確率・統計　模擬試験問題 2021 解答

問題 1

(1) $2^3 = 8$ 通り　　それらは、

(1) = (男，男，男)，　(2) = (女，男，男)，　(3) = (男，女，男)，　(4) = (男，男，女)，

(5) = (男，女，女)，　(6) = (女，男，女)，　(7) = (女，女，男)，　(8) = (女，女，女)，

(2) 8 通りの組み合わせのうち問題の組み合わせは、(5)，(6)，(7) の 3 通りであるから、$\dfrac{3}{8}$

(3) 8 通りのうち (1) は無いので、$\dfrac{3}{8-1} = \dfrac{3}{7}$

(4) 8 通りのうち (2), (6), (7), (8) の 4 通りが考えられるから、$\dfrac{4}{8} = \dfrac{1}{2}$

(5) (1) を除く 7 通りのうちの 4 通りだから、$\dfrac{4}{7}$

問題 2

(1) $\mathrm{P}(1_当) = \dfrac{4}{100} = \dfrac{1}{25}$

(2) $\mathrm{P}(1_外) = \dfrac{96}{100} = \dfrac{24}{25}$

(3) $\mathrm{P}(2_当|1_当) = \dfrac{3}{99} = \dfrac{1}{33}$

(4) $\mathrm{P}(2_当|1_外) = \dfrac{4}{99}$

(5) $\mathrm{P}(2_当) = \mathrm{P}(2_当|1_当)\mathrm{P}(1_当) + \mathrm{P}(2_当|1_外)\mathrm{P}(1_外)$

(6) $\mathrm{P}(2_当) = \dfrac{3}{99} \cdot \dfrac{1}{25} + \dfrac{4}{99} \cdot \dfrac{24}{25} = \dfrac{3 + 4 \times 24}{99 \cdot 25} = \dfrac{1}{25}$

問題 3

(1) (答)　$\mathrm{P}(青_目) = \mathrm{P}(青_目|\,青_走)\mathrm{P}(青_走) + \mathrm{P}(青_目|\,緑_走)\mathrm{P}(緑_走)$
$= 0.75 \times 0.1 + 0.25 \times 0.9 = 0.3$

(2) (答)　$\mathrm{P}(緑_目) = \mathrm{P}(緑_目|\,緑_走)\mathrm{P}(緑_走) + \mathrm{P}(緑_目|\,青_走)\mathrm{P}(青_走)$
$= 0.75 \times 0.9 + 0.25 \times 0.1 = 0.7$

(3) (答)　$\mathrm{P}(青_{事故}) = \dfrac{\mathrm{P}(青_目|\,青_走)\mathrm{P}(青_走)}{\mathrm{P}(青_目)} = \dfrac{0.75 \times 0.1}{0.3} = \dfrac{0.75}{3} = 0.25$

問題 4

[1]

(1) $\mathrm{P}(1_当) = \dfrac{1}{25}$

(2) $\mathrm{P}(1_外) = \dfrac{24}{25}$

(3) $\mathrm{P}(2_当|1_外) = \dfrac{1}{24}$

(4) $\mathrm{P}(2_当) = \mathrm{P}(2_当|1_外)\mathrm{P}(1_外) = \dfrac{1}{24} \times \dfrac{24}{25} = \dfrac{1}{25}$

(5) $\mathrm{P}(1_当) + \mathrm{P}(2_当) = \dfrac{2}{25} \left(= \dfrac{1 \times 24}{{}_{25}C_2} = \dfrac{2 \times 24}{25 \times 24} \right)$

(6) $\mathrm{P}(2_外 \cap 1_外) = \mathrm{P}(2_外|1_外)\mathrm{P}(1_外) = \dfrac{23}{24} \cdot \dfrac{24}{25} = \dfrac{23}{25} \left(= \dfrac{{}_{24}C_2}{{}_{25}C_2} \right)$

(7) $\mu = 0 \times \mathrm{P}(0) + 100 \times \mathrm{P}(100) = 0 \times \dfrac{23}{25} + 100 \times \dfrac{2}{25} = 8$ 円

(8) $\sigma^2 = 0 \times \mathrm{P}(0) + 100^2 \times \mathrm{P}(100) - \mu^2 = 100^2 \times \dfrac{2}{25} - 8^2 = 200 \times \dfrac{100}{25} - 64 = 800 - 64 = 736$

[2]

(1) $\mathrm{P}(2_{当} \cap 1_{当}) = \mathrm{P}(2_{当}|1_{当})\mathrm{P}(1_{当}) = \dfrac{1}{49} \cdot \dfrac{2}{50} = \dfrac{1}{1225} \left(= \dfrac{1}{{}_{50}C_2} \right)$

(2) $\mathrm{P}(2_{外} \cap 1_{当}) = \mathrm{P}(2_{外}|1_{当})\mathrm{P}(1_{当}) = \dfrac{48}{49}\dfrac{2}{50} = \dfrac{48}{1225}$

(3) $\mathrm{P}(2_{当} \cap 1_{外}) = \mathrm{P}(2_{当}|1_{外})\mathrm{P}(1_{外}) = \dfrac{2}{49} \cdot \dfrac{48}{50} = \dfrac{48}{1225}$

(4) $\mathrm{P}(2_{当} \cap 1_{外}) + \mathrm{P}(2_{当} \cap 1_{外}) = \dfrac{48 \times 2}{1225} = \dfrac{96}{1225} \left(= \dfrac{2 \times 48}{{}_{50}C_2} \right)$

(5) $\mathrm{P}(2_{外} \cap 1_{外}) = \mathrm{P}(2_{外}|1_{外})\mathrm{P}(1_{外}) = \dfrac{47}{49} \cdot \dfrac{48}{50} = \dfrac{1128}{1225} \left(= \dfrac{{}_{48}C_2}{{}_{50}C_2} \right)$

(6) $\mu = 0 \times \dfrac{1128}{1225} + 100 \times \dfrac{96}{1225} + 200 \times \dfrac{1}{1225} = \dfrac{100 \cdot (96 + 2)}{1225} = 8$

(7) $\sigma^2 = 100^2 \times \dfrac{96}{1225} + 200^2 \times \dfrac{1}{1225} - 8^2$

$$= 100^2 \times \dfrac{(96 + 4)}{1225} - 64 = 100^2 \times \dfrac{100}{25 \times 49} - 64$$

$$= \dfrac{40000}{49} - 64 = \dfrac{(4000 - 64 \times (50 - 1))}{49} = \dfrac{36864}{49} \approx 752.3265$$

問題 **5**

(1) $p = \dfrac{2}{6} = \dfrac{1}{3}$

(2) $\mathrm{P}(x) = \left(\dfrac{1}{3}\right) \cdot \left(\dfrac{2}{3}\right)^{x-1}$

(3) $\mu = \dfrac{1}{\frac{1}{3}} = 3$

(4) $\sigma^2 = \dfrac{\frac{2}{3}}{\left(\frac{1}{3}\right)^2} = 6$

(5) $\mathrm{Q}(x) = \left(\dfrac{2}{3}\right)^x$

(6) $\mathrm{P}(x) + \mathrm{Q}(x) = \left(\dfrac{2}{3}\right)^{x-1} = \mathrm{Q}(x-1)$

(7) $\mathrm{P}(1) + \mathrm{P}(2) + \cdots + \mathrm{P}(x) + \mathrm{Q}(x) = \mathrm{P}(1) + \mathrm{P}(2) + \cdots + \mathrm{Q}(x-1) = \cdots = 1$

(8) $1 - \mathrm{Q}(n) > \dfrac{1}{2}$ より、$\mathrm{Q}(n) < \dfrac{1}{2}$. $\left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9} < \dfrac{1}{2}$. ゆえに、$n = 2$.

問題6

(1) $p = \dfrac{1}{4}$

(2) $q = \dfrac{3}{4}$

(3) $\mathrm{P}(x) = {}_5C_x \left(\dfrac{1}{4}\right)^x \left(\dfrac{3}{4}\right)^{5-x}$

(4) $\mu = np = 5 \times \dfrac{1}{4} = \dfrac{5}{4}$

(5) $\sigma^2 = npq = 5 \times \dfrac{1}{4} \times \dfrac{3}{4} = \dfrac{15}{16}$

(6) $\mathrm{P}(5) + \mathrm{P}(4) + \mathrm{P}(3) = \left(\dfrac{1}{4}\right)^5 + 5\left(\dfrac{1}{4}\right)^4 \left(\dfrac{3}{4}\right) + 10\left(\dfrac{1}{4}\right)^3 \left(\dfrac{3}{4}\right)^2 = \dfrac{1 + 15 + 90}{1024} = \dfrac{106}{1024} = \dfrac{53}{512} \approx 10.\%$