Ex

Length and Distance in inner product space

Defination :-

If V is an inner product space then the norm (or length) of a vector u in V is denoted by $\|u\|$ and is defined by

$$\|u\| = \langle u, u \rangle^{\frac{1}{2}} = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}$$

The distance between two points (vectors) u and v is denoted by d(u, v) and is defined by

$$d(u, v) = \|u - v\|$$

$$d(u, v) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots (u_n - v_n)^2}$$

Q Use the inner Product find d(u, v) and $\|u\|$

For $u = (-1, 2)$, $v = (2, 5)$

$$\|u\| = \sqrt{(-1)^2 + (2)^2}$$
$$= \sqrt{1 + 4}$$
$$\boxed{\|u\| = \sqrt{5}}$$

$$d(u, v) = \sqrt{(-1-2)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \text{ Ans.}$$