$$\begin{bmatrix} \lambda-1 & -3 \\ -4 & \lambda-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If $\lambda = -2$

$$\begin{bmatrix} -2-1 & -3 \\ -4 & -2-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 \\ -4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = -t, \quad x_2 = t$$

So, the eigenvectors corresponding of $\lambda = -2$ are the nonzero solutions of the form

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix}$$

If $\lambda = 5$

$$\begin{bmatrix} 5-1 & -3 \\ -4 & 5-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = -\frac{3}{4}t, \quad x_2 = t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4}t \\ t \end{bmatrix}$$

$$\det(\lambda I - A) = 0$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -3 \\ -4 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda-2) - (-4)(-3) = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 - 12 = 0$$

$$\lambda^2 - 3\lambda - 10 = 0$$

$$\lambda^2 - 5\lambda + 2\lambda - 10 = 0$$

$$\lambda(\lambda-5) + 2(\lambda-5) = 0$$

$$(\lambda-5)(\lambda+2) = 0$$

$$\lambda - 5 = 0 \qquad \lambda + 2 = 0$$

$$\boxed{\lambda = 5} \qquad \boxed{\lambda = -2}$$