Formula:

① If $\{v_1, v_2, \ldots v_n\}$ is an orthonormal basis for W and u is any vector in V then

$$Proj_W u = \langle u, v_1\rangle v_1 + \langle u, v_2\rangle v_2 + \cdots \langle u, v_n\rangle v_n$$

② If $\{v_1, v_2, v_3, \ldots v_n\}$ is an orthogonal basis for W and u is any vector in V then

$$Proj_W u = \frac{\langle u, v_1\rangle}{\|v_1\|^2} v_1 + \frac{\langle u, v_2\rangle}{\|v_2\|^2} v_2 + \cdots + \frac{\langle u, v_n\rangle}{\|v_n\|^2} v_n$$

Example: Let R^3 have the Euclidean inner product and let W be the subspace spanned by the orthonormal vector $v_1 = (0, 1, 0)$ and $v_2 = (-4/5, 0, 3/5)$ The orthogonal Proj of $u = (1, 1, 1)$ on W is

$$Proj_W u = \langle u, v_1\rangle v_1 + \langle u, v_2\rangle v_2$$

$$= (1)(0, 1, 0) + \langle -1/5)(-4/5, 0, 3/5)$$

$$= (0, 1, 0) + (4/25, 0, -3/25)$$

$$\boxed{Proj_W u = \frac{4}{25}, 1, -\frac{3}{25}}$$