

Define $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(x, y, z) = (y+z, x+z, x-y)$
Assume that T is a linear Transformation

Write down the matrix representation A of T with respect to standard basis in domain and codomain.

Solution:

Let,

Standard basis

$$e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$$

$$T(x, y, z) = (y+z, x+z, x-y)$$

$$T(e_1) = (1, 0, 0) = (0, 1, 1) = 0(1, 0, 0) + 1(0, 1, 0) + 1(0, 0, 1)$$

$$T(e_2) = (0, 1, 0) = (1, 0, -1) = 1(1, 0, 0) + 0(0, 1, 0) + (-1)(0, 0, 1)$$

$$T(e_3) = (0, 0, 1) = (1, 1, 0) = 1(1, 0, 0) + 1(0, 1, 0) + 0(0, 0, 1)$$

The matrix representation of T is

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Linear $\mathbb{R}^3 \rightarrow \mathbb{R}^3$