上 海 交 通 大 学 试 卷

（ 2023~ 2024~1Academic Year/Fall Semester ）

Class No.________________ _ Name in English or Pinyin: _____________________

Student ID No._______________ Name in Hanzi(if applicable): _____________________

ECE2150J/VE215 Introduction to Circuits

Mid-term Exam with Answer

10:00 – 11:40 an 7th November 2023

The exam paper with answer has 15 pages in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

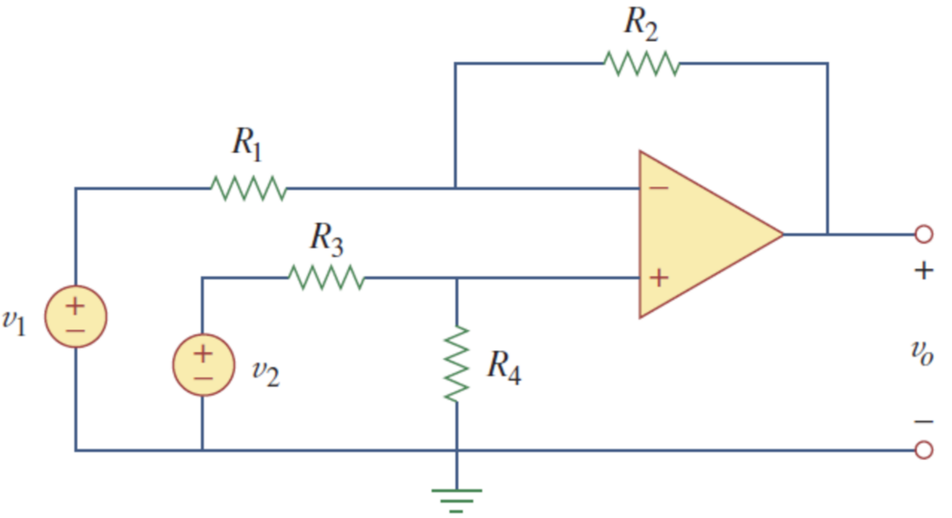
Signature:____________

Please enter grades here:

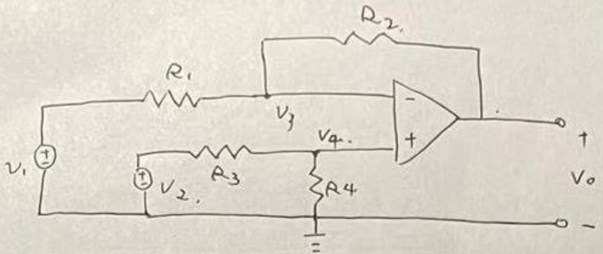
Exercises No. **题号**	**Points** **得分**	**Grader's Signature** **流水批阅人签名**
1		
2		
3		
4		
5		
6		
7		
Total 总分		

Q1. For the difference amplifier circuit, please derive the equation below. (8 points)

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$



Q.



$$\begin{cases} v_3 = v_4 & (2') \\ \dfrac{V_o - V_3}{R_2} + \dfrac{V_1 - V_3}{R_1} = 0 & (2') \\ \dfrac{V_2 - V_4}{R_3} + \dfrac{0 - V_4}{R_4} = 0 & (2') \end{cases} \Rightarrow v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}V_2 - \frac{R_2}{R_1}V_1 \quad (2')$$

If only write down "$V_o = (\frac{R_2}{R_1} + 1)\frac{R_4}{R_3+R_4}V_2 - \frac{R_2}{R_1}V_1$" with out any detailed deducing process. You will get 0 mark.

Q2. As a design engineer, you are asked to design a lighting system consisting of a 70 W power supply and two light bulbs as shown below. You must select the two bulbs from the following three available bulbs. The system should be designed for minimum cost such that lies within the range I = 1.2A ± 5%. (10 points)

$R_1 = 80\ \Omega$, cost ¥5; $R_2 = 90\ \Omega$, cost ¥7.5; $R_3 = 100\ \Omega$, cost ¥6.5

Q2. There are 3 possibilities. $R_1 \& R_2$. $R_2 \& R_3$ $R_1 \& R_3$.

range: $1.2 \pm 0.05 \cdot 1.2 = [1.14, 1.26]$ (1').

① $R_1 \& R_2$. $R = R_1 \| R_2 = 42.35\ \Omega$

$i = \sqrt{\frac{P}{R}} = 1.2857$ (not in range). (2')

② $R_2 \& R_3$ $R = R_2 \| R_3 = 47.37\ \Omega$. (2').

$i = \sqrt{\frac{P}{R}} = 1.2156$ (in the range). cost = ¥14

③ $R_1 \& R_3$ $R = R_1 \| R_3 = 44.44\ \Omega$. (2').

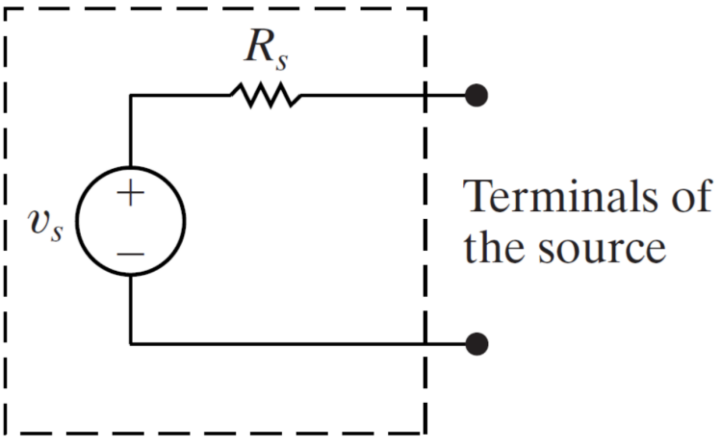
$i = \sqrt{\frac{P}{R}} = 1.2551$ (in the range) cost = ¥11.5

So, our choice is R_1 and R_3 (3').

Q3. The circuit model of a dc voltage source is shown below. The following voltage measurements are made at the terminals of the source: (1) With the terminals of the source open, the voltage is measured at 50 mV, and (2) with a 15 MΩ resistor connected to the terminals, the voltage is measured at 48.75 mV. All measurements are made with a digital voltmeter that has a meter resistance of 10 MΩ. (10 points)

(a) What is the internal voltage of the source v_s in millivolts?

(b) What is the internal resistance of the source R_s in kilo-ohms?



$$\begin{cases} ① \; 50 = \dfrac{10V_s}{10+R_s} & 2' \quad V_s \text{ in mV} \quad R_s \text{ in M}\Omega. \\ ② \; 48.75 = \dfrac{6V_s}{6+R_s} & 2' \end{cases}$$

(a) $50R_s = 10V_s - 500 \Rightarrow R_s = 0.2V_s - 10.$ 1'

plug in ② ↳ $48.75 = \dfrac{6V_s}{0.2V_s - 4} \Rightarrow V_s = 52\text{mV}$ 2'

(b) $50 = \dfrac{520}{10+R_s} \Rightarrow R_s = 400\text{k}\Omega$

1' 2'

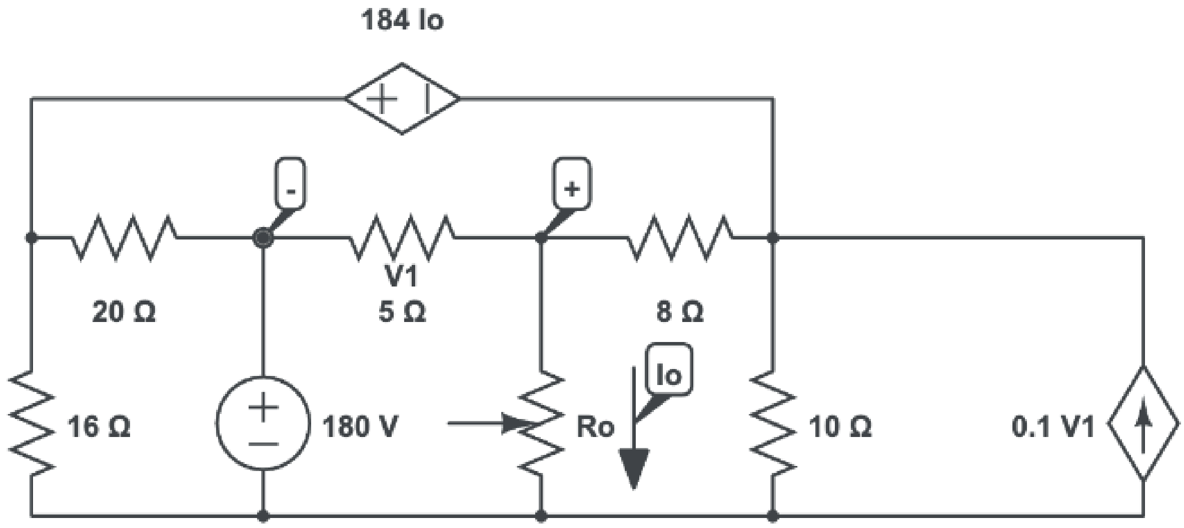
Note: Consider 10MΩ and 15MΩ as in series ⇒ maximum is 5'

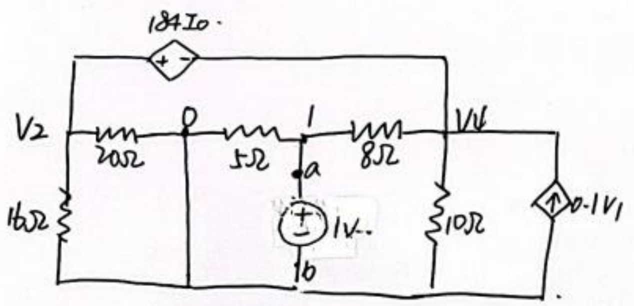
Wrong Unit ⇒ 1'

Q4. The rheostat R_o in the circuit below has been adjusted so that the maximum power is delivered to R_o. (18 points)

(a) Find the value of R_o.

(b) Find the maximum power delivered to R_o.





Rth:

For either mesh analysis or node analysis equation set, it worth three points (3 pts)

$$\begin{cases} \frac{V_2}{16}+\frac{V_2}{20}+\frac{V_4-1}{8}+\frac{V_4}{10}-0.1V_1=0 \\ V_2 = V_4+184\,I_o \\ I_o = -i_x \\ \frac{1-0}{5}+\frac{1-V_4}{8} = i_x \end{cases} \quad (3')$$

(test voltage) (test current)

Then for successfully obtaining the value of i_x or V_x.

It worth 2 points (pts) + 2 points for deriving any median values

By solving equation set.

$$\left(\frac{9}{80}\times 24+\frac{9}{40}\right)V_4 = \frac{9}{80}\times\frac{299}{5}+\frac{1}{8}-0.1$$

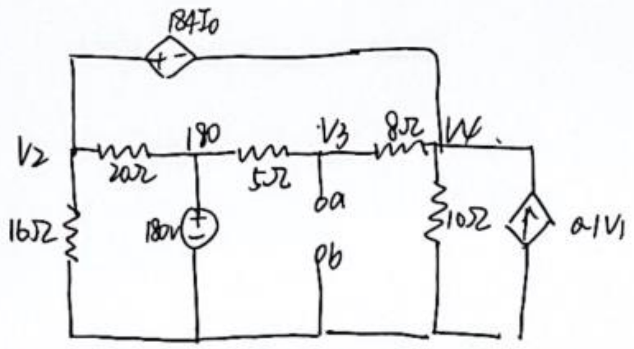
$$\boxed{V_4 = \frac{2701}{1170}\ V \text{ (median value)}} \quad (2')$$

$$\boxed{i_x = \frac{13}{40}-\frac{1}{8}V_4 = \frac{341}{9360}\ A} \quad (2')$$

$$\boxed{R_{th} = \frac{1}{i_x} = \frac{9360}{341} = 27.45\,\Omega} \quad \boxed{R_o = R_{th} = 27.45\,\Omega \quad (2')}$$

For successfully solving the value of Rth, you can get 2 points.

PS: There may be some bias in your answer if you approximately keep the value in some steps. If you find you actually get right answer but lose points for approximate issue, come to paper checking



Since it's similar to list equation set as question (1). this weighs 1 point.

$$\begin{cases} \frac{V_2}{16}+\frac{V_2-180}{20}+\frac{V_4-V_3}{8}+\frac{V_4}{10}-0.1V_1=0. \\ V_1=180-V_3 \\ \frac{V_3-180}{5}+\frac{V_3-V_4}{8}=0. \\ V_4+184I_0=V_2 \qquad (1') \\ I_0=0 \end{cases}$$

For successfully deriving Vab / Vth / V3. it weighs 4 points in total but if you separately calculate any median value, you get 2 pts. then after you derive the right value of Vab / Vth / V3 by applying the median value, you get 2 pts. (4' = 2' + 2')

$$V_3=\frac{1440}{13}+\frac{5}{13}V_4.$$

$$\left(\frac{1}{16}+\frac{1}{20}\right)V_4+\left(\frac{1}{8}+\frac{1}{10}\right)V_4-\frac{1}{8}\left(\frac{1440}{13}+\frac{5}{13}V_4\right)=9+0.1\left(180-\frac{1440}{13}-\frac{5}{13}V_4\right)$$

$$V_4=\frac{30960}{341}\,V \qquad (2')$$

$$V_{th}=V_3=\frac{1440}{13}+\frac{5}{13}\times\frac{30960}{341}=145.69V. \qquad (2')$$

$$\boxed{P=\frac{V_{th}^2}{4R_{th}}}^{\,2'}=\frac{145.69^2}{4\times27.45}\overset{(2')}{=}193.3\,w$$

This equation weighs 2 points as long as you write it.
The right answer weighs another 2 points.

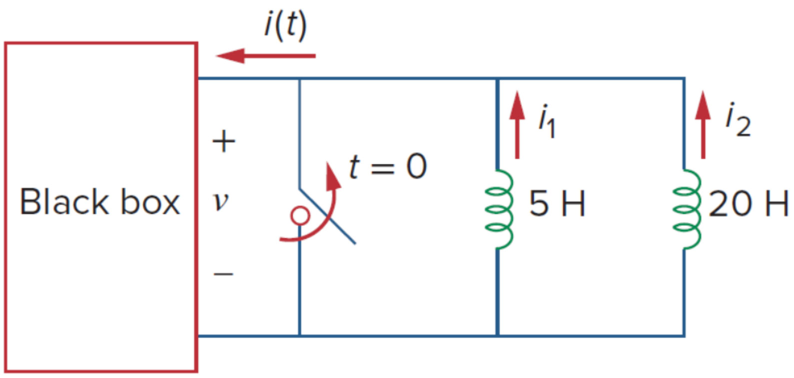
Q5. Inductors are initially charged and are connected to the black box at t = 0. If $i_1(0) = 6$ A, $i_2(0) = -3$ A, and $v(t) = 50e^{-100t}$ mV when $t \geq 0$, please answer (a)-(d). (16 points)

(a) The energy initially stored in each inductor.

(b) The total energy delivered to the black box from t = 0 to t = ∞.

(c) $i_1(t)$ and $i_2(t)$ when $t \geq 0$.

(d) $i(t)$, $t \geq 0$.



Note:

- The unit of $v(t)$ is mV.
- Pay attention to the minus sign of $v(t)$ in question(c) since $i(t)$ flows in from the positive side of $v(t)$.

Answer:

(a) (4') The energy initially stored in each inductor is:

$$W_1 = \tfrac{1}{2}L_1 i_1^2 = \tfrac{1}{2} \times 5 \times 6^2 = 90 \text{ J} \qquad (2')$$
$$W_2 = \tfrac{1}{2}L_2 i_2^2 = \tfrac{1}{2} \times 20 \times (-3)^2 = 90 \text{ J} \qquad (2')$$

(b) (2') *This question has a design error. Both answers below are considered correct when the paper is batched.

Answer 1:

All the energy is transferred to the black box.

$$W = W_1 + W_2 = 90 + 90 = 180 \text{ J} \qquad (2')$$

Answer 2:

After question(c) is finished, the energy delivered can be calculated from the difference between the initial total energy stored by the inductor and the final total energy.

$$\begin{aligned}
W &= W_i - W_f \\
&= (W_{1i} + W_{2i}) - (W_{1f} + W_{2f}) \\
&= (90 + 90) - [\frac{1}{2}L_1 i_1(\infty)^2 + \frac{1}{2}L_2 i_2(\infty)^2] \\
&= 180 - [\frac{1}{2} \times 5 \times 5.9999^2 + \frac{1}{2} \times 20 \times (-3.000025)^2] \\
&= 1.49996875 \times 10^{-3} \text{ J } (or\ 1.5 \times 10^{-3} \text{ J }) \qquad (2')
\end{aligned}$$

(c) (8')

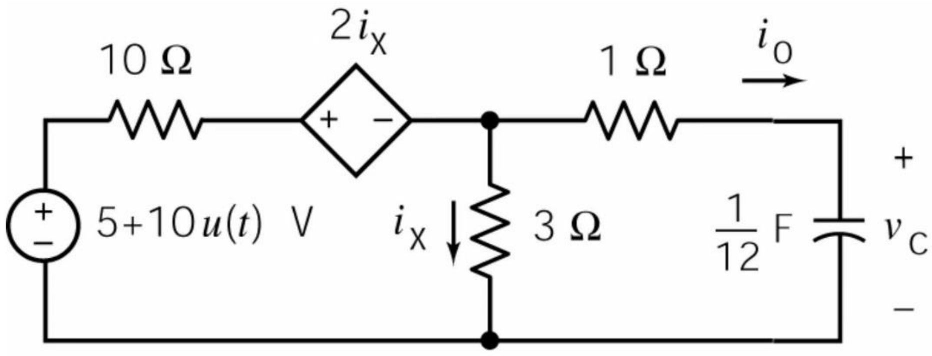
$$
\begin{aligned}
i_1(t) &= \frac{1}{L_1}\int_0^t -v(t)\times 10^{-3}dt + i_1(0) && (2') \\
&= \frac{1}{5}\int_0^t -50e^{-100t}\times 10^{-3}dt + 6 \\
&= \frac{1}{5}\left(\frac{1}{100}\right)\left[50e^{-100t}\times 10^{-3}\right]_0^t + 6 \\
&= 1\times 10^{-4}\left(e^{-100t}-1\right) + 6\ \mathrm{A} && (2')
\end{aligned}
$$

$$
\begin{aligned}
i_2(t) &= \frac{1}{L_2}\int_0^t -v(t)\times 10^{-3}dt + i_2(0) && (2') \\
&= \frac{1}{20}\int_0^t -50e^{-100}\times 10^{3}dt - 3 \\
&= \frac{1}{20}\left(\frac{1}{100}\right)\left[50e^{-100t}\times \left[10^{-3}\right]_0^t - 3 \\
&= 2.5\times 10^{-5}\left(e^{-100t}-1\right) - 3\ \mathrm{A} && (2')
\end{aligned}
$$

(d) (2')

$$
\begin{aligned}
i(t) &= i_1(t) + i_2(t) && (1') \\
&= 1.25\times 10^{-4}\left(e^{-100t}-1\right) + 3\ \mathrm{A} && (1')
\end{aligned}
$$

Q6. Please determine a capacitor current i_0 when $t > 0$. (18 points)



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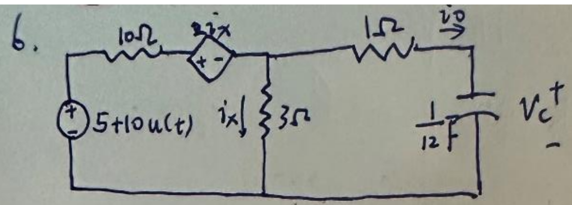


1° for $t<0$ (4+4)

$(10+2+3)i_x = 5 \Rightarrow i_x = \frac{1}{3}A$ --4'

$\Rightarrow v_c(0) = 3i_x = 1V$ ---4'

2° for $t \geq 0$ (10)

$i_x(10+2+3) = 15 \Rightarrow i_x = 1A$

$v(\infty) = 3V$ 2'

$\Rightarrow$ Then Thevenin.

$\Rightarrow$

$R_{TH} = 3\Omega$

$V_{TH} = 3V$

$\tau = RC = 3 \times \frac{1}{12} = \frac{1}{4}$ --- 4'

$\Rightarrow v_c(t) = 3 - 2e^{-4t}$

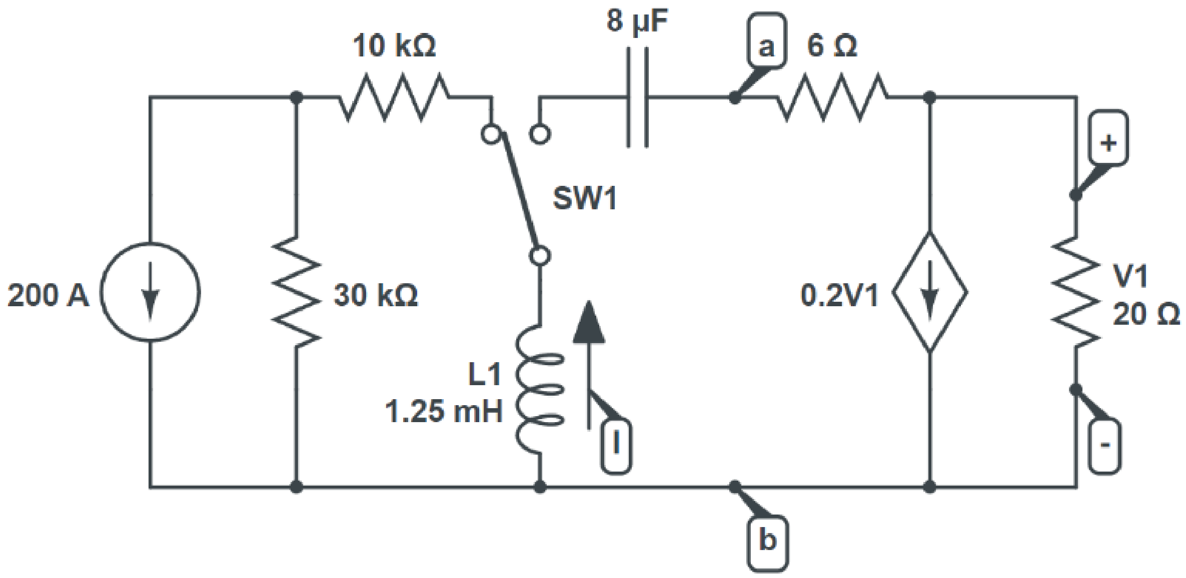
$i_0(t) = \frac{2}{3}e^{-4t}$ 4'

Q7. In the following circuit, the switch SW1 has been in left position for a long time. At t = 0, the switch moves instantaneously to the right position. Please answer the following questions. (20 points)

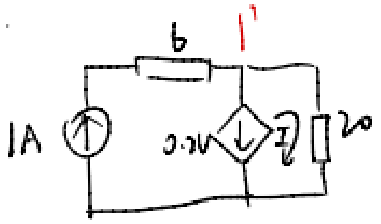
(a) Find the Norton equivalent resistance R_N between port a and b.

(b) Find $I(0+)$ and $I'(0+)$.

(c) Find $I(t)$ for t > 0.



(a) 5'



$1 - I = 0.2 \cdot 20 I$ 2'

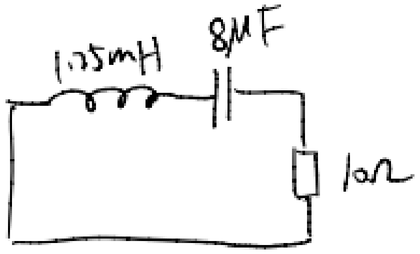
$I = 0.2A$

$V_{th} = I \cdot 20 + 1 \cdot 6 = 10V$ 1'

$R_N = 10\Omega$ 1'

(b) $I(0^+) = 150A$ 2'

5'



$L\frac{di}{dt} + V + 10i = 0$ 1'

$L\frac{d^2I}{dt^2} + \frac{I}{C} + 10\frac{dI}{dt} = 0$

$I'(0^+)$: $L \cdot I'(0^+) + 10I(0) = 0$

$I'(0^+) = -1.2\times10^6 A\cdot S^{-1}$ 2'

(c) $\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$ 1'

$S^2 + 8000S + 10^8 = 0$

10'

$S_{1,2} = -4000 \pm 9165.15139j$ 2'

$I(t) = e^{-4000t}(C_1\cos\omega t + C_2\sin\omega t)$ 2' $\omega = 9165.15$

$I'(t) = -4000e^{-4000t}(C_1\cos\omega t + C_2\sin\omega t)$

$+ \omega e^{-4000t}(-C_1\sin\omega t + C_2\cos\omega t)$

$I(0) = C_1 = 150$ 2'

$I'(0) = -4000C_1 + \omega C_2 = -1.2\times10^6$ 2'

$C_2 = -65.465367$

$I(t) = e^{-4000t}(150\cos(9165.15t) - 65.465\sin(9165.15t))$ 1'