This mapping of a mesh into a hypercube has certain useful properties. All nodes in the same row of the mesh are mapped to hypercube nodes whose labels have r identical most significant bits. We know that fixing any r bits in the node label of an (r + s)-dimensional hypercube yields a subcube of dimension s with 2^s nodes. Since each mesh node is mapped onto a unique node in the hypercube, and each row in the mesh has 2^s nodes, every row in the mesh is mapped to a distinct subcube in the hypercube. Similarly, each column in the mesh is mapped to a distinct subcube in the hypercube.

70) Explain the decomposition techniques for achieving concurrency.

Answer: One of the fundamental steps that we need to undertake to solve a problem in parallel is to split the computations to be performed into a set of tasks for concurrent execution defined by the task-dependency graph.

These techniques are broadly classified as **recursive decomposition**, **data-decomposition**, **exploratory decomposition**, and **speculative decomposition**. The recursive- and data-decomposition techniques are relatively general purpose as they can be used to decompose a wide variety of problems. On the other hand, speculative- and exploratory-decomposition techniques are more of a special purpose nature because they apply to specific classes of problems.

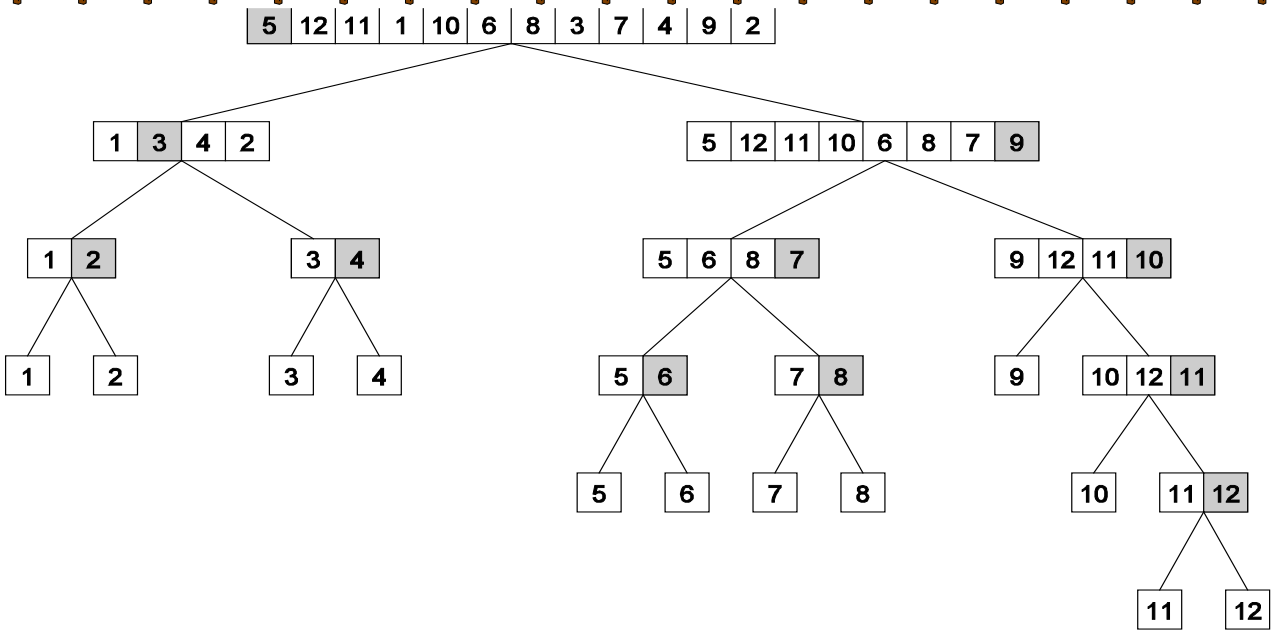
Recursive Decomposition

Recursive decomposition is a method for inducing concurrency in problems that can be solved using the divide-and-conquer strategy. In this technique, a problem is solved by first dividing it into a set of independent subproblems. Each one of these subproblems is solved by recursively applying a similar division into smaller subproblems followed by a combination of their results. The divide-and-conquer strategy results in natural concurrency, as different subproblems can be solved concurrently.

Example : Quicksort

Consider the problem of sorting a sequence A of n elements using the commonly used quicksort algorithm. Quicksort is a divide and conquer algorithm that starts by selecting a pivot element x and then partitions the sequence A into two subsequences A_0 and A_1 such that all the elements in A_0 are smaller than x and all the elements in A_1 are greater than or equal to x. This partitioning step forms the divide step of the algorithm. Each one of the subsequences A_0 and A_1 is sorted by recursively calling quicksort. Each one of these recursive calls further partitions the sequences. The recursion terminates when each subsequence contains only a single element.

Figure : The quicksort task-dependency graph based on recursive decomposition for sorting a sequence of 12 numbers.



In Figure, we define a task as the work of partitioning a given subsequence. Initially, there is only one sequence (i.e., the root of the tree), and we can use only a single process to partition it. The completion of the root task results in two subsequences (A_0 and A_1, corresponding to the two nodes at the first level of the tree) and each one can be partitioned in parallel. Similarly, the concurrency continues to increase as we move down the tree.

Data Decomposition

Data decomposition is a powerful and commonly used method for deriving concurrency in algorithms that operate on large data structures. In this method, the decomposition of computations is done in two steps.

In the first step, the data on which the computations are performed is partitioned, and in the second step, this data partitioning is used to induce a partitioning of the computations into tasks. The operations that these tasks perform on different data partitions are usually similar (e.g., matrix multiplication) or are chosen from a small set of operations (e.g., LU factorization)

The partitioning of data can be performed in many possible ways as discussed next. In general, one must explore and evaluate all possible ways of partitioning the data and determine which one yields a natural and efficient computational decomposition.

Partitioning Output Data : In many computations, each element of the output can be computed independently of others as a function of the input. In such computations, a partitioning of the output data automatically induces a decomposition of the problems into tasks, where each task is assigned the work of computing a portion of the output.

Example : Matrix multiplication

Consider the problem of multiplying two n x n matrices A and B to yield a matrix C. Figure shows a decomposition of this problem into four tasks. Each matrix is considered to be composed of four blocks or submatrices defined by splitting each dimension of the matrix into half. The four submatrices of C, roughly of size n/2 x n/2 each, are then independently computed by four tasks as the sums of the appropriate products of submatrices of A and B.

Figure : (a) Partitioning of input and output matrices into 2 x 2 submatrices. (b) A decomposition of matrix multiplication into four tasks based on the partitioning of the matrices in (a).

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

$$C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$$
$$C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$$
$$C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$$
$$C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$$

Most matrix algorithms, including matrix-vector and matrix-matrix multiplication, can be formulated in terms of block matrix operations. In such a formulation, the matrix is viewed as composed of blocks or submatrices and the scalar arithmetic operations on its elements are replaced by the equivalent matrix operations on the blocks.

Partitioning Input Data : Partitioning of output data can be performed only if each output can be naturally computed as a function of the input. In many algorithms, it is not possible or desirable to partition the output data. For example, while finding the minimum, maximum, or the sum of a set of numbers, the output is a single unknown value. In a sorting algorithm, the individual elements of the output cannot be efficiently determined in isolation. In such cases, it is sometimes possible to partition the input data, and then use this partitioning to induce concurrency. A task is created for each partition of the input data and this task performs as much computation as possible using these local data. Note that the solutions to tasks induced by input partitions may not directly solve the original problem. In such cases, a follow-up computation is needed to combine the results. For example, while finding the sum of a sequence of N numbers using p processes (N > p), we can partition the input into p subsets of nearly equal sizes. Each task then computes the sum of the numbers in one of the subsets. Finally, the p partial results can be added up to yield the final result.

Partitioning both Input and Output Data : In some cases, in which it is possible to partition the output data, partitioning of input data can offer additional concurrency.

Partitioning Intermediate Data : Algorithms are often structured as multi-stage computations such that the output of one stage is the input to the subsequent stage. A decomposition of such an algorithm can be derived by partitioning the input or the output data of an intermediate stage of the algorithm. Partitioning intermediate data can sometimes lead to higher concurrency than partitioning input or output data. Often, the intermediate data are not generated explicitly in the serial algorithm for solving the problem and some restructuring of the original algorithm may be required to use intermediate data partitioning to induce a decomposition.

The Owner-Computes Rule : A decomposition based on partitioning output or input data is also widely referred to as the **owner-computes rule**. The idea behind this rule is that each partition performs all the computations involving data that it owns. Depending on the nature of the data or the type of data-partitioning, the owner-computes rule may mean different things. For instance, when we assign partitions of the input data to tasks, then the owner-computes rule means that a task performs all the computations that can be done using these data. On the other hand, if we partition the output data, then the owner-computes rule means that a task computes all the data in the partition assigned to it.

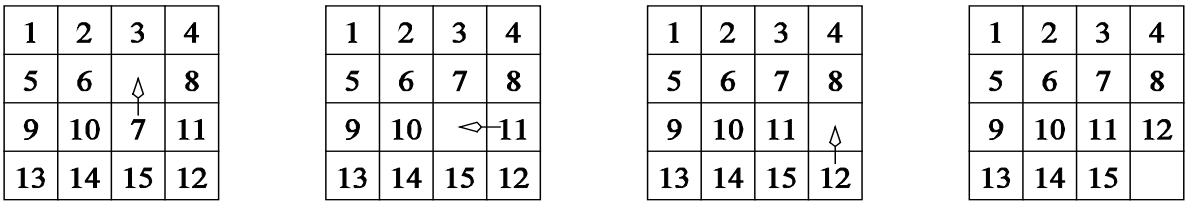
Exploratory Decomposition

Exploratory decomposition is used to decompose problems whose underlying computations correspond to a search of a space for solutions. In exploratory decomposition, we partition the search space into smaller parts, and search each one of these parts concurrently, until the desired solutions are found. For an example of exploratory decomposition, consider the 15-puzzle problem.

Example : The 15-puzzle problem

The 15-puzzle consists of 15 tiles numbered 1 through 15 and one blank tile placed in a 4 x 4 grid. A tile can be moved into the blank position from a position adjacent to it, thus creating a blank in the tile's original position. Depending on the configuration of the grid, up to four moves are possible: up, down, left, and right. The initial and final configurations of the tiles are specified. The objective is to determine any sequence or a shortest sequence of moves that transforms the initial configuration to the final configuration. The Figure illustrates sample initial and final configurations and a sequence of moves leading from the initial configuration to the final configuration.

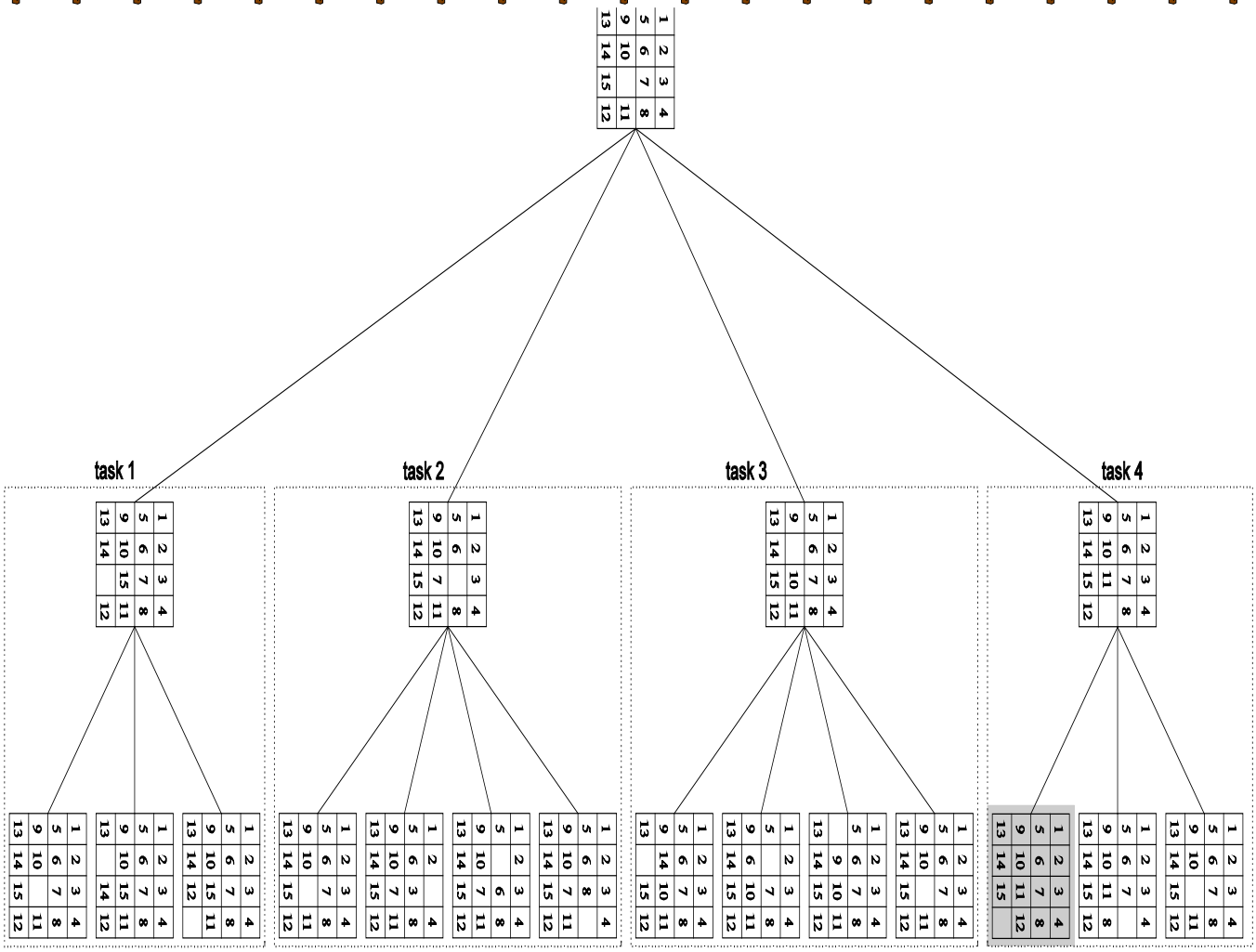
Figure. A 15-puzzle problem instance showing the initial configuration (a), the final configuration (d), and a sequence of moves leading from the initial to the final configuration.



The 15-puzzle is typically solved using tree-search techniques. Starting from the initial configuration, all possible successor configurations are generated. A configuration may have 2, 3, or 4 possible successor configurations, each corresponding to the occupation of the empty slot by one of its neighbors. The task of finding a path from initial to final configuration now translates to finding a path from one of these newly generated configurations to the final configuration. Since one of these newly generated configurations must be closer to the solution by one move (if a solution exists), we have made some progress towards finding the solution. The configuration space generated by the tree search is often referred to as a state space graph. Each node of the graph is a configuration and each edge of the graph connects configurations that can be reached from one another by a single move of a tile.

One method for solving this problem in parallel is as follows. First, a few levels of configurations starting from the initial configuration are generated serially until the search tree has a sufficient number of leaf nodes (i.e., configurations of the 15-puzzle). Now each node is assigned to a task to explore further until at least one of them finds a solution. As soon as one of the concurrent tasks finds a solution it can inform the others to terminate their searches.The following figure illustrates one such decomposition into four tasks in which task 4 finds the solution.

Figure : The states generated by an instance of the 15-puzzle problem.



Note that even though exploratory decomposition may appear similar to data-decomposition (the search space can be thought of as being the data that get partitioned) it is fundamentally different in the following way. The tasks induced by data-decomposition are performed in their entirety and each task performs useful computations towards the solution of the problem. On the other hand, in exploratory decomposition, unfinished tasks can be terminated as soon as an overall solution is found. Hence, the portion of the search space searched (and the aggregate amount of work performed) by a parallel formulation can be very different from that searched by a serial algorithm. The work performed by the parallel formulation can be either smaller or greater than that performed by the serial algorithm.

Speculative Decomposition

Speculative decomposition is used when a program may take one of many possible computationally significant branches depending on the output of other computations that precede it. In this situation, while one task is performing the computation whose output is used in deciding the next computation, other tasks can concurrently start the computations of the next stage. This scenario is similar to evaluating one or more of the branches of a switch statement in C in parallel before the input for the switch are available. While one task is performing the computation that will eventually resolve the switch, other tasks could pick up the multiple branches of the switch in parallel. When the input for the switch has finally been computed, the computation corresponding to the correct branch would be used while that corresponding to the other branches would be discarded. The parallel run time is smaller than the serial run time by the amount of time required evaluating the

condition on which the next task depends because this time is utilized to perform a useful computation for the next stage in parallel. However, this parallel formulation of a switch guarantees at least some wasteful computation. In order to minimize the wasted computation, a slightly different formulation of speculative decomposition could be used, especially in situations where one of the outcomes of the switch is more likely than the others. In this case, only the most promising branch is taken up a task in parallel with the preceding computation. In case the outcome of the switch is different from what was anticipated, the computation is rolled back and the correct branch of the switch is taken.

The speedup due to speculative decomposition can add up if there are multiple speculative stages. An example of an application in which speculative decomposition is useful is discrete event simulation. A detailed description of discrete event simulation is beyond the scope of this chapter; however, we give a simplified description of the problem.

Speculative decomposition is different from exploratory decomposition in the following way.

In speculative decomposition, the input at a branch leading to multiple parallel tasks is unknown, whereas in exploratory decomposition, the output of the multiple tasks originating at a branch is unknown.

In speculative decomposition, the serial algorithm would strictly perform only one of the tasks at a speculative stage because when it reaches the beginning of that stage, it knows exactly which branch to take. Therefore, by preemptively computing for multiple possibilities out of which only one materializes, a parallel program employing speculative decomposition performs more aggregate work than its serial counterpart. Even if only one of the possibilities is explored speculatively, the parallel algorithm may perform more or the same amount of work as the serial algorithm. On the other hand, in exploratory decomposition, the serial algorithm too may explore different alternatives one after the other, because the branch that may lead to the solution is not known beforehand. Therefore, the parallel program may perform more, less, or the same amount of aggregate work compared to the serial algorithm depending on the location of the solution in the search space.

Hybrid Decompositions

So far we have discussed a number of decomposition methods that can be used to derive concurrent formulations of many algorithms. These decomposition techniques are not exclusive, and can often be combined together. Often, a computation is structured into multiple stages and it is sometimes necessary to apply different types of decomposition in different stages.

71) What are the characteristics of Inter-Task Interactions?

Answer: In any nontrivial parallel algorithm, tasks need to interact with each other to share data, work, or synchronization information. Different parallel algorithms require different types of interactions among concurrent tasks. The nature of these interactions makes them more suitable for certain programming paradigms and mapping schemes, and less suitable for others. The types of inter-task interactions can be described along different dimensions, each corresponding to a distinct characteristic of the underlying computations.

Static versus Dynamic One way of classifying the type of interactions that take place among concurrent tasks is to consider whether or not these interactions have a static or dynamic pattern. An interaction pattern is static if for each task, the interactions happen at predetermined times, and if the set of tasks to interact with at these times is known prior to the execution of the algorithm. In other words, in a static interaction pattern, not only is the task-interaction graph known a priori, but the stage of the computation