

$$\begin{aligned}
 cu + du &= c(x, y) + d(x', y') \\
 &= (cx, cy) \oplus (dx', dy') \\
 &= (cx + dx' + 1, cy + dy' + 1) \\
 (c+d)u &\neq cu + du
 \end{aligned}$$

Q The set of all 2×2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with matrix addition and multiplication.

Solution

$$\text{Let, } u = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \quad v = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$

Axioms 01:

$$u + v \in V$$

$$\begin{aligned}
 u + v &= \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} \\
 &= \begin{bmatrix} a_1 + a_2 & 0 \\ 0 & b_1 + b_2 \end{bmatrix}
 \end{aligned}$$

Since u, v in R then $u + v$ in V , hence $u + v$ in V . V is closed under Addition.

Axiom 02:

$$u + v = v + u$$

$$u + v = \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix}$$