Md Shoaib Anwarr
2301010067.

<u>Assignments</u>

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ $\quad R_4 \to R_4 - 4R_1$ $\quad R_4 \to R_4 - R_3$ $\quad R_4 \to R_4 - R_2$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ no of non zero rows

rank = 3

2) $T: W \to P_2$

$T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

find the rank and nullity of T

$\to T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then.

$T(A) = (a-b) + (b-c)x + (c-a)x^2$

$= a - bx + c(x^2 - x + 1)$

∴ The image of T is the set of all polynomials of degree at most 2, donated as P_2

Rank of T :

The rank T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1, and x^2) the rank of T is 3.

The Null space of symmetric matrix

$$T(\lambda) = 0$$

This Leads to the system of Equation

$a-b=0$ $\quad$ $b-c=0$ $\quad$ $c-a=0$

$\therefore a=b=c$

$\therefore$ T is the set of symmetric matrices of the form

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scaler

$\therefore$ Dimension $=1$
(using only t)

$\therefore$ rank T is 3
The nullity of T is 1

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow \lambda^{-1} = \frac{1}{4-1} \begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & +\frac{1}{3} \\ \frac{1}{3} & 2/3 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} = \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$

$a^2-b^2 = (a-b)(a+b)$

$\left(\frac{1}{3}-\lambda\right)(1-\lambda) = 0$

$\boxed{\text{Eigenvalues } \lambda = 1, \frac{1}{3}}$

$\lambda = 1$

$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$= x\frac{1}{3} + y\left(\frac{1}{3}\right) = 0$

$\boxed{\text{Eigen vector} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$x = y = K$

$\lambda = \frac{1}{3} \quad \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$x = -y \quad$ Eigen vector $= \begin{bmatrix} K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$Z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

$Z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} \qquad a^2 - b^2 = (a-b)(a+b)$

$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$

$= 5-\lambda\ (7-\lambda) = 0$

$\boxed{\text{Eigen values } \lambda = 5, 7}$

$\lambda = 5 \qquad \boxed{\text{Eigen vector} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0$

$x = y \qquad K \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\text{Eigen vector} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

$\lambda = 7$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad x = -y \qquad K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Md Shoaib Anwar

4) $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$

$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$

$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$

Iteraction - 1 :-

$z = y = 0$

$x(1) = \frac{7.85}{3} = 2.61$

$z = 0$

$y = \frac{1}{7}\left(-19.3 - 0.1\left(\frac{7.85}{3}\right)\right) = 2.79$

$y(1) = 2.79$

$z = \frac{1}{10}[71.4 - 0.3(2.61) + 0.2(2.79)]$

$z = \frac{1}{10}(71.4 - 0.783 + 0.588)$

$z(1) = 7.1175$

Iteraction-2 :-

$x(2) = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$

$= 2.9255$

$y(2) = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$

$= 3.0123$

Shoaib

$$Z(2) = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10} = 7.0132$$

Iteraction - 3

$$x^{(3)} = \frac{7.85 \cdot 0.1(2.9255) - 0.2(7.0132)}{3} \simeq 3.0032$$

$$y^{(3)} = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} = 3.001$$

$$Z^{(3)} = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$$

$x = 3.0032$, $y = 3.0001$, $Z = 7.000$

5) Define consistent in consistent

$x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$

Sol^n) $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$ after performing row reduction we obtained Ehelon form.

$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ This corresponds to the system

$x + 3y + 2Z = 0$

$-7y - z = 0$

Now Lets express the variable in terms of parameters

Let $y = t$,

$x = -3t$, $z = -7t$

so, the system has infinitely many solution give by

$x=-3t$, $y=t$, $z=-7t$

The system is consistent and dependent

6) $T: P_2 \to P_2$ is Linear transformation

$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

$\rightarrow T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$

is a linear transformation, we need to check two properties

1) Additivity $(T(u+v) = T(u)+T(v))$

2) homogenity of degree 1:

$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

1) $T(u+v) = T(a_1+b_1x+c_1)+(a_2+b_2x+c_2)$

$= T(a_1+a_2)+(b_1+b_2)x+(c_1+c_2)$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a^2+1)+(b_2+1)x+(c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

so function is additive

$\therefore$ Homogenity of Degree 1:

$T(ku) = T(k(a+bx+c))$

$= T(ka+kbx+kc) = (ka+1)+(kb+1)x+(kc+1)x^2$

$= k(a+1)+k(b+1)x+k(c+1)x^2$

$= kT(a+bx+c)$

Shoaib Anwar

So, the function is homogenous of degree 1

$\therefore$ It indeed Linear transformed

7) $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determine subspace spanned by S.

$\rightarrow$ $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

can be arranged as a matrix.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now lets perform row reduction to obtained the Echelon form.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R$.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \rightarrow R_3 + \frac{9}{5}R_2$ $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Third row of zeros indicates that the vectors in S are linearly dependent

for basis of the subspace spanned by S

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

$(1,3,2)$ and $(0,-5,5)$ these vectors form a basis for the subspace spanned by S.

$\therefore$ Dimension of subspace spanned by $S = 2$

$\therefore$ set S is not a basis of R^3 because the row reduced form has a row of zeros

Shoaib Anwar

$\therefore$ The basis for the subspace spanned by S is $\{(1,3,-2);(0,-5,5)\}$

$\therefore$ The dimension of the subspace is 2.

8) Using Jacobis method (perform 3 iterations) solve.

$3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$

with initial values. $x_0 = 1$, $y_0 = 1$, $z_0 = 1$

$\rightarrow 3x - 6y + 2z = 23$

$-4x + y - z = -15$.

$x - 3y + z^{(7)} = 16$

with initial values

$x = 1$, $y = 1$, $z = 1$

Iteration -1

$$x(1) = \frac{23 + 6y(0) - 2(z)(0)}{3} \simeq 9.0$$

$$y(1) = \frac{-15 + 4(x)(0) + z^{(10)}}{1} \simeq -9.0$$

$$z(1) = \frac{16 - x(0) - 3(y^{(1)})}{7} = 2.0$$

Iteration -2

$$x(2) = \frac{23 + 6y(1) - 2(z)(1)}{3} = 5.0$$

$$y(2) = \frac{-15 + 4(x)(1) + z^{(1)}}{1} = -5.0$$

$$z^{(2)} = \frac{16 - x^{(1)} + 3y^{(1)}}{7} \simeq 3.0$$

Shoaib Anwore.

Iteration - 3

$$x^{(3)} = \frac{23 + 6y^{(2)} - 2z^{(2)}}{3} = 6.0$$

$$y^{(3)} = \frac{-15 + 4x^{(2)} + z^{(2)}}{1} \simeq -6.0$$

$$z^{(3)} = \frac{16 - x^{(2)} + 3y^{(2)}}{7} = \simeq 2.0$$