Question Determine whether the vector
$V_1 = (1, -2, 3), V_2 = (5, 6, -1), V_3 = (3, 2, 1)$
form a linearly dependent set or a linearly independent set.

Solution

In term of component the vector equation

$$K_1V_1 + K_2V_2 + K_3V_3 = 0$$

$$K_1(1, -2, 3) + K_2(5, 6, -1) + K_3(3, 2, 1) = 0$$

$$K_1 + 5K_2 + 3K_3 = 0$$

$$-2K_1 + 6K_2 + 2K_3 = 0$$

$$3K_1 + K_2 + K_3 = 0$$

Thus V_1, V_2, V_3 form a linearly dependent set if this system has a nontrivial solution. or a linearly independent set if it has only the trivial solution.

$$A = \begin{bmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

$$|A| = 1(6+2) - 5(-2-6) + 3(2-18)$$

$$|A| = 8 - 5(-8) + 3(-16)$$

$$|A| = 8 + 40 - 48$$

$$|A| = 48 - 48$$

$$\boxed{|A| = 0}$$ Nontrivial