VE215 RC5

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Overview

AC Power Analysis

- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Power Factor Correction
- Conservation of AC Power

Instantaneous Power

Definition:

$$p(t) = v(t) \cdot i(t)$$

Both v(t) and i(t) here are instantaneous values, not rms values.

Instantaneous Power for sinusoids: $v(t) = V_m \cos(\omega t + \theta_v)$ $i(t) = I_m \cos(\omega t + \theta_i)$

$$p(t) = V_m I_m cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \quad 1$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \underline{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)} \quad 2$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

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Average Power

Definition:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

For sinusoids:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Expressed in Phasor:

$$P = \frac{1}{2} Re(\tilde{V}\tilde{I}^*)$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$P = \frac{1}{2} V_m I_m \cdot Re[e^{j(\theta_v - \theta_i)}]$

$= \frac{1}{2} Re[V_m e^{j\theta_v} \cdot I_m e^{-j\theta_i}]$

$= \frac{1}{2} Re(\tilde{V}\tilde{I}^*)$

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Average Power

When $\theta_v = \theta_i$, we have a purely resistive load R:

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \underline{\frac{1}{2}\frac{V_m^2}{R}}$$ only in this case

When $\theta_v - \theta_i = \pm 90°$, we have a purely reactive load X:

$$P = \frac{1}{2}V_m I_m \cos(\pm 90°) = 0$$

We can conclude that only R absorbs average power. X absorbs no average power.

General Case:

$$P = \frac{1}{2}Re(\tilde{V}\tilde{I}^*) = \frac{1}{2}Re(\tilde{I}(R + jX)\,\tilde{I}^*) = \underline{\frac{1}{2}I_m^2 R}$$

$$\begin{cases} \tilde{I}Z \\ Z = R + jX \end{cases}$$

Maximum Average Power Transfer

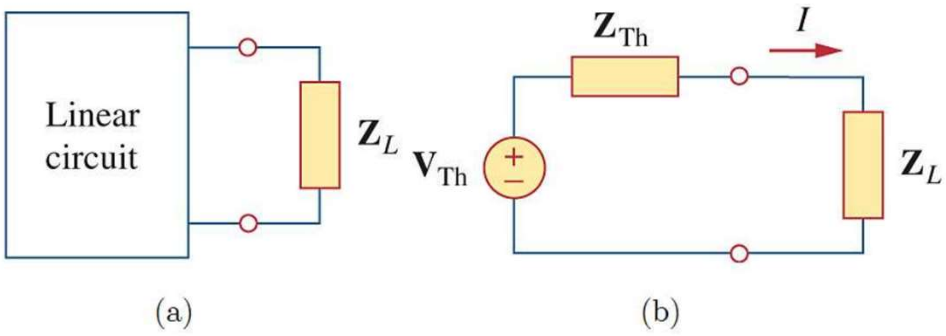

Figure 11.7 Finding the maximum average power transfer (a) circuit with a load, (b) the Thevenin equivalent.

If there is no restriction on Z_L,

$$R_L = R_{Th} \quad X_L = -X_{Th} \quad P_{max} = \frac{|V_{Th}^2|}{8R_{Th}}$$

If Z_L is purely resistive,

$X_L = 0$

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} \quad P_{max} = \frac{|V_{Th}^2|}{4(R_{Th} + \sqrt{R_{Th}^2 + X_{Th}^2})}$$

Proof for $P_{max} = \frac{V_{Th}^2}{8R_{Th}}$

$I = \frac{V_{Th}}{Z}$

$$P = \frac{1}{2} I^2 R_L = \frac{1}{2} \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

$$\begin{cases} \dfrac{\partial P}{\partial R_L} = \dfrac{V_{Th}^2 [R_{Th}^2 - R_L^2 + (X_{Th} + X_L)^2]}{2[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2} = 0 \\ \dfrac{\partial P}{\partial X_L} = -\dfrac{V_{Th}^2 R_L (X_{Th} + X_L)}{[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2} = 0 \end{cases}$$

$$\begin{cases} R_L = R_{Th} \\ X_L = -X_{Th} \end{cases} \quad \text{or } Z_L = Z_{Th}^*$$

Effective or RMS Value

Definition: The effective value of an ac current i is the dc current I_{eff} that delivers the same average power to a resistor as the ac current.

$$I_{eff}^2 R = \frac{R}{T}\int_0^T i^2 dt$$

Effective value = Root mean square (RMS) value:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = I_{eff}$$

$i(t)$ $v(t)$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = V_{eff}$$

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Effective or RMS Value

Avg power absorbed by a circuit element (General Case):

$$P = I_{rms}^2 R = V_{rms}^2 \frac{R}{R^2 + X^2}$$

$$P = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^*) = \mathrm{Re}(\tilde{V_{rms}}\tilde{I_{rms}^*}) = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{1}{2} V_m^2 \mathrm{Re}(\frac{1}{Z^*}) = V_{rms}^2 \frac{R}{R^2 + X^2}$$

For sinusoids:

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

Avg power absorbed by an element in a sinusoidal circuit:

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Recall $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= \underbrace{\frac{1}{\sqrt{2}} V_m}_{V_{rms}} \cdot \underbrace{\frac{1}{\sqrt{2}} I_m}_{I_{rms}} \cdot \cos(\theta_v - \theta_i)$

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Effective or RMS Value

Caution: from now on, unless specified, all values will be assumed to be RMS values.

Apparent Power and Power Factor

$$P = \underbrace{V_{rms} I_{rms}}_{|S|} \underbrace{cos(\theta_v - \theta_i)}_{pf}$$

Apparent power:

$$|S| = |V_{rms}||I_{rms}| = |I_{rms}|^2 |Z| = \sqrt{P^2 + Q^2}$$

Power factor (pf or $cos\theta$):

$$pf = cos\theta = \frac{P}{|S|} = cos(\theta_v - \theta_i)$$

Power factor angle (θ):

$$\theta = \theta_v - \theta_i$$

Leading/Lagging pf

Power factor:

$$pf = cos(\theta_v - \theta_i)$$

$\theta_v - \theta_i < 0$: leading pf

$\theta_v - \theta_i > 0$: lagging pf

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, the pf value only tells part of the story. Every time you are asked for a power factor, you must declare whether it is leading or lagging.

Complex Power

Apparent power:

$$|S| = |V_{rms}||I_{rms}| = |I_{rms}|^2|Z| = \sqrt{P^2 + Q^2}$$

Complex power:

$$\begin{aligned}\tilde{S} &= \tilde{V}_{rms}\,\tilde{I}^*_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i)\\ &= |S|\angle(\theta_v - \theta_i)\\ &= P + jQ\end{aligned}$$

Real power:

$$P = Re(\tilde{S}) = |S|\cos(\theta_v - \theta_i) = |I_{rms}|^2 R$$

Reactive power:

$$Q = Im(\tilde{S}) = |S|\sin(\theta_v - \theta_i) = |I_{rms}|^2 X$$

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Complex Power

$$Complex\ Power = \tilde{S} = \tilde{V}_{rms}\,\tilde{I}^{*}_{rms} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i)$$
$$= |S|\angle(\theta_v - \theta_i)\ (polar\ form)$$
$$= P\ +\ jQ\ (rectangular\ form)$$

Value	Name	Meaning	Unit
$\lvert S \rvert$	Apparent power	Magnitude of $\tilde{S}$	VA
$\cos(\theta_v - \theta_i)$	Power factor	Cosine of angle of $\tilde{S}$	/
P	Real power	Real part of $\tilde{S}$	W
Q	Reactive power	Imaginary part of $\tilde{S}$	VAR

Complex Power

We can use the sign of pf angle or Q to identify the property of the circuit and the loads:

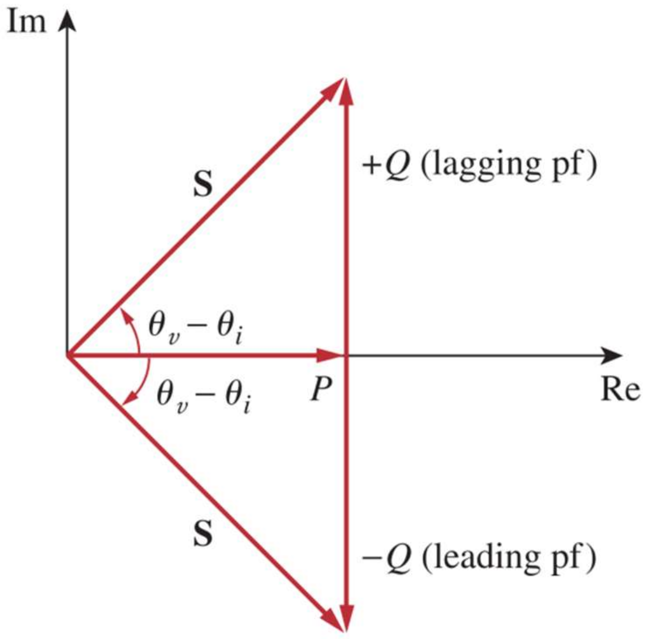
	(1)	(2)	(3)
pf Angle Sign of Q	$\theta_v - \theta_i = 0$ $Q = 0$	$\theta_v - \theta_i < 0$ $Q < 0$	$\theta_v - \theta_i > 0$ $Q > 0$
Properties	Unity pf I, V in phase $X = 0$ Resistive loads	Leading pf I leads V $X < 0$ Capacitive loads	Lagging pf I lags V $X > 0$ Inductive loads

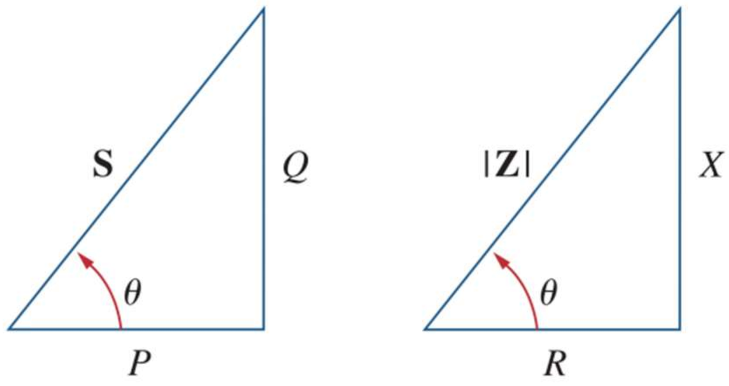
Complex Power

And observe that the power factor angle is equal to the angle of the impedance of that part of the circuit.

$S = P + jQ$

$Z = R + jX$

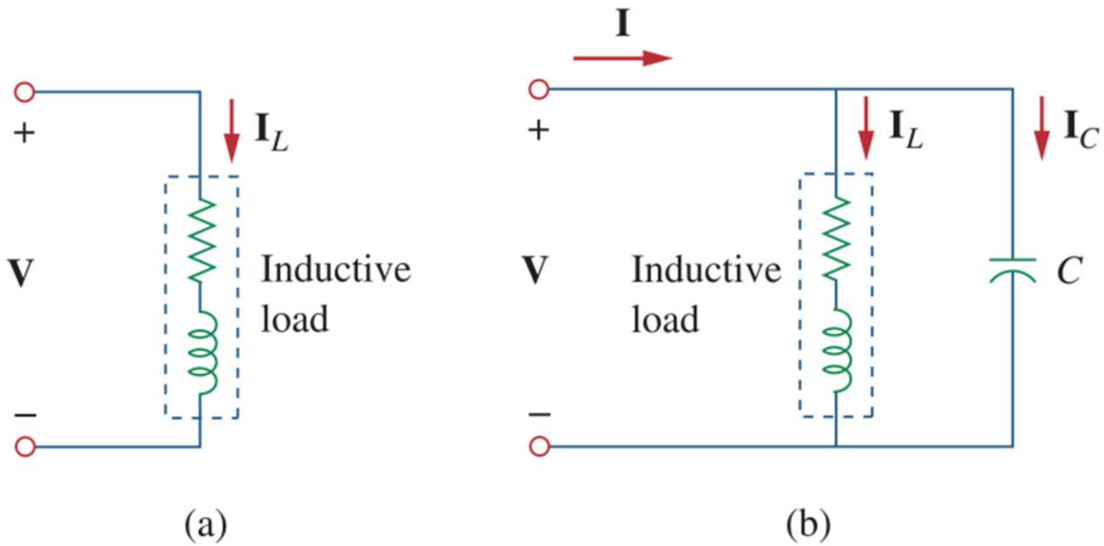




Power Factor Correction

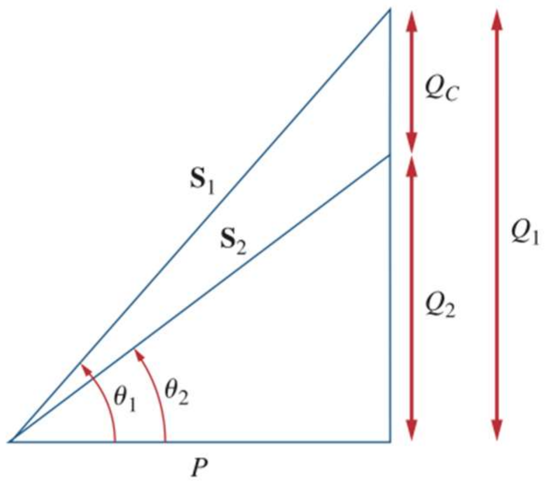
Goal: increase the pf of a load → make it less inductive → reduce energy loss

Solution: add a capacitor in parallel to the load



Power Factor Correction

Goal: increase the pf from $\cos\theta_1$ to $\cos\theta_2$.



Initial:

$$P = |S_1| cos\theta_1$$
$$Q = |Q_1| \sin\theta_1 = Ptan\theta_1$$

Expected outcome:

$$P = |S_2| cos\theta_2$$
$$Q = |Q_2| \sin\theta_2 = Ptan\theta_2$$

Since $Q_c\ (= Q_1 - Q_2) = \frac{V_{rms}^2}{X_c}$, then the value of the required capacitance C is:

$$C \Leftrightarrow \frac{1}{j\omega C} = \frac{1}{-\omega C} \cdot j \qquad C = \frac{Q_C}{\omega V_{rms}^2} = \frac{Q_2 - Q_1}{\omega V_{rms}^2} = \boxed{\frac{P(tan\theta_1 - tan\theta_2)}{\omega V_{rms}^2}}$$

$$X_C = -\frac{1}{\omega C} \Rightarrow Q_C = \frac{V_{rms}^2}{\frac{1}{-\omega C}}$$

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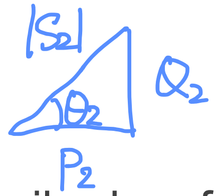
Conservation of AC Power

The Principle of Conservation of AC Power:

The complex, real, and reactive powers of the source equal the respective sums of the complex, real, and reactive powers of the individual loads.

Exercise 1



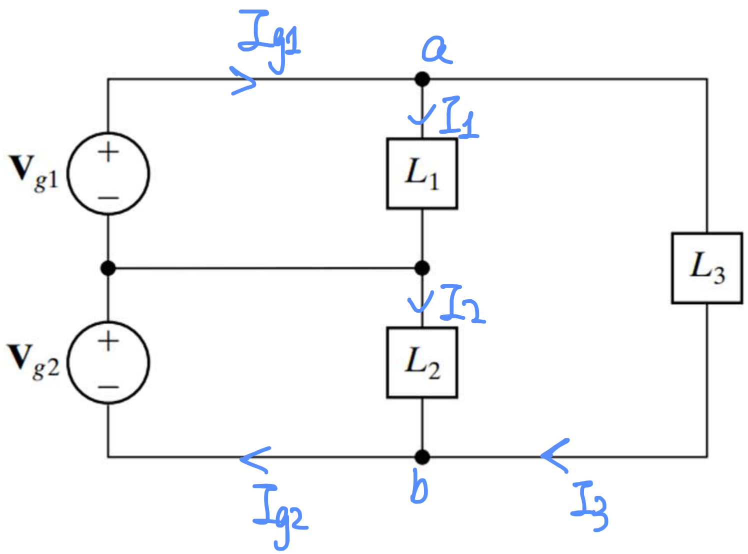
$\cos\theta_2 = 0.8$

$P_2 = 4.8k$

$Q_2 = 3.6k$

The three loads in the circuit seen in the figure are described as follows: Load 1 is absorbing $\underbrace{4.8\,kW}_{P_1}$ and delivering $\underbrace{2.4\,kVAR}_{Q_1}$; Load 2 is absorbing $\underbrace{6\,kVA}_{|S_2|}$ at a power factor of 0.8 lagging; Load 3 is a 24Ω resistor in parallel with an inductance whose reactance is 6Ω.

Calculate the average power and the magnetizing reactive power delivered by each source if $\boldsymbol{V}_{g1} = \boldsymbol{V}_{g2} = 120\angle 0°V$ (rms)



$S_1 = 4.8k - j2.4k$

$= \tilde{V}_1 \tilde{I}_1^*$

$\tilde{V}_1 = \tilde{V}_{g1} = 120\angle 0° V$

$\tilde{I}_1^* = 40 - j20\ [A]$

$\tilde{I}_1 = 40 + j20\ [A]$

$S_2 = 4.8k + j3.6k$

$= \tilde{V}_2 \cdot \tilde{I}_2^*$ $\quad \tilde{V}_2 = \tilde{V}_{g2} = 120\angle 0° V \quad \tilde{I}_2 = 40 - j30\ [A]$

$S_{g1} = -\tilde{V}_{g1} \cdot \tilde{I}_{g1}^*$

$S_{g2} = -\tilde{V}_{g2} \cdot \tilde{I}_{g2}^*$

a: $\tilde{I}_{g1} = \tilde{I}_1 + \tilde{I}_3$

b: $\tilde{I}_{g2} = \tilde{I}_2 + \tilde{I}_3$

$\tilde{I}_3 = \frac{\tilde{V}_3}{Z_3} = \frac{240V}{24\Omega} + \frac{240V}{j6\Omega} = 10 - j40\ [A]$

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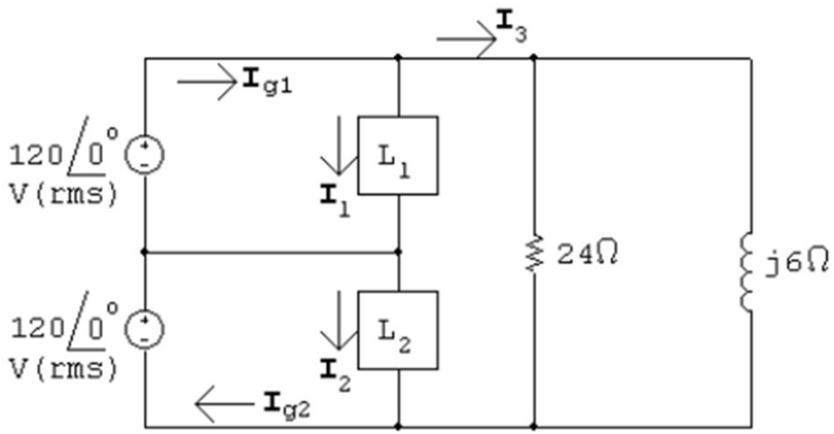
$\tilde{S}_{g1} = -6k - j2.4k$

$\tilde{S}_{g2} = -6k - j8.4k$

Exercise 1



$$120\mathbf{I}_1^* = 4800 - j2400; \qquad \therefore \quad \mathbf{I}_1 = 40 + j20\,\mathrm{A(rms)}$$

$$120\mathbf{I}_2^* = 4800 + j3600; \qquad \therefore \quad \mathbf{I}_2 = 40 - j30\,\mathrm{A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{24} + \frac{240}{j6} = 10 - j40\,\mathrm{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 50 - j20\,\mathrm{A}$$

$$S_{g1} = -120(50 + j20) = -6000 - j2400\,\mathrm{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 6 kW and 2.4 kvars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 50 - j70\,\mathrm{A(rms)}$$

$$S_{g2} = -120(50 + j70) = -6000 - j8400\,\mathrm{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 6 kW and 8.4 kvars.

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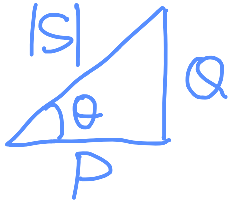
Exercise 2



$\cos\theta = 0.8$

$Q = 1.8k$

$Q_1 = 1.5k$

Two loads connected in parallel draw a total of 2.4 kW (P) at 0.8 pf lagging from a 120-V rms, 60-Hz line. One load absorbs 1.5 kW (P_1) at a 0.707 pf lagging.

Determine: (a) the pf of the second load, (b) the parallel element required to correct the pf to 0.9 lagging for the two loads

(a) $\tilde{S} = 2.4k + j1.8k$

$\tilde{S}_1 = 1.5k + j1.5k$

$\tilde{S}_2 = \tilde{S} - \tilde{S}_1$

$= 0.9k + j0.3k$

$pf_2 = \cos\theta_2 = \frac{3\sqrt{10}}{10} = 0.949$

(b). Apply the formula:

$$C = \frac{P(\tan\theta - \tan\theta_{new})}{\omega V_{rms}^2}$$

$$\tan\theta_{new} = \tan[\arccos 0.9] = \frac{\sqrt{19}}{9}$$

$$C = \frac{2.4kW \times [0.75 - \frac{\sqrt{19}}{9}]}{(2\pi \times 60Hz) \times (120V)^2}$$

$$= 117.46\ \mu F$$

Exercise 2

(a) $P = S\cos\theta_1 \longrightarrow S = \dfrac{P}{\cos\theta_1} = \dfrac{2.4}{0.8} = 3.0\text{ kVA}$

$pf = 0.8 = \cos\theta_1 \longrightarrow \theta_1 = 36.87^o$

$Q = S\sin\theta_1 = 3.0\sin 36.87^o = 1.8\text{ kVAR}$

Hence, $\mathrm{S} = 2.4 + \mathrm{j}1.8\text{ kVA}$

$S_1 = \dfrac{P_1}{\cos\theta} = \dfrac{1.5}{0.707} = 2.122\text{ kVA}$

$pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$Q_1 = P_1 = 1.5\text{ kVAR} \longrightarrow \mathrm{S}_1 = 1.5 + j1.5\text{ kVA}$

Since, $S = S_1 + S_2 \longrightarrow S_2 = S - S_1 = (2.4 + j1.8) - (1.5 + j1.5) = 0.9 + j0.3\text{ kVA}$

$S_2 = 0.9497 < 18.43^o$

$\mathrm{pf} = \cos 18.43^{\mathrm{o}} = \underline{0.9487}$

(b) $pf = 0.9 = \cos\theta_2 \longrightarrow \theta_2 = 25.84^o$

$C = \dfrac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \dfrac{2400(\tan 36.87 - \tan 25.84)}{2\pi x 60 x (120)^2} = \underline{117.5\ \mu\mathrm{F}}$

References

1. 2023 Fall VE215 slides, Sung-Liang Chen
2. 2023 Fall RC5, Shunyao Huang
3. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew

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THANK YOU!