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Exoplanet Demographics and Method Sensitivity using the NASA Exoplanet Archive

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1. INTRODUCTION

In the past couple decades the quantity of known exoplanets has increased near-exponentially. One of the main drivers for discovering planets is finding Earth's twin, a habitable terrestrial planet. With such a tough task ahead, understanding our means of detection can guide the decisions of future missions and technological advancements. With improvements in ground and space-based observatories, like the proposed *Habitable Worlds Observatory*, we may be able to catch sight of these faraway worlds. By understanding the biases of the methods we utilize, we can better constrain the parameter spaces these exoplanets occupy. We can also predict whether or not specific methods in their current state can detect an Earth-like planet around a solar-type star.

2. METHODS

2.1. *Transit*

The radius of a planet can be derived by using the signal-to-noise equation given by Howard et al. (2012) which defines the signal-to-noise integrated over all observed transits given a Kepler planet:

$$SNR = \frac{\delta}{\sigma_{CDPP}}\sqrt{\frac{n_{tr}t_{dur}}{3hr}} \tag{1}$$

Assuming Kepler criterion, we can assume a observational period of 90 days, allowing us to substitute:

$$n_{tr} = 90d/p \tag{2}$$

Additionally, we can express the transit depth as:

$$\delta = R_{PL}^2/R_*^2 \tag{3}$$

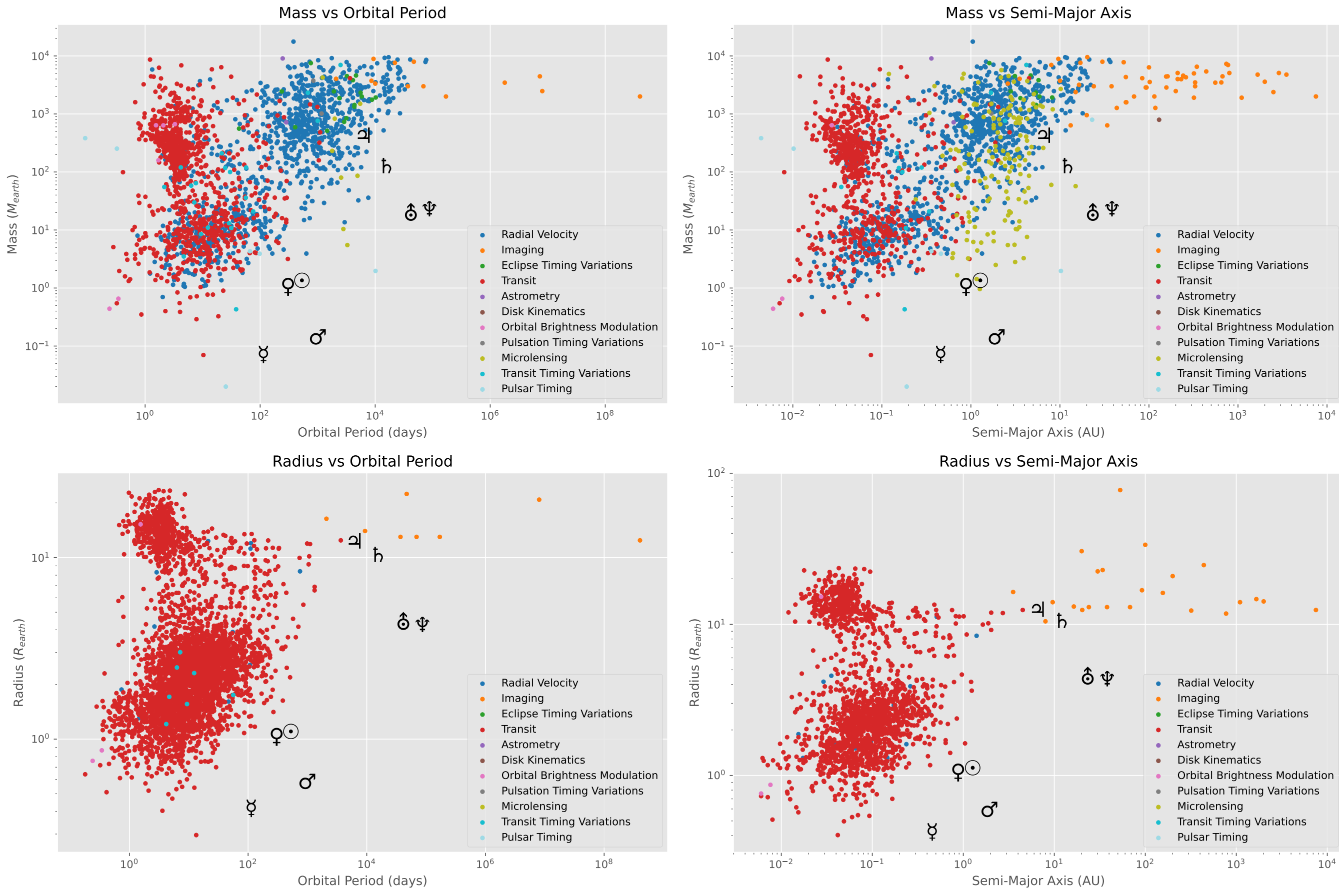

Figure 1. Exoplanet demographics for various methods.

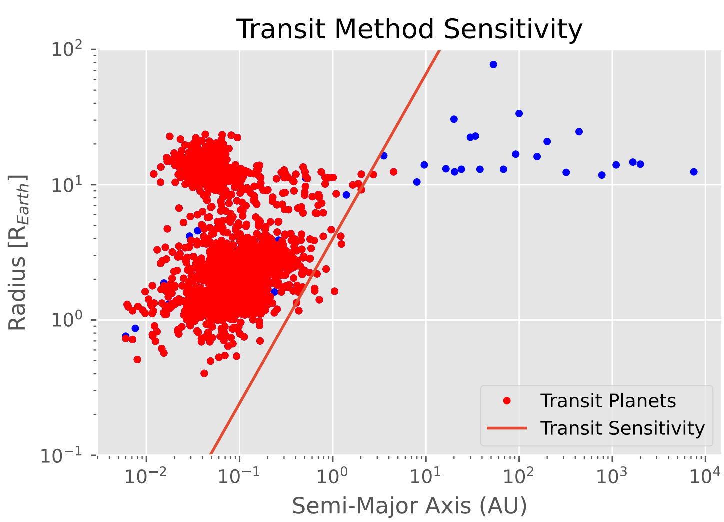

Figure 2. Sensitivity limit for planets found via the transit method.

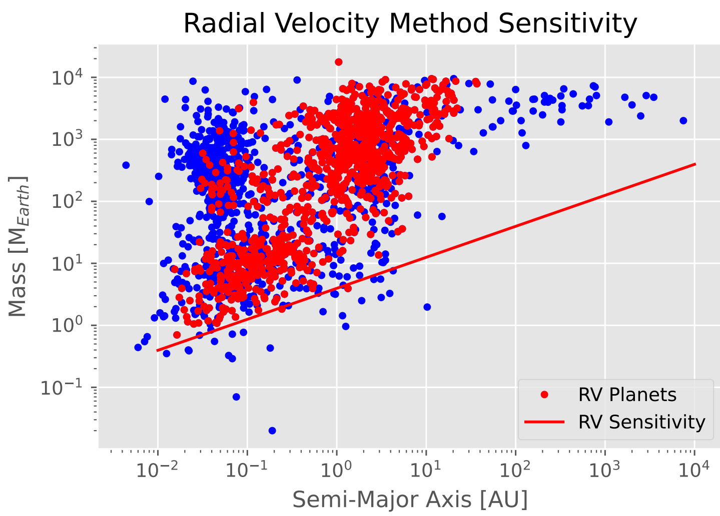

Figure 3. Sensitivity for detecting planets via Radial Velocity.

given by Winn (2010). If we assume a host star of solar density we can express our transit duration as

$$t_{dur} = 3.91hr(P/10d)^{1/3} \tag{4}$$

Substituting these into equation (1), we can solve for the radius of a given planet:

$$R_{PL} = a \cdot (P/d)^{1/6} \tag{5}$$

To find an Earth-like planet around a solar type star, a SNR value of at least 8[1] is required.

2.2. *Radial Velocity*

The Doppler shifted velocity and mass/volume relations are given by

$$K = V_* sin(i) \tag{6}$$

$$M_{PL}V_{PL} = M_* V_* \tag{7}$$

where K is the amplitude of the radial velocity signal. V_* can further be expressed as $\frac{M_p}{M_*} \cdot \sqrt{\frac{GM_*}{a}}$. If we the assume an edge-on orbit, for which i is 90 degrees, we can express M_{PL} as:

$$M_{PL} = K \cdot M_* \cdot \sqrt{\frac{a}{GM_*}} \tag{8}$$

[1] https://www.nasa.gov/kepler/overview/abouttransits

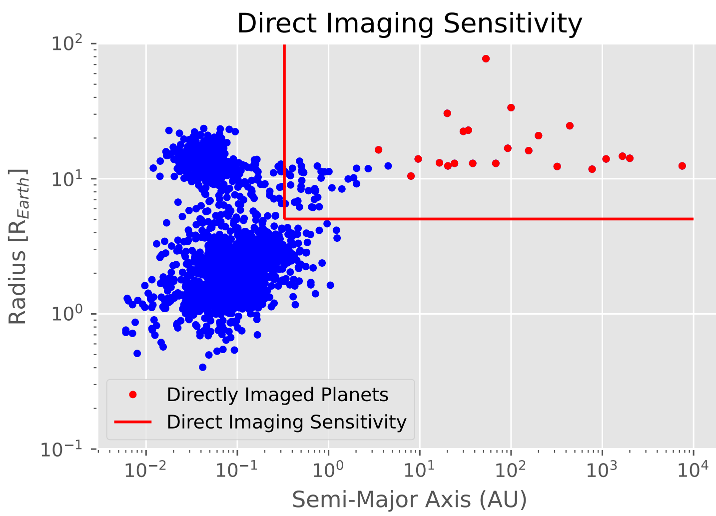

Figure 4. Sensitivity limits of directly imaged planets.

With state of the art technology, to detect planets using this method, we will need a radial velocity signal of at least 0.5 m/s. Using Eqn. 8, we can plot the sensitivity for detecting a planet with the radial velocity technique, as seen in Fig. 3.

2.3. *Direct Imaging*

In order to directly image exoplanets, there are roadblocks which become apparent. The most obvious of these issues is that in comparison to the host star, the planet is very dim. This requires the planet be on a long orbit, far from their host. We can calculate some of these limitations, making use of equations published by Traub & Oppenheimer (2010). The contrast in observed flux can be defined as:

$$C = \frac{f_\lambda(p)}{f_\lambda(s)} \tag{9}$$

Where $f_\lambda(p)$ is the incoming flux of the planet (and similarly for the host star). These can be directly related to the photon flux.

$$f_\lambda = \dot{N}_\lambda = \dot{n}_\lambda(T)\Omega \tag{10}$$

$$= \frac{6 \times 10^{26}}{\lambda_{\mu m}^4 (e^{14388/\lambda_{\mu m}T} - 1)} \frac{\pi R^2}{d^2} \tag{11}$$

Where Ω is the solid angle, R is the radius of an object of distance d away. T is the effective temperature, if treated as a blackbody. We then can plug this into our equation for contrast, Eqn. 9, for both the planet and host star.

$$C = \frac{f_\lambda(p)}{f_\lambda(s)} = \left(\frac{R_p}{R_*}\right)^2 \frac{e^{14388/\lambda T_*} - 1}{e^{14388/\lambda T_p} - 1} \tag{12}$$

This contrast needed to detect Earth-like planets is approximately 10^{-9} to 10^{-10}, but many state-of-the-art surveys have contrasts of order 10^{-7} (Liu et al. 2015; Li et al. 2021). By utilizing this contrast limit, we find that our detection sensitivity is roughly around 5.0 Earth Radii. This is done by utilizing the peak blackbody wavelength of $10\mu m$ as well as blackbody temperatures of $165K$ and $5700K$ for the Jupiter and Sun, respectively. We also try and place constraints on the semi-major axis (a) of detected exoplanets using direct imagining. We do so by finding the angular resolution required, assuming a standard telescopes mirror diameter of $D = 10m$ and using the closest system (d) to us according to the NEA.

$$\theta = \frac{a}{d} \approx 1.22\frac{\lambda}{D} \tag{13}$$

This resulted in a sensitivity limit of approximately $0.33au$, with both of these limits seen in Fig. 4

3. RESULTS

In the search for habitable life in the universe, one might aim to search for analogs to our own solar system. We utilized the limits above to test whether an Earth-like planet around a sun-like star is detectable using current telescopes and techniques.

We used Eqn. 1 to determine the detectability of this proposed system using the transit technique. According to Howard et al. (2012), a standard σ_{CDPP} is approximately 30ppm. Using this adopted value, along with standard values for the solar system, we are able to calculate a SNR for an Earth transit. This SNR would be approximately 2.9, far below the detection threshold set by the *Kepler* mission. By utilizing Eqn. 8, we can calculate the radial velocity signal for the Earth-Sun system. Our radial velocity signal amplitude, K, ends up approximating to $0.089m/s$, which is lower that the current state-of-the-art detection capabilities. For direct imaging, we utilized a blackbody peak wavelength of approximately $10\mu m$ for Earth and effective temperatures of $252K$ and $5700K$ for the Earth and Sun, respectively. When used alongside Eqn. 12, we result in a required contrast of 8.0×10^{-8} which is right around the boundary of current techniques.

4. DISCUSSION

Over 5,000 exoplanets have been detected to date using various methods. Of these methods, transit, radial velocity, and direct imaging stand out as discovering the most planets. With such a large population, we can understand the limits each technique has in their ability to make a significant detection. We also showed how an Earth-like planet is not able to be detected easily by with current technology and technique. With the advancement of the *James Webb Space Telescope* and *Nancy Grace Roman Space Telescope*, we may be able to push the sensitivity boundary and find planets like home.

5. ACKNOWLEDGEMENTS

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