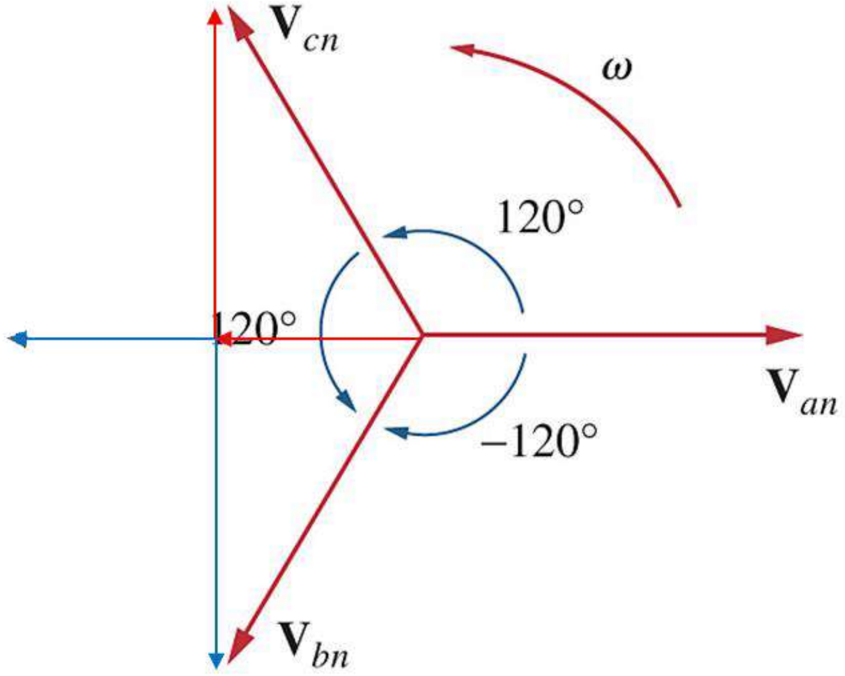
VE215 RC 6

Three-phase circuits and magnetically coupled circuits

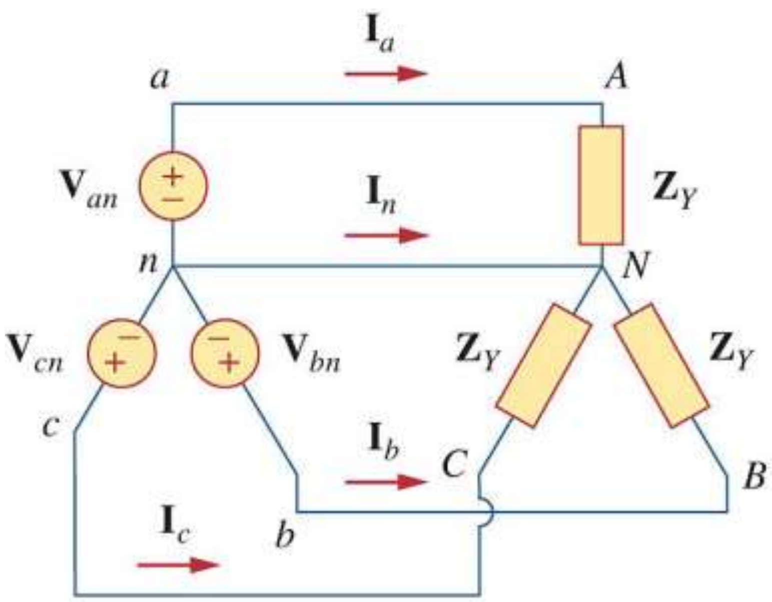
Physical illustration of three-phase



Balanced Y-Y

V_p is RMS here

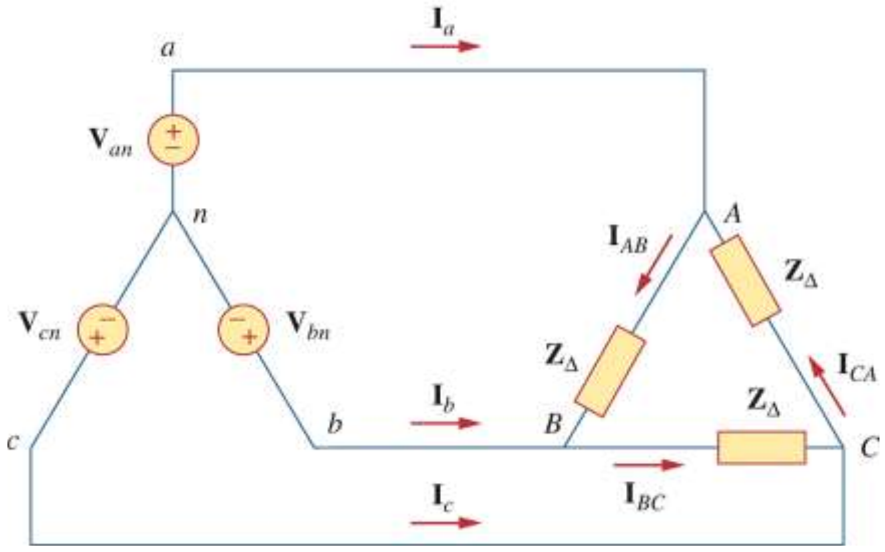
copy these 4 pages onto cheating paper



Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Balanced Y-Δ



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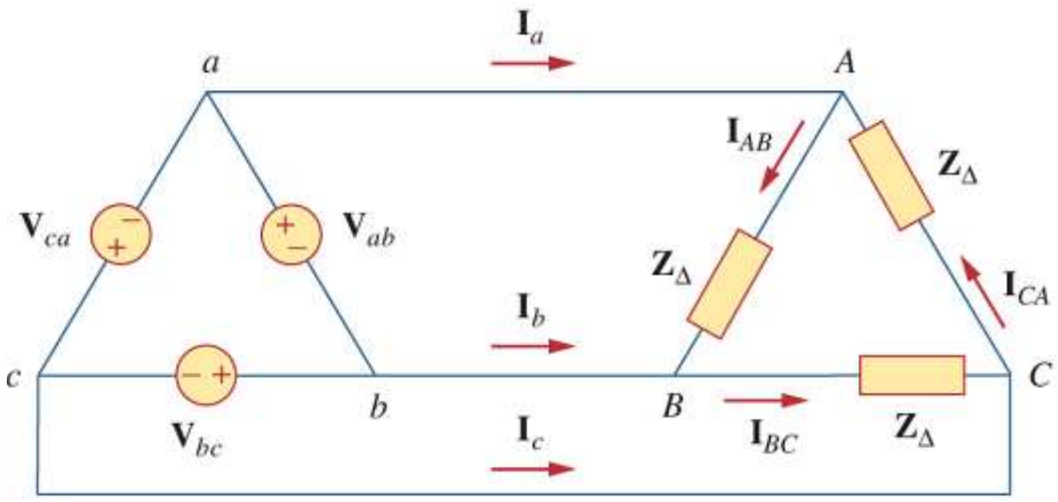
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Balanced Δ-Δ



Summary of phase and line voltages/currents for balanced three-phase systems.[1]

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Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
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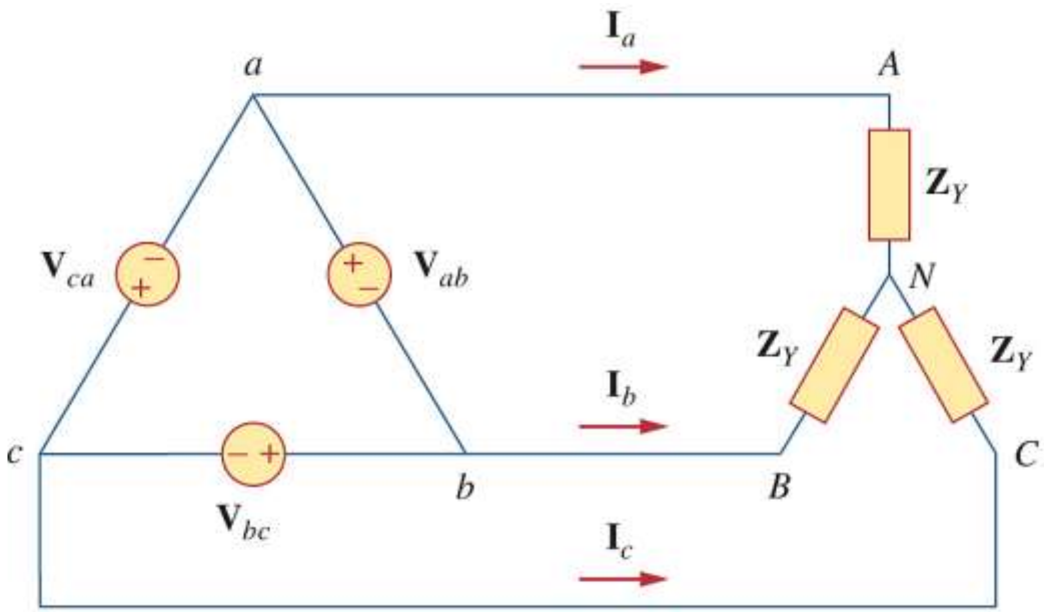
Balanced Δ-Y



Summary of phase and line voltages/currents for balanced three-phase systems.[1]

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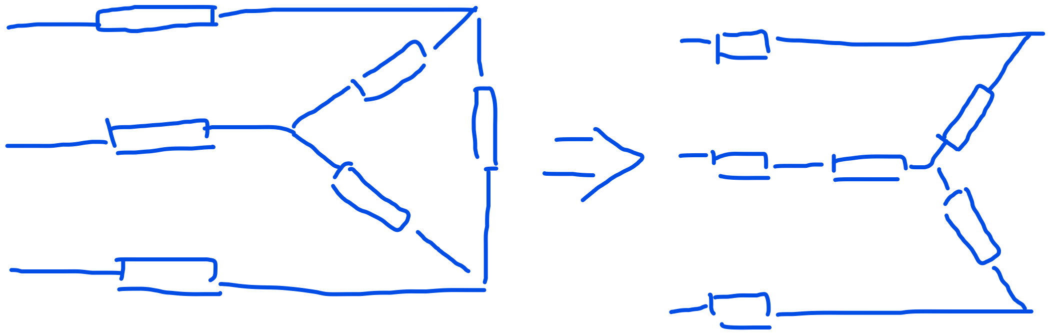
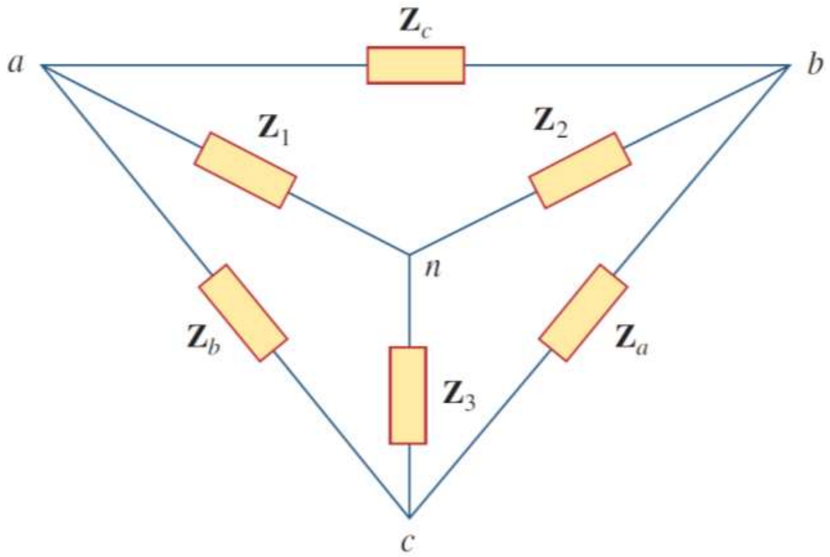
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Balanced Y-Δ transformation

- $Z_\Delta = 3Z_Y$
- A common trick when the lines also consume power in a Δ load circuit is to convert the Δ load into Y load.





Three-phase power analysis

- Total instantaneous/average power
- $P = 3V_{Prms}I_{Prms}\cos(\angle Z)$
- Total reactive power
- $Q = 3V_{Prms}I_{Prms}\sin(\angle Z)$
- Total complex power
- $S = 3\widetilde{V_{Prms}}\widetilde{I_{Prms}}^*$

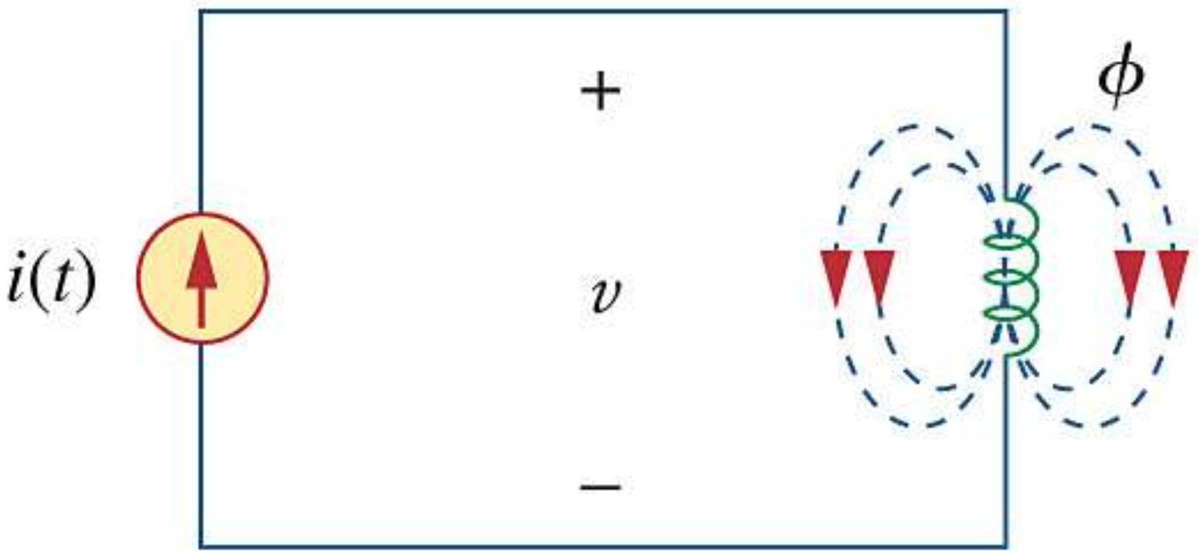
Three-phase power analysis

- In particular, for Y-Y and Δ-Δ structures:
- $P = 3V_{Prms}I_{Prms}\cos(\angle Z) = \sqrt{3}V_{Lrms}I_{Lrms}\cos(\angle Z)$

- The material to deliver the same power and tolerate the same loss for a three-phase system is 75% of that for a single-phase system.

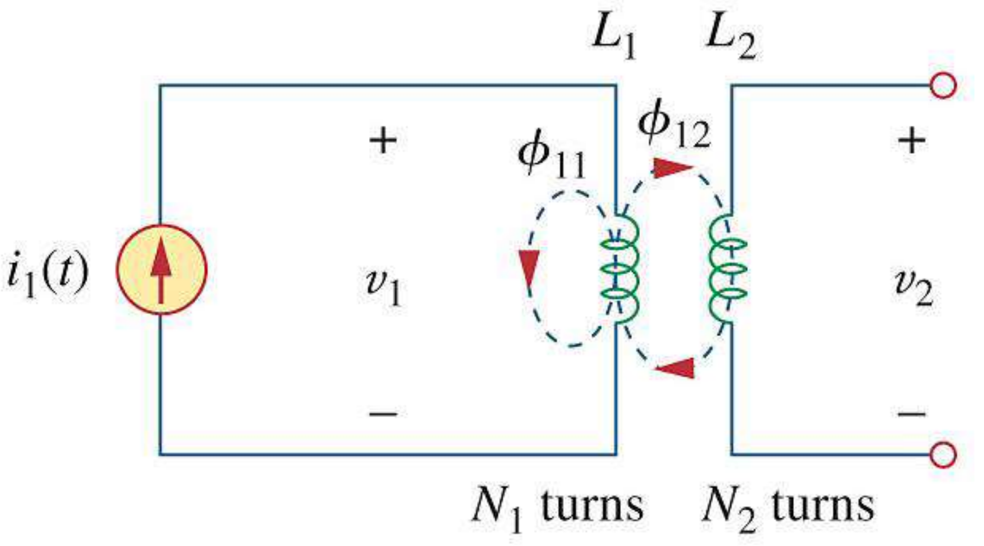
Self-inductance

- A single coil with N turns
- Current causes magnetic flux
- $v = N\frac{d\varphi}{dt}$
- $L = N\frac{d\varphi}{di}$



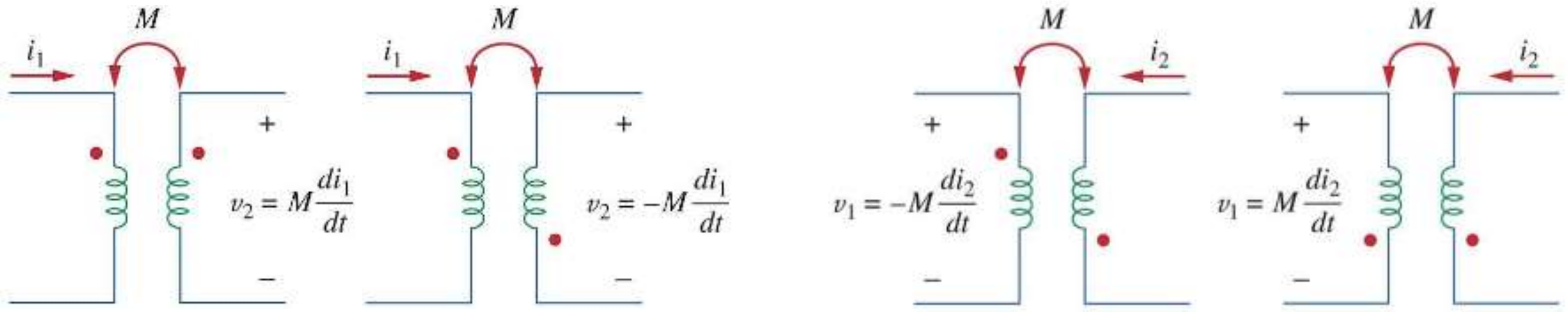
Mutual inductance

- $v_1 = N_1 \frac{d\varphi_1}{dt} = \underbrace{N_1 \frac{d\varphi_1}{di_1}}_{=L_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$
- $v_2 = N_2 \frac{d\varphi_{12}}{dt} = \underbrace{N_2 \frac{d\varphi_{12}}{di_1}}_{=M} \frac{di_1}{dt} = M \frac{di_1}{dt}$
- $M = N_2 \frac{d\varphi_{12}}{di_1}$



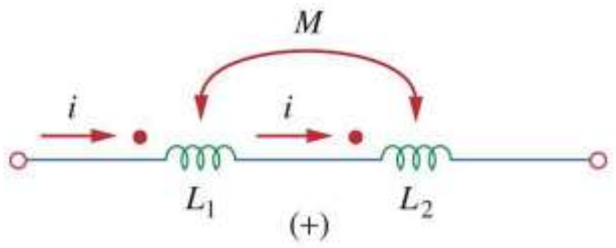
Dot convention

- If dots are on the same side, the direction of the induced voltage difference (from high to low) is the parallel to that of the current.
- If dots are on the opposite side, the direction of the induced voltage difference (from high to low) is antiparallel to that of the current.



Dot convention

- $L = L_1 + L_2 + 2M$



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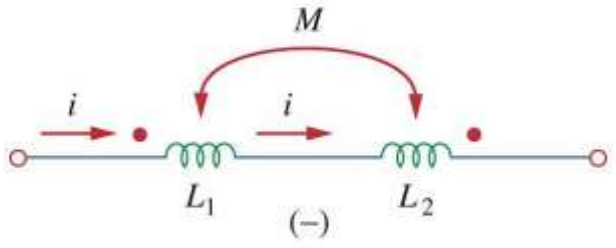
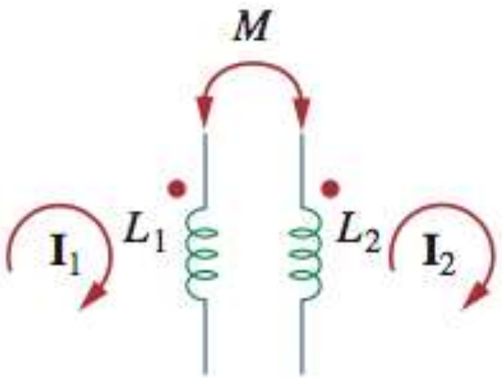
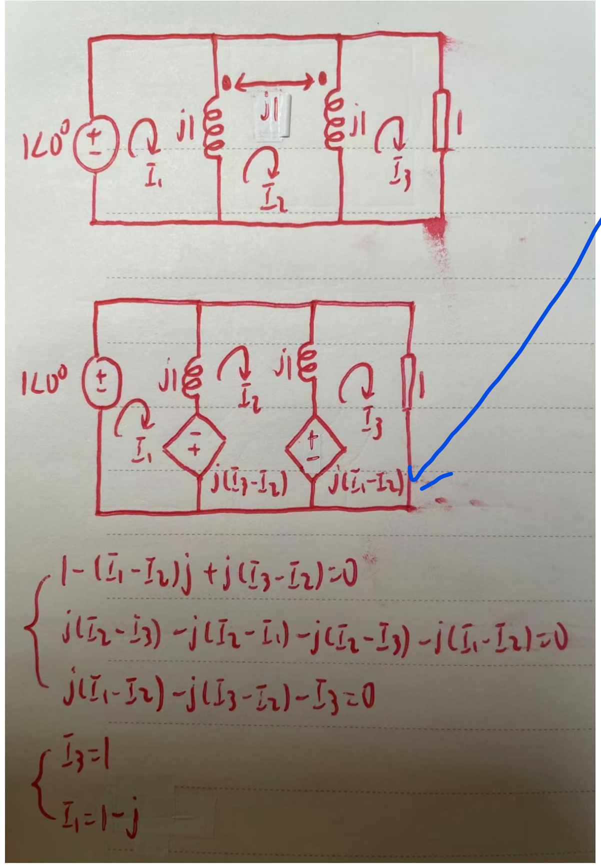


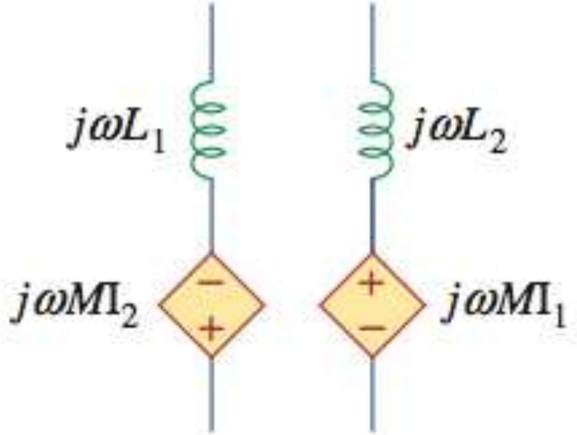
Example: Find I_1 and I_3

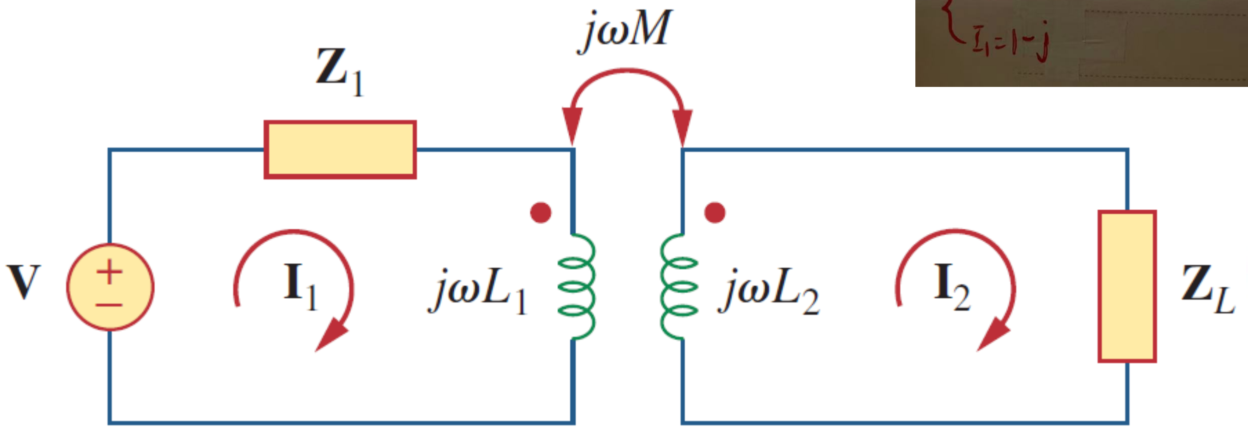
KVL application

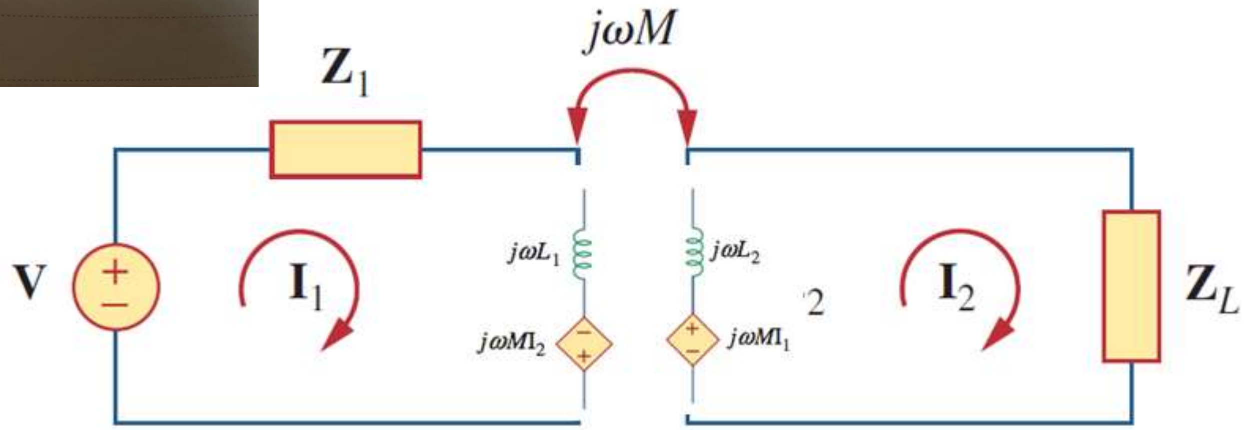




Do not forget to consider currents on both sides !!!

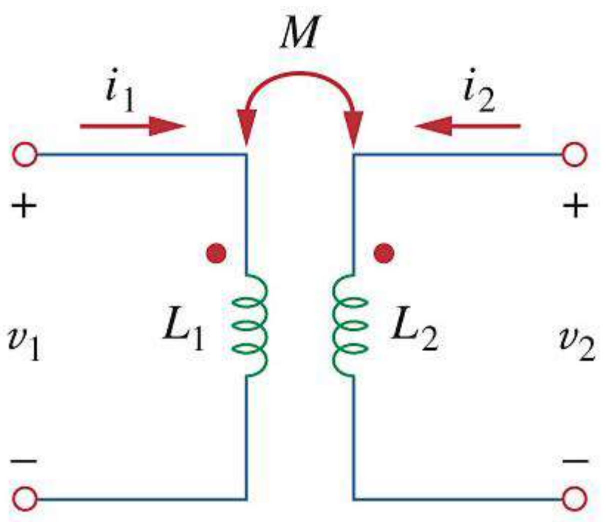




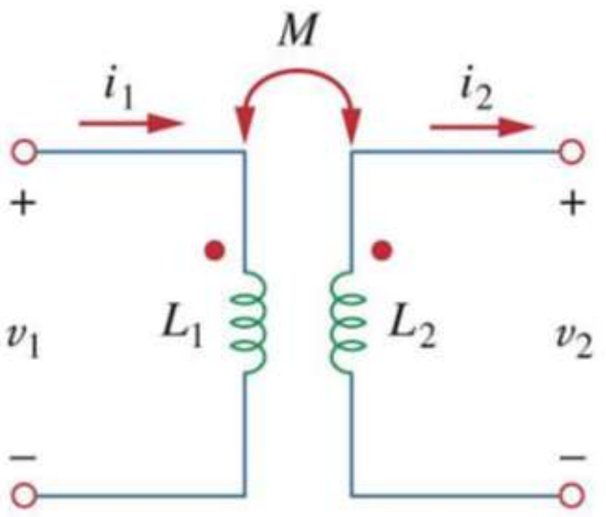


Energy

- $w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$



- $w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$



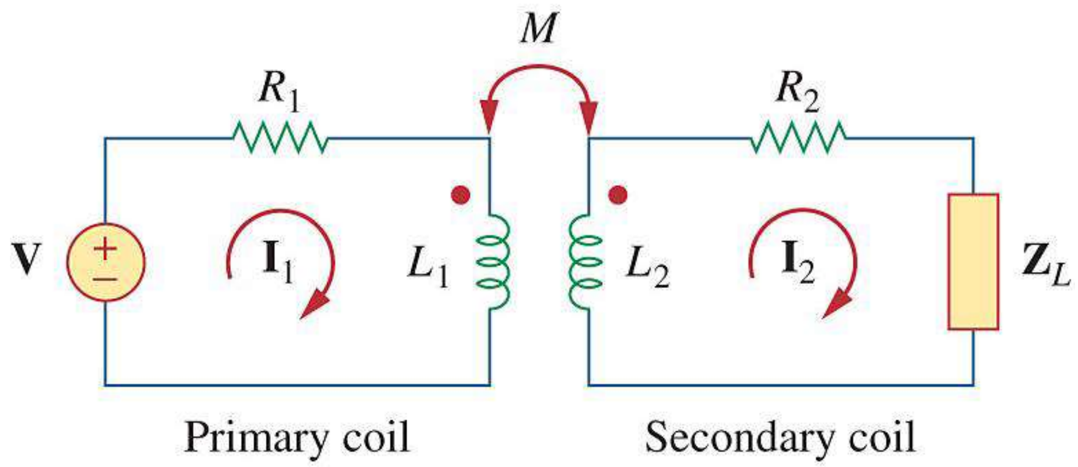
Coupling coefficient

- $k = \frac{M}{\sqrt{L_1 L_2}} \in [0,1]$
- $k < 0.5$, loosely coupled
- $k > 0.5$, tightly coupled

Linear transformer

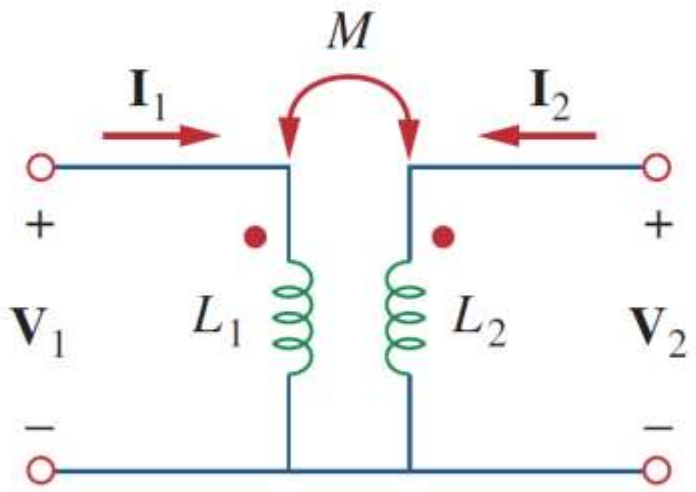
- $V = (R_1 + j\omega L_1)I_1 - j\omega M I_2$
- $0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2$

- $I_2 = \frac{j\omega M I_1}{R_2 + j\omega L_2 + Z_L}$
- $Z_{in} = \frac{V}{I_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$

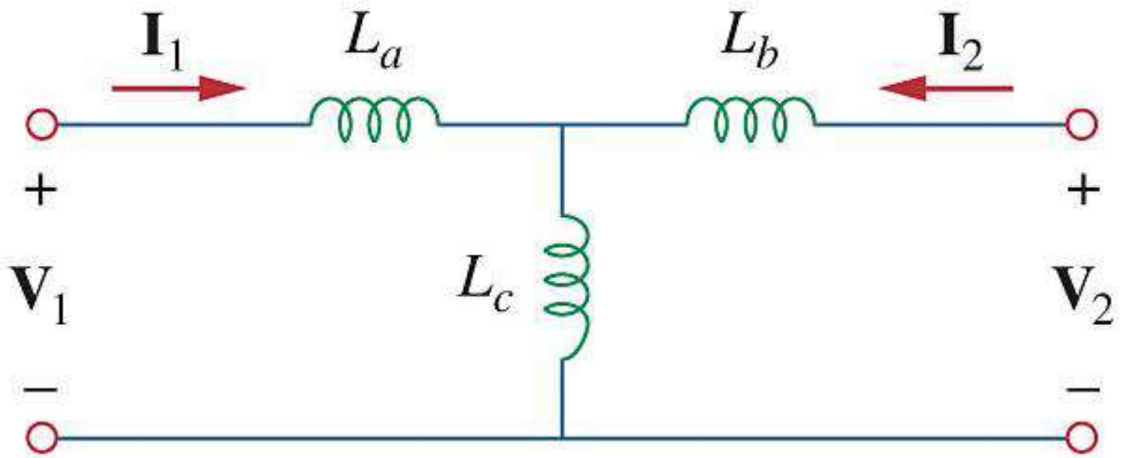


Linear transformer

- $V_1 = j\omega L_1 I_1 + j\omega M I_2$
- $V_2 = j\omega M I_1 + j\omega L_2 I_2$



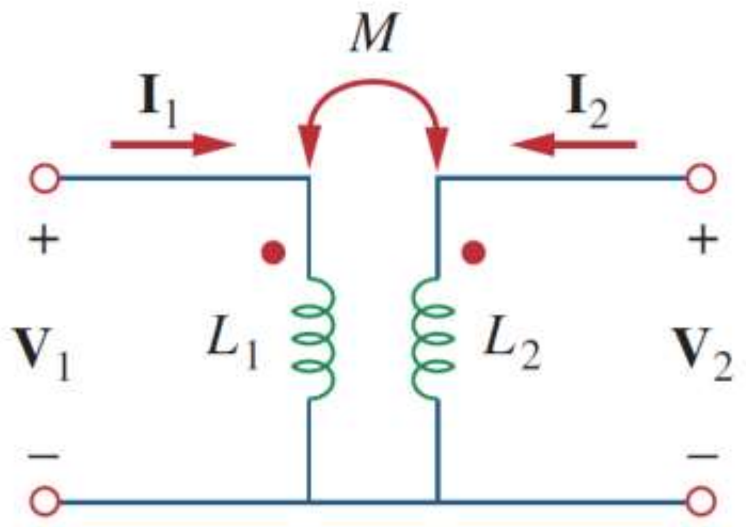
- Equivalent T circuit
- $L_a = L_1 - M$
- $L_b = L_2 - M$
- $L_c = M$

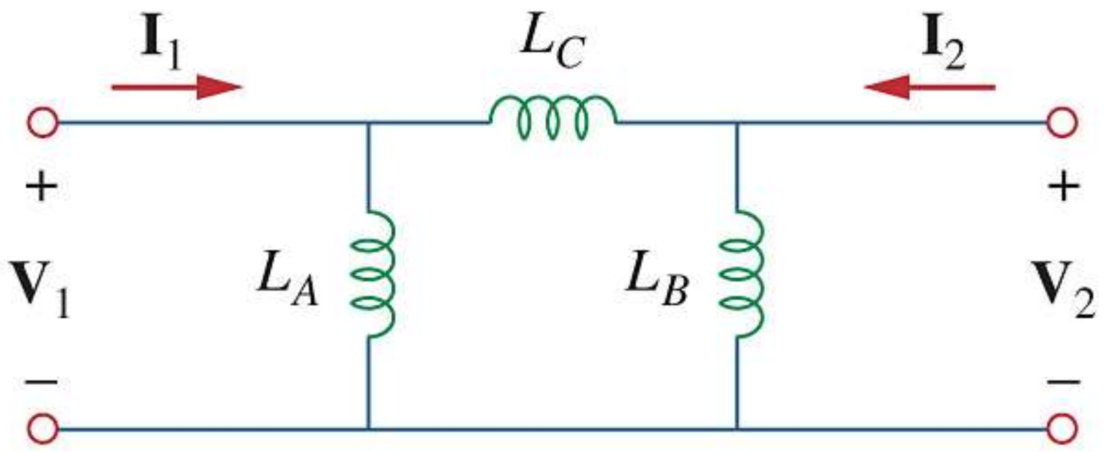


Linear transformer

- $V_1 = j\omega L_1 I_1 + j\omega M I_2$
- $V_2 = j\omega M I_1 + j\omega L_2 I_2$

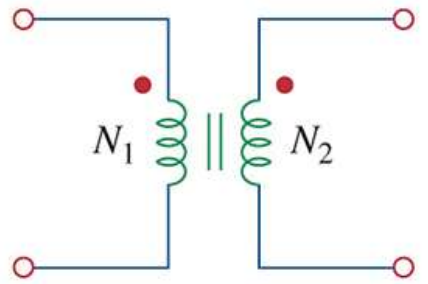
- Equivalent π circuit
- $L_A = \frac{L_1 L_2 - M^2}{L_2 - M}$
- $L_B = \frac{L_1 L_2 - M^2}{L_1 - M}$
- $L_C = \frac{L_1 L_2 - M^2}{M}$

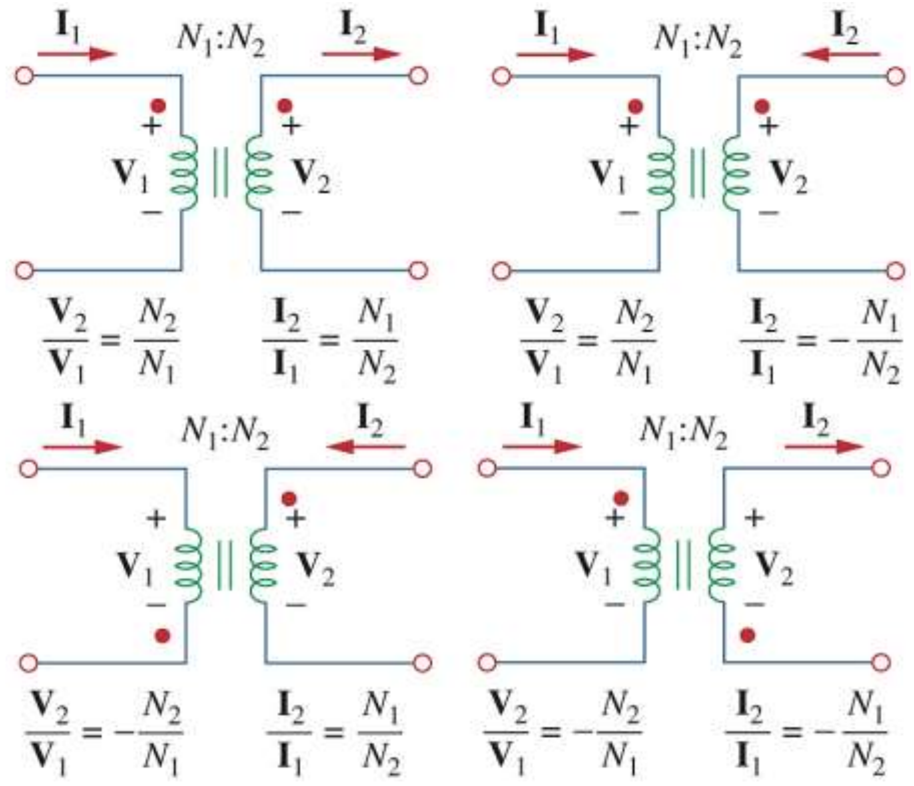




Ideal transformer

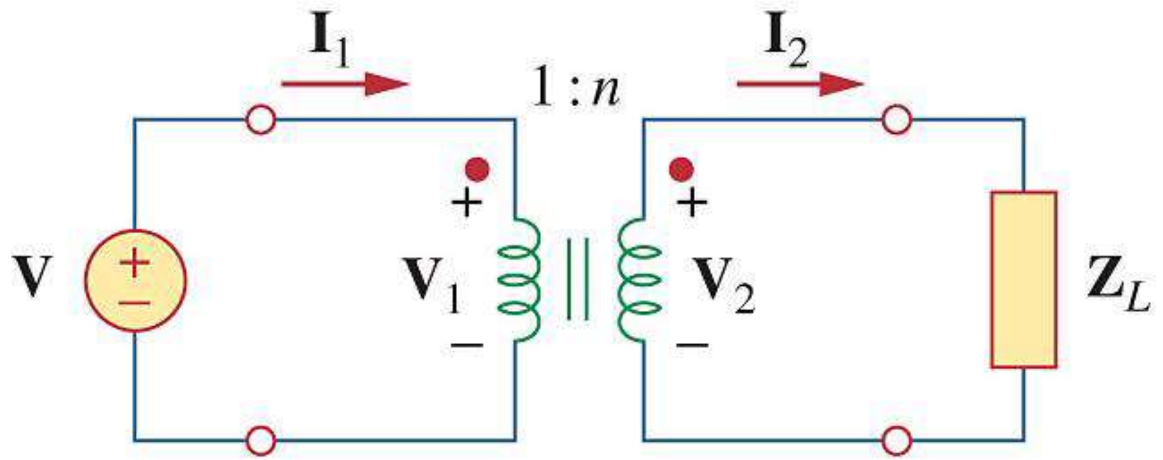
- $k = 1$
- $R_1 = R_2 = 0$
- $L_1 \to \infty$
- $L_2 \to \infty$
- $M \to \infty$





Ideal transformer

- $S_1 = \widetilde{V_1}\widetilde{I_1}^* = \frac{\widetilde{V_2}}{n}(n\widetilde{I_2})^* = \widetilde{V_2}\widetilde{I_2}^* = S_2$
- $Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_L}{n^2}$



Ideal transformer