COORDINATES RELATIVE TO ORTHONORMAL BASES :-

If $S = \{v_1, v_2, \ldots, v_n\}$ is an orthonormal basis for an inner product space V and u is any vector in V then.

$$u = \langle u, v_1\rangle v_1 + \langle u, v_2\rangle v_2 + \cdots \langle u, v_n\rangle v_n$$

$$(u)_s = (\langle u, v_1\rangle, \langle u, v_2\rangle, \ldots, \langle u, v_n\rangle)$$

is the coordinate vector of u relative to the basis.

Example : $V_1 = (0,1,0)$, $V_2 = (-4/5, 0, 3/5)$

$V_3 = (3/5, 0, +4/5)$

$u = (1, 1, 1)$

Find the coordinate vector $(u)_s$.

$$\langle u, v_1\rangle = \langle (1,1,1), (0,1,0)\rangle = 0+1+0 = 1$$

$$\langle u, v_2\rangle = \langle (1,1,1), (-4/5, 0, 3/5)\rangle = -\frac{4}{5} + 0 + \frac{3}{5} = -\frac{1}{5}$$

$$\langle u, v_3\rangle = \langle (1,1,1), (3/5, 0, +4/5)\rangle = \frac{3}{5} + 0 + \frac{4}{5} = +\frac{7}{5}$$

$$u = \langle u, v_1\rangle v_1 + \langle u, v_2\rangle v_2 + \langle u, v_3\rangle v_3$$

$$u = (1)V_1 + \left(-\frac{1}{5}\right)V_2 + \left(\frac{7}{5}\right)V_3$$

$$(1,1,1) = (0,1,0) - \frac{1}{5}(-4/5, 0, 3/5) + 7/5(3/5, 0, 4/5)$$

The coordinate vector of u relative to S is

$$(u)_s = (\langle u, v_1\rangle, \langle u, v_2\rangle, \langle u, v_3\rangle) = \left(1, -\frac{1}{5}, 7/5\right)$$