

INNER PRODUCT SPACE:-

An inner product on a real vector space V is a function that associates a real number $\langle u, v \rangle$ with each pair of vectors u and v in V in such a way that the following axioms are satisfied for all vectors u, v and w in V and all scalars k .

$$\textcircled{1} \quad \langle u, v \rangle = \langle v, u \rangle \quad [\text{Symmetric axiom}]$$

$$\textcircled{2} \quad \langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$$

$$\textcircled{3} \quad \langle ku, v \rangle = k \langle u, v \rangle$$

$$\textcircled{4} \quad \langle v, v \rangle \geq 0$$

$$\text{where } \langle v, v \rangle = 0$$

$$\text{if and only if } v=0.$$

$$\langle u, v \rangle = u \cdot v = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$

Ex 01 Let $\langle u, v \rangle$ be the Euclidean inner product on \mathbb{R}^2 , and let $u = (3, -2)$ $v = (4, 5)$,

$w = (-1, 6)$ and $k = -4$ Find

$$\textcircled{a} \quad \langle u, v \rangle = \langle v, u \rangle$$

L.H.S

$$\langle u, v \rangle = \langle (3, -2), (4, 5) \rangle$$

$$= (3)(4) + (-2)(5)$$

$$= 12 - 10$$

$$\boxed{\langle u, v \rangle = 2}$$