Ex

Suppose that u, v and w are vectors such that

$\langle u, v \rangle = 2$, $\langle v, w \rangle = -3$, $\langle u, w \rangle = 5$

$\|u\| = 1$, $\|v\| = 2$, $\|w\| = 7$.

Evaluate the given expression

(a) $\langle u+v, v+w \rangle$

$= \langle u, (v+w) + v, (v+w) \rangle$

$= \langle (u,v) + (u,w) + (v,v) + (v+w) \rangle$

$= \langle (u,v) + (u,w) + \|v\|^2 + (v+w) \rangle$

$= \langle 2 + (-3) + (2)^2 + 5 \rangle$

$= 2 - 3 + 4 + 5$

$= 8$ Ans.

(b) $\langle 2v - w, 3u + 2w \rangle$

$= \langle 2v, (3u+2w) - w, (3u+2w) \rangle$

$= \langle (2v, 3u) + (2v, 2w) - (w, 3u) - (w, 2w) \rangle$

$= \langle 6(u,v) + 4(v,w) - 3(u,w) - 2\|w\|^2 \rangle$

$= 6(2) + 4(-3) - 3(5) - 2(7)^2$

$= 12 - 12 - 15 - 98$

$= -113$