If $|A|$ give nontrivial solution it means that the set of vector is linearly dependent.

$$\underset{\sim}{R}\begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

$$R_2 + 2R_1 \;,\; R_3 - 3R_1$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & -16 & -8 \end{bmatrix}$$

$$R_3 + R_2$$

$$R\begin{bmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{16} R_2$$

$$R\begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - 5R_2$$

$$R\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$