Assignment - II

1) Big Omega Notation Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

solution:

To prove that exist Positive Constant on to such that for all $n > n_0$ to show $g(n)$ $= n^3 + 2n^2 + nm$ is $\Omega(n^3)$ $g(n) \geq (\cdot n)$

$g(n) = n^3 + 2n^2 + 4n$ we can chose $c=1$ and

$n_0 = 1$ those for all $n \geq 1$

$$n^3 + 2n^2 + 4n \geq 1 \cdot n^3$$

$$g(n) = n^3 + 2n^2 + 4n \text{ is } \Omega(n^3)$$

2) Big theta Notation. Determine whether $g(n) = 4n^2 + 3n$ is $O(n^2)$ or not.

Solution:

To determine whether $f(n) = 4n^2 + 3n$ is $O(n^2)$ we need to check if these exist Positive constant c_1 c_2 and n_0 such that for all $n \geq n_0$

$$n^2 \leq h(n) \leq c_2 \cdot n^2$$

$$h(n) - h(n) \leq c_2 \cdot n^2$$

$$h(n) = nn^2 + 3n^2 \leq nn^2 + 3n^2 = 7n^2$$

$$c_2 = 1$$

$h(n) = 4n^2 + 3n \, 7 \, 4n^2$

$c_1 = 4$

$4n^2 \leq nn^2 + 3n \leq 7n^2$

$4(n) - 4n^2 + 3n$ is $O(n^2)$

3. Let $f(n) = n^3 - 2n^2 + 2n$ and $g(n) - n^2$ show that $f(n) = \Omega\, g(n)$ is true or false Justify your answer?

$f(n) \geq c \cdot g(n)$

Substiting $f(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq (c - n^2)$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + (n^2 \geq 0$

$n^3 - (c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0 \; (n^3 \geq 0)$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$

$f(n) = n^3 - 2n^2 + n$ is $\Omega\,(g(n) = \Omega(-n^2))$

$f(n) = \Omega\, g((n))$ is True

Determine whether $h(n) = n\log n + n$ is $O(n\log n)$. Prove a rigorous proof for your conclusion

$$c_1 n\log n < h(n) \leq c_2 n\log n$$

$$h(n) \leq c_2 n\log n$$

$$h(n) \leq n\log n + n$$

$$n\log n + n \leq c_2 n\log n$$

$$1 + \frac{n}{n\log n} \leq 2$$

$$1 + \frac{1}{\log n} \leq 2$$

$h(n)$ is $O(n\log n)$

$$h(n) \geq C_1 A_1$$

$$h(n) = n\log n + n$$

$$n\log n + n \geq c_1 n\log n$$

$$1 + \frac{n}{n\log n} \geq c_1 \qquad \frac{1}{\log n} \geq 0$$

$$1 + \frac{1}{\log n} \geq c_1 \qquad h(n) \text{ is } \Omega(n\log n)$$

$h(n) = n\log n + n$ is $O(n\log n)$

$$1 + \frac{1}{\log n} \geq 1$$

5. Solve the following recurrence relations and find the order of growth of solutions

$T(n) = \cancel{2T}\,4T\left(n/2\right) + n^2, \quad T(1) = 1$

$T(n) = 4T\left(n/2\right) + n^2, \quad T(1) = 1$

$T(n) = 4T\left(n/2\right) + f(n)$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$T(n) = aT\left(n/b\right) + f(n)$

$f(n) = O\left(n^{\log_b a - c}\right)$

$f(n) = \Theta\left(n^{\log_b a}\right) \quad T(n) = \Theta\left(n^{\log_b a} \log n\right)$

$f(n) = \Omega\left(n^{\log_b a + E}\right)$ then $T(n) = f(n)$

$\log_b a$:

$\log_b a = \log_2 n = 2$

$f(n) = n^2 = O(n^2)$

$f(n) = O(n^2) = O\left(n^{\log_b a}\right)$

$T(n) = n + (n/2) + n^2$

$T(n) = O\left(n^{\log_b a}(\lg n)\right) = O(n^2 \log n)$

$T(n) = nT(n/2) + n^2$ with $T(1) = 1$ is $O(n^2 \log n)$