ECE2150J 2024 FA Assignment 6

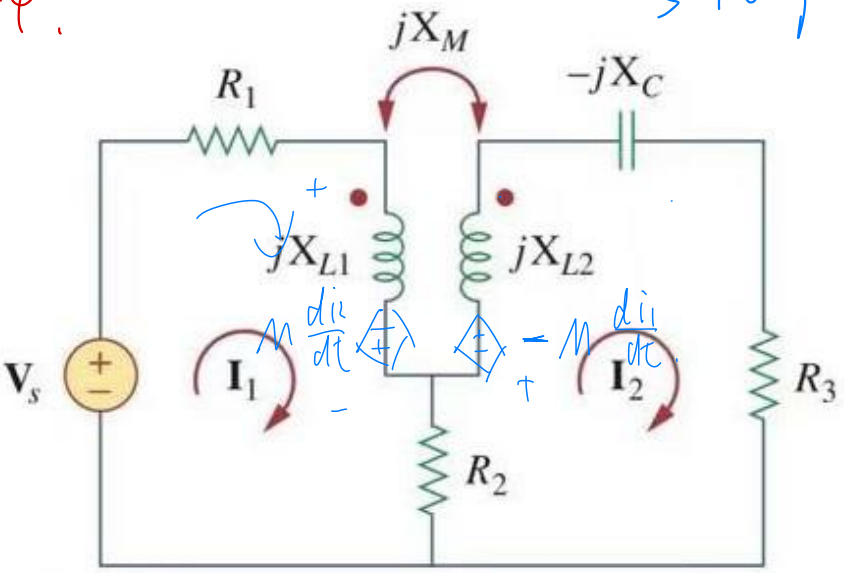
Due Date: 23:59 December 20th

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated.

Exercise 6.1 (25%)

$V_S = 10\cos(4t+\pi/4)$, $R1 = R2 = 5\Omega, R3 = 10\Omega, X_{L1} = 15\Omega, X_{L2} = 20\Omega, X_M = 2\Omega, X_C = 0.5\Omega$.

Find I_1 and I_2.



$$\begin{cases} -V_S + I_1 R_1 + I_1 \cdot jX_{L1} - jX_M \cdot I_2 + R_2(I_1 - I_2) = 0 \\ (I_2 - I_1)R_2 - jX_M \cdot I_1 + jX_{L2} \cdot I_2 - I_2 \cdot jX_C + I_2 \cdot R_3 = 0 \end{cases}$$

$$\begin{cases} (R_1 + X_{L1}j + R_2)I_1 + (-X_M j - R_2)I_2 = V_S \\ (-R_2 - X_M j)I_1 + (R_2 + X_{L2}j + R_3 - X_C j)I_2 = 0 \end{cases}$$

$$\begin{pmatrix} 10+15j & -5-2j \\ -5-2j & 15+20j \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 10\angle\phi \\ 0 \end{pmatrix}$$

$I_1 = 0.569\angle -14.8°$ $\quad I_2 = 0.125\angle -65.4°$

$i_1 = 0.569\cos(4t - 0.26)$. $\quad i_2 = 0.125\cos(4t - 80)$.

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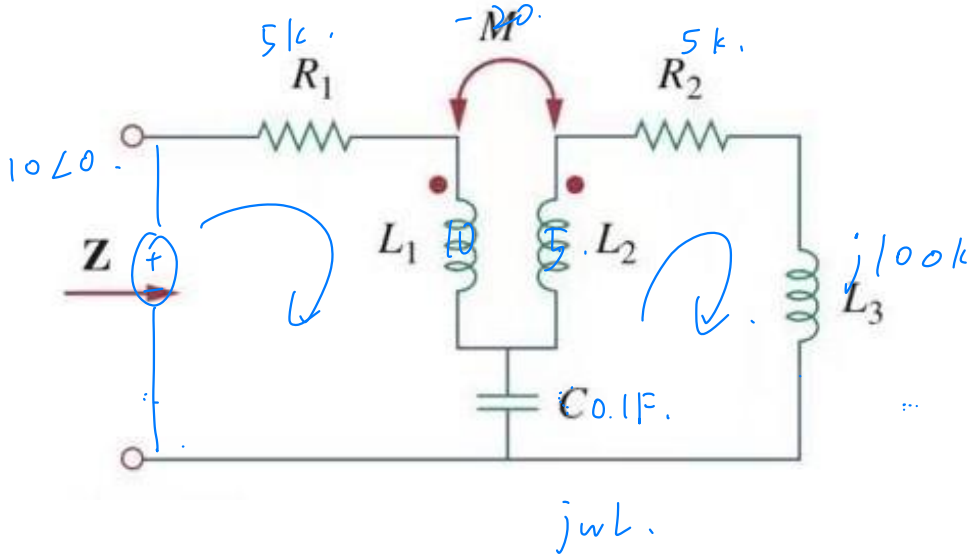
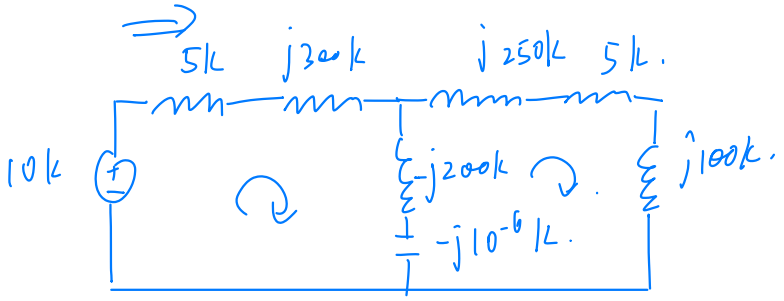
Exercise 6.2 (25%)

$R_1 = R_2 = 5\ k\Omega, M = 20\ H, L_1 = 10\ H, L_2 = 5\ H, L_3 = 10\ H, C = 0.1\ F$. Suppose frequency of source is $\omega = 10\ kHz$. Find the equivalent impedance Z. All currents flow clockwise.



jwL.



$$\begin{cases} -10k + i_1(5k + j300k) + (i_1 - i_2)(-j200k - j10^{-6}k) = 0 \\ (i_2 - i_1)(-j200k - j10^{-6}k) + i_2(5k + j350k) = 0 \end{cases}$$

$$\begin{pmatrix} 5k + j(100k - 10^{-6}k) & j10^{-6}k + j200k \\ j10^{-6}k + j200k & 5k + j(150k - 10^{-6}k) \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 10k \\ 0 \end{pmatrix}$$

$$i_1 = \frac{50 + (3500 - 10^{-5})j}{2525 + 1250j} = 0.0685.23^\circ\ A$$

$$Z = \frac{V}{i_1} = 1.67 \times 10^5 \angle -85.23^\circ\ \Omega$$

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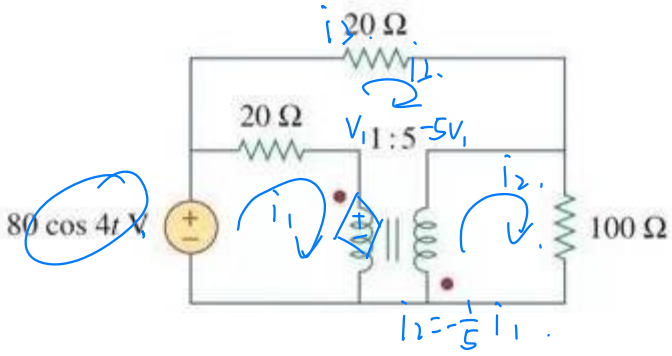
Exercise 6.3 (25%)

Determine the average power absorbed by each resistor in this circuit. All currents flow clockwise.



$$\begin{cases} -80 + 20(i_1 - i_3) + V_1 = 0 \\ -V_2 + i_2 \cdot 100 = 0 \\ (i_3 - i_1)\cdot 20 + i_3 \cdot 20 + V_2 - V_1 = 0 \\ (i_1 - i_3) = -5(i_2 - i_3) \\ V_2 = -5V_1 \end{cases}$$

$$\begin{cases} 20i_1 - 20i_3 + V_1 = 80 \\ 100i_2 + 5V_1 = 0 \\ -20i_1 + 40i_3 - 6V_1 = 0 \\ i_1 + 5i_2 - 6i_3 = 0 \end{cases}$$

$$\begin{cases} i_1 = 5.94 \\ i_2 = 0.52 \\ i_3 = 1.42 \end{cases}$$

for 20Ω on the top: $P = \frac{1}{2} i_3^2 R = 20.1$ W.

for 20Ω on the center: $P = \frac{1}{2}(i_1 - i_3)^2 R = 203.25$ W.

for 20Ω on the right: $P = \frac{1}{2} i_3^2 R = 13.3$ W.

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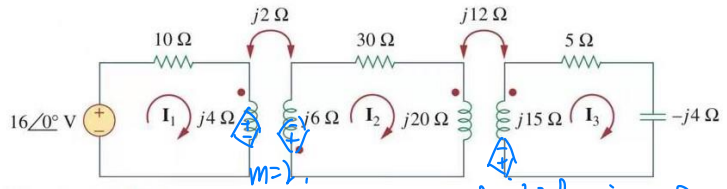
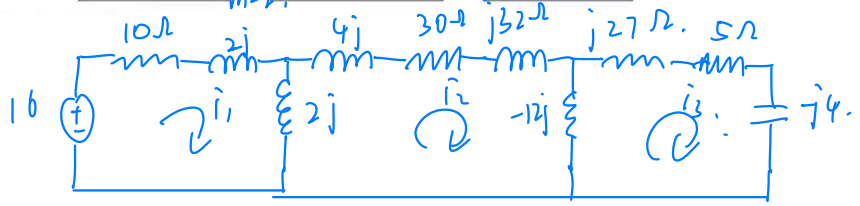
Exercise 6.4 (25%)
Determine I_1, I_2, I_3 in this circuit.





$$-16 + (10+2j)i_1 + 2j \cdot i_2 = 0.$$

$$2j \cdot i_1 + i_2(30+26j) - i_3 \cdot 12j = 0.$$

$$i_2 \cdot 12j + (5+j11) \cdot i_3 = 0.$$

$$\begin{pmatrix} (10+4j) & -2j & 0 \\ 2j & 30+26j & -12j \\ 0 & 12j & 5+11j \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ 0 \end{pmatrix}.$$

$i_1 = 1.48\angle -21.4^\circ$ A

$i_2 = 7.75 \times 10^{-2} \angle -134.7$ A

$i_3 = 7.70 \times 10^{-2} \angle -110.4$ A.