My Student number is 1004306985:

Student #	1	0	0	4	3	0	6	9	8	5
Letter				T	U	V	W	X	Y	Z

Material Parameters

Face Material	Density ρ_f	1600 + 30(43) = 2890 kg/m^3
	Young's Modulus E_f	40 + 06 = 46 GPa
	Compressive Strength σ_f	200 + 100(9) = 1100 MPa
Core Material	Density ρ_c	20 + 5(85) = 445 kg/m^3
	Compressive Strength σ_c	$0.5 + 6.5(0.85)^{3/2} = 5.593$ MPa
	Shear Strength $\boldsymbol{\tau}_c$	$0.5 + 4.5(0.85)^{3/2} = 4.026$ MPa

First thing we need to do is to plot a failure mechanism map. We'll do this by plotting the boundary lines first, where we set the failure loads equal to each other, and solve it as a function of c-bar (core thickness/ length, or c/L) , with units t-bar (face thickness/core thickness, or t/c).

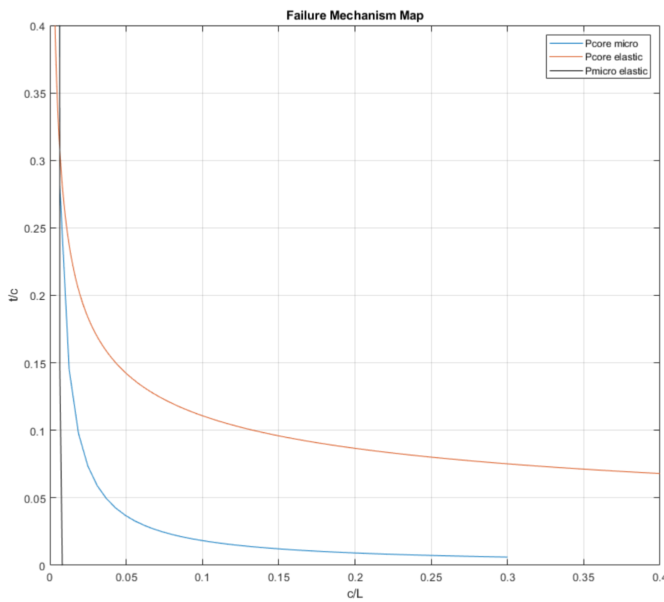

Figure 1: Boundary plot

Note that you can find the equation for the 3 boundaries plotted above in Appendix A1 . Because the way these equations are solved, only considering 2 of the 3 failure mechanisms at any one time, the lines plotted show “real” and “fictitious” boundaries. The fictitious boundary is where the boundary doesn’t make sense, as it’s in a failure region which is dominated by another failure mechanism that isn’t being considered in the boundary line. To account for this, we’ll make a grid of points on the x and y axis going from 0 to 0.3, and evaluate which is the active failure mechanism. This requires converting the equations of the loads of the failure mechanisms as a function of t-bar and c-bar, evaluating within the test grid, picking the lowest one, and assigning an integer to an empty matrix. When the matrix is filled, we can contour plot it onto the boundary plot, and see the active failure regions:

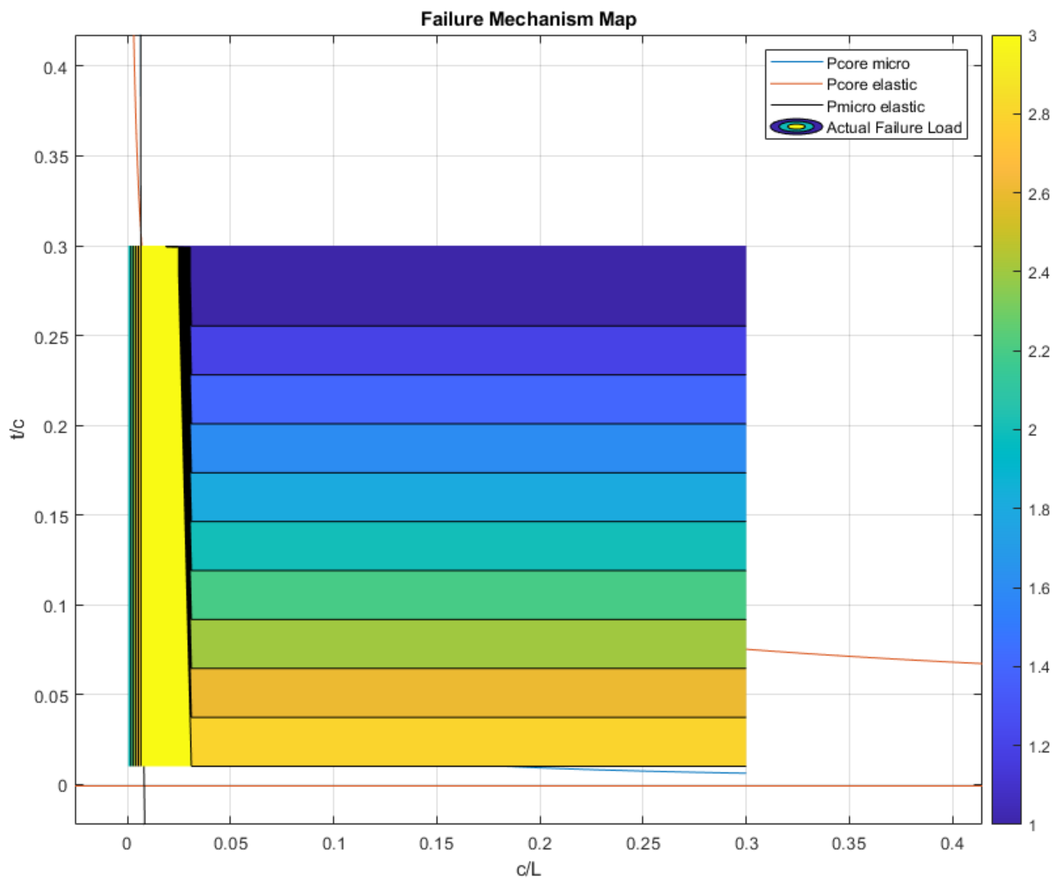

Figure 2: Contour filled plot

Note in the above figure the contourf command was used for a contour filled plot. While it blocks a lot of the boundary lines, it shows the active failure mechanisms. Here the dark blue (level 1), is the core shear region, the teal (level 2) is the microbuckling region, and the yellow (level 3) is the elastic indentation region. Note that in the $0.05 < x < 0.3$ region, there’s no microbuckling, despite the teal strip in between the dark blue and yellow region. This is just a quirk of how MATLAB plots contours. Anyway, if we just use the normal contour plot to get lines instead of colored regions, we can annotate the plot, pick out the real boundaries, and have a valid failure mechanism map:

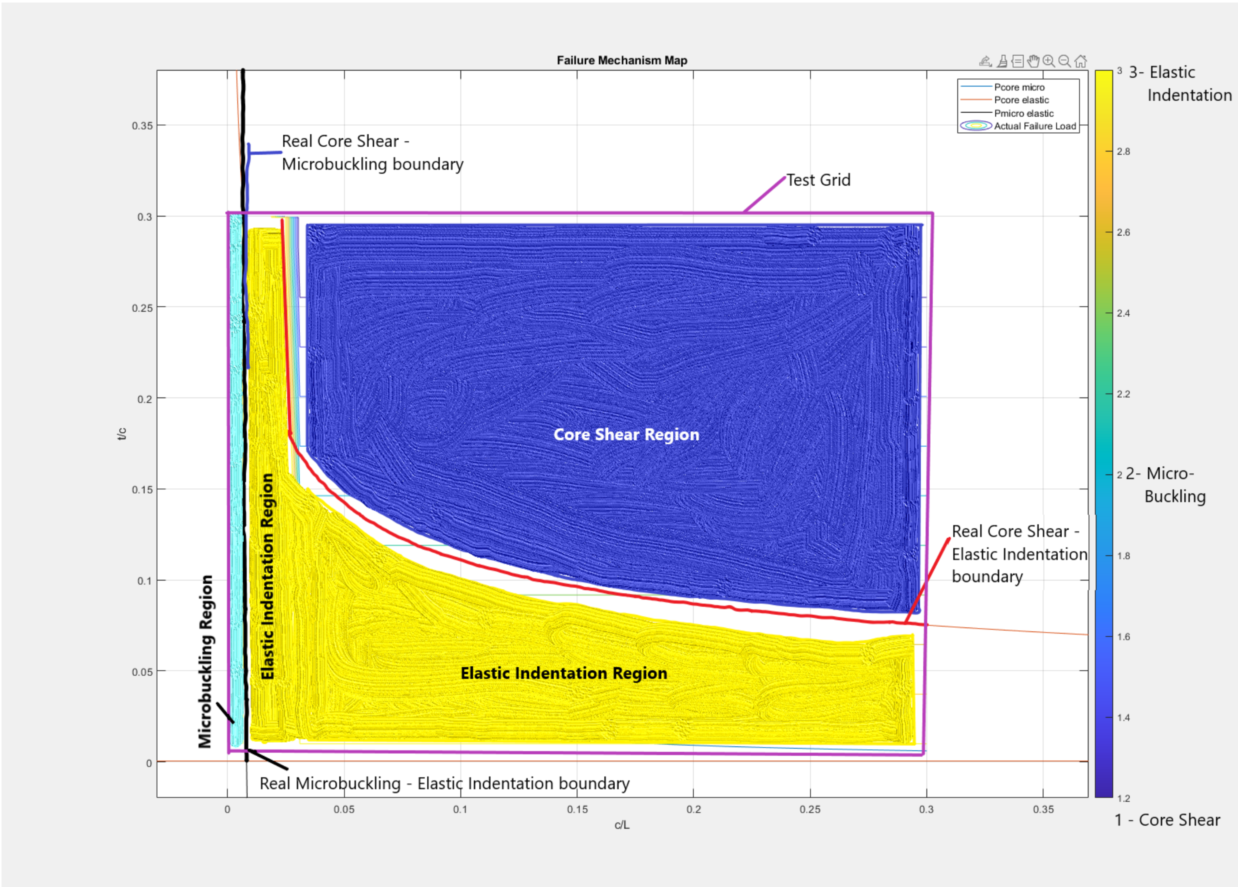

Figure 3: Annotated failure mechanism map

Let's talk through the above figure. The purple line outlines the test grid, which contains 50 points in the x and y direction, for a total of 2500 points. Starting at the bottom left, we see the microbuckling region in teal. The black line from figure 1 has been bolded, and we see that the real microbuckling-elastic indentation boundary is pretty close, at least for our test grid. What we don't see is the boundary from the core shear - microbuckling. This is because the test grid is too small here, it exists far in the top left of the graph, but we'll ignore it as it's a small sliver even if the test grid were to be expanded. Lastly, the bolded red line shows the real core shear and elastic indentation boundary. Notice that in figure 2, the contours in this region go on as straight strips, but they start out with a small slope, which looks black. This is the boundary going from steep to more level, and it would be more apparent if there were even more points in the test grid for a finer resolution. Nevertheless, the point of this annotation is to show the real boundaries and have a valid failure mechanism map, as programming languages like MATLAB can't show all this.

Now we need to plot the optimal trajectory, using the non dimensional performance indices:

$$\hat{M} = \frac{M}{bL^2\rho_f}; \quad \hat{P} = \frac{P}{bL\sigma_f};$$

Where M is the mass of the beam, and P is the load of one of the active failure mechanisms. These 2 variables are linked by the condition for optimality:

$$\nabla\hat{M} = \lambda\nabla\hat{P}$$

Here M(hat) is a function of both t-bar and c-bar, as is P(hat). When using the condition for optimality we get 2 equations with 3 unknowns (t-bar, c-bar, and **λ**). We can use one equation to solve for **λ**, and plug that into the other equation so we only have t-bar and c-bar. We can use this to plot the trajectory of the optimal design for each failure load. All of this can be found in Appendix A2. Note that we know from the lecture slides that the trajectory doesn't go in the core shear region, but rather follows the elastic indentation and core shear boundary. Therefore, we won't plot the core shear trajectory.

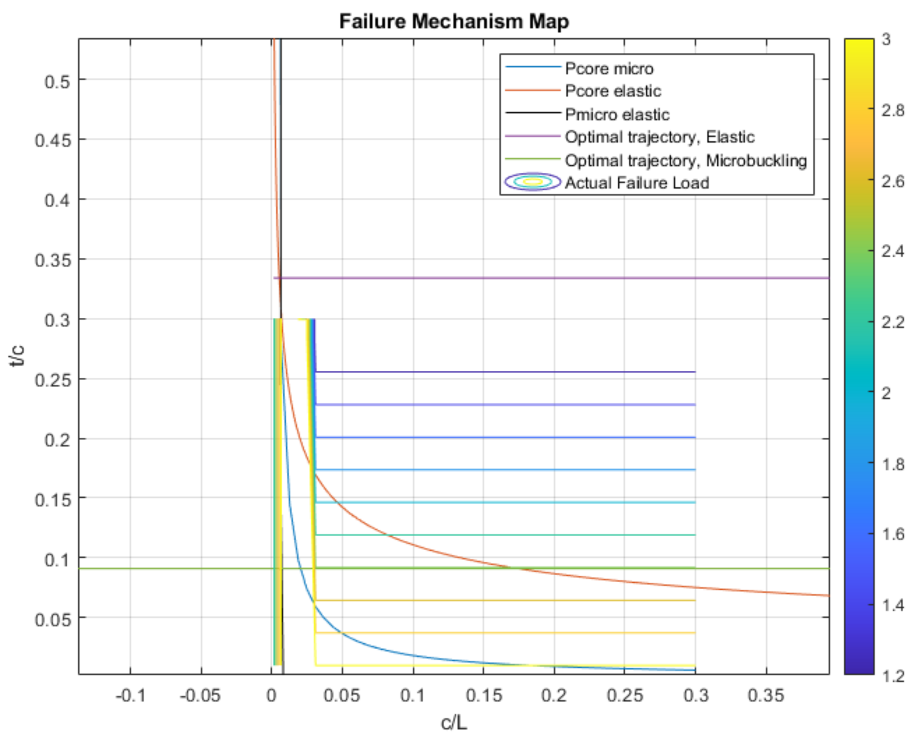

Figure 4: Optimal trajectory

In the above figure, we see a jump from the trajectory of the microbuckling (green), and the trajectory of the elastic indentation (purple). Unfortunately, for our material parameters, the elastic indentation optimal trajectory is very high, thus it has a very small travel horizontally from the microbuckling boundary to the core shear boundary. Perhaps the following figure will describe it in more detail.

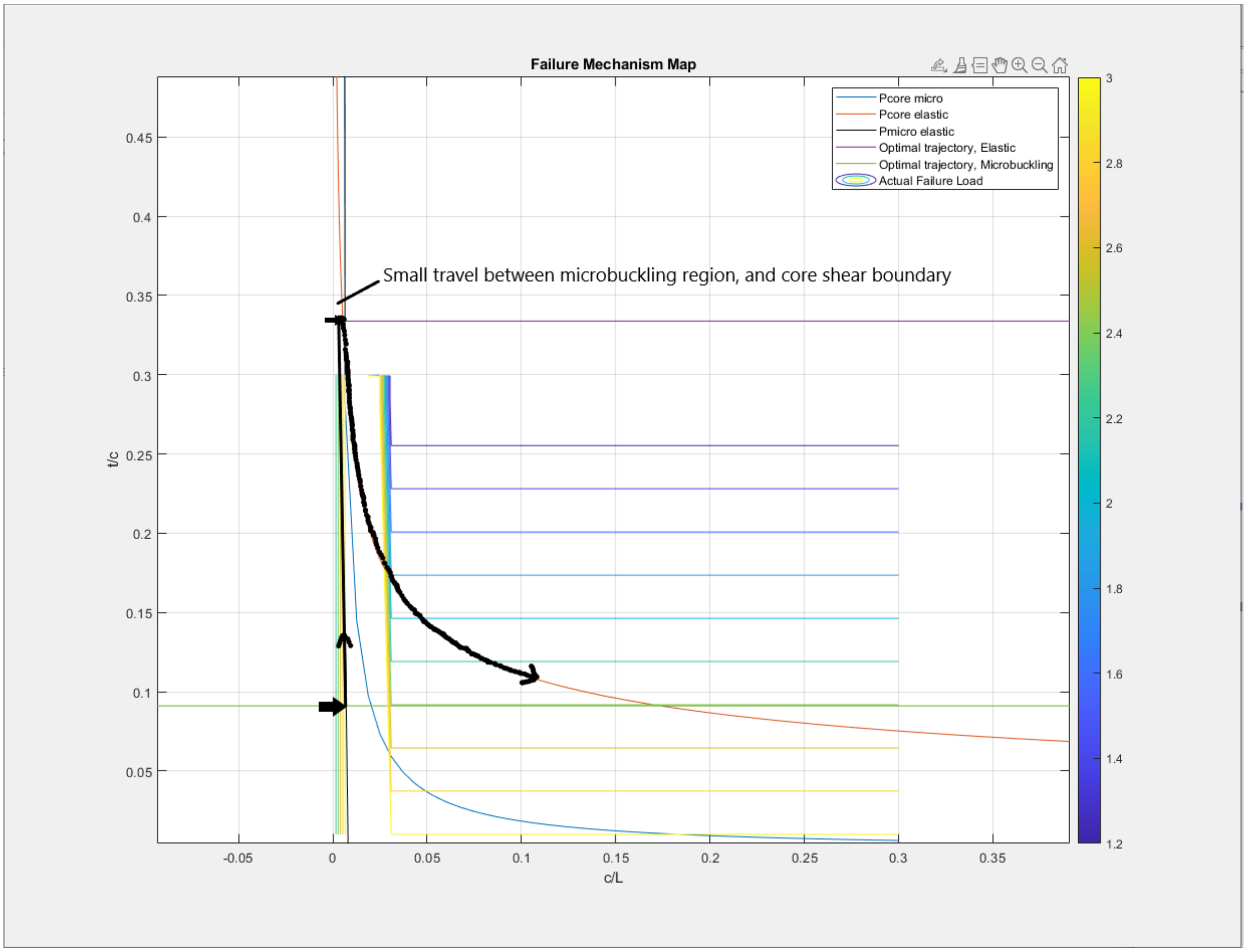

Figure 5: Annotated Optimal trajectory

In the above figure we can see the projected optimal trajectory (black line), as well as the small travel from the microbuckling and core shear boundary as mentioned before.

There's also the plot of the minimum mass as a function of the applied load, or M(hat) vs P(hat). We found the formulas for these when plotting the optimal trajectory (Appendix A2), so we know both are a function of t-bar and c-bar. Thus we can plot them accordingly:

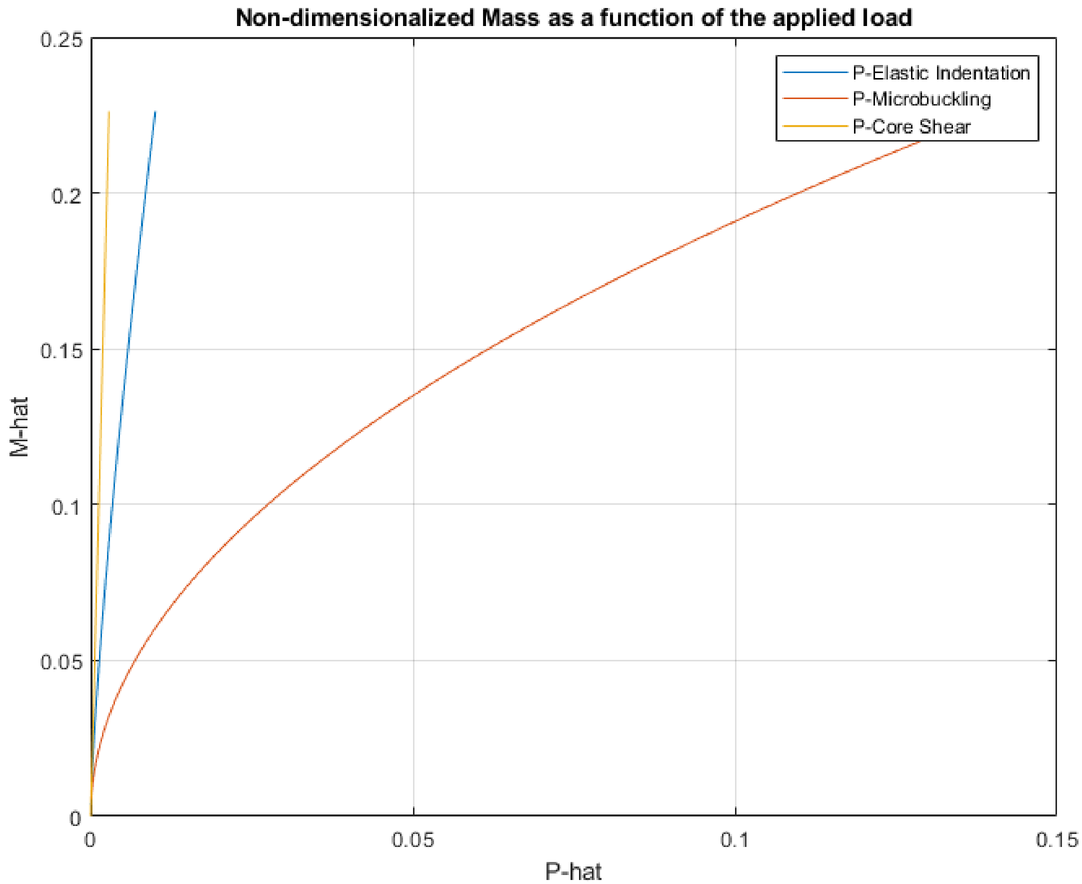

Figure 6: Non-dimensionalized mass as a function of the applied load

Note that there's a slight difference between M(hat), and M(hat), minimum. To get the latter, you'd need to isolate a variable in the corresponding P(hat), input it into the M(hat), isolate the remaining variable, set it to zero, and simplify. This doesn't yield that much a different result from Figure 6, and the core shear cannot be plotted, but because the wording of the project document is a little vague, we're choosing to present both. The formulas for this can be found at the end of Appendix A2.

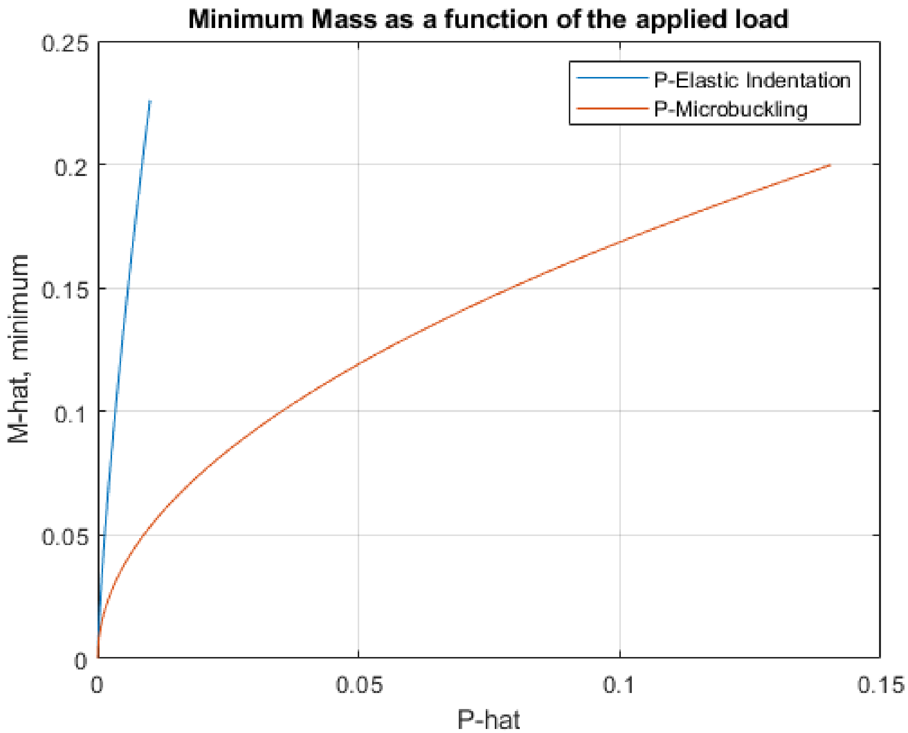

Figure 7: Minimum mass as a function of the applied load

Lastly, the optimal design has to be calculated if L = 2.5m, b =0.2m, and P = 75000 N. This was already done analytically, and can be found in Appendix A3. The minimum mass is 22.48 kg, where t = 0.004215m, and c = 0.04632m, and the sandwich beam fails due to microbuckling.

Matlab Code

```
clc, clearvars, close all

%face properties
pf = 2890; %kg/m^3
Ef = 46000E6; %Pa or N/m^2
sigmaf = 1100E6; %Pa or N/m^2
%core propetries
pc = 445; %kg/m^3 %Pa or N/m^2
sigmac = 5.593E6; %Pa or N/m^2
tauc = 4.026E6; %Pa or N/m^2

b=0.2;
t=linspace(0.00001,0.225,50);
L = 2.5;
c=linspace(0.001,0.75,50);

d = t + c;
%Failure loads

P_core = 2.*b.*d.*tauc;
P_indentation = b.*t.*((pi^2).*d.*Ef*(sigmac^2)/3*L).^(1/3);
P_microbuckling = 4*b.*d.*t*sigmaf/L;


y = t./c; %tbar
x = c./L; %cbar


[X,Y] = meshgrid(x,y);


%iterate through X and Y, state loads in terms of X and Y, and check P values
for i=1:50 %these are the contour green lines
        for j=1:50
        P_core_terms = L*(2*b*tauc.*(X(i,j)).*(Y(i,j)) + 2*b*tauc.*(X(i,j)));
        aa(i,j) = P_core_terms;
        P_micro_terms = L*(4*b*sigmaf.*(X(i,j).^2).*(Y(i,j).^2) + 4*b*sigmaf.*(X(i,j).^2).*Y(i,j));
        bb(i,j) = P_micro_terms;
        P_elastic_terms = ((L^3)*(((b^3)*(pi^2)*Ef*(sigmac^2))/3).*(Y(i,j).^4).*(X(i,j).^4) +
(((b^3)*(pi^2)*Ef*(sigmac^2))/3).*(X(i,j).^4).*(Y(i,j).^3)).^(1/3);
        cc(i,j) = P_elastic_terms;

        if (aa(i,j) < bb(i,j)) && (aa(i,j) < cc(i,j)) %if core shear wins

        p_win(i,j) = 1; %pwin is the matrix that collects data on the active failure mechanism
        else if (bb(i,j) < aa(i,j)) && (bb(i,j) < cc(i,j)) %if microbuckling wins
        p_win(i,j) = 2;
        else if (cc(i,j) < aa(i,j)) && (cc(i,j) < bb(i,j)) %if elastic indentation wins
        p_win(i,j) =3;
        end
        end
        end
        end
end


P_core_micro = (2*b*tauc)./(4*b*sigmaf.*x)
P_micro_elastic = ((((pi^2)*Ef*(sigmac^2))./(192*(sigmaf^3).*(x.^2))).^0.5) -1;


Mhat = 2.*x.*(y) + (pc/pf).*x;

Phat_elastic = ((((pi^2)*(sigmac^2)*Ef)/(3*(sigmaf^3))).^(1/3)).*(x.^(4/3)).*y.*((y+1).^(1/3));
Phat_microbuckling = 4.*y.*(y+1).*(x.^2);
Phat_core_shear = 2*(tauc/sigmaf).*(y+1).*x;
```

```
Mhat_min_microbuckling = ((pc/pf)*(2- (pc/pf)).*Phat_microbuckling).^(1/2);
Mhat_min_elastic = 4*((((pc/pf)*(2- (pc/pf))^3)/ (9*(pi^2)*((sigmac/sigmaf)^2)*(Ef/sigmaf) ) )^(1/4)).*(Phat_elastic.^(3/4));

lambda_elastic = ((2.*x.*((y+1).^(2/3)) ) / (0.20352.*(x.^(4/3)).*(y+0.75)));
z4 = (0.20352*(x.^(1/3)).*y.*((y+1).^(1/3)));

lambda_micro =  ( (2.*x)/ (4.*(x.^2).*(2.*y +1)) );
z5 = 8.*x.*y.*(y+1);



figure (1)
plot(x,P_core_micro)
grid on
hold on
z1 = fimplicit(@(x,y)  ((pi^2)*Ef*(sigmac^2)/(24*(tauc^3))).*(y.^2).*x - 1./y -2 - y) %core elastic, plotted as an implicit function
hold on
plot(x,P_micro_elastic,'k')
hold on
optimal_elastic = fimplicit(@(x,y) 2.*y + 0.154 -  ((2.*x.*((y+1).^(2/3)) ) ./ (0.20352*(x.^(4/3)).*(y+0.75))) .*(0.20352*(x.^(1/3)).*y.*((y+1).^(1/3)))  )
hold on
optimal_micro = fimplicit(@(x,y) 2.*y + 0.154 -  ( (2.*x)./ (4.*(x.^2).*(2.*y +1)) ).*8.*x.*y.*(y+1)  )
hold on

contour(X,Y,p_win) %contour of our test grid, use contourf for a filled plot

legend('Pcore micro','Pcore elastic','Pmicro elastic','Optimal trajectory, Elastic','Optimal trajectory, Microbuckling','Actual Failure Load')
xlabel('c/L')
ylabel('t/c')
title('Failure Mechanism Map')
colorbar;
xlim([0 0.4])
ylim([0 0.4])

figure (2)
grid on
plot(Phat_elastic,Mhat) % to change to Mhat, minimum, simply change the y axis here to the Mhat minimum
hold on
plot(Phat_microbuckling,Mhat)
hold on
plot(Phat_core_shear,Mhat)
legend('P-Elastic Indentation', 'P-Microbuckling', 'P-Core Shear')
xlabel('P-hat')
ylabel('M-hat')
title('Non-dimensionalized Mass as a function of the applied load')
```

Appendix A1

$\rightarrow P_{micro} = P_{elastic}$

$$\frac{4b\,d\,\cancel{t}\,\sigma_f}{L} = \cancel{b}\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3} = \left(\frac{4d\sigma_f}{L}\right)^3 = \frac{\pi^2 \cancel{d} E_f \sigma_c^2}{3L} = \frac{64 d^{\cancel{3}\,2} \sigma_f^3}{L^3}$$

$$\frac{d^2}{L^2} = \frac{\pi^2 E_f \sigma_c^2}{3L}\,\frac{1}{64\sigma_f^3} = \frac{c^2}{L^2} + \frac{2ct}{L^2} + \frac{t^2}{L^2} = \frac{\pi^2 E_f \sigma_c^2}{64\cdot 3\cdot \sigma_f^3} = cbar^2 + 2(cbar)(cbar)tbar + tbar^2 \cdot cbar^2$$

$$\frac{\pi^2 E_f \sigma_c^2}{64\cdot 3\cdot \sigma_f^3} = cbar^2\left(1 + 2tbar + tbar^2\right)$$

$$\frac{\pi^2 E_f \sigma_c^2}{64\cdot 3\,\sigma_f^3\,(cbar)^2} = (tbar + 1)^2 \rightarrow \sqrt{\frac{\pi^2 E_f \sigma_c^2}{64\cdot 3\cdot \sigma_f^3 (cbar)^2}} - 1 = tbar$$

$\rightarrow P_{micro} = P_{core\ shear}$

$$2bd\tau_c = \frac{4bdt\sigma_f}{L} \rightarrow 2b\tau_c = 4b\sigma_f\left(\frac{t}{L}\right) = 4b\sigma_f\left(\frac{t}{c}\right)\left(\frac{c}{L}\right)$$

$$tbar = \frac{2b\tau_c}{4b\sigma_f(cbar)} \leftarrow \frac{2b\tau_c}{4b\sigma_f(c/L)} = \frac{t}{c}$$

$P_{core\ shear} = P_{elastic\ indentation}$

$$2bd\tau_c = bt\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3}$$ → cube both sides to get rid of this.

$$8d^2\tau_c^3 = \frac{t^3\pi^2 E_f \sigma_c^2}{3L}$$

$$d^2 = \frac{t^3\pi^2 E_f \sigma_c^2}{24L\tau_c^3} = c^2 + 2ct + t^2$$

$$L\left(\frac{\pi^2 E_f \sigma_c^2}{24L\tau_c^3}\right) = \left(\frac{c^2}{t^3} + \frac{2c}{t^2} + \frac{1}{t}\right)L = \frac{L}{t}\frac{c^2}{t^2} + 2\frac{c}{t}\frac{L}{t} + \frac{L}{t}$$

$$\text{cbar}\left(\frac{1}{(\text{tbar})(\text{cbar})(\text{tbar})^2} + 2\frac{1}{\text{tbar}}\frac{1}{\text{tbar}\cdot\text{cbar}} + \frac{1}{(\text{tbar}\cdot)(\text{cbar})}\right) = \left(\frac{\pi^2 E_f \sigma_f^2}{24\tau_c^3}\right) = A \quad \cdot\text{cbar}$$

$$\text{tbar}^3\left(\frac{1}{\text{tbar}^3} + \frac{2}{\text{tbar}^2} + \frac{1}{\text{tbar}}\right) = \left(A\cdot(\text{cbar})\right)\text{tbar}^3$$

$$1 + 2\text{tbar} + \text{tbar}^2 = A\cdot\text{cbar}\cdot\text{tbar}^3$$

$$\frac{(\text{tbar}+1)^2}{\text{tbar}^3} = A\cdot\text{cbar} = \frac{\pi^2 E_f \sigma_f^2}{24\tau_c^3}(\text{cbar})$$

↓

graph as an implicit function

↓

$$\frac{(\text{tbar}+1)^2}{\text{tbar}^3} - \frac{\pi^2 E_f \sigma_f^2}{24\tau_c^3}(\text{cbar}) = 0$$

Appendix A2

Trajectory of the optimal design

$$\hat{M} = \frac{2bLt\rho_f + bLc\rho_c}{bL^2\rho_f} = \frac{2t}{L} + \frac{c}{L}\frac{\rho_c}{\rho_f} = 2(tbar)(cbar) + \frac{\rho_c}{\rho_f}(cbar)$$

$$\hat{P}_{microbuckling} = \frac{\frac{4bdt\sigma_f}{L}}{bL\sigma_f} = \frac{4dt}{L^2} = \frac{4t^2}{L^2} + \frac{4tc}{L^2}$$

$$= 4(tbar)(cbar)^2(tbar+1)$$

$$\hat{P}_{core\ shear} = \frac{2bd\tau_c}{bL\sigma_f} = \frac{2d}{L}\frac{\tau_c}{\sigma_f} = 2\left(\frac{\tau_c}{\sigma_f}\right)(cbar)(tbar+1)$$

equal to ≈ 0.15264

$$\hat{P}_{elastic\ indentation} = \frac{bt\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3}}{bL\sigma_f} = \left(\frac{\pi^2\sigma_c^2 E_f}{3\sigma_f^3}\right)^{1/3} cbar^{4/3}\, tbar(tbar+1)^{1/3}$$

→ Let's first start w/ the elastic indentation region

$$\nabla\hat{M} = \lambda\nabla\hat{P}_{elastic}$$

$$\nabla\hat{M} = \begin{bmatrix} \frac{\partial}{\partial cbar}\left(2(tbar)(cbar) + \frac{\rho_c}{\rho_f}(cbar)\right) \\ \frac{\partial}{\partial tbar}\left(2(tbar)(cbar) + \frac{\rho_c}{\rho_f}(cbar)\right) \end{bmatrix} = \begin{bmatrix} 2tbar + \frac{\rho_c}{\rho_f} \\ 2cbar \end{bmatrix}$$

$$\nabla \hat{P}_{elastic} = \begin{bmatrix} \frac{\partial}{\partial cbar}\left(0.15264(cbar^{4/3})(tbar)(tbar+1)^{1/3}\right) \\ \frac{\partial}{\partial tbar}\left(0.15264(cbar^{4/3})(tbar)(tbar+1)^{1/3}\right) \end{bmatrix} = \begin{bmatrix} 0.20352(cbar)^{1/3} tbar(tbar+1)^{1/3} \\ \frac{0.20352(cbar)^{4/3}(tbar+0.75)}{(tbar+1)^{2/3}} \end{bmatrix}$$

So we have

$$\begin{bmatrix} 2tbar + \frac{P_c}{P_f} \\ 2cbar \end{bmatrix} = \lambda_0 \begin{bmatrix} 0.20352(cbar)^{1/3} tbar(tbar+1)^{1/3} \\ \frac{0.20352(cbar)^{4/3}(tbar+0.75)}{(tbar+1)^{2/3}} \end{bmatrix}$$

or

$$① \quad 2tbar + \frac{P_c}{P_f} = \lambda_0\left(0.20352(cbar)^{1/3}(tbar)(tbar+1)^{1/3}\right)$$

$$② \quad 2cbar = \lambda_0\left(\frac{0.20352(cbar)^{4/3}(tbar+0.75)}{(tbar+1)^{2/3}}\right)$$

$$\lambda_0 = \frac{2(cbar)(tbar+1)^{2/3}}{0.20352(cbar)^{4/3}(tbar+0.75)}$$

plug into ①

$$2tbar + \frac{P_c}{P_f} - \lambda_0\left(0.20352(cbar)^{1/3}(tbar)(tbar+1)^{1/3}\right) = 0$$

→ Equation to plot in failure mechanism map.

→ We know the condition for optimality does not work in the core shear region, instead it follows the core shear – elastic indentation boundary, so $\hat{P}_{core\ shear}$ is neglected.

→ Let's finish w/ the microbuckling region

$$\nabla \hat{M} = \nabla \hat{P}_{microbuckling}$$

We know $\nabla \hat{M} = \begin{bmatrix} 2tbar + \frac{\rho_c}{\rho_f} \\ 2cbar \end{bmatrix}$

$$\nabla P_{microbuckling} = \begin{bmatrix} \frac{\partial}{\partial(cbar)}\left(4(tbar)(cbar^2)(tbar+1)\right) \\ \frac{\partial}{\partial(tbar)}\left(4(tbar)(cbar^2)(tbar+1)\right) \end{bmatrix} = \begin{bmatrix} 8\,cbar\,tbar\,(tbar+1) \\ 4cbar^2(2tbar+1) \end{bmatrix}$$

So we have

$$\begin{bmatrix} 2tbar + \frac{\rho_c}{\rho_f} \\ 2cbar \end{bmatrix} = \lambda_0 \begin{bmatrix} 8(cbar)(tbar)(tbar+1) \\ 4\,cbar^2(2tbar+1) \end{bmatrix}$$

or

① $2tbar + \rho_c/\rho_f = \lambda_0 \left(8cbar(tbar)(tbar+1)\right)$

② $2cbar = \lambda_0 \left(4cbar^2(2tbar+1)\right)$

$$\lambda_0 = \frac{2\text{cbar}}{4\text{cbar}^2(2\text{tbar}+1)}$$

$$2\text{tbar} + \frac{\rho_c}{\rho_f} - \lambda_0\left(8(\text{cbar})(\text{tbar})(\text{tbar}+1)\right) = 0$$

Equation to plot in failure mechanism map.

Additionally, if we want to find $\hat{M}_{min}$, we'd need to isolate for a variable in $\hat{P}$, input it into $\hat{M}$, isolate the other variable, set it to zero, & simplify.

→ For example, $\hat{P}_{microbuckling} = 4(\text{tbar})(\text{cbar})^2(\text{tbar}+1)$

$$\text{cbar} = \sqrt{\frac{(\hat{P}_{micro})}{4(\text{tbar}+1)(\text{tbar})}}$$

Input into

$$\hat{M} = 2(\text{tbar})(\text{cbar}) + \frac{\rho_c}{\rho_f}(\text{cbar})$$ → set to zero & solve for tbar.

↓ That's your $\hat{M}_{min}$

$$\hat{M}_{min} = \left(\left(\frac{\rho_c}{\rho_f}\right)\left(2-\left(\frac{\rho_c}{\rho_f}\right)\right)\hat{P}_{micro}\right)^{1/2}$$

→ Similarily for elastic indentation

$$\hat{M}_{min} = 4\left(\frac{\left(\frac{\rho_c}{\rho_f}\right)\left(2-\left(\frac{\rho_c}{\rho_f}\right)\right)^3}{9\pi^2\left(\frac{\sigma_c}{\sigma_f}\right)^2\left(\frac{E_f}{\sigma_f}\right)}\right)\hat{P}_{elastic}^{3/4}$$

→ Because the optimal trajectory doesn't go into the core shear region, there's no $\hat{M}_{min}$ for it.

Appendix A3

Optimum Design for the example beam

Given: $L = 2.5m$, $b = 0.2m$, $P = 75000N$

Material parameters: $\rho_f = 2890\ kg/m^3$, $E_f = 4.6 \times 10^{10}\ Pa$, $\sigma_f = 1.1 \times 10^9\ Pa$

$\rho_c = 445\ kg/m^3$, $\sigma_c = 5.593 \times 10^6\ Pa$, $\tau_c = 4.026 \times 10^6\ Pa$

$$M = 2bLt\rho_f + bLc\rho_c = 2890t\left(\frac{kg}{m}\right) + 222.5c\left(\frac{kg}{m}\right) = f(c,t)$$

1) Elastic indentation

$$P = 75000N = bt\left(\frac{\pi^2 d E_f \sigma_c^2}{3L}\right)^{1/3} = b\left(\frac{\pi^2 E_f \sigma_c^2}{3L}\right)^{1/3} + (d)^{1/3}$$

$$75000 = 24{,}743{,}358.44 + (c+t)^{1/3}$$

$$3.031E\text{-}3 = t(t+c)^{1/3} \longrightarrow t(t+c)^{1/3} - 3.031E\text{-}3 = 0 = g(c,t)$$

Apply condition of optimality

$$\nabla M = \lambda \nabla P \quad \text{or} \quad \nabla f = \lambda \nabla g$$

$$\nabla f = \begin{bmatrix} \frac{\partial}{\partial c}(2890t + 222.5c) \\ \frac{\partial}{\partial t}(2890t + 222.5c) \end{bmatrix} = \begin{bmatrix} 222.5 \\ 2890 \end{bmatrix}$$

$$\nabla g = \begin{bmatrix} \frac{\partial}{\partial c}\left(t(t+c)^{1/3} - 3.01E\text{-}3\right) \\ \frac{\partial}{\partial t}\left(t(t+c)^{1/3} - 3.01E\text{-}3\right) \end{bmatrix} = \begin{bmatrix} \frac{t}{3(c+t)^{2/3}} \\ \frac{3c+4t}{3(c+t)^{2/3}} \end{bmatrix}$$

So we have

$$\begin{bmatrix} 222.5 \\ 2890 \end{bmatrix} = \lambda_0 \begin{bmatrix} \frac{t}{3(c+t)^{2/3}} \\ \frac{3c+4t}{3(c+t)^{2/3}} \end{bmatrix}$$

or

$$① \quad 222.5 = \lambda_0 \frac{t}{3(c+t)^{2/3}}$$

$$② \quad 2890 = \lambda_0 \left(\frac{3c+4t}{3(c+t)^{2/3}} \right)$$

$+$

$$③ \quad t(t+c)^{1/3} - 3.081E{-3} = 0$$

$c \approx 0.02737\ m$

$t = 0.009136\ m$

$\lambda = 32.464$

$\rightarrow M = f(c,t) = 2890t + 222.5c \approx 32.49\ kg$

② Core Shear

$\rightarrow$ The condition of optimality does not apply in the core shear region, so it's neglected here.

③ Microbuckling

$$P = 75000N = \frac{4bdt\sigma_f}{L} = 3.52 \times 10^8\, t(c+t) \left(\frac{N}{m^2}\right)$$

$$t(c+t) = 2.13E{-4} \quad \text{or} \quad t(c+t) - 2.13E{-4} = 0 = g(c,t)$$

Again $M = f(c,t) = 2890t + 222.5c$

$\rightarrow$ Apply condition of optimality again!

$\nabla M = \lambda \nabla p$ or $\nabla f = \lambda \nabla g$

we know from before $\nabla f = \begin{bmatrix} 222.5 \\ 2890 \end{bmatrix}$

$$\nabla g = \begin{bmatrix} \frac{\partial}{\partial c}\left(t(t+c) - 2.13E\text{-}4\right) \\ \frac{\partial}{\partial t}\left(t(t+c) - 2.13E\text{-}4\right) \end{bmatrix} = \begin{bmatrix} t \\ 2t+c \end{bmatrix}$$

so we have

$$\begin{bmatrix} 222.5 \\ 2890 \end{bmatrix} = \lambda_0 \begin{bmatrix} t \\ 2t+c \end{bmatrix} \text{ or }$$

① $222.5 = \lambda_0 t$

② $2890 = \lambda_0 (2t+c)$

+

③ $t(c+t) - 2.13E\text{-}4 = 0$

$t = 0.004215\,m$
$c = 0.04632\,m$
$\lambda = 52787$

$M = f(c,t) = 2890t + 222.5c \doteq 22.48\ kg$

The actual minimum mass