

VE215 RC1

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October 9, 2024

Overview

General Information

Basic Concepts

Basic Laws

Methods of Analysis

Course Structure

- ▶ Goal: analyze the circuits, from simple to complex.
- ▶ Structure:
 1. Chap. 1-8: DC circuits (the circuits driven by constant current/voltage sources)
A variety of analysis tools
 - introducing some new circuits components
 - analyze circuits with those complex components added
 2. Chap. 9-14: AC circuits (the circuits driven by alternating current/voltage sources)

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Current

$$i \triangleq \frac{dq}{dt}$$

$$Q \triangleq \int_{t_0}^t i dt$$

Reference direction of current

In solving problems, it does not matter which direction we initially assume. If we obtain a result of negative current, it indicates that the actual direction is opposite to that we have initially assumed.



(a)



(b)

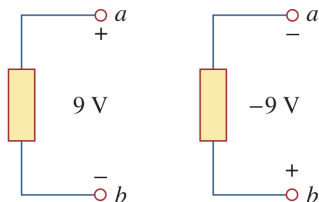
Voltage

$$V_{ab} = V_a - V_b$$

$$V_{ab} \triangleq \frac{dw}{dq}$$

Reference direction of voltage

In solving problems, it does not matter how we assign the “+/-” signs to two terminals of a circuit element. The two representations below are equivalent.

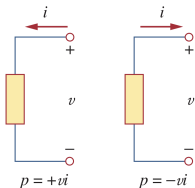


Power and Energy

$$p = \frac{dw}{dt} = vi \quad w = \int_{t_0}^t vidt$$

Passive sign convention w.r.t. power:

- ▶ Currents enter through the positive terminal: $p = +vi$
- ▶ Currents enter through the negative terminal: $p = -vi$

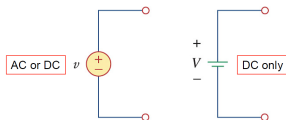


Power and energy consumption:

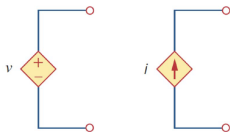
- ▶ $p > 0$, element consumes energy.
- ▶ $p < 0$, element generates energy.

Circuit Elements

- ▶ **Active elements:** can generate energy
e.g., generators, batteries, operational amplifiers
 - ▶ **independent source:** the source whose quantity is uninfluenced by its “surroundings”.



- ▶ **dependent source:** source quantity is controlled by another voltage or current in the circuit.



- ▶ **Passive elements:** cannot generate energy,
e.g., resistors, capacitors, inductors

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Nodes, Meshes and Loops

Branch: a single element, such as a voltage source or a resistor

Node: the point of connection between two or more branches

Loop: any closed path in a circuit

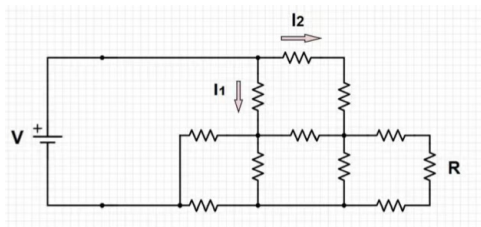
- ▶ **Mesh:** a loop that does not enclose any other loops, i.e., smallest loop
- ▶ **Independent loop:** a loop that contains at least one branch which is not a part of any other independent loop

Fundamental theorem of network topology:

$$b \text{ (branches)} = l \text{ (mesh)} + n \text{ (nodes)} - 1$$

Exercise 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?
2. Count the number of nodes, branches, meshes, loops in the following figure.



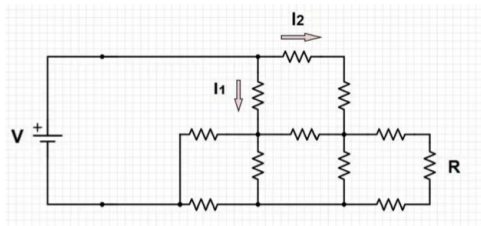
Exercise 1.

1. Suppose there are 3 meshes and 6 branches in one circuit. How many nodes in it?

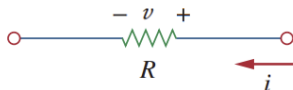
Answer: 4

2. Count the number of nodes, branches, meshes in the following figure.

Answer: 8,12,5



Ohm's Law



Ohm's law:

$$V = IR \quad R = \frac{v}{i}$$

Passive sign convention for Ohm's law:

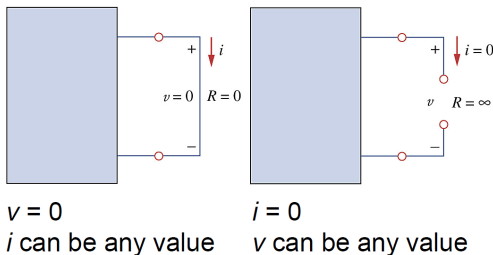
- ▶ i enters through the positive terminal: $v = iR$
- ▶ i enters through the negative terminal: $v = -iR$

Not all resistors obey Ohm's law!

A resistor that obeys Ohm's law is known as a **linear resistor**, i.e., a constant resistance.

Resistance with extreme values

1. Short circuit: a circuit element with resistance approaching zero.
2. Open circuit: a circuit element with resistance approaching infinity.



Conductance

$$G = \frac{1}{R} = \frac{i}{v}, \quad 1S = 1\mathcal{U} = 1A/V$$

where **G** is the conductance, **S** (siemens) is the SI unit of conductance and \mathcal{U} is the reciprocal ohm.

some useful formula:

$$i = Gv, p = vi = i^2 R = \frac{v^2}{R} = v^2 G = \frac{i^2}{G}$$

Kirchhoff's Law

Kirchhoff's Law	Expression	Based on
KCL	$\sum i_k = 0$ for a node	Conservation of charge
KVL	$\sum v_k = 0$ for a mesh	Conservation of energy

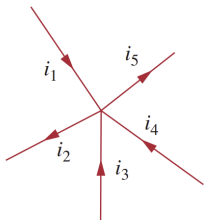


Figure: KCL

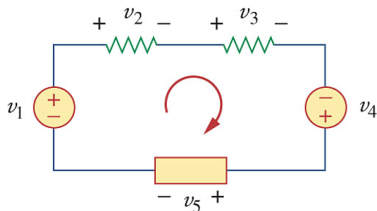


Figure: KVL

KCL: the algebraic sum of currents entering **a node (or a closed boundary)** is zero.

Steps of applying KCL:

1. Find out all branches connected to the node of interest.
2. Specify **reference** direction for current on each branch.
3. Find all $i_k (k = 1, 2, \dots, n)$ (Ohm's law $i = \frac{v_a - v_b}{R}$ for linear resistors).
4. List the KCL equation $\sum_k i_k = 0$.

KVL: the algebraic sum of all voltages around **a closed path (or loop)** is zero.

Steps of applying KVL:

1. Select reference KVL direction (clockwise by convention).
2. Confirm/specify the $+/-$ terminal of each branch.
3. Find $v_k (k = 1, 2, \dots, n)$ for each branch.
4. List the KVL equation $\sum_k v_k = 0$. Mind that by passive sign convention, the sign in front of a certain term v_k is
 - ▶ “+” if the reference KVL direction enters through the positive terminal of the branch.
 - ▶ “-” if the reference KVL direction enters through the negative terminal of the branch.

Series connection and Parallel connection

R_{eq} : the **equivalent resistance**

1. **Series** connection:

$$R_{eq} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^N R_n$$

Principle of voltage division: $v_n = \frac{R_n}{\sum_{n=1}^N R_n} v$

2. **Parallel** connection:

$$G_{eq} = \frac{1}{R_{eq}} = G_1 + G_2 + \dots + G_N = \sum_{n=1}^N G_n$$

Principle of voltage division: $i_n = \frac{G_n}{\sum_{n=1}^N G_n} i$

Wye-Delta Transformation

- Motivation: simplify the circuits for easier calculation.
- Two forms of special circuit connections:

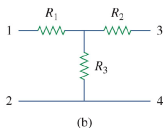
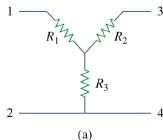


Figure: Wye (Y or T)

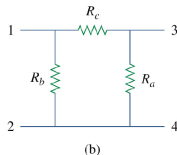
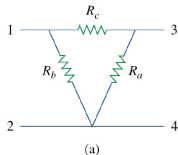


Figure: Delta (Δ or π)

- Goal: transform one type of connection into another.

Wye-Delta Transformation

$$\begin{cases} R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \\ R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \\ R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \end{cases}$$

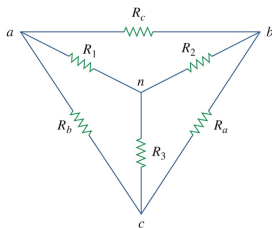


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: $\Delta - Y$

Intuition: parallel \rightarrow series, resistance for each element decreases.

Wye-Delta Transformation

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

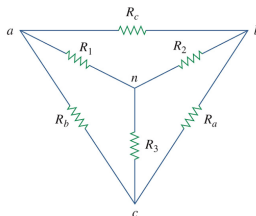


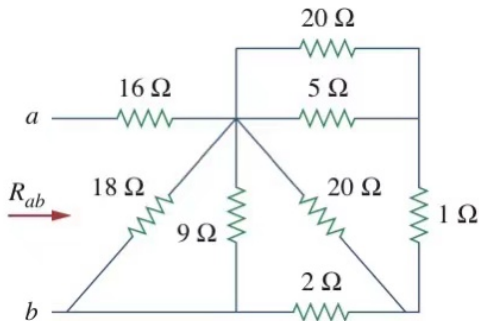
Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: Y-Δ

Intuition: series \rightarrow parallel, resistance for each element increases.

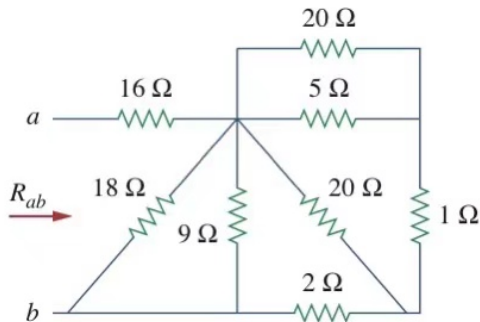
Exercise 2.

Calculate the equivalent resistance R_{ab} in the circuit



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Calculate the equivalent resistance R_{ab} in the circuit

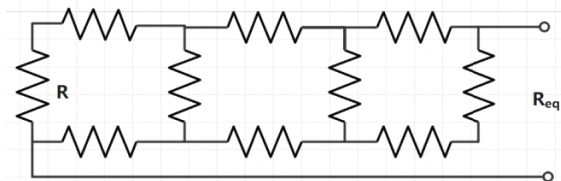


Answer: $19\ \Omega$

Exercise 3.

6. Suppose the resistance of all the resistors is R , what's the equivalent resistance R_{eq} ?

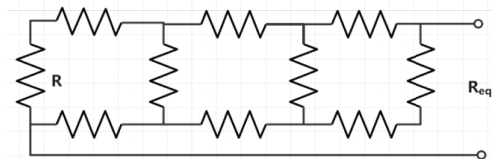
(4 marks)



Exercise 3.

6. Suppose the resistance of all the resistors is R , what's the equivalent resistance R_{eq} ?

(4 marks)



Answer: $0.5 + \frac{0.5 + 1 + 0.25}{2} + 0.5 = \frac{15}{8} R$

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Nodal Analysis:

1. Select a reference node (ground)
2. Apply KCL
3. Solve the equations

Mesh Analysis:

1. Mark the current of all the meshes
2. Apply KVL
3. Solve the equations

Analysis by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j , $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all **independent** current sources directly connected to node k , with currents entering the node treated as positive

For Nodal Analysis

(only current source in circuit)

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j , $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

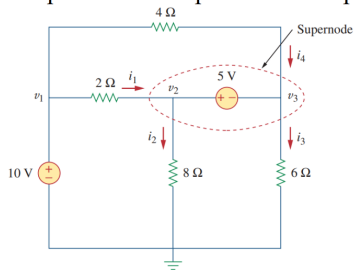
v_k = Sum taken clockwise of all **independent** voltage sources in mesh k , with voltage rise treated as positive

For Mesh Analysis

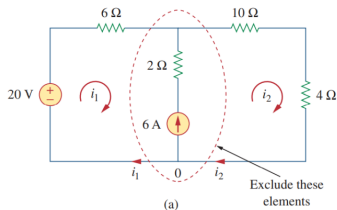
(only voltage source in circuit)

Supernode & Supermesh

- Supernode & Supermesh – simplify the equation

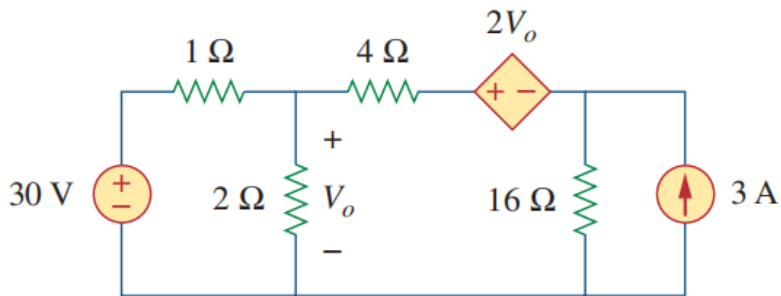


$$i_1 - i_2 - i_3 + i_4 = 0$$

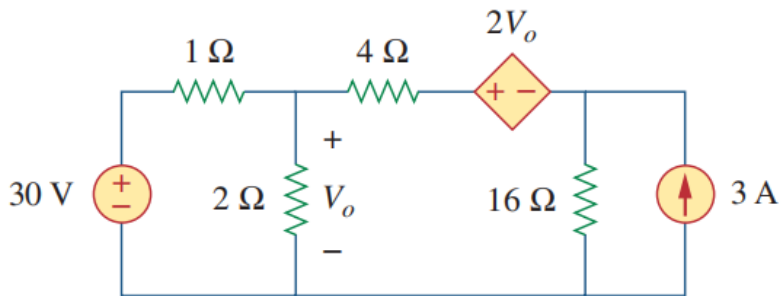


$$20 - 6i_1 - 14i_2 = 0$$

Exercise 4.



Exercise 4.



Answer: $\frac{648}{29}\text{ V} \approx 22.34\text{ V}$

References

1. 2024Fall VE215 slides
2. 2023Fall RC1,RC2, Chongye Yang

Thank you!