VE215 Mid RC C1-3

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Overview

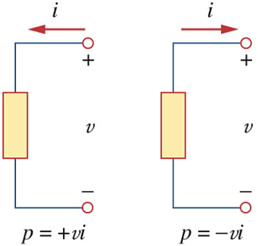
Basic Conventions

- $i = \frac{dq}{dt}$
- $Q = \int_{t0}^{t} idt$
- $v = \frac{dw}{dq}$
- $p = \frac{dw}{dt} = vi$

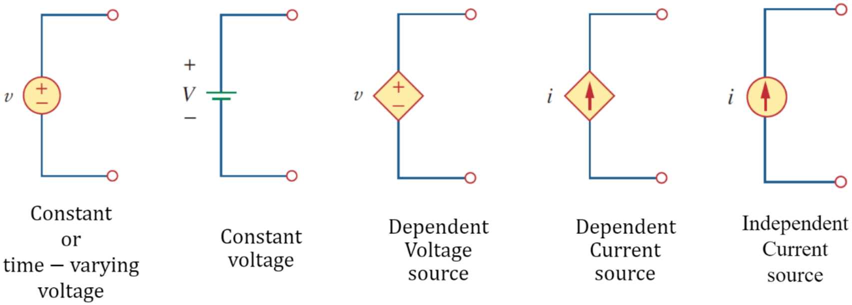
Just remember them!

Passive sign convention

- Passive sign convention:
  - When **the current enters through the positive terminal** of **an element**, $p = +vi$.
  - If the current enters through **the negative terminal**, $p = -vi$.



Symbols


v
+
−
Constant
or
time − varying
voltage
+
V
−
Constant
voltage
v
+
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Dependent
Voltage
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i
Dependent
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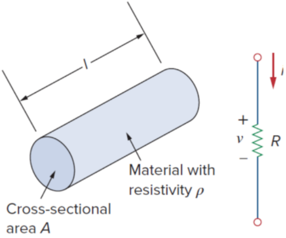
source
i
Independent
Current
source

Ohm's law



Resistance $R = \rho \frac{l}{A}$

The resistance of any material depends on a uniform cross-sectional area A and its length l, where ρ is known as the *resistivity* of the material in ohm-meters.

Ohm's law

$$V = IR, or\ R = \frac{v}{i}, \text{thus } 1\,\Omega = 1\,\text{V/A}$$

- Surprisingly, **not all resistors obey Ohm's law**.
- A resistor that obeys Ohm's law is known as **a linear resistor**, i.e. a constant resistance.

$$G = \frac{1}{R} = \frac{i}{v}, 1\,\text{S} = 1\,\mho = 1\,A/V$$

Nodes, Branches and Loops

(iii) A *loop* is any closed path in a circuit.

- *Independent loop:* the loop contains at least one branch which is not a part of any other independent loop.
- A *mesh* is a loop that does not enclose any other loops. (i.e., smallest loop)

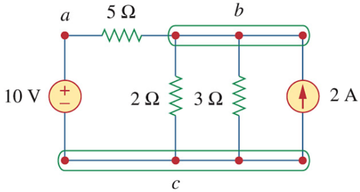
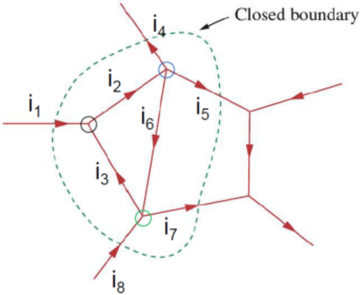


Figure 2.10 Nodes, branches, and loops.

b(branches)=l(independent loop)+n(nodes)-1

KCL



Node 1:
$i_1 + i_3 = i_2$

Node 2:
$i_2 = i_4 + i_5 + i_6$

Node 3:
$i_8 + i_6 = i_3 + i_7$

Node 1 + 2 + 3
$i_1 + i_2 + i_3 + i_8 + i_6 = i_2 + i_4 + i_5 + i_6 + i_3 + i_7$
$\rightarrow \mathbf{i_1 + i_8 = i_4 + i_4 + i_7}$
Current entering the closed boundary = current leaving the boundary

KVL

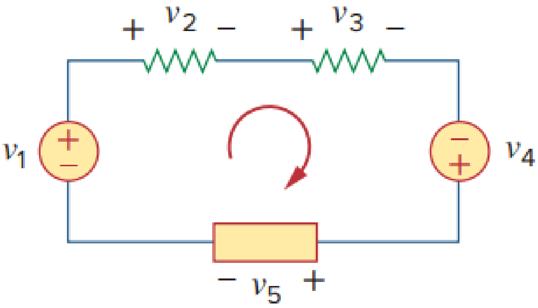


Figure 2.19
A single-loop circuit illustrating KVL.

Series and parallel connection

Series connection:

$$R_{eq} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^{N} R_n$$

Parallel connection:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \dots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

$$G_{eq} = G_1 + G_2 + \dots + G_N = \sum_{n=1}^{N} G_n$$

Wye-Delta Transformation

Y = T

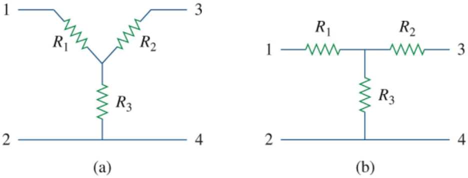


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

Δ = π

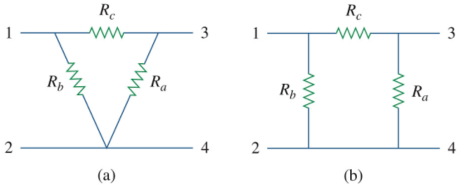


Figure 2.48 Two forms of the same network: (a) $\mathbf{\Delta}$, (b) $\mathbf{\Pi}$.

Wye-Delta Transformation

$$
\begin{cases}
R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\
R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\
R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}
\end{cases}
$$

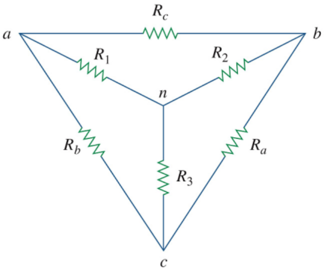


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Wye-Delta Transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

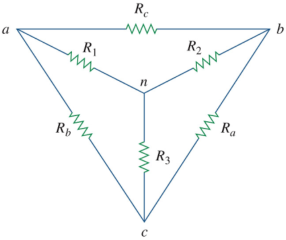
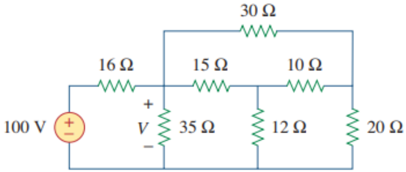


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Exercise

Determine V in the circuit



3. Chapter3 Methods of Analysis

Chapter 3 Methods of Analysis

Nodal Analysis:

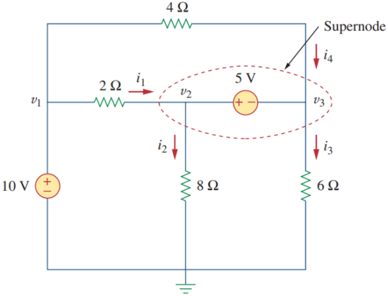
1. Select a reference node (ground)
2. Apply KCL
3. Solve the equations

Mesh Analysis:

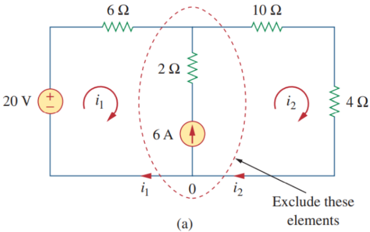
1. Mark the current of all the meshes
2. Apply KVL
3. Solve the equations

Supernode & Supermesh

- Supernode & Supermesh – simplify the equation



$$i_1 - i_2 - i_3 + i_4 = 0$$



(a)

$$20 - 6i_1 - 14i_2 = 0$$

Analysis by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and $j, k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all ~~independent~~ current sources directly connected to node k, with currents entering the node treated as positive

For Nodal Analysis

(only current source in circuit)

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and $j, k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all ~~independent~~ voltage sources in mesh k, with voltage rise treated as positive

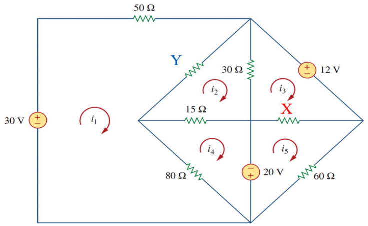
For Mesh Analysis

(only voltage source in circuit)

Exercise

1-2. Please determine resistance **X** [Ω] and **Y** [Ω]. What is **X** + **Y**?

[5 points]



$$\begin{bmatrix} a_{11} & a_{21} & 0 & a_{14} & 0 \\ a_{21} & 65 & -30 & a_{24} & 0 \\ a_{31} & a_{32} & 50 & a_{34} & -20 \\ a_{41} & a_{42} & 0 & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix}$$

(1) 40

(2) 45

(3) 50

(4) 55

(5) Cannot tell from the information given.

References

1. 2024FA VE215 slides
2. 2023FA Mid RC, Hengyi Cai

Thank you!