# 確率・統計 試験問題 2021 解答

### 問題1

 $(1) 2^3 = 8 通り$  それらは、

 $(1) = (\mathbb{H}, \mathbb{H}, \mathbb{H}), (2) = (\mathcal{L}, \mathbb{H}, \mathbb{H}), (3) = (\mathbb{H}, \mathcal{L}, \mathbb{H}), (4) = (\mathbb{H}, \mathbb{H}, \mathcal{L}),$ 

(2) 8 通りの組み合わせのうち問題の組み合わせは (8) の1 通りであるから、 $\frac{1}{8}$ 

(3) 8 通りの組み合わせのうち問題の組み合わせは、(5), (6), (7) の 3 通りであるから、 $\frac{3}{8}$ 

(4) 8 通りの組み合わせのうち問題の組み合わせは、(2), (3), (4) の 3 通りであるから、 $\frac{3}{8}$ 

(5) 8 通りのうち、3 人とも男である可能性は無いので、 $\frac{3}{8-1} = \frac{3}{7}$ 

(6)  $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 

## 問題2

(2)  $P(1_{5}) = \frac{195}{200} = \frac{39}{40}$ 

(3)  $P(2_{\sharp}|1_{\sharp}) = \frac{4}{199}$ 

(4)  $P(2_{\frac{1}{2}}|1_{\frac{1}{2}}) = \frac{5}{199}$ 

(5)  $P(2_{\sharp}) = P(2_{\sharp}|1_{\sharp})P(1_{\sharp}) + P(2_{\sharp}|1_{\hslash})P(1_{\hslash})$ 

(6)  $P(2_{\frac{1}{2}}) = \frac{4}{199} \cdot \frac{1}{40} + \frac{5}{199} \cdot \frac{39}{40} = \frac{4+5\times39}{199\times40} = \frac{1}{40}$ 

#### 問題3

(1) (答)  $P(\bar{\tau}_{\parallel}) = P(\bar{\tau}_{\parallel} | \bar{\tau}_{\pm}) P(\bar{\tau}_{\parallel}) + P(\bar{\tau}_{\parallel} | \bar{k}_{\pm}) P(\bar{k}_{\pm})$ =  $0.85 \times 0.2 + 0.15 \times 0.8 = 0.29$ 

(2) (答)  $P(k_{\parallel}) = P(k_{\parallel} | k_{\pm}) P(k_{\pm}) + P(k_{\parallel} | k_{\pm}) P(k_{\pm})$ =  $0.85 \times 0.8 + 0.15 \times 0.2 = 0.71$ 

(3) (答)  $P(f_{\pm}) = \frac{P(f_{\pm}|f_{\pm})P(f_{\pm})}{P(f_{\pm})} = \frac{0.85 \times 0.2}{0.29} = \frac{0.17}{0.29} = \frac{17}{29} \approx 0.586$ 

### 問題4

### [1]

(1) 
$$P(1_{\frac{\omega}{1}}) = \frac{1}{30}$$

(2) 
$$P(1_{5}) = \frac{29}{30}$$

(3) 
$$P(2 \leq |1_{4}|) = \frac{1}{29}$$

(4) 
$$P(2_{\underline{\exists}}) = P(2_{\underline{\exists}}|1_{\mathcal{H}})P(1_{\mathcal{H}}) = \frac{1}{29} \times \frac{29}{30} = \frac{1}{30}$$

(5) 
$$P(1_{\underline{\exists}}) + P(2_{\underline{\exists}}) = \frac{1}{15} \left( = \frac{1 \times 29}{30C_2} = \frac{2 \times 29}{30 \times 29} \right)$$

(6) 
$$P(2_{5} \cap 1_{5}) = P(2_{5} | 1_{5}) P(1_{5}) = \frac{28}{29} \frac{29}{30} = \frac{28}{30} = \frac{14}{15} \left( = \frac{29C_2}{30C_2} \right)$$

(7) 
$$\mu = 100 \times \frac{1}{15} = \frac{20}{3} \text{ PM}$$

(8) 
$$\sigma^2 = 100^2 \times \frac{1}{15} - \left(\frac{20}{3}\right)^2 = 100 \times \left(\frac{100}{15} - \frac{4}{9}\right)$$
  
=  $100 \times \left(\frac{20}{3} - \frac{4}{9}\right) = 100 \times \frac{(60 - 4)}{9} = \frac{5600}{9} \approx 622.2$ 

[2]

(1) 
$$P(2_{\sharp} \cap 1_{\sharp}) = P(2_{\sharp} | 1_{\sharp}) P(1_{\sharp}) = \frac{1}{59} \frac{2}{60} = \frac{1}{1770} \left( = \frac{1}{60C_2} \right)$$

(2) 
$$P(2_{\cancel{M}} \cap 1_{\cancel{\exists}}) = P(2_{\cancel{M}}|1_{\cancel{\exists}})P(1_{\cancel{\exists}}) = \frac{58}{59}\frac{2}{60} = \frac{58}{1770} = \frac{29}{885}$$

(3) 
$$P(2_{\sharp} \cap 1_{\Re}) = P(2_{\sharp} | 1_{\Re}) P(1_{\Re}) = \frac{2}{59} \frac{58}{60} = \frac{58}{1770} = \frac{29}{885}$$

(4) 
$$P(2_{\mbox{$\frac{1}{3}$}} \cap 1_{\mbox{$\frac{1}{3}$}}) + P(2_{\mbox{$\frac{1}{3}$}} \cap 1_{\mbox{$\frac{1}{3}$}}) = \frac{116}{1770} = \frac{58}{885} \left( = \frac{2 \times 58}{60C_2} \right)$$

(5) 
$$P(2_{\%} \cap 1_{\%}) = P(2_{\%}|1_{\%})P(1_{\%}) = \frac{57}{59}\frac{58}{60} = \frac{1653}{1770} = \frac{551}{590} \left( = \frac{58C_2}{60C_2} \right)$$

(6) 
$$\mu = 100 \times \frac{58}{885} + 200 \times \frac{1}{1770} = \frac{20}{3}$$

(7) 
$$\sigma^2 = 100^2 \times \frac{116}{1770} + 200^2 \times \frac{1}{1770} - \left(\frac{20}{3}\right)^2$$
  
 $= 100^2 \times \frac{(116+4)}{1770} - \frac{400}{9} = 100^2 \times \frac{120}{1770} - \frac{400}{9}$   
 $= 400 \times \left(\frac{100}{59} - \frac{1}{9}\right) = 400 \times \frac{(900 - 59)}{531} = \frac{336400}{531} \approx 633.5$ 

### 問題5

(1) 
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(2) 
$$P(x) = \left(\frac{1}{4}\right) \cdot \left(\frac{3}{4}\right)^{x-1}$$

(3) 
$$\mu = \frac{1}{\frac{1}{4}} = 4$$

(4) 
$$\sigma^2 = \frac{\frac{3}{4}}{\left(\frac{1}{4}\right)^2} = 12$$

(5) 
$$Q(x) = \left(\frac{3}{4}\right)^x$$

(6) 
$$P(x) + Q(x) = \left(\frac{3}{4}\right)^{x-1} = Q(x-1)$$

(7) 
$$P(1) + P(2) + \cdots + P(x) + Q(x) = P(1) + P(2) + \cdots + Q(x-1) = \cdots = 1$$

(8) 
$$1 - Q(n) > \frac{1}{2}$$
 より、 $Q(n) < \frac{1}{2}$ .  $\left(\frac{3}{4}\right)^3 = \frac{27}{64} < \frac{1}{2}$ . ゆえに、 $n = 3$ .

#### 問題6

(1) 
$$P(x) = {}_{5}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{5-x}$$

(2) 
$$\mu = \frac{5}{3}$$

(3) 
$$\sigma^2 = 5 \times \frac{1}{3} \times \frac{2}{3} = \frac{10}{9}$$

(4) 
$$P(3) + P(4) + P(5) = \frac{{}_{5}C_{3} \cdot 2^{2} + {}_{5}C_{4} \cdot 2 + {}_{5}C_{5}}{3^{5}} = \frac{51}{243} = \frac{17}{81} \approx 20.98\%$$