ECE2150J Mid-RC Part3

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Overview

Overview

Ideal Op-amp

Characteristics of ideal op-amp:

- Open circuit at two input terminals ($i_1 = i_2 = 0$)
- Same voltage at two input terminals ($v_1 = v_2$)
- Do not apply KCL to the output terminals!

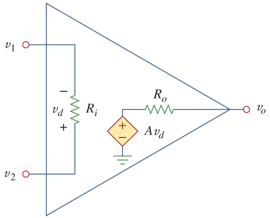


Figure: Op-amp's equivalent circuit

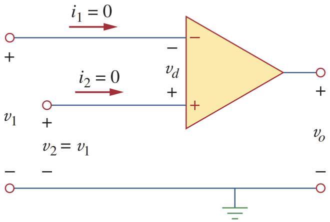
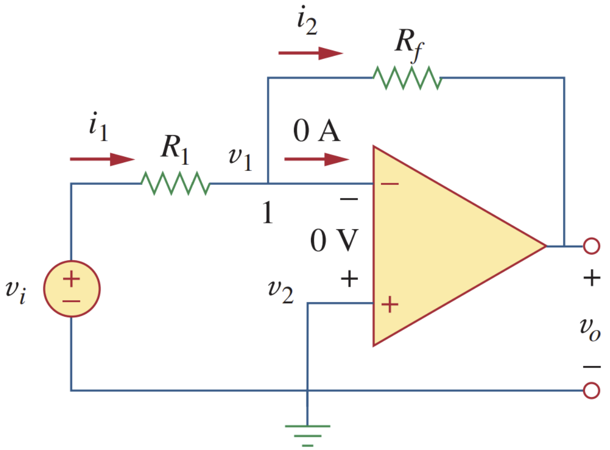


Figure: Symbol of ideal op-amp

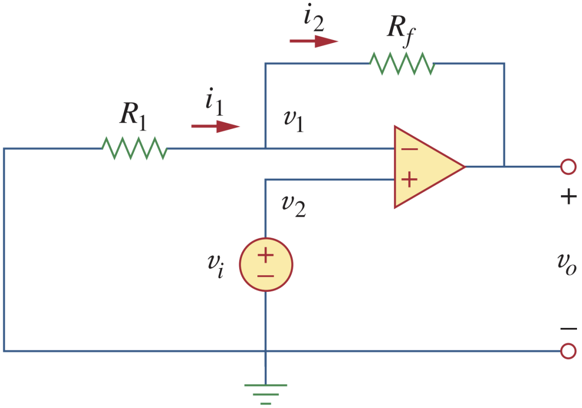
Inverting Amplifier

$$A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$



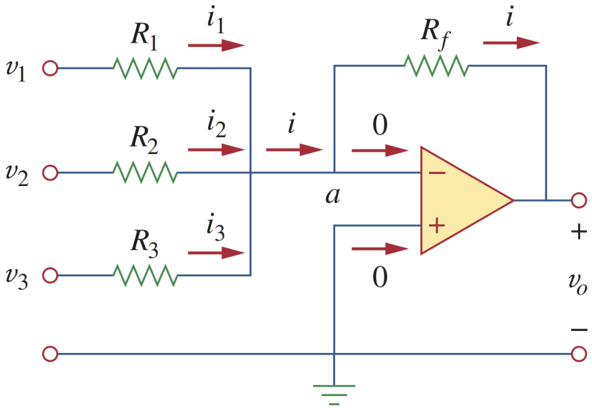
Non-inverting Amplifier

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$



Summing Amplifier

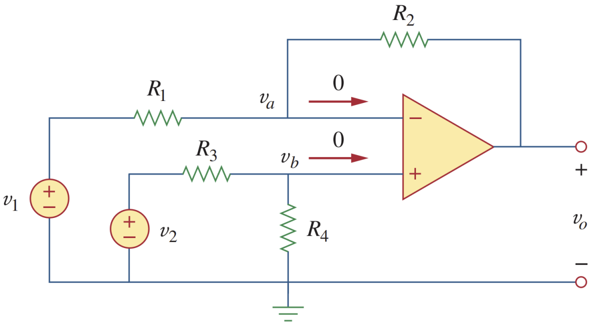
$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$



Difference Amplifier

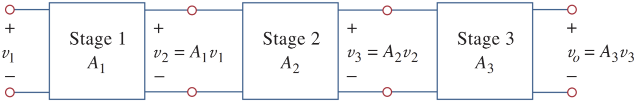
Check if $R_1 = R_2,\ R_3 = R_4$

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$



Cascaded Op Amps

Gain of Cascaded Op Amp



Original input signal is increased by the gain of the individual stage, and the final gain is the **product of all gains at each stage**.

$$A = \frac{v_o}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdot \frac{v_o}{v_3} = A_1 A_2 A_3$$

Basic Op-amp Circuits: Summary

For basic op-amp circuits:

Op-amp circuits	Input-output relationship
Inverting amplifier	$A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$
Non-inverting amplifier	$A = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$
Summing amplifier	$v_o = -(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3)$
Difference amplifier	$v_o = \left[(\frac{R_2}{R_1} + 1)(\frac{R_4/R_3}{1+R_4/R_3})\right] v_2 - \left[\frac{R_2}{R_1}\right] v_1$

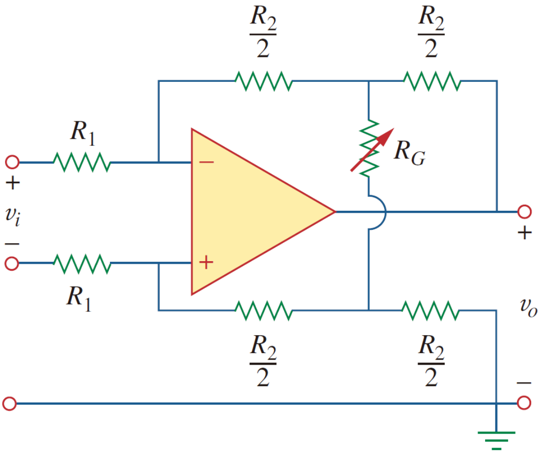
For complicated op-amp circuits:

- Identify basic op-amp circuits within it
- Use the formula for cascaded op-amp circuit
- Be proficient in listing **nodal analysis** equations to obtain v_o/v_i

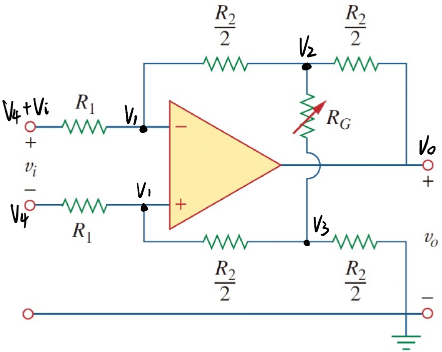
Exercise 1

Show that:

$$\frac{v_o}{v_i} = \frac{-R_2}{R_1}\left(1 + \frac{R_2}{2R_G}\right)$$



Exercise 1



$$\begin{cases} \dfrac{V_1-(V_4+V_i)}{R_1} + \dfrac{V_1-V_2}{\frac{R_2}{2}} = 0 & ① \\[2ex] \dfrac{V_1-V_4}{R_1} + \dfrac{V_1-V_3}{\frac{R_2}{2}} = 0 & ② \end{cases}$$

$$① - ② : \quad -\frac{V_i}{R_1} - \frac{2(V_2-V_3)}{R_2} = 0$$

$$\Rightarrow V_i = -\frac{2R_1}{R_2}(V_2-V_3)$$

$$\begin{cases} \dfrac{V_2-V_1}{\frac{R_2}{2}} + \dfrac{V_2-V_3}{R_G} + \dfrac{V_2-V_o}{\frac{R_2}{2}} = 0 \\[2ex] \dfrac{V_3-V_1}{\frac{R_2}{2}} + \dfrac{V_3-V_2}{R_G} + \dfrac{V_3}{\frac{R_2}{2}} = 0 \end{cases}$$

$$\frac{2V_2}{R_2} + \frac{V_2-V_3}{R_G} + \frac{2(V_2-V_o)}{R_2} = \frac{2V_3}{R_2} + \frac{V_3-V_2}{R_G} + \frac{2V_3}{R_2}$$

$$\Rightarrow V_o = \left(2+\frac{R_2}{R_G}\right)(V_2-V_3)$$

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\left(1+\frac{R_2}{2R_G}\right)$$

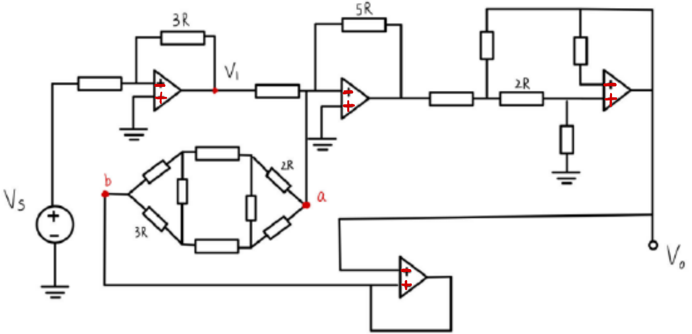
Exercise 2

4. Suppose all resistors without labels have resistance of R. Please answer the following questions. [16 points]

(1) V_1 in terms of V_s and R.

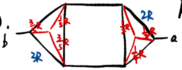
(2) The equivalent resistance R_{ab} between terminals a and b.

(3) The overall gain $G = \frac{V_o}{V_s}$.



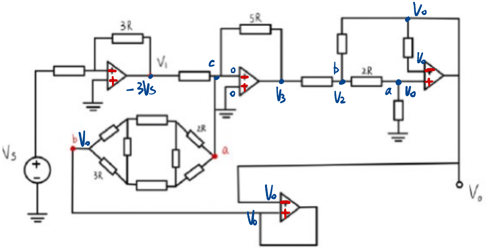
Exercise 2

(1). $V_1 = -\frac{3R}{R}V_s = -3V_s$

(2). 

$R_{ab} = \frac{3}{5}R + \left(\frac{1}{5}R + R + \frac{1}{2}R\right) \parallel \left(\frac{3}{5}R + R + \frac{1}{4}R\right) + \frac{1}{2}R = \frac{141}{71}R = 1.986R$

(3).



$$\begin{cases} \text{node } a: & \frac{V_o - V_2}{2R} + \frac{V_o}{R} = 0 \\ \text{node } b: & \frac{V_2 - V_3}{R} + \frac{V_2 - V_o}{R} + \frac{V_2}{2R+R} = 0 \\ \text{node } c: & \frac{3V_s}{R} - \frac{V_3}{5R} - \frac{V_o}{\frac{141}{71}R} = 0 \end{cases}$$

$\Rightarrow V_3 = 6V_o$

$3V_s - \frac{6}{5}V_o - \frac{71}{141}V_o = 0$

$\frac{V_o}{V_s} = \frac{2115}{1201} = 1.76$

Overview

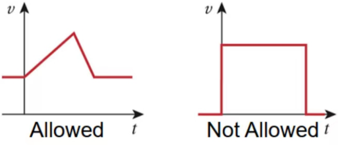
Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Capacitors

1. **Open Circuit Property** When the voltage across a capacitor is not changing with time (**DC steady state**), **the capacitor could be treated as an open circuit**.
2. **Continuity property** The voltage on a capacitor must be continuous.



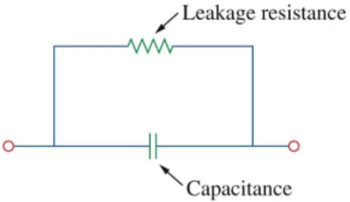
3. **Capacitors IV relationship**

$$i = C\frac{dv}{dt}$$

property 2 can be intuitively shown be property 3. If the voltage across the capacitor is not continuous, say $\frac{dv}{dt} = \infty$, which will cause i to be infinity.

Capacitors

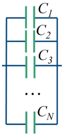
1. **An ideal capacitor will not dissipate energy.** It takes power from the circuit when storing energy in its electric field and returns previously stored energy when delivering power to the circuit.
2. **A real capacitor has a large leakage resistance**



Capacitors in parallel & in series

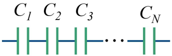
- **capacitors in parallel**

$\sim G = \frac{1}{R}$



$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N$$

- **capacitors in series**



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + ... + \frac{1}{C_N}$$

Energy stored in Capacitors

The instantaneous power delivered to the capacitor is

$$p = vi = v(C\frac{dv}{dt})$$

Therefore, **the total energy** stored in the capacitor is

$$w = \frac{1}{2}CV^2$$

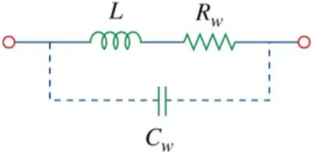
Inductors

1. **Short Circuit Property** When the current through an inductor is not changing with time (**DC steady state**), **the inductor could be treated as a short circuit in the circuit**.
2. **Continuity property** The current through a capacitor must be continuous.
3. **Inductor IV relationship**

$$v = L\frac{di}{dt}$$

Inductors

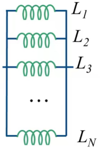
1. **An ideal inductor will not dissipate energy.** It takes power from the circuit when storing energy in its magnetic field and returns previously stored energy when delivering power to the circuit.
2. **A real inductor has a significant winding resistance and a small winding capacitance**



Inductors in parallel & in series

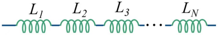
- **inductors in parallel**

∽ R



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \ldots + \frac{1}{L_N}$$

- **inductors in series**



$$L_{eq} = L_1 + L_2 + L_3 + L_4 + \ldots + L_N$$

Energy stored in Inductors

The instantaneous power delivered to the inductor is

$$p = vi = (L\frac{di}{dt})i$$

Therefore, **the total energy** stored in the inductor is

$$w = \frac{1}{2}Li^2$$

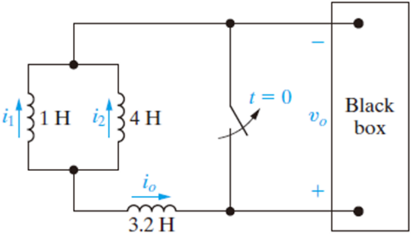
HW 3.1

Exercise 3.1 (20%)

The three inductors in the circuit are connected across the terminals of a black box at $t = 0$. The resulting voltage for $t > 0$ is known to be

$$v_o = 2000e^{-100t}\,\text{V}.$$

If $i_1(0) = -6\,\text{A}$ and $i_2(0) = 1\,\text{A}$, find
(a) $i_o(0)$;
(b) $i_o(t)$, $t \geq 0$; (Hint: Calculate the equivalent inductance L_{eq} first.)
(c) $i_1(t)$, $t \geq 0$;
(d) $i_2(t)$, $t \geq 0$;
(e) the initial energy stored in the three inductors;



HW 3.1

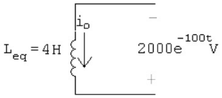
(a) (4%)

$$i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,A$$

(b) (4%) For the equivalent circuit with $L_{eq} = 4\,H$:

$$i_o = -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,A, \quad t \geq 0$$

$V = L\frac{di}{dt} \Rightarrow$



(c) (4%)

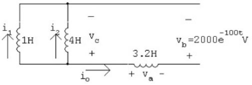
Using the values of v_a and v_c:

$$v_a = 3.2 \cdot (-500e^{-100t}) = -1600e^{-100t}\,V$$

$$v_c = v_a + v_b = -1600e^{-100t} + 2000e^{-100t} = 400e^{-100t}\,V$$

$$i_1 = \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 = -4e^{-100t} + 4 - 6$$

$$i_1 = -4e^{-100t} - 2\,A, \quad t \geq 0$$



(d) (4%)

$$i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1 = -e^{-100t} + 2\,A, \quad t \geq 0$$

(e) (4%)

$$w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,J$$

Overview

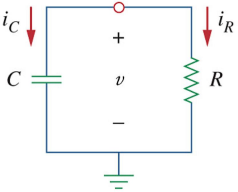
Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Source-Free Circuits (I) Response

Source-free RC



Voltage: $v = v_0 e^{-t/RC}$

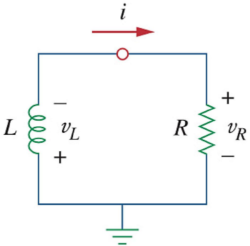
Time constant: $\tau = RC$

Current: $i_R = \frac{v}{R} = \frac{v_0}{R} e^{-t/\tau}$

Power: $p = v i_R = \frac{v_0^2}{R} e^{-2t/\tau}$

Energy: $w_R = \frac{1}{2} C V_0^2 \left(1 - e^{-\frac{2t}{\tau}}\right)$

Source-free RL



Current: $i = i_0 e^{-t/(L/R)}$

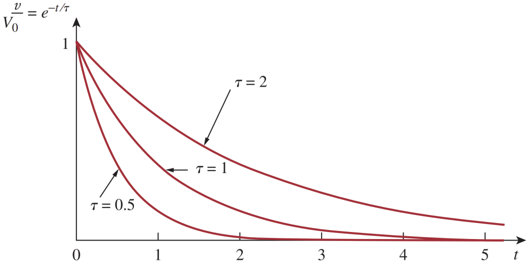
Time constant: $\tau = L/R$

Voltage: $v_R = iR = \frac{i_0}{R} e^{-t/\tau}$

Power: $p = v_R i = i_0^2 R e^{-2t/\tau}$

Energy: $w_R = \frac{1}{2} L I_0^2 \left(1 - e^{-\frac{2t}{\tau}}\right)$

Source-Free Circuits (II) Time Constant



	Source-free RC	Source-free RL
Time constant	$\tau = RC$	$\tau = L/R$
Relation to initial decay rate	$\frac{d}{dt}(\frac{v}{v_0}) = -1/\tau$	$\frac{d}{dt}(\frac{i}{i_0}) = -1/\tau$

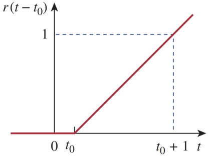
- Time required for the response to decay to a factor of $1/e$ or 36.8% of its initial value
- Indicates the initial decaying rate
- Assume complete decay after 5τ

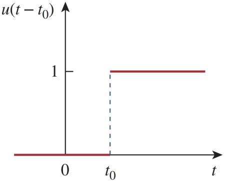
Source-Free Circuits (III) General Steps

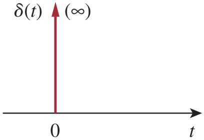
- Find the initial value v_0, i_0.
- Find the time constant, i.e., find R_{eq}. (R_{eq} is the Thevenin or Norton equivalent resistance at the capacitor and inductor terminals.)
- Use the equation in the previous slide to get results.
- You can always use Mesh/Nodal analysis to calculate if you are not familiar with the steps above.
- (Of course, you need to remember that $i = C\frac{dv}{dt}$ for capacitors and $v = L\frac{di}{dt}$ for inductors!)

Singularity Functions

Unit ramp	Unit step	Unit impulse
$r(t) = \begin{cases} 0, t \leq 0 \\ t, t > 0 \end{cases}$	$u(t) = \begin{cases} 0, t \leq 0 \\ 1, t > 0 \end{cases}$	$\delta(t) = \begin{cases} 0, t \neq 0 \\ \text{Undef.}, t = 0 \end{cases}$





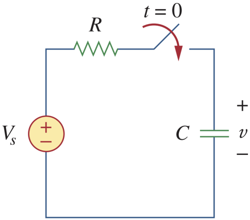


Give a nice way to represent "Switch on/off" of the sources/part of circuits.

$$\delta(t) \xrightarrow{\int} u(t) \xrightarrow{\int} r(t)$$

Circuits with Step Input (I) Response

Step-input RC

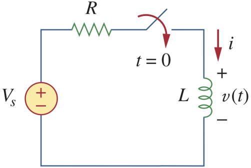


Initial condition:
$$v(0^+) = v(0^-) = V_0$$

Equation:
$$\text{(KVL)}\ (C\frac{dv}{dt}R + v = V_s)$$

Response:
$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$

Step-input RL



Initial condition:
$$i(0^+) = i(0^-) = I_0$$

Equation:
$$\text{(KCL)}\ iR + L\frac{di}{dt} = V_s$$

Response:
$$i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-t/\tau}$$

Circuits with Step Input (II) General Steps

- General form: $x(t) = x(\infty) + (x(0) - x(\infty))e^{-t/\tau}$.
- Step response = natural response + forced response, where natural response is $x(0)e^{-t/\tau}$, forced response is $x(\infty)(1 - e^{-t/\tau})$.
- Step response = transient state response + steady state response, where transient response is $(x(0) - x(\infty))e^{-t/\tau}$, steady state response is $x(\infty)$.
- Steps to find step response:
  - Find $x(0)$.
  - Find $x(\infty)$.
  - Find τ.
- Still, you can find step response using mesh/nodal analysis. By solving the differential equation, you will find that natural response is the homogeneous solution of the equation, while the forced response is the particular solution.

General Formula for First-Order Circuits

General formula for RC:

$$v(t) = v(\infty) + \left[v(0^+) - v(\infty)\right] e^{-t/\tau}$$

General formula for RL:

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}$$

HW 3.5.b

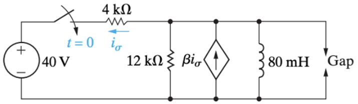
Exercise 3.5 (20%)

The gap in the circuit seen in the figure below will arc over whenever the voltage across the gap reaches 30 kV. The initial current in the inductor is zero. The value of β is adjusted so the Thévenin equivalent resistance with respect to the terminals of the inductor is -4 kΩ. Now please answer the following two questions below.

(a)What is the value of β ? $\frac{7}{3}$

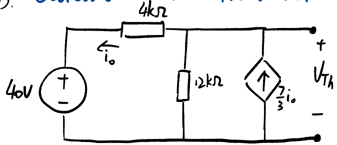
(b)After the switch has been closed, how long will it take for the gap to arc over?

Tip: For this question, the gap can be treated as an open circuit before the voltage across it reaches 30 kV, and you do not need to know what "arc over" really is .



HW 3.5.b

(b). Calculate the Thevenin equivalent first.

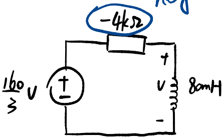


$$\begin{cases} \dfrac{V_{Th}-40}{4000} + \dfrac{V_{Th}}{12000} - \dfrac{7}{3}i_o = 0 \\ i_o = \dfrac{V_{Th}-40}{4000} \end{cases} \Rightarrow V_{Th} = \frac{160}{3}V$$

In hw 2.4, we have seen that circuit with negative R_{Th} is supplying power.

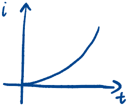
When calculating the time constant τ, we can directly substitute the negative R_{Th} into the formula

negative?



$$i(0^+) = i(0^-) = 0$$

$$i(\infty) = \frac{160/3}{-4000} = -\frac{40}{3}mA$$



$$\tau = \frac{L}{R} = \frac{0.08}{-4000} = -\frac{1}{50000}$$

$$i(t) = i(\infty) + (i(0) - i(\infty))\cdot e^{-t/\tau} = -\frac{40}{3} + \frac{40}{3}\cdot e^{50000t} \, mA$$

$$V = 0.08\frac{di}{dt} = 0.08\cdot\frac{2000}{3}\cdot e^{50000t} = \frac{160}{3}\cdot e^{50000t} = 30000$$

$$e^{50000t} = 562.5 \Rightarrow t = \frac{\ln 562.5}{50000} = 126.6\,\mu s$$

References

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Thank you!