VE215 RC1

Xinyue Mao

UM-SJTU JI

October 9, 2024

Overview

Course Structure

- Goal: analyze the circuits, from simple to complex.
- Structure:
  1. Chap. 1-8: DC circuits (the circuits driven by constant current/voltage sources)
     A variety of analysis tools
     $\rightarrow$ introducing some new circuits components
     $\rightarrow$ analyze circuits with those complex components added
  2. Chap. 9-14: AC circuits (the circuits driven by alternating current/voltage sources)

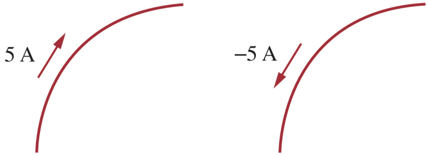
Overview

Current

$$\boxed{\begin{aligned} i &\triangleq \frac{\mathrm{d}q}{\mathrm{d}t} \\ Q &\triangleq \int_{t_0}^{t} i\mathrm{d}t \end{aligned}}$$

Reference direction of current

In solving problems, it does not matter which direction we initially assume. If we obtain a result of negative current, it indicates that the actual direction is opposite to that we have initially assumed.



(a) (b)

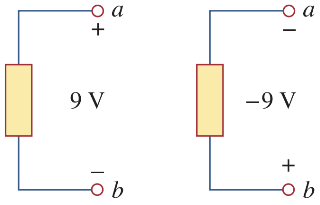
Voltage

$$\underbrace{v_{ab} = v_a - v_b}$$

$$v_{ab} \triangleq \frac{\mathrm{d}w}{\mathrm{d}q}$$

Reference direction of voltage

In solving problems, it does not matter how we assign the "+/-" signs to two terminals of a circuit element. The two representations below are equivalent.

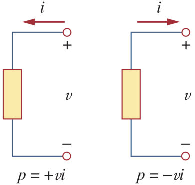


Power and Energy

$$\boxed{p = \frac{\mathrm{d}w}{\mathrm{d}t} = vi \qquad w = \int_{t_0}^{t} vidt}$$

Passive sign convention w.r.t. power:

- Currents enter through the positive terminal: $p = +vi$
- Currents enter through the negative terminal: $p = -vi$

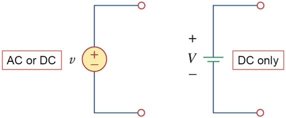


Power and energy consumption:

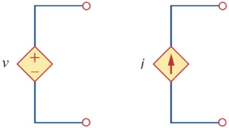
- $p > 0$, element consumes energy.
- $p < 0$, element generates energy.

Circuit Elements

- **Active elements**: can generate energy
  e.g., generators, batteries, operational amplifiers
  - **independent source**: the source whose quantity is uninfluenced by its "surroundings".



  - **dependent source**: source quantity is controlled by another voltage or current in the circuit.



- **Passive elements**: cannot generate energy,
  e.g., resistors, capacitors, inductors
  电容 C 电感 L

Overview

Nodes, Meshes and Loops

Branch: a single element, such as a voltage source or a resistor
Node: the point of connection between two or more branches
Loop: any closed path in a circuit

- **Mesh:** a loop that does not enclose any other loops, i.e., smallest loop
- **Independent loop:** a loop that contains at least one branch which is not a part of any other independent loop

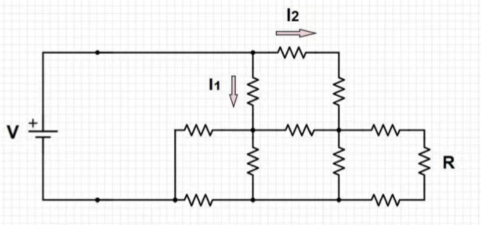
Fundamental theorem of network topology:

$$b \text{ (branches)} = l(\text{mesh}) + n \text{ (nodes)} - 1$$

Exercise 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?

2. Count the number of nodes, branches, meshes, loops in the following figure.



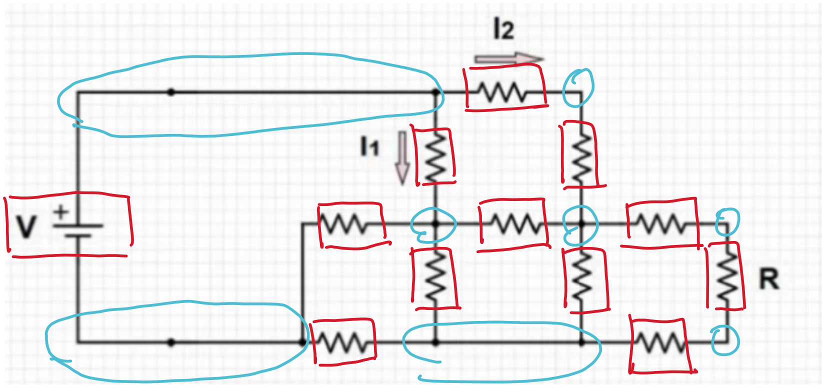
VE 215 24Fall RC1

Ex 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?

$b = l + n - 1 \Rightarrow 6 = 3 + n - 1 \Rightarrow n = 4$ 4 nodes.

2. Count the number of nodes, branches, meshes, ~~loops~~ in the following figure.



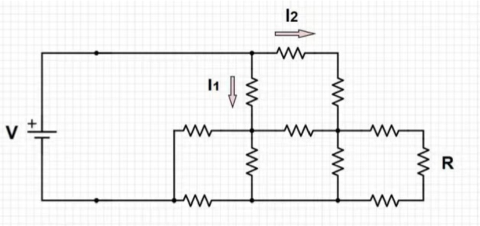
nodes : 8

branches : 12

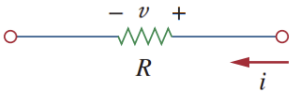
meshes : $12 = l + 8 - 1 \Rightarrow 5$

Exercise 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?
   Answer: 4
2. Count the number of nodes, branches, meshes, loops in the following figure.
   Answer: 8,12,5,21



Ohm's Law



Ohm's law:

$$V = IR \qquad R = \frac{v}{i}$$

Passive sign convention for Ohms's law:

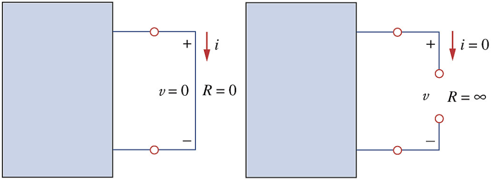
- i enters through the positive terminal: $v = iR$
- i enters through the negative terminal: $v = -iR$

Not all resistors obey Ohm's law!

A resistor that obeys Ohm's law is known as **a linear resistor**, i.e., a constant resistance.

Resistance with extreme values

1. Short circuit: a circuit element with resistance approaching zero.
2. Open circuit: a circuit element with resistance approaching infinity.



$v = 0$
i can be any value

$i = 0$
v can be any value

Conductance

$$G = \frac{1}{R} = \frac{i}{v},\ 1S = 1\mho = 1A/V$$

where **G** is the conductance, **S** (siemens) is the SI unit of conductance and $\mho$ is the reciprocal ohm.
some useful formula:

$$i = Gv, p = vi = i^2R = \frac{v^2}{R} = v^2G = \frac{i^2}{G}$$

Kirchhoff's Law

Kirchhoff's Law	Expression	Based on
KCL	$\sum i_k = 0$ for a node	Conservation of charge
KVL	$\sum v_k = 0$ for a mesh	Conservation of energy

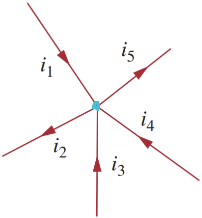


Figure: KCL

$i_1 - i_5 + i_4 + i_3 - i_2 = 0$

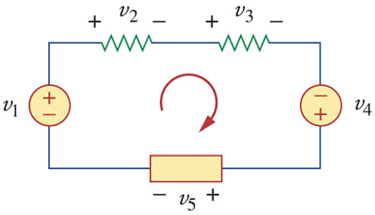


Figure: KVL

$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$

KCL

KCL: the algebraic sum of currents entering **a node (or a closed boundary)** is zero.
Steps of applying KCL:

1. Find out all branches connected to the node of interest.
2. Specify **reference** direction for current on each branch.
3. Find all $i_k (k = 1, 2, \cdots, n)$ (Ohm's law $i = \frac{v_a - v_b}{R}$ for linear resistors).
4. List the KCL equation $\sum_k i_k = 0$.

KVL

KVL: the algebraic sum of all voltages around **a closed path (or loop)** is zero.
Steps of applying KVL:

1. Select reference KVL direction (clockwise by convention).
2. Confirm/specify the +/- terminal of each branch.
3. Find $v_k (k = 1, 2, \cdots, n)$ for each branch.
4. List the KVL equation $\sum_k v_k = 0$. Mind that by passive sign convention, the sign in front of a certain term v_k is
   - "+" if the reference KVL direction enters through the positive terminal of the branch.
   - "-" if the reference KVL direction enters through the negative terminal of the branch.

Series connection and Parallel connection

R_{eq}: the **equivalent resistance**

1. **Series** connection:

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n$$

Principle of voltage division: $v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v$

2. **Parallel** connection:

$$G_{eq} = \frac{1}{R_{eq}} = G_1 + G_2 + \ldots + G_N = \sum_{n=1}^{N} G_n$$

Principle of voltage division: $i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i$

Wye-Delta Transformation

- Motivation: simplify the circuits for easier calculation.
- Two forms of special circuit connections:

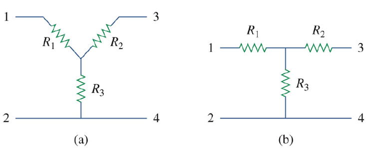


Figure: Wye (Y or T)

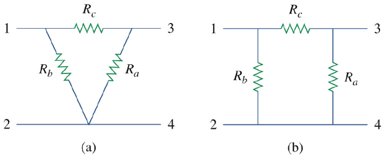


Figure: Delta (Δ or π)

- Goal: transform one type of connection into another.

Wye-Delta Transformation

$$
\begin{cases}
R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\[2ex]
R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\[2ex]
R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}
\end{cases}
$$

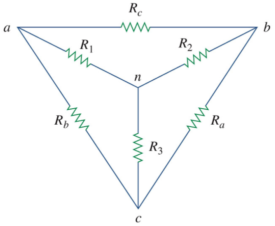


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: $\Delta - Y$

Intuition: parallel $\rightarrow$ series, resistance for each element decreases.

Wye-Delta Transformation

$$\left\{\begin{aligned} R_a &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \\ R_b &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ R_c &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \end{aligned}\right.$$

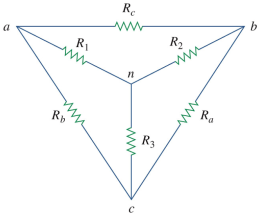


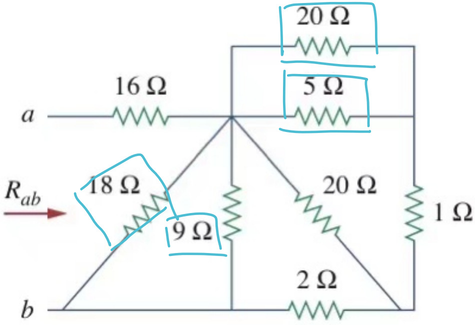
Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: Y-Δ

Intuition: series → parallel, resistance for each element increases.

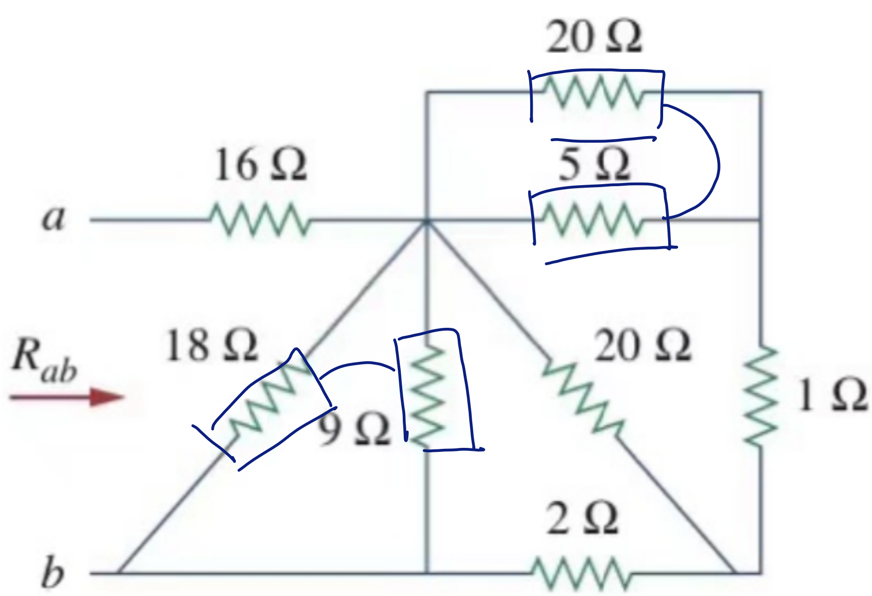
Exercise 2.

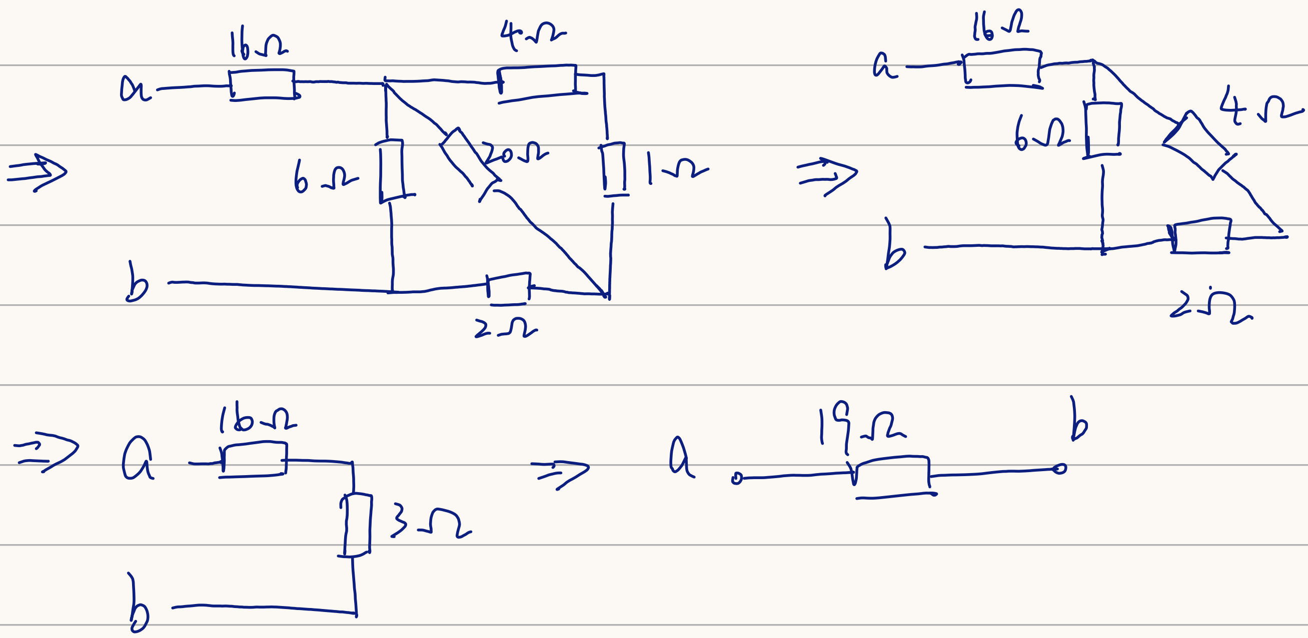
Calculate the equivalent resistance R_{ab} in the circuit



Ex2.

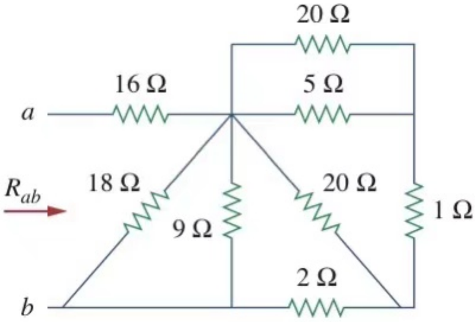
Calculate the equivalent resistance R_{ab} in the circuit





Exercise 2.

Calculate the equivalent resistance R_{ab} in the circuit

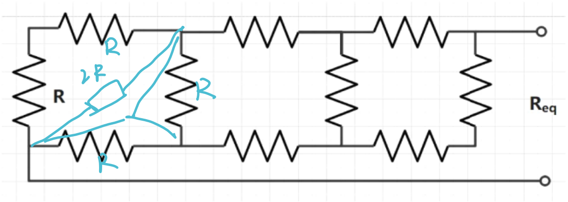


Answer: 19Ω

Exercise 3.

6. Suppose the resistance of all the resistors is **R**, what's the equivalent resistance $\mathbf{R_{eq}}$?

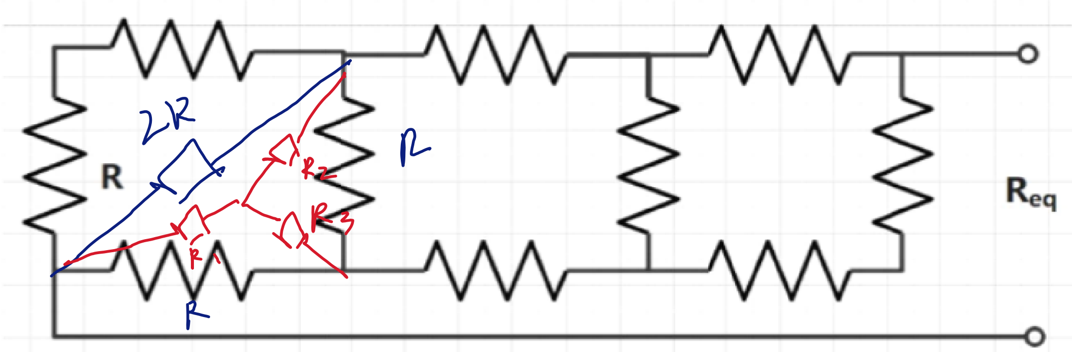
________________________________ (4 marks)



Ex 3.

6. Suppose the resistance of all the resistors is **R**, what's the equivalent resistance $\mathbf{R_{eq}}$?

_______________________________________ (4 marks)

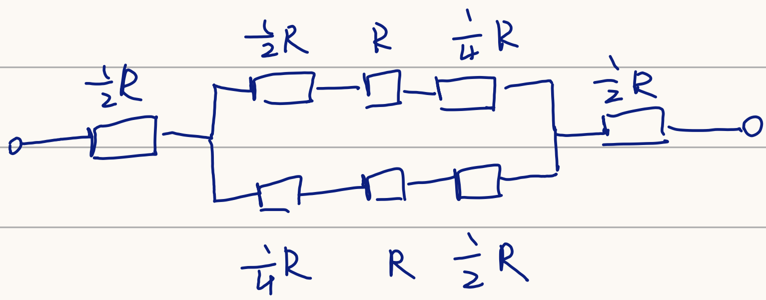


$$R_1 = \frac{1 \times 2}{1+2+1} = \frac{1}{2} R$$

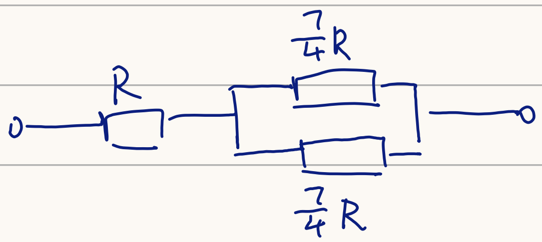
$$R_2 = \frac{1 \times 2}{1+2+1} = \frac{1}{2} R$$

$$R_3 = \frac{1 \times 1}{1+2+1} = \frac{1}{4} R$$

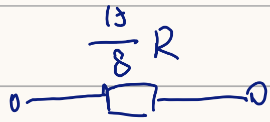
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$

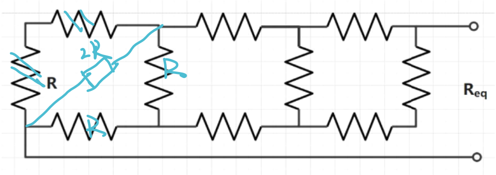


$R_{eq} = \frac{13}{8} R$

Exercise 3.

6. Suppose the resistance of all the resistors is **R**, what's the equivalent resistance $\mathbf{R_{eq}}$?

____________________________________ (4 marks)



Answer: $0.5 + \dfrac{0.5 + 1 + 0.25}{2} + 0.5 = \dfrac{15}{8}R$

Overview

Methods of Analysis

1. Nodal Analysis:
   1. Select a reference node (ground)
   2. Apply KCL
   3. Solve the equations
2. Mesh Analysis:
   1. Mark the current of all the meshes
   2. Apply KVL
   3. Solve the equations

Analysis by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all ~~independent~~ current sources directly connected to node k, with currents entering the node treated as positive

For Nodal Analysis

(only current source in circuit)

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

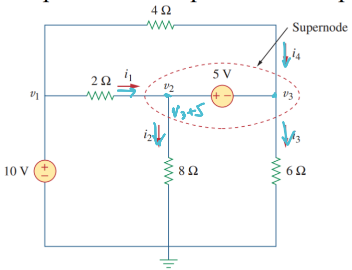
v_k = Sum taken clockwise of all ~~independent~~ voltage sources in mesh k, with voltage rise treated as positive

For Mesh Analysis

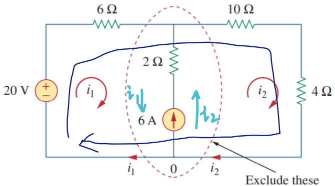
(only voltage source in circuit)

Supernode & Supermesh

- Supernode & Supermesh – simplify the equation



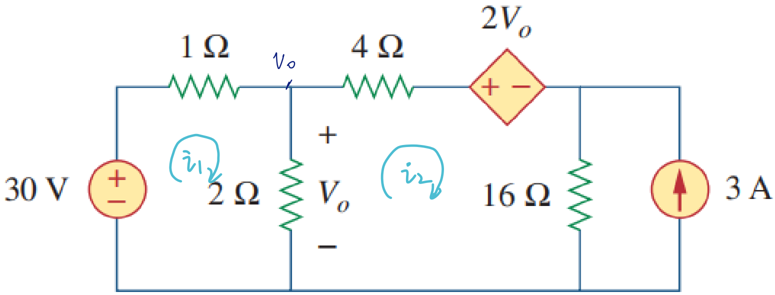
$$i_1 - i_2 - i_3 + i_4 = 0$$



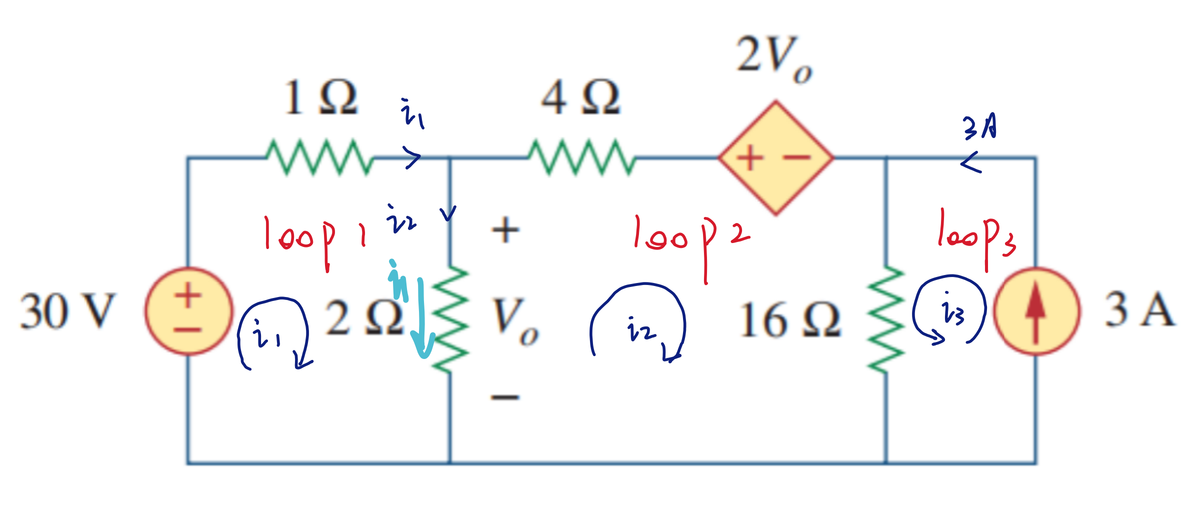
(a)

$$20 - 6i_1 - 14i_2 = 0$$

Exercise 4.



Ex 4.



$$\begin{cases} -30 + i_1 \cdot 1 + (i_1 - i_2) \cdot 2 = 0 & ① \\ -V_o + 4 i_2 + 2V_o + 16(i_2 + i_3) = 0 & ② \\ i_3 = 3 & ③ \\ V_o = (i_1 - i_2) \cdot 2 & ④ \end{cases}$$

loop 1

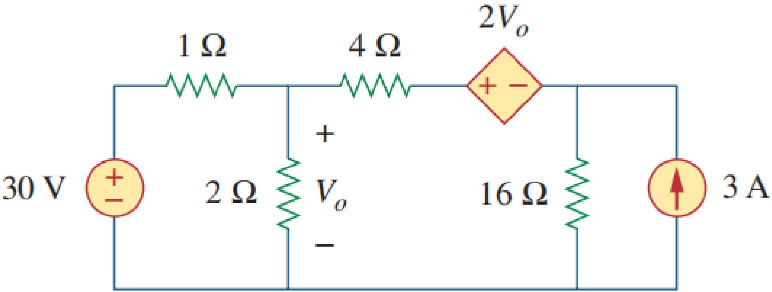
loop 2

loop 3

the 2Ω resistor.

$$\Rightarrow \begin{cases} i_1 = \frac{222}{29} \\ i_2 = -\frac{102}{29} \end{cases} \Rightarrow V_o = \frac{648}{29} \text{ V}$$

Exercise 4.



Answer: $\frac{648}{29} V \approx 22.34 V$

References

1. 2024Fall VE215 slides
2. 2023Fall RC1,RC2, Chongye Yang

Thank you!