# VE215 2024FA Assignment 3

## Due Date: 23:59, Nov.18th, 2024

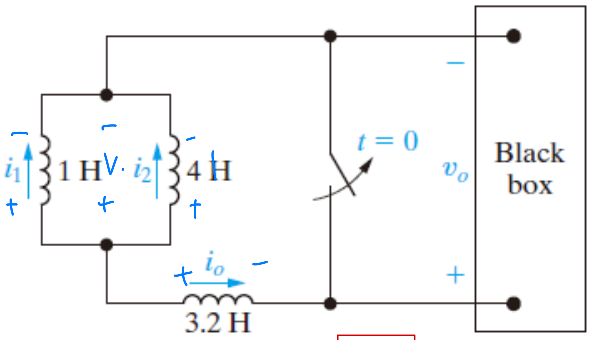
In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated. This assignment covers content from chapter 6 to 8.

**Exercise 3.1 (20%)**

The three inductors in the circuit are connected across the terminals of a black box at $t = 0$. The resulting voltage for $t > 0$ is known to be

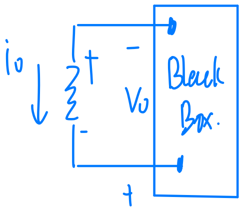
$$v_o = 2000e^{-100t}\,\text{V}.$$

If $i_1(0) = -6\,\text{A}$ and $i_2(0) = 1\,\text{A}$, find
(a) $i_o(0)$;
(b) $i_o(t)$, $t \geq 0$; (Hint: Calculate the equivalent inductance $L_{eq}$ first.)
(c) $i_1(t)$, $t \geq 0$;
(d) $i_2(t)$, $t \geq 0$;
(e) the initial energy stored in the three inductors;




(a) $i_0(0) = -(i_1(0) + i_2(0)) = 6A - 1A = \boxed{5A}$.

(b) $t \geq 0$.




$L_{eq} = 1\|4 + 3.2 = 4\,H$

$V_0 = -V_L = -L\frac{di_0}{dt}$

$di_0 = -\frac{1}{L}V_0\,dt$

$i_0 = -\frac{1}{4}\int 2000e^{-100t}\,dt$

$= 5\int e^{-100t}\,d(-100t)$

$= 5e^{-100t} + C$

$i_0(0) = 5 \quad C = 0$

$\boxed{i_0 = 5e^{-100t}}$

(c) when $t > 0$.

$V = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$

$V - V_0 - V_H = 0$

$V_H = L_H \cdot \frac{di_0}{dt}$

$= 3.2(-500)e^{-100t}$

$= -1600e^{-100t}$

$V = 400e^{-100t}$

$i_1 = 400\int e^{-100t}\,dt + C$

$= -4e^{-100t} + C$

$i_1(0) = -4 + C = -6$

$C = -2$

$\boxed{i_1 = -4e^{-100t} - 2}$

(d)

$i_2 = 100\int e^{-100t}\,dt + C$

$= -e^{-100t} + C$

$i_2(0) = -1 + C = 1$

$C = 2$

$\boxed{i_2(t) = -e^{-100t} + 2}$

(e) $W = \frac{1}{2}Li^2$

$= \frac{1}{2}\cdot 1\cdot 6^2 + \frac{1}{2}\cdot 4\cdot 1^2 + \frac{1}{2}\cdot 3.2\cdot 5^2$

$= \boxed{60\,J}$

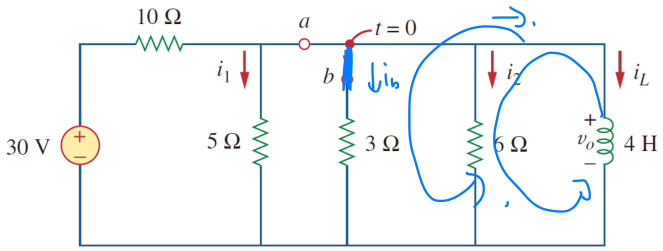
**Exercise 3.2 (20%)**

In the circuit below, assume that the switch has been in position $a$ for a long time, find:

(a) $i_1(0)$, $i_2(0)$, and $v_o(0)$

(b) $i_L(t)$

(c) $i_1(\infty)$, $i_2(\infty)$, and $v_o(\infty)$




(a) $\boxed{i_1(0) = i_2(0) = 0A. \quad V_o = L\cdot\frac{di}{dt} = 0V.}$

(b) $i_L(0) = \frac{30}{10} A = 3A.$

$i_b + i_2 + i_L = 0$

$\frac{V}{3} + \frac{V}{6} + i_L = 0.$

$\frac{4}{3}\cdot\frac{di}{dt} + \frac{2}{3}\frac{di}{dt} + i_L = 0.$

$\frac{di_L}{dt} = -\frac{1}{2} i_L.$

$\ln|i_L| = -\frac{1}{2}t + C.$

$i_L = e^{-\frac{1}{2}t + C}.$

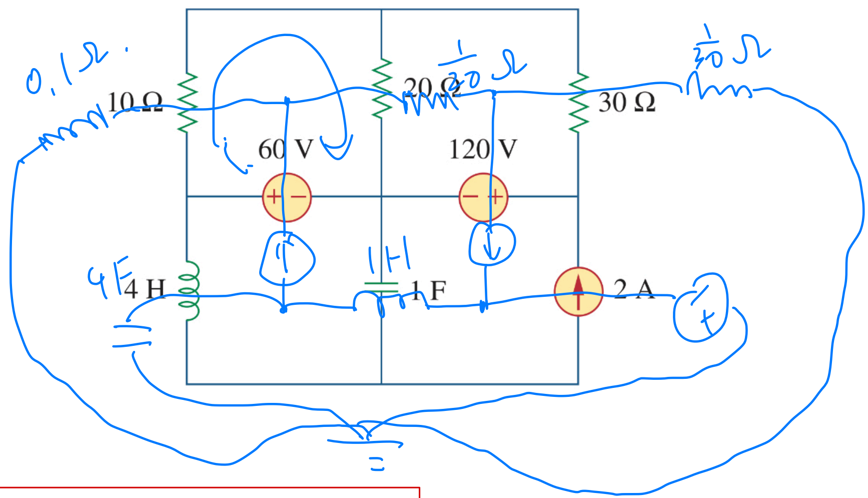
$e^C = 3.$

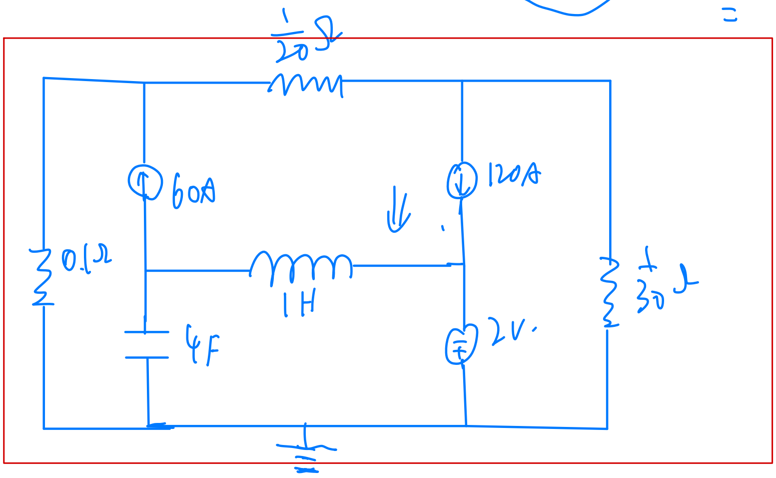
$\boxed{i_L = 3\cdot e^{-\frac{1}{2}t} \quad (t \geq 0)}.$

(c) $\boxed{i_1(\infty) = \frac{30}{15} = 2A, \quad i_2(\infty) = 0A, \quad V_o(\infty) = 0V.}$

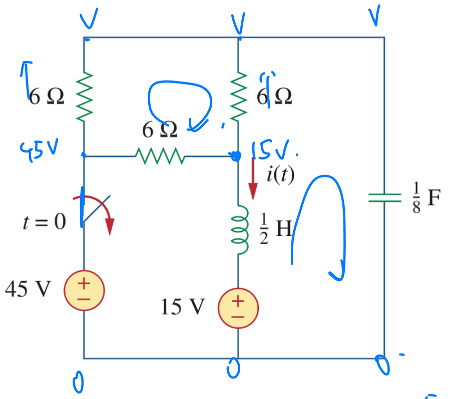
**Exercise 3.3 (10+10%)**

(a) (10%) Find the dual of the circuit below.



(b) (10%) For the network below, solve for $i(t)$ for $t > 0$.




when $t<0$.

$\frac{45-V_{(0)}}{6}+\frac{15-V_{(0)}}{6}=0.$

$V_{(0)}=30V.$

$i_0+\frac{15-30}{6}+\frac{15-45}{6}=0.$

$i_0=7.5A.$

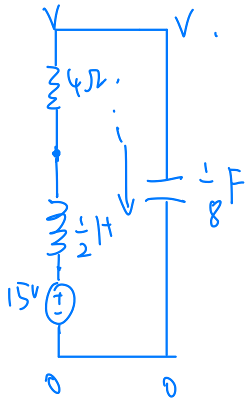
$-15+\frac{1}{2}i'_{(0)}-30+30=0$

$i'_c(0)=30$

$i_c=7.5e^{-4t}.$

$30e^{-4t}+C_2e^{-4t}-4C_1te^{-4t}=30$

when $t>0$.




$$\begin{cases} i_c=\frac{1}{8}\frac{dv}{dt}. \\ -15+\frac{1}{2}\cdot\frac{di_c}{dt}+i_c4+V=0. \end{cases}$$

$\frac{di_c}{dt}+8i_c+16\int_0 i\,dt=30.$

$\frac{d^2i_c}{dt^2}+8\frac{di_c}{dt}+16=0.$

$i_c(t)=C_1e^{-4t}+C_2te^{-4t}.$

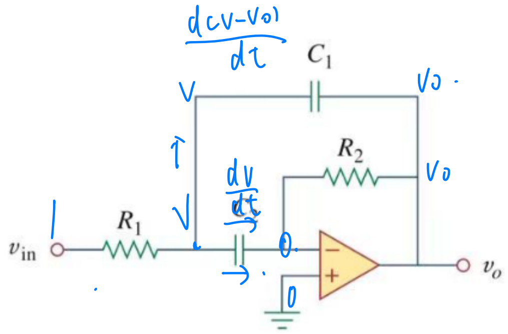
$C_2e^{-4t}=0$

$C_2=0.$

$$\boxed{i=-i_c=7.5e^{-4t}.}$$

**Exercise 3.4 (20%)**

In the op amp circuit of Fig. 3.5, determine $v_0(t)$ for t > 0. Let $v_in$ = u(t)V, $R_1$=$R_2$=10k ohm, $C_1$=$C_2$= 100$\mu$F.




when $t<0$.

$V_0=0.$

when $t>0$.

$$\begin{cases} \frac{V-1}{R_1}+C_2\frac{dv}{dt}+C_1\frac{d(V-V_0)}{dt}=0. \\ C_2\frac{dv}{dt}=-\frac{V_0}{R_2}. \end{cases}$$

$2C\frac{dv}{dt}$  $2\times\frac{-V_0}{R_2}$

$V=\frac{1}{C_2R_2}\int V_0\,dt.$

$C_1=0.$

$-\frac{1}{R_1}-C_1\frac{dv_0(0^+)}{dt}=0.$

$\frac{dv_0(0^+)}{dt.}=\frac{1}{C_1R_1}=-1.$

$-\frac{1}{C_2R_1R_2}\int V_0\,dt-\frac{1}{R_1}-\frac{2}{R_2}V_0-C_1\frac{dV_0}{dt}=0.$

$C_2e^{-t}-C_1te^{-t}=-1.$

when $t=0.$

$C_2=-1.$

$\frac{d^2V_0}{dt^2}+\frac{2}{R_2C}\frac{dV_0}{dt}+\frac{1}{C_1C_2R_1R_2}V_0=0$

$\frac{d^2V_0}{dt^2}+2\frac{dV_0}{dt}+V_0=0.$

$V_0=C_1e^{-t}+C_2te^{-t}.$

$$\boxed{V_0=-te^{-t}\cdot(t>0).}$$

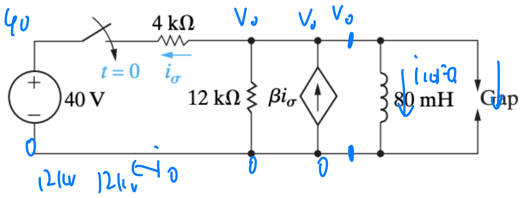
**Exercise 3.5 (20%)**

The gap in the circuit seen in the figure below will arc over whenever the voltage across the gap reaches 30 kV. The initial current in the inductor is zero. The value of $\beta$ is adjusted so the Thévenin equivalent resistance with respect to the terminals of the inductor is -4 kΩ. Now please answer the following two questions below.

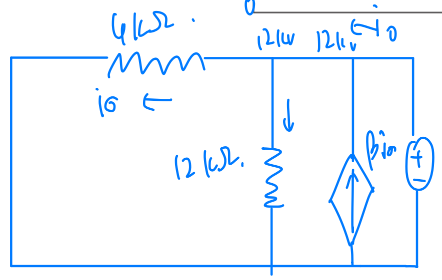
(a)What is the value of $\beta$ ?

(b)After the switch has been closed, how long will it take for the gap to arc over?

Tip: For this question, the gap can be treated as an open circuit before the voltage across it reaches 30 kV, and you do not need to know what "arc over" really is .




(a)




$$\begin{cases} 3+1 = i_0 + 3\beta \\ \frac{12k}{i_0} = -4k \end{cases}$$

$$\boxed{\begin{aligned} i_0 &= -3 \\ \beta &= \frac{7}{3} = 2.33 \end{aligned}}$$

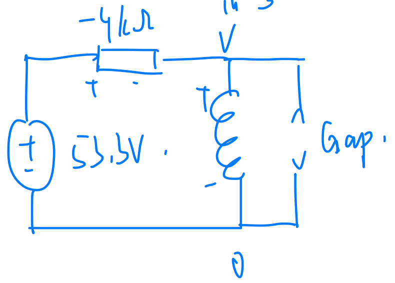
(b):

$$\frac{V_{Th}-40}{4k} + \frac{V_{Th}}{12k} = \frac{7}{3}\cdot\frac{V_{Th}-40}{4k}$$

$$\frac{4}{3}\frac{V_{Th}-40}{4k} = \frac{V_0}{12k}$$

$$4V_{Th} - 160 = V_{Th}$$

$$V_{Th} = \frac{160}{3} \approx 53.3\text{V}$$




$50\times 80$

$4000$

$$-53.3 - 4k\,i + 80\cdot 10^{-3}\cdot\frac{di}{dt} = 0$$

$$\frac{di}{dt} = 4k\cdot\frac{1}{8}\cdot 10^{2}\, i + \frac{160}{3}\cdot\frac{1}{8}\cdot 10^{2}$$

$$\int_0^{i_0}\frac{di}{i+\frac{1}{75}} = \int_0^t 5\times10^4\,dt$$

$$\ln\left(\frac{i_1+\frac{1}{75}}{\frac{1}{75}}\right) = 5\times10^4 t$$

$$75 i_1 + 1 = e^{5\times10^4 t}$$

$$i_1 = \frac{1}{75}e^{5\times10^4 t} - \frac{1}{75}$$

$$V = L\frac{di}{dt} = 80\cdot10^{-3}\cdot\frac{5\times10^4}{75}\cdot e^{5\times10^4 t} = \frac{160}{3}\cdot e^{5\times10^4 t} = 30000$$

$$\boxed{t = 126.6\,\mu s}$$