Impossibility Results

The best can be found using hamming bound proof: -

$$2^{k} \leq \frac{2^{n}}{\sum_{f=0}^{\frac{d-1}{2}} (\frac{n}{f})}$$
$$2^{k} (\sum_{f=0}^{\frac{d-1}{2}} (\frac{n}{f}) \leq 2^{n}$$

For 15: -

$$\begin{array}{c} 2^{10} \times (20C_0 + 20C_1 + 20C_2 + 20C_3 + 20C_4 + 20C_5) \leq 2^{20} \\ 2^{10} \times (1 + 20 + 190 + 1140 + 4845 + 15504) \leq 1,048,576 \\ 1024 \times (21,700) \leq 1,048,576 \\ 22,220,800 \not \leq 1,048,576 \end{array}$$

Possible

For 16: -

$$2^{10} \times (1 + 20 + 190 + 1140 + 4845) \le 1,048,576$$

 $1024 \times (6,196) \le 1,048,576$
 $6,344,704 \le 1,048,576$

Possible

For 17: -

$$2^{10} \times (1 + 20 + 190 + 1140) \le 1,048,576$$

 $1024 \times (1,351) \le 1,048,576$
 $1,383,424 \le 1,048,576$

Possible

For 18: -

$$2^{10} \times (1 + 20 + 190) \le 1,048,576$$

 $1024 \times (1,211) \le 1,048,576$
 $216,064 \le 1,048,576$

Impossible

The best Possible result is 17.