

問題 1

$$(1) \quad \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$(2) \quad 32 \text{ 通りのうち、5 人とも男である可能性は無いので、} \frac{1}{32-1} = \frac{1}{31}$$

$$(3) \quad \text{一番上が女の子であるのは、31 通りのうちの 16 通りだから、} \frac{16}{32-1} = \frac{16}{31}$$

$$(4) \quad \text{残りの 4 人が女の子である確率だから、} \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

問題 2

$$(1) \quad P(1_{\text{当}}) = \frac{2}{100} = \frac{1}{50}$$

$$(2) \quad P(1_{\text{外}}) = \frac{98}{100} = \frac{49}{50}$$

$$(3) \quad P(2_{\text{当}}|1_{\text{当}}) = \frac{1}{99}$$

$$(4) \quad P(2_{\text{当}}|1_{\text{外}}) = \frac{2}{99}$$

$$(5) \quad P(2_{\text{当}}) = P(2_{\text{当}}|1_{\text{当}})P(1_{\text{当}}) + P(2_{\text{当}}|1_{\text{外}})P(1_{\text{外}})$$

$$(6) \quad P(2_{\text{当}}) = \frac{1}{99} \cdot \frac{1}{50} + \frac{2}{99} \cdot \frac{49}{50} = \frac{1+98}{99 \cdot 50} = \frac{1}{50}$$

問題 3

$$(1) \quad (\text{答}) \quad P(\text{青}_{\text{目}}) = P(\text{青}_{\text{目}}|\text{青}_{\text{走}})P(\text{青}_{\text{走}}) + P(\text{青}_{\text{目}}|\text{緑}_{\text{走}})P(\text{緑}_{\text{走}}) \\ = 0.85 \times 0.2 + 0.15 \times 0.8 = 0.29$$

$$(2) \quad (\text{答}) \quad P(\text{緑}_{\text{目}}) = P(\text{緑}_{\text{目}}|\text{緑}_{\text{走}})P(\text{緑}_{\text{走}}) + P(\text{緑}_{\text{目}}|\text{青}_{\text{走}})P(\text{青}_{\text{走}}) \\ = 0.85 \times 0.8 + 0.15 \times 0.2 = 0.71$$

$$(3) \quad (\text{答}) \quad P(\text{青}_{\text{事故}}) = \frac{P(\text{青}_{\text{目}}|\text{青}_{\text{走}})P(\text{青}_{\text{走}})}{P(\text{青}_{\text{目}})} = \frac{0.85 \times 0.2}{0.29} = \frac{0.17}{0.29} = \frac{17}{29} \approx 0.586$$

問題 4

[1]

$$(1) \quad P(1_{\text{当}}) = \frac{1}{60}$$

$$(2) \quad P(1_{\text{外}}) = \frac{59}{60}$$

$$(3) \quad P(2_{\text{当}}|1_{\text{外}}) = \frac{1}{59}$$

$$(4) \quad P(2_{\text{当}}) = P(2_{\text{当}}|1_{\text{外}})P(1_{\text{外}}) = \frac{1}{59} \times \frac{59}{60} = \frac{1}{60}$$

$$(5) \quad P(1_{\text{当}}) + P(2_{\text{当}}) = \frac{1}{30} \left(= \frac{1 \times 59}{60 C_2} = \frac{2 \times 59}{60 \times 59} \right)$$

$$(6) \quad P(2_{\text{外}} \cap 1_{\text{外}}) = P(2_{\text{外}}|1_{\text{外}})P(1_{\text{外}}) = \frac{58}{59} \cdot \frac{59}{60} = \frac{58}{60} = \frac{29}{30} \left(= \frac{59 C_2}{60 C_2} \right)$$

$$(7) \quad \mu = 90 \times \frac{1}{30} = 3 \text{ 円}$$

$$(8) \sigma^2 = 90^2 \times \frac{1}{30} - 3^2 = 90 \times \frac{90}{30} - 9 = 270 - 9 = 261$$

[2]

$$(1) P(2_{\text{当}} \cap 1_{\text{当}}) = P(2_{\text{当}}|1_{\text{当}})P(1_{\text{当}}) = \frac{1}{119} \cdot \frac{2}{120} = \frac{1}{7140} \left(= \frac{1}{120C_2} \right)$$

$$(2) P(2_{\text{外}} \cap 1_{\text{当}}) = P(2_{\text{外}}|1_{\text{当}})P(1_{\text{当}}) = \frac{118}{119} \cdot \frac{2}{120} = \frac{118}{7140} = \frac{59}{3570}$$

$$(3) P(2_{\text{当}} \cap 1_{\text{外}}) = P(2_{\text{当}}|1_{\text{外}})P(1_{\text{外}}) = \frac{2}{119} \cdot \frac{118}{120} = \frac{118}{7140} = \frac{59}{3570}$$

$$(4) P(2_{\text{当}} \cap 1_{\text{外}}) + P(2_{\text{外}} \cap 1_{\text{外}}) = \frac{236}{7140} = \frac{118}{3570} = \frac{59}{1785} \left(= \frac{2 \times 118}{120C_2} \right)$$

$$(5) P(2_{\text{外}} \cap 1_{\text{外}}) = P(2_{\text{外}}|1_{\text{外}})P(1_{\text{外}}) = \frac{117}{119} \cdot \frac{118}{120} = \frac{6903}{7140} = \frac{2301}{2380} \left(= \frac{118C_2}{120C_2} \right)$$

$$(6) \mu = 90 \times \frac{236}{7140} + 180 \times \frac{1}{7140} = \frac{90 \cdot (236 + 2)}{7140} = 3$$

$$(7) \sigma^2 = 90^2 \times \frac{236}{7140} + 180^2 \times \frac{1}{7140} - 3^2$$

$$= 90^2 \times \frac{(236 + 4)}{7140} - 9 = 90^2 \times \frac{240}{7140} - 9$$

$$= 8100 \times \frac{4}{119} - 9 = \frac{(8100 \times 4 - 119 \times 9)}{119} = \frac{31329}{119} \approx 263.2689$$

問題 5

$$(1) p = \frac{1}{6}$$

$$(2) P(x) = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{x-1}$$

$$(3) \mu = \frac{1}{p} = 6$$

$$(4) \sigma^2 = \frac{1-p}{p^2} = 30$$

$$(5) Q(x) = \left(\frac{5}{6}\right)^x$$

$$(6) P(x) + Q(x) = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{x-1} + \left(\frac{5}{6}\right)^x = \left(\frac{5}{6}\right)^{x-1} = Q(x-1)$$

$$(7) P(1) + P(2) + \cdots + P(x-1) + (P(x) + Q(x)) = P(1) + P(2) + \cdots + P(x-1) + Q(x-1)$$

$$= \cdots = P(1) + Q(1) = 1$$

$$(8) (7) \text{ より、 } P(1) + P(2) + \cdots + P(n) = 1 - Q(n) \quad \text{だから、}$$

$$1 - Q(n) > \frac{1}{2}. \quad \text{したがって、} \frac{1}{2} > Q(n).$$

$$\left(\frac{5}{6}\right)^2 = \frac{25}{36}, \quad \left(\frac{5}{6}\right)^3 = \frac{75}{216}, \quad \left(\frac{5}{6}\right)^4 = \frac{625}{1296}, \quad \cdots \quad \text{なので、} \left(\frac{5}{6}\right)^4 < \frac{1}{2}. \quad \text{ゆえに、} n = 4$$

問題 6

$$(1) P(x) = {}_{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = \frac{{}_{10}C_x}{2^{10}}$$

$$(2) \mu = 10 \times \frac{1}{2} = 5$$

$$(3) \sigma^2 = 10 \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{2}$$

$$(4) P(6) + P(7) + P(8) + P(9) + P(10) = \frac{1}{2^{10}} \times ({}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10})$$

$$= \frac{1}{1024} \times (210 + 120 + 45 + 10 + 1) = \frac{386}{1024} = \frac{193}{512}$$