

Due Date: 23:59, Oct.28th, 2024

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated. This assignment covers content from chapter 1 to 3.

## Exercise 1.1 (20%)

The current entering the positive terminal of a device is  $i(t) = 2e^{-2t} \text{ mA}$  and the voltage across the device is  $v(t) = 15 \frac{di}{dt} \text{ V}$ .

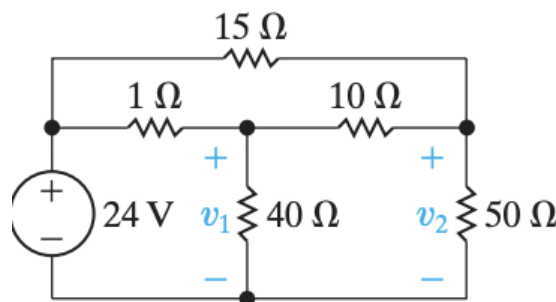
- Find the total charge in the device at  $t = 3 \text{ s}$ ,  $q(0)=0$ .
- Calculate the power absorbed  $P(t)$ .
- Determine the total energy absorbed in 5s.

$$\begin{aligned}
 (a) \quad q &= \int_0^3 2e^{-2t} dt + q_0 \\
 &= -\int_0^3 e^{-2t} d(2t) + 0 \\
 &= e^{-2t} \Big|_0^3 \\
 &= (-e^{-6} + 1) \\
 &= 0.9975 \text{ mC}
 \end{aligned}$$

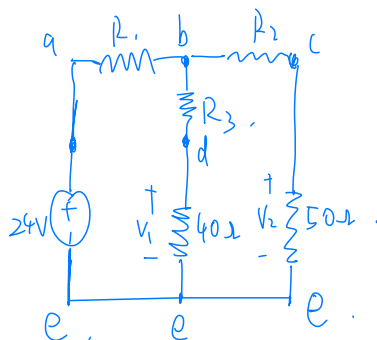
$$\begin{aligned}
 (2) \quad \frac{di}{dt} &= -4 \cdot e^{-2t} \\
 v(t) &= -60 \cdot e^{-2t} \\
 P(t) &= v(t) i(t) \\
 &= 0.12 e^{-2t} \cdot \text{W}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad W &= \int_0^5 P(t) dt \\
 &= \int_0^5 0.12 e^{-2t} dt \\
 &= 0.03 e^{-2t} \Big|_0^5 = 0.03(e^{-10} - 1) \approx -0.03 \text{ J}
 \end{aligned}$$

**Exercise 1.2 (20%)** Please find the voltage  $v_1$  and  $v_2$  in the circuit below by using delta-to-wye transformation.



We can get the equivalent, by delta to w



$$R_1 = \frac{15 \times 1}{15 + 1 + 10} = \frac{15}{26} \Omega$$

$$R_2 = \frac{15 \times 10}{15 + 1 + 10} = \frac{75}{13} \Omega$$

$$R_3 = \frac{1 \times 10}{15 + 1 + 10} = \frac{5}{13} \Omega$$

$$R_p = \frac{(\frac{5}{13} + 40)(\frac{75}{13} + 50)}{\frac{5}{13} + \frac{75}{13} + 90} = \frac{608}{26} \text{ V}$$

$$V_p = 24 \cdot \frac{\frac{608}{26}}{\frac{608}{26} + \frac{15}{26}} = \frac{608}{26} \text{ V}$$

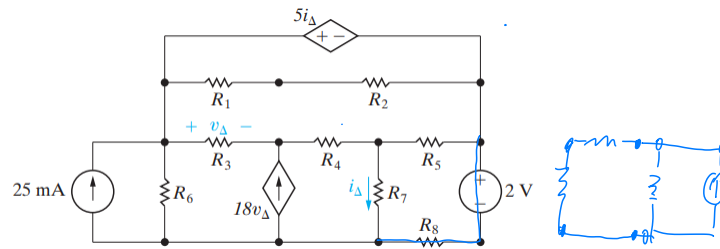
$$V_1 = \frac{608}{26} \cdot \frac{40}{\frac{5}{13} + 40} = 23.2 \text{ V}$$

$$V_2 = \frac{608}{26} \cdot \frac{50}{\frac{75}{13} + 50} = 21 \text{ V}$$

## Exercise 1.3 (20%)

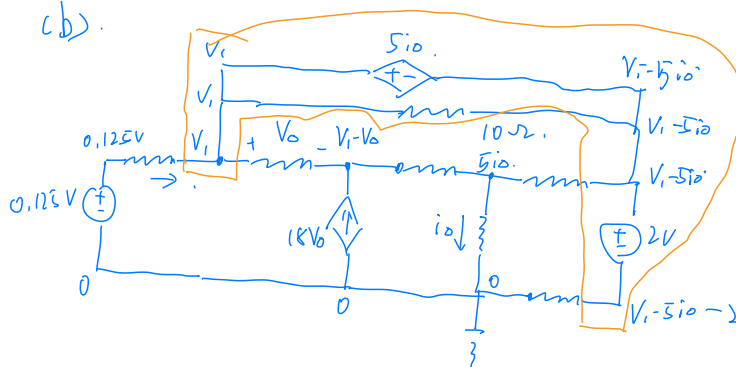
In the network graph below

- Determine the number of branches, nodes and meshes.
- Assuming all the resistor have the resistance of 5 ohm, calculate out  $v_{\Delta}$  and  $i_{\Delta}$



(a). Branches: 12 meshes: 6 nodes: 7.

(b).

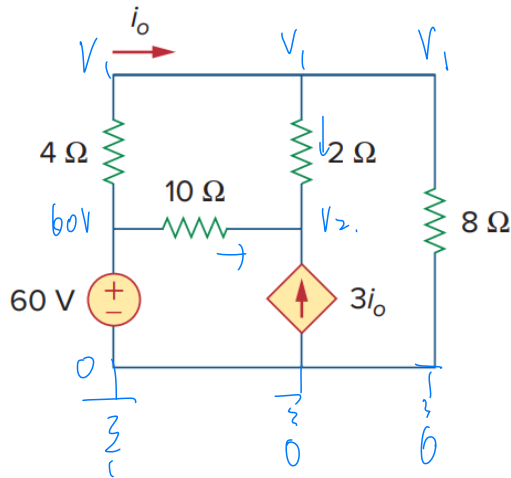


$$\begin{cases} \frac{0.125 - V_1}{5} + \frac{2 + 5i_{\Delta} - V_1}{5} - \frac{V_2}{5} + \frac{10i_{\Delta} - V_1}{5} = 0 \\ 18V_{\Delta} + \frac{V_2}{5} + \frac{5i_{\Delta} + V_2 - V_1}{5} = 0 \\ \frac{5i_{\Delta} + V_2 - V_1}{5} + i_{\Delta} + \frac{10i_{\Delta} - V_1}{5} = 0 \end{cases}$$

$$\begin{cases} V_2 = 7.67 \times 10^{-3} \text{ V} \\ V_1 = 1.4 \text{ V} \\ i_{\Delta} = 0.14 \text{ A} \end{cases}$$

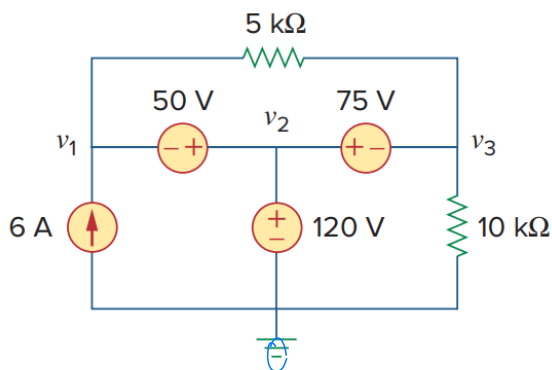
## Exercise 1.4 (10+10%)

(a) (10%) Using nodal analysis, find current  $i_0$  in the circuit below.



$$\begin{cases} \frac{60 - V_1}{4} + \frac{V_2 - V_1}{2} - \frac{V_1}{8} = 0 \\ \frac{V_1 - V_2}{10} + \frac{60 - V_2}{2} + 3i_0 = 0 \\ i_0 = \frac{V_1 - V_2}{2} + \frac{V_1}{8} \end{cases} \Rightarrow \begin{cases} V_1 = 50.3 \text{ V} \\ V_2 = 58.0 \text{ V} \\ i_0 = 2.43 \text{ A} \end{cases}$$

(b) (10%) Obtain the node voltages  $v_1$ ,  $v_2$ , and  $v_3$  in the circuit below.



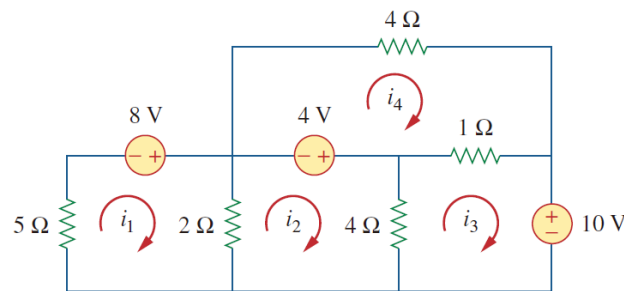
$$V_2 = 120 \text{ V}$$

$$V_1 = 70 \text{ V}$$

$$V_3 = 45 \text{ V}$$

## Exercise 1.5 (10+10%)

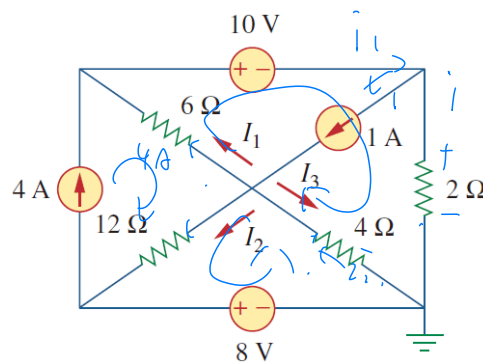
(a) (10%) Use mesh analysis **by inspection** to find current  $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$  in the circuit below.



$$\begin{cases} 5i_1 - 8 + (i_1 - i_2) \cdot 2 = 0 \\ 2(i_2 - i_1) - 4 + 4(i_2 - i_3) = 0 \\ 4(i_3 - i_2) + (i_3 - i_4) + 10 = 0 \\ i_4 - i_3 + 4 + 4i_4 = 0 \end{cases}$$

$$\begin{cases} \hat{i}_1 = 0.77 \text{ A} \\ \hat{i}_2 = -1.5 \text{ A} \\ \hat{i}_3 = -3.3 \text{ A} \\ \hat{i}_4 = -1.47 \text{ A} \end{cases}$$

(b) (10%) Find current  $I_1$ ,  $I_2$  and  $I_3$  in the circuit below.



$$\begin{cases} 6(i_1 - 4) + 10 - 2i_3 + 4(i_2 - i_3) = 0 \\ 12(i_2 + 4) + 8 + 4(i_2 - i_3) = 0 \\ i_1 + i_3 = 1 \end{cases}$$

$$\begin{cases} \hat{i}_1 = 3 \text{ A} \\ \hat{i}_2 = -4 \text{ A} \\ \hat{i}_3 = 2 \text{ A} \end{cases}$$

$$\begin{cases} I_1 = \hat{i}_1 - 4 = -1 \text{ A} \\ I_2 = \hat{i}_2 + 4 = 0 \text{ A} \\ I_3 = \hat{i}_3 - \hat{i}_2 = 2 \text{ A} \end{cases}$$