Cryptography and Network Security Chapter 10

Fourth Edition

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Chapter 10 – Key Management; Other Public Key Cryptosystems

No Singhalese, whether man or woman, would venture out of the house without a bunch of keys in his hand, for without such a talisman he would fear that some devil might take advantage of his weak state to slip into his body.

—*The Golden Bough*, Sir James George Frazer

Key Management

- public-key encryption helps address key distribution problems
- have two aspects of this:
  - distribution of public keys
  - use of public-key encryption to distribute secret keys

Distribution of Public Keys

- can be considered as using one of:
  - public announcement
  - publicly available directory
  - public-key authority
  - public-key certificates

Public Announcement

- users distribute public keys to recipients or broadcast to community at large
  - eg. append PGP keys to email messages or post to news groups or email list
- major weakness is forgery
  - anyone can create a key claiming to be someone else and broadcast it
  - until forgery is discovered can masquerade as claimed user

Figure 10.1 Uncontrolled Public Key Distribution

Publicly Available Directory

- can obtain greater security by registering keys with a public directory
- directory must be trusted with properties:
  - contains {name,public-key} entries
  - participants register securely with directory
  - participants can replace key at any time
  - directory is periodically published
  - directory can be accessed electronically
- still vulnerable to tampering or forgery

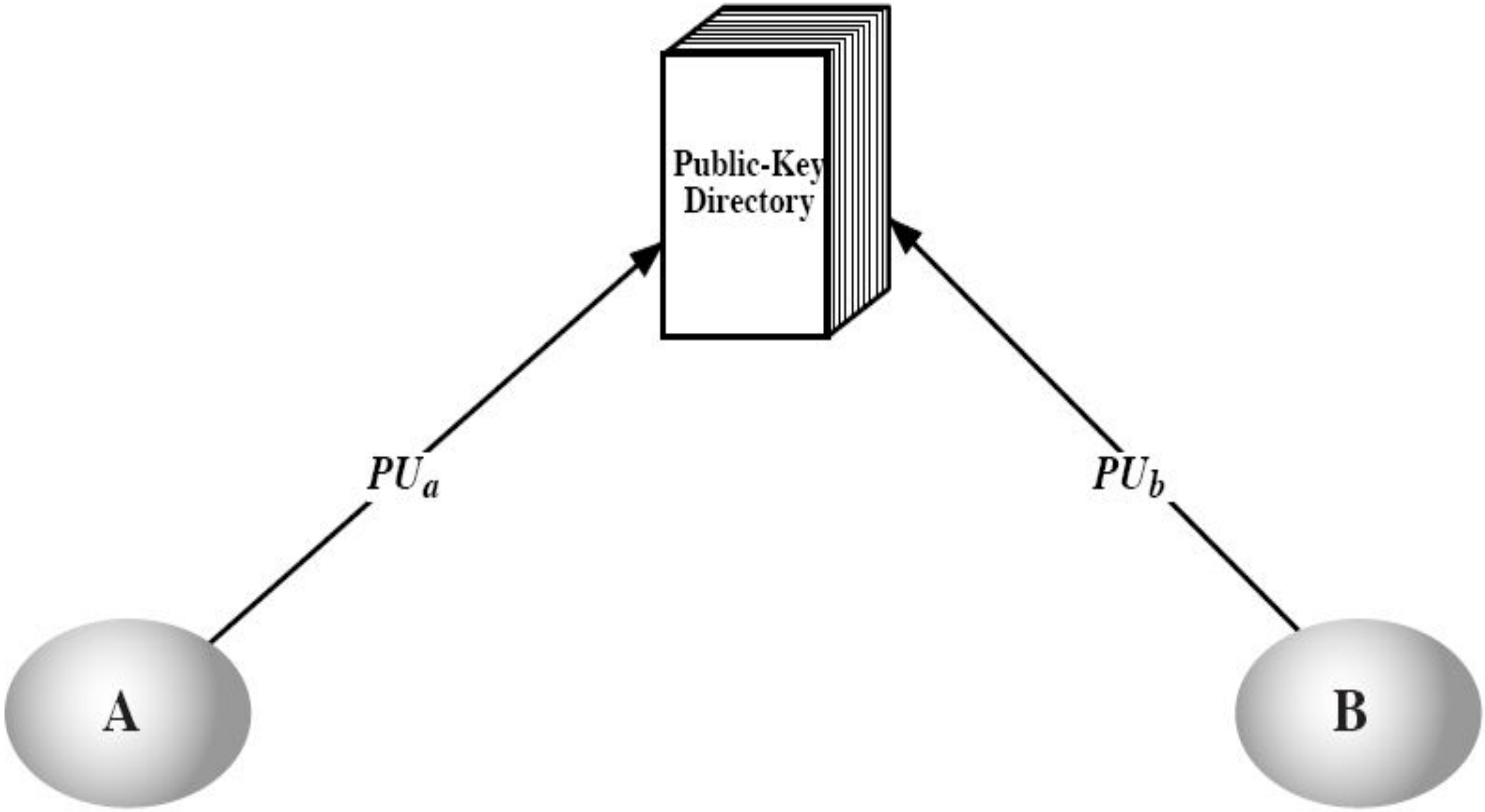
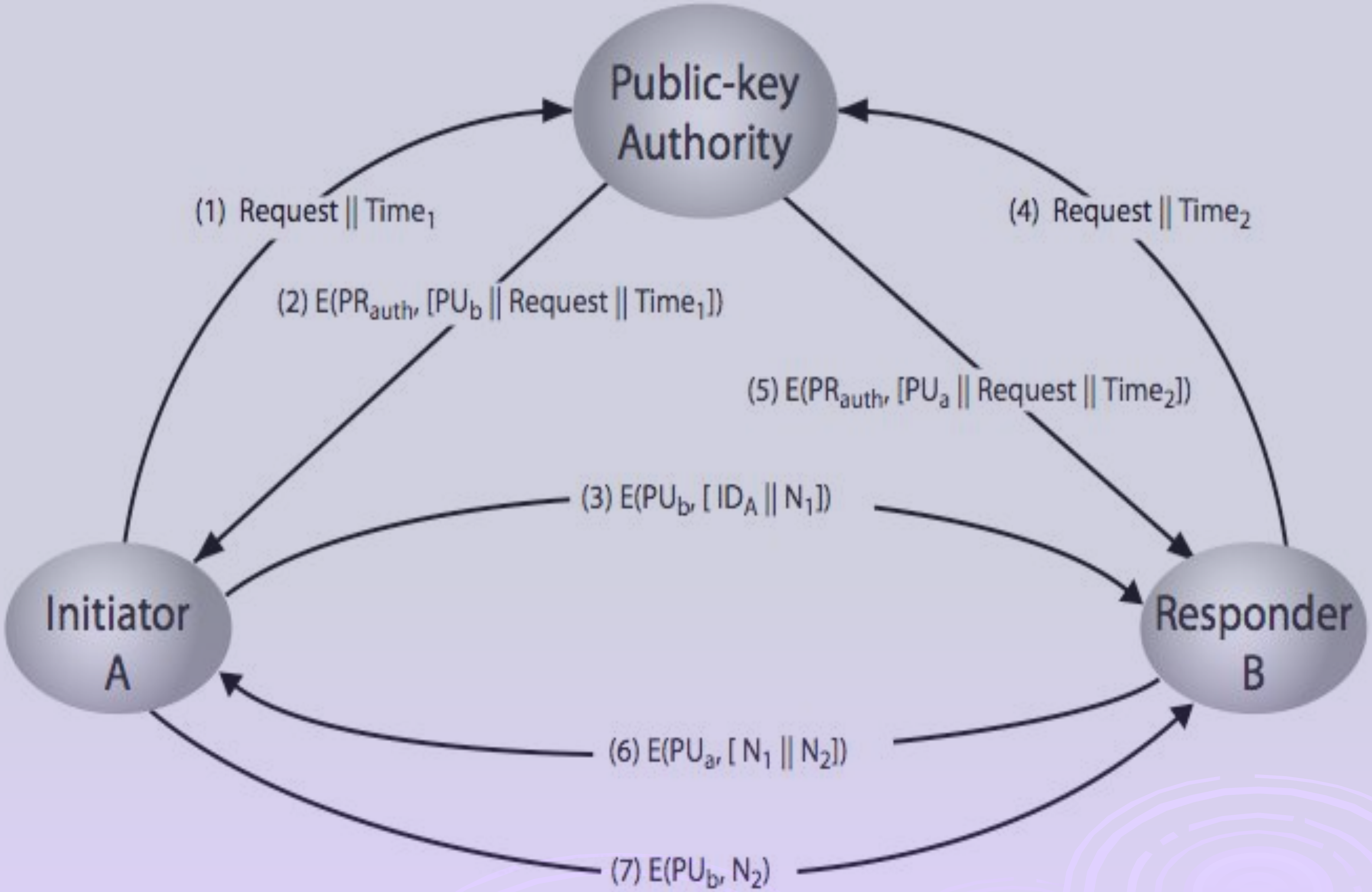


Figure 10.2 Public Key Publication

Public-Key Authority

- improve security by tightening control over distribution of keys from directory
- has properties of directory
- and requires users to know public key for the directory
- then users interact with directory to obtain any desired public key securely
  - does require real-time access to directory when keys are needed

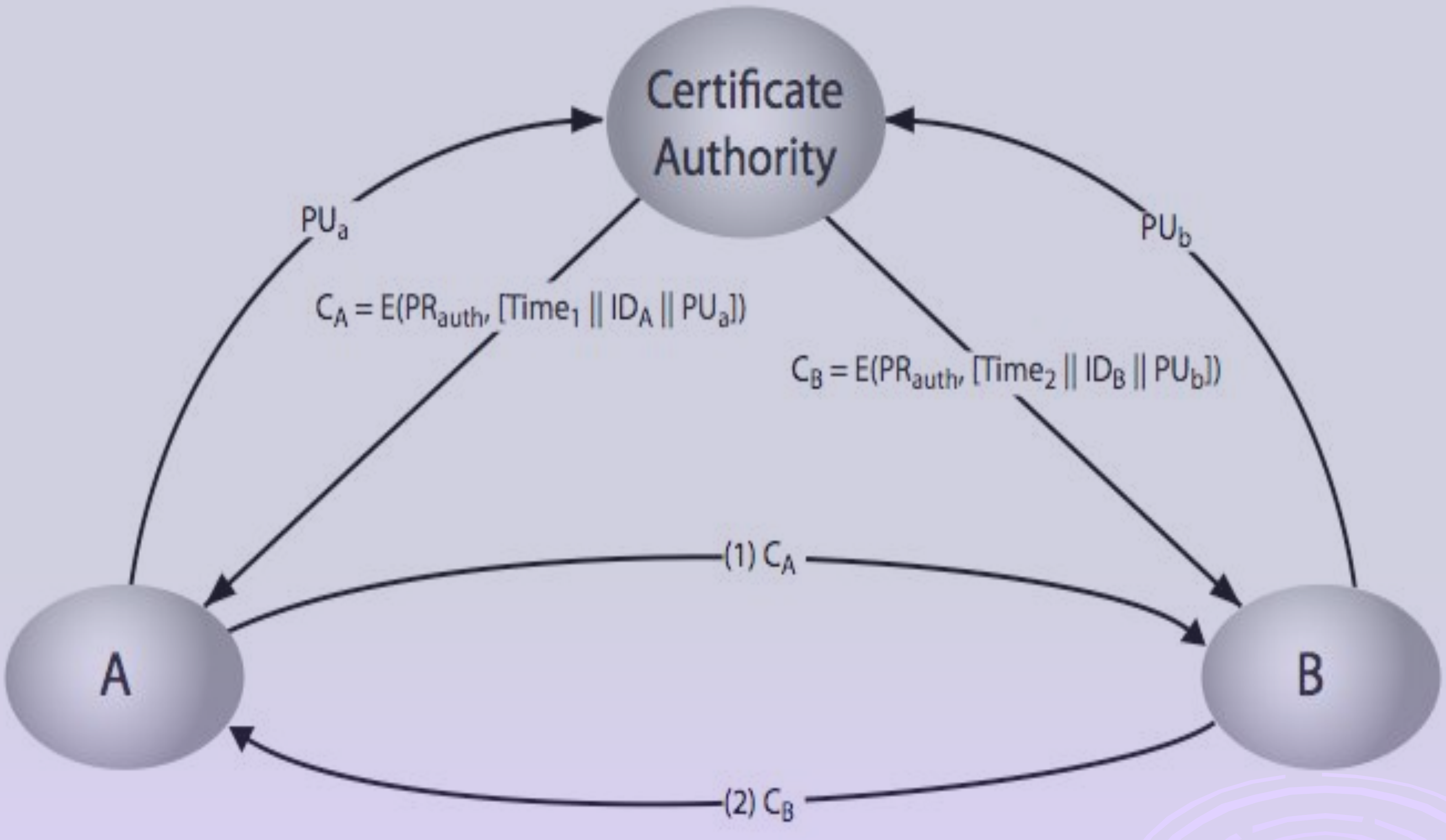
Public-Key Authority



Public-Key Certificates

- certificates allow key exchange without real-time access to public-key authority
- a certificate binds **identity** to **public key**
  - usually with other info such as period of validity, rights of use etc
- with all contents **signed** by a trusted Public-Key or Certificate Authority (CA)
- can be verified by anyone who knows the public-key authorities public-key

Public-Key Certificates



Public-Key for Distribution of Secret Keys

- use previous methods to obtain public-key
- can use for secrecy or authentication
- but public-key algorithms are slow
- so usually want to use private-key encryption to protect message contents
- hence need a session key
- have several alternatives for negotiating a suitable session

Simple Secret Key Distribution

- proposed by Merkle in 1979
  - A generates a new temporary public key pair
  - A sends B the public key and their identity
  - B generates a session key K sends it to A encrypted using the supplied public key
  - A decrypts the session key and both use
- problem is that an opponent can intercept and impersonate both halves of protocol

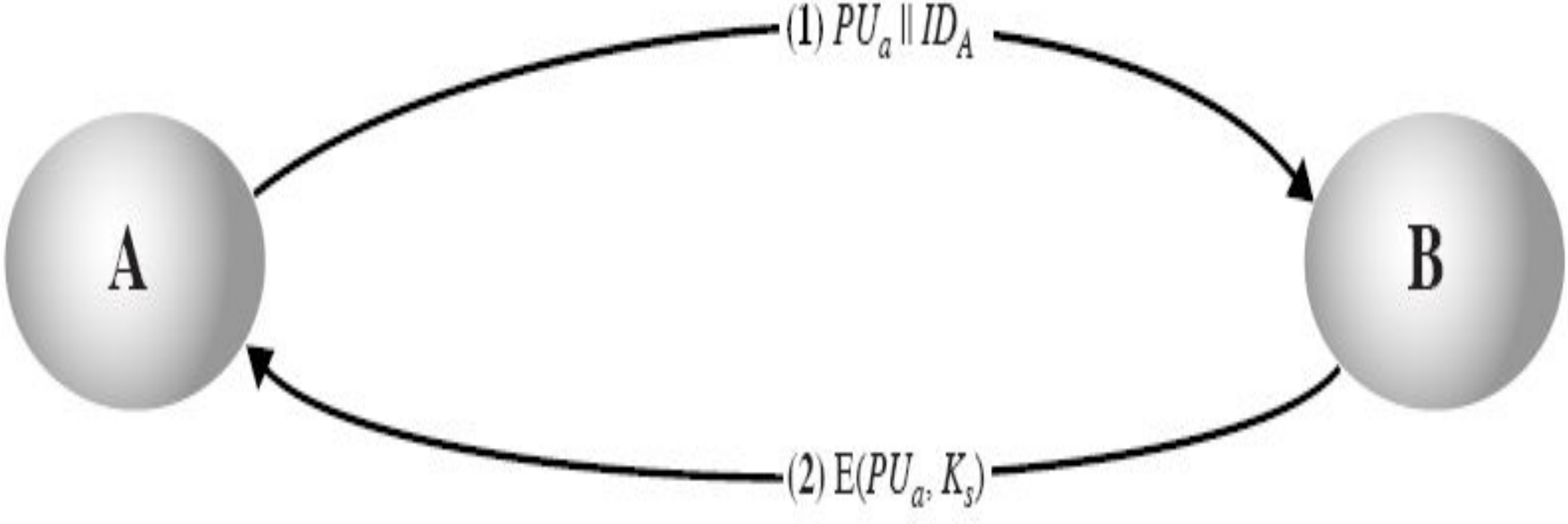
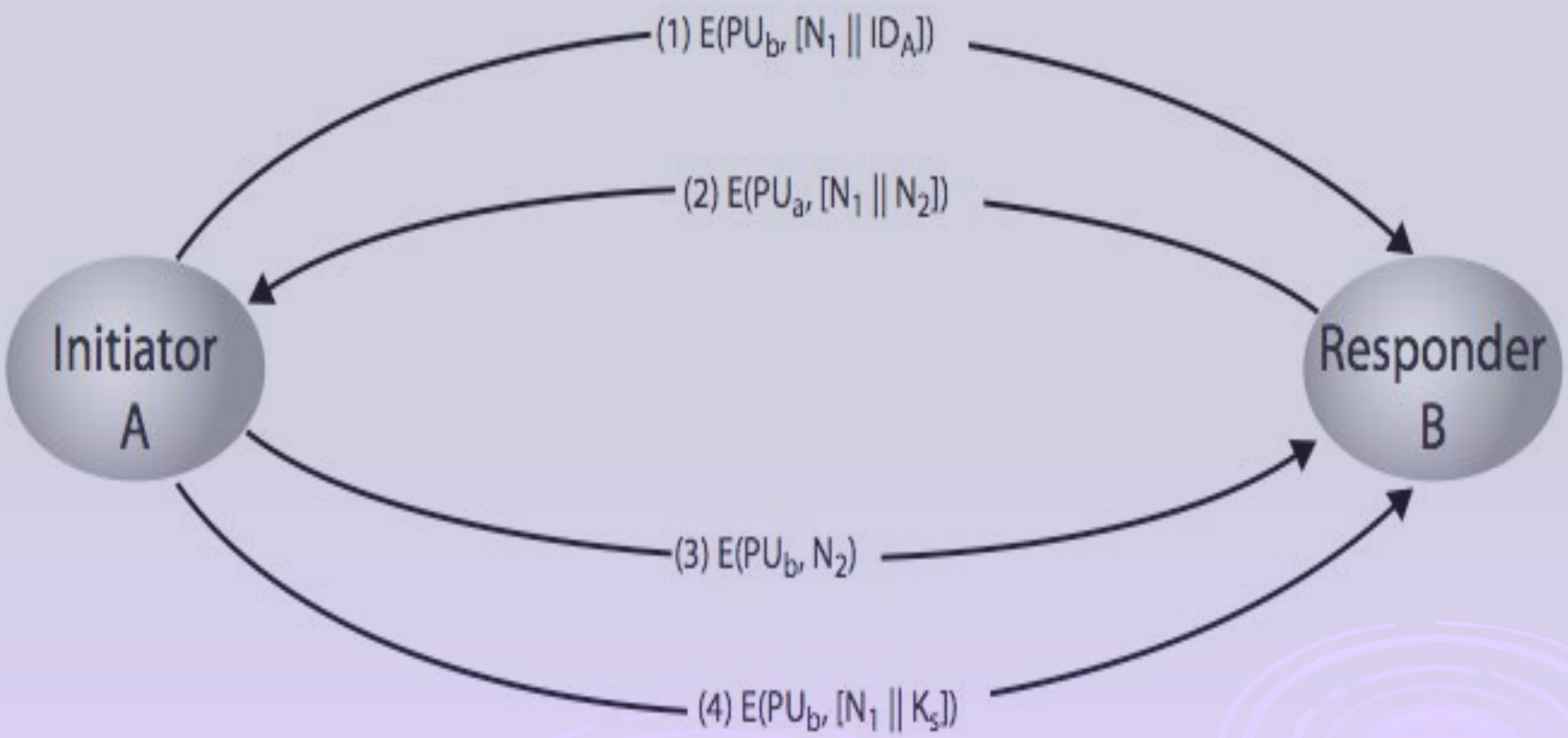


Figure 10.5 Simple Use of Public-Key Encryption to Establish a Session Key

Public-Key Distribution of Secret Keys

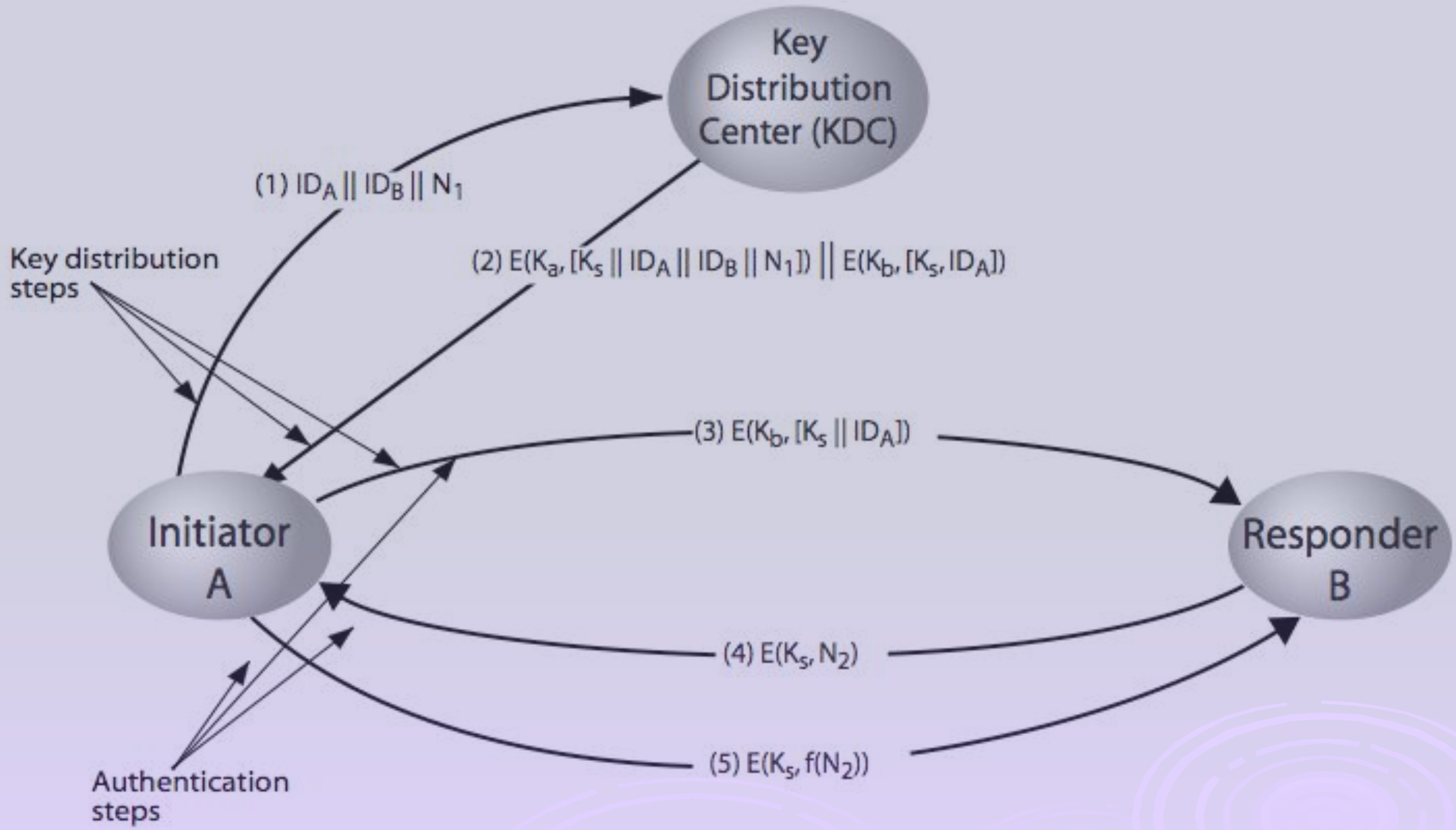
- if have securely exchanged public-keys:



Hybrid Key Distribution

- retain use of private-key KDC
- shares secret master key with each user
- distributes session key using master key
- public-key used to distribute master keys
  - especially useful with widely distributed users
- rationale
  - performance
  - backward compatibility

Key Distribution Scenario



Diffie-Hellman Key Exchange

- first public-key type scheme proposed
- by Diffie & Hellman in 1976 along with the exposition of public key concepts
  - note: now know that Williamson (UK CESG) secretly proposed the concept in 1970
- is a practical method for public exchange of a secret key
- used in a number of commercial products

Diffie-Hellman Key Exchange

- a public-key distribution scheme
  - cannot be used to exchange an arbitrary message
  - rather it can establish a common key
  - known only to the two participants
- value of key depends on the participants (and their private and public key information)
- based on exponentiation in a finite (Galois) field (modulo a prime or a polynomial) - easy
- security relies on the difficulty of computing discrete logarithms (similar to factoring) – hard

Diffie-Hellman Setup

- all users agree on global parameters:
  - large prime integer or polynomial q
  - a being a primitive root mod q
- each user (eg. A) generates their key
  - chooses a secret key (number): $x_A < q$
  - compute their **public key**: $y_A = a^{x_A} \bmod q$
- each user makes public that key y_A

Primitive roots of small primes

n	g(n)
2	1
3	2
4	3
5	2, 3
6	5
7	3, 5
9	2, 5
10	3, 7
11	2, 6, 7, 8
13	2, 6, 7, 11

Diffie-Hellman Key Exchange

- shared session key for users A & B is K_{AB}:

$$K_{AB} = a^{xA.xB} \bmod q$$
$$= y_A^{xB} \bmod q \quad \text{(which } \mathbf{B} \text{ can compute)}$$
$$= y_B^{xA} \bmod q \quad \text{(which } \mathbf{A} \text{ can compute)}$$

- K_{AB} is used as session key in private-key encryption scheme between Alice and Bob
- if Alice and Bob subsequently communicate, they will have the **same** key as before, unless they choose new public-keys
- attacker needs an x, must solve discrete log

Global Public Elements	
q	prime number
α	$\alpha < q$ and α a primitive root of q

User A Key Generation	
Select private X_A	$X_A < q$
Calculate public Y_A	$Y_A = \alpha^{X_A} \bmod q$

User B Key Generation	
Select private X_B	$X_B < q$
Calculate public Y_B	$Y_B = \alpha^{X_B} \bmod q$

Calculation of Secret Key by User A

$K = (Y_B)^{X_A} \bmod q$

Calculation of Secret Key by User B

$K = (Y_A)^{X_B} \bmod q$

Figure 10.7 The Diffie-Hellman Key Exchange Algorithm

Diffie-Hellman Example

- users Alice & Bob who wish to swap keys:
- agree on prime $q=353$ and $a=3$
- select random secret keys:
  - A chooses $x_A=97$, B chooses $x_B=233$
- compute respective public keys:
  - $y_A=3^{97} \bmod 353 = 40$ (Alice)
  - $y_B=3^{233} \bmod 353 = 248$ (Bob)
- compute shared session key as:
  - $K_{AB}= y_B^{xA} \bmod 353 = 248^{97} = 160$ (Alice)
  - $K_{AB}= y_A^{xB} \bmod 353 = 40^{233} = 160$ (Bob)

Key Exchange Protocols

- users could create random private/public D-H keys each time they communicate
- users could create a known private/public D-H key and publish in a directory, then consulted and used to securely communicate with them
- both of these are vulnerable to a meet-in-the-Middle Attack
- authentication of the keys is needed

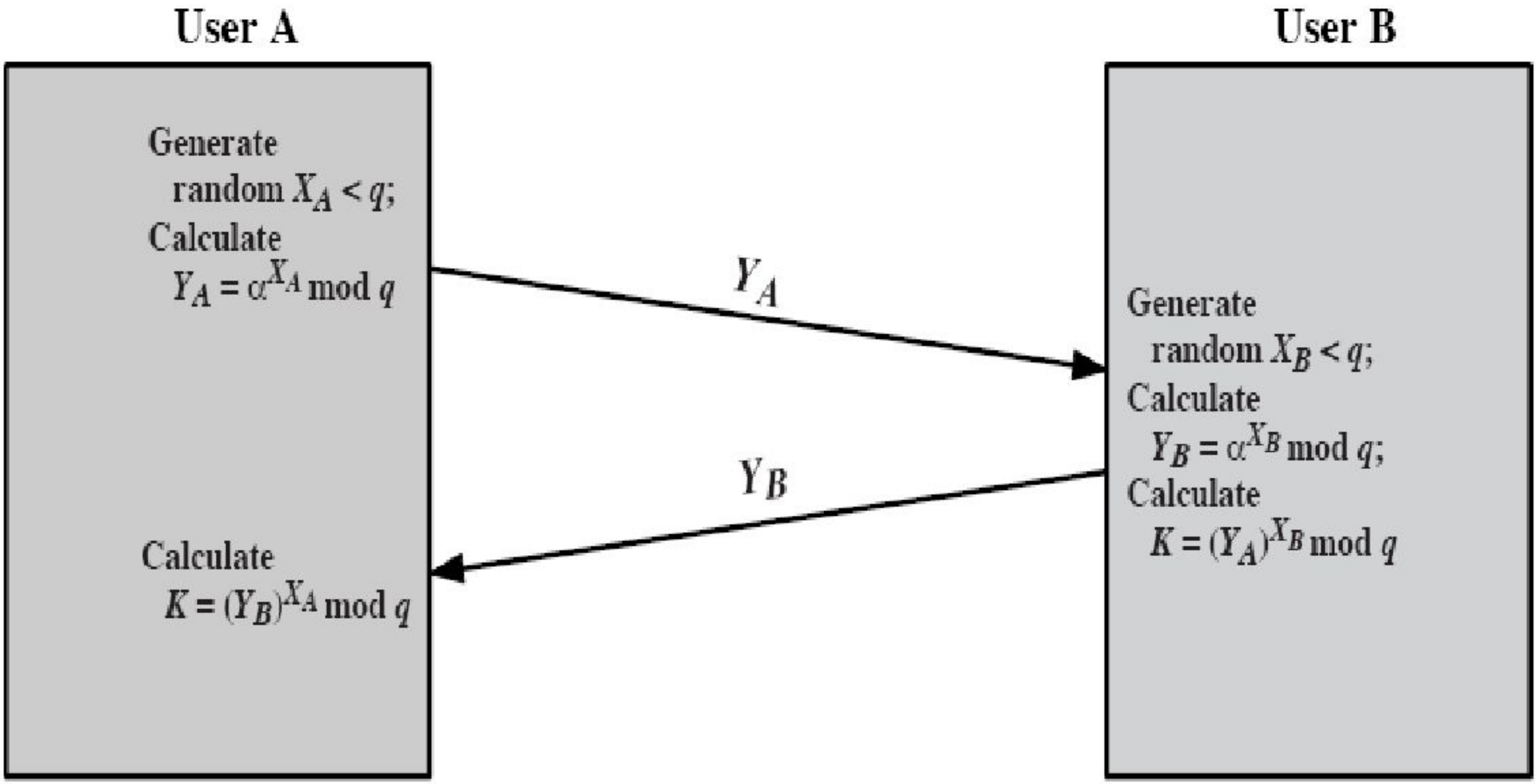


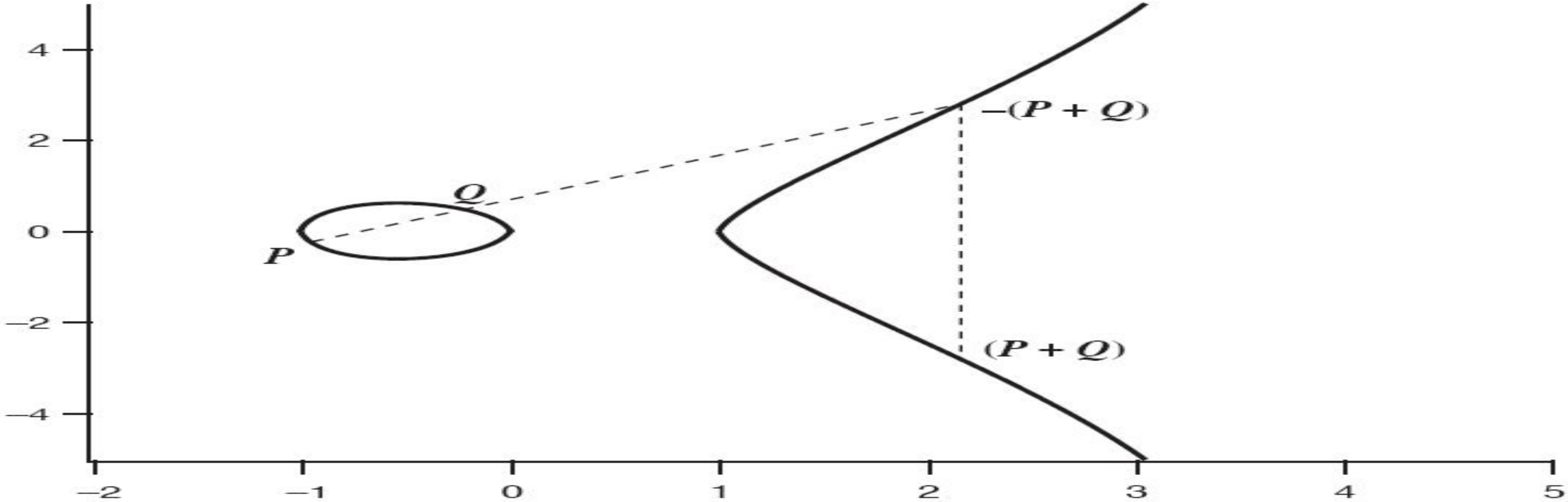
Figure 10.8 Diffie-Hellman Key Exchange

Elliptic Curve Cryptography

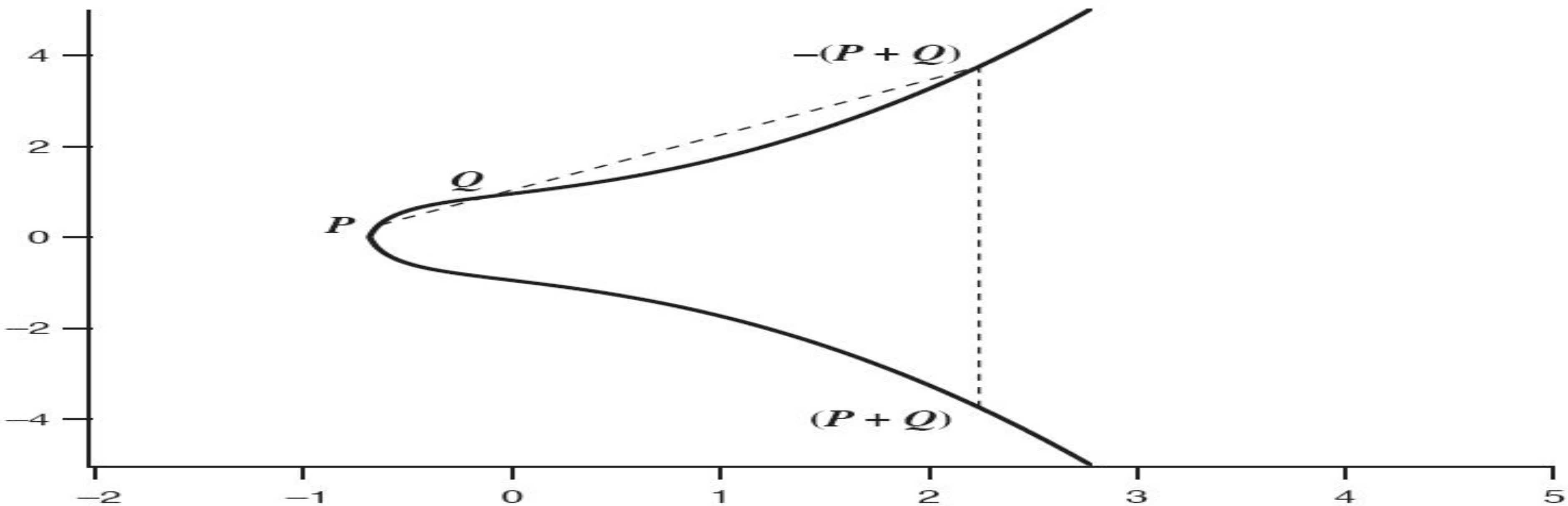
- majority of public-key crypto (RSA, D-H) use either integer or polynomial arithmetic with very large numbers/polynomials
- imposes a significant load in storing and processing keys and messages
- an alternative is to use elliptic curves
- offers same security with smaller bit sizes
- newer, but not as well analysed

Real Elliptic Curves

- an elliptic curve is defined by an equation in two variables x & y, with coefficients
- consider a cubic elliptic curve of form
  - $y^2 = x^3 + ax + b$
  - where x,y,a,b are all real numbers
  - also define zero point O
- have addition operation for elliptic curve
  - geometrically sum of Q+R is reflection of intersection R



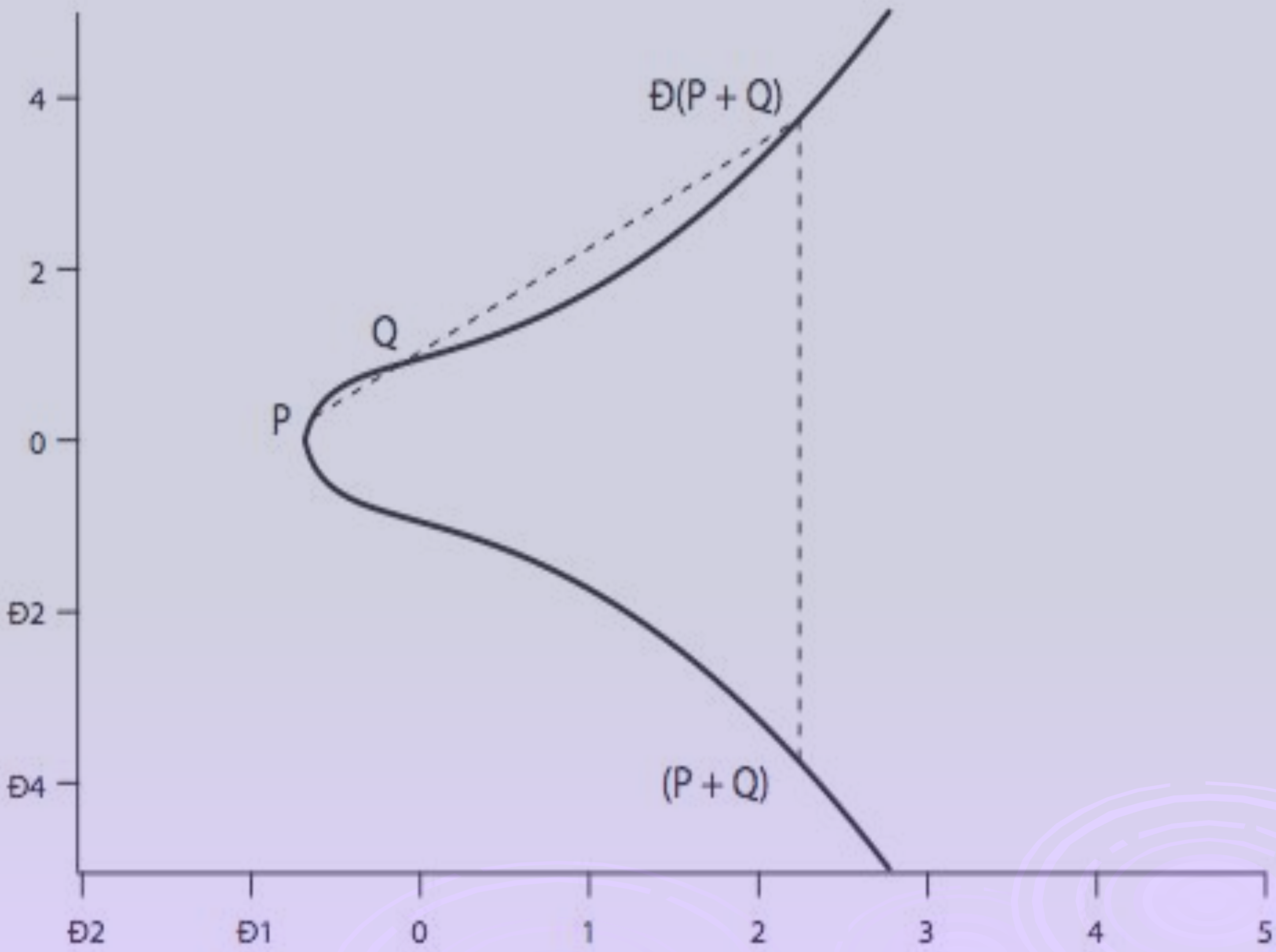
(a) $y^2 = x^3 - x$



(b) $y^2 = x^3 + x + 1$

Figure 10.9 Example of Elliptic Curves

Real Elliptic Curve Example



(b) $y^2 = x^3 + x + 1$

Finite Elliptic Curves

- Elliptic curve cryptography uses curves whose variables & coefficients are finite
- have two families commonly used:
  - prime curves $E_p(a,b)$ defined over Z_p
    - use integers modulo a prime
    - best in software
  - binary curves $E_{2m}(a,b)$ defined over $GF(2^n)$
    - use polynomials with binary coefficients
    - best in hardware

Elliptic Curve Cryptography

- ECC addition is analog of modulo multiply
- ECC repeated addition is analog of modulo exponentiation
- need “hard” problem equiv to discrete log
  - $Q=kP$, where Q,P belong to a prime curve
  - is “easy” to compute Q given k,P
  - but “hard” to find k given Q,P
  - known as the elliptic curve logarithm problem
- Certicom example: $E_{23}(9,17)$

$Y^2 \bmod 23 = x^3+9x+17 \bmod 23$

Q=(4,5)

P=(16,5)

What is value of k ?

P=(16,5)

2P=(20,20)

3P=(14,14)

4P=(19,20)

5P=(13,10)

6P=(7,3)

7P=(8,7)

8P=(12,17)

9P=(4,5)

K=9 Answer

ECC Diffie-Hellman

- can do key exchange analogous to D-H
- users select a suitable curve $E_p(a,b)$
- select base point $G=(x_1,y_1)$
  - with large order n s.t. $nG=O$
- A & B select private keys $n_A<n, \ n_B<n$
- compute public keys: $P_A=n_AG, \ P_B=n_BG$
- compute shared key: $K=n_AP_B, \ K=n_BP_A$
  - same since $K=n_An_BG$

ECC Encryption/Decryption

- several alternatives, will consider simplest
- must first encode any message M as a point on the elliptic curve P_m
- select suitable curve & point G as in D-H
- each user chooses private key $n_A < n$
- and computes public key $P_A = n_A G$
- to encrypt P_m : $C_m = \{kG, \; P_m + kP_b\}$, k random
- decrypt C_m compute:

$$P_m + kP_b - n_B(kG) = P_m + k(n_B G) - n_B(kG) = P_m$$

ECC Security

- relies on elliptic curve logarithm problem
- fastest method is “Pollard rho method”
- compared to factoring, can use much smaller key sizes than with RSA etc
- for equivalent key lengths computations are roughly equivalent
- hence for similar security ECC offers significant computational advantages

Comparable Key Sizes for Equivalent Security

Symmetric scheme (key size in bits)	ECC-based scheme (size of n in bits)	RSA/DSA (modulus size in bits)
56	112	512
80	160	1024
112	224	2048
128	256	3072
192	384	7680
256	512	15360

Summary

- have considered:
  - distribution of public keys
  - public-key distribution of secret keys
  - Diffie-Hellman key exchange
  - Elliptic Curve cryptography