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# VE215 Final RC Part 1

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SJTU

# Contents



# Alternating Current(AC) Circuits:

Circuits driven by sinusoidal current or voltage sources

$$v(t) = V_m \sin(\omega t + \phi)$$

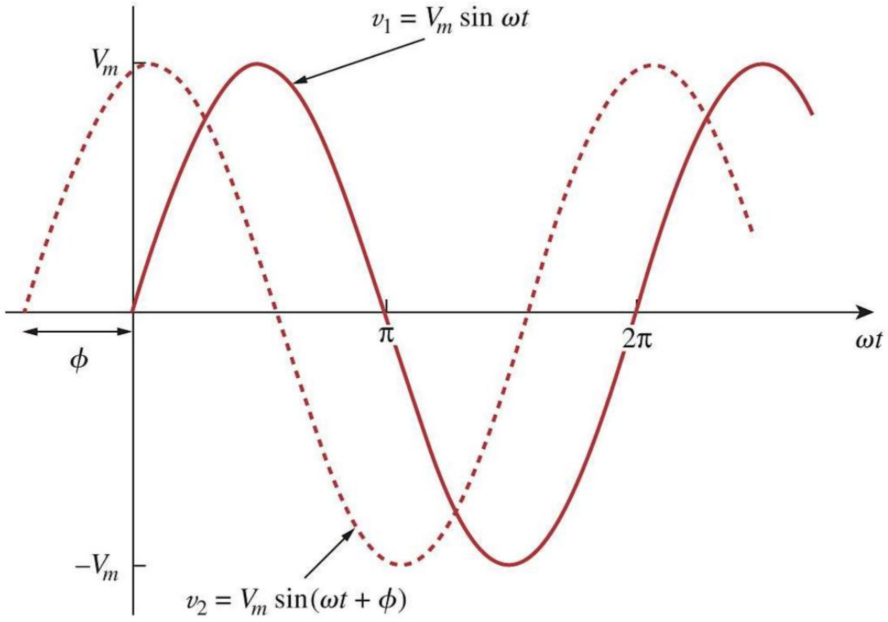
where

$V_m$ : amplitude

$\omega$: angular frequency

$\phi$: initial phase

Leads & Lags:



# Phasor Diagram:

# Phasors:

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(\widetilde{V} e^{j\omega t})$$

where

$$\widetilde{V} = V_m e^{j\phi} = V_m \angle \phi$$

is the **phasor** representation, polar form

while V= a+bj, is teh so called rectangular form

memorize, or copy

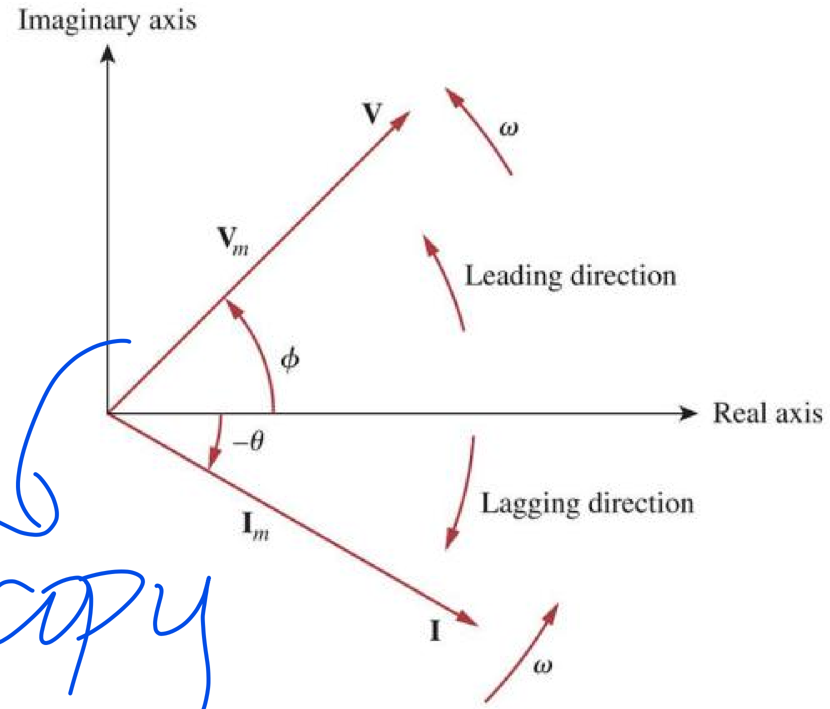


Figure 9.8 A phasor diagram.

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$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \widetilde{V} = V_m \angle \phi$$

# A little more about lag, lead, and in phase: sinusoidal

Let us examine the two sinusoids

$$v_1(t) = V_m \sin \omega t$$

$$v_2(t) = V_m \sin(\omega t + \phi)$$

shown in Fig. 9.2.

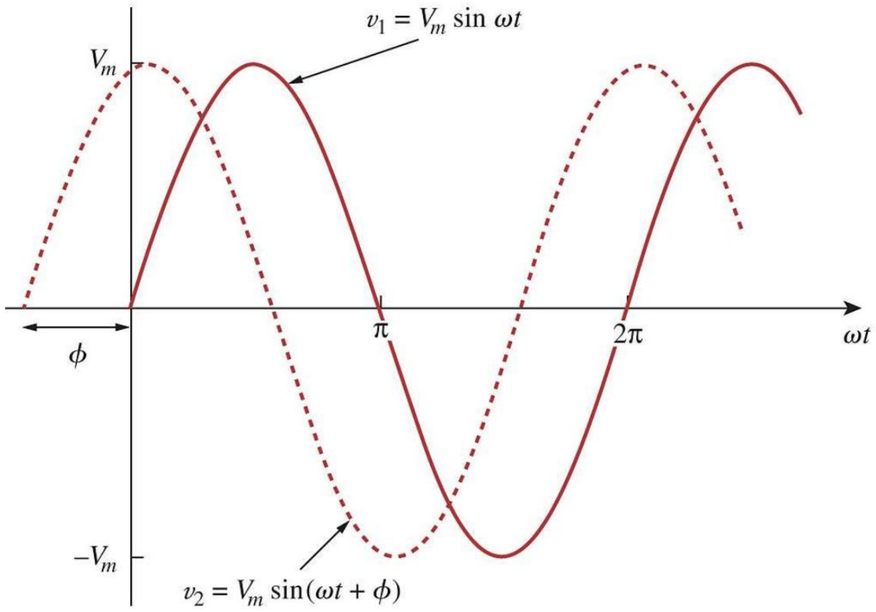


Figure 9.2 Two sinusoids with different phases.

The starting point of $v_2$ in Fig. 9.2 occurs first in time. Therefore, we say that $v_2$ leads $v_1$ by $\phi$ or that $v_1$ lags $v_2$ by $\phi$. If $\phi \neq 0$, we say that $v_1$ and $v_2$ are *out of phase*. If $\phi = 0$, then $v_1$ and $v_2$ are said to be *in phase*.

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Recall:

$$sin(x) = cos(x - \pi/2)$$

No need to differentiate sine/cosine forms here!

**TABLE 9.2**

**Summary of voltage - current relationships**

| Element | Time domain | Frequency domain |
|---|---|---|
| $R$ | $v = Ri$ | $\tilde{V} = R\tilde{I}$ |
| $L$ | $v = L\dfrac{di}{dt}$ | $\tilde{V} = j\omega L\tilde{I}$ |
| $C$ | $i = C\dfrac{dv}{dt}$ | $\tilde{V} = \dfrac{1}{j\omega C}\tilde{I}$ |

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## About admittance and impedance:

$$\underbrace{Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{Z}}_{\text{copy}}$$

The admittance can be written as

$Y = G + jB$

where

$G$ : conductance

$B$ : susceptance

The admittance, conductance, and susceptance are all measured in siemens.

$$Z = R + jX = |Z|\angle\theta$$

where

$R$ : resistance

$X$ : reactance

copy

If $X > 0$, we say that the impedance is inductive or lagging since current lags voltage; If $X < 0$, we say that the impedance is capacitive or leading because current leads voltage.

The impedance, resistance, and reactance are all measured in ohms.

# Equivalence Circuits

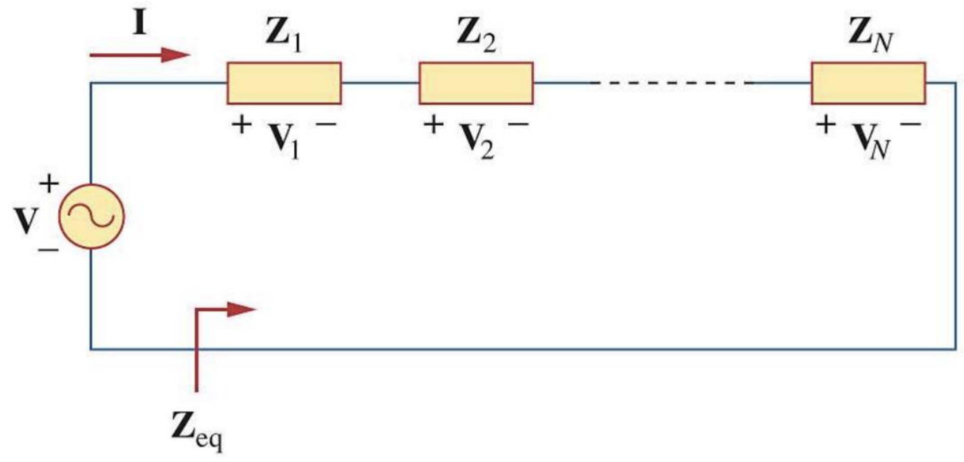


Figure 9.18 *N* impedances in series.

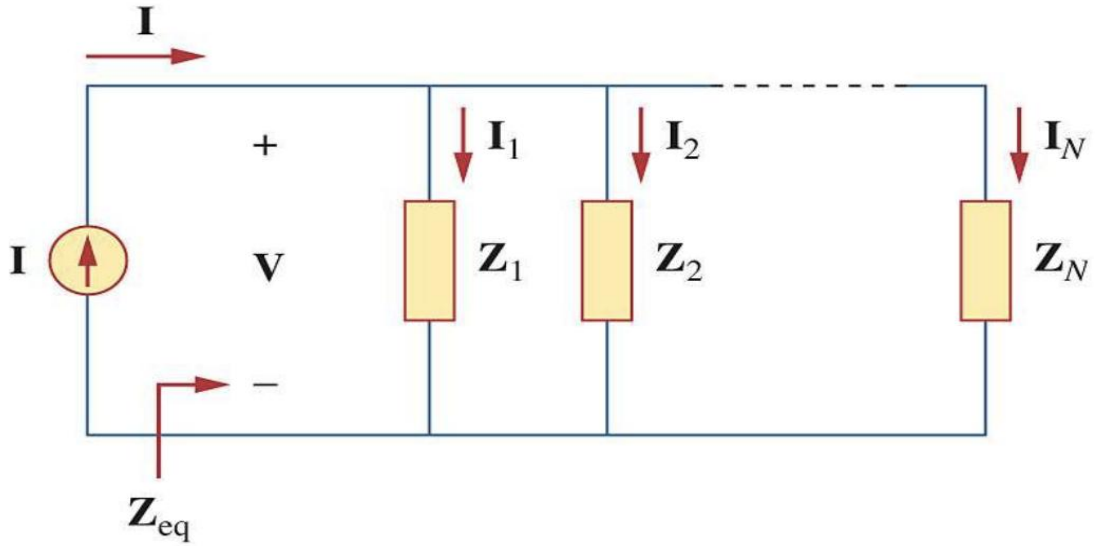


Figure 9.20 *N* impedances in parallel.

$$Z_{eq} = \frac{\tilde{V}}{\tilde{I}} = \frac{\boxed{\text{KVL}}\ \sum_{i=1}^{N} \tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} \frac{\tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} Z_i$$

$$Y_{eq} = \frac{\tilde{I}}{\tilde{V}} = \frac{\boxed{\text{KCL}}\ \sum_{i=1}^{N} \tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} \frac{\tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} Y_i$$

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# Exercise 1

Suppose we have following elements: 600 Ω resistors, 0.1μF capicitances,  10mH inductances. Please use the above elements  to create an impedance of $300 - 400j\ \Omega$ at a frequency of 10,000 rad/s. The number of each elements is adequate.

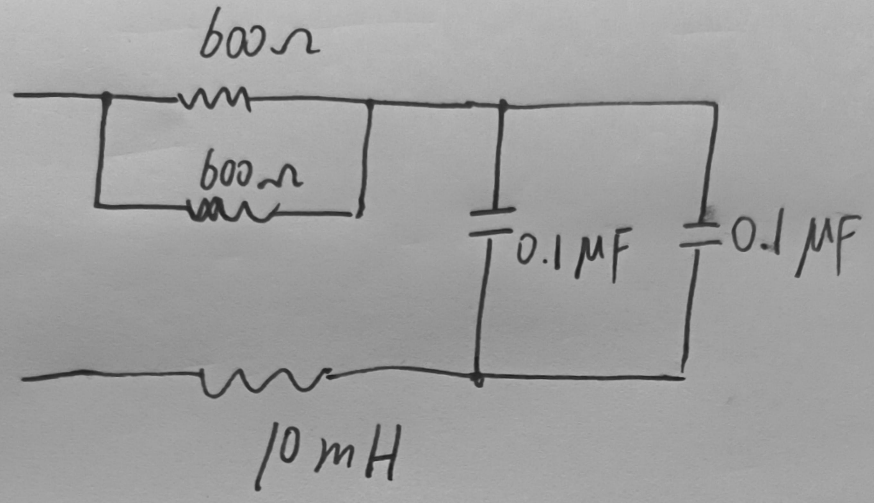
$$R = 300\,\Omega = \frac{600^2}{600+600}\,\Omega$$

$$wL - \frac{1}{wC} = 10000L - \frac{1}{10^4 C} = -400$$

$$L = 10\,mH$$

$$\Rightarrow \frac{1}{10^4 C} = 100 + 400$$

$\Rightarrow C = 0.2\,\mu F$, two $0.1\,\mu F$ in parallel.



# Source Transformation and Equivalence Circuit:

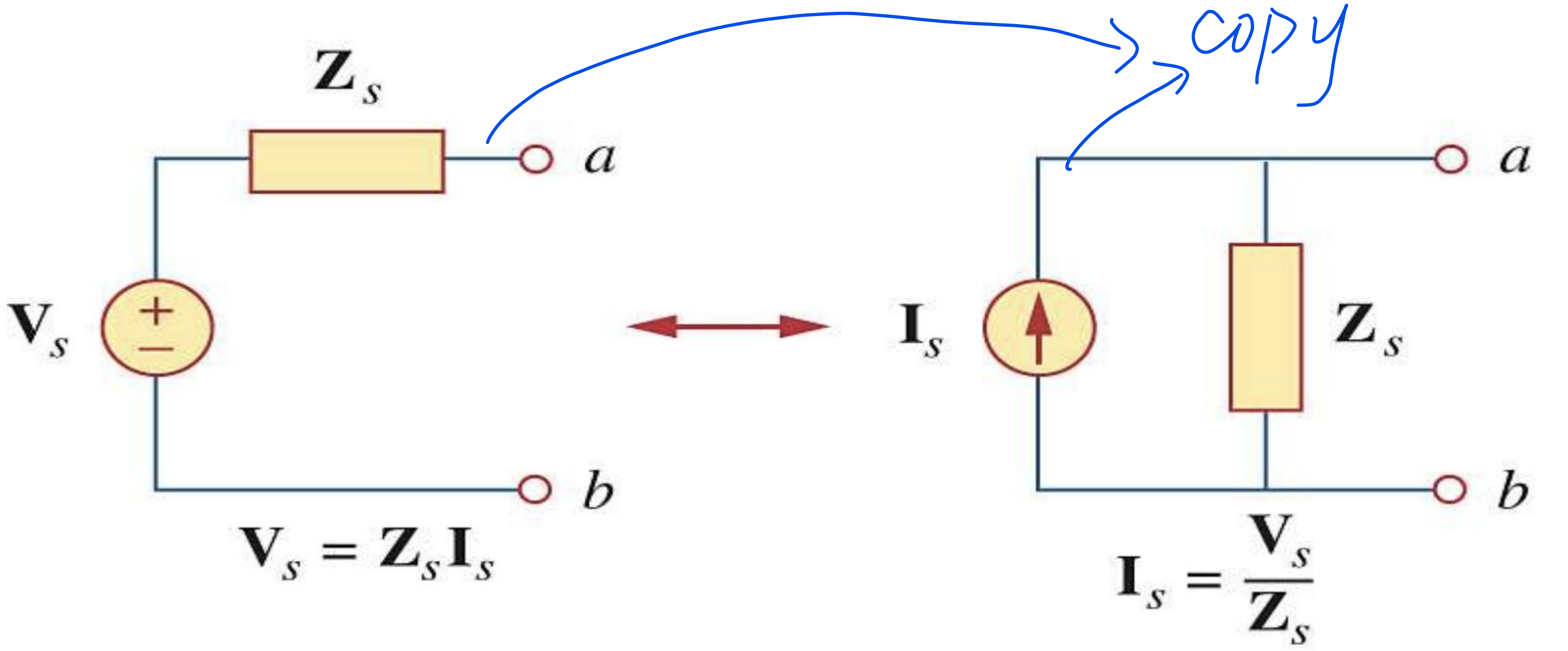
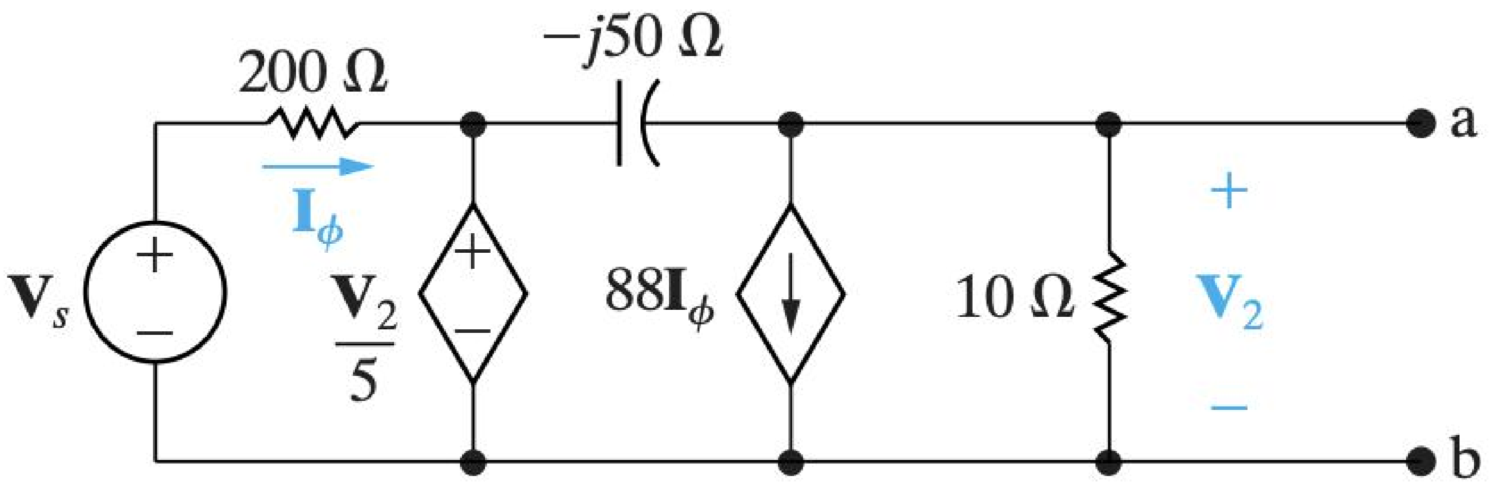



Figure 10.16 Source transformation.

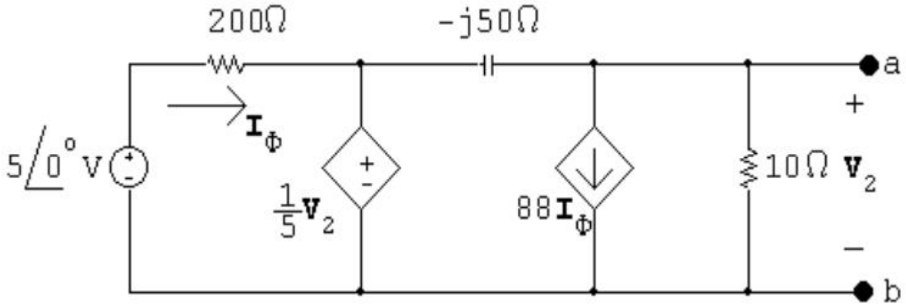
# Exercise 2

Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.50 when $\mathbf{V}_s = 5\angle 0^\circ$ V.

**Figure P9.50**



P 9.50 Open circuit voltage:



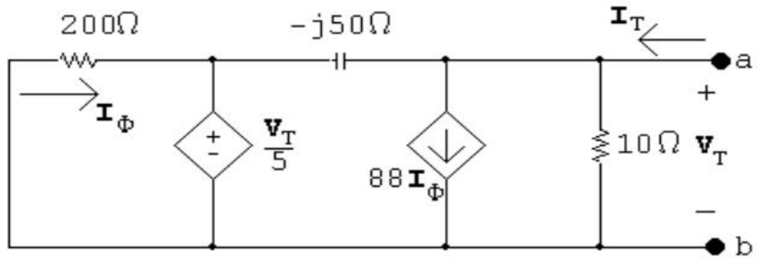

$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\mathrm{V} = \mathbf{V}_{\mathrm{Th}}$$

Find the Thévenin equivalent impedance using a test source:




$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

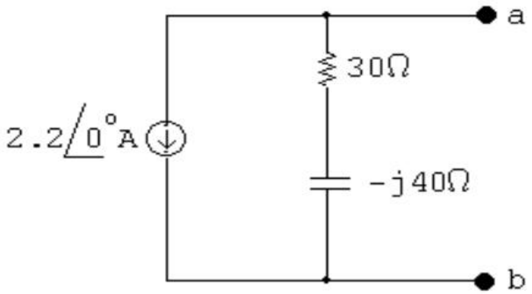
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\mathrm{Th}}$$

$$\mathbf{I}_{\mathrm{N}} = \frac{\mathbf{V}_{\mathrm{Th}}}{Z_{\mathrm{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\mathrm{A}$$

The Norton equivalent circuit:



# Superposition in AC Circuits:

☆ Follow these three steps

1. For DC voltage/current sources: take capacitance as an open and inductance as a short circuit.
2. For AC voltage/current sources: transform capacitance and inductance into impedance in frequency domain.
3. In AC circuit, when analyzing one specific source:
(1)Handle the capacitance and inductance following 1.&2. above, based on what type of the source is being analyzed.
(2)Then, take other voltage sources as a short circuit while other current sources as an open circuit.

# Exercise 3

**10.48** Find $i_o$ in the circuit of Fig. 10.93 using superposition.

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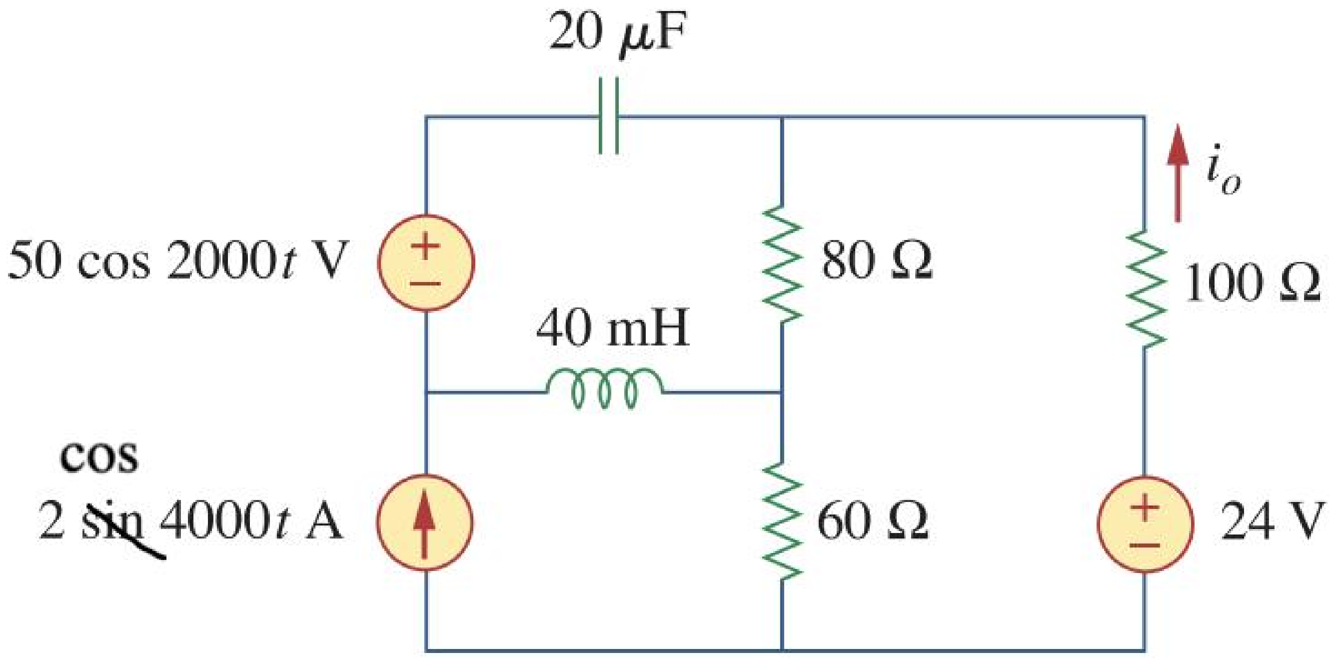
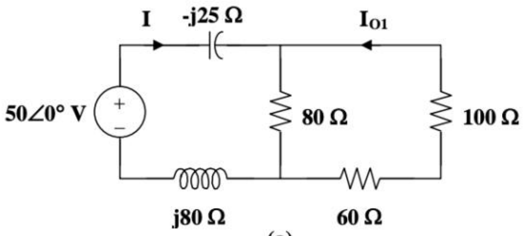


**Chapter 10, Solution 48.**

Let $i_O = i_{O1} + i_{O2} + i_{O3}$, where $i_{O1}$ is due to the ac voltage source, $i_{O2}$ is due to the dc voltage source, and $i_{O3}$ is due to the ac current source. For $i_{O1}$, consider the circuit in Fig. (a).

$$\omega = 2000$$
$$50\cos(2000t) \longrightarrow 50\angle 0°$$
$$40\text{ mH} \longrightarrow j\omega L = j(2000)(40\times10^{-3}) = j80$$
$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times10^{-6})} = -j25$$



(a)

$$80 \,||\, (60+100) = 160/3$$
$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

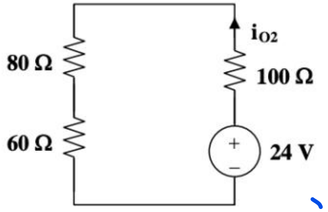
Using current division,

$$\mathbf{I}_{O1} = \frac{-80\,\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{10\angle 180°}{46\angle 45.9°}$$
$$\mathbf{I}_{O1} = 0.217\angle 134.1°$$

Hence, $i_{O1} = 0.217\cos(2000t + 134.1°)$ A

when there are multiple $w_s$, conserve the "$w$" part by using $\sin/\cos(wt+\cdots)$ in time domain.

For $i_{O2}$, consider the circuit in Fig. (b).

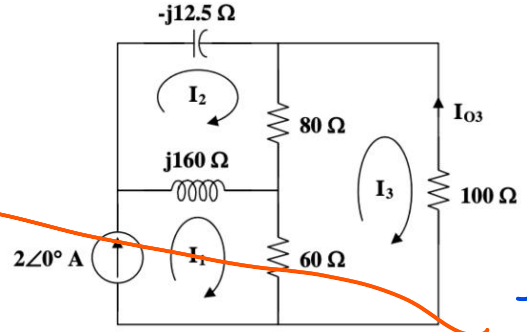


(b)

$$i_{O2} = \frac{24}{80 + 60 + 100} = 0.1\text{ A}$$

For $i_{O3}$, consider the circuit in Fig. (c).

$$\omega = 4000$$
$$2\cos(4000t) \longrightarrow 2\angle 0°$$
$$40\text{ mH} \longrightarrow j\omega L = j(4000)(40\times10^{-3}) = j160$$
$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times10^{-6})} = -j12.5$$



(c)

For mesh 1,
$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,
$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields
$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \qquad (2)$$

For mesh 3,
$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields
$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \qquad (3)$$

Substituting (3) into (2) yields
$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$
$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$
$$\mathbf{I}_{O3} = -\mathbf{I}_3 = -1.1782\angle 7.38°$$

Hence, $i_{O3} = -1.1782\,\cancel{\sin}(4000t + 7.38°)$ A cos

Therefore,

cos

$$i_O = \{0.1 + 0.217\cos(2000t + 134.1°) - 1.1782\,\cancel{\sin}(4000t + 7.38°)\}\text{ A}$$

$w_1$: $I_1 = a_1 + b_1 j = c_1\angle\theta_1$

$w_2$: $I_2 = a_2 + b_2 j = c_2\angle\theta_2$

If $w_1 \neq w_2$ Don't directly sum $I_1, I_2$ up in phasor or rect form!

# Three-phase Circuits

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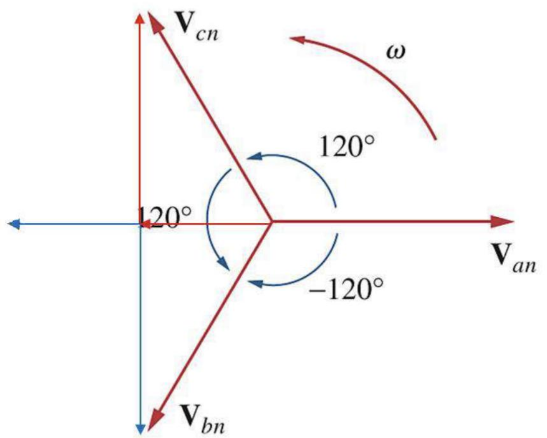
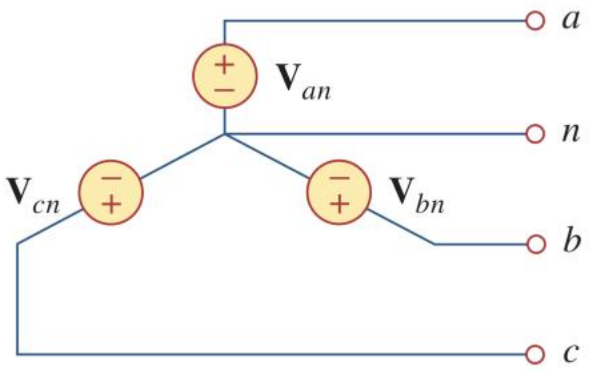
A three-phase system is produced by a generator consisting of three sources having **the same amplitude and frequency** but **out of phase with each other by 120 degrees.** **Voltage sources meeting this condition is called balanced sources.**

i.e., $$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = 0$$

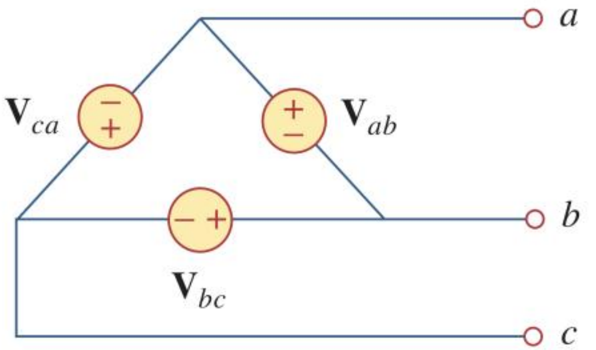
Physical illustration of three-phase




- Balanced Three-Phase Voltage Sources
  - Diffrent Types
    - Y-Type




    - Δ-Type



A load is said to be balanced if the phase impedances **are equal in magintude and phase**

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For a balanced wye-connected load,

$$Z_1 = Z_2 = Z_3 = Z_Y$$

For a balanced delta-connected load,

$$Z_A = Z_B = Z_C = Z_\Delta$$

If loads are balanced, the relationship below holds:

$$Z_\Delta = 3Z_Y \text{ or } Z_Y = \frac{1}{3} Z_\Delta$$

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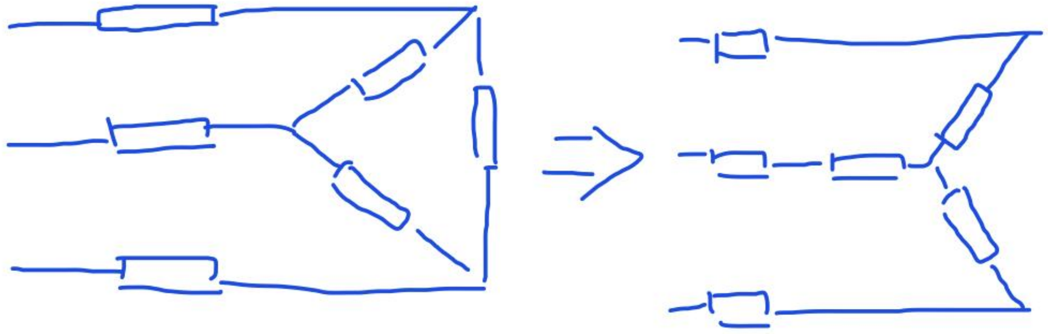


Some concepts to be emphasized:

**Line:**
Conductors connecting loads and sources
**Line voltage:**
voltage measured between two different lines
**Line currents:**
The current passing along each line

**Phase:**
Connected between any pair of line terminals (an element)
**Phase voltage:**
The voltage measured across any phase. In delta-delta system, VP=VL.
**Phase currents:**
The current passing through the phase. In Y-Y system, IP=IL.

ignore this if you find difficult to understand

Just refer to Page 20-21 is enough.

**Also, abc/acb, positive/negative sequence, check them in your slides!**

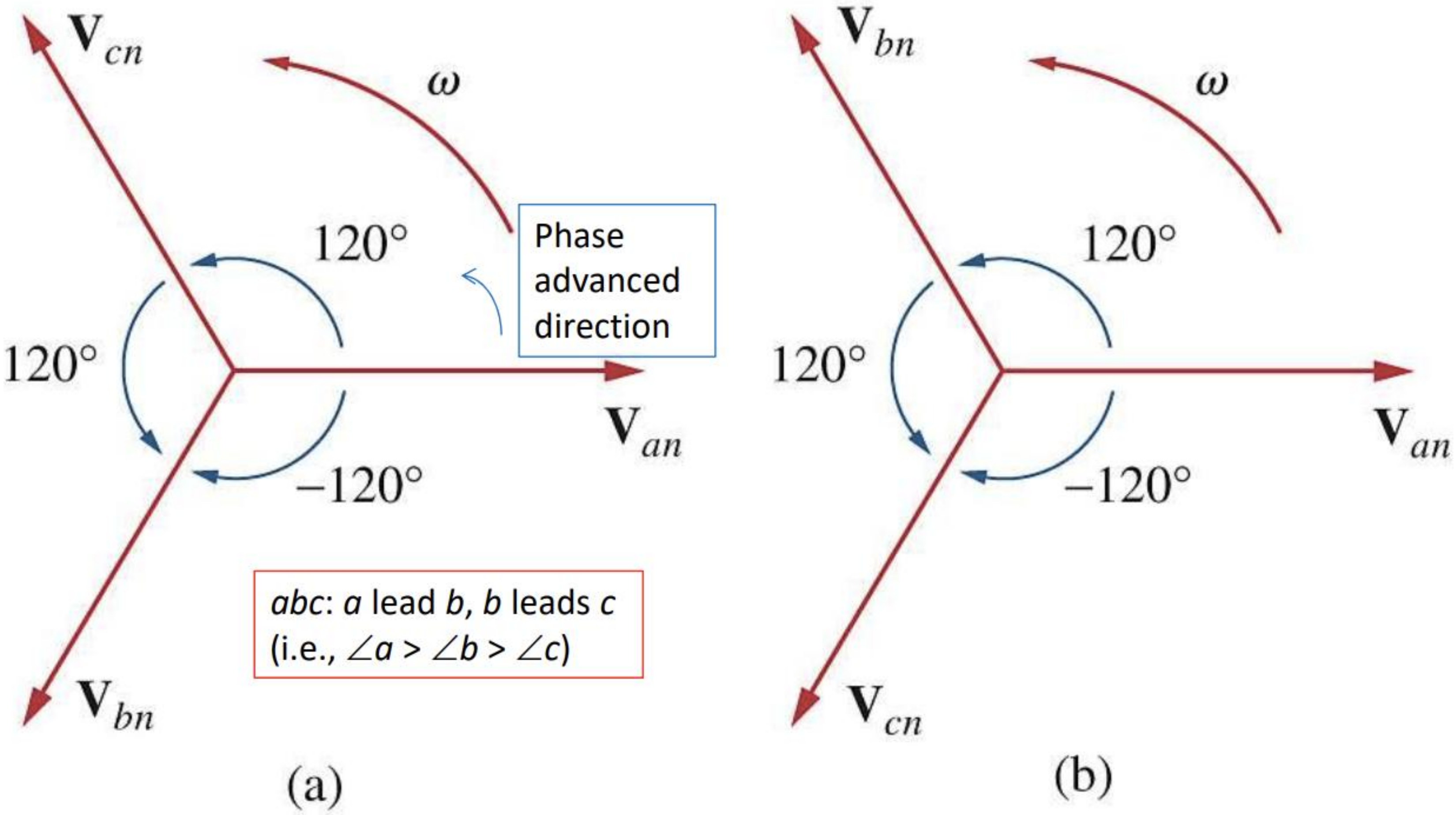



Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

Take-aways: Vp is the effective or rms value of phase voltage

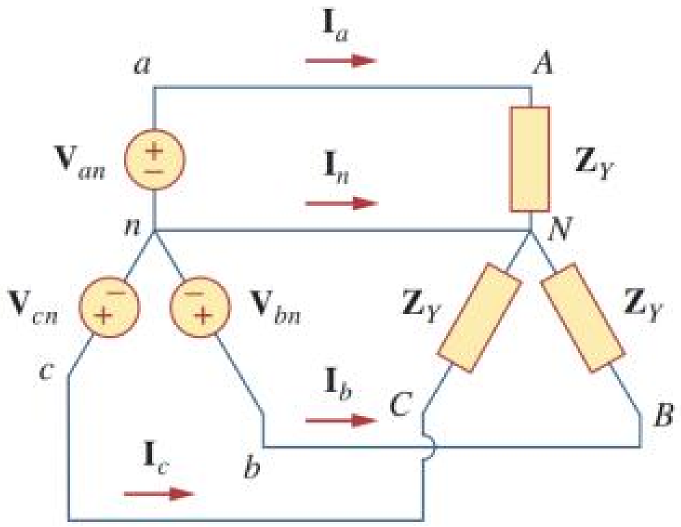
COPY

Y-Y

$\mathbf{V}_{an} = V_p\angle 0^\circ$
$\mathbf{V}_{bn} = V_p\angle -120^\circ$
$\mathbf{V}_{cn} = V_p\angle +120^\circ$
Same as line currents

$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

magnitude: $V_L = \sqrt{3}\,V_P$
$I_L = I_P$

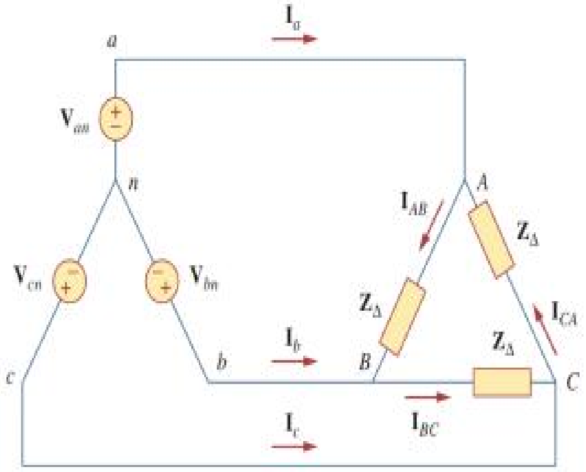



Y-Δ

$\mathbf{V}_{an} = V_p\angle 0^\circ$
$\mathbf{V}_{bn} = V_p\angle -120^\circ$
$\mathbf{V}_{cn} = V_p\angle +120^\circ$
$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$
$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$
$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$

$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

magnitude
$V_L = \sqrt{3}\,V_P$
$I_L = \sqrt{3}\,I_P$




for Page 20-21: assume all are balanced.

Take-aways: Vp is the effective or rms value of phase voltage

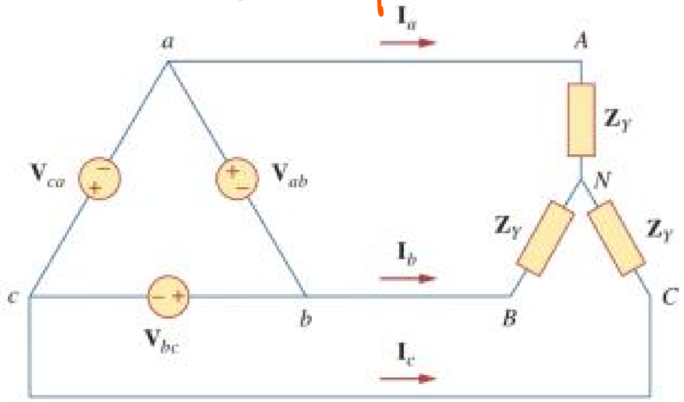
COPY

| | | |
|---|---|---|
| Δ-Y | $\mathbf{V}_{ab} = V_p\angle 0^\circ$<br>$\mathbf{V}_{bc} = V_p\angle -120^\circ$<br>$\mathbf{V}_{ca} = V_p\angle +120^\circ$ | Same as phase voltages |
| | Same as line currents | $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$<br>$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$<br>$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$ |

magnitude

$I_L = I_p$

$V_L = V_p$

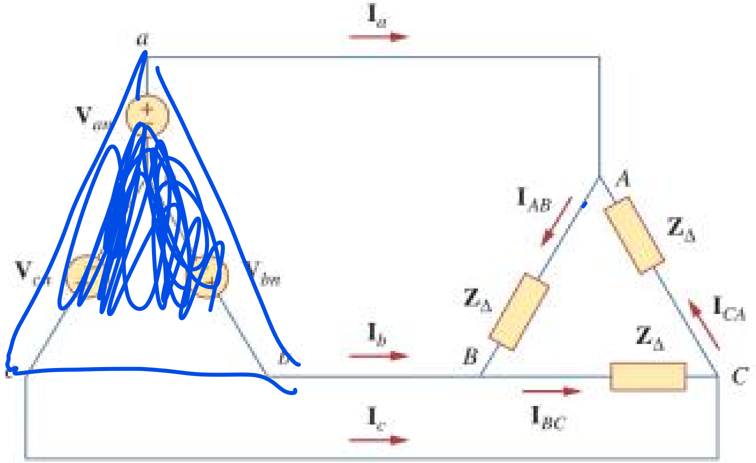



| | | |
|---|---|---|
| Δ-Δ | $\mathbf{V}_{ab} = V_p\angle 0^\circ$<br>$\mathbf{V}_{bc} = V_p\angle -120^\circ$<br>$\mathbf{V}_{ca} = V_p\angle +120^\circ$ | Same as phase voltages |
| | $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$ | $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$<br>$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$<br>$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$ |

magnitude:

$V_L = V_p$

$I_L = \sqrt{3} I_p$



Take-aways:
Power analysis in Three-phase circuits

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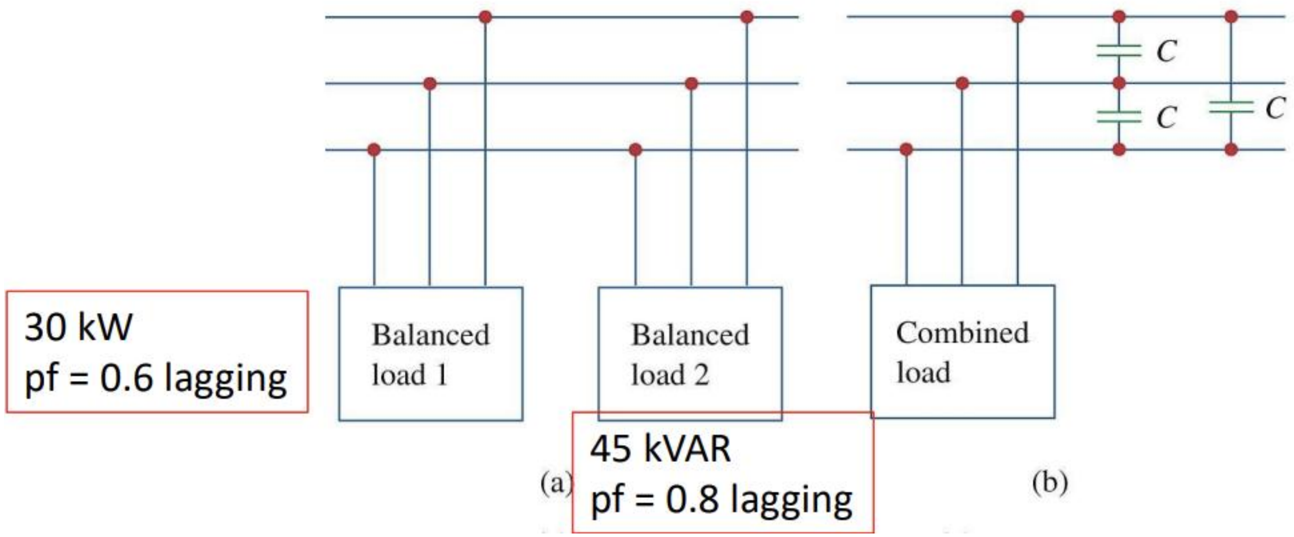
- Total instantaneous/average power
- $P = 3V_{Prms}I_{Prms}\cos(\angle Z)$ → You can always use this
- Total reactive power
- $Q = 3V_{Prms}I_{Prms}\sin(\angle Z)$
- Total complex power
- $S = 3\widetilde{V_{Prms}}\widetilde{I_{Prms}}^{*}$
- In particular, for Y-Y and Δ-Δ structures:
- $P = 3V_{Prms}I_{Prms}\cos(\angle Z) = \sqrt{3}V_{Lrms}I_{Lrms}\cos(\angle Z)$

If you want to use this make sure it's Y-Y / Δ-Δ.

# Exercise 4

(Prof.Chen’s slides, chapter 12, example 12.8)
Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig. 12.22(a).
Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging.
Assuming the **abc** sequence, determine:
(a) the complex, real, and reactive powers absorbed by the combined load,
(b) the line current
(c) the kVAR rating of the three capacitors -connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.



recall $pf = \frac{P}{|S|}$

(a) $V_L = 240\ kV$

$\cos\theta_1 = 0.6 \Rightarrow \tan\theta_1 = \frac{4}{3}$

$S_1 = P_1 + jQ_1$

$= P_1 + jP_1\tan\theta_1$

$= 30 + j40\ (kVA)$

Similarly, $S_2 = P_2 + jQ_2 = 60 + j45\ (kVA)$

$S = S_1 + S_2 = 90 + j85\ (kVA)$

$S = 90 + j85\ (kVA)$

$P = 90\ (kW)$

$Q = 85\ (kVAR)$

pay attention to the units!

(b) phase as whole: $\theta_1 = \tan^{-1}\left(\frac{85}{90}\right) = 43.36°$ ☆ clarify degrees & radians

$I_L = \frac{P}{\sqrt{3}V_L\cos\theta_1} = \frac{90\times10^3}{\sqrt{3}\times240\times10^3\times\cos 43.36°} = 0.298\ (A)$ → Assume Y−Y or Δ−Δ connected.

(c) new phase: $\theta_2 = \cos^{-1}0.9 = 25.84°$

$\Delta Q$ contributed by capacitors:

$\Delta Q = P(\tan\theta_1 - \tan\theta_2) = 90\times(\tan 43.36° - \tan 25.84°) =$

$= 41.40\ (kVAR)$

for a single Capacitor: $\Delta Q' = \Delta Q/3 = 13.80\ (kVAR)$

Recall $|Q_C| = |I^2X| = |V^2/X^*| = \omega CV^2$, if purely capacitive.

$\Rightarrow C = \frac{|\Delta Q'|}{\omega V_L^2} = \frac{13.8\times10^3}{2\pi\times60\times(240\times10^3)^2} = 635.51\ pF$

pay attention to the difference between f and w

Tips for exams:

1. Get familiar with basic concepts

2. Get familiar with your calculator

## Reference

[1] 2024 Fall VE215 slides, Songliang Chen

[2] 2024 Fall RC6, Yuting Cao

[3] 2023 Fall Final RC

[4] Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew

THANK YOU