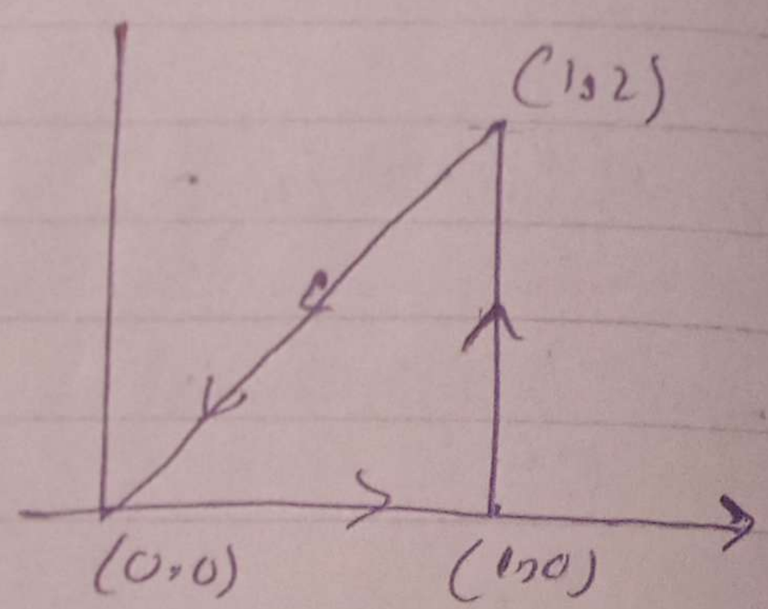
Exercise No 15.4

Use Green Theorm to evaluto $\int$?

$$\int_c x^2 y\, dx + x\, dy$$

$f(x,y) = x^2 y$ and $g(x,y) = x$

$\frac{\partial f}{\partial y} = x^2$ and $\frac{\partial g}{\partial x} = 1$



$$\int_c x^2 y\, dx + x\, dy =$$

$$\iint_R \left(\left(\frac{\partial g}{\partial x}\right) - \left(\frac{\partial f}{\partial y}\right)\right) dA$$

$$\Rightarrow \int_0^1 \int_0^{2x} (1 - x^2)\, dx\, dy$$

$$\Rightarrow \int_0^1 \left(x - \frac{x^3}{3}\right)_0^{2x} dy$$

$$\cancel{\Rightarrow \int_0^{2x} \left(1 - \frac{1}{3}\right) dy \Rightarrow \left(y - \frac{y}{3}\right)_0^{2x}}$$

$$\Rightarrow \int_0^1 \left(2x^2 - \frac{2x^4}{3}\right) dx$$

$\Rightarrow \left(x^2 - \frac{x^4}{2}\right) = \frac{1}{2}$

Exampl (2)

Find The work done by Force field.

$F(x,y) = (e^x - y^3)i + (\cos y + x^3)j$

Particle moves in a Circular path
$x^2 + y^2 = 1$

$w = \oint_c F\,dr.$

$\oint (e^x - e^y)\,dx + (\cos y + x^3)\,dy$

$\Rightarrow \frac{\partial I}{\partial x} = 3x^2$

$\frac{\partial J}{\partial y} = -3y^2$

$w = \oint_0^{2\pi}\int_0^1 3x^2 + 3y^2 \, dA$

$w = \int_0^{2\pi}\int_0^1 3(x^2+y^2)\, r\,dr\,d\theta$

$\cancel{\int_0^{2\pi}\left[\frac{r^3}{3}\right]_0^1} \Rightarrow \int_0^{2\pi}\int_0^1 3(r^2)\, r\,dr$

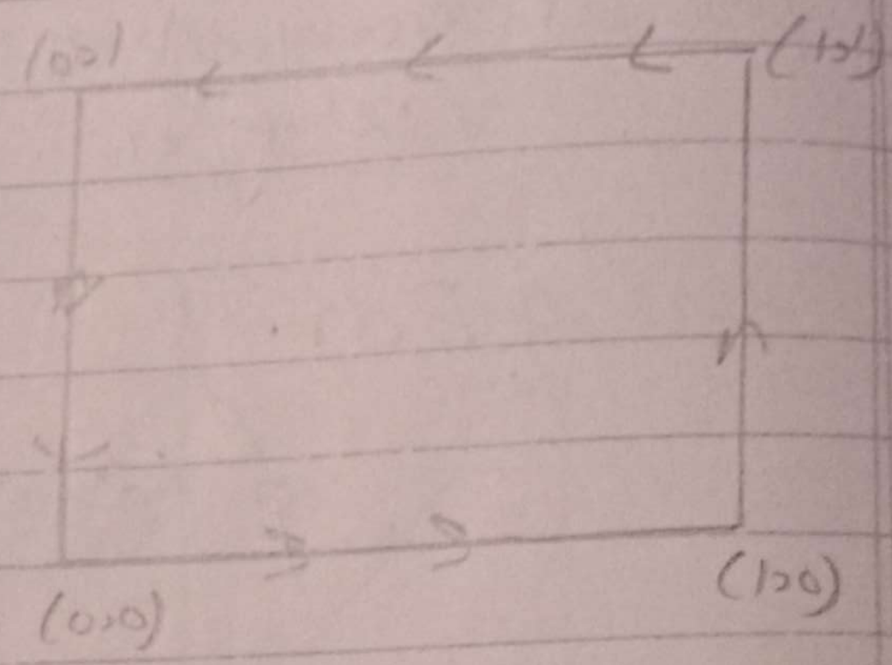
$\Rightarrow \int_0^{2\pi}\left(\int_0^1 \frac{3r^4}{4}\right) \Rightarrow \frac{6\pi}{4} \Rightarrow \frac{3\pi}{2}$

Ans

Exercise Q (15.4)

Evalue using Green Theorm

Q① $\oint_c y\,dx + x^2\,dy$ where c, is the square with vertexes $(0,0)$ $(1,0)$ $(1,1)$ $(0,1)$ oriented counterclockwise.



From the figer it is clear to that

$0 \le x \le 1$

$0 \le y \le 1$

So

Ale To Grem Theorm

$$\iint_D \left(\frac{\partial x}{\partial x} - \frac{\partial f}{\partial y} \right) dA \rightarrow Ⓐ$$

$$\frac{\partial x}{\partial x} = \partial x$$

$$\frac{\partial f}{\partial y}, \partial y$$

Ⓐ becomes

$$\Rightarrow \int_0^1 \int_0^1 (\partial x - \partial y)\, dx\, dy$$

$$\Rightarrow \int_0^1 \left(\frac{\partial x^2}{\partial} - \partial y x \right)_0^1 dy$$

$$\int_0^1 y - 2y \Rightarrow \left(\frac{y^2}{2} - \frac{2y^2}{2}\right)_0^1$$

$$\Rightarrow \left(\frac{1}{2} - 1\right) \Rightarrow \left(\frac{1-2}{2}\right) \Rightarrow -1/2$$

$\hookrightarrow$ Ans

$$\Rightarrow \int_0^1 (1 - 2y)\,dy \Rightarrow \left(y - \frac{2y^2}{2}\right)_0^1$$

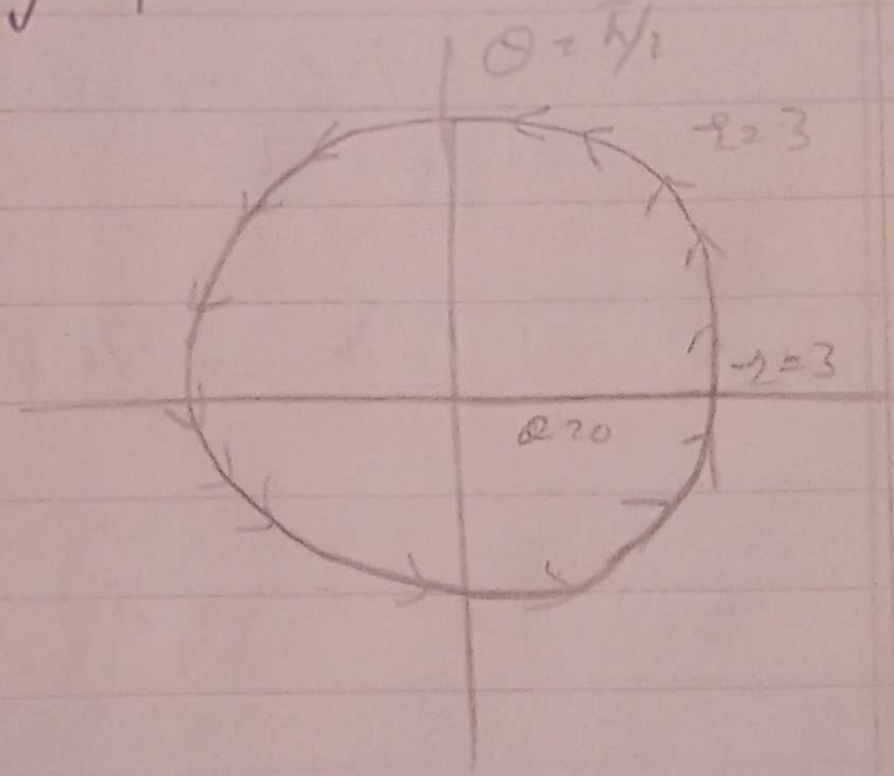
$$\Rightarrow (1-1) \Rightarrow 0 \Rightarrow \text{Ans}$$

Q4 $\oint_C (x^2 - y^2)\,dx + x\,dy$ where C is The circle $x^2 + y^2 = 9$

Given $x^2 + y^2 = 9$

$0 \le r \le 3$

$0 \le \theta \le 2\pi$



$f(x,y) = (x^2 - y^2)$

$g(x,y) = x$

$\frac{\partial f}{\partial y} = -2y$

$\frac{\partial g}{\partial x} = 1$

Ale to Green Theorm:

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$\Rightarrow$ So it is clear to That

$$\int_0^{2\pi}\int_0^{3} (1 + 2y)\, dA$$

$$\int_0^{2\pi}\int_0^{3} (1 + 2r\sin\theta)\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\int_0^{3} (r + 2r^2\sin\theta)\, dr\right) d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\frac{r^2}{2} + \frac{2r^3}{3}\sin\theta\right)_0^3 d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\frac{9}{2} + 18\sin\theta\right) d\theta$$

$$\Rightarrow \left(\frac{9}{2}\theta - 18\cos\theta\right)_0^{2\pi}$$

$$\Rightarrow \frac{18\pi}{2} - 18(1-1)$$

$$\Rightarrow (9\pi) \Rightarrow \text{Ans.}$$

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(7) $(x^2 - y)\,dx + x\,dy$ where C is the circle line

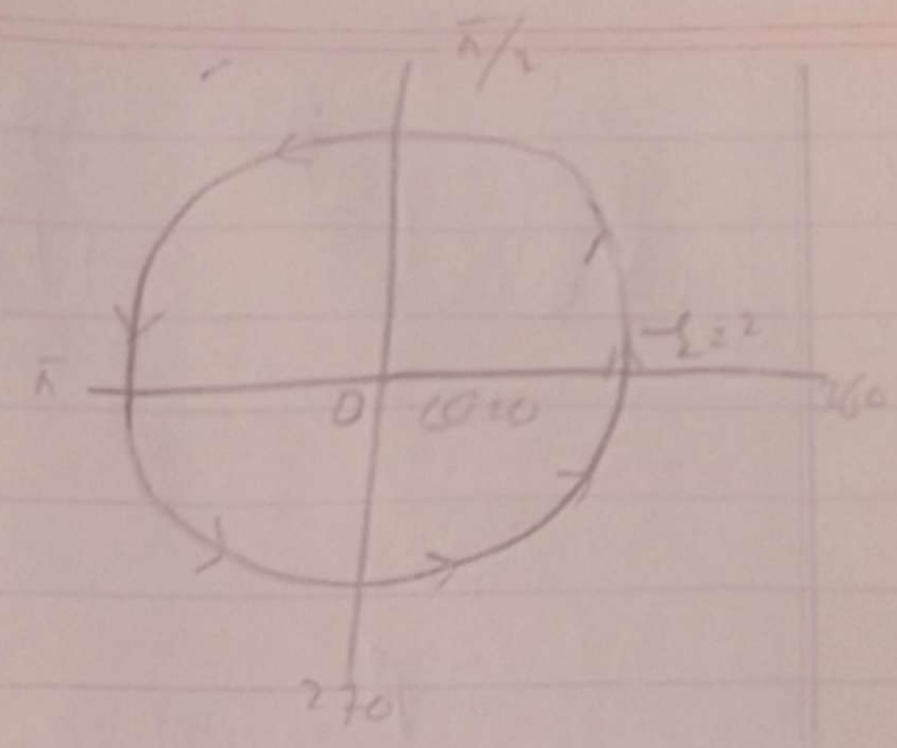
$$x^2 + y^2 = 4$$

$$f(x,y) = (x^2 - y)$$

$$g(x,y) = x$$

$$\frac{\partial f}{\partial y} = -1 \qquad \frac{\partial g}{\partial x} = 1$$

$x^2+y^2=4$



$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{2}(1+1)\,r\,dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\int_0^{2} 2r\,dr\right)d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\frac{2r^2}{2}\right)_0^2 d\theta \Rightarrow \int_0^{2\pi}(4-0)\,d\theta$$

$$\Rightarrow \int_0^{2\pi}4\theta \Rightarrow (8\pi) \Rightarrow \text{Ans}$$

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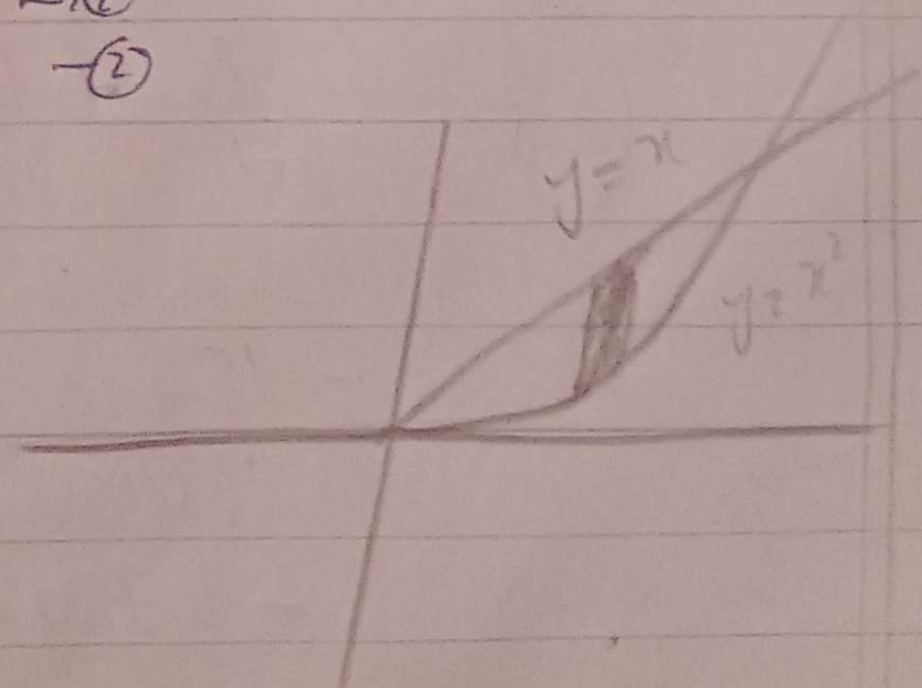
(8) $\oint_C (e^x+y^2)\,dx+(e^y+x^2)\,dy$ where C is The boundry region b/w $y=x^2$ & $y=x$

Given $y=x^2$ —(1)

& $y=x$ —(2)



$x \le y \le x^2$

$0 \le x \le 1$

$\Rightarrow x^2-x=0$

$x(1-x)=0$

$x=0$

$$\int_0^1 \int_{x^2}^{x} \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

$$\Rightarrow \frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\Rightarrow \int_0^1 \int_{x^2}^{x} (2x - 2y)\, dy\, dx$$

$$\Rightarrow \int_0^1 \left(2xy - \cancel{2}y^2/\cancel{2} \right)_{x^2}^{x} dx$$

$$\Rightarrow \int_0^1 2xy(x - x^2 - (x^2 - x^4)\, dx$$

$$\Rightarrow \int 2x^2 - 2x^3 - x^2 + x^4 \quad dx$$

$$\Rightarrow \int_0^1 x^4 - 2x^3 + x^2\, dx$$

$$\Rightarrow \left(\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right)_0^1$$

$$\Rightarrow \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3} \right)$$

$$\begin{array}{c|c} 2 & 5 - 2 - 3 \\ \hline 3 & 5 - 1 - 3 \\ \hline 5 & 5 - 1 - 3 \\ & 1 - 1 \end{array}$$

$$\frac{6 - 15 + 10}{30} \Rightarrow$$

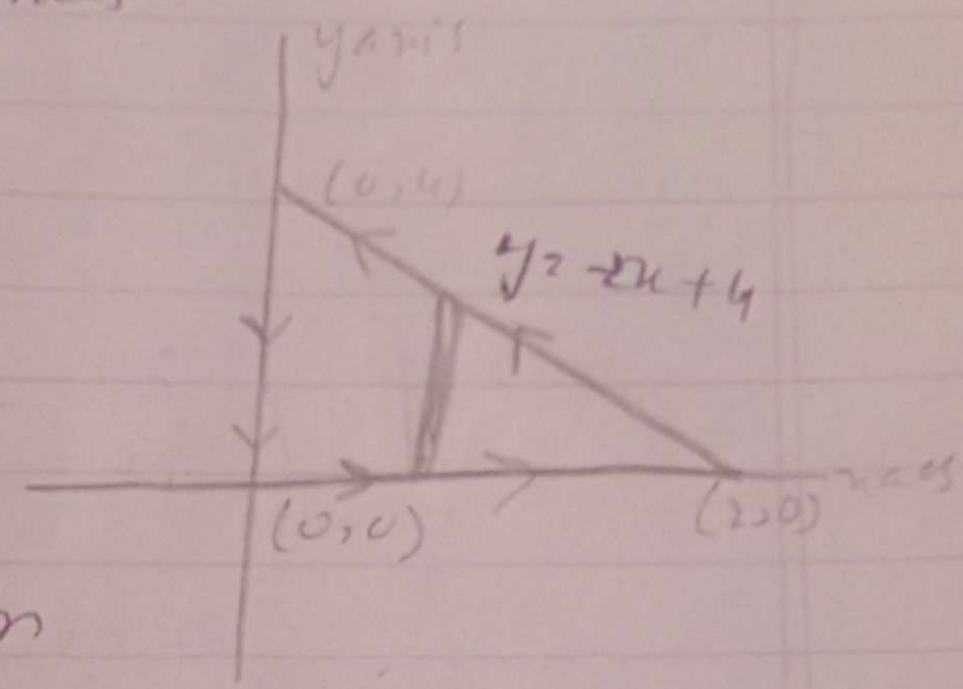
$$\Rightarrow \frac{-9 + 10}{30} \Rightarrow \left(1/30 \right) \Rightarrow \text{Ans}$$

(1) $\oint \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$ (is

The Triangular Region with vertices

(0,0) (2,0) and (0,4)

Given The Triangle vertices



It is clear to Thele from a drag ram.

$$0 \le x \le 2$$

There two point (2,0) & (0,4) a line segment using for The purpose of eq.

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-0}{4-0} = \frac{x-2}{0-2}$$

$$\Rightarrow -2y = 4x-8$$

$$\Rightarrow y = \frac{\cancel{4}x-8}{-\cancel{2}}$$

$$\Rightarrow y = -2x+4$$

So Ale Green theorm

$f(x,y) = \ln(1+y)$

$g(x,y) = -\frac{xy}{1+y}$

$\frac{\partial f}{\partial y} = \frac{1}{1+y}$

$\frac{\partial g}{\partial x} = \frac{-y}{1+y}$ So,

$$\Rightarrow \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} \left(\frac{-y}{1+y} - \frac{1}{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} \left(\frac{-y-1}{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \int_0^{-2x+4} -1 \left(\frac{y+1}{1+y} \right) dy\, dx$$

$$\Rightarrow \int_0^2 \left(\int_0^{-2x+4} -1 \cdot dy \right) dx$$

$$\Rightarrow \int_0^2 2x - 4\, dx$$

$$\left(\frac{8x^2}{x} - 4x\right)_0^2$$

$$\Rightarrow 4 - 4(2) \Rightarrow 4 - 8$$

$$\Rightarrow -4 \Rightarrow \text{Ans}$$

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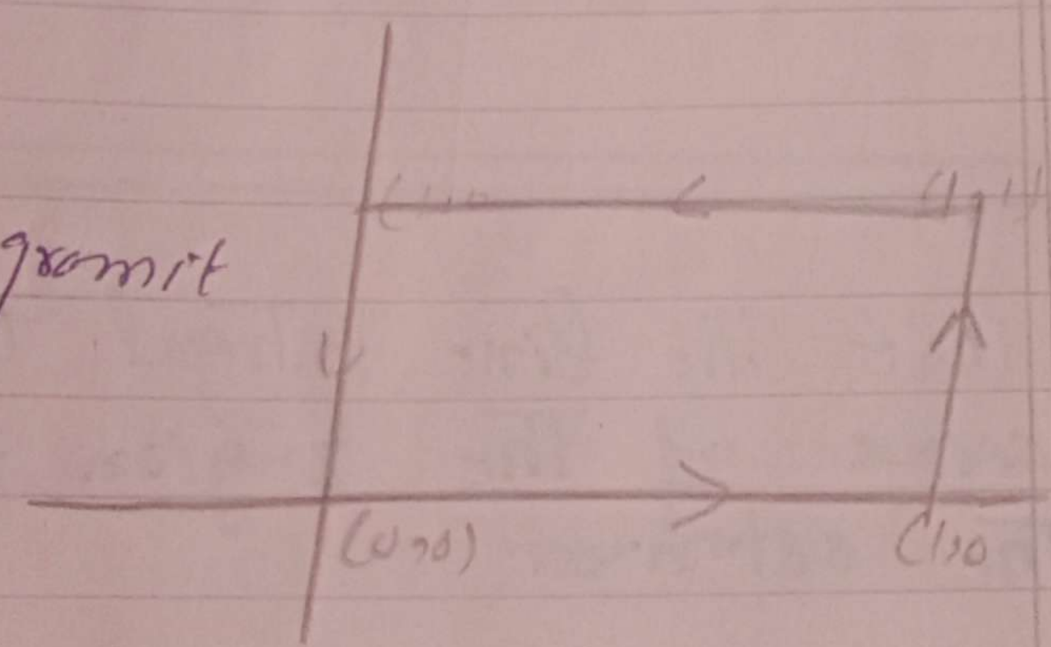
(11) $\oint \tan^{-1} y\, dx - \frac{y^2 x}{1+y^2}\, dy$, C is the square with vertical (0,0), (1,0) (1,1) and (0,1)



From The diagram it is clear to That

$0 \le x \le 1$

$0 \le y \le 1$

So.

$$f(x,y) = \tan^{-1} y$$

$$g(x,y) = -\frac{y^2 x}{1+y^2}$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{1}{1+y^2}$$

$$\frac{\partial g}{\partial x} = -\frac{y^2}{1+y^2}$$

$$\int_0^1\int_0^1 \left(\frac{-y^2}{1+y^2} - \frac{1}{1+y^2} \right) dx\,dy$$

$$\Rightarrow \int_0^1\int_0^1 \frac{-y^2-1}{1+y^2}\, dx\, dy$$

$$\Rightarrow \int_0^1 \left(\int_0^1 -1\, dx \right) dy$$

$$\Rightarrow \int_0^1 -1\, dy$$

$$\Rightarrow -1 \Rightarrow \text{Ans.}$$

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(21) use the line intrgal to find the area of the region enclosed by the astroid

$$x = a\cos^3\phi \;,\quad y = a\sin^3\phi$$

$$0 \le \phi \le 2\pi$$

using $A = \frac{1}{2}\oint -y\,dx + x\,dx \rightarrow (A)$

$$x = a\cos^3\phi$$
$$dx = 3a\cos^2\phi\,(-\sin\phi)$$
$$dx = -3a\sin\phi\cos^2\phi$$

$$y = a\sin^3\phi$$

$dy = 3a \sin^2\phi \cos\phi$

$dy = 3a \cos\phi \sin^2\phi$

So, (A) becomes

$$\Rightarrow A = \frac{1}{2}\int_0^{2\pi} (-a\sin^3\phi)(-3a\sin\phi\cos^2\phi) + (a\cos^3\phi)(3a\cos\phi\sin^2\phi)$$

$$A = \frac{1}{2}\int_0^{2\pi} 3a^2\sin^4\phi\cos^2\phi\, d\phi + 3a^2\cos^4\phi\sin^2\phi\, d\phi$$

$$A = \frac{1}{2}\int_0^{2\pi} 3a^2\sin^2\phi\cos^2\phi\left[\sin^2\phi + \cos^2\phi\right] d\phi$$

$$\Rightarrow A = \frac{1}{2}\int_0^{2\pi} 3a^2\sin^2\phi\cos^2\phi\, d\phi$$

$$A = \frac{3a^2}{2}\int_0^{2\pi} \frac{1}{2}(1-\cos 2\phi)\frac{1}{2}(1+\cos 2\phi)\, d\phi$$

$$\cancel{A = \frac{3a^2}{2}\left[\frac{1}{2}\left(\phi - \frac{\sin 2\phi}{2}\right)\frac{1}{2}\left(\phi + \frac{\sin 2\phi}{2\phi}\right)\right]}$$

$$\cancel{A = \left(\frac{3a^2}{2}\right)\left[\left(\frac{2\pi}{2}\right) - 0\ \frac{1}{2}(2\pi + 0)\right]}$$

$$\cancel{A = \left(\frac{3a^2}{2}\right)\left(\right.}$$

$$A = \frac{3a^2}{8}\left(1 + \cos 2\phi\right)\left(1 + \cos 2\phi\right)$$

$$A = \frac{3a^2}{8}\int_0^{2\pi}\left(1 + \cos 2\phi - \cos 2\phi - \cos^2 2\phi\right) d\theta$$

$$A = \frac{3a^2}{8}\left[\phi + \frac{\sin 2\phi}{2} - \frac{\sin 2\phi}{2}\right]_0^{2\pi} - \left(\frac{1}{2}\left(1 + \frac{\cos 4\phi}{2}\right)\right) d\theta$$

$$A = \frac{3a^2}{8}\left[2\pi + 0 - 0\right] - \frac{1}{2}\left(\phi + \frac{\sin 4\phi}{2}\right)$$

$$A = \frac{3a^2}{8}\left[2\pi\right] - \frac{1}{2}\left(2\pi\right) + 0\Big]$$

$$A = \frac{3a^2}{8}\left(2\pi - \pi\right) \Rightarrow$$

$$A = \frac{3a^2}{8}\left(\pi\right)$$

$$A = \left(\frac{3a^2\pi}{8}\right) \Rightarrow \text{Ans}$$

33

Given $x = a\cos t$

$y = b\sin t$

$0 \le t \le 2\pi$

$dx = -a\sin t$

$dy = b\cos t$

using

$$A = \frac{1}{2}\oint -y\,dx + x\,dy$$

$$\Rightarrow \frac{1}{2}\oint_0^{2\pi} (-b\sin t)(-a\sin t) + (a\cos t)(b\cos t)$$

$$\Rightarrow \frac{1}{2}\oint_0^{2\pi} ab\sin^2 t + ab\cos^2 t.$$

$$\Rightarrow \frac{1}{2}\oint_0^{2\pi} ab(\sin^2 t + \cos^2 t)$$

$$\Rightarrow \frac{ab}{2}\int_0^{2\pi} 1\,dt = \frac{2\pi ab}{2}$$

$$(ab\pi) \Rightarrow \text{Ans}$$

Exercise No 15.5

Examples:-

Evalute the ~~line~~ surface intregal

$\iint_S x^2 ds$ over the $x^2+y^2+z^2=1$

The parametric eq of surface is

$r(\phi,\theta) = \sin\phi\cos\theta\, i + \sin\phi\sin\theta\, j + \cos\phi\, k$

$\Rightarrow \frac{\partial r}{\partial \phi} = \cos\phi\cos\theta + \cos\phi\sin\theta - \sin\phi$

$\frac{\partial r}{\partial \theta} = -\sin\phi\sin\theta + \sin\phi\cos\theta + 0k$

$$\frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} = \begin{vmatrix} i & j & k \\ \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ -\sin\phi\sin\theta & \sin\phi\cos\theta & 0 \end{vmatrix}$$

$\Rightarrow i(\sin^2\phi\cos\theta) - j(-\sin^2\phi\sin\theta) + k$

$(\cos^2\theta\cos\phi\sin\phi + \sin^2\phi\sin\phi\cos\phi)$

$\Rightarrow (\sin^2\phi\cos\theta)i + (\sin^2\phi\sin\theta)$
$+ \sin\phi\cos\phi\, k$

$\left\| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right\|$

$$\Rightarrow \sqrt{(\sin^2\phi\cos\theta)^2+(\sin^2\phi\sin\theta)^2+(\sin\phi\cos\phi)}$$

$$\Rightarrow \sqrt{\sin^4\phi\cos^2\theta+\sin^4\phi\sin^2\theta+\sin^2\phi\cos^2\phi}$$

$$\Rightarrow \cancel{\sin^2\phi\cos^2\theta}\left(\cancel{\sin^2\phi\cos^2\theta+}\sin^2\phi\right.$$

$$\sqrt{\left[\sin^4\phi\cos^2\theta+\sin^4\phi\sin^2\theta\right]+\sin^2\phi\cos^2\phi}$$

$$\Rightarrow \sqrt{\sin^4\phi(\cos^2\theta+\sin^2\theta)+\sin^2\phi\cos^2\phi}$$

$$\Rightarrow \sqrt{\sin^4\phi+\sin^2\phi\cos^2\phi}$$

$$\Rightarrow \sqrt{\sin^2\phi(\sin^2\phi+\cos^2\phi)}$$

$$\Rightarrow = \sin\phi.$$

So, $\left\|\frac{\partial r}{\partial\phi}\times\frac{\partial r}{\partial\theta}\right\|$

using 9

$$\Rightarrow \iint_R x^2\,ds \Rightarrow \iint_R(\sin^2\phi\cos^2\theta)\sin\phi\,d\phi\,d\theta$$

$$\Rightarrow \iint(\sin^3\phi\cos^2\theta)\,d\phi\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} (\sin^3\phi \cos^2\theta)\, d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\left(\int_0^{\pi} \sin^3\phi\, d\phi\right) \cos^2\theta\, d\theta$$

so;

$$\int_0^{\pi} \sin^3\phi\, d\phi$$

$$\Rightarrow \int_0^{\pi} (1-\cos^2\phi)\sin\phi\, d\phi$$

$$\Rightarrow \int_0^{\pi} \sin\phi - \cos^2\phi \sin\phi\, d\phi$$

$$\Rightarrow \int_0^{\pi}(\sin\phi) - \int_0^{\pi} \cos^2\phi \sin\phi\, d\phi$$

$$\Rightarrow -\cos\phi\Big|_0^{\pi} \Rightarrow -(-1-1) \Rightarrow 2.$$

let $u = \cos\phi$

$du = -\sin\phi\, d\phi$

$\frac{du}{-\sin\phi} = d\phi$

when $\phi \to \pi$, $u \to -1$

$\phi \to 0$ $u \to 1$

$$\Rightarrow 2 - \int_{-1}^{1} (u)^2\, du$$

$$2 - \left(\frac{(u)^3}{3}\right)_{-1}^{1} \Rightarrow 2 - \left(\frac{1}{3} + 1\right)$$

$$2 - \frac{2}{3} \Rightarrow 6 - \frac{2}{3} \Rightarrow 4/3$$

So $\int_0^{2\pi} \frac{4}{3} \cos^2\theta \, d\theta$

$\frac{4}{3} \int_0^{2\pi} \frac{1}{2}(1 - \sin 2\theta) \, d\theta$

$\frac{4}{3} \left[\frac{1}{2}\left(\theta + \frac{\cos 2\theta}{2}\right) \right]_0^{2\pi}$

$\Rightarrow \frac{4}{3}\left(\frac{1}{2}(2\pi)\right) - (1-1)$

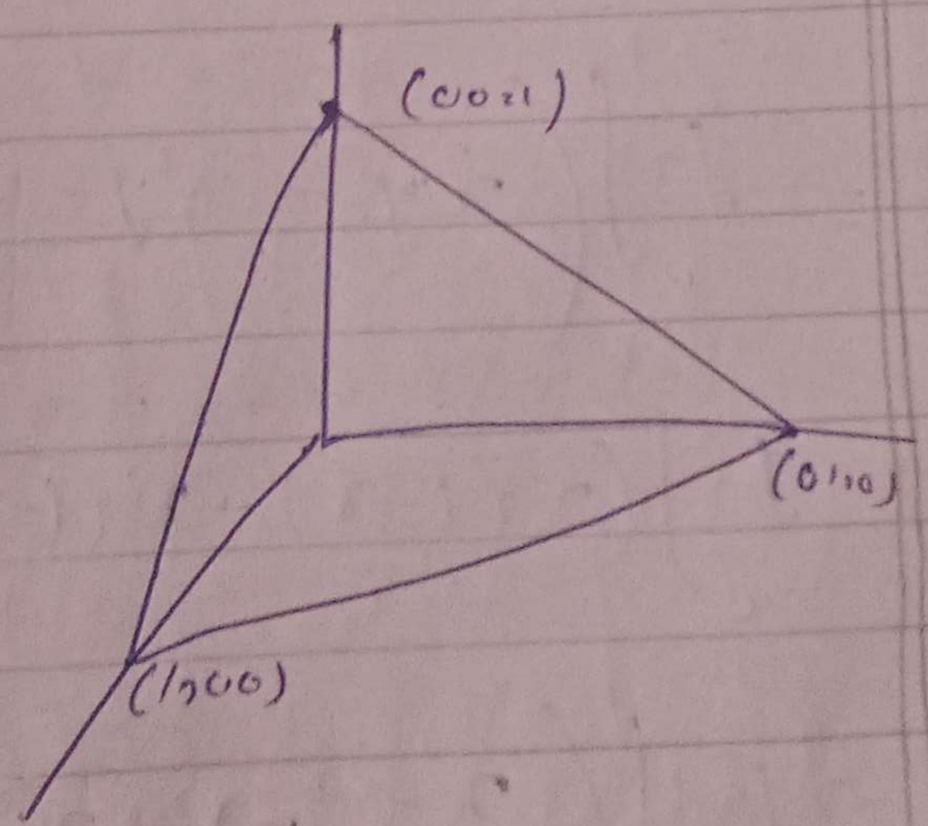
$\Rightarrow \left(\frac{4\pi}{3}\right) \Rightarrow$ Ans

Evalute the Surface intregal

$$\iint_\sigma xz \, ds$$

wher σ is the Portion of the plane $x+y+z=1$ that lies in first octant.

$z = 1 - x - y$



using

So,

$$\iint_R f(x,y) \cdot g(x,y) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = -1$$

$$\frac{\partial z}{\partial y} = -1$$

$$z = 1 - x - y$$

$$\text{or} \quad x + y + z = 1$$

$$0 \le x \le 1$$

$$0 \le y \le 1 - x$$

$$\int_0^1 \int_0^{1-x} f(x,y) \sqrt{(-1)^2 + (-1)^2 + 1}\, dy\, dx$$

$$\Rightarrow \int_0^1 \int_0^{1-x} x(1-x-y)\, \sqrt{3}\, dy\, dx$$

$$\Rightarrow \sqrt{3} \int_0^1 \int_0^{1-x} x - x^2 - xy\, dy\, dx$$

$$\Rightarrow \sqrt{3} \int_0^1 \left(xy - x^2 y - \frac{xy^2}{2} \right)_0^{1-x} dx$$

$$\sqrt{3} \int_0^1 \left(x(1-x) - x^2(1-x) - \frac{x}{2}(1-x)^2 \right)$$

$$\Rightarrow \sqrt{3} \int_0^1 \left(x - x^2 - x^2 + x^3 - \frac{x}{2}(1 + x^2 - 2x) \right)$$

dA

$$\sqrt{3}\int_0^1 \left(x - x^2 - x^2 + x^3 - \frac{x}{2} \frac{1}{2}\left(-x - x^3 + 2x^2\right)\right) dx$$

$$\Rightarrow \sqrt{3}\int_0^1 \left(x - 2x^2 + x^3 - \frac{x}{2} - \frac{x^3}{2} + x^2\right) dx$$

$$\Rightarrow \sqrt{3}\int_0^1 x - \frac{x}{2} + x^3 - \frac{x^3}{2} - 2x^2 + x^2 \quad dx$$

$$\Rightarrow \sqrt{3}\int_0^1 \left(\frac{x}{2} + \frac{x^3}{2} - x^2\right) dx$$

$$\Rightarrow \sqrt{3}\int \left(\frac{x^2}{4} + \frac{x^4}{8} - \frac{x^3}{3}\right)_0^1$$

$$\Rightarrow \sqrt{3}\left(\frac{1}{4} + \frac{1}{8} - \frac{1}{3}\right)$$

2	4 - 8 - 3
2	2 - 4 - 3
2	1 - 2 - 3
3	1 - 1 - 3
	1 - 1

$$\Rightarrow \sqrt{3}\left(\frac{6 + 3 - 8}{24}\right) \Rightarrow \sqrt{3}\left(\frac{1}{24}\right)$$

$$\Rightarrow \sqrt{3}/24 \Rightarrow \text{Ans.}$$

$z = \sqrt{x^2 + y^2}$ an lims b/w $z = 1$

& $z = 2$

Evalue $\iint_S y^2 z^2 \, ds$

Given $z = \sqrt{x^2+y^2}$ $\quad f(x,y,z) = y^2 z^2$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

and $z=1$ & $z=2$

$$r^2 = 1$$
$$r^2 = 2$$
$$1 \le r \le 2$$
$$0 \le \theta \le 2\pi$$

&

$$= \iint_R f(x,y)\, g(x,y) \sqrt{\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right) + 1}\, dA$$

$$\iint_R y^2 \left(\sqrt{x^2+y^2}\right)^2 \sqrt{\frac{x^2+y^2}{x^2+y^2} + 1}\, dA$$

$$\Rightarrow \iint_R y^2 (r^2) \sqrt{1+1}\, dA$$

$$\sqrt{2} \int_0^{2\pi}\int_1^2 r^2 \sin^2\theta\, (r^2)\, r\, dr\, d\theta$$

$$\int_0^{2\pi}\int_1^2 r^5 \sin^2\theta\, dr\, d\theta$$

$$\Rightarrow \frac{\sqrt{6}}{6} \sin^2\theta \Big|_1^2 d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{6}(64-1) \sin^2\theta \Big] d\theta$$

$$\Rightarrow \sqrt{2} \int_0^{2\pi} \frac{63}{6} \sin^2\theta \, d\theta$$

$$\Rightarrow \frac{63\sqrt{2}}{6} \left(\int_0^{2\pi} \frac{1}{2}(1-\cos 2\theta) \, d\theta \right.$$

$$\Rightarrow \frac{63\sqrt{2}}{6} \left(\frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi} \right.$$

$$\Rightarrow \frac{63\sqrt{2}}{6} \left(\frac{1}{\cancel{2}}(\cancel{2}\pi - 0) \right)$$

$$\Rightarrow \frac{21\sqrt{2}}{2} \left(\pi \right) \Rightarrow$$

$$\left(\frac{21\sqrt{2}\,\pi}{2} \right) \Rightarrow \text{Ans}$$

Exercise Question

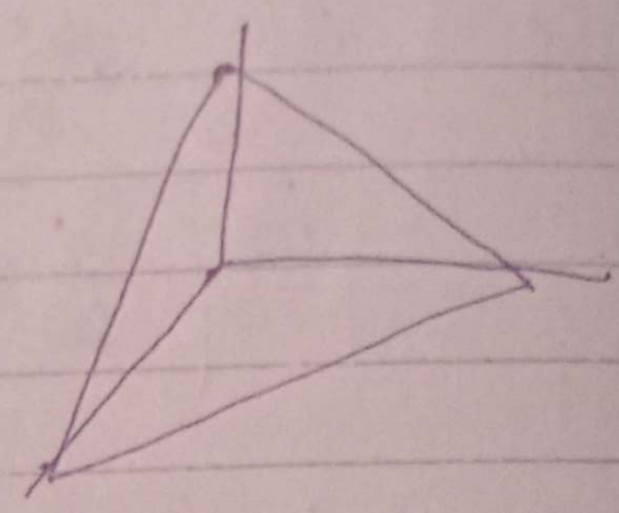
(Q5) $f(x,y,z) = x-y-z$; σ is the portion is The plane $x+y=1$ in The first octant b/w $z=0$ & $z=1$ find The surface integral.

=> Given

$x = 1-y$ & $f(x,y,z) = x-y-z$

=> $\frac{\partial x}{\partial y} = -1$

$\frac{\partial x}{\partial z} = 0$



so, $x+y=1$

$0 \le x \le 1$

~~$0 \le y \le 1-x$~~

$0 \le y \le 1$

=> using

$$\iint_R \cancel{g(x,y)}\ z(x,y) \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial y}\right)^2 + 1}\ dA$$

$$\iint_R (1-y)-y-z \sqrt{(-1)^2 + (0)^2 + 1}\ dA$$

$$\iint_R 1-2y-z\ \sqrt{2}\ dA$$

$$\sqrt{2} \int_0^1 \int_0^{1-1} (1-2y-z)\ dy\ dz$$

$$\Rightarrow \sqrt{2}\int_0^1 \left(y - \frac{2y^2}{2} - z \right) dz$$

$$\sqrt{2}\int_0^1 \left(\cancel{1} - \cancel{1} - z \right) dz$$

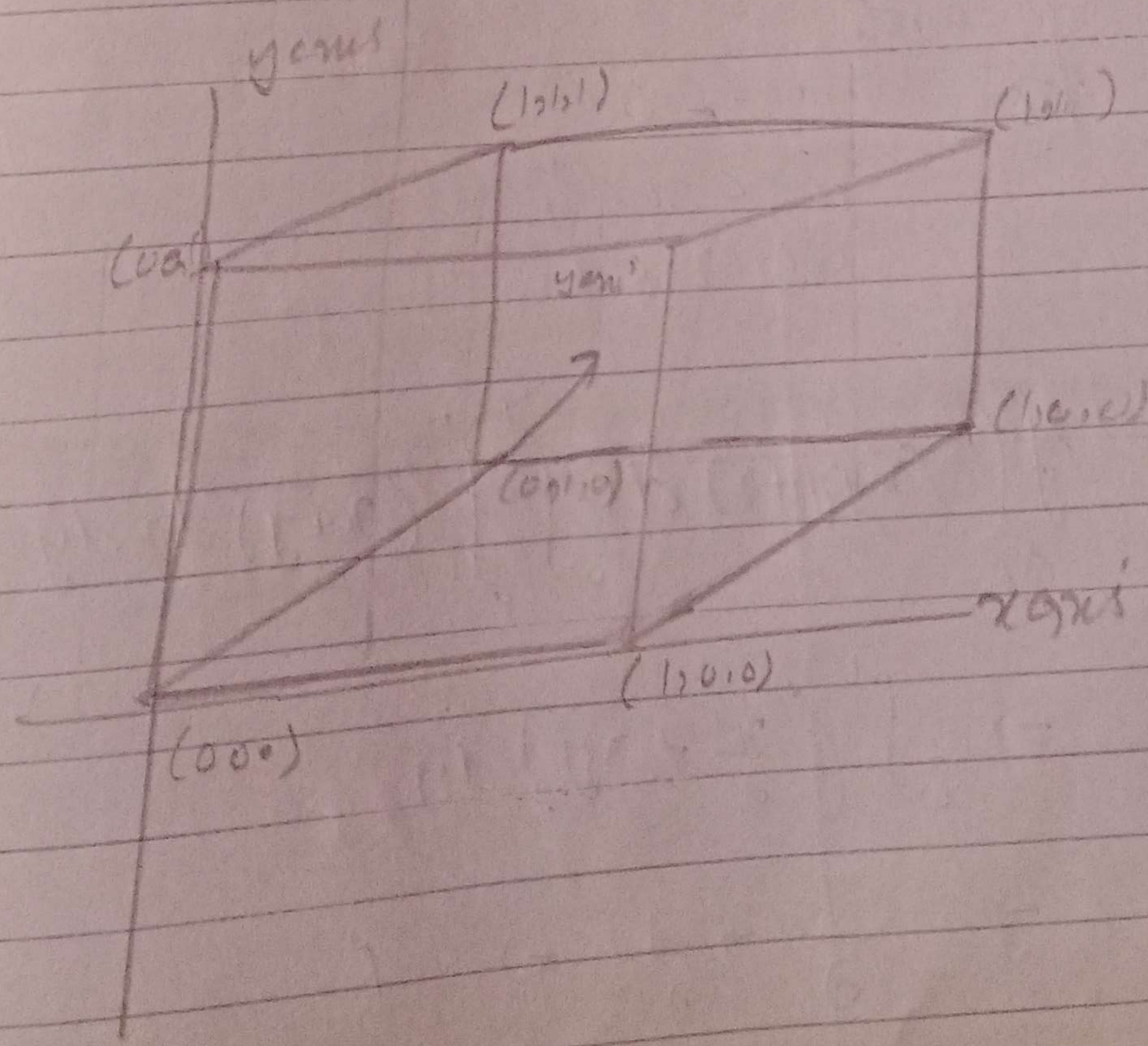
$$\Rightarrow \sqrt{2} \left(\frac{-z^2}{2} \right)_0^1 \Rightarrow \sqrt{2} \left(-\frac{1}{2} \right)$$

$$\Rightarrow \left(-\frac{\sqrt{2}}{2} \right) \Rightarrow \text{Ans.}$$

---

7) $f(x, y, z)$ Evale Surface intrgal $f(x,y,z) = x+y+z$; σ is The surface of cube determine The inequalities.

$$0 \le x \le 1$$
$$0 \le y \le 1$$
$$0 \le z \le 1$$



solution

σ_1 onto xy plane

$z=0$

$0 \le x \le 1 \quad 0 \le y \le 1$

$\sigma_2 = z=1.$

$0 \le y \le 1 \quad 0 \le x \le 1$

Onto xz plane

$\sigma_3;$ $y=0$

$0 \le x \le 1$

$0 \le z \le 1$

$\sigma_4;$ $x=1$

$1 \le x \le 1$

$0 \le z \le 1$

onto σ_5 y = plane $x=0$

$0 \le z \le 1 \quad 0 \le y \le 1$

onte

σ_6 $y=0$

$0 \le \le x \le 1$

$0 \le z \le 1$

$$\iint_{\sigma_1} (x+y+z)\, ds = \int_0^1\int_0^1 (x+y)\, dx\, dy$$

$$\Rightarrow \int_0^1 \left(\frac{x^2}{2} + xy\right) dy$$

$$\Rightarrow \frac{1}{2} + \frac{1}{0} \Rightarrow 1$$

σ_2

8

$$\sigma_2 \int_0^1\int_0^1 (x+y+1)\,dx\,dy$$

$$\int_0^1 \left(\frac{1}{2}+y+1\right) dy$$

$$\Rightarrow \frac{1}{2}+\frac{1}{2}+1 \Rightarrow 2.$$

$$\sigma_3 = \int_0^1\int_0^1 (x+1+z)\,dx\,dz = 3$$

$$\sigma_4 = \int_0^1\int_0^1 (x+1+z) = 2$$

$$\sigma_5 = \int_0^1\int_0^1 (y+z)\,dy\,dz = 1$$

$$\sigma_6 = \int_0^1\int_0^1 (x+y+z)\,ds = \int_0^1\int_0^1 (1+y+z)\,dy\,dz = 2$$

$$\iint_S (x+y+z)\,ds = (3)(1)+2(3)$$

$$= 9$$

---

(8) $f(x,y,z) = x^2+y^2$; σ is the surface of the sphere

$$x^2+y^2+z^2 = a^2$$

As we know that the parametric eq of sphere is

$$r(\phi, \theta) = a f \sin\phi \cos\theta\, i + a f \sin\phi \sin\theta\, j + a f \cos\phi\, k.$$

$$\frac{\partial r}{\partial \phi} = a f \cos\phi \cos\theta\, i + a f \cos\phi \sin\theta\, j - a f \sin\phi\, k$$

$$\frac{\partial r}{\partial \theta} = -a f \sin\phi \sin\theta\, i + a f \sin\phi \cos\theta\, j + 0k.$$

$$\frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} =$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ a f \cos\phi \cos\theta & a f \cos\phi \sin\theta & -a f \sin\phi \\ -a f \sin\phi \sin\theta & a f \sin\phi \cos\theta & 0 \end{vmatrix}$$

$$\Rightarrow i\left(a^2 f^2 \sin^2\phi \cos\theta\right) - j\left(-a^2 f^2 \sin^2\phi \sin\theta\right) + k\left(a^2 f^2 \cos^2\theta \cos\phi \sin\phi + a^2 f^2 \sin^2\theta \cos\phi \sin\phi\right)$$

$$= a^2 f^2 \cos\phi \sin\phi \left(\cos^2\theta + \sin^2\theta\right) = 1$$

$$\Rightarrow$$

$$\left(a^2 f^2 \sin^2\phi \cos\theta\right) i + j\left(a^2 f^2 \sin^2\phi \sin\theta\right) + k\left(a^2 f^2 \cos\phi \sin\phi\right.$$

$$\left\| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right\|$$

$$\Rightarrow \sqrt{(a^2 \rho^2 \sin^2\phi \cos\theta)^2 + (a^2 \rho^2 \sin^2\phi \sin\theta)^2 + (a^2 \rho^2 \sin\phi \cos\phi)^2}$$

$$\Rightarrow \sqrt{a^4 \rho^4 \sin^4\phi \cos^2\theta + a^4 \rho^4 \sin^4\phi \sin^2\theta + a^4 \rho^4 \sin^2\phi \cos^2\phi}$$

$$\Rightarrow \sqrt{a^4 \rho^4 \sin^4\phi (\cos^2\theta + \sin^2\theta) + a^4 \rho^4 \sin^2\phi \cos^2\phi}$$

$$\Rightarrow \sqrt{a^4 \rho^4 \sin^4\phi + a^4 \rho^4 \sin^2\phi \cos^2\phi}$$

$$\Rightarrow \sqrt{a^4 \rho^4 \sin^2\phi (\sin^2\phi + \cos^2\phi)}$$

$$a^2 \rho^2 \sin\phi \qquad \because \rho = a$$

$\Rightarrow$ b/c ρ is radius in spherical coordinates so, in this a is the radius. so $\rho = a$ and ρ is neglegel.

$$\left| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right|^2 \quad a^2 \sin\phi$$

So using

$$0 \le \phi \le \pi$$

$$0 \le \theta \le 2\pi$$

$$\iint_R f(x,y,z) \left\| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right\| dA$$

$$\int_0^{2\pi}\int_0^{\pi} x^2y^2 \left| \frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} \right| d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} \left(a^2\sin^2\phi\cos^2\theta \; a^2\sin^2\phi\sin^2\theta\right)\left(a^2\sin\phi\right) d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} a^2\sin^2\phi\left(\cos^2\theta+\sin^2\theta\right)\left(a^2\sin\phi\right) d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} a^4\sin^3\phi \; d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi} a^4 \left[\int_0^{\pi} \sin\phi\left(1-\cos^2\phi\right) d\phi\right] d\theta$$

$$\Rightarrow \int_0^{\pi} \sin\phi - \cos^2\phi\sin\phi \; d\phi$$

$$\Rightarrow -\cos\phi \Big|_0^{\pi} -$$

let $u = \cos\phi$

$\frac{du}{-\sin\phi} = d\theta$

37

when $\phi \to \pi \Rightarrow u \to -1$

& $\phi \to 0 \Rightarrow u \to 1$

$$\Rightarrow 2 - \left(\frac{u^3}{3}\right)_{-1}^{1}$$

$$\Rightarrow 2 - \frac{(1+1)}{3} \Rightarrow 2 - \frac{2}{3} \Rightarrow 4/3$$

$$\Rightarrow a^4 \int_0^{2\pi} 4/3 \, d\theta \Rightarrow \frac{a^4 4}{3}(2\pi)$$

$$\Rightarrow \frac{a^4 8\pi}{3}$$

---

37) $$f(x,y,z) = \frac{1}{\sqrt{1+4x^2+4y^2}}$$

$$r(u,v) = u\cos v\, i + u\sin v\, j + \cancel{2\cos u}\, u^2 k$$

$$0 \le u \le \sin v$$
$$0 \le v \le \pi$$

$$r(u,v) = u\cos v\, i + u\sin v\, j \cancel{+ 2\cos u}\, + u^2 k$$

$$\frac{\partial r}{\partial u} = \cos v\, i + \sin v\, j + 2u k$$

$$\frac{\partial r}{\partial v} = -u\sin v\, i + u\cos v\, j + 0k$$

$$\left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| =$$

$$\begin{vmatrix} i & j & k \\ \cos v & \sin v & 2u \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$$i(-2u^2\cos v) - j(+2u^2\sin v) + k(u\cos v^2 + u\sin v^2$$

$$= u(\cos^2 v + \sin^2 v$$

$$= u\,k$$

$$-2u^2\cos v - 2u^2\sin v + u\,k$$

$$\sqrt{(-2u^2\cos v)^2 + (2u^2\sin v)^2 + (u)^2}$$

$$\Rightarrow \sqrt{4u^4\cos^2 v + 4u^4\sin^2 + u^2}$$

$$\Rightarrow \sqrt{4u^4(\cos^2 v + \sin^2 v) + u^2}$$

$$\Rightarrow \sqrt{4u^4 + u^2}$$

So using

$$\iint_R F(x,y,z) \left\| \frac{\partial r}{\partial u} \quad \frac{\partial r}{\partial v} \right\| dA$$

$$\iint_R \frac{1}{\sqrt{1+4x^2+4y^2}} \left(\sqrt{4u^4+u^2}\right) dA$$

$$\iint_R \frac{1}{\sqrt{1+4(u\cos v)^2+4(u\sin v)^2}}\left(\sqrt{4u^2+u^2}\right) dA$$

$$\iint_R \frac{1}{\sqrt{1+4u^2}}\sqrt{4u^2+u^2}\Big) dA$$

$$\Rightarrow \iint_R \frac{\sqrt{u^2}}{\sqrt{1+4u^2}}\sqrt{\cancel{1+4u^2}}\; dA$$

$$\Rightarrow \int_0^{\pi}\int \frac{u^2}{2}\Big)_0^{\sin v} du\, dv$$

$$\Rightarrow \frac{1}{2}(\sin v^2)\, dv$$

$$\int_0^{\pi} \frac{\sin v^2}{2} dv \Rightarrow \int_0^{\pi} \frac{1}{2}(1-\cos 2v)$$

$$\Rightarrow \frac{1}{4}\left(v - \frac{\sin 2v}{2}\right)_0^{\pi}$$

$$\Rightarrow \frac{1}{4}(\pi - 0) \Rightarrow \pi/4$$

$\Rightarrow$ Ans

(Q38) $\Rightarrow$

$f(x,y,z) = e^{-z}$

$r(u,v) = 2\sin u \cos v\, i + 2\sin u \sin v\, j + 2\cos u\, k$

$0 \le u \le \pi/2$

$0 < v \le 2\pi$

$\frac{\partial r}{\partial u} = 2\cos u \cos v\, i + 2\cos u \sin v\, j - 2\sin u\, k$

$\frac{\partial r}{\partial v} = -2\sin u \sin v\, i + 2\sin u \cos v\, j + 0k$

$\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} =$

$$\begin{vmatrix} i & j & k \\ 2\cos u \cos v & 2\cos u \sin v & -2\sin u \\ -2\sin u \sin v & 2\sin u \cos v & 0 \end{vmatrix}$$

$i(4\sin u^2 \cos v) - j(-4\sin^2 u \sin v)$

$+ k(4\cos^2 v \sin u \cos u + 4\sin u \cos u \sin v^2)$

$\Rightarrow k(4\sin u \cos u(\cos^2 v + \sin^2)$

$= 1$

$4\sin^2 u^2 \cos v\, i + 4\sin^2 u \sin v\, j$

$+ 4\sin u \cos u\, k.$

$\left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| =$

$$\sqrt{16\sin^4 u\cos^2 v + 16\sin^4 u\sin^2 v + 16\sin^4 u\cos^2 u}$$

$$\Rightarrow \sqrt{16\sin^4 u(\cos^2 v + \sin^2 v) + 16\sin^2 u\cos^2 u}$$

$$= \sqrt{16\sin^2 u(\sin^2 u + \cos^2 u)}$$

$$\Rightarrow 4\sin u = \left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\|$$

So, using

$$\iint_R f(x,y,z) \left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| dA$$

$$\Rightarrow \iint_R e^{-2\cos u} (4\sin u)\, dA$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi} e^{-2\cos u}\, 4\sin u \, du\, dv$$

$$\int_0^{2\pi}\left(\int_0^{\pi} e^{-2\cos u} \sin u \, du\right) dv$$

Let $u = -2\cos u$ when $u \to \pi/2$

$du = +2\sin u$ $\quad u \to -2(-1)2)$

$\frac{du}{2\sin u} = du$ $\quad u \to -2(1) = -2$

$u \to 0$

$u \to -2$

$$\Rightarrow \frac{4}{2}\int_0^{2\pi}\left(\int_{-2}^{0} e^{u} \cancel{\sin u}\, \frac{du}{\cancel{\sin u}}\right) du$$

$$2\int_0^{2\pi}\left(e^{0} - e^{-2}\right) du$$

$$\Rightarrow 2\int_0^{2\pi}\left(1 - e^{2}\right) du$$

$$\Rightarrow 2\left(2\pi\left(1-e^{2}\right)\right)$$

$$\Rightarrow 4\pi\left(1-e^{2}\right)$$

$\Rightarrow$ Ans

Exercise No 15.5 Complete

"Exercise No 15.6"

Find the flux of a vector field $F(x,y,z) = zk$ across the outward orientation sphere $x^2+y^2+z^2=a^2$

The Parametric eq of sphere is

$$r(\phi,\theta) = a\sin\phi\cos\theta\, i + a\sin\phi\sin\theta\, j + a\cos\phi\, k$$

$$\frac{\partial r}{\partial \phi} = a\cos\phi\cos\theta\, i + a\cos\phi\sin\theta\, j - a\sin\phi\, k$$

$$\frac{\partial r}{\partial \theta} = -a\sin\phi\sin\theta\, i + a\sin\phi\cos\theta\, j + 0k$$

$$\frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta} =$$

$$\begin{vmatrix} i & j & k \\ a\cos\phi\cos\theta & a\cos\phi\sin\theta & -a\sin\phi \\ -a\sin\phi\sin\theta & a\sin\phi\cos\theta & 0 \end{vmatrix}$$

$$\Rightarrow i\left(a^2\sin^2\phi\cos\theta\right) - j\left(-a^2\sin^2\phi\sin\theta\right) + k$$

$$\left(a^2\sin\phi\cos\phi\cos^2\theta + a^2\sin^2\theta\cos\phi\sin\phi\right.$$

$$\Rightarrow (a^2 \sin^2\phi \cos\theta)\,i + (a^2 \sin^2\phi \sin\theta)\,j$$
$$+ k(a^2 \sin\phi \cos\phi)$$

$$\Rightarrow \phi = \iint_R F\left(\frac{\partial r}{\partial \phi} \times \frac{\partial r}{\partial \theta}\right) dA$$

$$\Rightarrow \phi = \int_0^{2\pi}\int_0^{\pi} 2k\left(a^2 \sin\phi \cos\theta\, i + a^2 \sin^2\phi \sin\theta\right) j$$
$$+ k(a^2 \sin\phi \cos\phi)$$

$$\phi = \int_0^{2\pi}\int_0^{\pi} a\cos\phi\, k\,(a^2 \sin\phi \cos\phi)\, k.$$

$$\phi = \int_0^{2\pi}\int_0^{\pi} a^3 \cos^2\phi \sin\phi \; d\phi \; d\theta$$

$$\Rightarrow \phi = a^3 \int_0^{2\pi}\int_0^{\pi} \cos^2\phi \sin\phi \; d\phi \, d\theta$$

let $u = \cos\phi$

$du = -\sin\phi$

$\frac{du}{-\sin\phi} = d\phi$

when $\phi \to \pi$ $u \to -1$

$\phi \to 0$ $u \to 1$

$$\Rightarrow \int_{-1}^{1} (u)^2 \cancel{\sin\phi} \frac{du}{\cancel{\sin\phi}} \, d\phi \, d\theta$$

$$\left(\frac{(u)^3}{3}\right]_{-1}^{1} = \frac{1}{3}(1-(-1))$$

$$\Rightarrow \frac{1}{3}(1+1) = 2/3$$

$$a^3\int_0^{2\pi} 2/3 \, d\theta \Rightarrow \frac{4\pi}{3}$$

$$= \frac{4\pi a^3}{3} \Rightarrow \text{Ans.}$$

Examples:

$$z = 1 - x^2 - y^2$$

$$F(x,y,z) = xi + yj + zk$$

The orientation is upword so using

$$\phi = \iint_R \left(-\frac{\partial z}{\partial x} i - \frac{\partial z}{\partial y} j + k\right) dA$$

$$\frac{\partial z}{\partial x} = -2x$$

$$\partial z/\partial y = -2y$$

~~$\phi = \int_0^{2\pi}$~~

~~$\phi = \pi/2$~~

Using above result.

$$\phi = \iint_R xi + yj + zk \left(+ 2xi + 2yi + k \right) dA$$

$$\phi = \iint_R \left(2x^2 + 2y^2 + 1 \right) dA$$

$$\phi = \iint_R 2(r^2) + 1 \quad dA$$

Since $x^2 + y^2 = 1$

$$0 \leq r \leq 1$$
$$0 \leq \theta \leq 2\pi$$

$$\phi = \int_0^{2\pi} \int_0^1 (r^2 + 1) r \, dr \, d\theta$$

$$\phi = \int_0^1 (r^3 + r) \, dr$$

$$\Rightarrow \left(\frac{r^4}{4} + \frac{r^2}{2} \right)_0^1 \quad d\theta$$

$$\Rightarrow \left(\frac{1}{4} + \frac{1}{2} \right) \Rightarrow 6/8 \Rightarrow 3/4$$

$$\int_0^{2\pi} \left(3/4 \right) d\theta$$

$$6\pi/4 \Rightarrow \left(3\pi/2 \right)$$

$\Rightarrow$ Ans

(13)

Exercise Question

find the flux of vector field F across σ.

(3) $F(x,y,z) = xi + yj + 2zk$; σ is the portion of cone $z^2 = x^2+y^2$ b/w plane $z=1$ and $z=2$ oriented by upward orientation.

Since the orientation of $f(x)$ is upward so using

$$\iint_R F\cdot\left(\frac{-\partial z}{\partial x}i - \frac{\partial z}{\partial y} + k\right) dA$$

$$z = \sqrt{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \iint_R xi+yj+2zk\left(\frac{-x}{\sqrt{x^2+y^2}}i + \frac{-y}{\sqrt{x^2+y^2}} + k\right)$$

$$\iint_R \left(\frac{-x^2}{\sqrt{x^2+y^2}} + \frac{-y^2}{\sqrt{x^2+y^2}} + 2k\right) dA$$

$$\iint \frac{(x^2+y^2)}{\sqrt{x^2+y^2}} + 2k \quad dA$$

$$\iint_R \left(\frac{-(x^2+y^2) + 2\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} \right) dA$$

$$\cancel{\iint_R -(x^2+y^2)(x^2+y^2)^{-1/2} + \frac{2\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}}$$

$$\cancel{\iint_R -(x^2+y^2)^{1/2} + \frac{2\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}}$$

$$\cancel{\sqrt{x^2+y^2}}$$

$$\iint_R -(x^2+y^2)(x^2+y^2)^{-1/2} + 2(x^2+y^2)^{1/2}$$

$$\iint_R -(x^2+y^2) + 2(x^2+y^2)^{1/2}$$

$$\iint_R (x^2+y^2)^{1/2} dA$$

we also know that $z=1$

$r=1$

$\xi \quad z=2 \quad r=2$

$1 \le r \le 2$

$0 \le \theta \le 2$

$$\int_0^{2\pi}\int_1^2 (r)(r)\, dr\, d\theta$$

$$\int_0^{2\pi} \left(\frac{-z^3}{3}\right)^2 d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{3}(z-1)\, d\theta \Rightarrow \int_0^{2\pi} \left(\frac{\pi}{3}\right) d\theta$$

$$14\pi/3 \Rightarrow \text{Ans}$$

---

(19) $F(x,y,z) = \sqrt{x^2+y^2}\, k$ σ is The portion of Coine $r(u,v) = u\cos v\, i + u\sin v\, j + 2u\, k$

$$\frac{\partial r}{\partial u} = \cos v\, i + \sin v\, j + 2k$$

$$\frac{\partial r}{\partial v} = -u\sin v\, i + u\cos v\, j + 0k$$

So using

$$\phi = \iint_R F\cdot n = \iint_R F\left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right) dA$$

$\rightarrow$ A

$$\left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right) =$$

$$\Rightarrow \begin{vmatrix} i & j & k \\ \cos v & \sin v & 2 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$i(2u\cos v) - j(2u\sin v) + k(u\cos v^2 + u\sin^2 v)$

$\Rightarrow 2u\cos v\, i - 2u\sin v\, j + uk$

$\Rightarrow$ So using $\rightarrow$ (A)

$$\phi = \iint_R \sqrt{(u\cos v)^2 + (u\sin v)^2 k}\left(2u\cos v\, i - 2u\sin v\, j + uk\right) dA$$

$$\phi = \iint_R uk\left(2u\cos v\, i - 2u\sin v\, j + uk\right) dA$$

$$\phi = \iint_R u^2 \, dA$$

$$\int_0^{\pi}\int_0^{\sin v} \left(\frac{u^3}{3}\right)_0^{\sin v} dA$$

$$\Rightarrow \int_0^{\pi} \frac{1}{3}\sin^3 v \, dv$$

$$\Rightarrow \frac{1}{3}\int_0^{\pi} (1-\cos^2 v)\sin v \, dv$$

$$\Rightarrow \frac{1}{3}\int_0^{\pi} (\sin v - \cos^2 v \sin v)\, dv$$

$$\frac{1}{3}\int_0^{\pi} -\cos v$$

$$\Rightarrow -(-1-1)$$

$$\Rightarrow -(-2)$$

$$= 2$$

$u = \cos v$

$du = -\sin v \, dv$

$\frac{du}{-\sin v} = dv$

27

$\Rightarrow$ when $\leftarrow \rightarrow \pi$ & $u \rightarrow -1$

" $v \rightarrow \pi$ & $u \rightarrow 1$

$$\int_{-1}^{1} (u)^2 du \Rightarrow \left(\frac{u^3}{3}\right)_{-1}^{1}$$

$$\Rightarrow \frac{1}{3}(1-(-1)) \Rightarrow 2/3$$

$$\frac{1}{3}\left[2 - 2/3\right] \Rightarrow \frac{1}{3}\left(4/3\right)$$

$$\Rightarrow (4/9) \Rightarrow 4/9 \Rightarrow \text{Ans}$$

---

(27) $F(x,y,z) = i + j + k$; The σ is the portion of the cone $z=\sqrt{x^2+y^2}$ b/w The plane $z=1$ and $z=2$ oriented downward unit normale.

Find The Flux of F across σ by expressing σ parametically.

we also know That The parametric eq of cone

$x = u\cos v$

$y = u\sin v$

$z = -r$

$r(u,v) = u\cos v\, i + u\sin v\, j + uk$

$$\frac{\partial r}{\partial u} = \cos v\, i + \sin v\, j + k$$

$$\frac{\partial r}{\partial v} = -u\sin v\, i + u\cos v\, j + 0k$$

So using result.

$$\phi = \iint_R f \cdot n \, dA$$

$$\phi = \iint_R F \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right) dA \longrightarrow (A)$$

$$\left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right)$$

$$\begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$$i(-u\cos v) - j(u\sin v) + k(u\cos^2 v + u\sin^2 v)$$

$$-u\cos v\, i - j\, u\sin v\, j + uk$$

So

$$\phi = \iint_R (i + j + k)(-u\cos v\, i - j\, u\sin v + u)$$

$$\phi = \iint_R \left(-u\cos v - u\sin v + u\right) dA$$

Since $z=1$ & $z=2$

$1 \leq r \leq 2$

$0 \leq \theta \leq 2\pi$

$$\phi = \int_0^{2\pi}\int_1^2 -u\left(\cos v + \sin v + 1\right) du\,dv$$

$$\left(-\frac{u^2}{2}\right)_1^2 \Rightarrow -\frac{1}{2}\left(4-1\right) \Rightarrow -3/2$$

$$\phi = -3/2 \int_0^{2\pi} \left(\cos v + \sin v + 1\right) dv$$

$$-3/2\left(\sin v\right)_0^{2\pi} + \left(-\cos v\right)_0^{2\pi} \cdot \left(v\right)_0^{2\pi}$$

$$\Rightarrow \phi = -3/2\left(0\right) - \left(1-1\right) - \left(2\pi\right)$$

$$\phi = \frac{-6\pi}{2} \Rightarrow \left(+3\pi\right) \Rightarrow \text{Ans}$$

Exercie No 14.7

Use divage Therum to find The out words flux of The vector field $F(x,y,z) = zk$

across The sphere $x^2+y^2+z^2=a^2$

using result

$$\iint_{\sigma} F \cdot n \, ds = \iiint_{G} \operatorname{div} F \, dv$$

$$\operatorname{div} F = \left(\frac{\partial f}{\partial x} i + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right)$$

$$\text{div of } F = \frac{\partial z}{\partial z} = 1$$

$$\Rightarrow \quad \begin{array}{l} 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le a \end{array}$$

So;

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} k(\ 1\ )\, dv$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} 1 \, dv$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\Rightarrow \left(\rho^3/3\right)_0^a \sin\phi \, d\phi \, d\theta$$

$$\Rightarrow \int_0^{\pi} a/3 \sin\phi \, d\phi$$

$$\Rightarrow \frac{a^3}{3}\left(-\cos\right)_0^{\pi} \Rightarrow -(-1-1)$$

$$\int_0^{2\pi} \frac{2a^3}{3} \, d\theta \Rightarrow \frac{4a^3\pi}{3}$$

$$\left(4a^3\pi/3\right) \Rightarrow \text{Ans}$$

Use diverge Theorum find The out flux of The vector field

$$F(x,y,z) = x^3 i + y^3 j + z^2 k$$

use diverge Theorm A/c to diverge Theorum

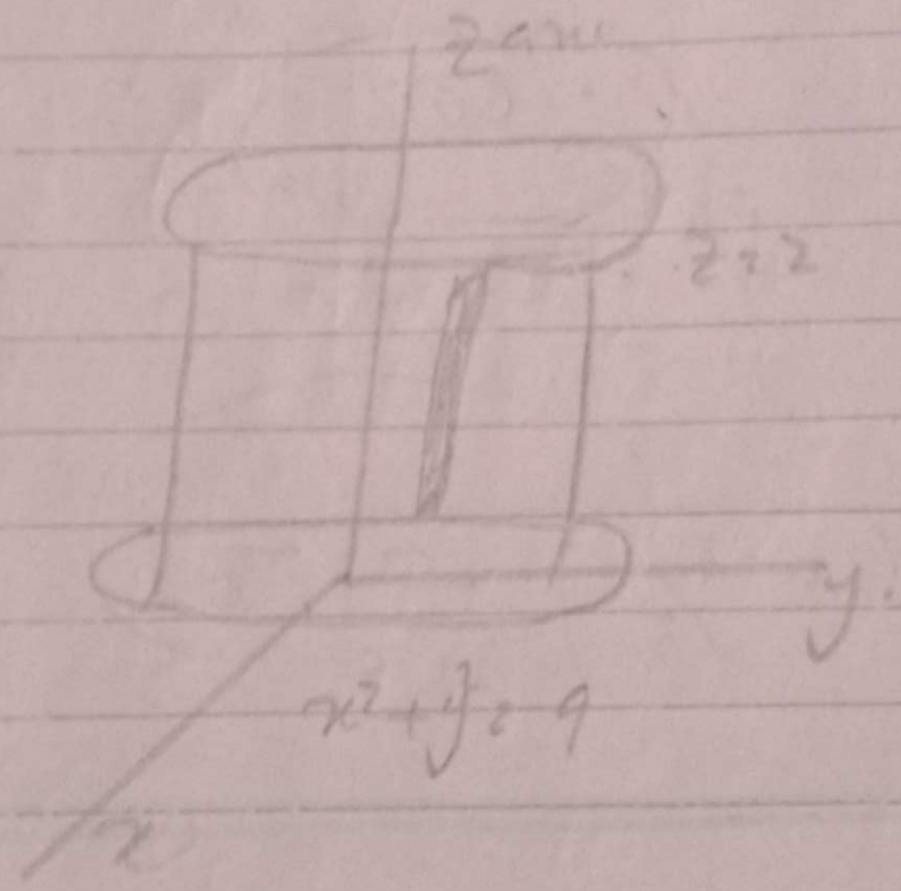
$$\iiint_G F \, \text{diveg} \, dv$$

S°

$$F = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}\right)$$

$$F(x, y, z) = 3x^2 + 3y^2 + z$$

Given $x^2 + y^2 = 9$
and plane $z = 0$ and $z = 2$



From the figure it is clear to that

$$x^2 + y^2 = 9$$

$$0 \le r \le 3$$
$$0 \le \theta \le 2\pi$$
$$0 \le z \le 2$$

$$\int_0^{2\pi}\int_0^{3}\int_0^{2} (3x^2 + 3y^2 + z)\, dz\, r\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{3} (3(r^2) + z)\, r\, dr\, d\theta$$

$$\int_0^2 (3r^3 + 2r)\, dr$$

$$\Rightarrow \left(3r^3 z + \frac{z^2}{2} r\right)_0^2 \Rightarrow 3r^3(2) - 2r)$$

$$\int_0^3 (6r^3 + 2r)\, dr$$

$$\Rightarrow \left(\frac{6r^4}{4} - \frac{2r^2}{2}\right)_0^3 \Rightarrow \left(\frac{3}{2}(81) + 18\right)$$

$$\left(\frac{243}{2} + 18\right) \Rightarrow \left(\frac{243+36}{2}\right)$$

$$\Rightarrow \int_0^{2\pi} \left(\frac{279}{2}\right)$$

$$\Rightarrow (279)\pi \Rightarrow \text{Ans.}$$

---

use divergence Theorem to find the out word flux of vector field.

$$F(x,y,z) = x^3 i + y^3 j + z^3 k$$

across hemisphere

$$z = \sqrt{a^2 - x^2 - y^2}$$

and plan $z = 0$

Ale to divergence Theorem

$$\iiint_G \operatorname{div} F$$

$$\iiint_G \operatorname{div} F = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right)$$

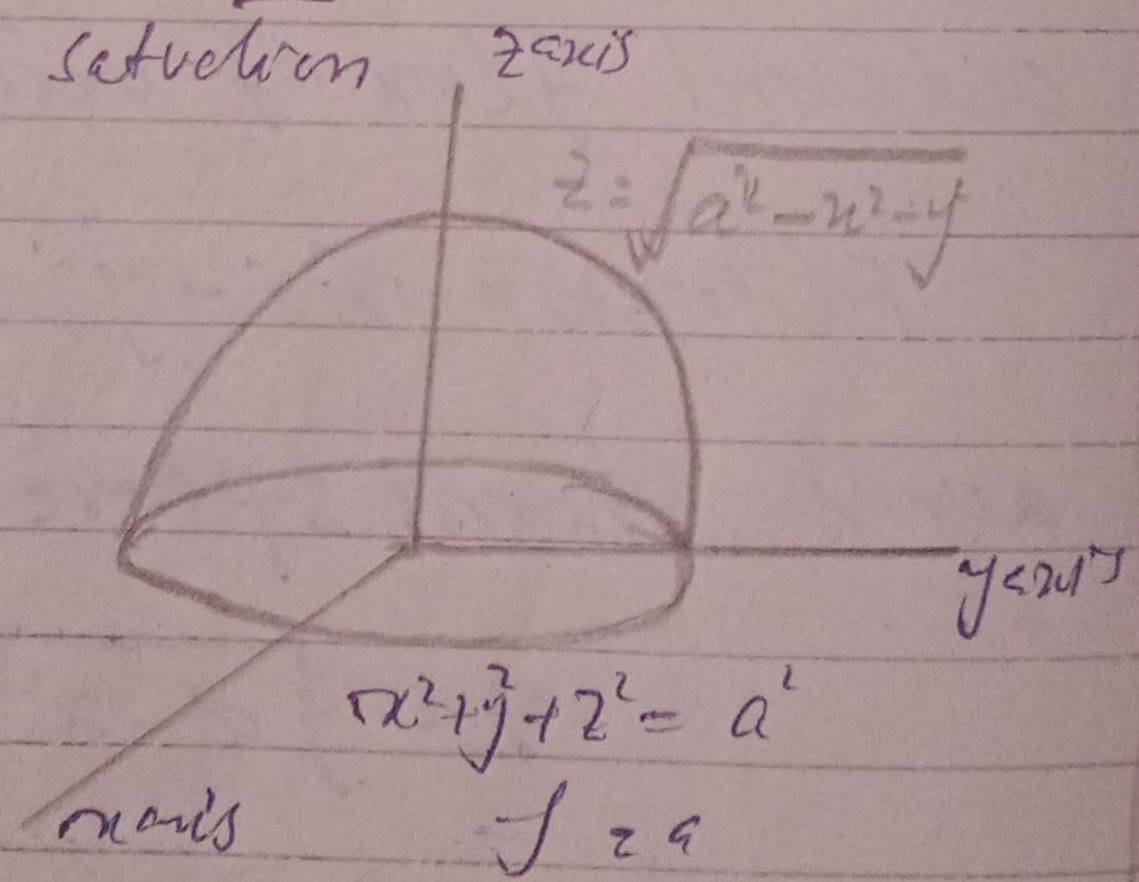
$$\Rightarrow \iiint_G \left(\frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial y}(y^3) + \frac{\partial}{\partial z}(z^3) \right)$$

$$\Rightarrow \iiint_G (3x^2 + 3y^2 + 3z^2)\, dv$$

$$\Rightarrow \iiint_G 3(x^2 + y^2 + z^2)\, dv$$

$$\Rightarrow \iiint_G 3(\rho^2)\, dv$$

But Ale to satuction



$0 \le \theta \le 2\pi$

$0 \le \phi \le \phi$

$$3\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \left(\rho^2 \sin\phi \, d\phi \, d\rho \, d\theta\right)$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \left(\rho^4 \sin\phi \, d\phi \, d\rho \, d\theta\right)$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi} \left(\rho^5/5\right) \sin\phi \, d\phi \, d\theta$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi} \sin\phi \, d\phi$$

$$\Rightarrow \frac{3a^5}{5}\int_0^{\pi} \sin\phi \, d\phi$$

$$\Rightarrow \frac{3a^5}{5}\left(-\cos\phi\right)_0^{\pi/2}$$

$$\Rightarrow \frac{3a^5}{5}\left(-(0-1)\right)$$

$$\Rightarrow \int_0^{2\pi} \frac{3a^5}{5} \, d\theta \Rightarrow \left(\frac{6\pi a^5}{5}\right)$$

$$\Rightarrow \left(\frac{6\pi a^5}{5}\right) \Rightarrow \text{Ans.}$$

(6) USE divergence Theorem to find the out word flux of the vector field.

$F(x,y,z) = 2x\hat{i} + 3y\hat{j} + z^2\hat{k}$

$\frac{\partial}{\partial x} = 2 \quad \frac{\partial}{\partial y} = 3 \quad \frac{\partial}{\partial z} = 2z$

$\Rightarrow \int_0^1\int_0^1\int_0^1 (5 + 2z)\, dx\, dy\, dz$

$\Rightarrow \int_0^1\int_0^1\int_0^1 (5 + 2z)\, dz\, dy\, dx$

$\Rightarrow \left(\int_0^1 5z + \frac{2z^2}{2}\right)_0^1 dx\, dy$

$\Rightarrow 5 + 1 \Rightarrow \int_0^1 6 \Rightarrow \int_0^1 6 \Rightarrow 6$

$6 \Rightarrow$ Ans

Exercise No 15.7"

Ex Question

19) $F(x,y,z) = (x^2+y)i + z^2 j + (e^y - z)k$; σ is the surface of the rectangular solid bounded by the coordinates planes and $x=3$ $y=1$ and $z=2$ using divege Theorum

$$0 \le x \le 3$$
$$0 \le y \le 1$$
$$0 \le z \le 2$$

$$\int_0^3\int_0^1\int_0^2 \frac{\partial}{\partial x}(x^2+y)i \; \frac{\partial}{\partial y} z^2 j + \frac{\partial}{\partial z}(e^y - z)$$

$$\int_0^3\int_0^1\int_0^2 (2x - 1)\, dv$$

$$\Rightarrow \int_0^3\int_0^1\int_0^2 (2x-1)\, dz\, dy\, dx$$

$$\Rightarrow \left(\frac{2x^2}{2} - x\right)_0^3 \Rightarrow (9-3) = 6$$

$$\int_0^1 6\, dy \Rightarrow \int_0^2 6\, dx$$

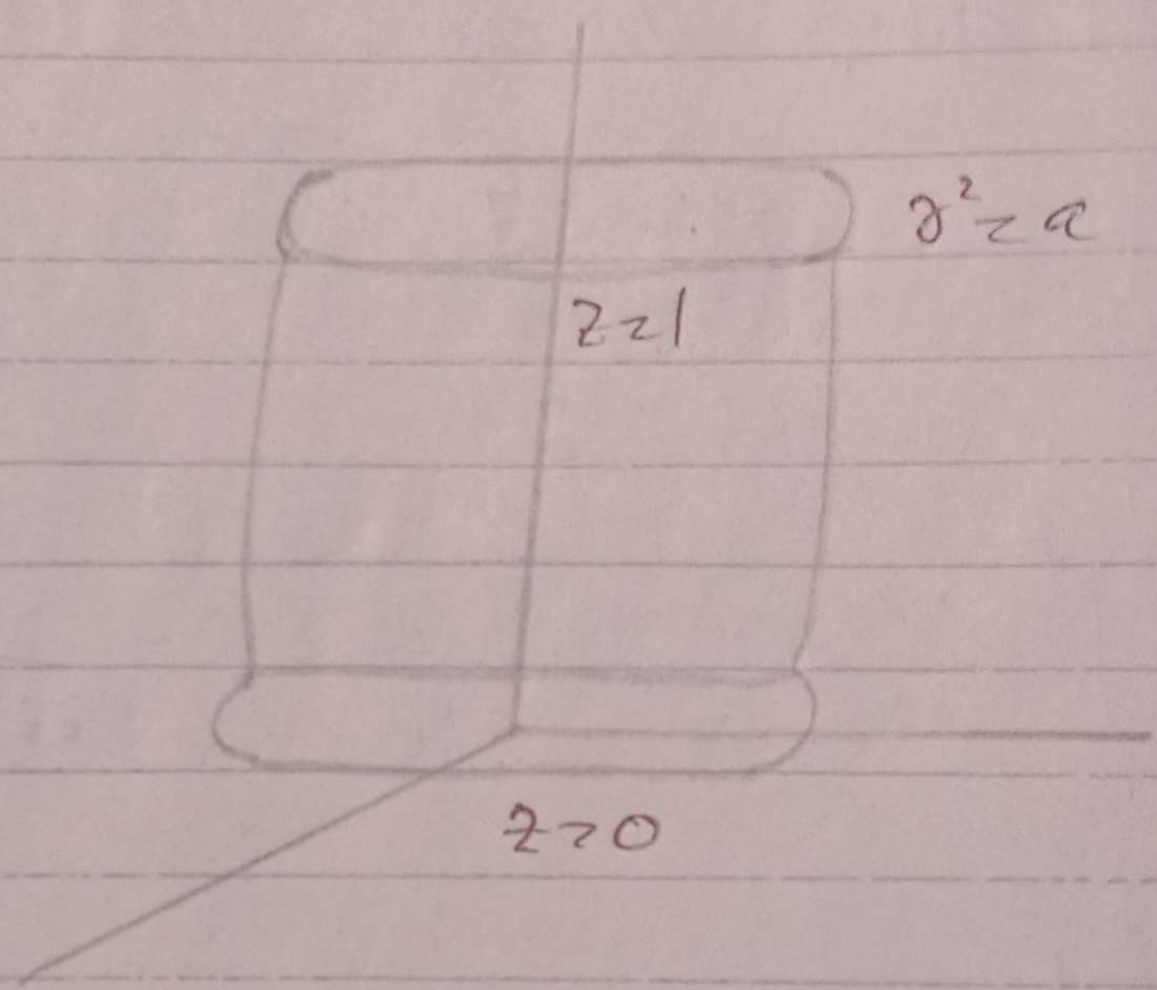
$$12 \Rightarrow \text{Ans}$$

(11) $F(x,y,z) = (x-z)i + (y-x)j + (z-y)k$; σ

σ is the cylinder solid bounded by

$$x^2+y^2 = a^2$$

and $z=0$ and $z=1$



$0 \le r \le a$
$0 \le \theta \le 2\pi$
$0 \le z \le 1$

So,

$$\frac{\partial}{\partial x}(x-z) = 1$$

$$\frac{\partial}{\partial y}(y-x) = 1$$

$$\frac{\partial}{\partial z}(z-y) = 1$$

$$\int_0^{2\pi}\int_0^{a}\int_0^{1}\left(1+1+1\right) dz\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{a}\int_0^{1}\left(3\right) dz\, r\, dr\, d\theta$$

$$\left(3rz\right)_0^1 dr\, d\theta$$

$$\Rightarrow \int_0^1 3r\, d\theta$$

$$\Rightarrow 3r^2/2 \Rightarrow \int_0^{2\pi} \frac{3a^2}{2}\, d\theta$$

$$\frac{6\pi a^2}{2} \Rightarrow \left(3\pi a^2\right)$$

---

(15) $\Rightarrow$

$$F(x,y,z) = (x^3 e^y)i + (y^3 + \sin z)j + (z^3 - xy)k$$

where σ is The Surface of solid bounded above by

$z = \sqrt{4-x^2-y^2}$ and below by The xy plane

Solution

Ak to divge Theorm.

$$\iiint_G f(x,y,z)\left(\frac{\partial x}{\partial u} \times \frac{\partial z}{\partial u}\right)$$

$$\text{div } F = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}\right)$$

$$\Rightarrow \frac{\partial}{\partial x}\left(x^3 - e^y\right) = 3x^2$$

$$\frac{\partial}{\partial y}\left(y^3 + \sin z\right) = 3y^2$$

$$\frac{\partial}{\partial z}\left(z^3 - xy\right) = 3z^2$$

So It is clear to That

$$z = \sqrt{4 - x^2 - y^2}$$

$$\Rightarrow x^2 + y^2 + z^2 = 4$$

$$\rho^2 = 4$$

$$\rho = 2$$

$$0 \leq \theta \leq 2\pi$$

and ϕ ?

The σ is The surface of solid bounded above by

$z = \sqrt{4 - x^2 - y^2}$ and below The xy plane - we also know That

at xy plane $z=0$

in spherical coordinates $z = \rho\cos\phi$.

$$\rho\cos\phi = 0$$

$$\Rightarrow \cos\phi = 0 \Rightarrow \phi = \cos^{-1}(0)$$

$$\phi = \pi/2$$

$$0 \le \phi \le \pi/2$$

A/c to divgence Theorm.

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2} (3x^2+3y^2+3z^2)\,\rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi/2}\left(\int_0^{2} \rho^4 d\rho\right)\sin\phi\, d\phi\, d\theta$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi/2}\left(\frac{\rho^5}{5}\right)_0^2 \sin\phi\, d\phi\, d\theta$$

$$\Rightarrow 3\int_0^{2\pi}\int_0^{\pi/2}\left(\frac{32}{5}\right)\sin\phi\, d\phi\, d\theta$$

$$= \frac{96}{5}\int_0^{2\pi}\left(-\cos\phi\right)_0^{\pi/2} d\theta$$

$$\Rightarrow \frac{96}{5}\left(-(0-1)\right) \Rightarrow \int_0^{2\pi} 96/5\, d\theta$$

$$\left(\frac{192\pi}{5}\right) \Rightarrow \text{Ans}$$

"Exercise 15.8"

Verified Stokes Theorem

$F(x,y,z) = 2z\,i + 3x\,j + 5y\,k$

and σ is the portion $\sigma = 4 - x^2 - y^2$ for which up wood (Orientation.

$x^2 + y^2 = 4$

The parametric eq of surface is

$x = 2\cos t \qquad y = 2\sin t \qquad z = 0$

$(0 \le t \le 2\pi)$

$$\oint F \cdot dr = \oint 2z\,dx + 3x\,dy + 5y\,dz$$

$$= \int_0^{2\pi} [0 + (6\cos t)(2\cos t) + 0]\,dt$$

$$= \int_0^{2\pi} 12\cos^2 t\,dt = 12\left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_0^{2\pi}$$

$$= 12\pi$$

Find a Curl.

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z & 3x & 5y \end{vmatrix}$$

$5i + 2j + 3k$

Since σ is Oriented up word.

$z = g(x,y) = 4 - x^2 - y^2$

$$\iint_{\sigma} (\text{curl} F) \left(-\frac{\partial z}{\partial x} i - \frac{\partial z}{\partial y} j + k \right)$$

$$\Rightarrow \iint_{R} (5i + 2j + 3k) \cdot (2x i + 2y j + k)$$

$$\iint_{R} (10x + 4y + 3)\, dA$$

$$\int_0^{2\pi} \int_0^{2} \left(\frac{10 r^3}{3} \cos\theta + \frac{4 r^3}{3} \sin\theta + \frac{3 r^2}{2} \right]_0$$

$$\Rightarrow \int_0^{2\pi} \left(\frac{80}{3} \cos\theta + \frac{32}{3} \sin\theta + 6 \right) d\theta$$

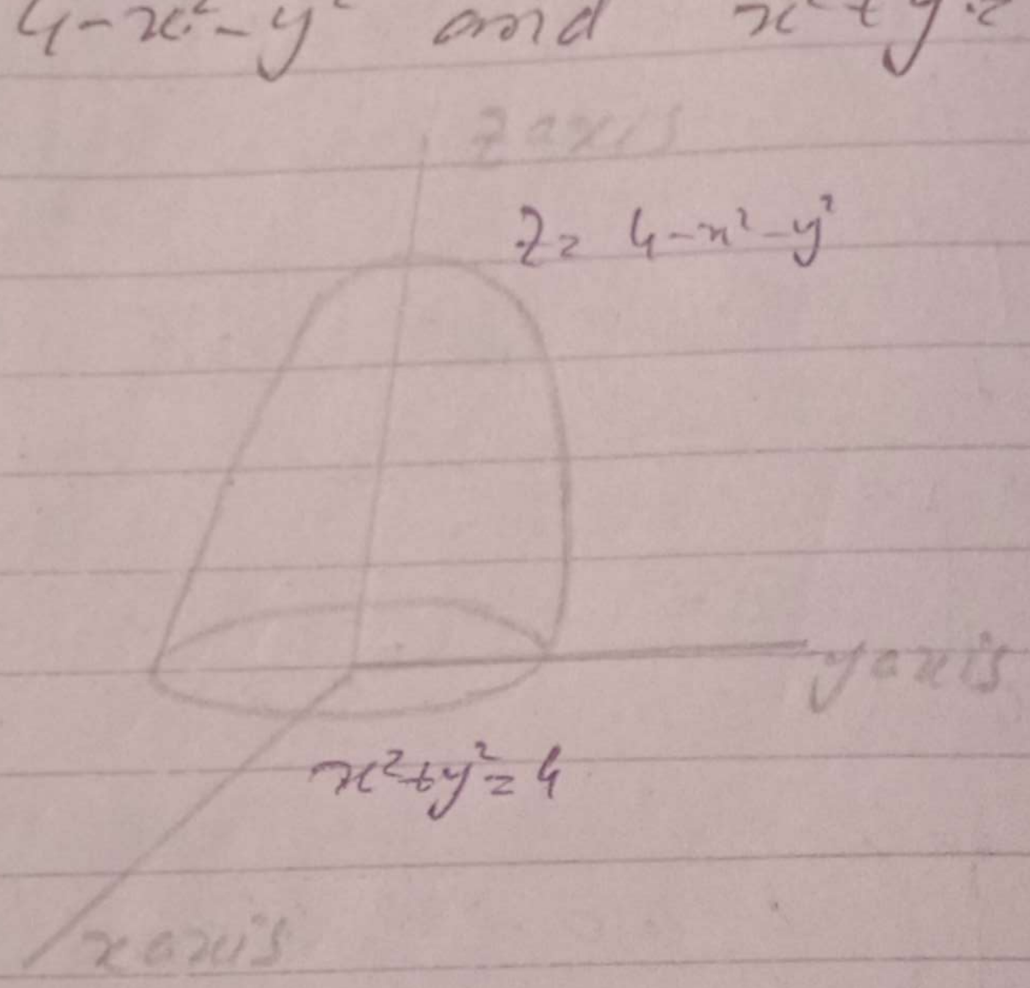
$$\Rightarrow \left(\frac{80}{3} \sin\theta - \frac{32}{3} \cos\theta + 6\theta \right)_0^{2\pi}$$

$\Rightarrow 12\pi \Rightarrow$ Hence verified.

Exercise Question

(7) $F(x,y,z) = 3z\,i + 4x\,j + 2y\,k$; C is the boundary of parabolide

$z = 4 - x^2 - y^2$ and $x^2 + y^2 = 4$



A/c to stokes Theorem

$$\oint F \cdot dr = \iint_S \text{curl } F \cdot n\, ds = \iint_R \text{curl } F \left(-\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} + k \right)$$

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3z & 4x & 2y \end{vmatrix}$$

$$\Rightarrow \quad i\left(\frac{\partial}{\partial y}(2y) - \frac{\partial}{\partial z}(4x)\right) - j\left(\frac{\partial}{\partial x}(2y) - \frac{\partial}{\partial z} 3z\right)$$

$$+ k\left(\frac{\partial}{\partial x} 4x - \frac{\partial}{\partial y} 3z\right)$$

$(n) = 2i + 3j + 4k$

So, σ is the portion of surface

$n\,ds =$

$$z = 4 - x^2 - y^2$$

$$-\frac{\partial z}{\partial x} = -2x$$

$$-\frac{\partial z}{\partial y} = -2y$$

So

$$\iint_{\sigma} (2i + 3j + 4k)(2xi + 2yj + k)\, dA$$

$$\iint_{R} (2x + 6y + 4)\, dA$$

$$\int_0^{2\pi}\int_0^{2} (2r\cos\theta + 6r\sin\theta + 4)\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2} \left(\underline{2r^2\cos\theta} + 6r^2\sin\theta + 4r\right) dr$$

$$\Rightarrow \left(\frac{2r^3\cos\theta}{3} + \frac{6r^3\sin\theta}{3} + \frac{4r^2}{2}\right)\Big]_0^2$$

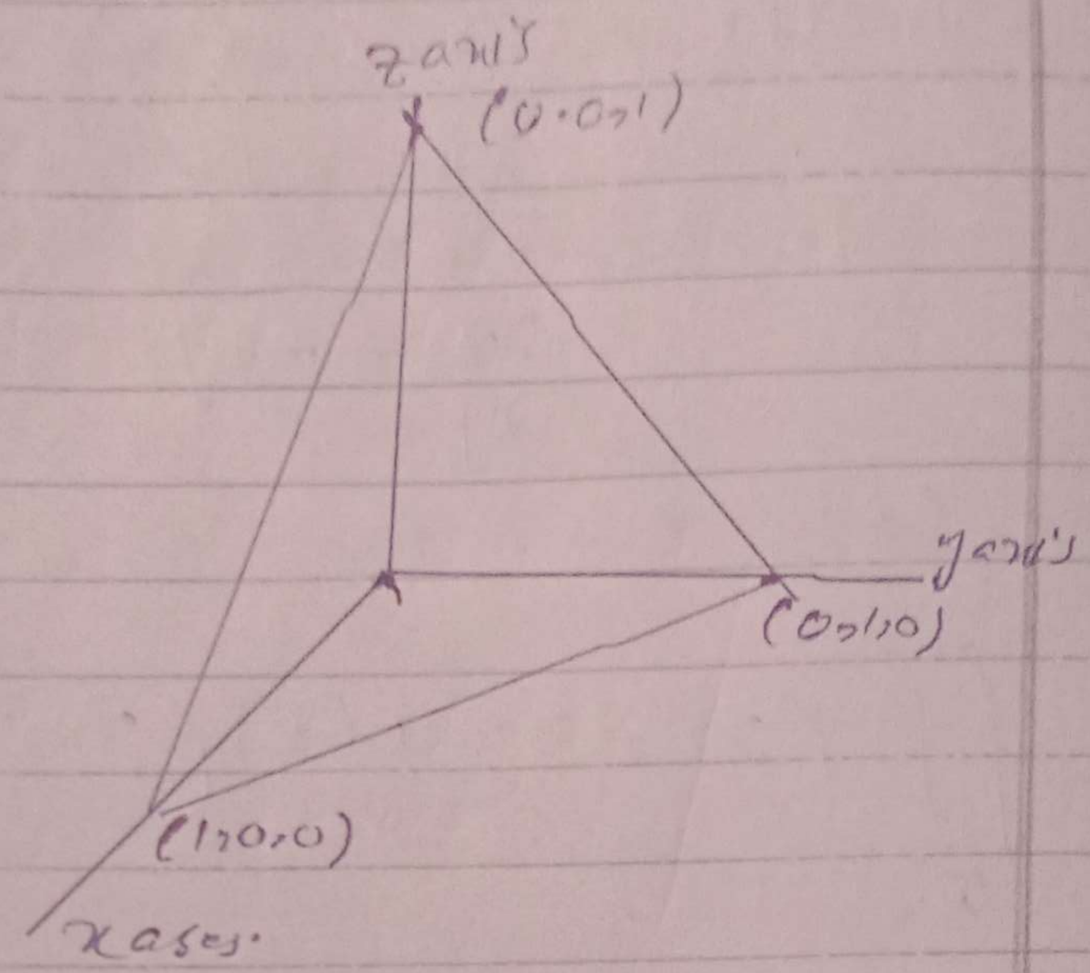
$$\Rightarrow \frac{2\cos\theta\,(2)^3}{3} + 2\sin\theta\,(2)^3 + 2(2)^2$$

$$\Rightarrow \int_0^{2\pi} \left(\frac{16}{3}\cos\theta + 16\sin\theta + 8\right) d\theta$$

$$\frac{16}{3}\sin\theta\Big]_0^{2\pi} + \left(-\frac{16}{3}\cos\theta\right)\Big]_0^{2\pi} + 8\Big]_0^{2\pi} = 16\pi$$

Ans.

(b) $F(x,y,z) = xyi + yzj + zxk$ is the triangle in the plane $x+y+z=1$ with vertices $(1,0,0)$ $(0,1,0)$ and $(0,0,1)$ octant



Acc to Stokes Theorem

$$\oint F \cdot dr = \iint_S \text{curl } F \cdot n \, dS$$

curl F =

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix}$$

$$\left(\frac{\partial}{\partial y}(zx) - \frac{\partial}{\partial z}(yz) i \right) + \left(\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial z}(xy) + k\left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy) \right) \right.$$

$$\Rightarrow \quad 0 - yi + zj - xk)$$

$-y\hat{i} + z\hat{j} - x\hat{k} = \text{Curl } F$

so σ is the oriented up

$n\,ds =$

$z = 1 - x - y$

$$-\frac{\partial z}{\partial x} = -1$$

$$-\frac{\partial z}{\partial y} = -1$$

$\Rightarrow n\,ds = \hat{i} + \hat{j} + k$

$x + y + z = 1$

$0 \le x \le 1$

$0 \le y \le 1 - x$

$$\oint F \cdot d\Sigma = \iint_R (-y\hat{i} - z\hat{j} - x\hat{k}) \cdot (\hat{i} + \hat{j} + k)\, dA$$

$$\Rightarrow \int_0^1 \int_0^{1-x} (-y - z - x)\, dy\, dx$$

$$\Rightarrow -\int_0^1 \int_0^{1-x} (x + y + z +)\, dy\, dx$$

$x + y + z = 1$

$$-\int_0^1 \int_0^{1-x} dy\, dx$$

$$\Rightarrow -\int_0^1 (1 - x)\, dx$$

$$\Rightarrow -\left(x - \frac{x^2}{2}\right)\Big|_0^1 \Rightarrow -\left(\frac{1}{2}\right) \Rightarrow \left(-\frac{1}{2}\right)$$

$\rightarrow$ Ans

(11) $F(x,y,z) = (x-y)i + (y-z)j + (z-x)k$
C is the circle $x^2+y^2=a$ in the xy plane with counter clock wise orientation looking down the +ive ~~x~~ z-axis

Solution

$$\oint F\cdot dr = \iint_R \text{curl } F\cdot n\, ds$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-y & y-z & z-x \end{vmatrix}$$

$$= i + j + k$$

and σ is the oriented up.

$$= \iint_R (i+j+k)\cdot(0i+0j+k)\, dA$$

$$\iint_R dA$$

if C is the circle $x+j=a$ in the xy plane the we use a polar coordinates.

$$\oint F\cdot dr = \int_0^{2\pi}\int_0^{a} r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{r^2}{2}\right)_0^{a} d\theta$$

$$= \frac{a^2}{2}\left[\theta\right]_0^{2\pi} \Rightarrow a^2\pi$$

plane
looking

$F(x,y,z) = (z+\sin x)i + (x+y^2)j + (y+e^z)k$.
C is the intersection of the sphere $x^2+y^2+z^2=1$ and the cone $z=\sqrt{x^2+y^2}$ with counter clock orientation looking down the positive z axis.

Sol

$$z = \sqrt{1-x^2-y^2} \rightarrow (i)$$
$$z = \sqrt{x^2+y^2} \rightarrow (ii)$$

Comparing (i) & (2) we get

$$x^2+y^2 = 1-x^2-y^2$$
$$\Rightarrow x^2+y^2+x^2+y^2=1$$
$$2r^2 = 1 \Rightarrow r = 1/\sqrt{2}$$

$$0 \le r \le 1/\sqrt{2}$$

$$\oint_\sigma F\cdot dr = \iint \text{curl } F\cdot n\, ds$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z+\sin & x+y^2 & (y+e^z) \end{vmatrix}$$

$$\Rightarrow \text{curl } F = i + j + k$$

Since σ is oriented up
So it is clear to that z is the intersection point b/w sphere and cone so $z = 1/\sqrt{2}$

So σ is oriented up word direction

$$n\,ds = \left(\frac{-\partial z}{\partial x} - \frac{\partial z}{\partial y} + k\right) du \text{ using}$$

$r^2 = 1/\sqrt{2} \Rightarrow r = 1/2$

$$\Rightarrow \frac{-\partial z}{\partial x} = 0$$

$$\frac{-\partial z}{\partial y} = 0$$

So Ak to Stokes theorm.

$$\oint F \cdot dr = \int_0^{2\pi}\int_0^{1/\sqrt{2}} (i+j+k)\cdot(0i+0j+k)\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{1/\sqrt{2}} 1\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(\frac{r^2}{2}\right)_0^{1/2} d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{1}{2}\left(\frac{1}{4}\right) d\theta$$

$$2\pi/8 \Rightarrow (\pi/4)$$

$\Rightarrow$ Ans.