Example Find QR - Decompositions of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The column vector A are

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Applying Gram-Schmidt process with subsequent normalizations to these column vectors yields the orthonormal vectors.

$$q_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \quad q_2 = \begin{bmatrix} -2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \quad q_3 = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

For R:

$$R = \begin{bmatrix} 3/\sqrt{3} & 2/\sqrt{3} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

Thus the QR - Decomposition of A is

$$\underset{A}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} = \underset{Q}{\begin{bmatrix} 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{bmatrix}} \underset{R}{\begin{bmatrix} 3/\sqrt{3} & 2/\sqrt{3} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{6} \\ 0 & 0 & 1/\sqrt{2} \end{bmatrix}}$$