COORDINATES RELATIVE TO ORTHOGONAL BASES &

If $S = \{v_1, v_2, \ldots, v_n\}$ is an orthogonal basis for a vector space V then normalizing each of these vectors yields the orthonormal basis

$$S' = \left\{ \frac{v_1}{\|v_1\|}, \frac{v_2}{\|v_2\|}, \frac{v_3}{\|v_3\|} \right\}$$

Thus if u is any vector V then

$$u = \left\langle u, \frac{v_1}{\|v_1\|} \right\rangle \frac{v_1}{\|v_1\|} + \left\langle u, \frac{v_2}{\|v_2\|} \right\rangle \frac{v_2}{\|v_2\|} + \cdots + \left\langle u, \frac{v_n}{\|v_n\|} \right\rangle \frac{v_n}{\|v_n\|}$$

$$u = \frac{\langle u, v_1 \rangle}{\|v_1\|^2} v_1 + \frac{\langle u, v_2 \rangle}{\|v_2\|^2} v_2 + \cdots \frac{\langle u, v_n \rangle}{\|v_n\|^2}$$