1. Solve the following recurrence relations

(a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

$x(1) = 0$

$x(n) = x(n-1) + 5$

$x(1) = 0$

$x(2) = x(1) + 5 = 0 + 5 = 5$

$x(3) = x(2) + 5 = 5 + 5 = 10$

$x(4) = x(3) + 5 = 10 + 5 = 15$

$x(5) = x(4) + 5 = 15 + 5 = 20$

$x(n) = 5(n-1)$

$n = 1,\ x(1) = 0 = 5(1-1)$

$x(k) = 5(k-1) \qquad k \geq 1$

$n = k + 1$

$x(k+1) = x(k) + 5$

$= 5(k-1) + 5 = 5k$

$x(n) = 5(n-1)$

$x(n) = 5n - 5$

(b) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

$x(1) = 4$

$x(n) = 3x(n-1)$

$x(1) = 4$

$x(2) = 3x(1) = 3.4 = 12$

$x(3) = 3x(2) = 3.12 = 36$

$x(4) = 3x(3) = 3.36 = 108$

$x(n) = 3x(n-1)$

$x(n) = 4 \cdot 3^{n-1}$

$n = 1 \quad x(1) = 4 = 4.3^{1-1} = 4$

$x(k) = 4 \cdot 3^{k-1} \quad k \geq 1$

$n = k+1$

$x(k+1) = 3x(k) = 3.4.3^{k-1} = 4 \cdot 3^{k}$

$x(n) = 4 \cdot 3^{n-1}$

d) $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$ (solve for $n = 3^k$)

$n = 3^k$

$x(1) = 1$

$x(3^k) = x(3^{k-1}) + 1$

$x(3^0) = x(1) = 1$

$x(3^1) = x(3^0) + 1 = 1 + 1 = 2$

$x(3^2) = x(3^1) + 1 = 2 + 1 = 3$

$x(3^3) = x(3^2) + 1 = 3 + 1 = 4$

$x(3^4) = x(3^2) + 1 = 4 + 1 = 5$

$x(3^k) = x(3^{k-1}) + 1$

$x(3^k) = k + 1$

$k = 0$, $x(3^0) = x(1) = 1 = 0 + 1$

Assum $x(3^j) = j + 1$ $\quad j \geq 0$

$k = j + 1$

$x(3^{j+1}) = x(3^j) + 1 = (j+1) + 1 = j + 2$

$x(3^k) = k + 1$

Evalutte the following recurrences completely

i) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$

$$T(n) = T(n/2) + 1$$

$n = 2^k$

$$T(n) = T(n/2) + 1$$
$$= T(n/4) + 1 + 1$$
$$= T(n/8) + 1 + 1$$
$$= T(n/2^i) + i$$

$n/2^i = 1 \quad 2^i = n \quad i = \log_2 n$

$T(1) = T(1) = 0 \quad$ assume $T(1) = 0$

$T(n) = T(1) + \log_2 n = 0 + \log_2 n = \log_2 n$

$$T(n) = \log_2 n$$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a \geq 1$ $b > 1$ $f(n)$ is asymptocialy

$a = 2$, $b = 3$, $f(n) = cn$

$f(n)$ with $n^{\log_b a}$

$\log_b a = \log_3 2$

$f(n) = O(n^c)$ where $c < \log_b a$ Then $T(n) = \Theta(n^{\log_b a})$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^c)$ $c > \log_b a$ and $af\left(\frac{n}{b}\right) \leq kf(n)$

for som $k < 1$

$T(n) = O(f(n))$

$f(n) = cn = O(n)$

$\log_b a = \log_3 2$.

$T(n) = \Theta(n)$

Conside the following recursion algorithm

MIN1[A[0 - - - n-1]])

if $n = 1$ return A[0]

Else temp = Min1[A[0 -- n-2]]

if temp <= A[n-1] return temp

Else

Return A[n-1]

(a) what does algorithm compute?

$n = 1$

it return the single element A[0]

A[0, .. n-2] case and then compares the value to A[n-1] returning the smaller of two.

(b) Setup a recurrence relation for the algorithms basic operation count and solve it

$$T(1) = 0$$

$$T(n) = T(n-1) + 1$$

$$= (T(n-2) + 1) + 1$$

$$(T(n-3) + 1) + 1) + 1$$

$$= T(1) + (n-1)$$

$$= 0 + (n-1)$$

$$= n-1$$

$$T(n) = n - 1$$

$$T(n) = T(n-1) + 1$$

$$T(n) = n - 1$$

4. Analyse the order of growth

(i) $f(n) = 2n^2 + 5$ and $g(n) = 7n$

use $\Omega(g(n))$ notation

$$f(n) = 2n^2 + 5$$

$$g(n) = 7n$$

$$f(n) = \Omega(g(n))$$

$$2n^2 + 5 \geq c$$

$$2n^2 + 5 \geq 7$$

$$2n + \frac{5}{n} \geq 7c$$

$$2n \geq 7c$$

$$c \leq \frac{2n}{7}$$

$$c = 1$$

$$2n \geq 7$$

$$n \geq \frac{7}{2}$$

$$2n^2 + 5 \geq 7n \quad \boxed{c=1}$$

$f(n) = 2n^2 + 5 = \Omega(7n)$

$f(n) = \Omega(g(n))$. $g(n) = 7n$