Basis For a vector Space :-

Defination: If V is any vector space and $S = \{v_1, v_2, \ldots, v_n\}$ is a set of vectors in V then S is called a basis for V if the following two Conditions hold

① S is linearly independent.

② S span V.

Question $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ show that the set $S = \{v_1, v_2, v_3\}$ is a basis for R^3

Solution:

In term of component the vector equation

$$k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$$

$$k_1(1, 2, 1) + k_2(2, 9, 0) + k_3(3, 3, 4) = 0$$

$$k_1 + 2k_2 + 3k_3 = 0 \rightarrow ①$$

$$2k_1 + 9k_2 + 3k_3 = 0 \rightarrow ②$$

$$k_1 + 0k_2 + 4k_3 = 0 \rightarrow ③$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix}$$