarrow z = x^2 - y^2 + 2xy.$

$\dfrac{\partial z}{\partial x} = 2x + 0 + 2y.$

$\Rightarrow \dfrac{\partial z}{\partial x} = 2x + 2y \rightarrow (1)$

again P. diff w.r.t x (1)

$\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) = \dfrac{\partial}{\partial x}\left(2x + 2y\right)$

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$$\Rightarrow \frac{\partial^2 z}{\partial x^2} = 2 \longrightarrow (A)$$

Now

we find $\frac{\partial^2 z}{\partial y^2}$

$$\Rightarrow z = x^2 - y^2 + 2xy.$$

$$\Rightarrow \frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(x^2 - y^2 + 2xy\right)$$

$$\Rightarrow \frac{\partial z}{\partial y} = (0 - 2y + 2x)$$

$$\Rightarrow \frac{\partial z}{\partial y} = -2y + 2x \longrightarrow (ii)$$

again diff w.r.t y. (ii)

$$\Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(-2y + 2x\right)$$

$$\Rightarrow \frac{\partial^2 z}{\partial y^2} = -2 + 0$$

$$\Rightarrow \frac{\partial^2 z}{\partial y^2} = -2 \longrightarrow (B)$$

put (A) & (B) in $\boxed{5}$

$$\Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

$$2 + (-2) = 0$$

$$\Rightarrow \quad 2 - 2 = 0$$

$$0 = 0$$

Hence $Z = x^2 - y^2 + 2xy$ is
Satisfied Laplace equation

Practis Problem

Complete ex 13.3.
with example 9 page (932)