

$$\begin{aligned}
 u+v &= \begin{bmatrix} a_1+a_2 & 0 \\ 0 & b_1+b_2 \end{bmatrix} \\
 &= \begin{bmatrix} a_2+a_1 & 0 \\ 0 & b_2+b_1 \end{bmatrix} \\
 &= \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \\
 &= v + u
 \end{aligned}$$

$$\boxed{u+v = v+u}$$

Hence Axioms ② is satisfied.

Axioms 03:

The next Axioms is Associative

If $u, v, w \in V$

$$u + (v+w) = (u+v) + w$$

$$\begin{aligned}
 u + (v+w) &= \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \left(\begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} a_3 & 0 \\ 0 & b_3 \end{bmatrix} \right) \\
 &= \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2+a_3 & 0 \\ 0 & b_2+b_3 \end{bmatrix}
 \end{aligned}$$

$$= \begin{bmatrix} a_1+a_2+a_3 & 0 \\ 0 & b_1+b_2+b_3 \end{bmatrix}$$