Problem: What conditions must b_1, b_2 and b_3 satisfy in order for the system of equations

$$x_1 + 2x_2 + 3x_3 = b_1$$
$$2x_1 + 5x_2 + 3x_3 = b_2$$
$$x_1 + \qquad + 8x_3 = b_3$$

is Consistent?

Soln:

The Augmented Matrix

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & b_1 \\ 2 & 5 & 3 & b_2 \\ 1 & 0 & 8 & b_3 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & b_1 \\ 0 & 1 & -3 & b_2 - 2b_1 \\ 0 & -2 & 5 & b_3 - b_1 \end{array}\right]$$

$R_1 - 2R_2$, $R_3 + 2R_2$

$$\left[\begin{array}{ccc:c} 1 & 0 & 9 & 5b_1 - 2b_2 \\ 0 & 1 & -3 & b_2 - 2b_1 \\ 0 & 0 & -1 & -5b_1 + 2b_2 + b_3 \end{array}\right]$$

$-R_3$

$$\left[\begin{array}{ccc:c} 1 & 0 & 9 & 5b_1 - 2b_2 \\ 0 & 1 & -3 & b_2 - 2b_1 \\ 0 & 0 & 1 & 5b_1 - 2b_2 - b_3 \end{array}\right]$$

$R_1 - 9R_3$, $R_2 + 3R_3$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & -40b_1 + 16b_2 + 9b_3 \\ 0 & 1 & 0 & 13b_1 - 5b_2 - 3b_3 \\ 0 & 0 & 1 & 5b_1 - 2b_2 - b_3 \end{array}\right]$$

In this case there are no restrictions on b_1, b_2 and b_3 that is the given system $Ax = b$ is Unique Solution.

$$x_1 = -40b_1 + 16b_2 + 9b_3$$
$$x_2 = 13b_1 - 5b_2 - 3b_3$$
$$x_3 = 5b_1 - 2b_2 - b_3$$