2020 年度 確率・統計　試験問題略解

問題 1

(1) $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

(2) 32 通りのうち、5 人とも男である可能性は無いので、$\frac{1}{32-1} = \frac{1}{31}$

(3) 一番上が女の子であるのは、31 通りのうちの 16 通りだから、$\frac{16}{32-1} = \frac{16}{31}$

(4) 残りの 4 人が女の子である確率だから、$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

問題 2

(1) $\mathrm{P}(1_{\text{当}}) = \frac{2}{100} = \frac{1}{50}$

(2) $\mathrm{P}(1_{\text{外}}) = \frac{98}{100} = \frac{49}{50}$

(3) $\mathrm{P}(2_{\text{当}}|1_{\text{当}}) = \frac{1}{99}$

(4) $\mathrm{P}(2_{\text{当}}|1_{\text{外}}) = \frac{2}{99}$

(5) $\mathrm{P}(2_{\text{当}}) = \mathrm{P}(2_{\text{当}}|1_{\text{当}})\mathrm{P}(1_{\text{当}}) + \mathrm{P}(2_{\text{当}}|1_{\text{外}})\mathrm{P}(1_{\text{外}})$

(6) $\mathrm{P}(2_{\text{当}}) = \frac{1}{99}\cdot\frac{1}{50} + \frac{2}{99}\cdot\frac{49}{50} = \frac{1+98}{99\cdot 50} = \frac{1}{50}$

問題 3

(1) (答)
$$\begin{aligned}\mathrm{P}(\text{青}_{\text{目}}) &= \mathrm{P}(\text{青}_{\text{目}}|\,\text{青}_{\text{走}})\mathrm{P}(\text{青}_{\text{走}}) + \mathrm{P}(\text{青}_{\text{目}}|\,\text{緑}_{\text{走}})\mathrm{P}(\text{緑}_{\text{走}}) \\ &= 0.85 \times 0.2 + 0.15 \times 0.8 = 0.29\end{aligned}$$

(2) (答)
$$\begin{aligned}\mathrm{P}(\text{緑}_{\text{目}}) &= \mathrm{P}(\text{緑}_{\text{目}}|\,\text{緑}_{\text{走}})\mathrm{P}(\text{緑}_{\text{走}}) + \mathrm{P}(\text{緑}_{\text{目}}|\,\text{青}_{\text{走}})\mathrm{P}(\text{青}_{\text{走}}) \\ &= 0.85 \times 0.8 + 0.15 \times 0.2 = 0.71\end{aligned}$$

(3) (答) $\mathrm{P}(\text{青}_{\text{事故}}) = \frac{\mathrm{P}(\text{青}_{\text{目}}|\,\text{青}_{\text{走}})\mathrm{P}(\text{青}_{\text{走}})}{\mathrm{P}(\text{青}_{\text{目}})} = \frac{0.85 \times 0.2}{0.29} = \frac{0.17}{0.29} = \frac{17}{29} \approx 0.586$

問題 4

[1]

(1) $\mathrm{P}(1_{\text{当}}) = \frac{1}{60}$

(2) $\mathrm{P}(1_{\text{外}}) = \frac{59}{60}$

(3) $\mathrm{P}(2_{\text{当}}|1_{\text{外}}) = \frac{1}{59}$

(4) $\mathrm{P}(2_{\text{当}}) = \mathrm{P}(2_{\text{当}}|1_{\text{外}})\mathrm{P}(1_{\text{外}}) = \frac{1}{59} \times \frac{59}{60} = \frac{1}{60}$

(5) $\mathrm{P}(1_{\text{当}}) + \mathrm{P}(2_{\text{当}}) = \frac{1}{30}\left(= \frac{1 \times 59}{{}_{60}C_2} = \frac{2 \times 59}{60 \times 59}\right)$

(6) $\mathrm{P}(2_{\text{外}} \cap 1_{\text{外}}) = \mathrm{P}(2_{\text{外}}|1_{\text{外}})\mathrm{P}(1_{\text{外}}) = \frac{58}{59}\cdot\frac{59}{60} = \frac{58}{60} = \frac{29}{30}\left(= \frac{{}_{59}C_2}{{}_{60}C_2}\right)$

(7) $\mu = 90 \times \frac{1}{30} = 3$ 円

(8) $\sigma^2 = 90^2 \times \frac{1}{30} - 3^2 = 90 \times \frac{90}{30} - 9 = 270 - 9 = 261$

[2]

(1) $\mathrm{P}(2_{当} \cap 1_{当}) = \mathrm{P}(2_{当}|1_{当})\mathrm{P}(1_{当}) = \frac{1}{119} \cdot \frac{2}{120} = \frac{1}{7140} \left(= \frac{1}{{}_{120}C_2} \right)$

(2) $\mathrm{P}(2_{外} \cap 1_{当}) = \mathrm{P}(2_{外}|1_{当})\mathrm{P}(1_{当}) = \frac{118}{119} \frac{2}{120} = \frac{118}{7140} = \frac{59}{3570}$

(3) $\mathrm{P}(2_{当} \cap 1_{外}) = \mathrm{P}(2_{当}|1_{外})\mathrm{P}(1_{外}) = \frac{2}{119} \cdot \frac{118}{120} = \frac{118}{7140} = \frac{59}{3570}$

(4) $\mathrm{P}(2_{当} \cap 1_{外}) + \mathrm{P}(2_{外} \cap 1_{当}) = \frac{236}{7140} = \frac{118}{3570} = \frac{59}{1785} \left(= \frac{2 \times 118}{{}_{120}C_2} \right)$

(5) $\mathrm{P}(2_{外} \cap 1_{外}) = \mathrm{P}(2_{外}|1_{外})\mathrm{P}(1_{外}) = \frac{117}{119} \cdot \frac{118}{120} = \frac{6903}{7140} = \frac{2301}{2380} \left(= \frac{{}_{118}C_2}{{}_{120}C_2} \right)$

(6) $\mu = 90 \times \frac{236}{7140} + 180 \times \frac{1}{7140} = \frac{90 \cdot (236+2)}{7140} = 3$

(7) $\sigma^2 = 90^2 \times \frac{236}{7140} + 180^2 \times \frac{1}{7140} - 3^2$

$$= 90^2 \times \frac{(236+4)}{7140} - 9 = 90^2 \times \frac{240}{7140} - 9$$

$$= 8100 \times \frac{4}{119} - 9 = \frac{(8100 \times 4 - 119 \times 9)}{119} = \frac{31329}{119} \approx 263.2689$$

問題 5

(1) $p = \frac{1}{6}$

(2) $\mathrm{P}(x) = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{x-1}$

(3) $\mu = \frac{1}{p} = 6$

(4) $\sigma^2 = \frac{1-p}{p^2} = 30$

(5) $\mathrm{Q}(x) = \left(\frac{5}{6}\right)^{x}$

(6) $\mathrm{P}(x) + \mathrm{Q}(x) = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{x-1} + \left(\frac{5}{6}\right)^{x} = \left(\frac{5}{6}\right)^{x-1} = \mathrm{Q}(x-1)$

(7) $\mathrm{P}(1) + \mathrm{P}(2) + \cdots + \mathrm{P}(x-1) + (\mathrm{P}(x) + \mathrm{Q}(x) = \mathrm{P}(1) + \mathrm{P}(2) + \cdots + \mathrm{P}(x-1) + \mathrm{Q}(x-1)$
$= \cdots = \mathrm{P}(1) + \mathrm{Q}(1) = 1$

(8) (7) より、$\mathrm{P}(1) + \mathrm{P}(2) + \cdots + \mathrm{P}(n) = 1 - \mathrm{Q}(n)$ だから、
$1 - Q(n) > \frac{1}{2}$. したがって、$\frac{1}{2} > Q(n)$.
$\left(\frac{5}{6}\right)^2 = \frac{25}{36}, \left(\frac{5}{6}\right)^3 = \frac{75}{216}, \left(\frac{5}{6}\right)^4 = \frac{625}{1296}, \cdots$ なので、$\left(\frac{5}{6}\right)^4 < \frac{1}{2}$. ゆえに、$n = 4$

問題 6

(1) $\mathrm{P}(x) = {}_{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = \frac{{}_{10}C_x}{2^{10}}$

(2) $\mu = 10 \times \frac{1}{2} = 5$

(3) $\sigma^2 = 10 \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{2}$

(4) $\mathrm{P}(6) + \mathrm{P}(7) + \mathrm{P}(8) + \mathrm{P}(9) + \mathrm{P}(10) = \frac{1}{2^{10}} \times ({}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10})$
$= \frac{1}{1024} \times (210 + 120 + 45 + 10 + 1) = \frac{386}{1024} = \frac{193}{512}$