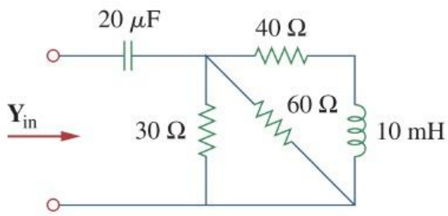
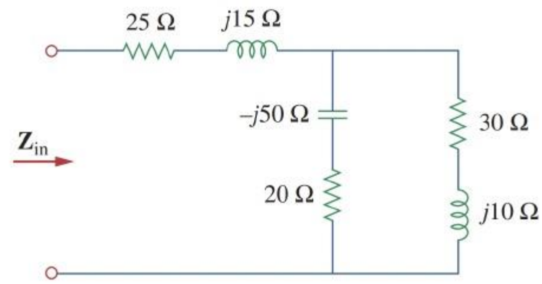
Exercise 4.1 (30%)

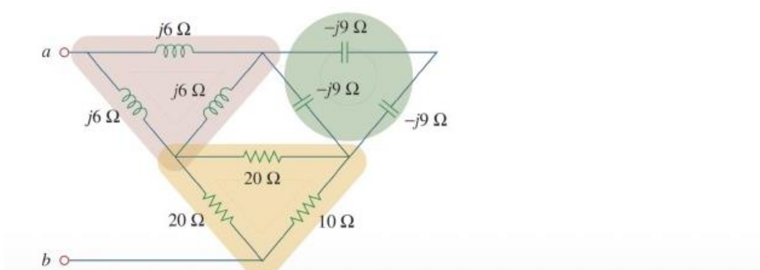
(a) (10%) Obtain the equivalent admittance Y_{in} of the circuit at $\omega = 100\ rad/s$.
(b) (10%) Obtain the equivalent impedance Z_{in} of the circuit.
(c) (10%) Obtain the equivalent impedance Z_{ab} of the circuit.



(a)



(b)



(c)

(a)

$10\,mH \longrightarrow j\omega L = j \times 100 \times (10 \times 10^{-3}) = j$ (2')

$20\,\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j \times 100 \times (20 \times 10^{-6})} = -500j$ (2')

$30 \parallel 60 = 20\,\Omega$

$Z_{in} = -500j + 20 \parallel (40+j)$ (2')

$= -500j + (13.36 + 0.11j)$

$= 13.36 - 499.89j\ \Omega$ (or $500.07\angle -88.47°\ \Omega$) (2')

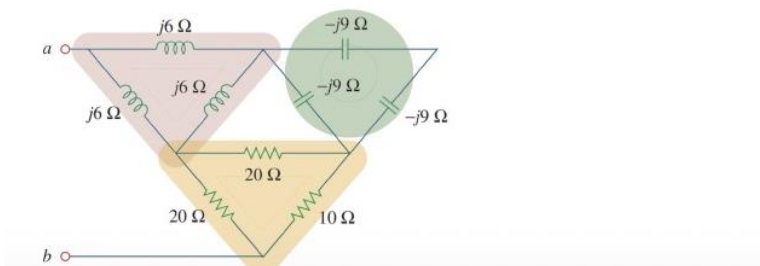
$Y_{in} = \frac{1}{Z_{in}} = 5.33 \times 10^{-5} + 2.00 \times 10^{-3} j\ S$ (or $2.00 \times 10^{-3} \angle 88.47°$) (2')

(b)

$$Z_{in} = (20+15j) + (20-50j) \parallel (30+10j) \quad (3')$$

$$= (20+15j) + (26.10 - 5.12j) \quad (3')$$

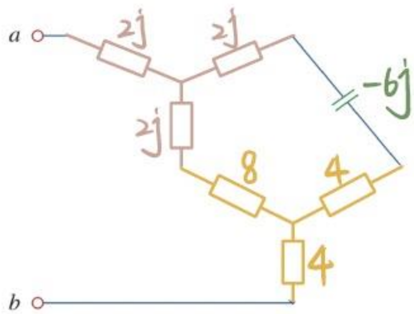
$$= 46.10 + 9.88j\ \Omega \quad (\text{or } 47.15\angle 12.10^\circ\ \Omega) \quad (4')$$



(c)

$\nabla \to Y$

$\nabla \to Y$



$$(-9j-9j) \parallel (-9j) = -6j$$

$$\frac{6j \times 6j}{6j+6j+6j} = 2j$$

$$\frac{20 \times 20}{20+20+10} = 8 \qquad \frac{20 \times 10}{20+20+10} = 4$$

$$Z_{ab} = 2j + (2j - 6j + 4) \parallel (2j + 8) + 4$$

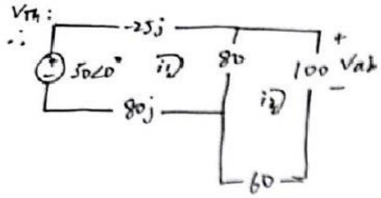
$$= 2j + (3.57 - 1.41j) + 4$$

$$= 7.57 + 0.59j\ \Omega \qquad (\text{or } 7.59\angle 4.49^\circ\ \Omega) \quad (10')$$

(a) 1' $w = 2000 \text{rad/s}$ $\therefore Z_C = -25j\Omega, Z_L = 80j\Omega$ (2')

V_{Th}:

$\therefore$



$\therefore \begin{cases} (80+55j)i_1 - 80i_2 = 50\angle 0^\circ \\ 240i_2 - 80i_1 = 0 \end{cases}$

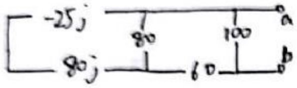
$\therefore i_2 = \frac{320}{2113} - \frac{330}{2113}j = 0.1514 - 0.1562j = 0.2175\angle -45.88^\circ$

$\therefore V_{ab} = 21.755\angle -45.88^\circ V = (15.144 - 15.618j)V = V_{Th}$ (10')

($V_{Th} = 21.755\cos(2000t - 45.88^\circ)$ V 也行)

OK

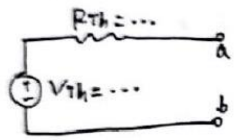
R_{Th}:



$\therefore R_{Th} = 100 // (60 + 80 // 55j) = (48.2371 + 10.4117j)\Omega$ (3')

~~49.348∠12.18~~

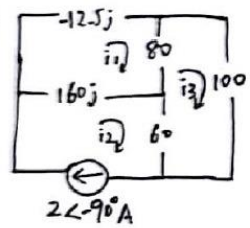
$\therefore$



(图 2')

2' $w = 4000 \text{rad/s}$ $\therefore Z_C = -12.5j\Omega, Z_L = 160j\Omega$. (2')

V_{Th}:



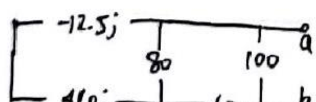
$\therefore \begin{cases} (80+147.5j)i_1 - 160j\, i_2 - 80i_3 = 0 \\ i_2 = 2\angle -90^\circ \\ 240i_3 - 80i_1 - 60i_2 = 0 \end{cases}$

$\therefore i_3 = 1.178\angle -82.62^\circ$ A $= (0.1513 - 1.1685j)$ A

$V_{Th} = V_{ab} = 100i_3 = 117.821\angle -82.62^\circ V = (15.13 - 116.85j)$ V (10')

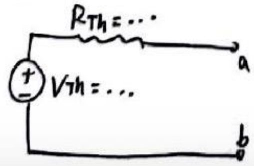
($V_{Th} = 117.821\cos(4000t - 82.62^\circ)$ V 也行)

$\therefore R_{Th}$:

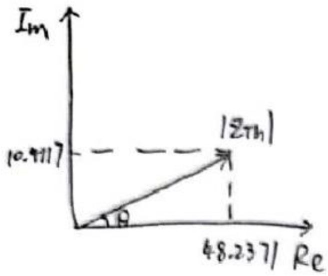


$\therefore R_{Th} = 100 // (60 + 80 // 147.5j) = (55.9245 + 6.6620j)\Omega$ (3')

$\therefore$



(图 2')



∴1' $w = 2000\,rad/s$

∴ $R_{Th} = (48.2371 + 10.4117j)\,\Omega\ (= 49.348\angle 12.18^\circ)$

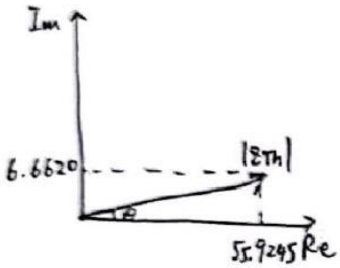
∴ $|Z_{Th}| = 49.348\,\Omega$, $\theta = 12.18^\circ$

(图上 Re, Im 坐标 2×1')

$|Z_{Th}|$值 1'

θ值 1'

图上有$|Z_{Th}|$和θ标注 1'



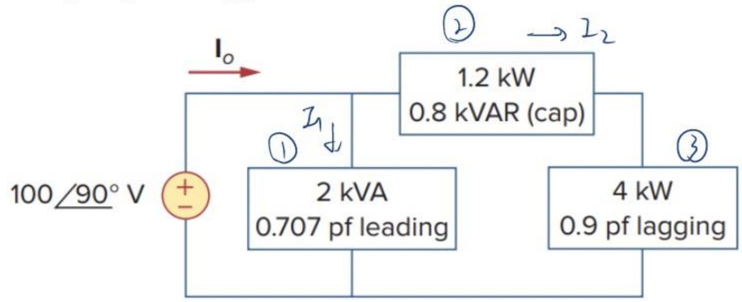
2' $w = 4000\,rad/s$

∴ $R_{Th} = (55.9245 + 6.6620j)\,\Omega\ (= 56.3199\angle 6.79^\circ)$

∴ $|Z_{Th}| = 56.3199\,\Omega$, $\theta = 6.79^\circ$

(同上)

Exercise 6.3 (30%)
Given the circuit below, calculate the equivalent impedance in each of the blocks. Also find I_o and the overall complex power supplied.



$S_1 = 1.414 - 1.414j$ kVA (2')

$S_2 = 1.2 - 0.8j$ kVA (2')

$S_3 = 4 + j4 \cdot \tan(\cos^{-1}(0.9)) = 4 + 1.937j$ kVA (2')

$I_1^* = \frac{S_1}{V} = \frac{1.414 - 1.414j\ k}{j100} = -14.14 - 14.14j$ A $\Rightarrow I_1 = -14.14 + 14.14j$ (2')

$I_2^* = \frac{S_2 + S_3}{V} = 11.37 - 52j$ A $\Rightarrow I_2 = 11.37 + 52j$ (2')

$I_o = I_1 + I_2 = -2.77 + 66.14j = 66.198 \angle 92.398$ A. (4')

$S = S_1 + S_2 + S_3 = 6.614 - 0.277j$ kVA. (4')

$Z_1 = \frac{V}{I_1} = 3.54 - 3.54j = 5 \angle -45° \ \Omega$ (4')

$Z_2 = \frac{S_2}{|I_2|^2} = 0.424 - 0.282j = 0.509 \angle -33.690° \ \Omega$ (4')

$Z_3 = \frac{S_3}{|I_2|^2} = 1.412 + 0.684j = 1.569 \angle 25.839° \ \Omega$ (4')