# ECE2150J RC3

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# Overview

# Overview

# Ideal Op-amp

Assumption:

- Infinite open-loop gain ($A = \infty$)
- Infinite input resistance ($R_i = \infty$)
- Zero output resistance ($R_0 = 0$)
- (Does not mean that $v_0 = \infty$)

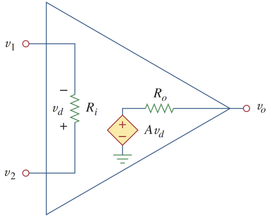



Figure: Op-amp's equivalent circuit

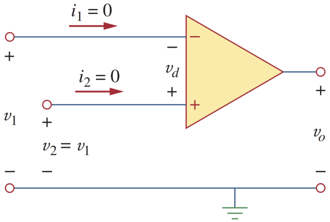



Figure: Symbol of ideal op-amp

# Ideal Op-amp

Characteristics of ideal op-amp:

- Open circuit at two input terminals ($i_1 = i_2 = 0$)
- Same voltage at two input terminals ($v_1 = v_2$)
- (**Does not mean that** $i_o = 0$**!**)

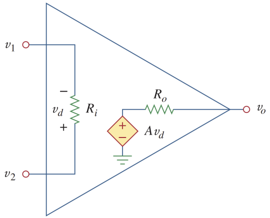



Figure: Op-amp's equivalent circuit

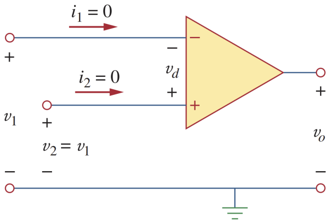
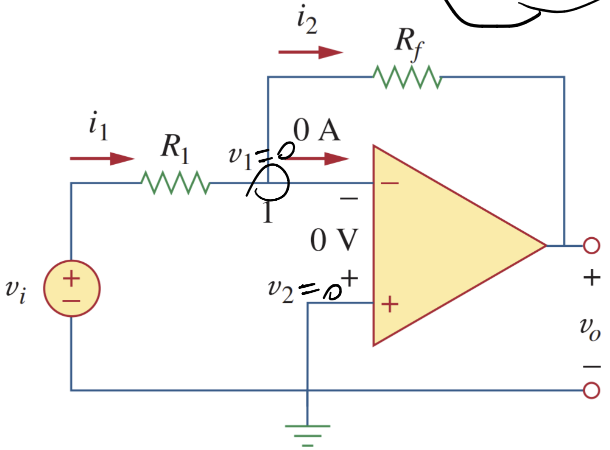



Figure: Symbol of ideal op-amp

# Inverting Amplifier

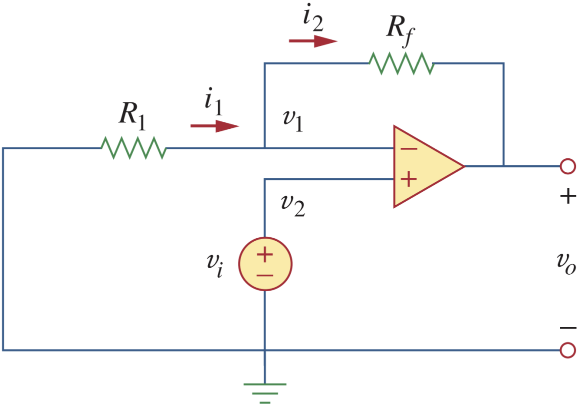
$$A_v = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

$$\frac{0 - V_i}{R_1} + \frac{0 - V_o}{R_f} = 0$$



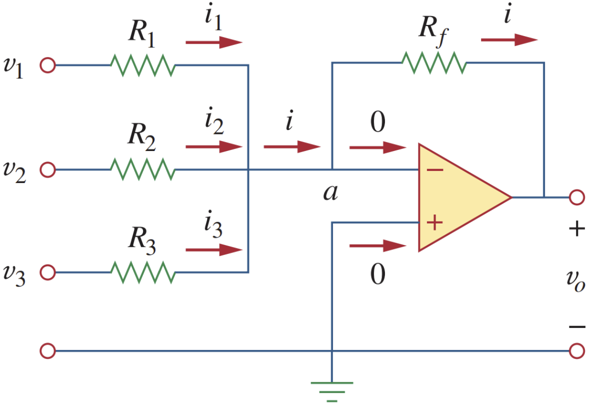
# Non-inverting Amplifier

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_1}$$



# Summing Amplifier

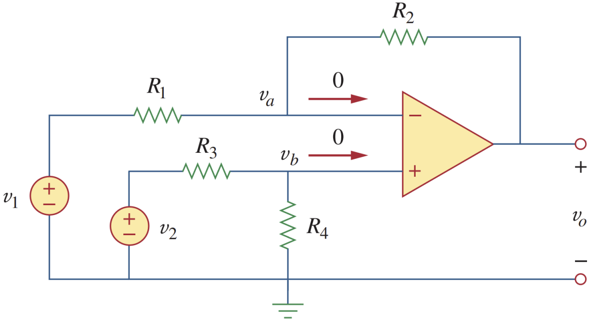
$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$



# Difference Amplifier

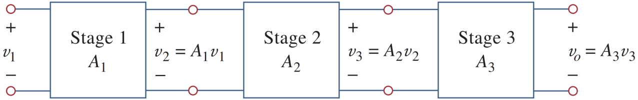
Check if $R_1 = R_2,\ R_3 = R_4$

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$



# Cascaded Op Amps

## Gain of Cascaded Op Amp




Original input signal is increased by the gain of the individual stage, and the final gain is the **product of all gains at each stage**.

$$A = \frac{v_o}{v_1} = \frac{v_2}{v_1} \cdot \frac{v_3}{v_2} \cdot \frac{v_o}{v_3} = A_1 A_2 A_3$$

# Basic Op-amp Circuits: Summary

For basic op-amp circuits:

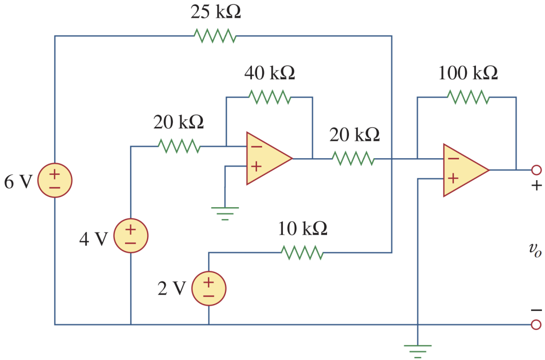
| Op-amp circuits | Input-output relationship |
| --- | --- |
| Inverting amplifier | $A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$ |
| Non-inverting amplifier | $A = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$ |
| Summing amplifier | $v_o = -(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3)$ |
| Difference amplifier | $v_o = \left[ (\frac{R_2}{R_1} + 1)(\frac{R_4/R_3}{1+R_4/R_3}) \right] v_2 - \left[ \frac{R_2}{R_1} \right] v_1$ |

For complicated op-amp circuits:

- Identify basic op-amp circuits within it
- Use the formula for cascaded op-amp circuit
- Be proficient in listing nodal analysis equations to obtain $v_o/v_i$

# Exercise 1

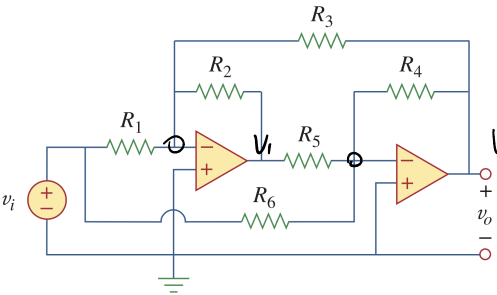
Find $v_o$.




$$V_o = -\frac{100}{25}\cdot 6 - \frac{100}{20}\cdot\left(-\frac{40}{20}\right)\cdot 4 - \frac{100}{10}\cdot 2 = -24 - 20 + 40 = -4V$$

# Exercise 2

Determine the gain $v_o/v_i$ of the circuit.




$$\frac{0-V_1}{R_2} + \frac{0-V_o}{R_3} + \frac{0-V_i}{R_1} = 0$$

$$V_1 = -\frac{R_2}{R_1} V_i - \frac{R_2}{R_3} V_o$$

$$\frac{0-V_1}{R_5} + \frac{0-V_i}{R_6} + \frac{0-V_o}{R_4} = 0$$

$$V_o = -\frac{R_4}{R_5} V_1 - \frac{R_4}{R_6} V_i$$

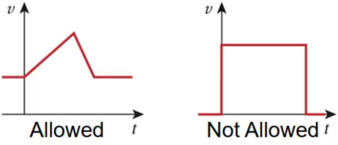
$$V_o = \frac{R_4}{R_5} \cdot \frac{R_2}{R_1} \cdot V_i + \frac{R_4}{R_5} \cdot \frac{R_2}{R_3} \cdot V_o - \frac{R_4}{R_6} V_i$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{\frac{R_2 R_4}{R_1 R_5} - \frac{R_4}{R_6}}{1 - \frac{R_2 R_4}{R_3 R_5}}$$

# Overview

# Capacitors

1. **Open Circuit Property** When the voltage across a capacitor is not changing with time (**DC steady state**), **the capacitor could be treated as an open circuit**.
2. **Continuity property** The voltage on a capacitor must be continuous.



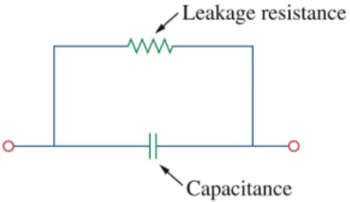

3. **Capacitors IV relationship**

$$i = C\frac{dv}{dt}$$

property 2 can be intuitively shown be property 3. If the voltage across the capacitor is not continuous, say $\frac{dv}{dt} = \infty$, which will cause $i$ to be infinity.

# Capacitors

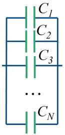
1. **An ideal capacitor will not dissipate energy.** It takes power from the circuit when storing energy in its electric field and returns previously stored energy when delivering power to the circuit.
2. **A real capacitor has a large leakage resistance**



# Capacitors in parallel & in series

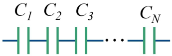
- **capacitors in parallel**

$G(S) = \frac{1}{R}$




$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_N$$

- **capacitors in series**




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots + \frac{1}{C_N}$$

# Energy stored in Capacitors

**The instantaneous power** delivered to the capacitor is

$$p = vi = v(C\frac{dv}{dt})$$

Therefore, **the total energy** stored in the capacitor is

$$w = \frac{1}{2}CV^2$$

$$W = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} v\left(C\frac{dv}{dt}\right)dt = C\int_{v(-\infty)}^{v} v\,dv$$

$$= \frac{1}{2}Cv^2\Big|_{\underbrace{v(-\infty)}_{=0}}^{v} = \frac{1}{2}Cv^2$$

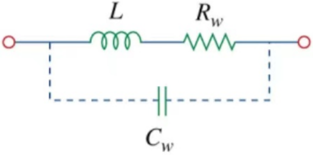
# Inductors

1. **Short Circuit Property** When the current through an inductor is not changing with time (**DC steady state**), **the inductor could be treated as a short circuit in the circuit**.
2. **Continuity property** The current through a capacitor must be continuous.
3. **Inductor IV relationship**

$$v = L\frac{di}{dt}$$

# Inductors

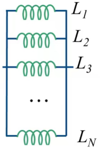
1. **An ideal inductor will not dissipate energy.** It takes power from the circuit when storing energy in its magnetic field and returns previously stored energy when delivering power to the circuit.
2. **A real inductor has a significant winding resistance and a small winding capacitance**



# Inductors in parallel & in series

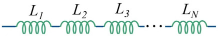
- **inductors in parallel**

R




$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + ... + \frac{1}{L_N}$$

- **inductors in series**




$$L_{eq} = L_1 + L_2 + L_3 + L_4 + ... + L_N$$

# Energy stored in Inductors

**The instantaneous power** delivered to the inductor is

$$p = vi = (L\frac{di}{dt})i$$

Therefore, **the total energy** stored in the inductor is

$$w = \frac{1}{2}Li^2$$

# Summary of Capacitors and Inductors

| | Capacitor | Inductor |
|---|---|---|
| Electric/magnetic | $q$ | $\psi$ |
| | $q=Cv$ | $\psi=Li$ |
| i-v (or v-i) relation | $i=C\times dv/dt$ | $v=L\times di/dt$ |
| energy | $1/2Cv^2$ | $1/2Li^2$ |

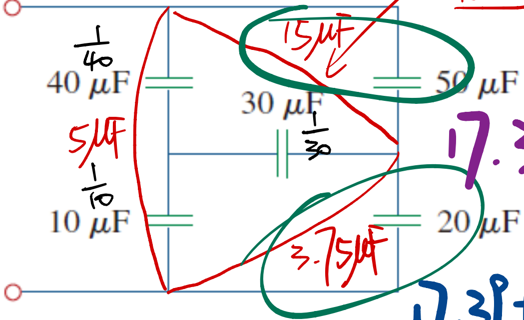
# Exercise 3

Y-Δ

Obtain the equivalent capacitance of the network below.

$$\frac{\frac{1}{40}\times\frac{1}{30}+\frac{1}{30}\times\frac{1}{10}+\frac{1}{10}\times\frac{1}{40}}{\frac{1}{10}}$$

$= \frac{1}{15}$



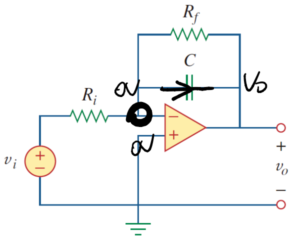

17.39

$17.39 + 5$

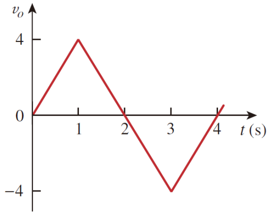
$= \underline{22.39}$

# Exercise 4

The output $v_o$ of the op amp circuit in Fig.(a) is shown in Fig.(b). Let $R_i = R_f = 1\,\text{M}\Omega$ and $C = 1\,\mu\text{F}$. Determine the input voltage waveform and sketch it.



(a)

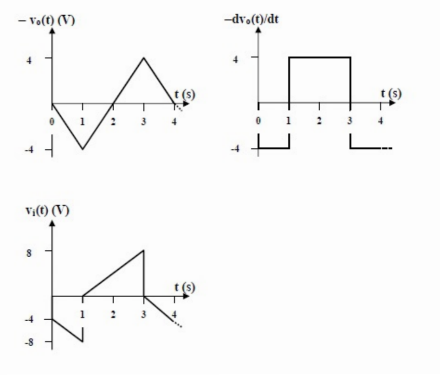


(b)

# Exercise 4

$$\frac{0-V_i}{R_i} + \frac{0-V_o}{R_f} + \underbrace{C\frac{d}{dt}(0-V_o)}_{} = 0$$

$$\Rightarrow \quad V_i = -\left(V_o + \frac{dV_o}{dt}\right)$$



# Overview

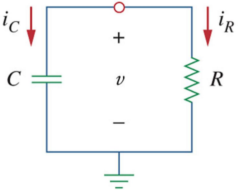
Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

# Source-Free Circuits (I) Response

**Source-free RC**

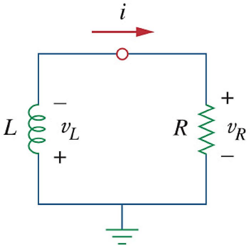



Voltage: $v = v_0 e^{-t/RC}$

Time constant: $\tau = RC$

Current: $i_R = \frac{v}{R} = \frac{v_0}{R} e^{-t/\tau}$
Power: $p = v i_R = \frac{v_0^2}{R} e^{-2t/\tau}$
Energy: $w_R = \int_0^t p dt = \frac{1}{2} C v_0^2$

**Source-free RL**

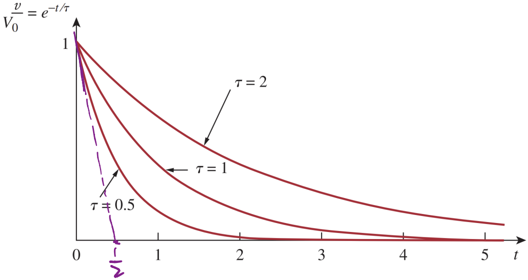



Current: $i = i_0 e^{-t/(L/R)}$

Time constant: $\tau = L/R$

Voltage: $v_R = iR = \frac{i_0}{R} e^{-t/\tau}$
Power: $p = v_R i = i_0^2 R e^{-2t/\tau}$
Energy: $w_R = \int_0^t p dt = \frac{1}{2} L i_0^2$

# Source-Free Circuits (II) Time Constant




| | Source-free RC | Source-free RL |
|---|---|---|
| Time constant | $\tau = RC$ | $\tau = L/R$ |
| Relation to initial decay rate | $\frac{d}{dt}(\frac{v}{v_0}) = -1/\tau$ | $\frac{d}{dt}(\frac{i}{i_0}) = -1/\tau$ |

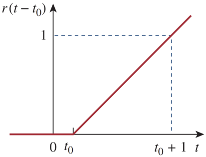
- Time required for the response to decay to a factor of $1/e$ or 36.8% of its initial value
- Indicates the initial decaying rate
- Assume complete decay after $5\tau$

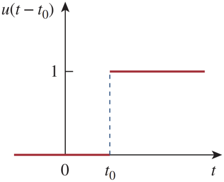
# Source-Free Circuits (III) General Steps

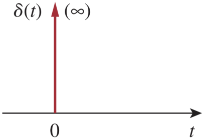
- Find the initial value $v_0$, $i_0$.
- Find the time constant, i.e., find $R_{\text{eq}}$. ($R_{\text{eq}}$ is the Thevenin or Norton equivalent resistance at the capacitor and inductor terminals.)
- Use the equation in the previous slide to get results.
- You can always use Mesh/Nodal analysis to calculate if you are not familiar with the steps above.
- (Of course, you need to remember that $i = C\frac{dv}{dt}$ for capacitors and $v = L\frac{di}{dt}$ for inductors!)

# Singularity Functions

| Unit ramp | Unit step | Unit impulse |
|---|---|---|
| $\mathrm{r(t)}= \begin{cases} 0, t \leq 0 \\ t, t > 0 \end{cases}$ | $\mathrm{u(t)}= \begin{cases} 0, t \leq 0 \\ 1, t > 0 \end{cases}$ | $\delta(t) = \begin{cases} 0, t \neq 0 \\ \text{Undef.}, t = 0 \end{cases}$ |




Give a nice way to represent "Switch on/off" of the sources/part of circuits.

$$\delta(t) \xrightarrow{\int} u(t) \xrightarrow{\int} r(t)$$

# Exercise 5

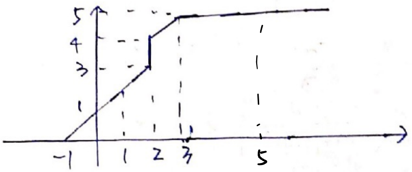
Please plot the function diagram of the following singular function

$$f(t) = r(t+1) + u(t-2) - r(t-3)$$

and calculate

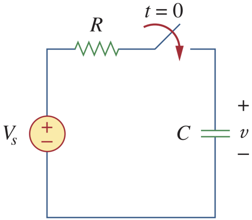
$$\int_{-\infty}^{+\infty} f'(t)\delta(t-5)\,dt$$

$$= f'(5) = 0$$



# Circuits with Step Input (I) Response

**Step-input RC**

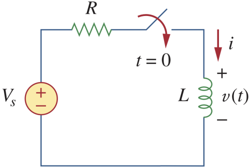



Initial condition:
$v(0^+) = v(0^-) = V_0$

Equation:
(KVL) $(C\frac{dv}{dt}R + v = V_s)$

Response:
$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$

**Step-input RL**




Initial condition:
$i(0^+) = i(0^-) = I_0$

Equation:
(KCL) $iR + L\frac{di}{dt} = V_s$

Response:
$i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-t/\tau}$

# Circuits with Step Input (II) General Steps

- General form: $x(t) = x(\infty) + (x(0) - x(\infty))e^{-t/\tau}$.
- Step response = natural response + forced response, where natural response is $x(0)e^{-t/\tau}$, forced response is $x(\infty)(1 - e^{-t/\tau})$.
- Step response = transient state response + steady state response, where transient response is $(x(0) - x(\infty))e^{-t/\tau}$, steady state response is $x(\infty)$.
- Steps to find step response:
  - Find $x(0)$.
  - Find $x(\infty)$.
  - Find $\tau$.
- Still, you can find step response using mesh/nodal analysis. By solving the differential equation, you will find that natural response is the homogeneous solution of the equation, while the forced response is the particular solution.

# General Formula for First-Order Circuits

General formula for RC:

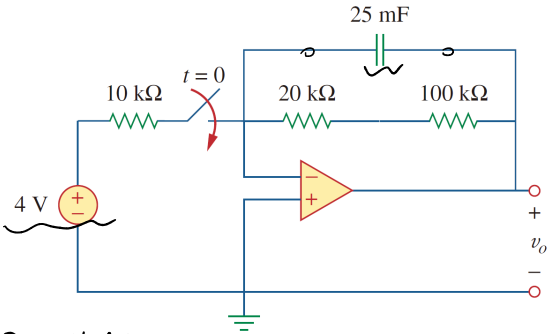
$$v(t) = v(\infty) + \left[v(0^+) - v(\infty)\right] e^{-t/\tau}$$

General formula for RL:

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}$$

# Exercise 6

For the op amp circuit below, find $v_o(t)$ for $t > 0$.




① For $t<0$, $V_o(0) = 0$

②. For $t>0$, $V_o(\infty) = -\frac{20+100}{10} \cdot 4 = -48$

# Exercise 6

③. $R_{Th} = 20 + 100 = 120k\Omega$

$\tau = R_{th}C = (120 \times 10^3) \cdot (25 \times 10^{-3}) = 3000$

$V_o(t) = V_o(\infty) + [V_o(0) - V_o(\infty)] \cdot e^{-t/\tau}$

$\Rightarrow$

$V_o(t) = -48(1 - e^{-t/3000})\,V$

# References


1. 2024 Fall VE215 slides, Sung-Liang Chen
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2023 Summer RC3, Erdao Liang, Chongye Yang
4. 2024 Summer RC3, Runting Zhang

Thank you!