ORTHONORMAL BASES ; GRAM-SCHMIDT PROCESS OR-DECOMPOSITION:-

ORTHOGONAL AND ORTHONORMAL BASES:-

Definition:

A set of vectors in an inner product space is called orthogonal set if all pairs of distinct vectors in the set are orthogonal. An orthogonal set in which each vector has norm 1 is called orthonormal.

Example

$u_1 = \langle 0, 1, 0 \rangle$, $u_2 = \langle 1, 0, 1 \rangle$, $u_3 = \langle 1, 0, -1 \rangle$

$S = \langle u_1, u_2, u_3 \rangle$ $\langle u_1, u_2 \rangle = \langle u_2, u_3 \rangle = \langle u_1, u_3 \rangle = 0$

$\frac{1}{\|v\|} v$ has norm 1 since

$$\left\| \frac{1}{\|v\|} v \right\| = \left| \frac{1}{\|v\|} \right| \|v\| = \frac{1}{\|v\|} \|v\| = 1$$

Normalizing:

The process of multiplying a nonzero vector v by the reciprocal of its length to obtain a vector of norm 1 is called normalizing.