Find the vector form of the general solution
of the given linear system $[Ax = b]$; then
use that results to find the vector form
of the general solution of $[Ax = 0]$

$$x_1 + x_2 + 2x_3 = 5$$
$$x_1 \quad\quad + x_3 = -2$$
$$2x_1 + x_2 + 3x_3 = 3$$

Solution:

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$$

R
$$\begin{bmatrix} 1 & 1 & 2 & \vdots & 5 \\ 1 & 0 & 1 & \vdots & -2 \\ 2 & 1 & 3 & \vdots & 3 \end{bmatrix}$$

$R_2 - R_1 \quad , \quad R_3 - 2R_1$

R
$$\begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & -1 & -1 & -7 \\ 0 & -1 & -1 & -7 \end{bmatrix}$$

$R_3 - R_2$

R
$$\begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & -1 & -1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$