

VE215 RC2

Gong Yuhan

UM-SJTU JI

October 23, 2024

Overview

Linearity Property

Superposition

Source Transformation

Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

Linearity Property

homogeneous: if $x \rightarrow y$, then $kx \rightarrow ky$

additive: if $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$, then $x_1 + x_2 \rightarrow y_1 + y_2$

linear circuit: homogeneous and additive

Exercise

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

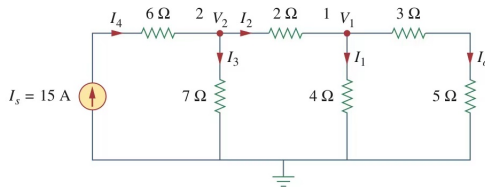
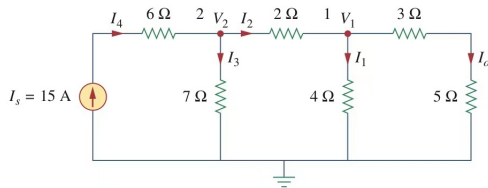


Figure 4.4

Exercise



Answer: $I_o = 3\text{ A}$

Overview

Linearity Property

Superposition

Source Transformation

Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

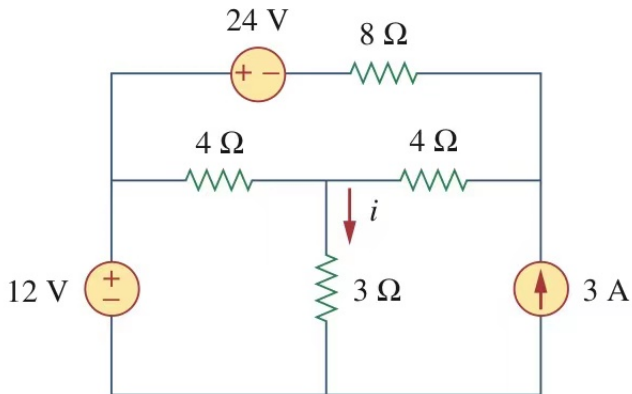
Superposition

Steps

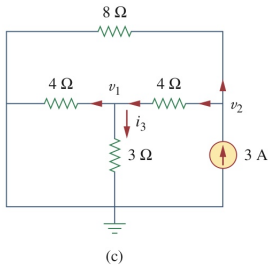
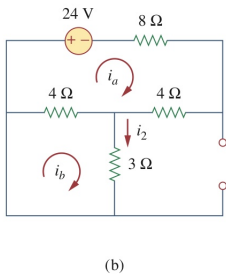
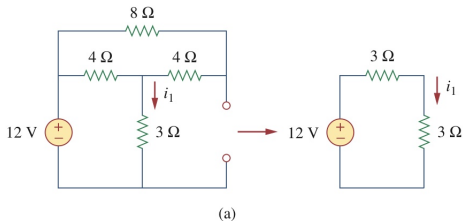
1. Only consider one **independent** source.
 - ▶ voltage source: short circuit
 - ▶ current source: open circuit
2. Use additivity.

Exercise

Find i in the circuit.



Exercise



Answer: 2A

Overview

Linearity Property

Superposition

Source Transformation

Thevenin's Theorem

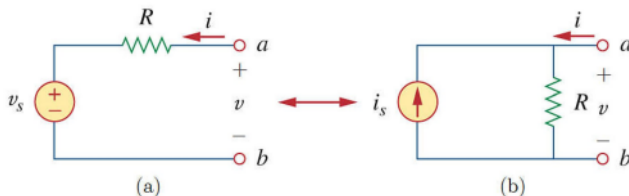
Norton's Theorem

Maximum Power Transfer

Source Transformation

We can replace a voltage source with a resistance with a corresponding current source with the same resistance to simplify the circuit.

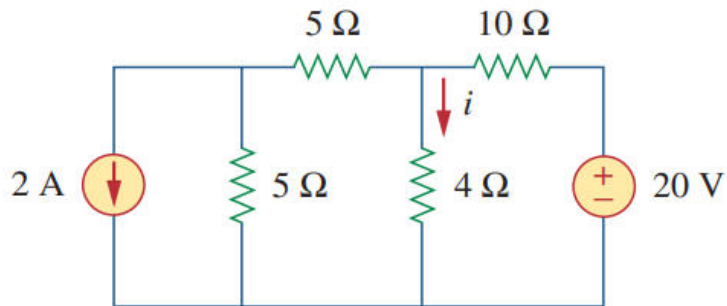
In the case shown below, $v_s = i_s \times R$



For dependent sources, the source transformation is also valid.

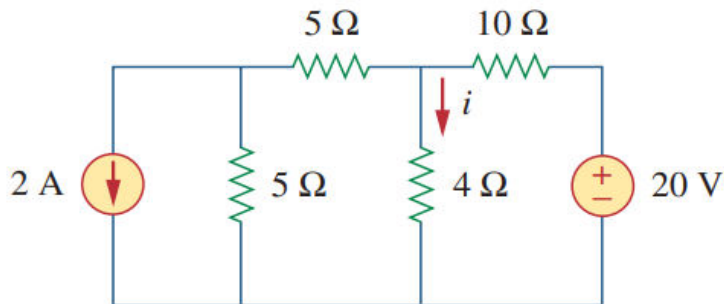
Exercise

Find i .



Exercise

Find i .



Answer: $5/9\text{ A}$

Overview

Linearity Property

Superposition

Source Transformation

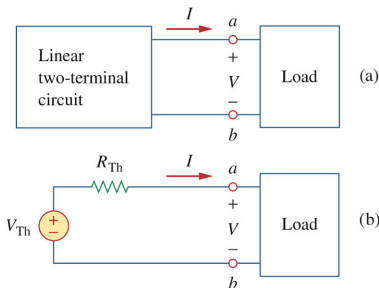
Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

Thevenin's Theorem

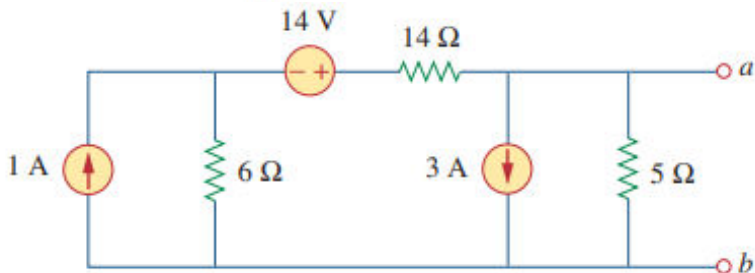
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} .



- ▶ V_{Th} : the open-circuit voltage at the terminals.
- ▶ R_{Th} : the equivalent resistance at the terminals when all the **independent sources** are turned off.

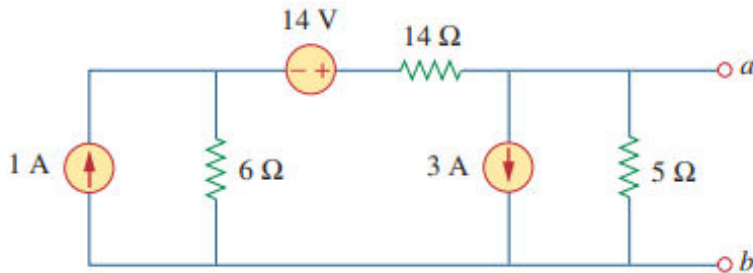
Exercise

Obtain the Thevenin equivalent circuit of this circuit with respect to terminal a and b.



Exercise

Obtain the Thevenin equivalent circuit of this circuit with respect to terminal a and b.



Answer: $V_{Th} = -8\text{ V}$

$R_{Th} = 4\Omega$

pay attention to the sign of the voltage !

Overview

Linearity Property

Superposition

Source Transformation

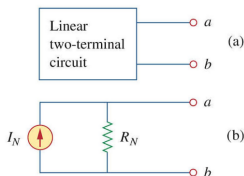
Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

Norton's Theorem

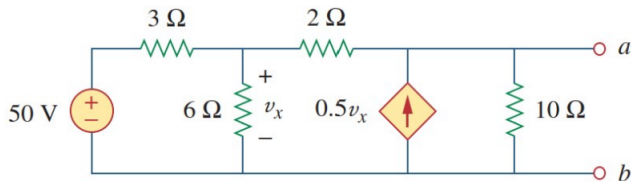
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_{Th} in parallel with a resistor R_{Th} .



- ▶ I_{Th} : the short-circuit current at the terminals.
- ▶ R_{Th} : the equivalent resistance at the terminals when all the **independent sources** are turned off.

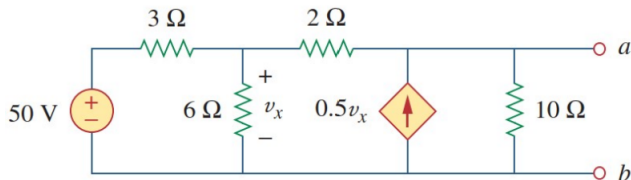
Exercise

Obtain the Norton equivalent circuit of this circuit with respect to terminal a and b.



Exercise

Obtain the Norton equivalent circuit of this circuit with respect to terminal a and b.



Answer: $I_N = 16.67\text{ A}$
 $R_N = 10\ \Omega$

Overview

Linearity Property

Superposition

Source Transformation

Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

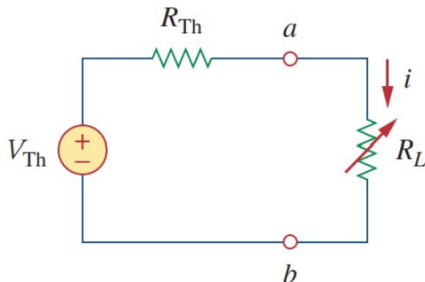
Maximum Power Transfer

A circuit is usually designed to provide power to a load. For different kinds of circuits, we have different concerns

- ▶ **Maximum Power Efficiency:** In power utility systems, the amount of electricity is very large. Therefore, how to **increase the efficiency of power transfer** becomes an important problem.
- ▶ **Maximum Power Transfer:** In communication and instrumental systems, the amount of electricity is small so the problem of efficiency is not so important. Instead, we want to **transfer as much of power as possible to the load**.

Maximum Power Transfer

The Thevenin's equivalent circuit is useful in finding the maximum power delivered to a load. In the circuit below, R_L represents the load.



Maximum Power Theorem

Since

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

Let $\frac{dP}{dR_L} = V_{Th}^2 \frac{R_{Th} - R_L}{(R_{Th} + R_L)^3} = 0$, we have $R_L = R_{Th}$.

And when $R_L = R_{Th}$, $\frac{d^2P}{dR_L^2} = V_{Th}^2 \frac{2R_L - 4R_{Th}}{(R_{Th} + R_L)^4} = -\frac{V_{Th}^2}{8R_{Th}^2} < 0$.

Thus p reaches maximum at $R_L = R_{Th}$. $p_{max} = \frac{V_{Th}^2}{4R_{Th}}$

References

1. 2024 Fall VE215 slides
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2023 Fall RC2 Chongye Yang

Thank you!