VE215 Mid RC Part 2

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Overview

Linearity Property

Superposition

Source Transformation

Thevenin's Theorem

Norton's Theorem

Maximum Power Transfer

Linearity Property

homogeneous: if $x \to y$, then $kx \to ky$
additive: if $x_1 \to y_1$ and $x_2 \to y_2$, then $x_1 + x_2 \to y_1 + y_2$
linear circuit: homogeneous and additive

Exercise

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

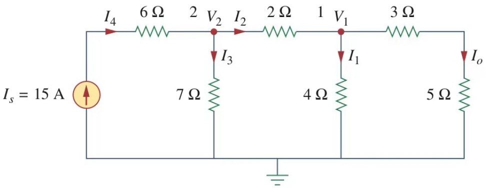
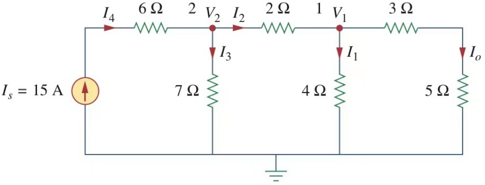


Figure 4.4

Exercise



Answer: $I_0 = 3A$

Overview

Superposition

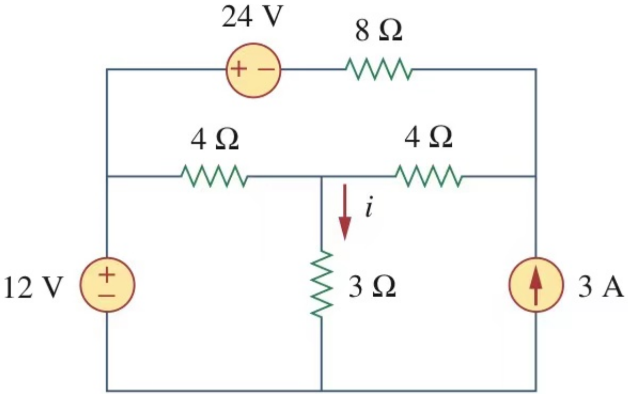
Steps

1. Only consider one **independent** source.
   - voltage source: short circuit
   - current source: open circuit

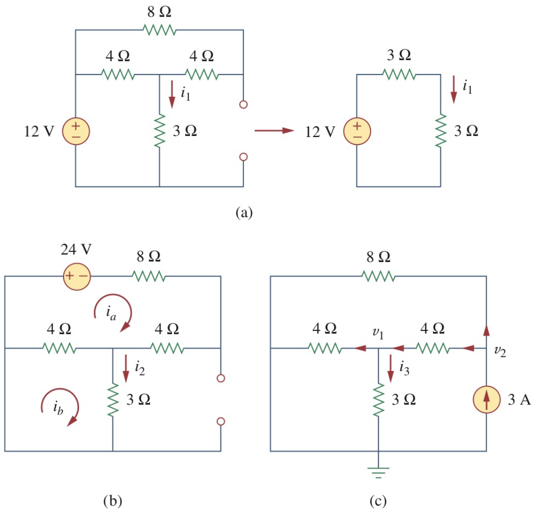
2. Use additivity.

Exercise

Find i in the circuit.



Exercise

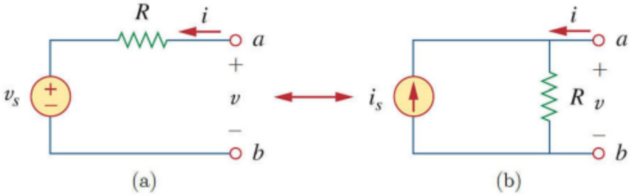


Answer: 2A

Overview

Source Transformation

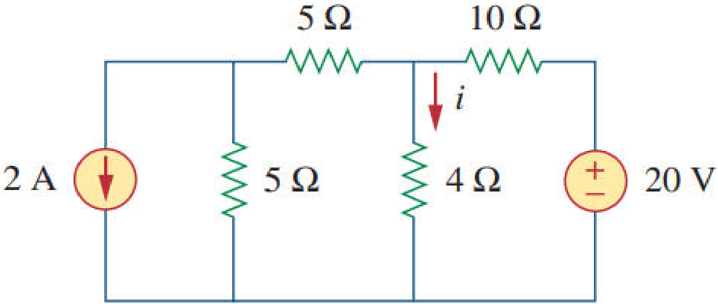
We can replace a voltage source with a resistance with a corresponding current source with the same resistance to simplify the circuit.
In the case shown below, $v_s = i_s \times R$



For dependent sources, the source transformation is also valid.

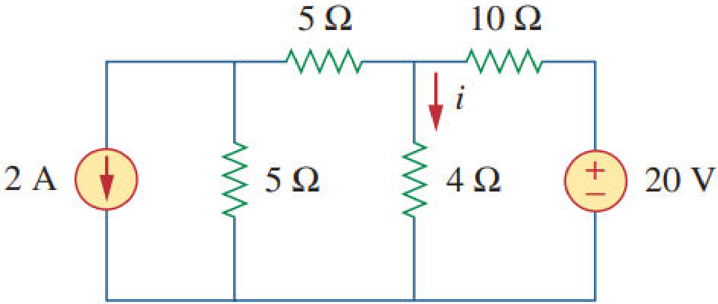
Exercise

Find i.



Exercise

Find i.

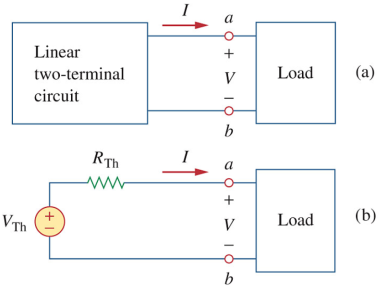


Answer: 5/9A

Overview

Thevenin's Theorem

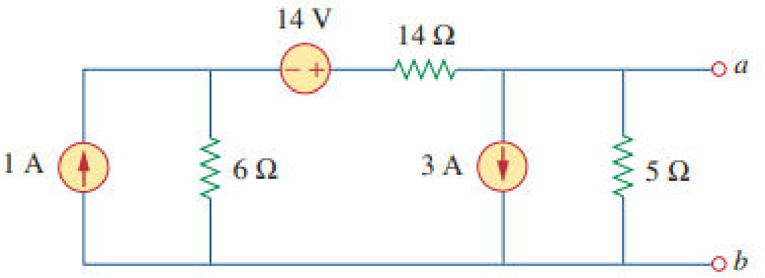
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}.



- V_{Th}: the open-circuit voltage at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

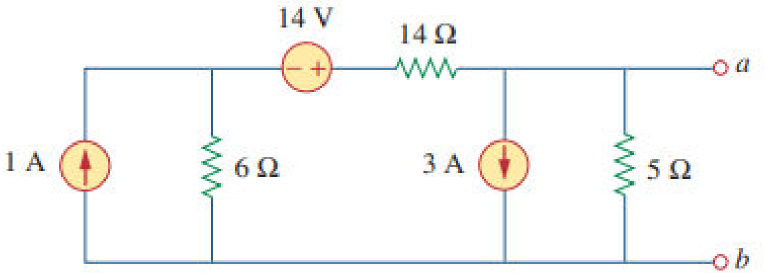
Exercise

Obtain the Thevenin equivalent circuit of this circuit with respect to terminal a and b.



Exercise

Obtain the Thevenin equivalent circuit of this circuit with respect to terminal a and b.



Answer: $V_{Th} =$ - 8 V

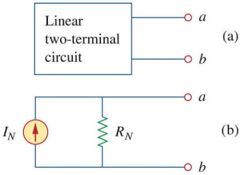
$R_{Th} =$ 4Ω

pay attention to the sign of the voltage !

Overview

Norton's Theorem

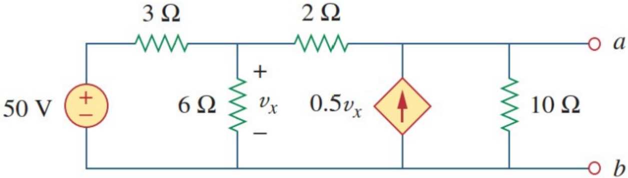
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_{Th} in parallel with a resistor R_{Th}.



- I_{Th}: the short-circuit current at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

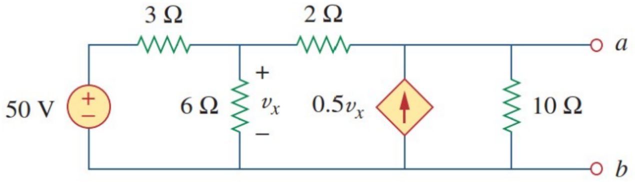
Exercise

Obtain the Norton equivalent circuit of this circuit with respect to terminal a and b.



Exercise

Obtain the Norton equivalent circuit of this circuit with respect to terminal a and b.



Answer: $I_N = 16.67A$
$R_N = 10\Omega$

Overview

Maximum Power Transfer

A circuit is usually designed to provide power to a load. For different kinds of circuits, we have different concerns

- **Maximum Power Efficiency:** In power utility systems, the amount of electricity is very large. Therefore, how to increase the efficiency of power transfer becomes an important problem.
- **Maximum Power Transfer:** In communication and instrumental systems, the amount of electricity is small so the problem of efficiency is not so important. Instead, we want to transfer as much of power as possible to the load.

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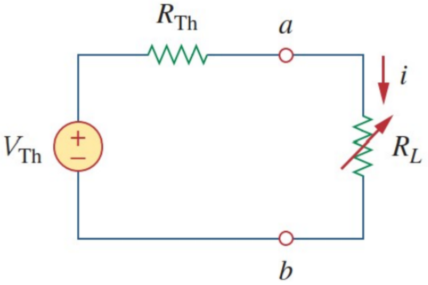
Maximum Power Transfer

The Thevenin's equivalent circuit is useful in finding the maximum power delivered to a load. In the circuit below, R_L represents the load.



Maximum Power Theorem

Since

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Let $\frac{dP}{dR_L} = V_{Th}^2 \frac{R_{Th} - R_L}{(R_{Th}+R_L)^3} = 0$, we have $R_L = R_{Th}$.
And when $R_L = R_{Th}$, $\frac{d^2P}{dR_L^2} = V_{Th}^2 \frac{2R_l - 4R_{Th}}{(R_{Th}+R_L)^4} = -\frac{V_{Th}^2}{8R_{Th}^2} < 0$.
Thus p reaches maximum at $R_L = R_{Th}$. $p_{max} = \frac{V_{Th}^2}{4R_{Th}}$

References

1. 2024 Fall VE215 slides
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2023 Fall RC2 Chongye Yang

Thank you!