Assignment 2
FitzHugh-Nagumo model

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1 Introduction

FitzHugh-Nagumo model is a simplifcation of the Hodgkin-Huxley model. The differential equation corresponding 'm' and 'j' are approximated to be constants and hence the model becomes a 2-variable model with the following equations.

$$\frac{dV}{dt} = f(v) - w + I_m$$

$$\frac{dw}{dt} = bv - rw$$

where, $f(v) = v(a-v)(v-1)$. Also, V_m, n from HH-model becomes the V, w variables in FN-model.

2 Case 1: $I_{ext} = 0$

2.1 Phase plot of v-nullcline and w-nullcline

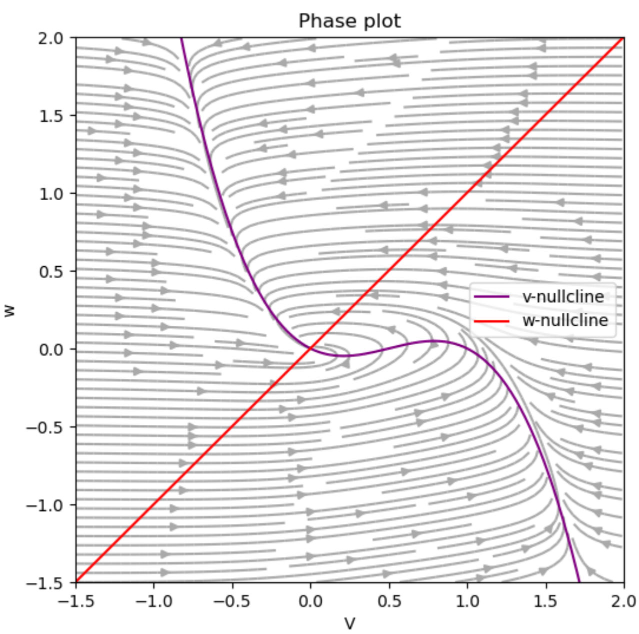

Figure 1: Super imposed phase plot v-nullcline and w-nullcline at $I_{ext} = 0$

Phase plot as shown in Figure 1, was plotted using stream plot from matplotlib library in Python. The grey arrows in plot depict the flow around the nullclines.

2.2 Time plots for V(t) and W(t)

Time plots for V(t) and W(t) are shown below for different conditions along with their phase plane trajectories.

2.2.1 $V(0) < 0; V(0) = 0.4; W(0) = 0$

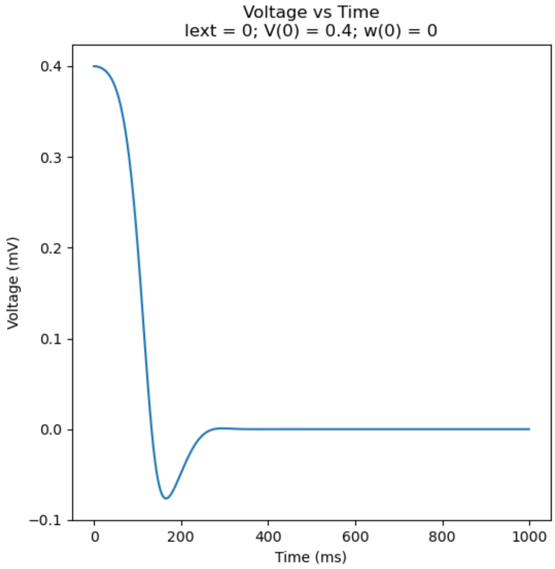

Figure 2: V(t) vs t plot for $I_{ext} = 0$ with V(0) = 0.4

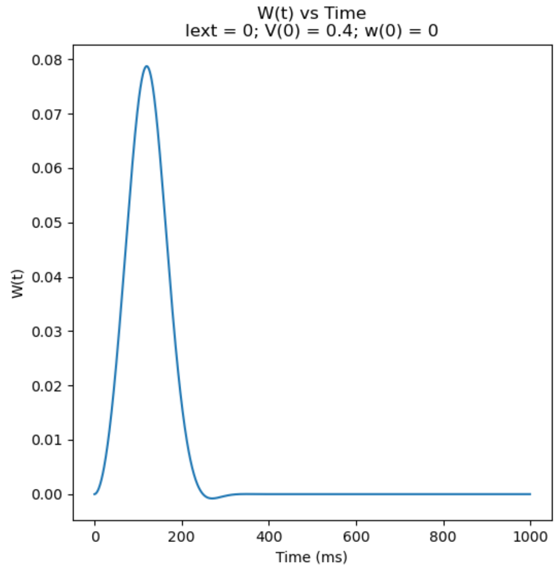

Figure 3: W(t) vs t plot for $I_{ext} = 0$ with V(0) = 0.4

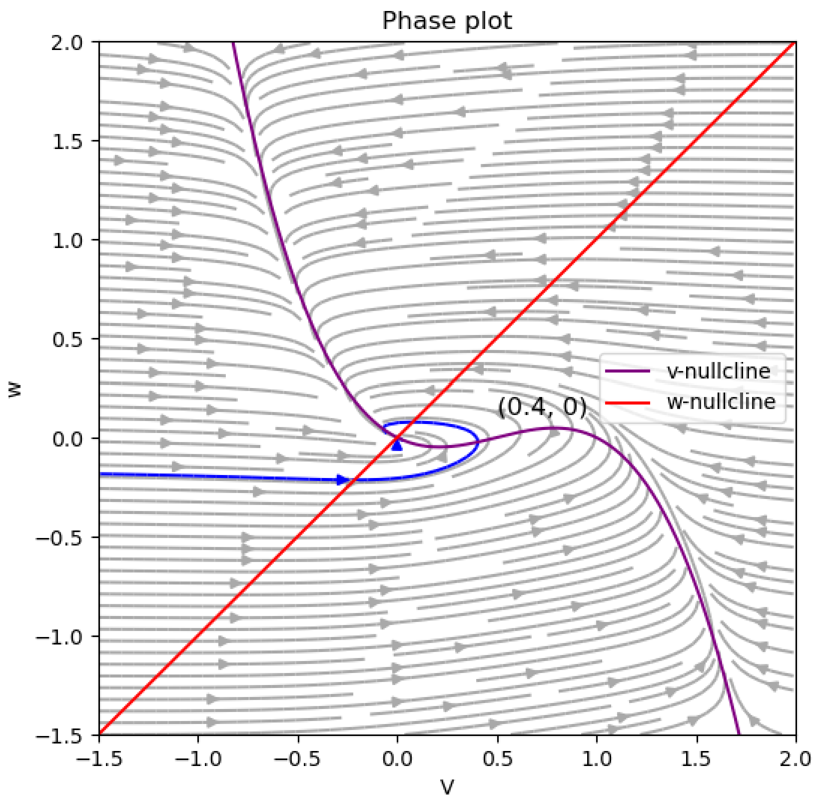

Figure 4: Phase plot of the trajectory of V(0) = 0.4 & W(0) = 0

2.2.2 $V(0) < 0; V(0) = 0.4; W(0) = 0$

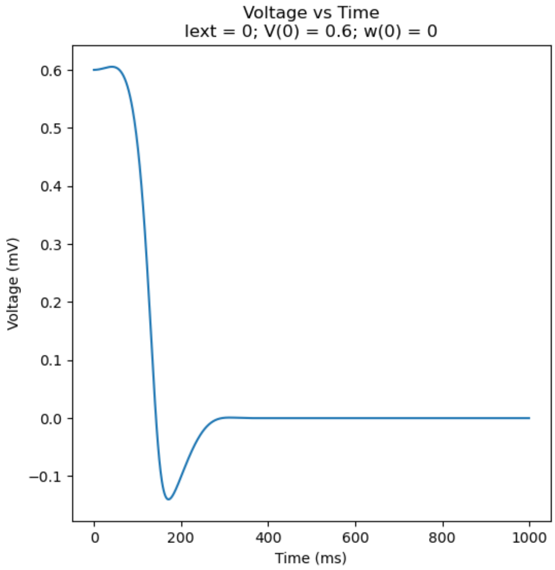

Figure 5: V(t) vs t plot for $I_{ext} = 0$ with V(0) = 0.6

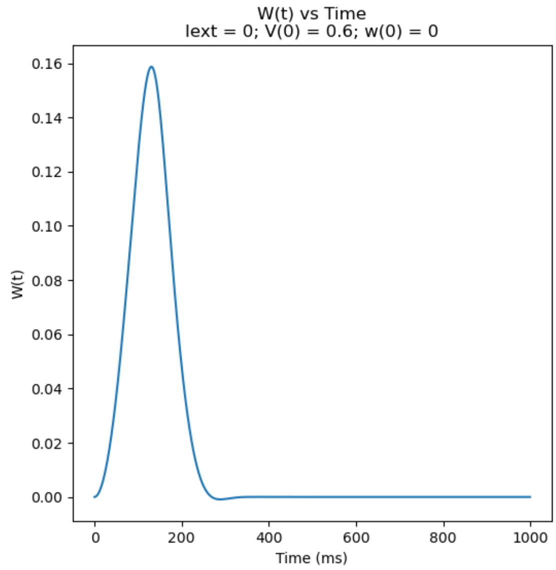

Figure 6: W(t) vs t plot for $I_{ext} = 0$ with V(0) = 0.6

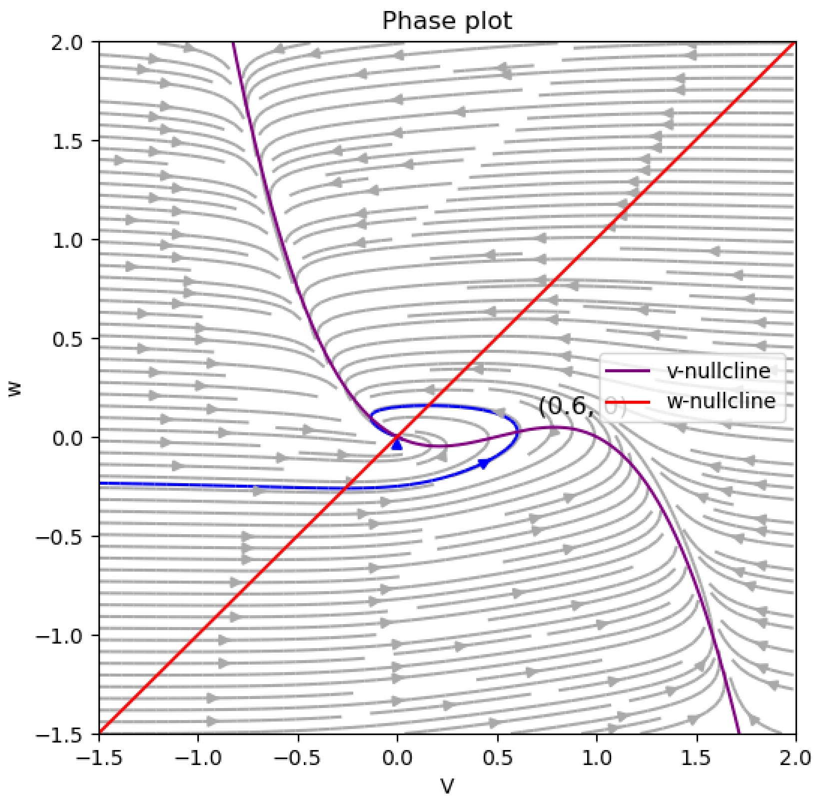

Figure 7: Phase plot of the trajectory of V(0) = 0.6 & W(0) = 0

3 Case 2: $I_1 < I_{ext} < I_2$

In order to find I_1 & I_2, the I_{ext} was varied such that after I_1, there were oscillations and after I_2 there are no oscillations. So, I_1 came out to be 0.23 and I_2 came out to be 0.80.

3.1 I_1 & I_2 plots; V(0) < a; V(0) = 4;

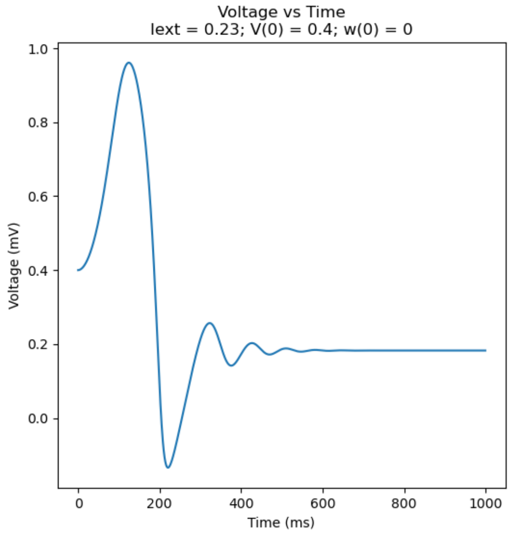

Figure 8: V(t) vs t at $I_{ext} = 0.23$

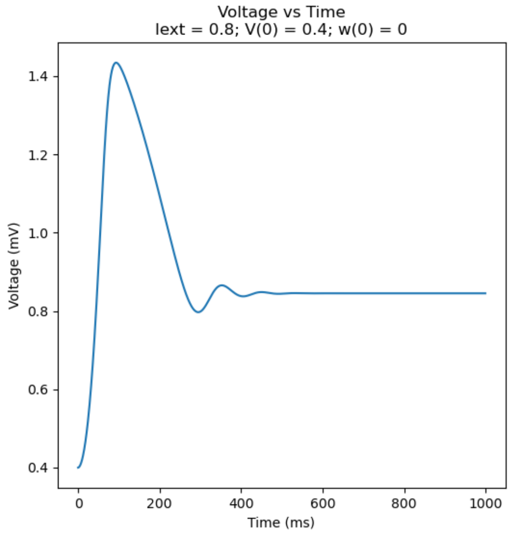

Figure 9: V(t) vs t at $I_{ext} = 0.80$

The figures 8 and 9 shows the plots at $I_1 = 0.23mV$ & $I_2 = 0.80mV$ respectively at the V(0) = 0.4 mV. So after 0.23 mV, there will oscillations and after 0.80 mV, there will be no oscillations at all.

3.2 I_1 & I_2 plots; V(0) < a; V(0) = 4;

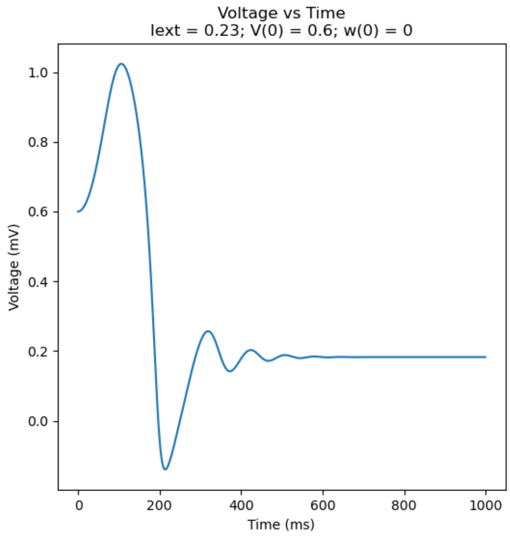

Figure 10: V(t) vs t at $I_{ext} = 0.23$

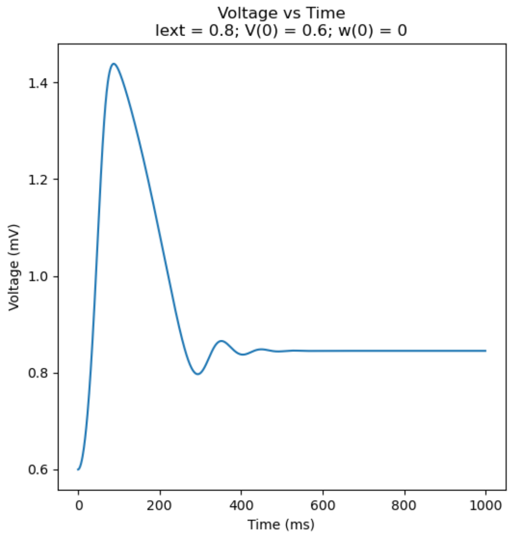

Figure 11: V(t) vs t at $I_{ext} = 0.80$

The figures 10 and 11 shows the plots at $I_1 = 0.23mV$ & $I_2 = 0.80mV$ respectively at the V(0) = 0.6 mV. So after 0.23 mV, there will oscillations and after 0.80 mV, there will be no oscillations at all.

3.3 Phase plot for $I_{ext} = 0.5$

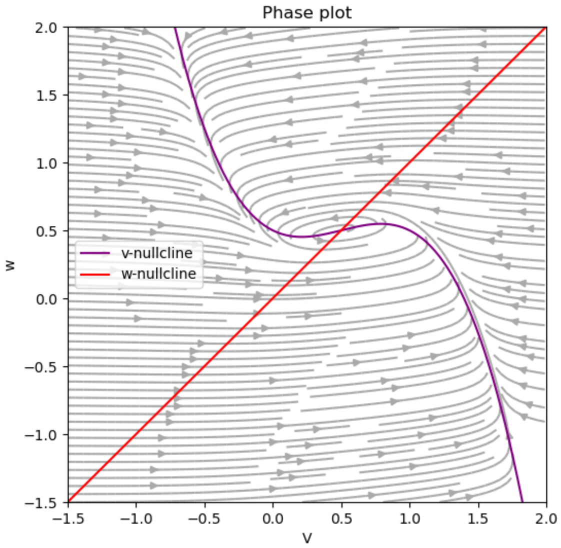

Figure 12: Phase plot of $I_{ext} = 0.5$

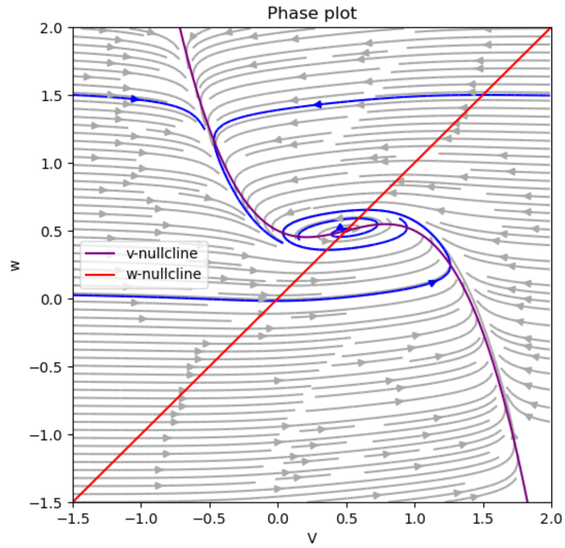

Figure 13: Trajectories showing the fixed point produces stable limit cycle

As shown in figure 13, there is a stable limit cycle which is produced by the unstable node at (0.5, 0.5). From wherever you start, either from inside the limit cycle i.e. very close to the unstable node or from outside i.e. totally away from the nullclines, the trajectory follows and comes into the limit cycle.

3.4 Time plots for V(t) and W(t) for $I_{ext} = 0.5$

Time plots for V(t) and W(t) are shown below for different conditions at an $I_{ext} = 0.5$

3.4.1 $V(0) < 0; V(0) = 0.4; W(0) = 0$

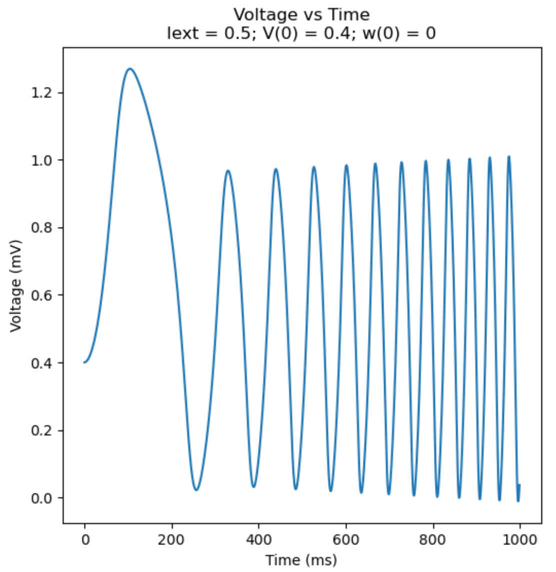

Figure 14: V(t) vs t plot for $I_{ext} = 0.5$ with V(0) = 0.4

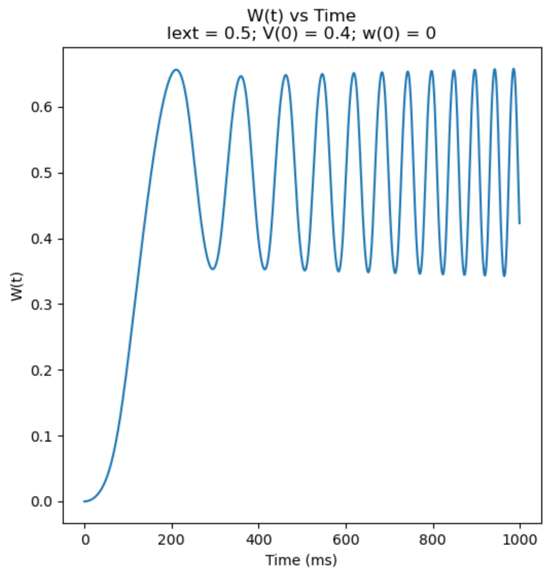

Figure 15: W(t) vs t plot for $I_{ext} = 0.5$ with V(0) = 0.6

3.4.2 $V(0) > 0; V(0) = 0.6; W(0) = 0$

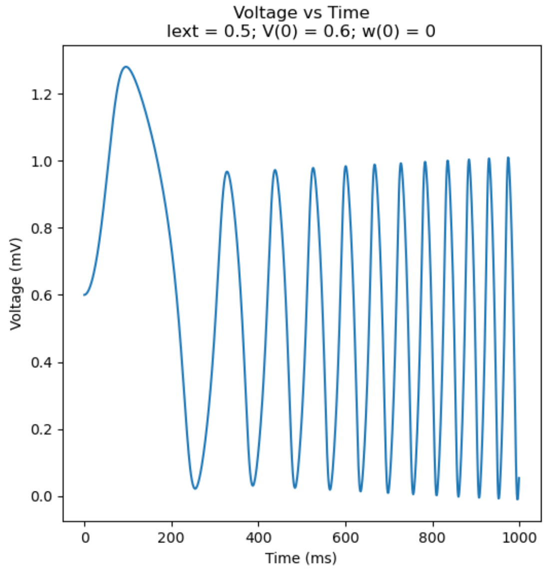

Figure 16: V(t) vs t plot for $I_{ext} = 0.5$ with V(0) = 0.6

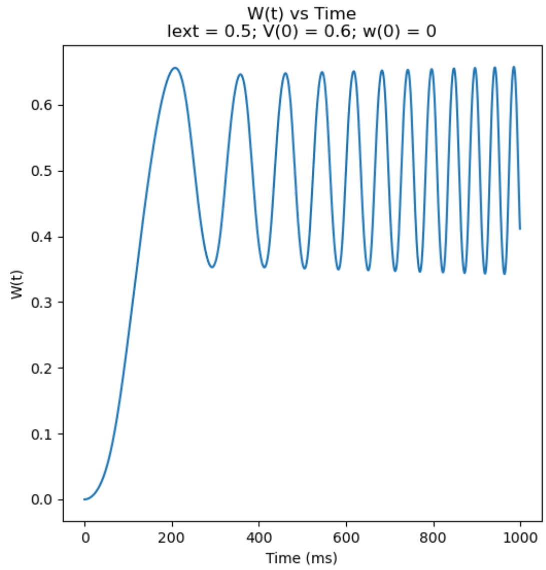

Figure 17: W(t) vs t plot for $I_{ext} = 0.5$ with V(0) = 0.6

For both initial values, V(0) = 0.4 and V(0) = 0.6 there are a lot of oscillations. W(t) vs t plots for both the inital value of V as shown in figures 15 and 17 look very alike.

4 Case 3: $I_{ext} > I_2$

4.1 Phase plot for $I_{ext} = 0.5$

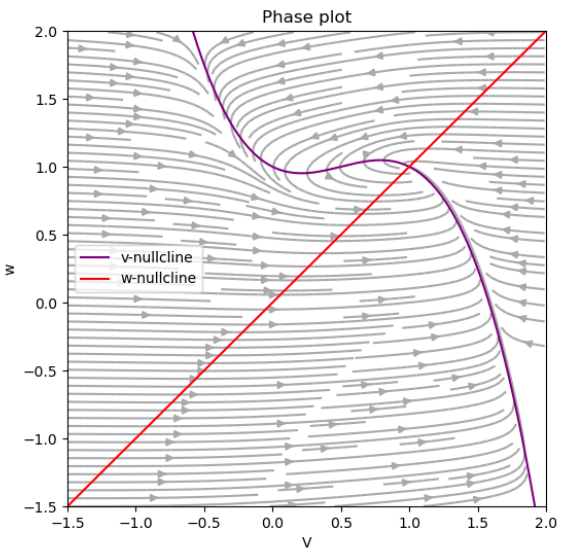

Figure 18: Phase plot of $I_{ext} = 1.0$

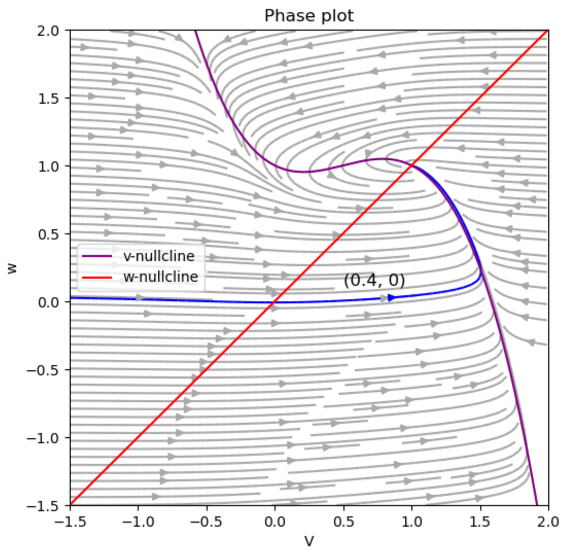

Figure 19: Trajectories showing the fixed point produces unstable limit cycle

4.2 Time plots for V(t) and W(t) for $I_{ext} = 1.0$

Time plots for V(t) and W(t) are shown below for different conditions at an $I_{ext} = 1.0$

4.2.1 $V(0) < 0; V(0) = 0.4; W(0) = 0$

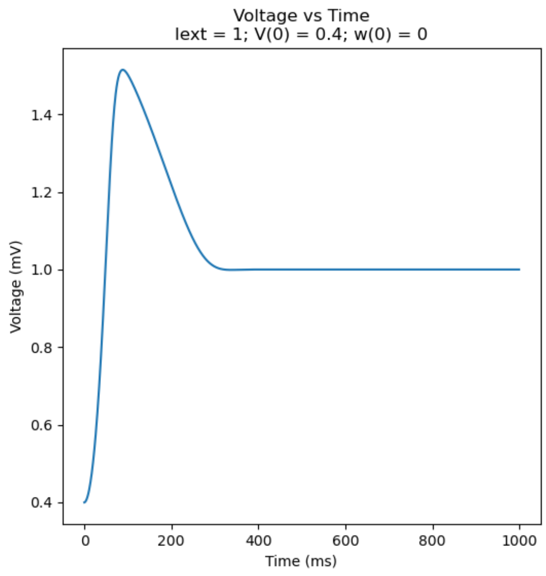

Figure 20: V(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.4

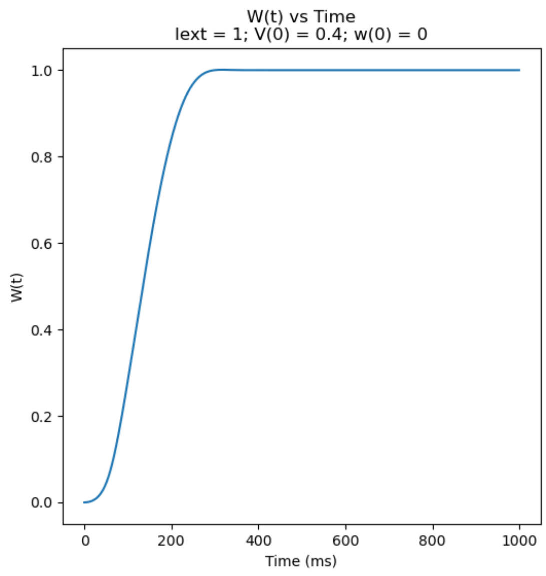

Figure 21: W(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

4.2.2 $V(0) > 0; V(0) = 0.6; W(0) = 0$

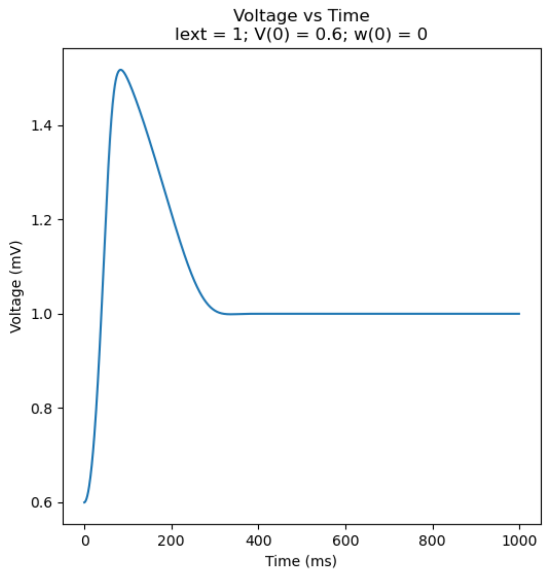

Figure 22: V(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

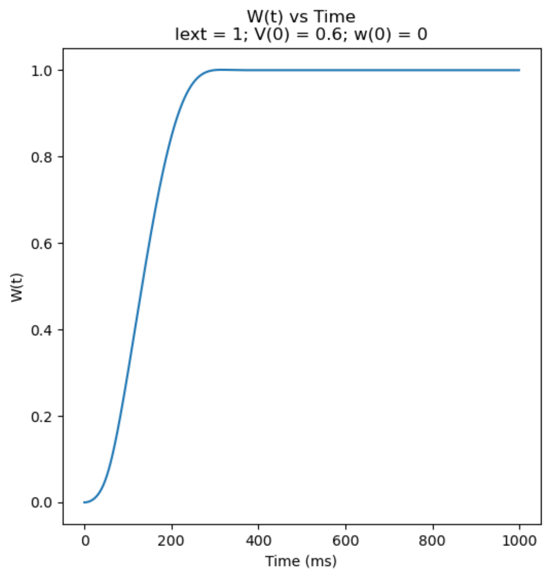

Figure 23: W(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

5 Case 4: $I_{ext} = 0;\ \frac{b}{r} = 0;$

5.1 Bistability Phase plot

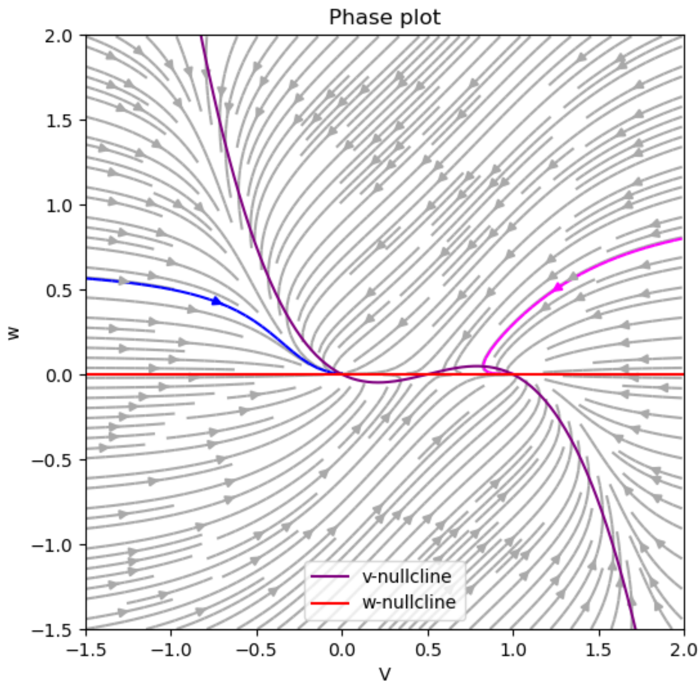

Figure 24: Phase plot of $I_{ext} = 0;\ \frac{b}{r} = 0;$

5.2 Time plots for V(t) and W(t) for $I_{ext} = 0$ & $\frac{b}{r} = 0$

Time plots for V(t) and W(t) are shown below for different conditions at an $I_{ext} = 0$ & $\frac{b}{r} = 0$

5.2.1 $V(0) < 0; V(0) = 0.4; W(0) = 0$

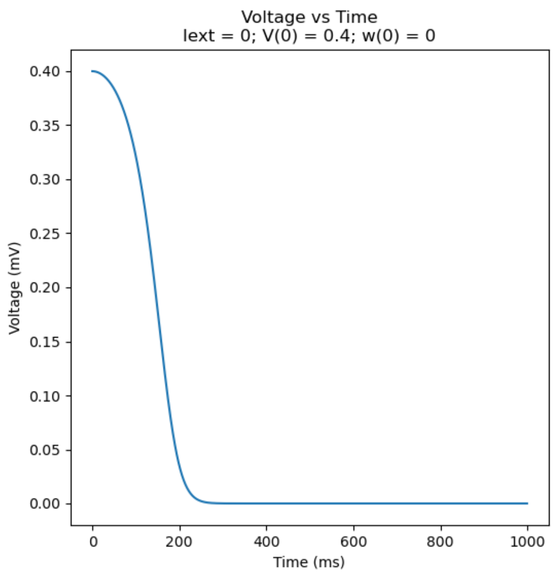

Figure 25: V(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.4

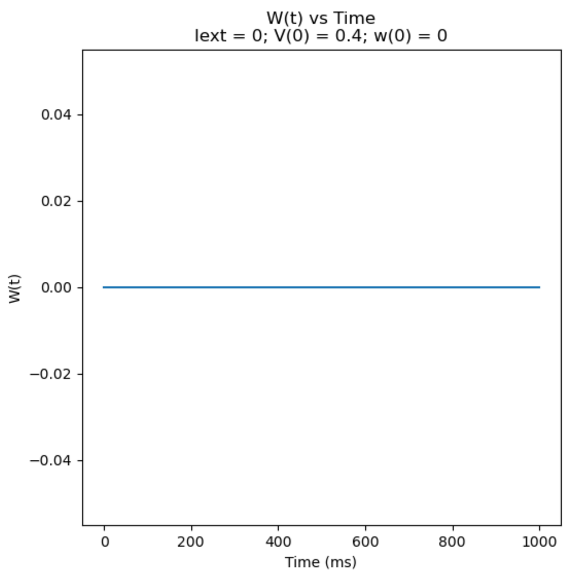

Figure 26: W(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

5.2.2 $V(0) > 0; V(0) = 0.6; W(0) = 0$

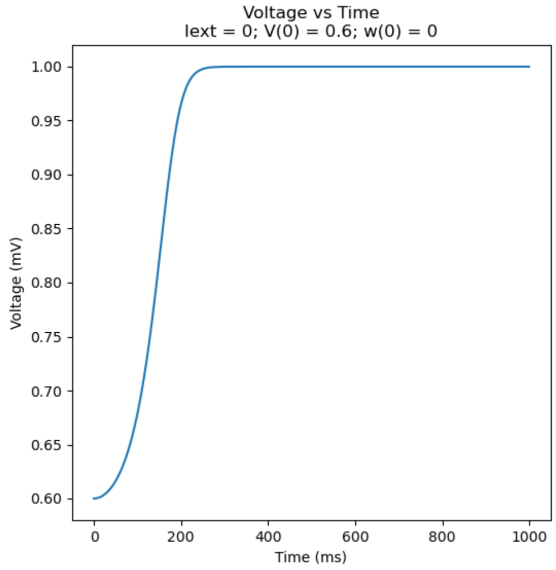

Figure 27: V(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

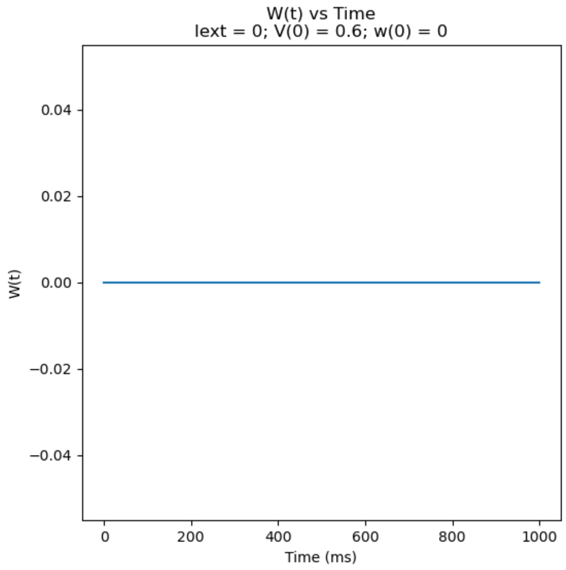

Figure 28: W(t) vs t plot for $I_{ext} = 1.0$ with V(0) = 0.6

From figure 24, Let P1, P2, P3 be the points of intersection of the W nullcline and V nullcline from left to right. For P1, a small perturbation will bring back the point to P1. Hence, P1 is stable. This is the same case with P3 too. Since there are 2 stable points, this condition is called as bistability.

In the case of P2, any perturbation will result in moving away from P2 and it will move into one of the stable fixed points.