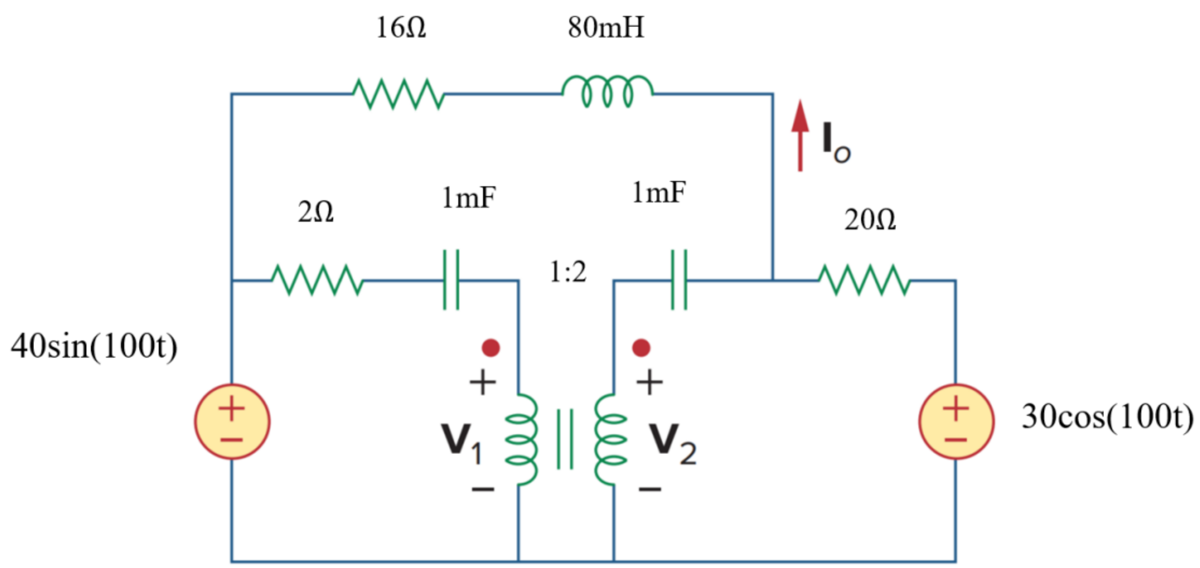
2. Please analyze an **ideal transformer** circuit below. Assume all values in the question below are **RMS** values. [Total 16 points]

(1) Transform the circuit below to the phasor domain, e.g. L → jωL. [2 points]

(2) Please assign currents at the transformer terminals (any direction as you prefer). Determine polarities of turn ratio *n* of both voltage and current. [2 points]

(3) Please get values for V_1, V_2 and I_o respectively in the circuit. [12 points]



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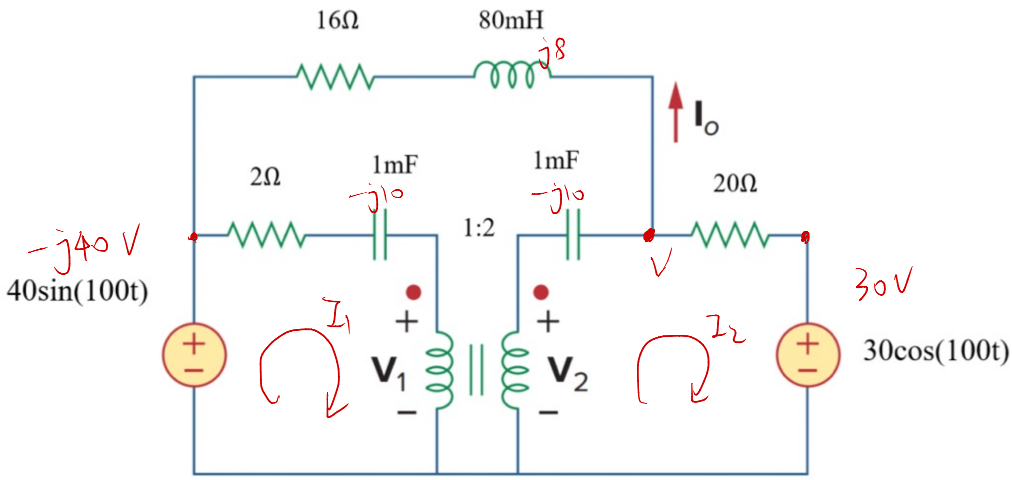


(1) $\omega = 100$

$80mH \rightarrow j8\,\Omega$ (1')

$1\,mF \rightarrow -j10\,\Omega$ (1')

(2) $\frac{V_1}{V_2} = \frac{1}{2}$ (1')

$\frac{I_1}{I_2} = \frac{2}{1}$ (depending on the assigned direction) (1')

(3)
$$I_1 = \frac{-j40 - V_1}{2 - j10} \quad (2')$$

$$I_2 = \frac{V_2 - V}{-j10} \quad (2')$$

$$\frac{I_1}{I_2} = 2, \quad \frac{V_1}{V_2} = \frac{1}{2}$$

$$\frac{V - V_2}{-j10} + \frac{V + j40}{16 + j8} + \frac{V - 30}{20} = 0. \quad (2')$$

$$I_o = \frac{V + j40}{16 + j8} \quad (2')$$

Any other partial procedure (1')

$\Rightarrow$

$V_1 = 7.56 - 15.80j$ V (1')

$= 17.52 \angle -64.43$ V

$V_2 = 15.12 - 31.60j$ V (1')

$= 35.03 \angle -64.43$ V

$I_o = 1.54 + 0.44j$ A (1')

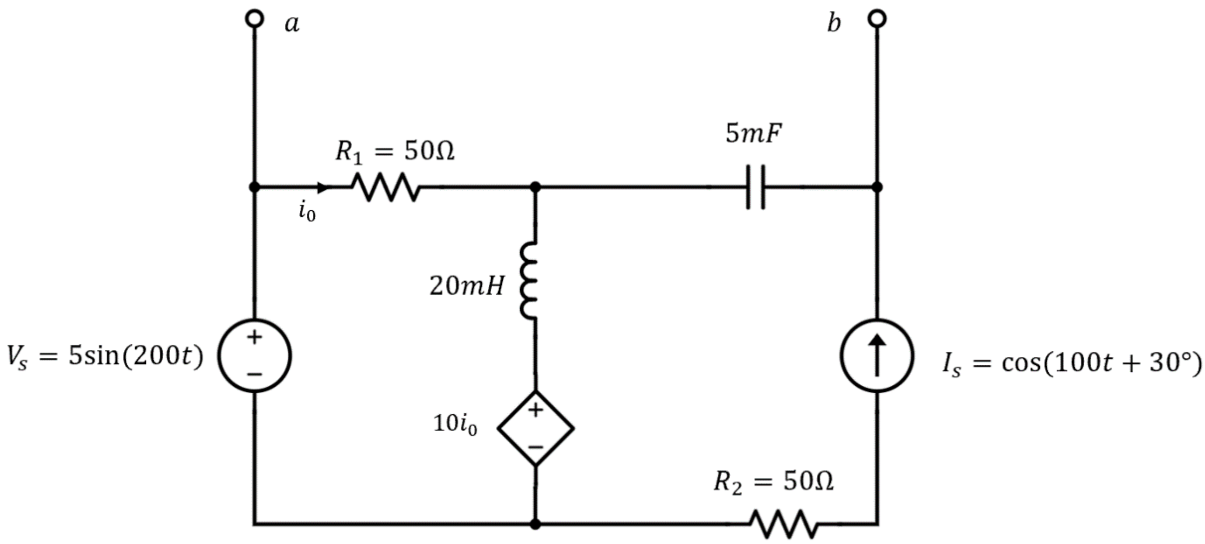
$= 1.60 \angle 15.95$ A

3. Below shows an **AC circuit** with an independent voltage and current source. [18 points]

(1) Please find the V_{TH} (Thevenin voltage) and Z_{TH} (Thevenin impedance) and draw Thevenin equivalent circuits between terminal a and b under $\boldsymbol{\omega = 100\,rad/s}$ and $\boldsymbol{\omega = 200\,rad/s}$. [12 points]

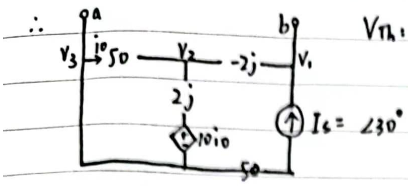
(2) Please draw a phasor diagram of the Thevenin equivalent impedance under (a) $\boldsymbol{\omega = 100\,rad/s}$ and (b) $\boldsymbol{\omega = 200\,rad/s}$. [6 points]

Hint: There should be 2 separate phasor diagrams in total. Please label all the angles, coordinates and magnitudes of the phasors on your phasor diagrams.



(1) 1' $w = 100 rad/s$ $\therefore Z_L = jwL = 2j$, $Z_C = \frac{1}{jwC} = -2j$

$\therefore$



V_{Th}: $\therefore \begin{cases} i_0 = \frac{V_3 - V_2}{50} \\ \frac{V_2 - V_3}{50} + \frac{V_2 - 10i_0}{2j} = I_s = \angle 30^\circ \\ V_1 - V_2 = -2jI_s = -2j \cdot \angle 30^\circ \end{cases}$

$\therefore V_1 = (2j+10)i_0$, $V_2 = (2j+10)i_0 + 2jI_s$

$V_3 = (2j+60)i_0 + 2jI_s$

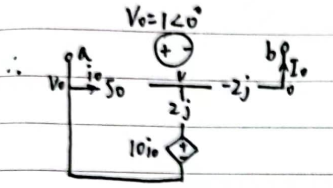
$\therefore V_{ab} = V_3 - V_1 = 50i_0 + 2jI_s$

$\therefore 50i_0 + 2j(i_0 + I_s) + 10i_0 = 0$, $i_0 = -\frac{j}{30+j} I_s$

$\therefore V_{Th} = V_{ab} = (2j - \frac{50j}{30+j})I_s = -0.216 + 0.263j = 0.340\angle 129.40^\circ$

(3')

$\therefore$



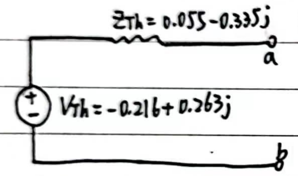
Z_{Th}: $\begin{cases} V_0 - V = 50i_0 \\ \frac{V - V_0}{50} + \frac{V - (V_0 + 10i_0)}{2j} + \frac{V}{-2j} = 0 \end{cases}$ (Equation max 2')

$\therefore V = \frac{151 - 25j}{26} V_0$

$\therefore I_0 = \frac{V}{-2j} = \frac{151 - 25j}{-52j} V_0$

$\therefore Z_{Th} = \frac{V_0}{I_0} = \frac{-52j}{151 - 25j} = 0.055 - 0.335j = 0.340\angle -80.60^\circ$ (2')

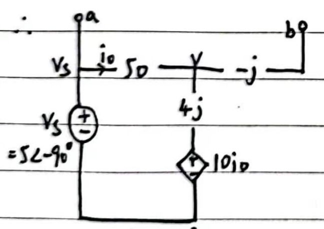
$\therefore$



(No Figure/Wrong -1)

2' $w = 200 rad/s$ $\therefore Z_L = jwL = 4j$, $Z_C = \frac{1}{jwC} = -j$, $V_s = 5\angle -90^\circ$

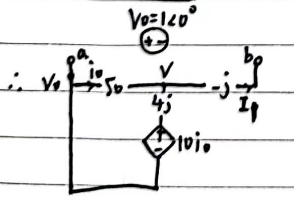
$\therefore$



V_{Th}: $\therefore \begin{cases} V_s - V = 50i_0 \\ \frac{V - 10i_0}{4j} = i_0 \end{cases}$ $\therefore V_s = (60+4j)i_0$, $V = (10+4j)i_0$

$\therefore V_{Th} = V_s - V = 50i_0 = \frac{50}{60+4j} V_s = -0.277 - 4.148j$

$= 4.157\angle -93.81^\circ$ (3')

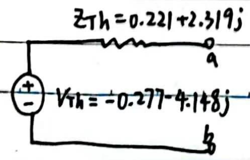


Z_{Th}: $\therefore \begin{cases} V_0 - V = 50i_0 \\ \frac{V - V_0}{50} + \frac{V - (V_0 + 10i_0)}{4j} + \frac{V}{-j} = 0 \end{cases}$

$\therefore V = -\frac{262 + 25j}{613} V_0$ $\therefore I_0 = \frac{V}{-j} = \frac{262 + 25j}{613j} V_0$

$\therefore Z_{Th} = \frac{V_0}{I_0} = \frac{613j}{262 + 25j} = 0.221 + 2.319j = 2.329\angle 84.55^\circ$ (2')

$\therefore$



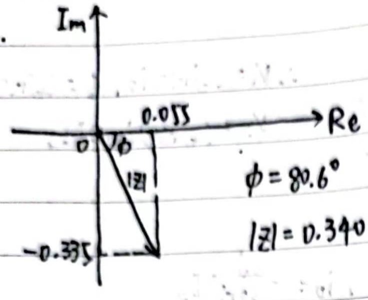
(1')

No.

Date

(2) 1' $w = 100$ rad/s $\therefore Z_{Th} = 0.340\angle\cancel{129.40}\ -80.60^\circ$

∴



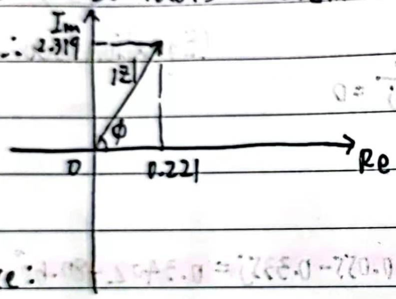
$\phi = 80.6^\circ$

$|Z| = 0.340\ \Omega$

2' $w = 200$ rad/s $\therefore Z_{Th} = 2.329\angle 84.55^\circ$

$\therefore \phi = 84.55^\circ$

$|Z| = 2.329\ \Omega$



Notice:

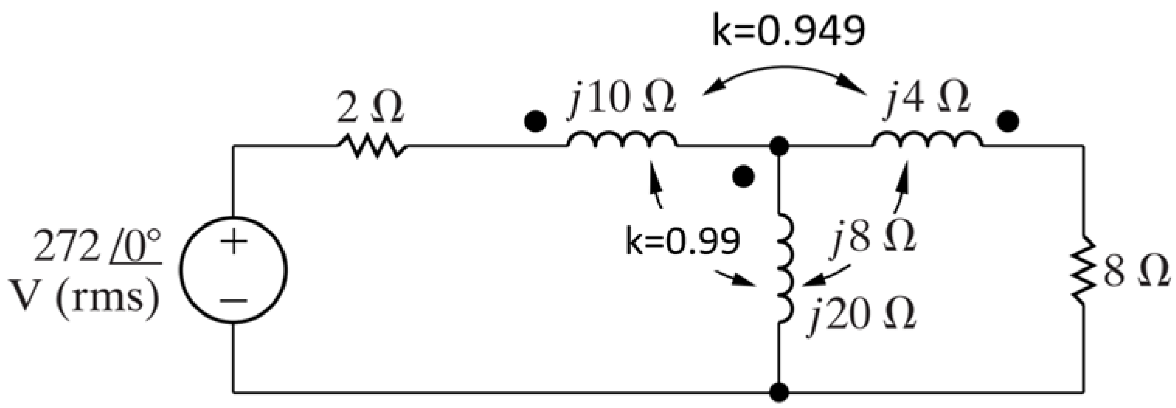
1. magnitude $|Z|$ 1' ; coordinates (Re, Im) 1' ; angle ϕ 1'

4. Please answer the following questions with the circuit shown below. [18 points]

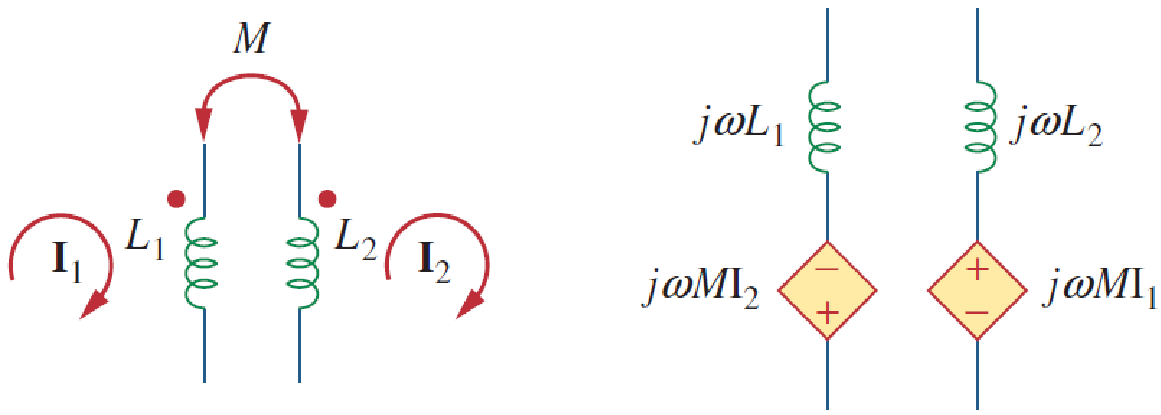
(1) Find the mutual inductance (assume $\boldsymbol{\omega} = \mathbf{1000}$) between j10 Ω, j4 Ω and j10 Ω, j20 Ω, are they tightly coupled or loosely coupled? [2 points]

(2) Please draw the equivalent circuit using the dependent voltage sources (example below). [8 points]

(3) Verify the conservation of average power in this circuit. [8 points]



Example: Magnetically coupled circuit and its equivalent circuit with dependent voltage sources.



4 (I)

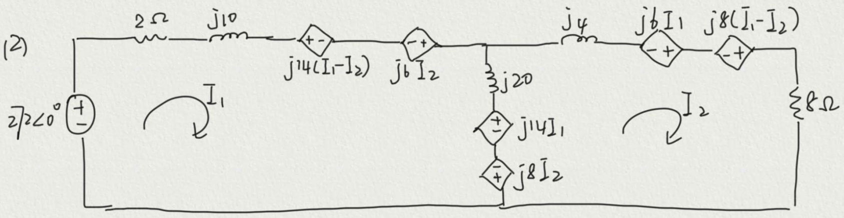
4

(1) $j10, j4$. $M = \dfrac{\sqrt{10\times4}\times0.949}{1000} = 6\ \mathrm{mH}$ (0.5')

$j10, j20$. $M = \dfrac{\sqrt{10\times20}\times0.99}{1000} = 14\ \mathrm{mH}$. (0.5')

They are both tightly coupled. (1')

(2)



All correct 8'

one dependent source wrong $-1'$: (start from 6')

(3)

$$\begin{cases} -272 + 2I_1 + j10I_1 + j20(I_1 - I_2) + j14(I_1 - I_2) - j6I_2 + j14I_1 - j8I_2 = 0 & (1') \\ j8I_2 - j14I_1 + j20(I_2 - I_1) + j4I_2 - j6I_1 - j8(I_1 - I_2) + 8I_2 = 0 & (1') \end{cases}$$

$$\begin{cases} I_1 = 20 - j4 & (1') \\ I_2 = 24 & (1') \end{cases}$$

$P_s = \mathrm{Re}(V_{rms} I_1^*) = 272 \times 20 = 5440\,W$ (1')

$P_{2\Omega} = |I_1|^2 \cdot R = 832W$ (1')

$P_{8\Omega} = |I_2|^2 \cdot R = 4608W$ (1')

No average power in inductence. $5440 = 832 + 4608$

$\Rightarrow$ Conservation of average powr. (1')