

# CS321 - Homework 1

Trevor Bramwell

October 6, 2013

## Problems from 1.2

- 6** Let  $L$  be any language on a non-empty alphabet. Show that  $L$  and  $\overline{L}$  cannot both be finite.
- 7** Are there language for which  $\overline{L^*} = (\overline{L})^*$ ?

## Additional Problems

For the additional problems, describe via an English sentence each of the languages from the following Exercises in Section 1.2.

- 14(b)** Let  $\Sigma = \{a, b\}$ . For each of the following languages, find a grammar that generates it.

$$L_2 = \{a^n b^{2n} : n \geq 0\}.$$

- 15(a)** Find grammars for the following languages on  $\Sigma = \{a\}$ .

$$L = \{w : |w| \bmod 3 = 0\}.$$

- 18(b,d)** Using the notation from Example 1.13:

$$S \rightarrow aA,$$

$$A \rightarrow bS,$$

$$S \rightarrow \lambda$$

find grammars for the languages below. Assume  $\Sigma = \{a, b\}$ .

(b)  $L = \{w : n_a(w) = n_b(w) + 1\}.$

(d)  $L = \{w \in \{a, b\}^* : |n_a(w) - n_b(w)| = 1\}.$

## Problems from 2.1

**2(d)** For  $\Sigma = \{a, b\}$ . construct dfa's that accept the sets containing of all strings with at least one  $a$  and exactly two  $b$ 's.

**7(c)** Find dfa's for the following languages on  $\Sigma = a, b$ .

$$L = \{w : |w| \bmod 3 > 1\}.$$

**9(c)** Consider the set of strings on  $\{0, 1\}$  defined by the requirements below. For each, construct an accepting dfa.

The leftmost symbol differs from the rightmost one.

**12** Show that  $L = \{a^n : n \geq 4\}$  is regular.

**17** Show that if  $L$  is regular, so is  $L - \{\lambda\}$ .

*Hint:* We know that  $L$  is regular. Thus there is a DFA  $M$  such that  $L = L(M)$ . To solve this problem you need to describe how to form a new DFA  $M'$  from  $M$  such that  $L(M') = L - \{\lambda\}$ . You can construct such an  $M'$  by adding a single state to  $M$  with the appropriate arcs. You need to provide an informal argument as to why your construction is correct.