VE215 Final RC Part 2

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Overview

AC Power Analysis

- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Complex Power
- Power Factor Correction

Instantaneous Power

Definition:

$$p(t) = v(t) \cdot i(t)$$

Both v(t) and i(t) here are instantaneous values, not rms values.

Instantaneous Power for sinusoids: $v(t) = V_m \cos(\omega t + \theta_v)$ $i(t) = I_m \cos(\omega t + \theta_i)$

$$p(t) = V_m I_m cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$
$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

Average Power

Definition:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

For sinusoids:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Expressed in Phasor:

$$P = \frac{1}{2}Re(\tilde{V}\tilde{I}^*)$$

$e^{j\theta} = \cos\theta + j\sin\theta$

Average Power

When $\theta_v = \theta_i$, we have a purely resistive load R:

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} \frac{V_m^2}{R}$$

Only valid in this case

When $\theta_v - \theta_i = \pm 90°$, we have a purely reactive load X:

$$P = \frac{1}{2} V_m I_m \cos(\pm 90°) = 0$$

We can conclude that only R absorbs average power. X absorbs no average power.

General Case:

$\tilde{V} = \tilde{I} Z$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} Re\left(\tilde{V} \tilde{I}^*\right) = \frac{1}{2} Re((\tilde{I} Z) \tilde{I}^*)$$

$$= \frac{1}{2} Re(\tilde{I}(R + jX)\, \tilde{I}^*) = \boxed{\frac{1}{2} I_m^2 R}$$

Maximum Average Power Transfer

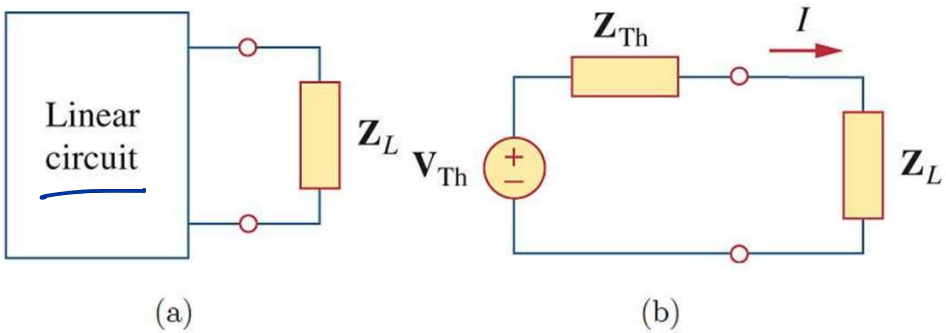

Figure 11.7 Finding the maximum average power transfer (a) circuit with a load, (b) the Thevenin equivalent.

If there is no restriction on Z_L,

$Z_L = Z_{Th}^*$

$$R_L = R_{Th} \quad X_L = -X_{Th} \quad P_{max} = \frac{|V_{Th}^2|}{8R_{Th}}$$

If Z_L is purely resistive,

$X_L = 0$

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} \quad P_{max} = \frac{|V_{Th}^2|}{4(R_{Th} + \sqrt{R_{Th}^2 + X_{Th}^2})}$$

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Effective or RMS Value

Definition: The effective value of an ac current i is the dc current I_{eff} that delivers the same average power to a resistor as the ac current.

$$I_{eff}^2 R = \frac{R}{T}\int_0^T i^2 dt$$

Effective value = Root mean square (RMS) value:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = I_{eff}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = V_{eff}$$

Effective or RMS Value

Avg power absorbed by a circuit element (General Case):

$$P = I_{rms}^2 R = V_{rms}^2 \frac{R}{R^2 + X^2}$$

$$P = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^*) = \mathrm{Re}(\tilde{V}_{rms}\tilde{I}_{rms}^*) = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{1}{2} V_m^2 \mathrm{Re}(\frac{1}{Z^*}) = V_{rms}^2 \frac{R}{R^2 + X^2}$$

For sinusoids:

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

Avg power absorbed by an element in a sinusoidal circuit:

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

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Effective or RMS Value

Caution: from now on, unless specified, all values will be assumed to be RMS values.

$P = \underbrace{V_{rms} I_{rms}}_{|S|} \underbrace{\cos(\theta_v - \theta_i)}_{pf}$

Complex Power

$$
\begin{aligned}
Complex\ Power &= \tilde{S} = \tilde{V}_{rms}\, \tilde{I}^*_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) \\
&= |S|\angle(\theta_v - \theta_i)\ (polar\ form) \\
&= P + jQ\ (rectangular\ form)
\end{aligned}
$$

Value	Name	Meaning	Unit
$\lvert S\rvert$	Apparent power	Magnitude of $\tilde{S}$	VA
$\cos(\theta_v - \theta_i)$	Power factor	Cosine of angle of $\tilde{S}$	/
P	Real power	Real part of $\tilde{S}$	W
Q	Reactive power	Imaginary part of $\tilde{S}$	VAR

Complex Power

Complex power:

$$\tilde{S} = \tilde{V}_{rms}\,\tilde{I}^*_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) = |S|\angle(\theta_v - \theta_i) = P + jQ$$
$$= |I_{rms}|^2 Z = \frac{|V_{rms}|^2}{Z^*}$$

Apparent power:

$$|S| = |V_{rms}||I_{rms}| = |I_{rms}|^2|Z| = \sqrt{P^2 + Q^2}$$

Real power:

$$P = Re(\tilde{S}) = |S|\cos(\theta_v - \theta_i) = |I_{rms}|^2 R$$

Reactive power:

$$Q = Im(\tilde{S}) = |S|\sin(\theta_v - \theta_i) = |I_{rms}|^2 X$$

Power factor (pf or $cos\theta$):

$$pf = \frac{P}{|S|} = cos(\theta_v - \theta_i)$$

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Complex Power

Power factor:

$$pf = cos(\theta_v - \theta_i)$$

$\theta_v - \theta_i < 0$: leading pf

$\theta_v - \theta_i > 0$: lagging pf

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, the pf value only tells part of the story. Every time you are asked for a power factor, you must declare whether it is leading or lagging.

Complex Power

We can use the sign of pf angle or Q to identify the property of the circuit and the loads:

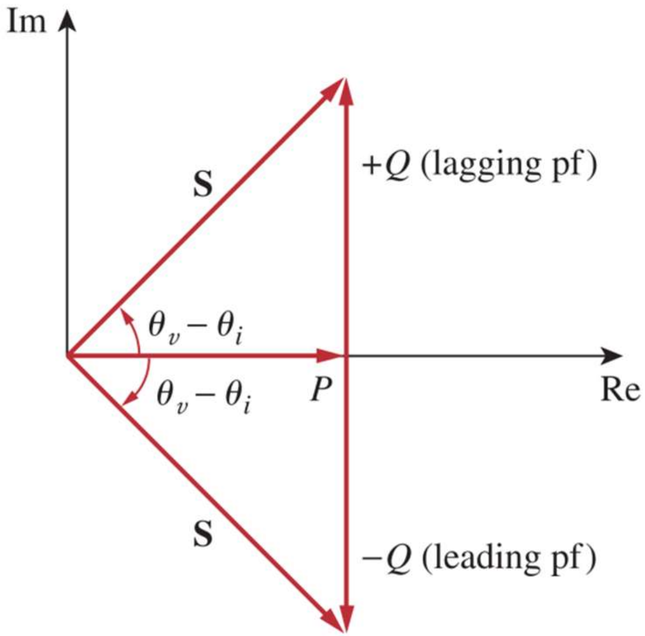
	(1)	(2)	(3)
pf Angle Sign of Q	$\theta_v - \theta_i = 0$ $Q = 0$	$\theta_v - \theta_i < 0$ $Q < 0$	$\theta_v - \theta_i > 0$ $Q > 0$
Properties	Unity pf I, V in phase $X = 0$ Resistive loads	Leading pf I leads V $X < 0$ Capacitive loads	Lagging pf I lags V $X > 0$ Inductive loads

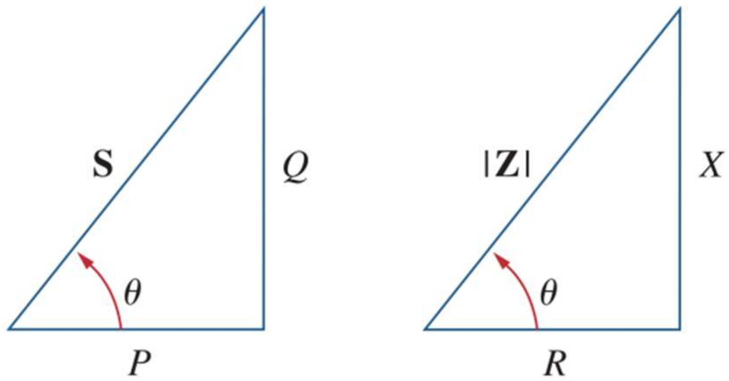
Complex Power

$S = P + jQ$

$Z = R + jX$

And observe that the power factor angle is equal to the angle of the impedance of that part of the circuit.

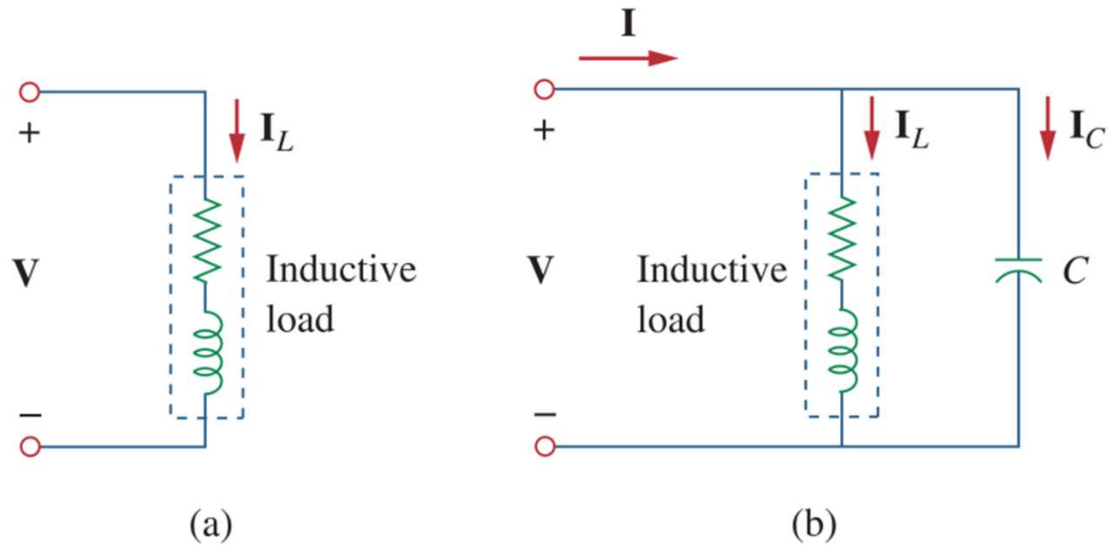




Power Factor Correction

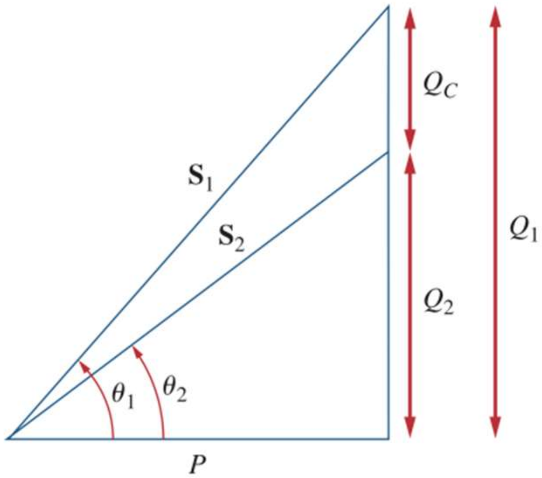
Goal: increase the pf of a load → make it less inductive → reduce energy loss

Solution: add a capacitor in parallel to the load



Power Factor Correction

Goal: increase the pf from $\cos\theta_1$ to $\cos\theta_2$.



Initial:

$$\begin{cases} P = |S_1| cos\theta_1 \\ Q = |Q_1| \sin\theta_1 = P tan\theta_1 \end{cases}$$

Expected outcome:

$$\begin{cases} P = |S_2| cos\theta_2 \\ Q = |Q_2| \sin\theta_2 = P tan\theta_2 \end{cases}$$

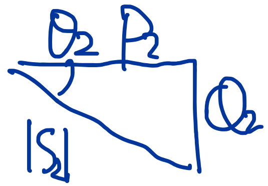
Since $Q_c \ (= Q_1 - Q_2) = \frac{V_{rms}^2}{X_C}$, then the value of the required capacitance C is:

$$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{Q_2 - Q_1}{\omega V_{rms}^2} = \frac{P(tan\theta_1 - tan\theta_2)}{\omega V_{rms}^2}$$

Conservation of AC Power

The Principle of Conservation of AC Power:

The complex, real, and reactive powers of the source equal the respective sums of the complex, real, and reactive powers of the individual loads.



$\cos\theta_2 = 0.8$

$P_2 = 4kW$

$Q_2 = -3kVAR$

Exercise 1

Three loads are connected in parallel across a 300 V(rms) line, as shown in the figure. Load 1 absorbs 3 kW at unity $Q_1 = 0$ power factor; Load 2 absorbs 5 kVA at 0.8 leading; Load 3 absorbs 5 kW and delivers 6 kvars. a) Find the impedance that is equivalent to the three parallel loads. b) Find the power factor of the equivalent load as seen from the line's input terminals.

a) $S_1 = 3kW + j0\,kVAR$

$S_2 = 4kW - j3kVAR$

$S_3 = 5kW - j6kVAR$

$S = S_1 + S_2 + S_3$

$= 12kW + j9kVAR$

$= V_{rms} I_{rms}^*$

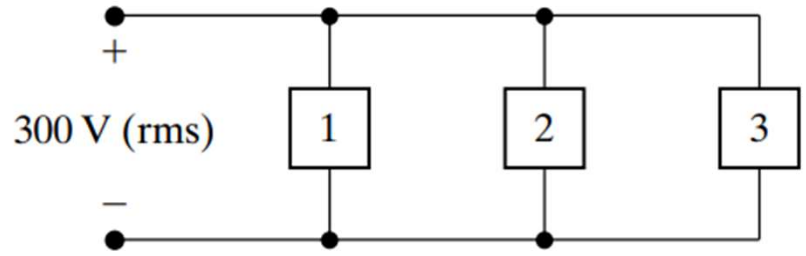
$\Rightarrow I_{rms}^* = 40 - j30\,A$

$I_{rms} = 40 + j30\,A$

$Z = \dfrac{V_{rms}}{I_{rms}} = 4.8 - j3.6\,\Omega = 6\angle -36.87^\circ\,\Omega$

b) $pf = \cos(-36.87^\circ)$

$= 0.8$ leading



Exercise 1

[a] $S_1 = 3 + j0\,\text{kVA}; \qquad S_2 = 4 - j3\,\text{kVA}; \qquad S_3 = 5 - j6\,\text{kVA}$

$$S_T = S_1 + S_2 + S_3 = 12 - j9\,\text{kVA}$$

$$300\mathbf{I}^* = (12 - j9) \times 10^3; \qquad \therefore \quad \mathbf{I} = 40 + j30\,\text{A}$$

$$Z = \frac{300}{40 + j30} = 4.8 - j3.6\,\Omega = 6\underline{/-36.87^\circ}\,\Omega$$

[b] pf $= \cos(-36.87^\circ) = 0.8$ leading

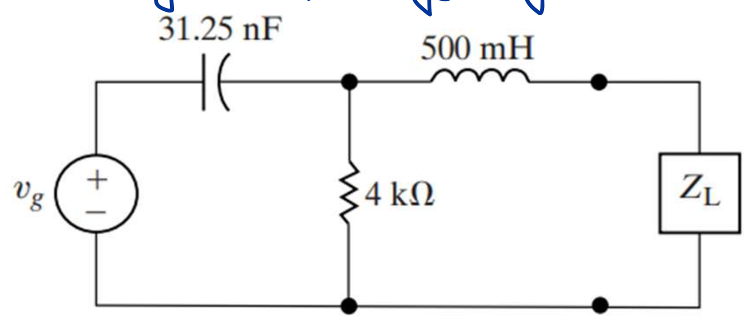
Exercise 2

a) Determine the load impedance for the circuit shown in the figure that will result in maximum average power being transferred to the load if $\omega = 8\ krad/s$.

b) Determine the maximum average power delivered to the load from part (a) if

$v_g = 10 \cos 8000t$ V.

$\frac{1}{j\omega C} = -j4k$ $j\omega L = j4k$



a) $Z_{Th} = -j4k // 4k + j4k$

$= 2k + j2k\ \Omega$

$Z_L = Z_{Th}^* = 2k - j2k\ \Omega$

b) $V_{Th} = 4k\Omega \cdot \frac{V_G}{-j4k + 4k}$

$V_G = 10\angle 0° V$

$V_{Th} = 5 + j5\ V$

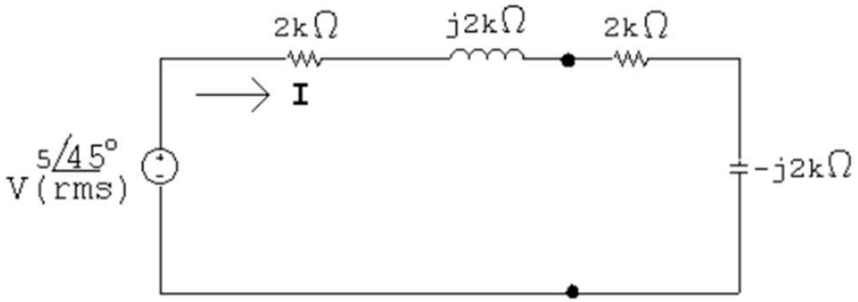
$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = 3.125\ mW$

Exercise 2

[a] $Z_{\mathrm{Th}} = j4000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2000 + j2000\,\Omega$

$\therefore \quad Z_{\mathrm{L}} = Z_{\mathrm{Th}}^{*} = 2000 - j2000\,\Omega$

[b] $\mathbf{V}_{\mathrm{Th}} = \dfrac{10\underline{/0^\circ}(4000)}{4000 - j4000} = 5 + j5 = 5\sqrt{2}\underline{/45^\circ}\,\mathrm{V}$



$\mathbf{I} = \dfrac{5\sqrt{2}\underline{/45^\circ}}{4000} = 1.25\sqrt{2}\underline{/45^\circ}\,\mathrm{mA}$

$|\mathbf{I}_{\mathrm{rms}}| = 1.25\,\mathrm{mA}$

$P_{\mathrm{load}} = (0.00125)^2(2000) = 3.125\,\mathrm{mW}$

Exercise 3

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

$$= 310.44\mu F$$

Exercise 3

Solution:

If the pf = 0.8, then

$$\cos\theta_1 = 0.8 \qquad \Rightarrow \qquad \theta_1 = 36.87°$$

where θ_1 is the phase difference between voltage and current. obtain the apparent power from the real power and the pf as

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95,

$$\cos\theta_2 = 0.95 \qquad \Rightarrow \qquad \theta_2 = 18.19°$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

References

1. 2023 Fall VE215 slides, Sung-Liang Chen
2. 2023 Fall RC5, Shunyao Huang
3. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew

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