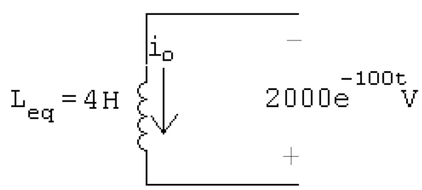
3.1

(a) (4%)

$$i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,A$$

(b) (4%) For the equivalent circuit with $L_{eq} = 4\,H$:

$$i_o = -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,A, \quad t \geq 0$$



(c) (4%)

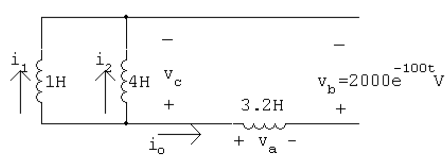
Using the values of v_a and v_c:

$$v_a = 3.2 \cdot (-500e^{-100t}) = -1600e^{-100t}\,V$$

$$v_c = v_a + v_b = -1600e^{-100t} + 2000e^{-100t} = 400e^{-100t}\,V$$

$$i_1 = \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 = -4e^{-100t} + 4 - 6$$

$$i_1 = -4e^{-100t} - 2\,A, \quad t \geq 0$$



(d) (4%)

$$i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1 = -e^{-100t} + 2\,A, \quad t \geq 0$$

(e) (4%)

$$w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,J$$

3.2

(a) (6%) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$i_1(0) = i_2(0) = v_o(0) = 0$$

but the current through the 4-H inductor is $i_L(0) = \frac{30}{10} = 3\,A$.

(b) (8%) When the switch is in position B,

$$R_{Th} = \frac{3}{1+6} = 2\,\Omega, \quad \tau = \frac{L}{R_{Th}} = \frac{4}{2} = 2\,s$$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\,e^{-t/\tau} = 0 + 3e^{-t/2} = 3e^{-t/2}\,A$$

(c) (6%)

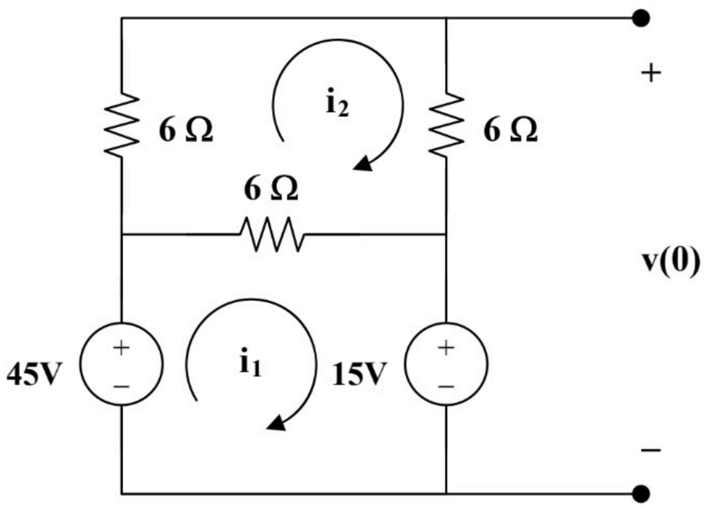
$$i_1(\infty) = \frac{30}{10+5} = 2\,A$$

$$i_2(\infty) = -\frac{3}{9}i_1(\infty) = 0\,A$$

$$v_o(\infty) = L\frac{di_L}{dt} \Rightarrow v_o(\infty) = 0\,V$$

3.3

For $t = 0^-$, the equivalent circuit is shown below.



$$18i_2 - 6i_1 = 0 \text{ or } i_1 = 3i_2 \qquad (1)$$

$$-45 + 6(i_1 - i_2) + 15 = 0 \text{ or } i_1 - i_2 = 30/6 = 5 \qquad (2)$$

From (1) and (2), $(2/3)i_1 = 5$ or $i_1 = 7.5$ and $i_2 = i_1 - 5 = 2.5$

$$i(0) = i_1 = 7.5\text{A}$$

$$-15 - 6i_2 + v(0) = 0$$

$$v(0) = 15 + 6\text{x}2.5 = 30$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2\text{x}(1/2)) = 4$$

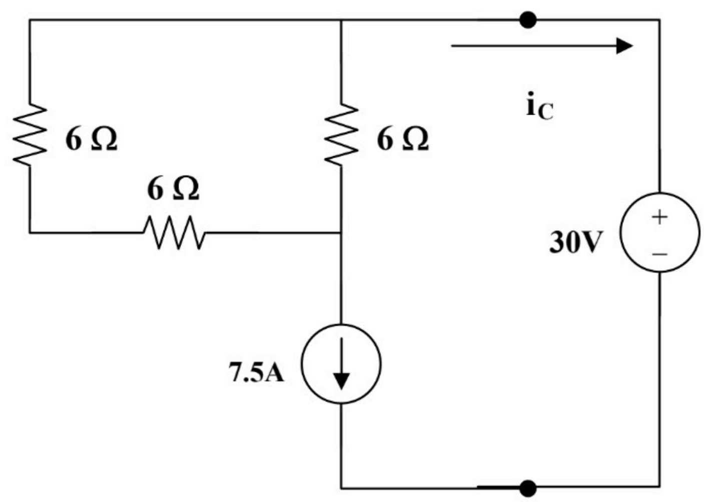
$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = 15$$

$$v(0) = 30 = 15 + A, \text{ or } A = 15$$

$$i_C = Cdv/dt = C[-4(15 + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

To find $i_C(0)$ we need to look at the circuit right after the switch is opened. At this time, the current through the inductor forces that part of the circuit to act like a current source and the capacitor acts like a voltage source. This produces the circuit shown below. Clearly, $i_C(0+)$ must equal $-i_L(0) = -7.5$A.



$$i_C(0) = -7.5 = C(-60 + B) \text{ which leads to } -60 = -60 + B \text{ or } B = 0$$

$$i_C = Cdv/dt = (1/8)[-4(15 + 0t)e^{-4t}] + (1/8)[(0)e^{-4t}]$$

$$i_C(t) = [-(1/2)(15)e^{-4t}]$$

$$i(t) = -i_C(t) = \mathbf{7.5e^{-4t}\ A}$$

3.4 Solution

At node 1,

$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

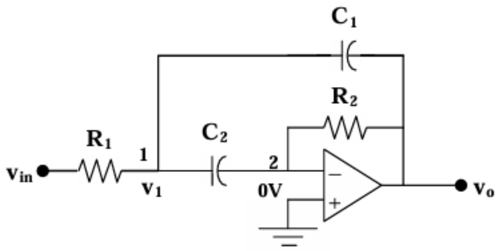
At node 2,

$$C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \text{ or } \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$



From (2) and (3),

$$-\frac{v_o}{C_2 R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{d^2 v_o}{dt^2} + \frac{R_1}{R_2}\frac{dv_o}{dt}$$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{C_1 C_2 R_2 R_1} = -\frac{1}{R_1 C_1}\frac{dv_{in}}{dt}$$

$$\text{But } C_1 C_2 R_1 R_2 = 10^{-4} \text{x} 10^{-4} \text{x} 10^{4} \text{x} 10^{4} = 1$$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2}{R_2 C_1} = \frac{2}{10^4 \text{x} 10^{-4}} = 2$$

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s + 1)^2 = 0$ and $s = -1, -1$

Therefore, $v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B - Bt)e^{-t}] \qquad (4)$$

From (2),

$$\frac{dv_o(0+)}{dt} = -\frac{v_o(0+)}{C_2R_2} = 0$$

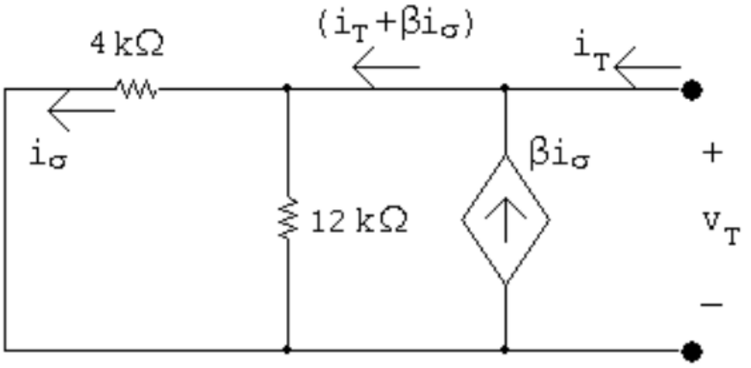
From (1) at $t = 0+$,

$$\frac{1-0}{R_1} = -C_1\frac{dv_o(0+)}{dt} \text{ which leads to } \frac{dv_o(0+)}{dt} = -\frac{1}{C_1R_1} = -1$$

Substituting this into (4) gives $B = -1$

Thus, $v(t) = \mathbf{-te^{-t}u(t)\ V}$

[a]



Using Ohm's law,

$v_T = 4000 i_\sigma$

Using current division,

$$i_\sigma = \frac{12{,}000}{12{,}000 + 4000}(i_T + \beta i_\sigma) = 0.75 i_T + 0.75\beta i_\sigma$$

Solve for i_σ:

$$i_\sigma(1 - 0.75\beta) = 0.75 i_T$$

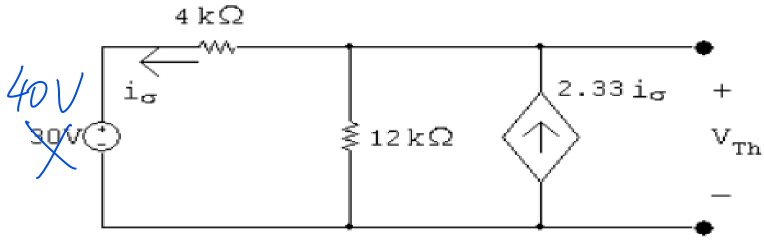
$$i_\sigma = \frac{0.75 i_T}{1 - 0.75\beta}; \qquad v_T = 4000 i_\sigma = \frac{3000 i_T}{(1 - 0.75\beta)}$$

Find β such that $R_{\text{Th}} = -4\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{3000}{1 - 0.75\beta} = -4000$$

$$1 - 0.75\beta = -0.75 \qquad \therefore \quad \beta = 2.33$$

[b] Find V_{Th};



Write a KCL equation at the top node:

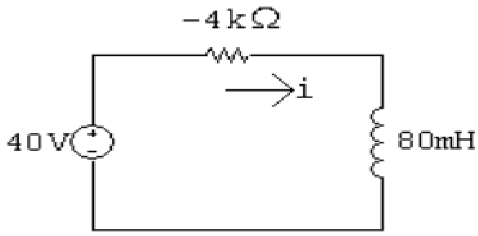
$$\frac{V_{\text{Th}} - \overset{4}{\cancel{3}}0}{4000} + \frac{V_{\text{Th}}}{12{,}000} - 2.33 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - \overset{4}{\cancel{3}}0)}{4000} = 0$$

Solving,

$$V_{\text{Th}} = 40\,\text{V}$$



Write a KVL equation around the loop:

$$40 = -4000 i + 0.08\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 500 + 50{,}000 i = \overset{50,000}{\cancel{25{,}000}}(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 50{,}000\, dt$$

$$\int_0^i \frac{dx}{x + 0.01} = \int_0^t 50{,}000\, dx$$

$$\therefore \quad i = -10 + 10e^{50{,}000t}\,\text{mA}$$

$$\frac{di}{dt} = (10 \times 10^{-3})(50{,}000)e^{50{,}000t} = 500e^{50{,}000t}$$

Solve for the arc time:

$$v = 0.08\frac{di}{dt} = 40e^{50{,}000t} = 30{,}000; \qquad e^{50{,}000t} = 750$$

$$\therefore \quad t = \frac{\ln 750}{50{,}000} = 132.4\,\mu\text{s}$$