LINEAR SYSTEMS OF THE FORMS $Ax = \lambda x$:

$$Ax = \lambda x$$

where λ is a scalar.

$$\lambda x - Ax = 0$$

$$(\lambda - A)x = 0$$

$$\boxed{(\lambda I - A)x = 0}$$

<u>Problem:</u> The linear system

$$x_1 + 3x_2 = \lambda x_1$$

$$4x_1 + 2x_2 = \lambda x_2$$

can be written in matrix form

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

This system can be written as

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 1 & -3 \\ -4 & \lambda - 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 1 & -3 \\ -4 & \lambda - 2 \end{bmatrix}$$

The value of λ is called a characteristic value or an eigenvalue of A.

$$\det(\lambda I - A) = 0$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 1 & -3 \\ -4 & \lambda - 2 \end{vmatrix} = 0$$

$$(\lambda - 1)(\lambda - 2) - (-4)(-3) = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\lambda^2 - 5\lambda + 2\lambda - 10 = 0$$

$$\lambda(\lambda - 5) + 2(\lambda - 5) = 0$$

$$(\lambda - 5)(\lambda + 2) = 0$$

$$\lambda - 5 = 0 \qquad \lambda + 2 = 0$$

$$\boxed{\lambda = 5} \qquad \boxed{\lambda = -2}$$