

Question The set of all pairs of real number (x, y) with the operation.

$$(x, y) + (x', y') = (x + x' + 1, y + y' + 1)$$

$$\text{and } K(x, y) = (Kx, Ky)$$

Solution

$$u = (x, y), \quad v = (x', y')$$

Axioms ⑦

$$K(u+v) = Ku + Kv$$

$$\begin{aligned} K \odot (u+v) &= K \odot ((x, y) + (x', y')) \\ &= K \odot (x + x' + 1, y + y' + 1) \\ &= K(x + x' + 1, y + y' + 1) \end{aligned}$$

$$\begin{aligned} Ku + Kv &= K \odot (x, y) \oplus K \odot (x', y') \\ &= (Kx, Ky) \oplus (Kx', Ky') \\ &= (Kx + Kx' + 1, Ky + Ky' + 1) \end{aligned}$$

$$K(u+v) \neq Ku + Kv$$

Axioms ⑧

$$(c+d)u = cu + du$$

$$\begin{aligned} (c+d)u &= (c+d) \odot (x, y) \\ &= (c+d)x + (c+d)y \\ &= (cx + dx, cy + dy) \end{aligned}$$