

Due Date: Wednesday September 29, 2021, 11:59pm

Instructions

Answer all questions on paper or a tablet using your own handwriting. **Please number each page** Include a cover page with your name, student ID number and a list of the questions you have answered.

Textbook Reading

- Sections: 11.1, 11.2, 3.4, 3.5, 3.6

Definitions, Concepts & Keywords

- Understand more terminology for graphs and trees.
- Know when two paths, cycles, graphs, and trees are not the same.
- Can calculate basic probabilities for discrete events.

Exercises

A. Textbook Questions

Section 11.1 Exercises 5.

Section 11.2 Exercises 9(a), 12(b).

Section 3.4 Exercises 6, 9.

Section 3.5 Exercises 10 (omit $Pr(A \triangle B)$).

Section 3.6 Exercises 1, 11

B. Instructors Questions

1. For a positive integer n , what is the value of the counter after the following code has been executed. (Both C and Python code are included, use the language you are familiar with.)

C code:

```
int i, j, k, counter;
counter = 0;
for( i=1; i<=n; i++ )
    for( j=1; j<=i; j++ )
        for( k=1; k<=j; k++ )
            counter ++;
```

python code:

```
counter = 0
for i in range(1, n+1):
    for j in range(1, i+1):
        for k in range(1, j+1):
            counter = counter + 1
```

2. Consider the graph G with vertices $V = \{1, 2, 3, 4, 5\}$ and edges $E = \{\{1, 2\}, \{2, 3\}, \{3, 1\}, \{3, 4\}, \{4, 5\}, \{5, 1\}\}$.
 - (a) Draw G .
 - (b) How many cycles does G have? List them.
 - (c) Is G bipartite? Explain.
 - (d) Give a spanning subgraph of G that is a tree.
 - (e) Draw the subgraph of G induced by the vertices $\{2, 4, 5\}$.
3. A graph $G = (V, E)$ is called regular if every vertex in G has the same degree. Draw a graph with 6 vertices where
 - (a) every vertex has degree 1
 - (b) every vertex has degree 2
 - (c) every vertex has degree 3
 - (d) every vertex has degree 4
4. Up to isomorphism, find all unlabelled graphs on 5 vertices with 5 edges.

5. Let n and k be integers with $3 \leq k \leq n$.
How many cycles of length k are there in the complete graph K_n ?
Hint: we can select a cycle of length k from K_n by choosing a sequence of k distinct vertices v_1, v_2, \dots, v_k and then selecting the cycle with vertex set $\{v_1, \dots, v_k\}$ and edge set $\{\{v_1, v_2\}, \dots, \{v_{k-1}, v_k\}, \{v_k, v_1\}\}$. It will help to determine (to avoid over-counting) for each cycle of length k how many sequences select this cycle.
6. How many subgraphs of K_n are isomorphic to $K_{3,3}$?
How many subgraphs of $K_{n,n}$ are isomorphic to $K_{3,3}$?
7. Let K_n^- be a graph obtained from K_n by deleting one edge.
How many subgraphs of K_n^- are isomorphic to K_5^- ?
8. We say that two distinct edges in a graph are **adjacent** if there is a vertex incident to both (the two edges share exactly one vertex.) For the graph $K_{n,n}$, determine the number of sets of two edges $\{e, f\}$ with the property that e, f are
 - (a) adjacent and
 - (b) not adjacent.
9. Suppose 6 men and 6 women attend a party.
The host wants to select some of the 12 guests to play a game.
 - (a) If the host chooses two guests at random, what is the probability of choosing two women?
 - (a) If the host chooses two guests at random, what is the probability of choosing one man and one woman?
 - (c) If the host chooses four guests at random, what is the probability of choosing two women and two men? You should get $5/11$.
10. At a university there are 300 first year computing majors. Suppose 100 can program in Python, 60 can program in Java, 80 can program in C++, 20 can program in Python and Java, 20 can program in Java and C++, 10 can program in Python and C++ and none can program in all three languages.
 - (a) If a student is selected at random, what is the probability that they can program in exactly 2 languages.
 - (b) If a student is selected at random, what is the probability that they can only program in Python?
 - (c) If two students are picked at random, what is the probability they can both program in Java?
Hint: use a Venn diagram.
11. Let S be a finite sample space and A, B and C be subsets of S .
 - (a) In class we showed that

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B).$$
 Generalize this to find a formula for $Pr(A \cup B \cup C)$.
 - (b) Let set $S = \{1, 2, \dots, 60\}$. Using the formula from part (a), find the probability of choosing $x \in S$ that is divisible by 2 **or** 3 **or** 5. You should get $11/15$.
12. A US company has developed an antibody test for whether a person has had COVID. If a person has had COVID, the probability that the test is positive is 0.98. If a person has NOT had COVID, the probability that the test is positive is 0.03. Suppose 10% of Americans have had COVID – which will probably be true by the time you are reading this. Use Bayes' theorem to determine the probability that an American who tests positive has had COVID.