VE215 Final RC Part 2

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Overview

AC Power Analysis

- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Complex Power
- Power Factor Correction

Instantaneous Power

Definition:

$$p(t) = v(t) \cdot i(t)$$

Both v(t) and i(t) here are instantaneous values, not rms values.

Instantaneous Power for sinusoids:

$$\begin{aligned} p(t) &= V_m I_m cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \\ &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \end{aligned}$$

Average Power

Definition:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

For sinusoids:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Expressed in Phasor:

$$P = \frac{1}{2}Re(\tilde{V}\tilde{I}^*)$$

Average Power

When $\theta_v = \theta_i$, we have a purely resistive load R:

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} \frac{V_m^2}{R}$$

When $\theta_v - \theta_i = \pm 90°$, we have a purely reactive load X:

$$P = \frac{1}{2} V_m I_m \cos(\pm 90°) = 0$$

We can conclude that only R absorbs average power. X absorbs no average power.

General Case:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} Re\left(\tilde{V} \tilde{I}^*\right) = \frac{1}{2} Re((\tilde{I} Z) \tilde{I}^*)$$
$$= \frac{1}{2} Re(\tilde{I}(R + jX)\, \tilde{I}^*) = \frac{1}{2} I_m^2 R$$

Maximum Average Power Transfer

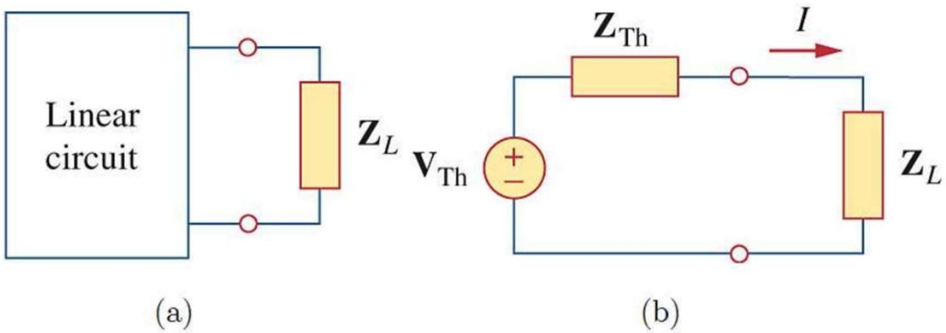

Figure 11.7 Finding the maximum average power transfer (a) circuit with a load, (b) the Thevenin equivalent.

If there is no restriction on Z_L,

$$R_L = R_{Th} \quad X_L = -X_{Th} \quad P_{max} = \frac{V_{Th}^2}{8R_{Th}}$$

If Z_L is purely resistive,

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} \quad P_{max} = \frac{V_{Th}^2}{4(R_{Th} + \sqrt{R_{Th}^2 + X_{Th}^2})}$$

Effective or RMS Value

Definition: The effective value of an ac current i is the dc current I_{eff} that delivers the same average power to a resistor as the ac current.

$$I_{eff}^2 R = \frac{R}{T}\int_0^T i^2 dt$$

Effective value = Root mean square (RMS) value:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = I_{eff}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = V_{eff}$$

Effective or RMS Value

Avg power absorbed by a circuit element (General Case):

$$P = I_{rms}^2 R = V_{rms}^2 \frac{R}{R^2 + X^2}$$

$$P = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^*) = \mathrm{Re}(\tilde{V}_{rms}\tilde{I}_{rms}^*) = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{1}{2} V_m^2 \mathrm{Re}(\frac{1}{Z^*}) = V_{rms}^2 \frac{R}{R^2 + X^2}$$

For sinusoids:

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

Avg power absorbed by an element in a sinusoidal circuit:

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Effective or RMS Value

Caution: from now on, unless specified, all values will be assumed to be RMS values.

Complex Power

$$\begin{aligned} Complex\ Power &= \tilde{S} = \tilde{V}_{rms}\,\tilde{I}^*_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) \\ &= |S|\angle(\theta_v - \theta_i)\ (polar\ form) \\ &= P + jQ\ (rectangular\ form) \end{aligned}$$

Value	Name	Meaning	Unit
$\lvert S\rvert$	Apparent power	Magnitude of $\tilde{S}$	VA
$\cos(\theta_v - \theta_i)$	Power factor	Cosine of angle of $\tilde{S}$	/
P	Real power	Real part of $\tilde{S}$	W
Q	Reactive power	Imaginary part of $\tilde{S}$	VAR

Complex Power

Complex power:

$$\tilde{S} = \tilde{V}_{rms}\ \tilde{I}^*_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) = |S|\angle(\theta_v - \theta_i) = P + jQ$$
$$= \ |I_{rms}|^2 Z \ = \frac{|V_{rms}|^2}{Z^*}$$

Apparent power:

$$|S| = |V_{rms}||I_{rms}| = |I_{rms}|^2|Z| = \sqrt{P^2 + Q^2}$$

Real power:

$$P = Re(\tilde{S}) = |S|\cos(\theta_v - \theta_i) = |I_{rms}|^2 R$$

Reactive power:

$$Q = Im(\tilde{S}) = |S|\sin(\theta_v - \theta_i) = |I_{rms}|^2 X$$

Power factor (pf or $cos\theta$):

$$pf = \frac{P}{|S|} = cos(\theta_v - \theta_i)$$

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Complex Power

Power factor:

$$pf = cos(\theta_v - \theta_i)$$

$\theta_v - \theta_i < 0$: leading pf

$\theta_v - \theta_i > 0$: lagging pf

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, the pf value only tells part of the story. Every time you are asked for a power factor, you must declare whether it is leading or lagging.

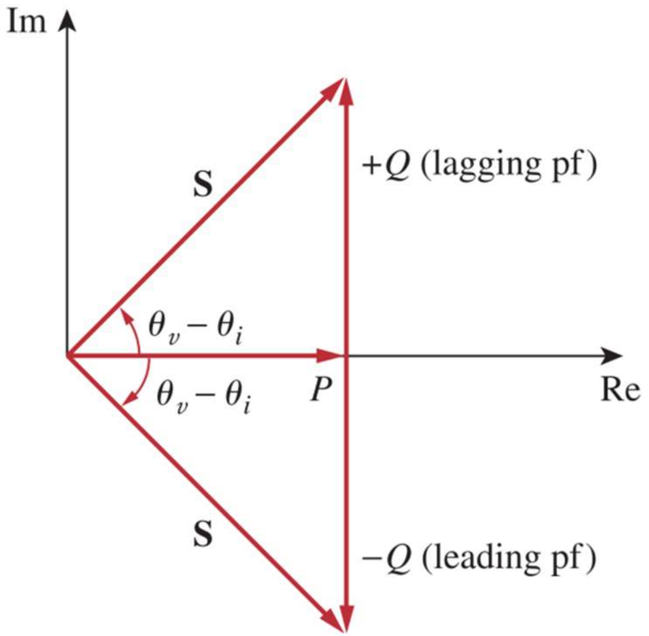
Complex Power

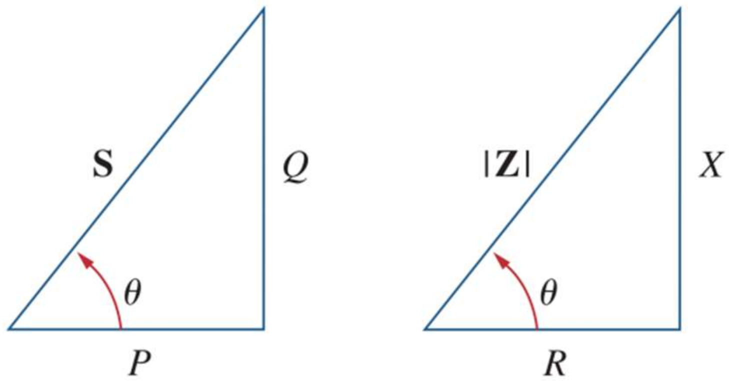
We can use the sign of pf angle or Q to identify the property of the circuit and the loads:

	(1)	(2)	(3)
pf Angle Sign of Q	$\theta_v - \theta_i = 0$ $Q = 0$	$\theta_v - \theta_i < 0$ $Q < 0$	$\theta_v - \theta_i > 0$ $Q > 0$
Properties	Unity pf I, V in phase $X = 0$ Resistive loads	Leading pf I leads V $X < 0$ Capacitive loads	Lagging pf I lags V $X > 0$ Inductive loads

Complex Power

And observe that the power factor angle is equal to the angle of the impedance of that part of the circuit.

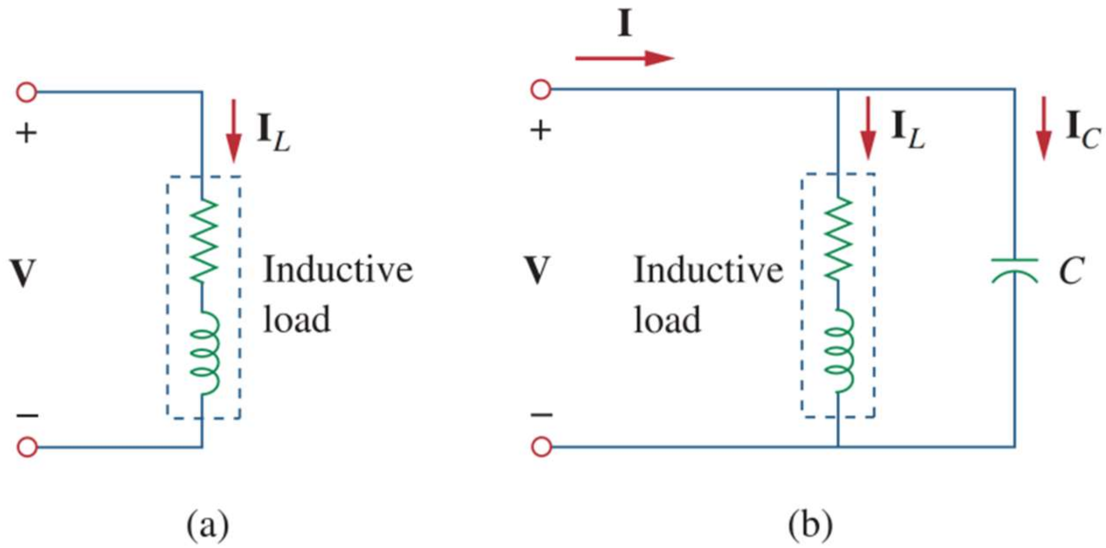




Power Factor Correction

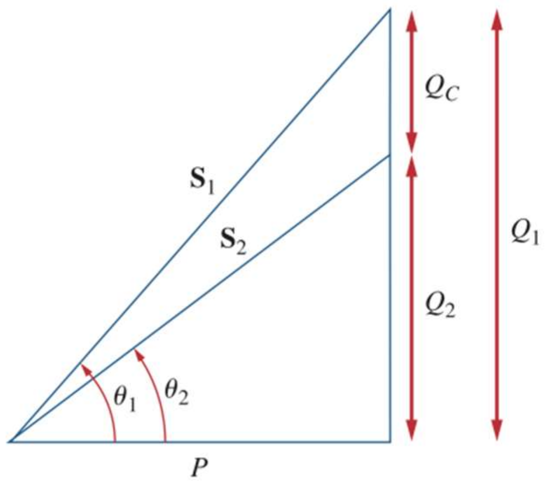
Goal: increase the pf of a load → make it less inductive → reduce energy loss

Solution: add a capacitor in parallel to the load



Power Factor Correction

Goal: increase the pf from $\cos\theta_1$ to $\cos\theta_2$.



Initial:

$$P = |S_1| cos\theta_1$$
$$Q = |Q_1| \sin\theta_1 = P tan\theta_1$$

Expected outcome:

$$P = |S_2| cos\theta_2$$
$$Q = |Q_2| \sin\theta_2 = P tan\theta_2$$

Since $Q_c \left(= Q_1 - Q_2\right) = \frac{V_{rms}^2}{X_c}$, then the value of the required capacitance C is:

$$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{Q_2 - Q_1}{\omega V_{rms}^2} = \frac{P(tan\theta_1 - tan\theta_2)}{\omega V_{rms}^2}$$

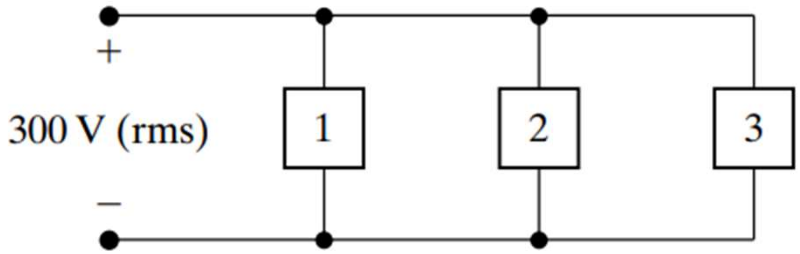
Conservation of AC Power

The Principle of Conservation of AC Power:

The complex, real, and reactive powers of the source equal the respective sums of the complex, real, and reactive powers of the individual loads.

Exercise 1

Three loads are connected in parallel across a 300 V(rms) line, as shown in the figure. Load 1 absorbs 3 kW at unity power factor; Load 2 absorbs 5 kVA at 0.8 leading; Load 3 absorbs 5 kW and delivers 6 kvars. a) Find the impedance that is equivalent to the three parallel loads. b) Find the power factor of the equivalent load as seen from the line's input terminals.

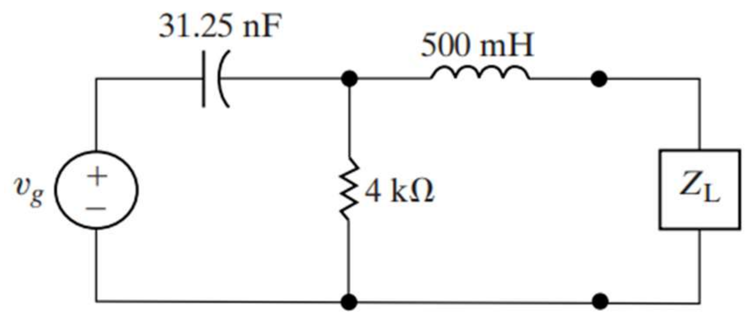


Exercise 2

a) Determine the load impedance for the circuit shown in the figure that will result in maximum average power being transferred to the load if $\omega = 8\ krad/s$.

b) Determine the maximum average power delivered to the load from part (a) if

$v_g = 10 \cos 8000t$ V.



Exercise 3

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

References

1. 2023 Fall VE215 slides, Sung-Liang Chen
2. 2023 Fall RC5, Shunyao Huang
3. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew

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Good luck for your exam!