CS321 - Homework 1

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Problems from 1.2

6 Let L be any language on a non-empty alphabet. Show that L and $\overline{L}$ cannot both be finite.

Proof of L and $\overline{L}$ cannot both be fininte. Let L be any language on a non-empty alphabet. Assume $L = \overline{L}$ and both L and $\overline{L}$ are finite.

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7 Are there languages for which $\overline{L^*} = (\overline{L})^*$?

Additional Problems

For the additional problems, describe via an English sentence each of the languages from the following Exercises in Section 1.2.

14(b) Let $\Sigma = \{a, b\}$. For each of the following languages, find a grammar that generates it.

$L_2 = \{a^n b^{2n} : n \geq 0\}$.

L_2 consists of two non-empty strings of a's and b's concatenated together, where the second string of b's has twice as many symbols as the first string of a's.

15(a) Find grammars for the following languages on $\Sigma = \{a\}$.

$L = \{w : |w| \bmod 3 = 0\}$.

L consists of all strings - including the empty string - with lengths that are multiples of 3.

18(b,d) Using the notation from Example 1.13:

$$
\begin{aligned}
S \to &\quad aA, \\
A \to &\quad bS, \\
S \to &\quad \lambda
\end{aligned}
$$

find grammars for the languages below. Assume $\Sigma = \{a, b\}$.

(b) $L = \{w : n_a(w) = n_b(w) + 1\}$.

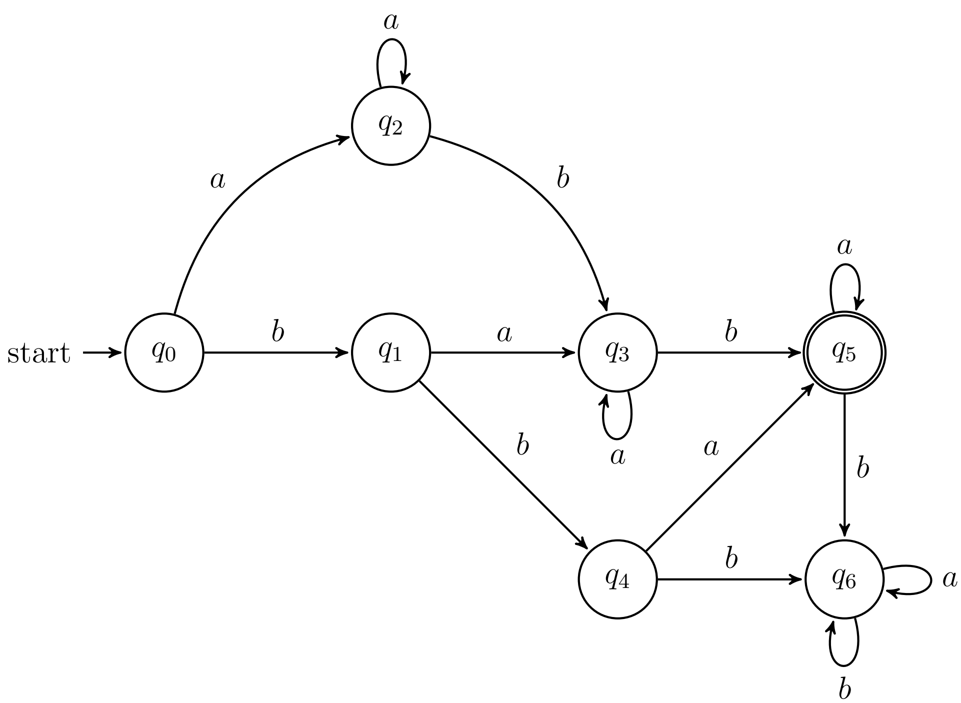
(d) $L = \{w \in \{a, b\}^* : |n_a(w) - n_b(w)| = 1\}$.

(b) Number b's in the string is 1 more then the number of a's.

(d) The number of a's and b's in a string differ by 1.

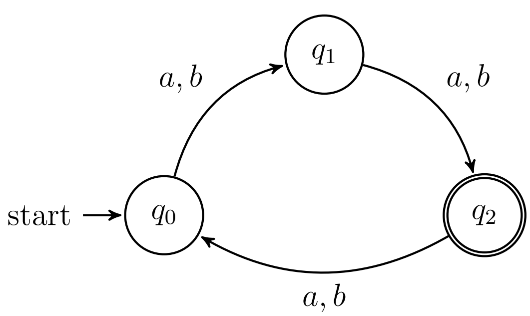
Problems from 2.1

2(d) For $\Sigma = \{a, b\}$. construct dfa's that accept the sets containing of all strings with at least one a and exactly two b's.



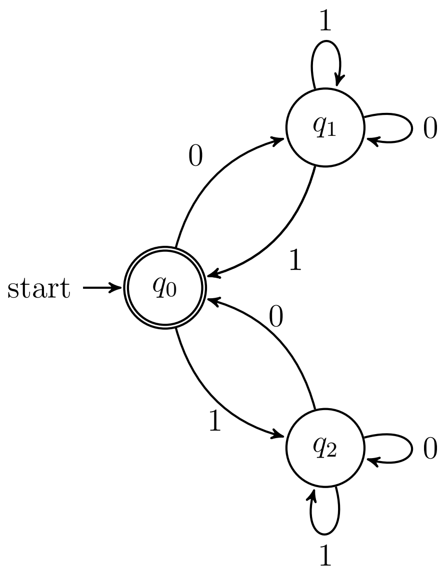
7(c) Find dfa's for the following languages on $\Sigma = a, b$.

$L = \{w : |w| \bmod 3 > 1\}$.

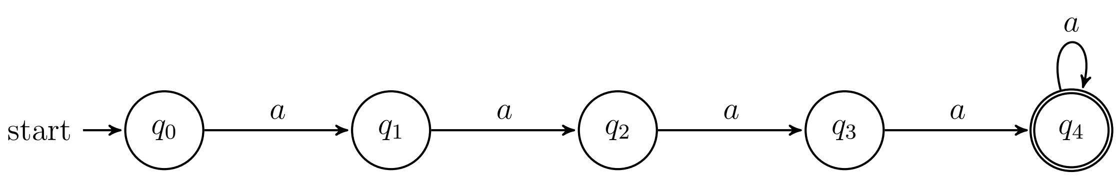


9(c) Consider the set of strings on $\{0, 1\}$ defined by the requirements below. For each, construct an accepting dfa.

The leftmost symbol differs from the rightmost one.



12 Show that $L = \{a^n : n \geq 4\}$ is regular.



17 Show that if L is regular, so is $L - \{\lambda\}$.

Hint: We know that L is regular. Thus there is a DFA M such that $L = L(M)$. To solve this problem you need to describe how to form a new DFA M' from M such that $L(M') = L - \{\lambda\}$. You can construct such an M' by adding a single state to M with the appropriate arcs. You need to provide an informal argument as to why your construction is correct.