上 海 交 通 大 学 试 卷

（ 2021~ 2021~1 Academic Year/Fall Semester ）

Class No.____________________ Name in English or Pinyin: ______________________

Student ID No.________________ Name in Hanzi (if applicable): ______________________

VE215 Introduction to Circuits

Final Exam

8:00 – 9:40 am 13th December

The exam paper has **16 pages** in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

Signature:_____________

Please enter grades here:

Exercises No. 题号		Points 得分	Grader's Signature 流水批阅人签名
1	1-1		
	1-2		
	1-3		
	1-4		
	1-5		
	1-6		
2			
3			
4			
5			
Total 总分			

Instructions:

- **Chapter 9-14** are covered in this exam.
- Closed book, scientific calculator and one page of summary in A4 size, double sided is allowed.
- There are **16 pages** in total.
- There are **5 questions** in total.
- Please write your **partial steps** and that will be counted.
- Please manage your time properly. If you encounter some questions and feel hard, please move on and go back after finishing all the rest.
- **No tolerance to cheating!** Any intentional violations of the SJTU exam rule of a vile nature will be assigned an "F" as course grade.

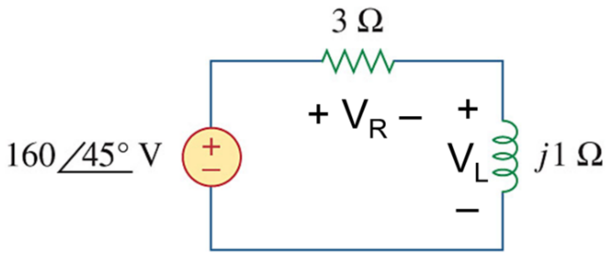
Fingers crossed!

Exam questions A type:

Q1. Please answer Questions 1-1 to 1-6. [Total 34 points]

1-1. Input voltage (V_i) is $160\angle 45°$. Please choose **one right** answer. [5 points]

Your answer: (a)(d)(e). Due to an error both (a) and (d) are right answers. And, we gave full marks for those who chose (e) by thinking that there is no one right answer.



(a) V_i leads V_R.

(b) V_i lags V_R.

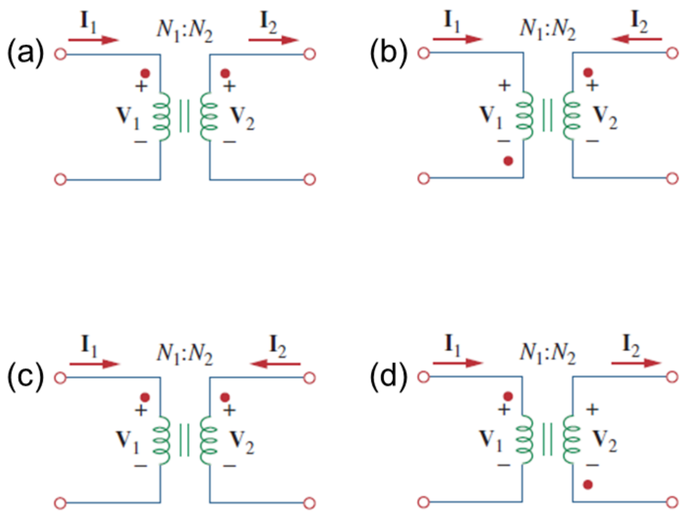
(c) V_i leads V_L.

(d) V_i lags V_L.

(e) None of the above.

1-2. Below show polarities of turns ratio n with respect to voltage and current in an ideal transformer. Choose **one wrong** description. [5 points]

Your answer: (d)



(a) $\frac{V_2}{V_1} = \frac{N_2}{N_1}, \frac{I_2}{I_1} = \frac{N_1}{N_2}$

(b) $\frac{V_2}{V_1} = -\frac{N_2}{N_1}, \frac{I_2}{I_1} = \frac{N_1}{N_2}$

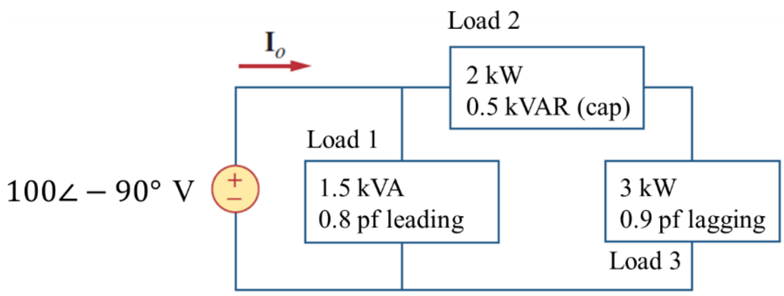
(c) $\frac{V_2}{V_1} = \frac{N_2}{N_1}, \frac{I_2}{I_1} = -\frac{N_1}{N_2}$

(d) $\frac{V_2}{V_1} = -\frac{N_2}{N_1}, \frac{I_2}{I_1} = \frac{N_1}{N_2}$

(e) None of the above

1-3. Choose **one wrong** description about the circuit below. All values are in **rms**. Please allow $\pm$**10%** calculation differences. [6 points]

Your answer: (c)



(a) Magnitude of I_o is 65.04.

(b) Angle of I_o is −88.02°.

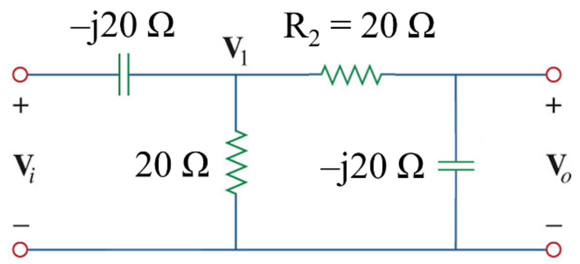
(c) Current at the Load 1 is $18.75\angle 53.13°$.

(d) Current at the Load 2 is $50.9\angle -100.79°$.

(e) None of the above.

1-4. Please choose **one wrong** description about the circuit below. Please allow $\pm$**10%** calculation differences. [6 points]

Your answer: (d)



(a) Phase of voltage V_1 leads V_i.

(b) Phase of output voltage V_o lags V_1.

(c) The total phase shift in this circuit is 0 degree.

(d) If we change the value of R_2 from 20 to 10 Ω, V_o leads V_i approximately by +18.4°.

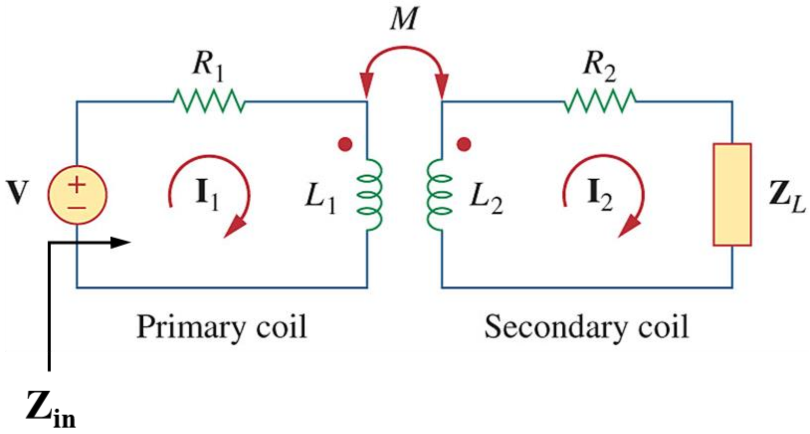
(e) None of the above.

1-5. When connected to a 240 V_{rms}, 60 Hz power line, a load has apparent power of 6 kVA and a lagging pf of 0.750. Find the value of capacitance necessary to raise the pf to 0.980. [6 points].

Your answer: 1.41×10^{-4} [F]

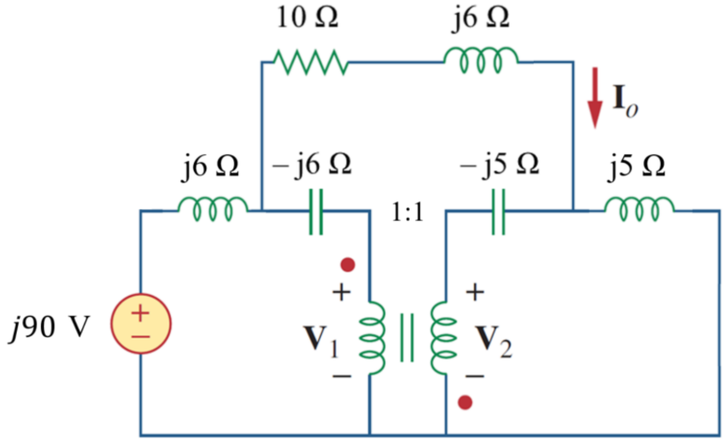
1-6. Find the input impedance Z_{in} in the linear transformer circuit below. [6 points]

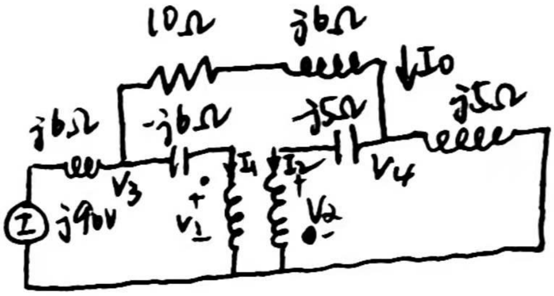
Your answer: $R_1 + jwL_1 + \dfrac{w^2M^2}{R_2 + jwL_2 + Z_L}$



***Q2-Q4. Please write phasor forms for all answers.**

Q2. Find $\mathbf{I_o}$ of the circuit below. The transformer is ideal. [Total 12 points]





$$\begin{cases} V_1 = -V_2 & 2' \\ \dfrac{V_3 - j90}{j6} + \dfrac{V_3 - V_4}{10+j6} + \dfrac{V_3 - V_1}{-j6} = 0 & 2' \\ \dfrac{V_4 - V_2}{-j5} + \dfrac{V_4 - V_3}{10+j6} + \dfrac{V_4}{j5} = 0 & 2' \end{cases}$$

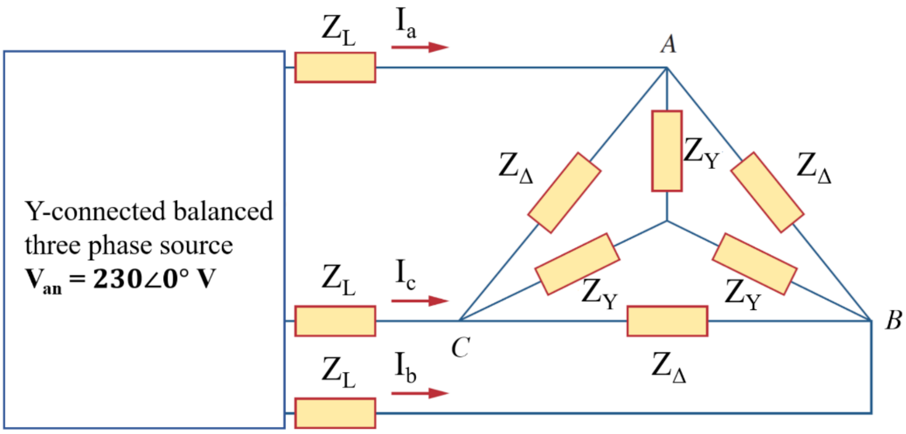
$$\frac{V_1 - V_3}{-j6} + I_1 = 0 \qquad 1'$$

$$\frac{V_2 - V_4}{-j5} + I_2 = 0 \qquad 1'$$

$$V_3 - V_4 = 900 + j540 \qquad 2'$$

$$I_o = \frac{V_3 - V_4}{10+j6} = 90 = 90\angle 0^\circ \qquad 2'$$

Q3. Please find (1) Line current $\mathbf{I_a}$, $\mathbf{I_c}$, and $\mathbf{I_b}$ [10 points] and (2) **total complex power** at the loads, (combined $Z_\Delta + Z_Y$). [6 points] Take $Z_L = 1 + j0.6\ \Omega$, $Z_\Delta = 6 - j9\ \Omega$, $Z_Y = 3 + j4\ \Omega$. All values are in **rms**. Assume **acb sequence**. [Total 16 points]



Summary of phase and line V and I for balanced three-phase systems (**abc sequence**).

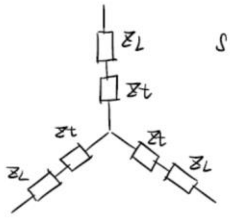
Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

(1) Δ Load and Y load are in parallel.

Apply Δ-Y transform on Δ load: $Z_{Ye} = \frac{Z_\Delta}{3} = 2-3j$ 2'

$Z_t = Z_{Ye} \| Z_Y = (2-3j) \| (3+4j) = \frac{89}{26} - \frac{23}{26}j$ 2' $(3.423-0.885j \,/\, 3.5355\angle-14.49° \,/\, \frac{5\sqrt{2}}{2}\angle-14.49°)$

we get



since acb sequence: $V_{an} = 230\angle0°$, $V_{bn} = 230\angle120°$, $V_{cn} = 230\angle-120°$

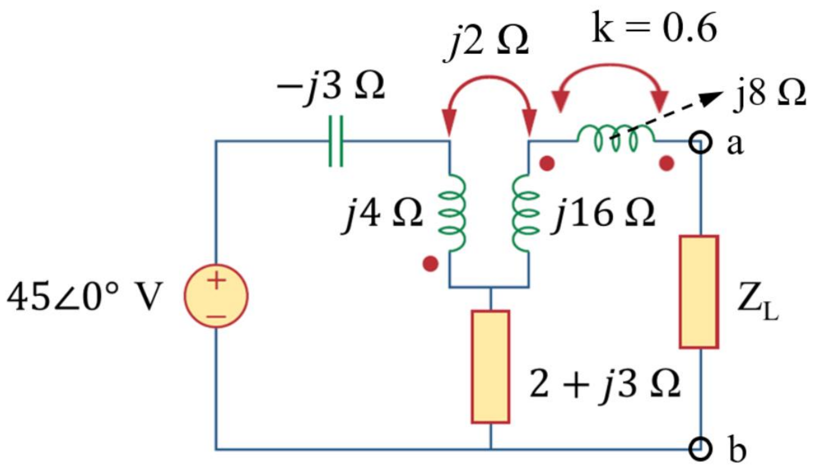
$I_a = \frac{V_{an}}{Z_L + Z_t} = 51.7856 + 3.3323j$ $(51.8927\angle3.6818°)$ 2'

$I_b = -28.7786 + 43.1815j$ $(51.8927\angle123.6818°)$ 2'

$I_c = -23.0069 - 46.5138j$ $(51.8927\angle-116.3182°)$ 2'

(2) $S = 3|I_p|^2 Z_t = 27653.49675 - 7146.40927j$ $(28561.9861\angle-14.4898°)$ 6'

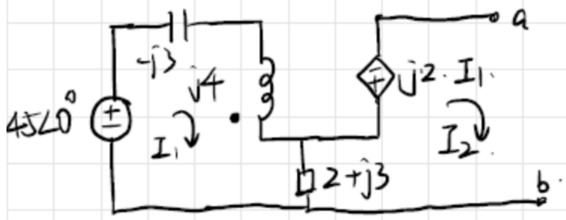
Q4. Below is the circuit with the magnetically coupled inductors. (1) Find the **Thevenin Equivalent** circuit between terminals a and b [10 points], and (2) $\mathbf{Z_L}$ to transfer maximum average power. [2 points] (3) If $Z_L = 6 + j4$, what is the **complex power** at the load Z_L? Please draw the **power triangle** and define **pf** at the $\mathbf{Z_L}$. [8 points] Please note that the values are **NOT in rms**. [Total 20 points]



Q4:

1) $j\omega M = j\cdot\sqrt{\omega L_1 \omega L_2} = j\cdot 0.6\sqrt{8\times16} = \underline{j\ 6.79}$ 1'

To find V_{Th}, make terminal a-b open circuit.

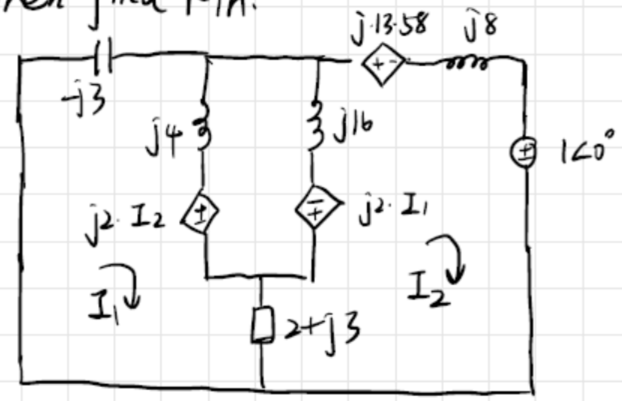


$\underline{I_1\cdot(-j3+j4+2+j3)=45\angle0^\circ}$ 1'

$I_1\ (2+j4)=45\angle0^\circ \Rightarrow \underline{I_1 = 10\angle-63^\circ}$ 1'

$\Rightarrow V_{Th} = (2+j3)I_1 - j2I_1 = (2+j)I_1 = \underline{22.5\angle-36.9^\circ}$ 1'

Then find R_{Th}.



1'

From mesh analysis:

$$\begin{cases}(-j3+j4)\cdot I_1 + j2\cdot I_2 + (I_1-I_2)(2+j3) = 0 & ① \\ (2+j3)(I_2-I_1) + j2I_1 + j16\cdot I_2 + j13.58 I_2 + I_2 j8 + 1 = 0 & ② \end{cases}$$

1'

From ① $I_1 = \frac{2+j}{2+j4} I_2$. 1'

From ② $(2+j40.58)I_2 - (2+j)I_1 + 1 = 0$. 1'

$\Rightarrow Z_{th} = \frac{1}{-I_2} = \underline{40.78\angle 88.7^\circ}$ 2'

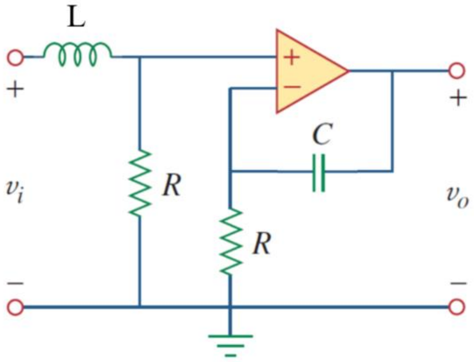
2) $Z_L = \underline{Z_{th}^*} = \underline{40.78\angle -88.7^\circ}$ 1' 1'

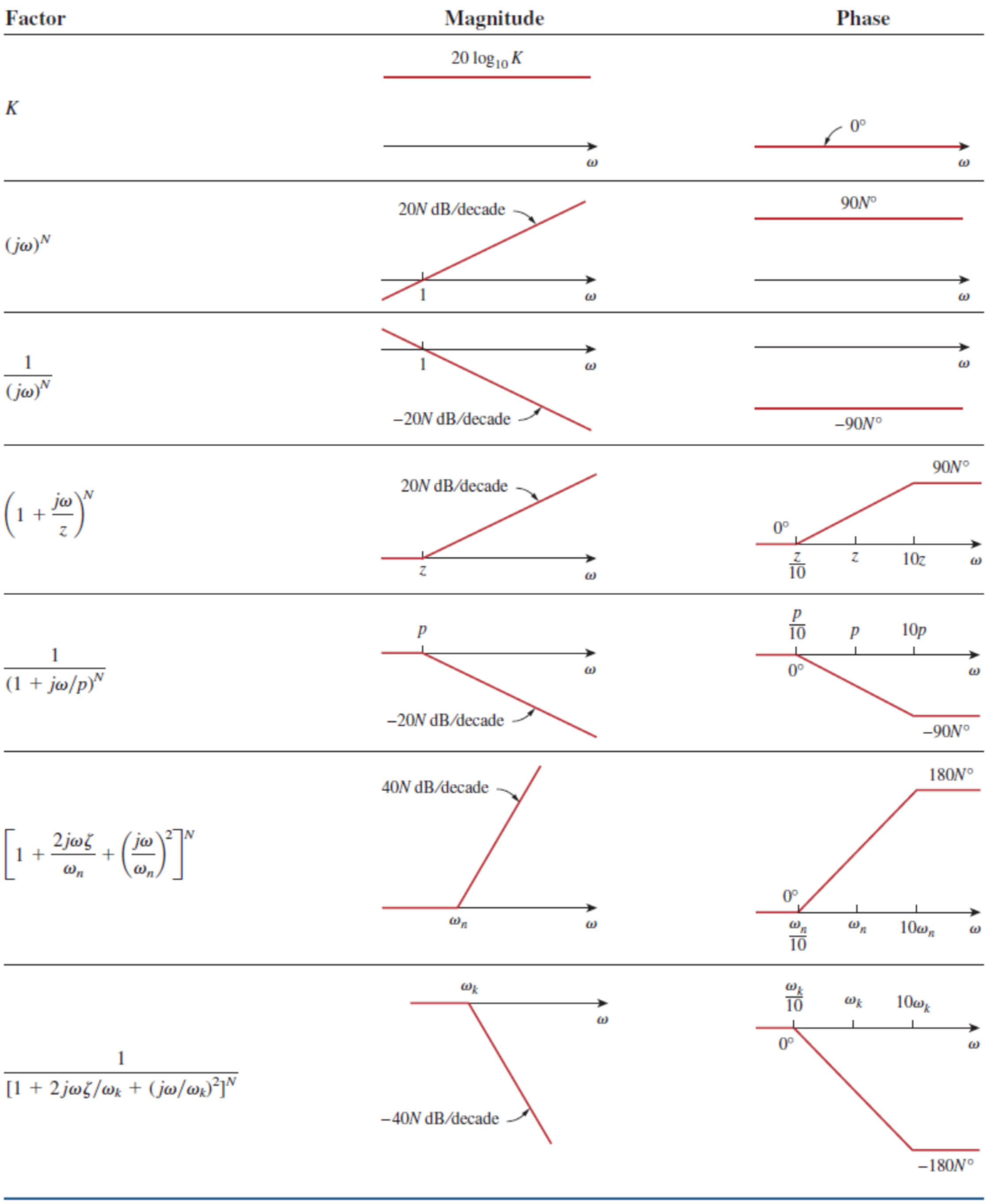
3) $S = \underline{\frac{1}{2}I^2 Z} = \frac{1}{2}\left|\frac{22.5}{6+j4+40.78\angle 88.7}\right|^2 (6+j4)$. 1'

$= 0.5 \cdot \frac{22.5^2}{45.3^2}(6+j4) = 0.88\angle 33.7^\circ$ 3'

$pf = 0.83$. 4'

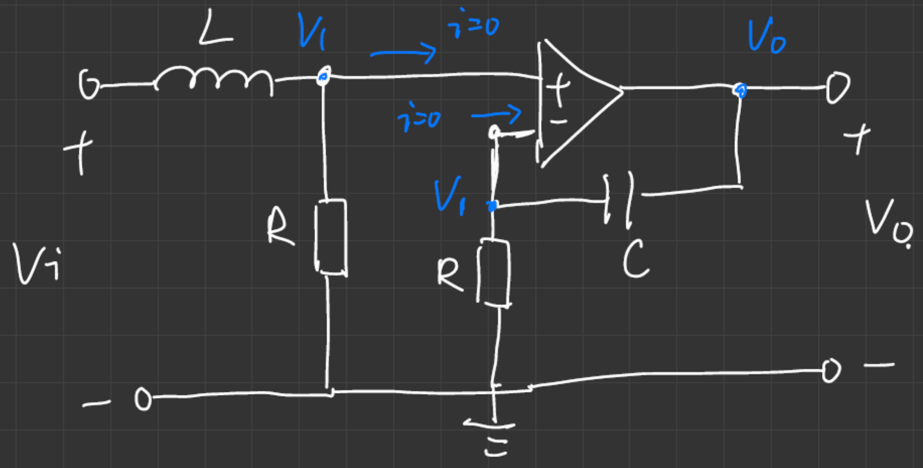
Q5. Please draw a **Bode plot** of the transfer function $H(\omega) = V_o/V_i$ [Total 18 points]





Question 5

13'



$$\begin{cases} \dfrac{V_1 - V_i}{j\omega L} + \dfrac{V_1}{R} = 0 \\[2ex] \dfrac{V_1}{R} + \dfrac{V_1 - V_o}{\frac{1}{j\omega C}} = 0 \end{cases} \Rightarrow \begin{aligned} V_i &= V_1 + \frac{j\omega L}{R} V_1 . \\ V_o &= V_1 + \frac{V_1}{j\omega CR} . \end{aligned}$$

6'

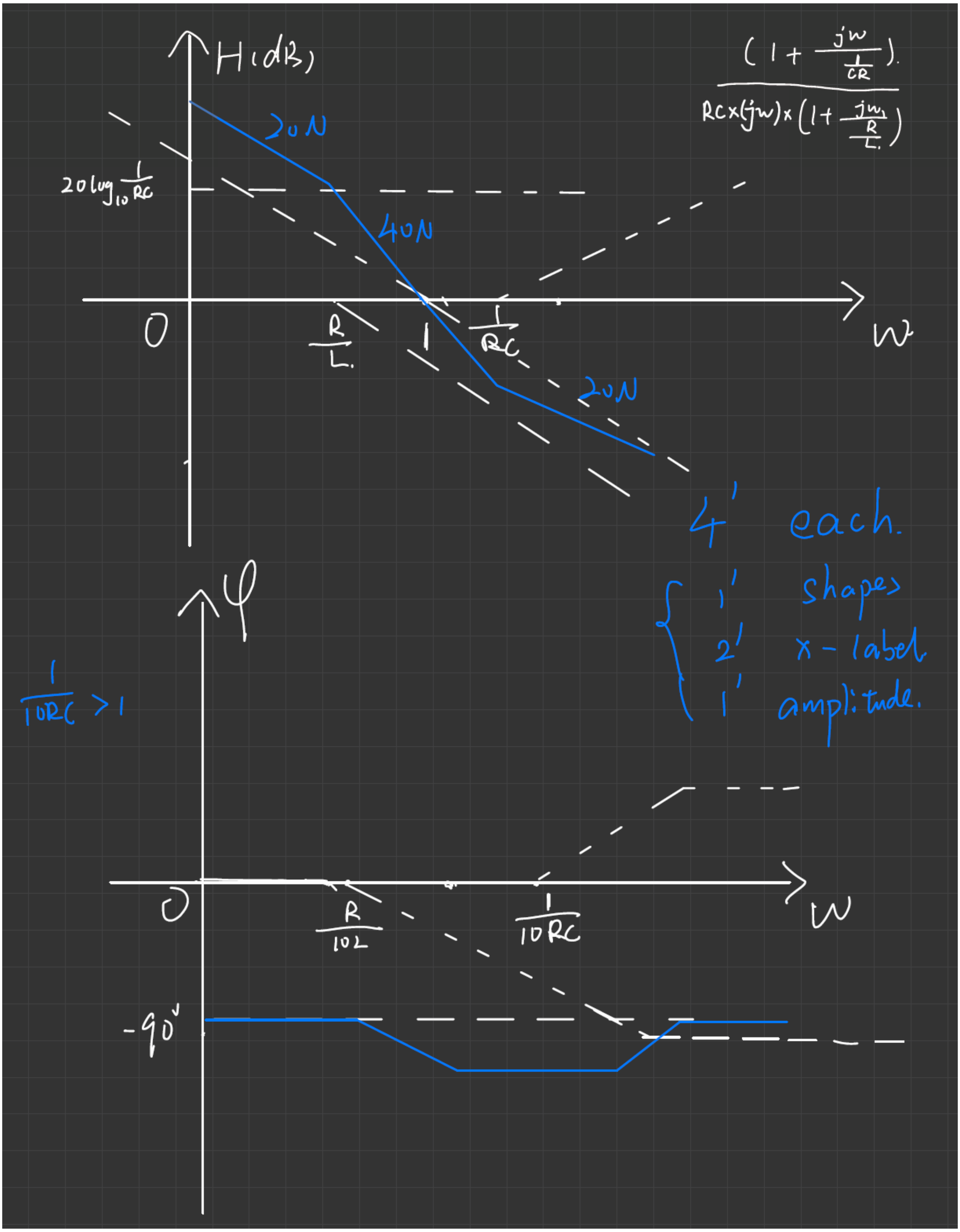
$$H(\omega) = \frac{V_o}{V_i} = \frac{V_1 + \frac{V_1}{j\omega CR}}{V_1 + \frac{j\omega L}{R} V_1} = \frac{1 + \frac{1}{j\omega CR}}{1 + \frac{j\omega L}{R}}$$

4'

$$= \frac{j\omega CR + 1}{j\omega CR + (j\omega)^2 CL} = \frac{\left(1 + \frac{j\omega}{\frac{1}{CR}}\right)}{RC \times (j\omega) \times \left(1 + \frac{j\omega}{\frac{R}{L}}\right)}$$

$$H_{dB}(\omega) = 20\log_{10} \frac{1}{RC} - 20\log_{10} |j\omega| - 20\log_{10} \left|1 + \frac{j\omega}{\frac{R}{L}}\right| + 20\log_{10} \left|1 + \frac{j\omega}{\frac{1}{CR}}\right|$$

$$\phi(\omega) = \arctan CR - 90^\circ - \arctan \frac{L}{R}$$


H(dB)
$\frac{\left(1+\frac{j\omega}{\frac{1}{CR}}\right)}{RC\times(j\omega)\times\left(1+\frac{j\omega}{\frac{R}{L}}\right)}$
20N
$20\log_{10}\frac{1}{RC}$
40N
0
$\frac{R}{L}$
1
$\frac{1}{RC}$
ω
20N
4' each.
1' shapes
2' x-label
1' amplitude.
φ
$\frac{1}{10RC}>1$
0
$\frac{R}{10L}$
$\frac{1}{10RC}$
ω
-90°

Mathematics Formula Sheet:

- **Phasor and Complex number**

$$j^2 = -1$$

$$\frac{a + jb}{c + jd} = \frac{(a + jb)(c - jd)}{(c + jd)(c - jd)} = \frac{(ac + bd) + j\,(bc - ad)}{c^2 + d^2}$$

$$A + jB = \sqrt{A^2 + B^2}\angle\left[\arctan\frac{B}{A}\right] = \sqrt{A^2 + B^2}\, e^{j\arctan\frac{B}{A}}$$

$$\frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2), \quad A\angle\theta_1 \times B\angle\theta_2 = AB\angle(\theta_1 + \theta_2)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Introduction to CASIO fx-991 calculator:

***Complex number calculation**: You may press "**menu**" button and select the "complex number" to do the complex number calculation.*

*For the unit imaginary "j", the button "**ENG**" stands for that.*

*For the phasor angle "∠", please press "**SHIFT**", and then press "**ENG**" button.*

Mention: It's invalid to input $\angle\theta$, *like* ∠60*! You must input a number before that, such as "1∠60" or "2∠60"*

Complex number and Phasor Transformation: *Please press "**SHIFT**" and then press "**menu**", you can select the "Complex" and then choose its form (" a+bi " or "r∠θ ")*

Angle unit: *Please press "**SHIFT**" and then press "**menu**", you can select the "angle unit" and then choose the unit of angle ("degrees" or "rad")*