Q Determine the dimension of and a basis for the solution space of the system.

(ii) $x_1 + x_2 - x_3 = 0$

$-2x_1 - x_2 + 2x_3 = 0$

$-x_1 + x_3 = 0$

Solution

$$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

which can be written in argumented form as

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & 1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right]$$

$R_2 + 2R_1 \,,\; R_3 + R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$