上 海 交 通 大 学 试 卷

（ 2024~ 2025~1 Academic Year/Fall Semester ）

Class No.______________________ Name in English or Pinyin: ______________________

Student ID No.________________ Name in Hanzi(if applicable): ______________________

ECE2150J and Intro to Circuits

Mid-term Exam

12th November 10:00 – 11:40 am

The exam paper has 13 pages in total.

You are to abide by the University of Michigan-Shanghai Jiao Tong University Joint Institute (UM-SJTU JI) honor code. Please sign below to signify that you have kept the honor code pledge.

THE UM-SJTU JI HONOR CODE

I accept the letter and spirit of the honor code:

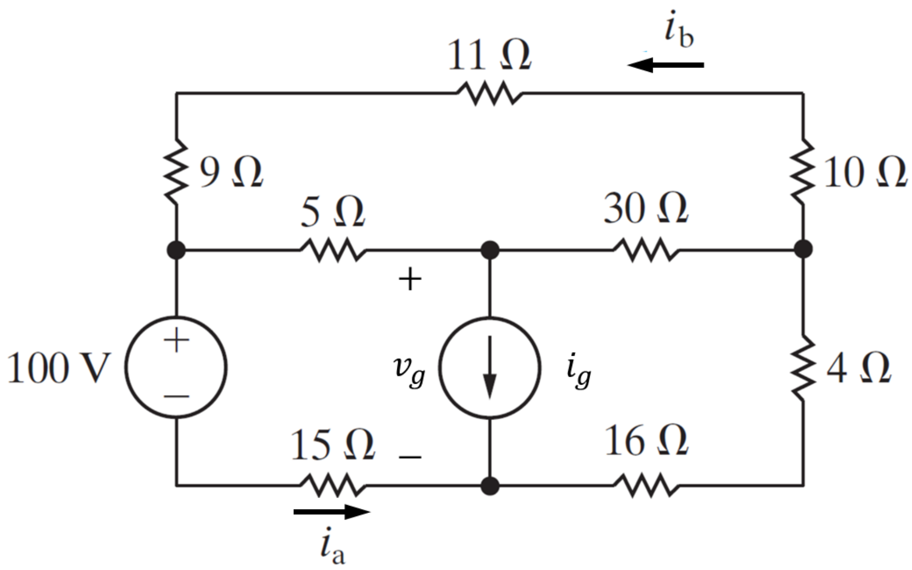
I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code by myself or others.

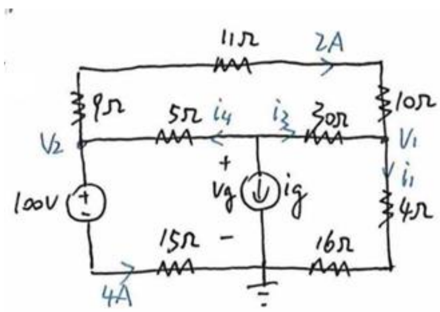
Signature:____________

Please enter grades here:

Exercises No. 题号	Points 得分	Grader's Signature 流水批阅人签名
1		
2		
3		
4		
5		
6		
Total 总分		

1. The currents i_a and i_b in the circuit are 4 A and -2 A respectively. Find i_g and v_g. [8 points]





$V_2 = 100 + 4\times 15 = 160\,V$ 1'

$V_1 = 160 - (9+11+10)\times 2 = 100\,V$ 1'

$i_1 = \dfrac{V_1}{4+16} = \dfrac{100}{20} = 5A$ 1'

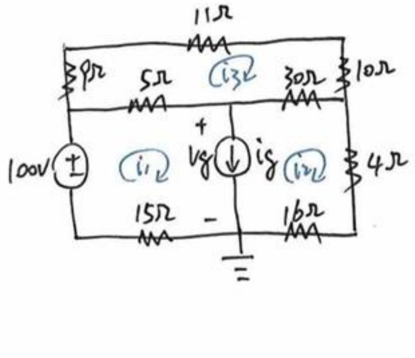
$i_3 = i_1 - 2 = 5 - 2 = 3A$ 1'

$V_g = V_1 + 30 i_3 = 100 + 30\times 3 = 190\,V$ 2'

$i_4 = \dfrac{V_g - V_2}{5} = \dfrac{190-160}{5} = 6A$

$i_g = -i_4 - i_3 = -6 - 3 = -9A$ 2'

or mesh analysis:

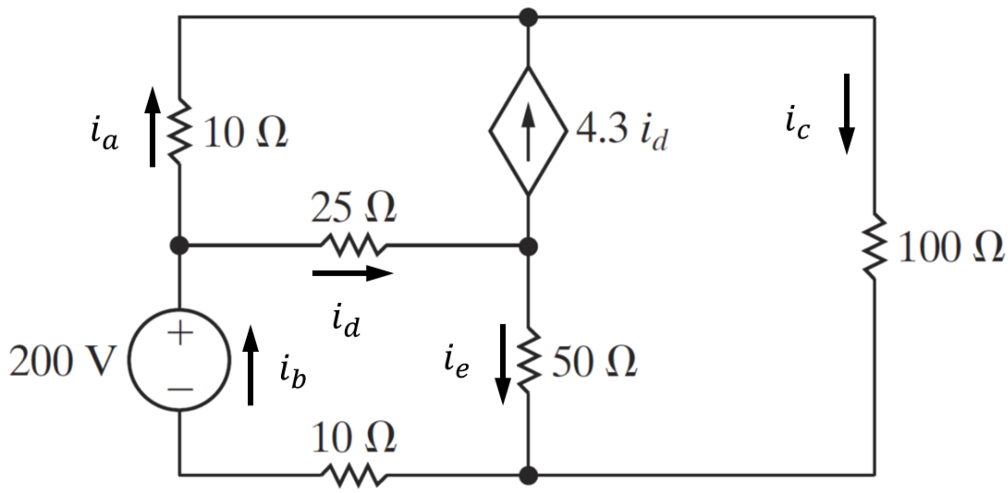


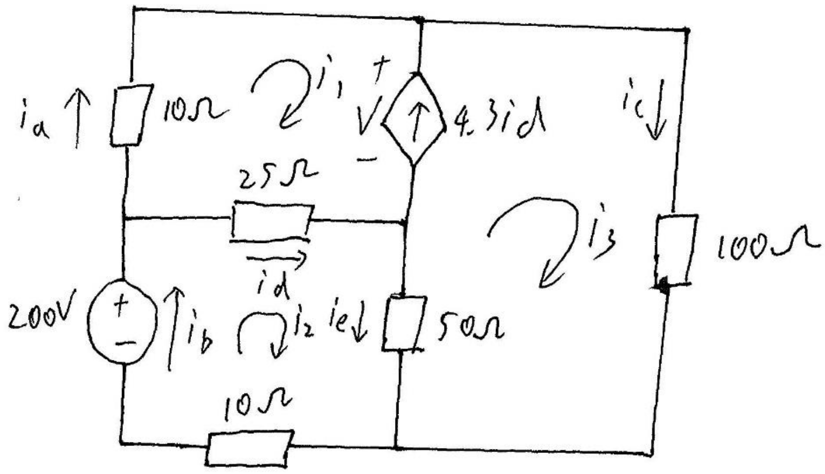
$$\begin{cases} i_1 = -4A,\ i_3 = 2A \\ 15i_1 + 5(i_1 - i_3) + V_g = 100 \quad 1' \\ (4+16)i_2 + 30(i_2 - i_3) = V_g \quad 1' \\ (9+11+10)\cdot i_3 + 30(i_3 - i_2) + 5(i_3 - i_1) = 0 \quad 1' \\ i_g = i_1 - i_2 \quad 1' \end{cases}$$

$V_g = 190\,V$ 2' $i_g = -9A$ 2'

2. (a) Use the mesh-current method to find the branch currents in $i_a \sim i_e$ in the circuit below. [8 points]

(b) Check your solution by showing that the total power generated in the circuit equals the total power consumed. [8 points]





(a) $i_a = i_1 \quad i_b = i_2 \quad i_c = i_3 \quad i_e = i_2 - i_3 \quad i_d = i_2 - i_1$ (1pts)

Mesh analysis

①
$$\begin{cases} 200 - 25(i_2 - i_1) - 50(i_2 - i_3) - 10 i_2 = 0 \\ i_3 - i_1 = 4.3 i_d \\ i_d = i_2 - i_1 \\ 200 - 10 \cdot i_1 - 100 \cdot i_3 - i_2 \cdot 10 = 0 \text{ (outer loop)} \end{cases}$$ (4pt)

②
$$\begin{cases} 200 - 25(i_2 - i_1) - 50(i_2 - i_3) - 10 i_2 = 0 \\ i_3 - i_1 = 4.3 i_d \\ i_d = i_2 - i_1 \\ 10 \cdot i_1 + 100 \cdot i_3 - (i_2 - i_3) \cdot 50 - 25(i_2 - i_1) = 0 \text{ (super mesh)} \end{cases}$$

$\Rightarrow$ ①
$$\begin{cases} 25 i_1 - 85 i_2 + 50 i_3 = -200 \\ 3.3 i_1 - 4.3 i_2 + i_3 = 0 \\ 10 i_1 + 10 i_2 + 100 i_3 = 200 \end{cases} \Rightarrow \quad i_1 = 5.7A \quad i_2 = 4.6A \quad i_3 = 0.97A$$ (1pt)

②
$$\begin{cases} 25 i_1 - 85 i_2 + 50 i_3 = -200 \\ 3.3 i_1 - 4.3 i_2 + i_3 = 0 \\ 35 i_1 - 75 i_2 + 150 i_3 = 0. \end{cases}$$

$\Rightarrow i_a = 5.7A \quad i_b = 4.6A \quad i_c = 0.97A \quad i_d = -1.1A \quad i_e = 3.63A$ (2pts)

(b) Power develop $\Rightarrow$ Voltage source, Current soure

Power Dissipate $\Rightarrow$ All resistor

$$\sum P_{Dis} = \sum I^2 R = (5.7)^2 10 + (1.1)^2 25 + (0.97)^2 100 + (4.6)^2 \cdot 10 + (3.63)^2 \cdot 50 = \underline{\underline{1319.685\,W}}$$

(2pts with formula)

(2pts)

$P_{voltage} = -(200 \cdot i_b) = -920\,W$ (2pts)

$10 i_a + V_{Current} + 25(1.1) = 0 \Rightarrow V_{current} = 84.5\,V$ (1pt)

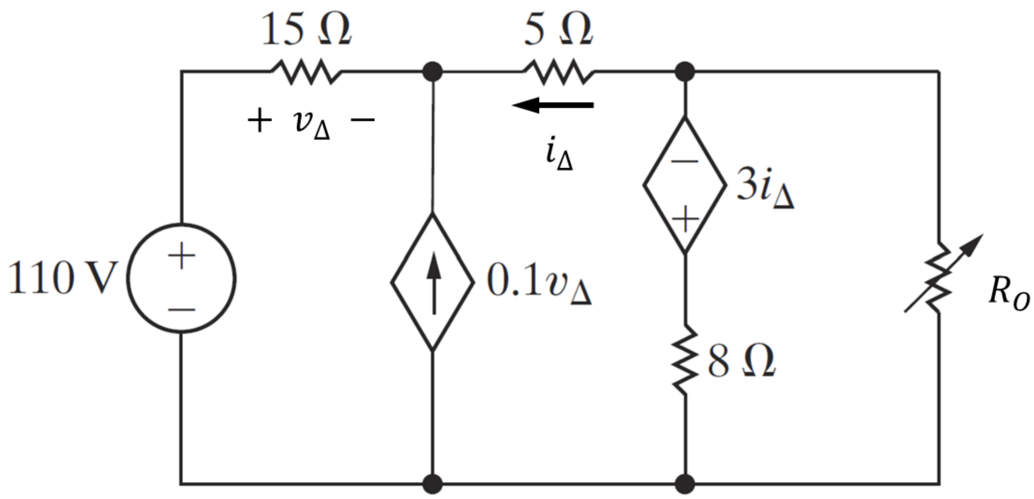
$P_{current} = -VI = -399.685\,W$ (2pts)

$P_{Dev} = P_{voltage} + P_{current} = 1319.685\,W = P_{Dis}$

3. The variable resistor (R_O) in the circuit is adjusted until it absorbs maximum power from the circuit.

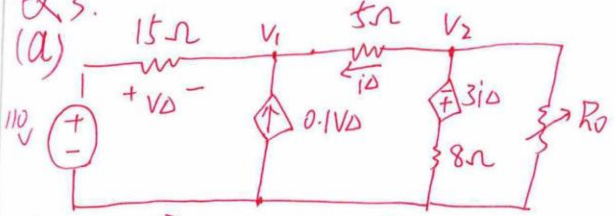
(a) Find the value of R_O. [12 points]

(b) Find the maximum power delivered to R_O. [4 points]



解3.

(a)



PS: 第一问若只求了 R_o
结果对: $R_o = 4\Omega$ 直接2', 第
Vth
不对: 给 R_o 过程分
看第二问是否求了
相应的在第二问给分

If one gets full mark in (a) we do not offer credit in (b) for Vth calculation.

full mark is offered for each of the sub-question as long as the result is correct

Suppose R_o is open: $R_o \to \infty$

$$\frac{V_\Delta}{15} + 0.1V_\Delta + i_\Delta = 0 \qquad 1'$$

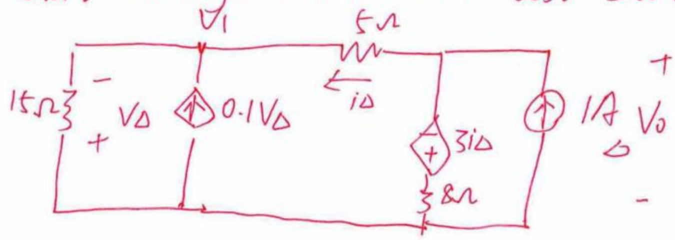
$$\begin{cases} V_\Delta = 110 - V_1 \\ 5i_\Delta = V_2 - V_1 \\ \dfrac{V_2 - V_1}{5} + \dfrac{V_2 + 3i_\Delta}{8} = 0 \qquad 1' \end{cases}$$

$$\Rightarrow \begin{cases} V_1 = 80V \\ V_2 = 55V = V_{Th} \end{cases} \qquad 3'$$

Now, calculate R_{th}

Recall that P_{max} is achieved when $R_o = R_{th}$. 1'

Short voltage source, add test source $i = 1A$



$$\begin{cases} 1 = i_\Delta + \dfrac{V_o + 3i_\Delta}{8} \qquad 1' \\ \dfrac{V_\Delta}{15} + 0.1V_\Delta + i_\Delta = 0 \\ V_o - 5i_\Delta = -V_\Delta \qquad 1' \\ V_\Delta = -V_1 \end{cases}$$

$V_o = 4V \quad R_{th} = \frac{V_o}{1} = 4\Omega$

$\Rightarrow R_o = R_{th} = 4\Omega \qquad 4'$

(b)

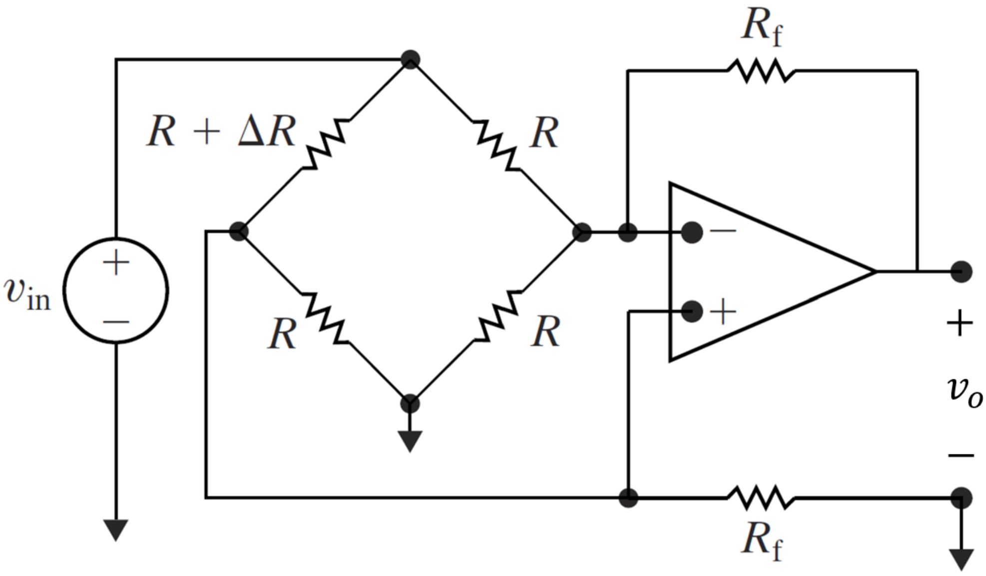
$$P_{max} = \frac{(27.5V)^2}{R_o} \Rightarrow \frac{V_{Th}^2}{4R_o} = 189.0625\,W.$$

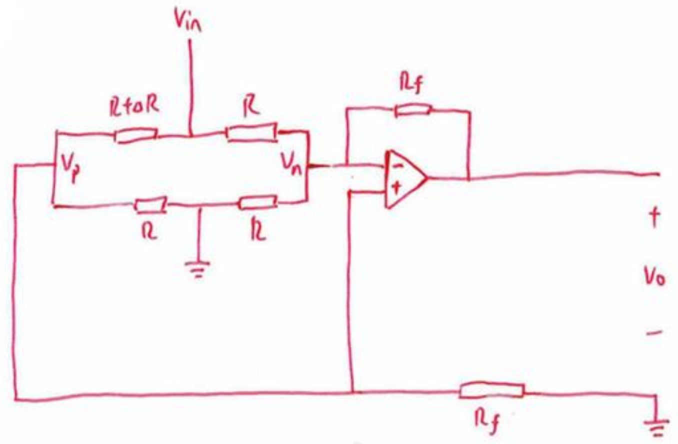
1' 3'

4. Show that if $\Delta R \ll R$, the output voltage of the op amp is approximately

$$v_o \approx \frac{R_f\left(R+R_f\right)}{R^2\left(R+2R_f\right)}(-\Delta R)v_{in}$$

for the circuit below. [20 points]





$$\frac{V_p}{R_f}+\frac{V_p}{R}+\frac{V_p-V_{in}}{R+\Delta R}=0 \quad (KCL) \quad \cdots\ 3'$$

$$V_p\left(\frac{1}{R_f}+\frac{1}{R}+\frac{1}{R+\Delta R}\right)=V_{in}\frac{1}{R+\Delta R}$$

$$\frac{V_n}{R}+\frac{V_n-V_{in}}{R}+\frac{V_n-V_o}{R_f}=0 \quad (KCL) \quad \cdots\ 3'$$

$$V_n\left(\frac{2}{R}+\frac{1}{R_f}\right)-\frac{1}{R_f}V_o=\frac{1}{R}V_{in}$$

$$V_n\left(\frac{2}{R}+\frac{1}{R_f}\right)-\frac{1}{R}V_{in}=\frac{1}{R_f}V_o$$

$$V_p=V_n \quad \text{(property of op amp)} \quad \cdots\ 1'$$

$$V_p=V_{in}\frac{\frac{1}{R+\Delta R}}{\frac{1}{R_f}+\frac{1}{R}+\frac{1}{R+\Delta R}}$$

$$=\frac{1}{\frac{R+\Delta R}{R_f}+\frac{R+\Delta R}{R}+1}V_{in}$$

$$=\frac{R_fR}{R^2+R\Delta R+2RR_f+R_f\Delta R}V_{in} \quad \cdots\ 3'$$

$$\frac{R_fR}{R^2+R\Delta R+2RR_f+R_f\Delta R}\times\frac{2R_f+R}{RR_f}V_{in}-\frac{1}{R}V_{in}=\frac{1}{R_f}V_o$$

$$\left(\frac{2R_f+R}{R^2+R\Delta R+2RR_f+R_f\Delta R}-\frac{1}{R}\right)V_{in}=\frac{1}{R_f}V_o$$

$$\frac{-R\Delta R-R_f\Delta R}{R(R^2+R\Delta R+2RR_f+R_f\Delta R)}V_{in}=\frac{1}{R_f}V_o \quad \cdots\ 8'$$

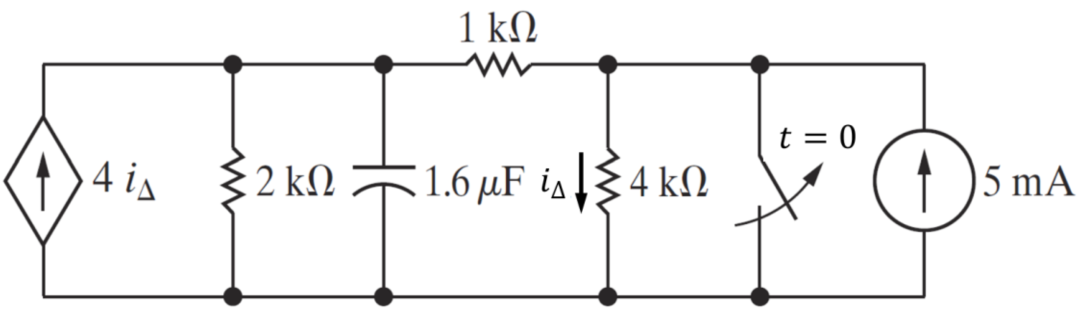
$$V_o=\frac{-\Delta R\,R_f(R+R_f)}{R((R+\Delta R)(R+R_f)+RR_f)}V_{in}\approx\frac{-\Delta R\,R_f(R+R_f)}{R(R(R+R_f)+RR_f)}V_{in}=\frac{-\Delta R\,R_f(R+R_f)}{R^2(R+2R_f)}V_{in} \quad \cdots\ 2'$$

5. The switch in the circuit below has been closed for a long time. The maximum voltage rating of the 1.6 μF capacitor is 14.4 kV. How long after the switch is opened does the voltage across the capacitor reach the maximum voltage rating? Please follow steps below.

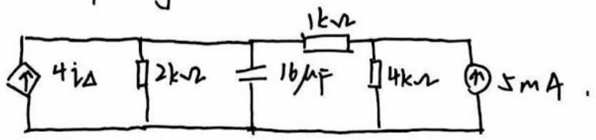
(a) Find Thevenin equivalent circuit seen from the capacitor. [10 points]

(b) Derive a capacitor voltage in the Thevenin circuit, i.e. *RC* circuit. [6 points]

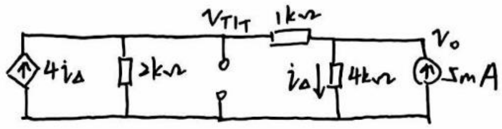
(c) Find time for the capacitor to reach the maximum voltage rating. [4 points]



(a) after opening the switch

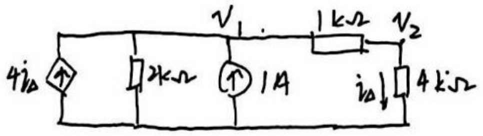


① V_{TH} :



$$\begin{cases} -4i_\Delta + \dfrac{V_{TH}}{2000} + \dfrac{V_{TH}-V_0}{1000} = 0 & 1' \\ \dfrac{V_0 - V_{TH}}{1000} + i_\Delta - 5\times10^{-3} = 0 & 1' \\ i_\Delta = \dfrac{V_0}{4000} & 1' \end{cases} \Rightarrow \begin{array}{l} V_0 = -60V \\ V_{TH} = -80V. \quad 2' \end{array}$$

② R_{TH} :

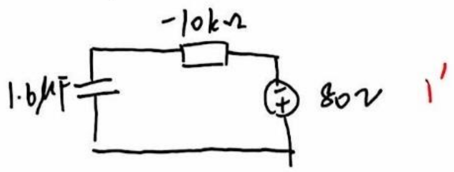


we add a 1A current source here.

$$\begin{cases} -4i_\Delta + \dfrac{V_1}{2000} - 1 + \dfrac{V_1 - V_2}{1000} = 0 & 1' \\ \dfrac{V_2 - V_1}{1000} + \dfrac{V_2}{4000} = 0 & 1' \\ i_\Delta = \dfrac{V_2}{4000} \end{cases} \Rightarrow \begin{cases} V_1 = -10kV \\ V_2 = -8kV \end{cases}$$

$$R_{TH} = \frac{V_1}{1A} = -10k\Omega. \quad 2'$$

Therenin equivalent:



(b) $v_c(0)=0$ 1' $v_c(\infty)=-80V$.

$\tau = RC = -10000 \times 1.6\times10^{-6} = -16\,ms$. 1' 1'

$\frac{1}{\tau} = -62.5$.

$v(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$ 1'

$= -80 + 80e^{62.5t}$ 2'

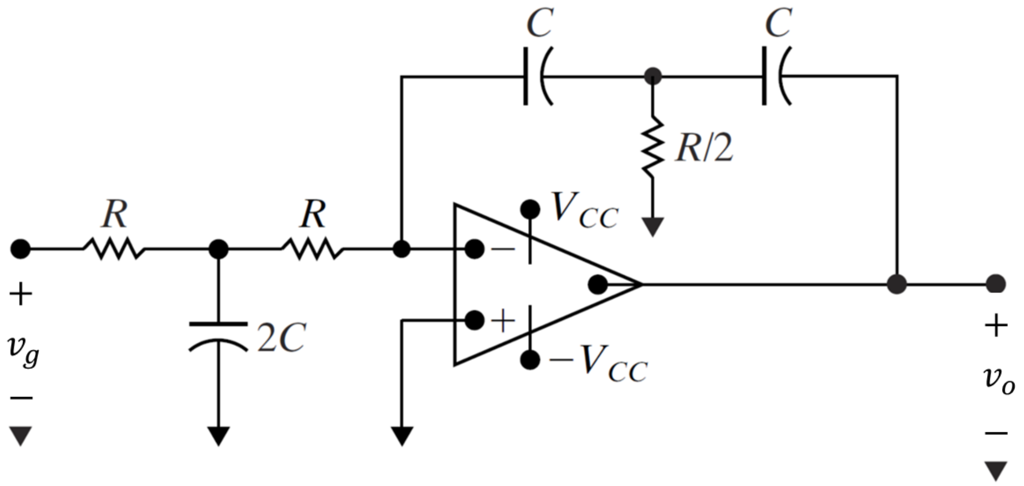
(c) $v_c(t_0) = 14400$.

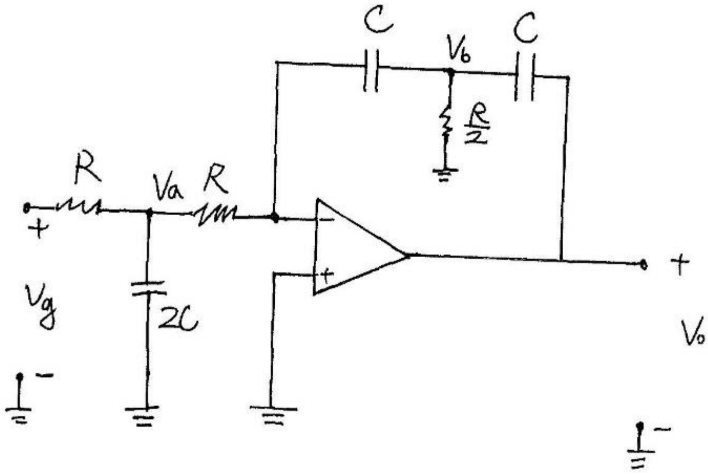
$e^{62.5t} = 181$ 1'

$62.5t = \ln 181$ 1'

$t = 83.09\,ms$. 2'

6. Derive the differential equation that relates the output voltage to the input voltage for the circuit shown below. [20 points]





$$\frac{V_g - V_a}{R} = 2C\frac{dV_a}{dt} + \frac{V_a}{R} \quad ① \quad 3'$$

$$\frac{V_a}{R} = C\frac{d(-V_b)}{dt} \quad ② \quad 3'$$

$$C\frac{d(-V_b)}{dt} = \frac{V_b}{\frac{R}{2}} + C\frac{d(V_b - V_o)}{dt} \quad ③ \quad 3'$$

from ①, $V_g = 2RC\frac{dV_a}{dt} + 2V_a$ ④ $2'$

from ②, $V_a = -RC\frac{dV_b}{dt}$ ⑤ $2'$

from ③, $\frac{dV_o}{dt} = 2\frac{dV_b}{dt} + \frac{2V_b}{RC} \Rightarrow \frac{d^2V_o}{dt^2} = 2\frac{d^2V_b}{dt^2} + \frac{2dV_b}{dt}\frac{1}{RC}$ $2'$

$$\Rightarrow \frac{d^2V_o}{dt^2} = -\frac{2}{RC}\frac{dV_a}{dt} + \frac{V_a}{-RC}\frac{2}{RC}$$

$$= -\frac{1}{(RC)^2}\cancel{\left[2\frac{dV_a}{dt} + 2V_a\right]}\left[2RC\frac{dV_a}{dt} + 2V_a\right]$$

$$= -\frac{V_g}{R^2C^2} \quad 5'$$

$$\Rightarrow \boxed{\frac{d^2V_o}{dt^2} = -\frac{V_g}{R^2C^2}}$$