VE215 RC WEEK 10

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Contents

Alternating Current(AC) Circuits:

Circuits driven by sinusoidal current or voltage sources

$$v(t) = V_m \sin(\omega t + \phi)$$

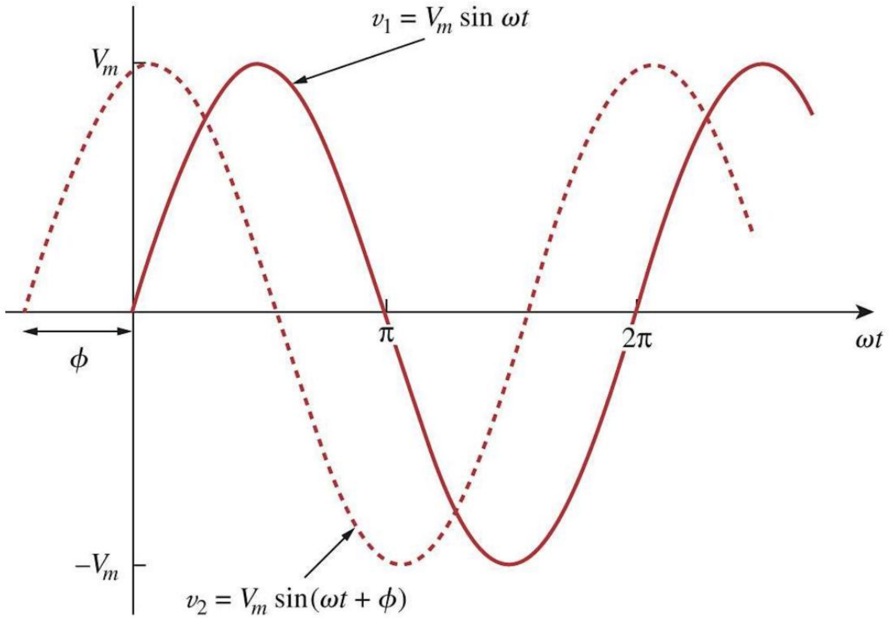
where

V_m : amplitude

ω: angular frequency

ϕ: initial phase

Leads & Lags:



Phasors:

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(\widetilde{V} e^{j\omega t})$$

where

$$\widetilde{V} = V_m e^{j\phi} = V_m \angle \phi$$

is the phasor representation of the sinusoidal signal

Phasor Diagram:

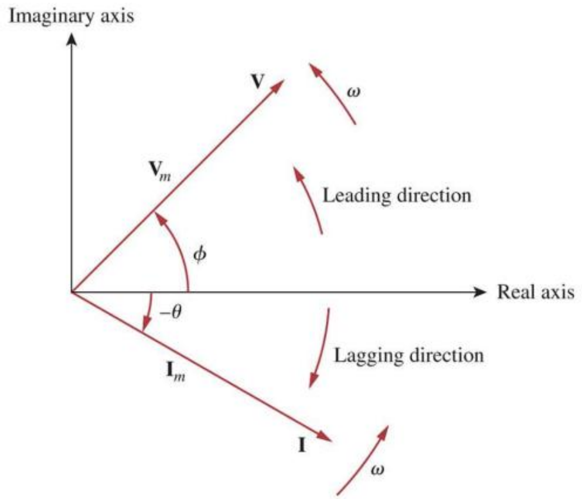


Figure 9.8 A phasor diagram.

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \widetilde{V} = V_m \angle \phi$$

TABLE 9.2

Summary of voltage - current relationships

Element	Time domain	Frequency domain
R	$v = Ri$	$\tilde{V} = R\tilde{I}$
L	$v = L\dfrac{di}{dt}$	$\tilde{V} = j\omega L\tilde{I}$
C	$i = C\dfrac{dv}{dt}$	$\tilde{V} = \dfrac{1}{j\omega C}\tilde{I}$

$$Z = R + jX = |Z|\angle\theta$$

where

R : resistance

X : reactance

If $X > 0$, we say that the impedance is inductive or lagging since current lags voltage; If $X < 0$, we say that the impedance is capacitive or leading because current leads voltage.

The impedance, resistance, and reactance are all measured in ohms.

About admittance and impedance:

$$Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{Z}$$

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The admittance can be written as

$Y = G + jB$

where

G: conductance

B: susceptance

The admittance, conductance, and susceptance are all measured in siemens.

About admittance and impedance:

$$Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{Z}$$

Equivalence Circuits

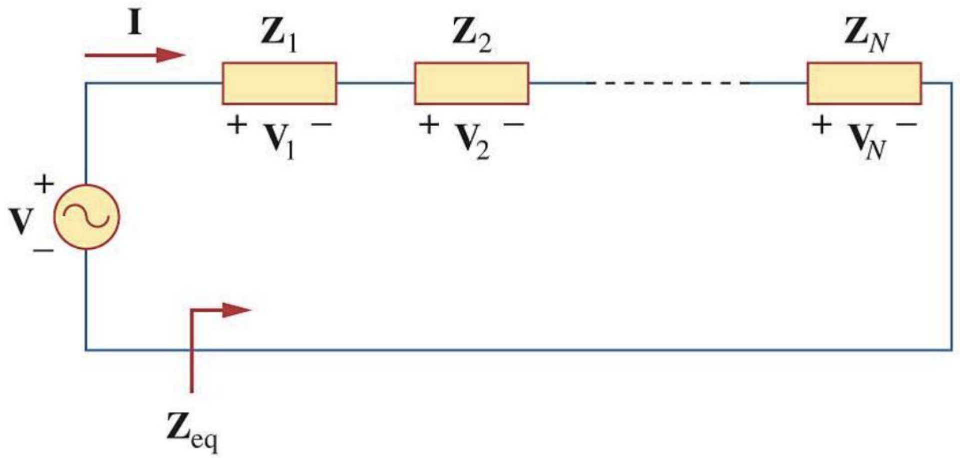


Figure 9.18 N impedances in series.

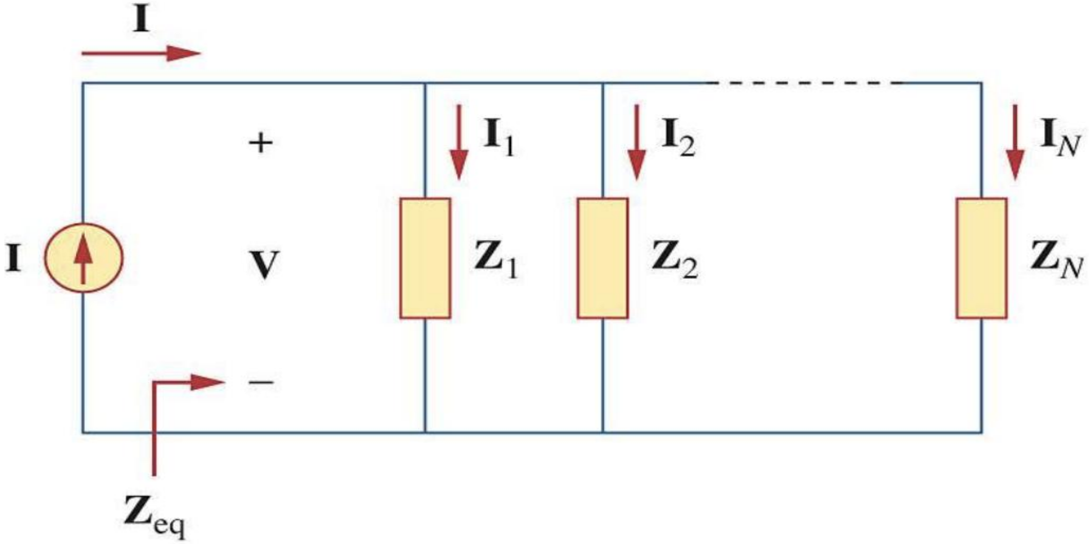


Figure 9.20 N impedances in parallel.

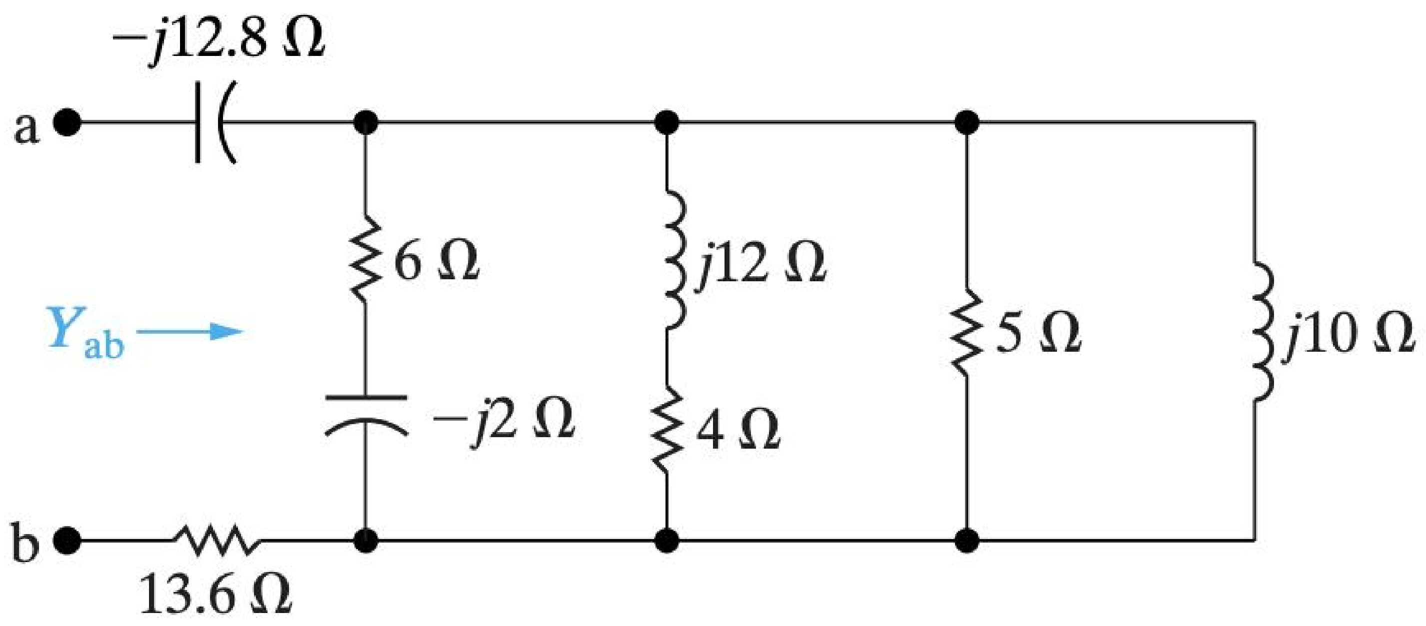
$$Z_{eq} = \frac{\tilde{V}}{\tilde{I}} = \frac{\boxed{\text{KVL}}\sum_{i=1}^{N} \tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} \frac{\tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} Z_i$$

$$Y_{eq} = \frac{\tilde{I}}{\tilde{V}} = \frac{\boxed{\text{KCL}}\sum_{i=1}^{N} \tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} \frac{\tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} Y_i$$

Exercise 1

9.23 Find the admittance Y_{ab} in the circuit seen in Fig. P9.23. Express Y_{ab} in both polar and rectangular form. Give the value of Y_{ab} in millisiemens.

Figure P9.23



First, find the admittance of paraller branches

$$Y_P = \frac{1}{6-2j} + \frac{1}{4+12j} + \frac{1}{5} + \frac{1}{10j} = 0.375 - 0.125j \ (S)$$

$$Z_P = \frac{1}{Y_P} = 2.4 + 0.8j \ (\Omega)$$

$$Z_{ab} = Z_P - 12.8j + 13.6 = 16 - 12j \ (\Omega)$$

$$Y_{ab} = \frac{1}{Z_{ab}} = [16 - 12j \ (\Omega)]^{-1} = 0.04 + 0.03j \ (S)$$

$$= 50 \angle 36.87^\circ \ (mS)$$

be careful with units

Exercise 2

Suppose we have following elements: 600 Ω resistors, 0.1μF capicitances, 10mH inductances. Please use the above elements to create an impedance of $300 - 400j\ \Omega$ at a frequency of 10,000 rad/s. The number of each elements is adequate.

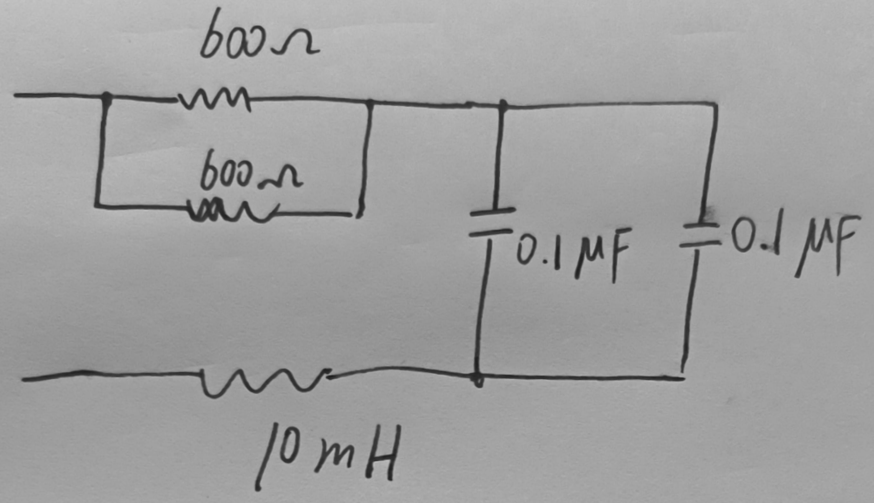
$R = 300\,\Omega = \dfrac{600^2}{600+600}\,\Omega$

$\omega L - \dfrac{1}{\omega C} = 10000L - \dfrac{1}{10^4 C} = -400$

$L = 10\,mH$

$\Rightarrow \dfrac{1}{10^4 C} = 100 + 400$

$\Rightarrow C = 0.2\,\mu F$, two $0.1\,\mu F$ in parallel.



Superposition in AC Circuits:

1. For DC voltage/current sources: take capacitance as an open and inductance as a short circuit.
2. For AC voltage/current sources: transform capacitance and inductance into impedance in frequency domain.
3. In AC circuit, when analyzing one specific source:

(1)Handle the capacitance and inductance following 1.&2. above, based on what type of the source is being analyzed.

(2)Then, take other voltage sources as a short circuit while other current sources as an open circuit.

Source Transformation and Equivalence Circuit:

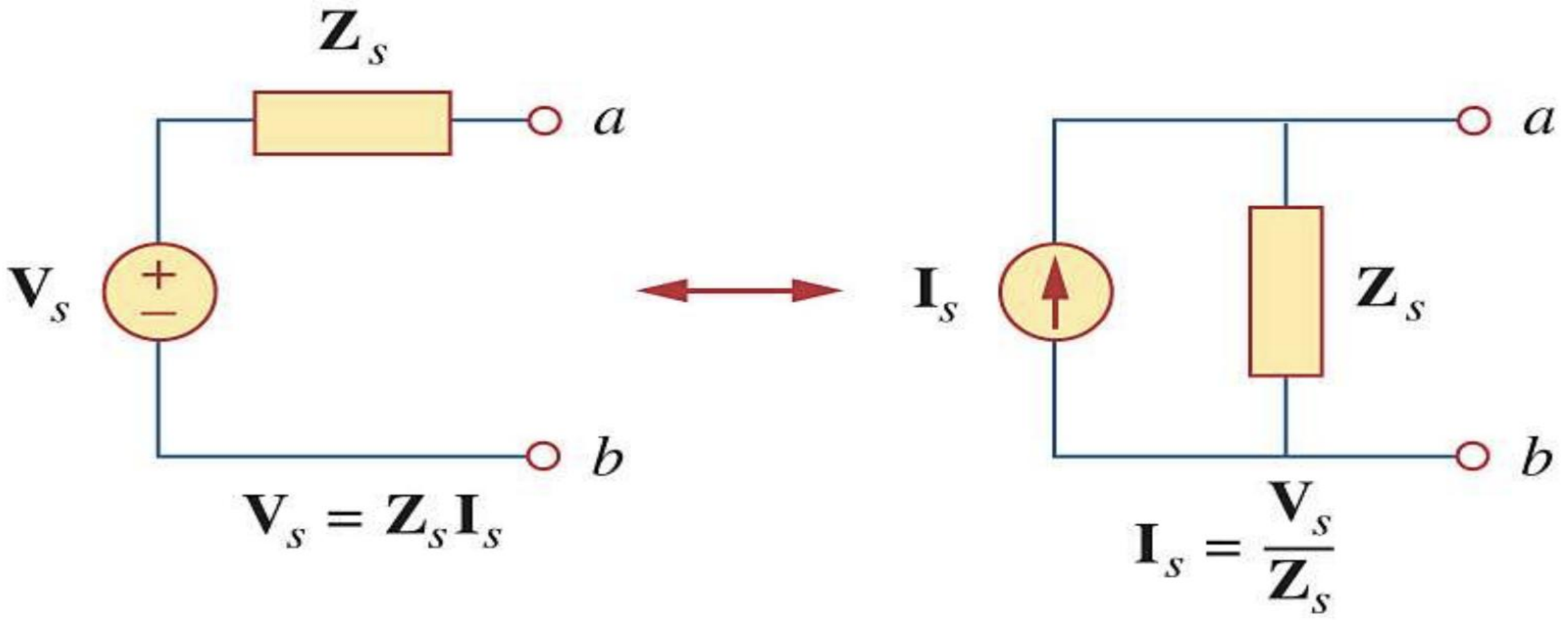
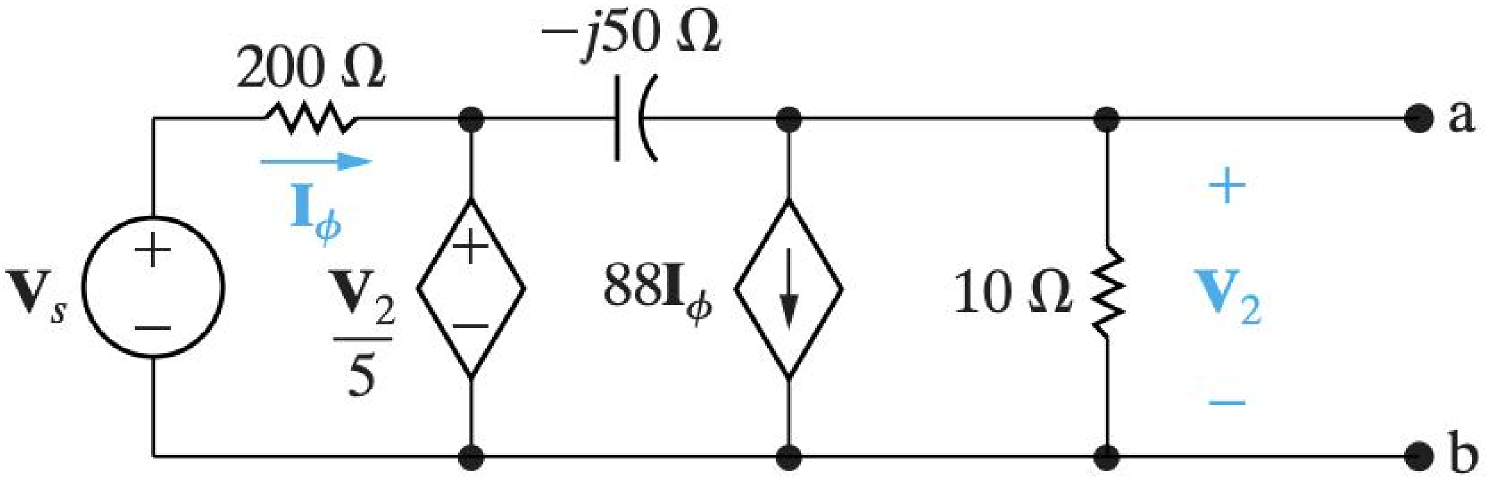


Figure 10.16 Source transformation.

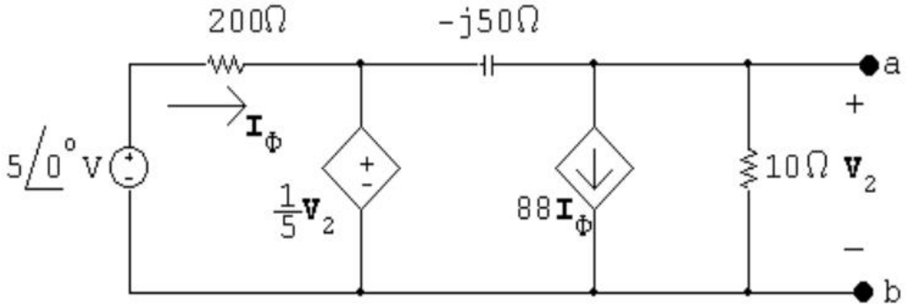
Exercise 3

Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.50 when $\mathbf{V}_s = 5\angle 0^\circ$ V.

Figure P9.50



P 9.50 Open circuit voltage:



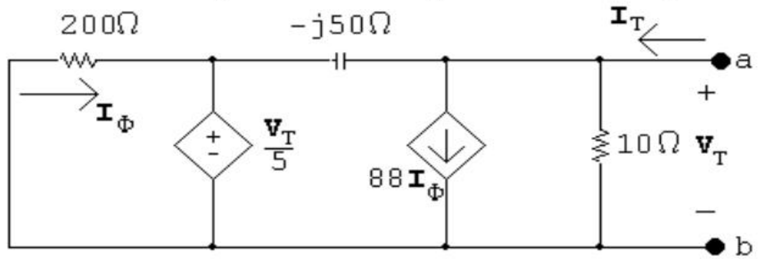
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\mathrm{V} = \mathbf{V}_{\mathrm{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

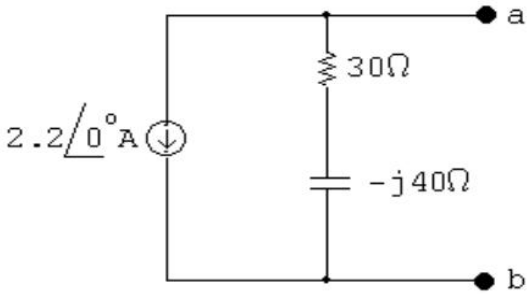
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\mathrm{Th}}$$

$$\mathbf{I}_{\mathrm{N}} = \frac{\mathbf{V}_{\mathrm{Th}}}{Z_{\mathrm{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\mathrm{A}$$

The Norton equivalent circuit:



Exercise 4

10.48 Find i_o in the circuit of Fig. 10.93 using superposition.

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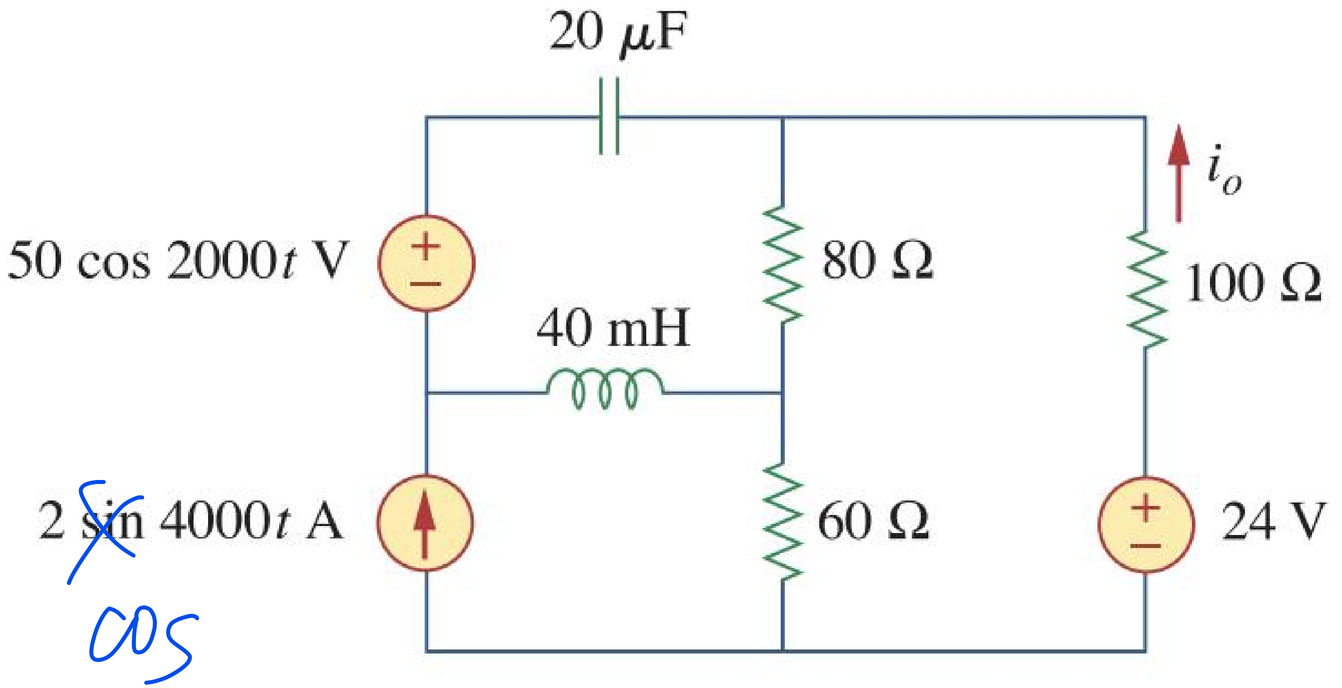
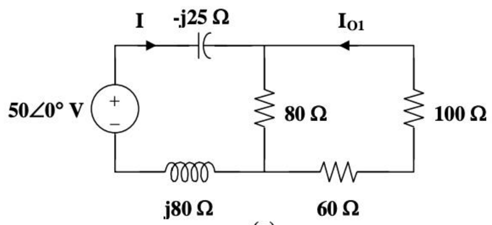


Chapter 10, Solution 48.

Let $i_o = i_{O1} + i_{O2} + i_{O3}$, where i_{O1} is due to the ac voltage source, i_{O2} is due to the dc voltage source, and i_{O3} is due to the ac current source. For i_{O1}, consider the circuit in Fig. (a).

$$\omega = 2000$$
$$50\cos(2000t) \longrightarrow 50\angle 0°$$
$$40\text{ mH} \longrightarrow j\omega L = j(2000)(40\times 10^{-3}) = j80$$
$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times 10^{-6})} = \text{-}j25$$

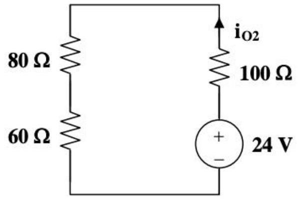


(a)

$$80 \,||\, (60+100) = 160/3$$
$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

Using current division,
$$\mathbf{I}_{O1} = \frac{\text{-}80\mathbf{I}}{80+160} = \frac{\text{-}1}{3}\mathbf{I} = \frac{10\angle 180°}{46\angle 45.9°}$$
$$\mathbf{I}_{O1} = 0.217\angle 134.1°$$
Hence, $\quad i_{O1} = 0.217\cos(2000t + 134.1°)$ A

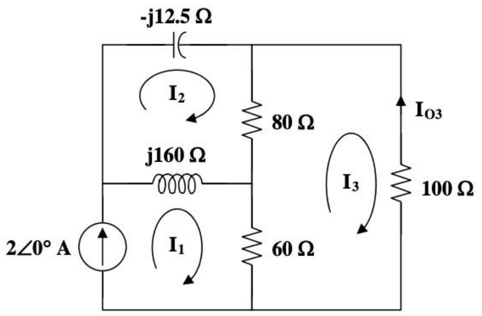
For i_{O2}, consider the circuit in Fig. (b).



(b)

$$i_{O2} = \frac{24}{80+60+100} = 0.1\text{ A}$$

For i_{O3}, consider the circuit in Fig. (c).
$$\omega = 4000$$
$$2\cos(4000t) \longrightarrow 2\angle 0°$$
$$40\text{ mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$$
$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = \text{-}j12.5$$



(c)

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For mesh 1,
$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,
$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields
$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \qquad (2)$$

For mesh 3,
$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields
$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \qquad (3)$$

Substituting (3) into (2) yields
$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$
$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$

$$\mathbf{I}_{O3} = \text{-}\mathbf{I}_3 = \text{-}1.1782\angle 7.38°$$
Hence, $\quad i_{O3} = \text{-}1.1782\,\cancel{\sin}(4000t + 7.38°)$ A

cos

Therefore,
$$i_o = \{\mathbf{0.1 + 0.217\cos(2000t + 134.1°) - 1.1782\,\cancel{\sin}(4000t + 7.38°)}\}\text{ A}$$

cos

Tips for this part:

1. Get familiar with basic concepts

2. Get familiar with your calculator

THANK YOU