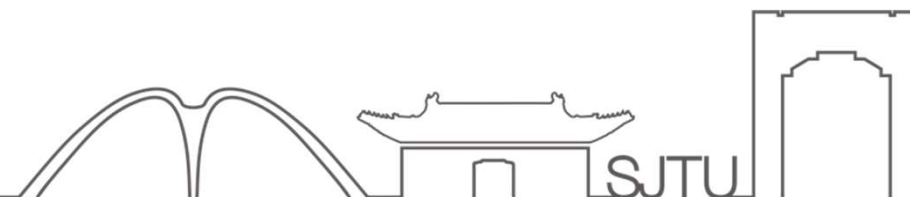




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VE215 RC5

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Overview

AC Power Analysis

Instantaneous and Average Power

Maximum Average Power Transfer

Effective or RMS Value

Apparent Power and Power Factor

Complex Power

Power Factor Correction

Conservation of AC Power

Instantaneous Power

Definition:

$$p(t) = v(t) \cdot i(t)$$

Both $v(t)$ and $i(t)$ here are instantaneous values, **not rms values**.

Instantaneous Power:

$$\begin{aligned} p(t) &= V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \\ &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \end{aligned}$$

Average Power

Definition:

$$P = \frac{1}{T} \int_0^T p(t) dt$$

For **sinusoids**:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Expressed in Phasor:

$$P = \frac{1}{2} \operatorname{Re}(\tilde{V} \tilde{I}^*)$$

Average Power

When $\theta_v = \theta_i$, we have a purely resistive load R :

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} \frac{V_m^2}{R}$$

When $\theta_v - \theta_i = \pm 90^\circ$, we have a purely reactive load X :

$$P = \frac{1}{2} V_m I_m \cos(\pm 90^\circ) = 0$$

We can conclude that only R absorbs average power. X absorbs no average power.

General Case:

$$P = \frac{1}{2} \operatorname{Re}(\tilde{V} \tilde{I}^*) = \frac{1}{2} \operatorname{Re}(\tilde{I}(R + jX) \tilde{I}^*) = \frac{1}{2} I_m^2 R$$

Maximum Average Power Transfer

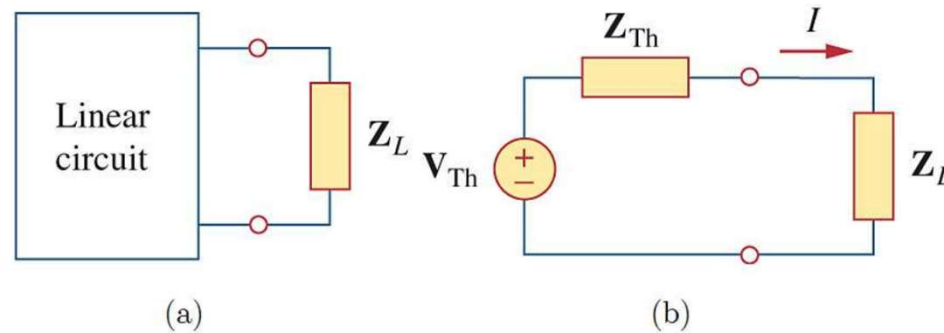


Figure 11.7 Finding the maximum average power transfer
(a) circuit with a load, (b) the Thevenin equivalent.

If there is no restriction on Z_L ,

$$R_L = R_{Th} \quad X_L = -X_{Th} \quad P_{max} = \frac{V_{Th}^2}{8R_{Th}}$$

If Z_L is purely resistive,

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} \quad P_{max} = \frac{V_{Th}^2}{4(R_{Th} + \sqrt{R_{Th}^2 + X_{Th}^2})}$$

Proof for $P_{max} = \frac{V_{Th}^2}{8R_{Th}}$

$$P = \frac{1}{2} I^2 R_L = \frac{1}{2} \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$
$$\begin{cases} \frac{\partial P}{\partial R_L} = \frac{V_{Th}^2 [R_{Th}^2 - R_L^2 + (X_{Th} + X_L)^2]}{2[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2} = 0 \\ \frac{\partial P}{\partial X_L} = -\frac{V_{Th}^2 R_L (X_{Th} + X_L)}{[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2} = 0 \end{cases}$$
$$\begin{cases} R_L = R_{Th} \\ X_L = -X_{Th} \end{cases} \quad \text{or } Z_L = Z_{Th}^*$$



Effective or RMS Value

Definition: The effective value of an ac current i is the dc current I_{eff} that delivers the same average power to a resistor as the ac current.

$$I_{eff}^2 R = \frac{R}{T} \int_0^T i^2 dt$$

Effective value = Root mean square (RMS) value:

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = I_{eff}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2 dt} = V_{eff}$$

Effective or RMS Value

Avg power absorbed by a circuit element (General Case):

$$P = I_{rms}^2 R = V_{rms}^2 \frac{R}{R^2 + X^2}$$

$$P = \frac{1}{2} \operatorname{Re}(\tilde{V} \tilde{I}^*) = \operatorname{Re}(\tilde{V}_{rms} \tilde{I}_{rms}^*) = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{1}{2} V_m^2 \operatorname{Re}\left(\frac{1}{Z^*}\right) = V_{rms}^2 \frac{R}{R^2 + X^2}$$

For **sinusoids**:

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

Avg power absorbed by an element in a **sinusoidal** circuit:

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Effective or RMS Value

Caution: from now on, unless specified, all values will be assumed to be RMS values.

Apparent Power and Power Factor

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Apparent power:

$$|S| = |V_{rms}| |I_{rms}| = |I_{rms}|^2 |Z| = \sqrt{P^2 + Q^2}$$

Power factor (pf or $\cos\theta$):

$$pf = \cos\theta = \frac{P}{|S|} = \cos(\theta_v - \theta_i)$$

Power factor angle (θ):

$$\theta = \theta_v - \theta_i$$

Leading/Lagging pf

Power factor:

$$pf = \cos(\theta_v - \theta_i)$$

$\theta_v - \theta_i < 0$: leading pf

$\theta_v - \theta_i > 0$: lagging pf

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, the pf value only tells part of the story. Every time you are asked for a power factor, **you must declare whether it is leading or lagging.**

Complex Power

Apparent power:

$$|S| = |V_{rms}| |I_{rms}| = |I_{rms}|^2 |Z| = \sqrt{P^2 + Q^2}$$

Complex power:

$$\begin{aligned}\tilde{S} &= \tilde{V}_{rms} \tilde{I}_{rms}^* = |I_{rms}| |V_{rms}| \angle(\theta_v - \theta_i) \\ &= |S| \angle(\theta_v - \theta_i) \\ &= P + jQ\end{aligned}$$

Real power:

$$P = \operatorname{Re}(\tilde{S}) = |S| \cos(\theta_v - \theta_i) = |I_{rms}|^2 R$$

Reactive power:

$$Q = \operatorname{Im}(\tilde{S}) = |S| \sin(\theta_v - \theta_i) = |I_{rms}|^2 X$$



Complex Power

$$\begin{aligned}
 \text{Complex Power} &= \tilde{S} = \tilde{V}_{rms} \tilde{I}_{rms}^* = |I_{rms}| |V_{rms}| \angle(\theta_v - \theta_i) \\
 &= |S| \angle(\theta_v - \theta_i) \text{ (polar form)} \\
 &= P + jQ \text{ (rectangular form)}
 \end{aligned}$$

Value	Name	Meaning	Unit
$ S $	Apparent power	Magnitude of \tilde{S}	VA
$\cos(\theta_v - \theta_i)$	Power factor	Cosine of angle of \tilde{S}	/
P	Real power	Real part of \tilde{S}	W
Q	Reactive power	Imaginary part of \tilde{S}	VAR

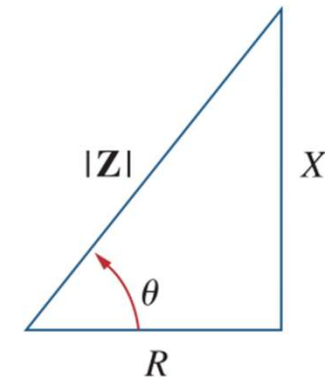
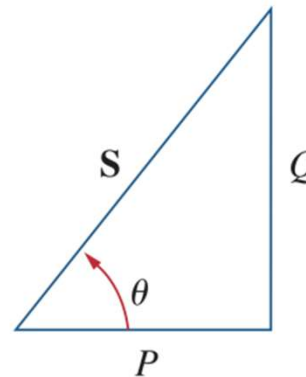
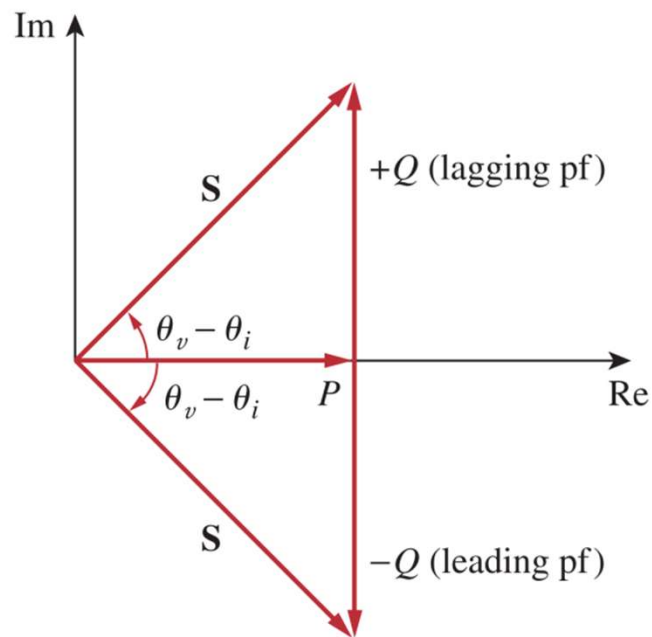
Complex Power

We can use the sign of pf angle or Q to identify the property of the circuit and the loads:

	(1)	(2)	(3)
pf Angle	$\theta_v - \theta_i = 0$	$\theta_v - \theta_i < 0$	$\theta_v - \theta_i > 0$
Sign of Q	$Q = 0$	$Q < 0$	$Q > 0$
Properties	Unity pf I, V in phase $X = 0$ Resistive loads	Leading pf I leads V $X < 0$ Capacitive loads	Lagging pf I lags V $X > 0$ Inductive loads

Complex Power

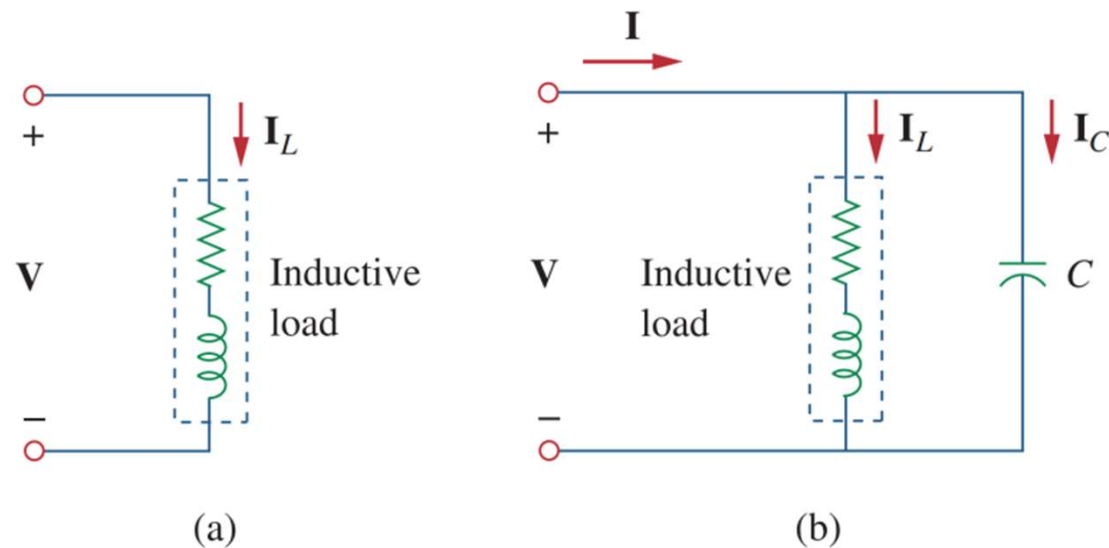
And observe that the power factor angle is equal to the angle of the impedance of that part of the circuit.



Power Factor Correction

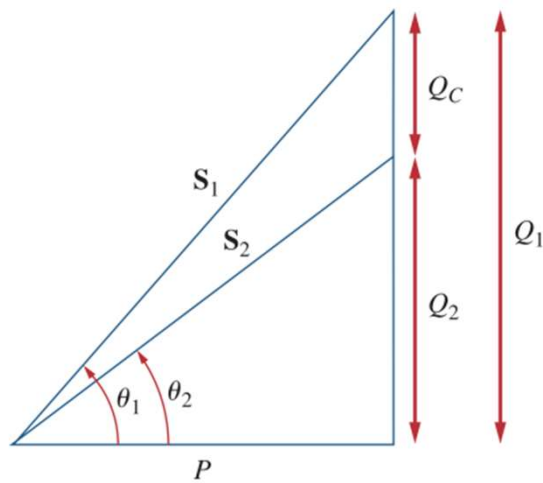
Goal: increase the pf of a load \rightarrow make it less inductive \rightarrow reduce energy loss

Solution: add a capacitor in parallel to the load



Power Factor Correction

Goal: increase the pf from $\cos\theta_1$ to $\cos\theta_2$.



Initial:

$$P = |S_1| \cos\theta_1$$
$$Q = |Q_1| \sin\theta_1 = P \tan\theta_1$$

Expected outcome:

$$P = |S_2| \cos\theta_2$$
$$Q = |Q_2| \sin\theta_2 = P \tan\theta_2$$

Since $Q_c (= Q_1 - Q_2) = \frac{V_{rms}^2}{X_c}$, then the value of the required capacitance C is:

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{Q_1 - Q_2}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

Conservation of AC Power

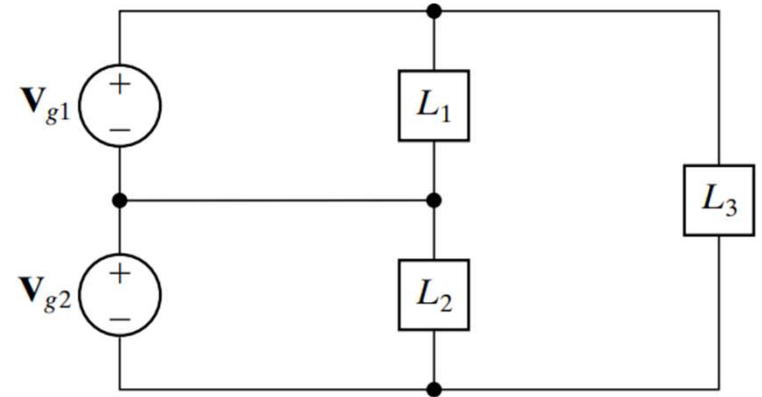
The Principle of Conservation of AC Power:

The complex, real, and reactive powers of the source equal the respective sums of the complex, real, and reactive powers of the individual loads.

Exercise 1

The three loads in the circuit seen in the figure are described as follows: Load 1 is absorbing 4.8 kW and delivering 2.4 kVAR ; Load 2 is absorbing 6 kVA at a power factor of 0.8 lagging; Load 3 is a 24Ω resistor in parallel with an inductance whose reactance is 6Ω .

Calculate the average power and the magnetizing reactive power delivered by each source if $V_{g1} = V_{g2} = 120\angle 0^\circ \text{V}$ (rms)



Exercise 2

Two loads connected in parallel draw a total of 2.4 kW at 0.8 pf lagging from a 120-V rms, 60-Hz line. One load absorbs 1.5 kW at a 0.707 pf lagging.

Determine: (a) the pf of the second load, (b) the parallel element required to correct the pf to 0.9 lagging for the two loads

References

1. 2023 Fall VE215 slides, Sung-Liang Chen
2. 2023 Fall RC5, Shunyao Huang
3. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew



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THANK YOU!

