Answer for HW1

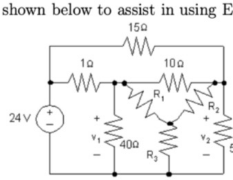
Exercise 1.1 (a) $Q = \int_0^3 2e^{-2t} \cdot 10^{-3} dt = -e^{-2t} \cdot 10^{-3} \Big|_0^3 = 1 - e^{-6} \cdot 10^{-3} \approx 0.998 \cdot 10^{-3} C$

(b) $P(t) = V(t)\, i(t) = 15 \cdot 10^{-3} \cdot -4e^{-2t} \cdot 2e^{-2t} \cdot 10^{-3} = 1.2 \cdot 10^{-4} e^{-4t}$

(c) $W(t) = \int_0^3 P(t)\, dt = 3 \times 10^{-5} W$

Exercise 1.2

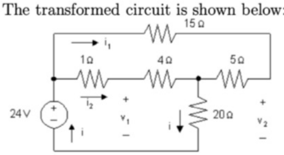
P 3.59 Begin by transforming the Δ-connected resistors ($10\,\Omega, 40\,\Omega, 50\,\Omega$) to Y-connected resistors. Both the Y-connected and Δ-connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 – 3.46 to calculate the values of the Y-connected resistors:

$$R_1 = \frac{(40)(10)}{10+40+50} = 4\,\Omega; \quad R_2 = \frac{(10)(50)}{10+40+50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10+40+50} = 20\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{eq} = (15+5)\|(1+4) + 20 = 20\|5 + 20 = 4 + 20 = 24\,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24\text{ V}}{24\,\Omega} = 1\text{ A}$$

Use current division to calculate the currents i_1 and i_2. Note that the current i_1 flows in the branch containing the $15\,\Omega$ and $5\,\Omega$ series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the $1\,\Omega$ and $4\,\Omega$ resistors:

$$i_1 = \frac{4}{15+5}(i) = \frac{4}{20}(1\text{ A}) = 0.2\text{ A}, \quad \text{and} \quad i_2 = 1\text{ A} - 0.2\text{ A} = 0.8\text{ A}$$

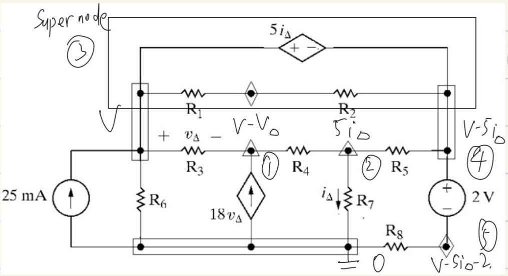
Now use KVL and Ohm's law to calculate v_1. Note that v_1 is the sum of the voltage drop across the $4\,\Omega$ resistor, $4i_2$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_1 = 4i_2 + 20i = 4(0.8\text{ A}) + 20(1\text{ A}) = 3.2 + 20 = 23.2\text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2. Note that v_2 is the sum of the voltage drop across the $5\,\Omega$ resistor, $5i_1$, and the voltage drop across the $20\,\Omega$ resistor, $20i$:

$$v_2 = 5i_1 + 20i = 5(0.2\text{ A}) + 20(1\text{ A}) = 1 + 20 = 21\text{ V}$$

Ex 1.3



(a) 12 branches, 7 nodes

$12-7+1=6 \Rightarrow 6$ meshes

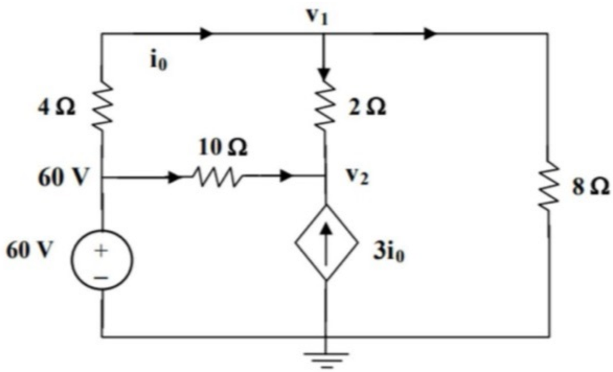
(b) Using nodal analysis, assuming 0 V for nodal 4. Assume voltage at ③ is V. Then $V_1 = V - V_\Delta$, $V_2 = 5i_\Delta$, $V_4 = V - 5i_\Delta$, $V_5 = V - 5i_\Delta - 2$

Then we get three equation by nodal analysis from ①, ②, ③

$$\begin{cases} \dfrac{V_\Delta}{5} + 18V_\Delta + \dfrac{5i_\Delta - (V - V_\Delta)}{5} = 0 \\ i_\Delta + \dfrac{5i_\Delta - (V - V_\Delta)}{5} + \dfrac{5i_\Delta - (V - 5i_\Delta)}{5} = 0 \\ \dfrac{V}{5} + \dfrac{V_\Delta}{5} - 0.025 + \dfrac{V - 5i_\Delta - 5i_\Delta}{5} + \dfrac{V - 5i_\Delta - 2}{5} = 0 \end{cases} \Rightarrow \begin{cases} i_\Delta = 0.14\,A \\ V_\Delta = 0.0077\,V \end{cases}$$

Ex1.4 Solution

(a)



At node 1, $\frac{60 - v_1}{4} = \frac{v_1}{8} + \frac{v_1 - v_2}{2}$ $\qquad 120 = 7v_1 - 4v_2 \qquad (1)$

At node 2, $3i_0 + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0$

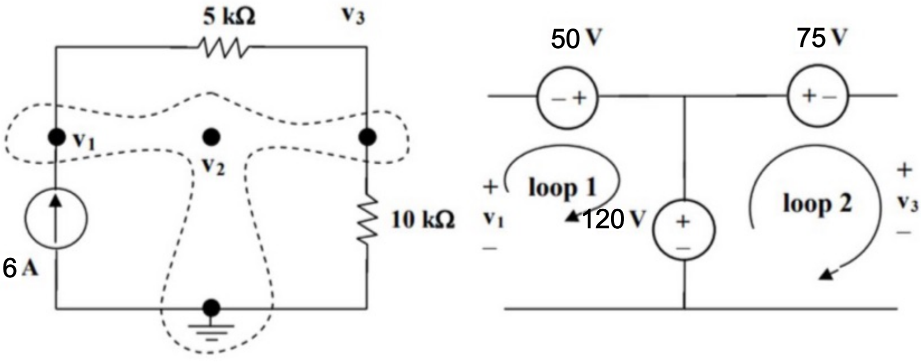
But $i_0 = \frac{60 - v_1}{4}$.

Hence

$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \qquad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \frac{60 - v_1}{4} =$ **1.73 A**

(b)



(a) (b)

We have a supernode as shown in figure (a). It is evident that $v_2 = 12$ V, Applying KVL to loops 1and 2 in figure (b), we obtain,

$$-v_1 - 50 + 120 = 0 \text{ or } v_1 = 70 \text{ and} -120 + 75 + v_3 = 0 \text{ or } v_3 = 45\text{ V}$$

Thus, $v_1 = 70\text{V}, \; v_2 = 120\text{V}, \; v_3 = 45\text{V}.$

1.5.a

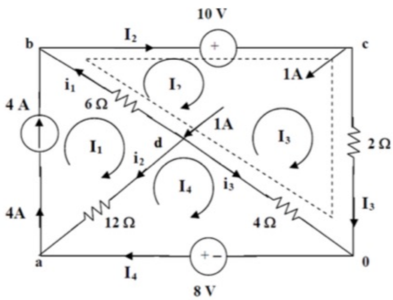
$$\begin{bmatrix} 7 & -2 & 0 & 0 \\ -2 & 6 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ 0 & 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -10 \\ -4 \end{bmatrix}$$

The solution is:

$$i_1 = 0.773, \quad i_2 = -1.295, \quad i_3 = -3.330, \quad i_4 = -1.466$$

1.5.b

For the circuit shown:



It is evident that:

$$I_1 = 4$$

For mesh 4:

$$12(I_4 - I_1) + 4(I_4 - I_3) - 8 = 0$$

For the supermesh:

$$6(I_2 - I_1) + 10 + 2I_3 + 4(I_3 - I_4) = 0 \quad \text{or} \quad -3I_1 + 3I_2 + 3I_3 - 2I_4 = -5$$

At node c:

$$I_2 = I_3 + 1$$

Solving these equations yields:

$$I_1 = 4A, \quad I_2 = 3A, \quad I_3 = 2A, \quad I_4 = 4A$$

Currents at the nodes are:

$$\text{At node b:} \quad i_1 = I_2 - I_1 = -1A$$

$$\text{At node a:} \quad i_2 = 4 - I_4 = 0A$$

$$\text{At node 0:} \quad i_3 = I_4 - I_3 = 2A$$