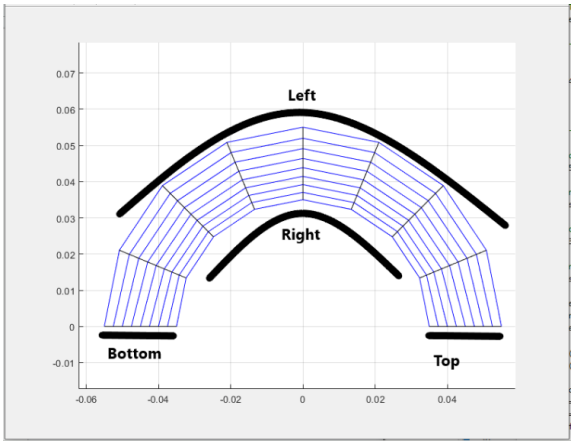
Code Explanation

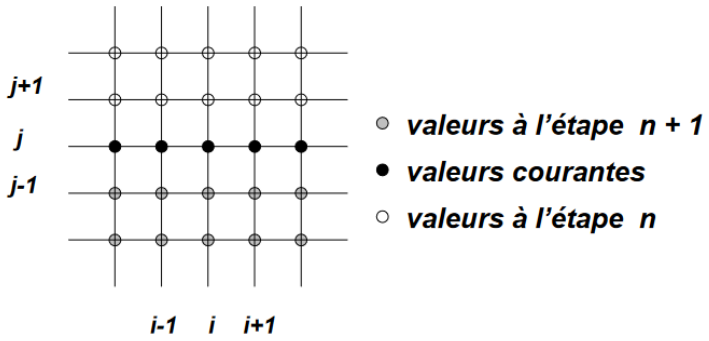
The mesh grid I formed by using an elliptic mesh generation paper [1] that defined the x and y coordinates of the nodes, and the boundaries as top, bottom, left, and right:



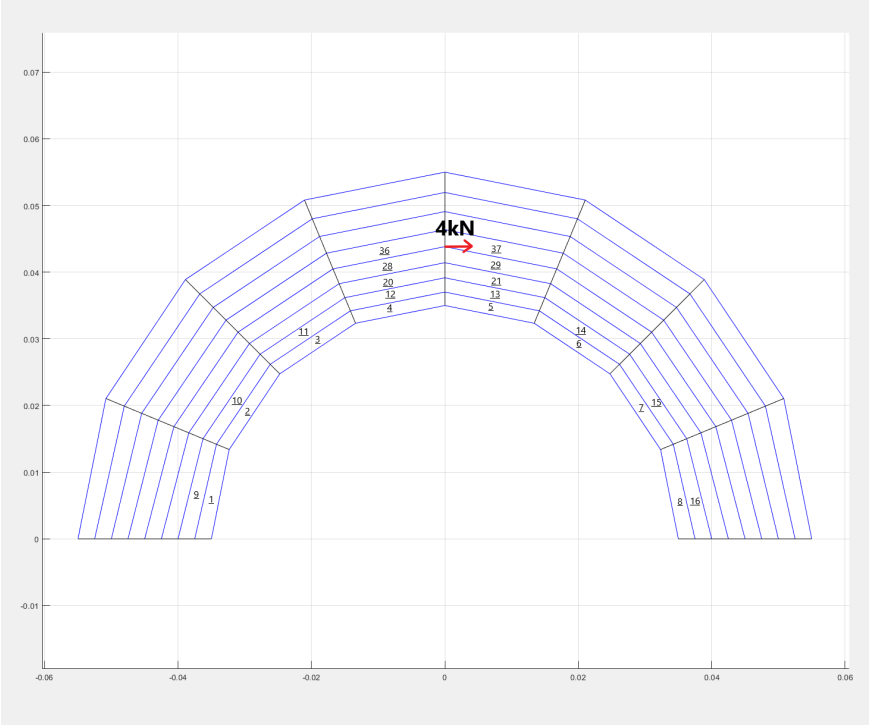
Even though it's straight and not on its side, I'm able to add more elements to create a refined mesh. The boundaries cover the first and last row and column of the X and Y variables, and point to the node locations. The paper then shows ways to fill in the remaining coordinates using variables alpha, beta, and gamma, which are a part of the gradient operator to solve PDE's.

$$\alpha = \left(\frac{\partial \xi_1}{\partial x_1}\right)^2 + \left(\frac{\partial \xi_1}{\partial x_2}\right)^2 \quad , \quad \beta = \frac{\partial \xi_1}{\partial x_1}\frac{\partial \xi_2}{\partial x_1} + \frac{\partial \xi_1}{\partial x_2}\frac{\partial \xi_2}{\partial x_2} \quad , \quad \gamma = \left(\frac{\partial \xi_2}{\partial x_1}\right)^2 + \left(\frac{\partial \xi_2}{\partial x_2}\right)^2$$

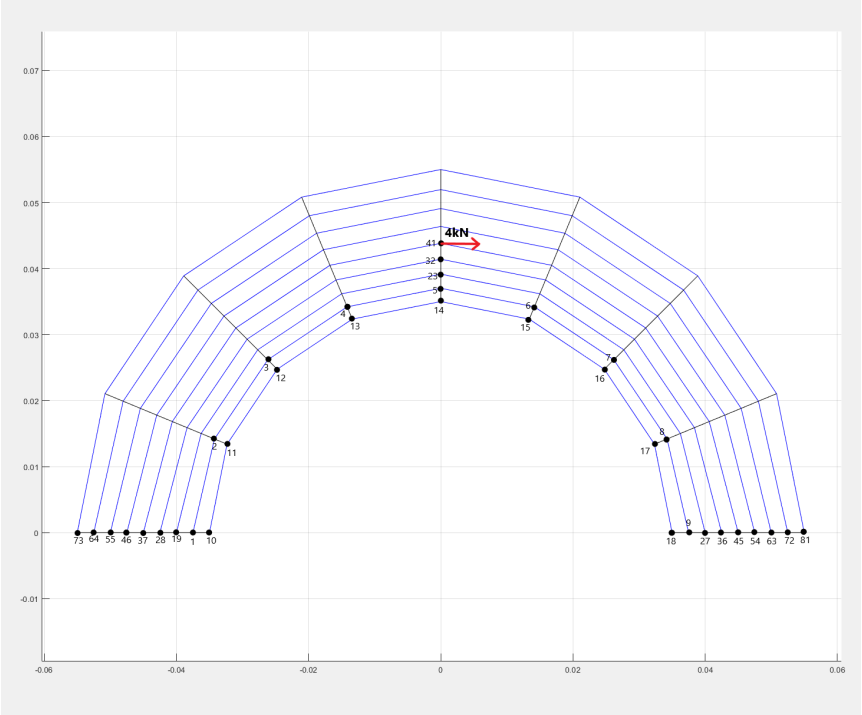
These are used to solve Poisson's equation, and fill in the elliptic coordinates inside the boundary. Because there's a x_1, x_2 ,ζ_1 ,and ζ_2 , there's 2 nested for loops that go through them. In MATLAB, i goes down from to bottom, and j from left to right. If i is constant, ζ_1, and j+1 = x_1, and j-1 = x_2. Similarly, if j is constant, ζ_2, and i+1 = x_1, and i-1 = x_2.



Also nx and ny are chosen as 9 initially, to show 9x9 = 81 nodes, and (9-1)(9-1) = 64 elements. This is how they're named:



Element numbers

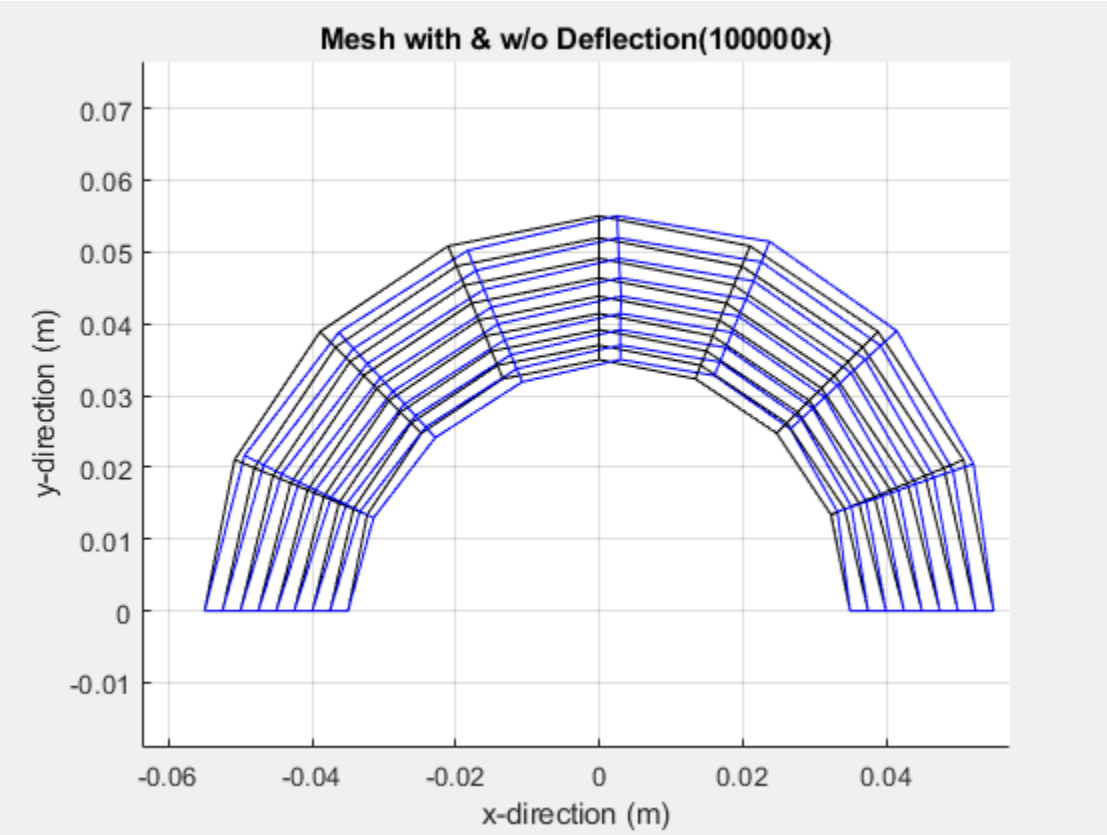


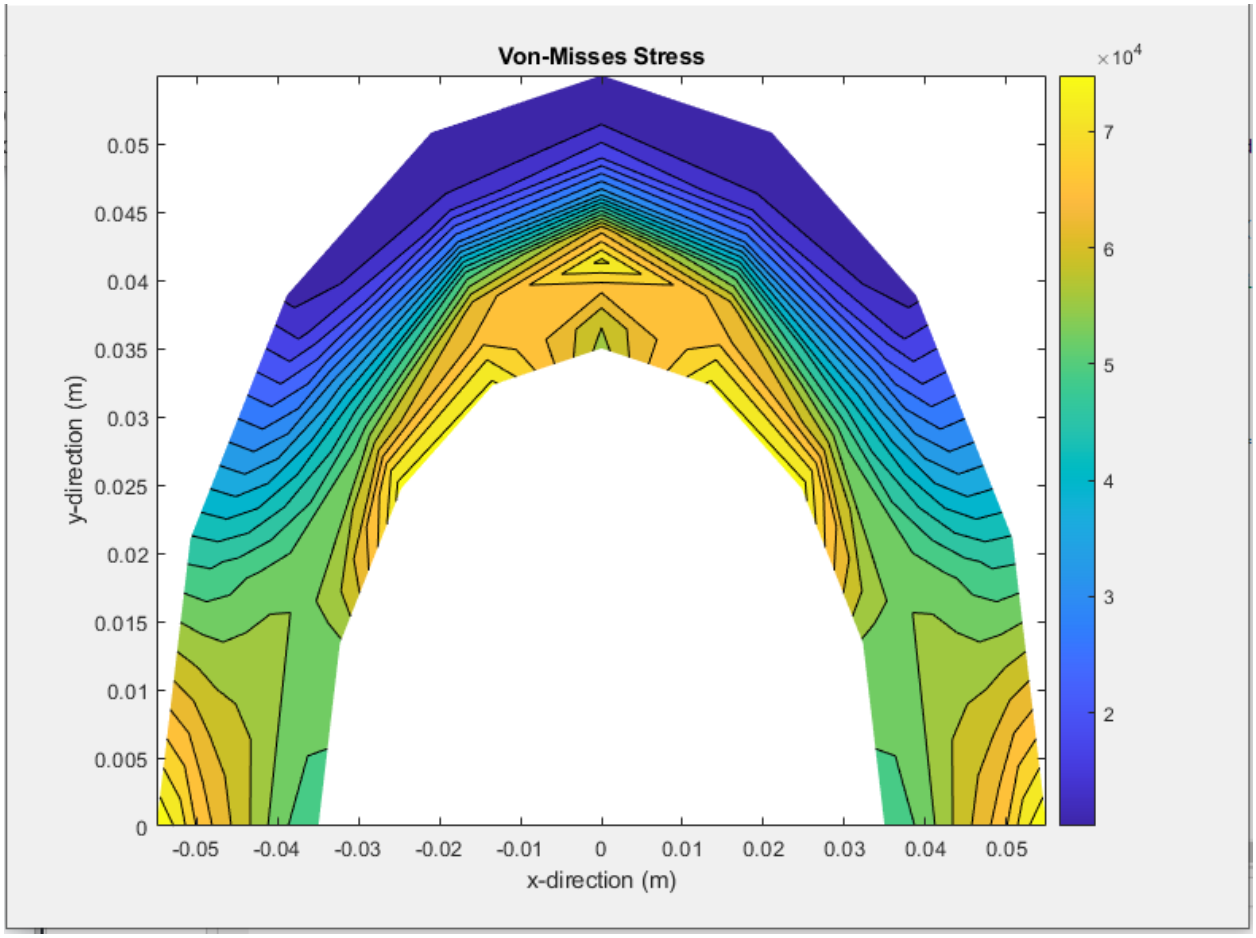
Node numbering

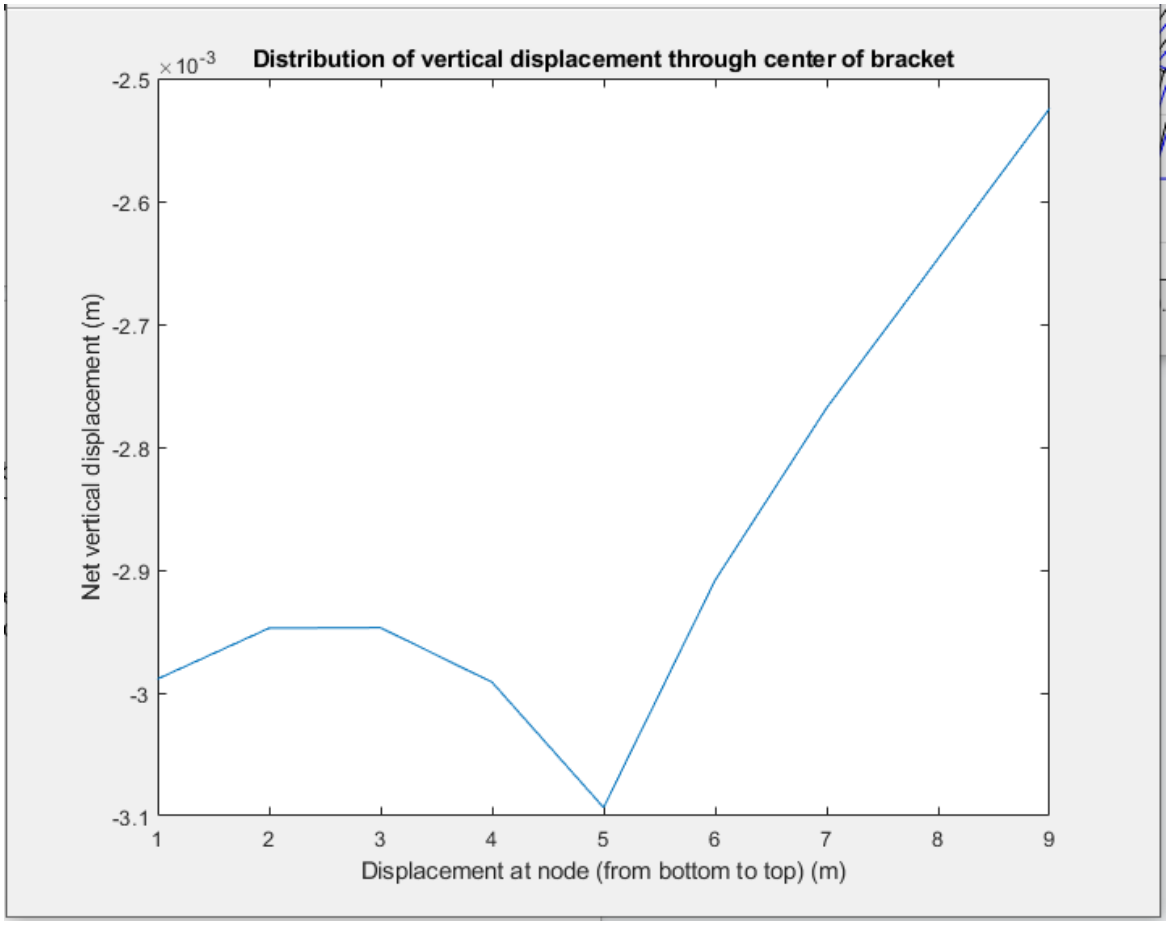
Future design considerations

While these results are good, they can slightly be better with a more refined mesh, specifically where the point force is located. In fact, if the force could be distributed to a few other nearby nodes, that can help give better results.

Also I wasn't able to contour plot the strains, but I did get values for epsilon based on the element displacement coordinates, which you can see in the workspace.




Von-Misses Stress
×10^4
7
6
5
4
3
2
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
y-direction (m)
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
x-direction (m)



References

1) Remacle, J.-F. (n.d.). *An introduction to mesh generation Part IV : elliptic meshing*. pdfcoffee.com. https://pdfcoffee.com/eliptic-grid-generation-pdf-free.html