

① If $u, v \in V$ then

$$\begin{aligned} u+v &= (u_1, 0, 0) + (v_1, 0, 0) \\ &= (u_1+v_1, 0, 0) \in W \end{aligned}$$

Hence, $u+v$ is in W . The given set is closed under addition.

②
$$\begin{aligned} ku &= k(u_1, 0, 0) \\ &= (ku_1, 0, 0) \end{aligned}$$

Thus, the given set is closed under addition and scalar multiplication and hence is a subspace of \mathbb{R}^3 .

④ All vectors of the form (a, b, c) where $b = a + c + 1$.

Let $u = (u_1, u_2, u_3)$, $u_2 = u_1 + u_3 + 1$
 $v = (v_1, v_2, v_3)$, $v_2 = v_1 + v_3 + 1$
be two space vectors in \mathbb{R}^3 and k is scalar, then

$$u+v = (u_1, u_2, u_3) + (v_1, v_2, v_3)$$

$$\begin{aligned} u+v &= (u_1, u_1+u_3+1, u_3) + (v_1, v_1+v_3+1, v_3) \\ &= (u_1+v_1, u_1+u_3+1+v_1+v_3+1, u_3+v_3) \\ &= (u_1+v_1, u_1+v_1+u_3+v_3+2, u_3+v_3) \end{aligned}$$

Hence $u+v$ is not in W . The given set is not subspace.