Question :- Consider the vector space R^3 with the euclidean inner product! Apply the Gram Schmidt process to transform the basis vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ $u_3 = (0,0,1)$ into an orthogonal vectors basis $\{v_1, v_2, v_3\}$ the normalizing the orthogonal basis vector to obtain an orthonormal basis $\{q_1, q_2, q_3\}$.

Solution

Step 01 $v_1 = u_1$

$$\boxed{v_1 = (1,1,1)}$$

Step 02 $$v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} v_1$$

$$\langle u_2, v_1 \rangle = \langle (0,1,1) \cdot (1,1,1) \rangle = 0+1+1 = 2$$

$$\|N_1\|^2 = \left(\sqrt{1+1+1}\right)^2 = 3$$

$$v_2 = (0,1,1) - \frac{2}{3}(1,1,1)$$

$$v_2 = (0,1,1) - \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$$