ECE2150J Homework 4

Deadline: 7th December 2024

Problem 1

(a) (10%) Obtain the equivalent admittance Y_{in} of the circuit at $\omega = 100\,\text{rad/s}$.

(b) (10%) Obtain the equivalent impedance Z_{in} of the circuit.

(c) (10%) Obtain the equivalent impedance Z_{ab} of the circuit.

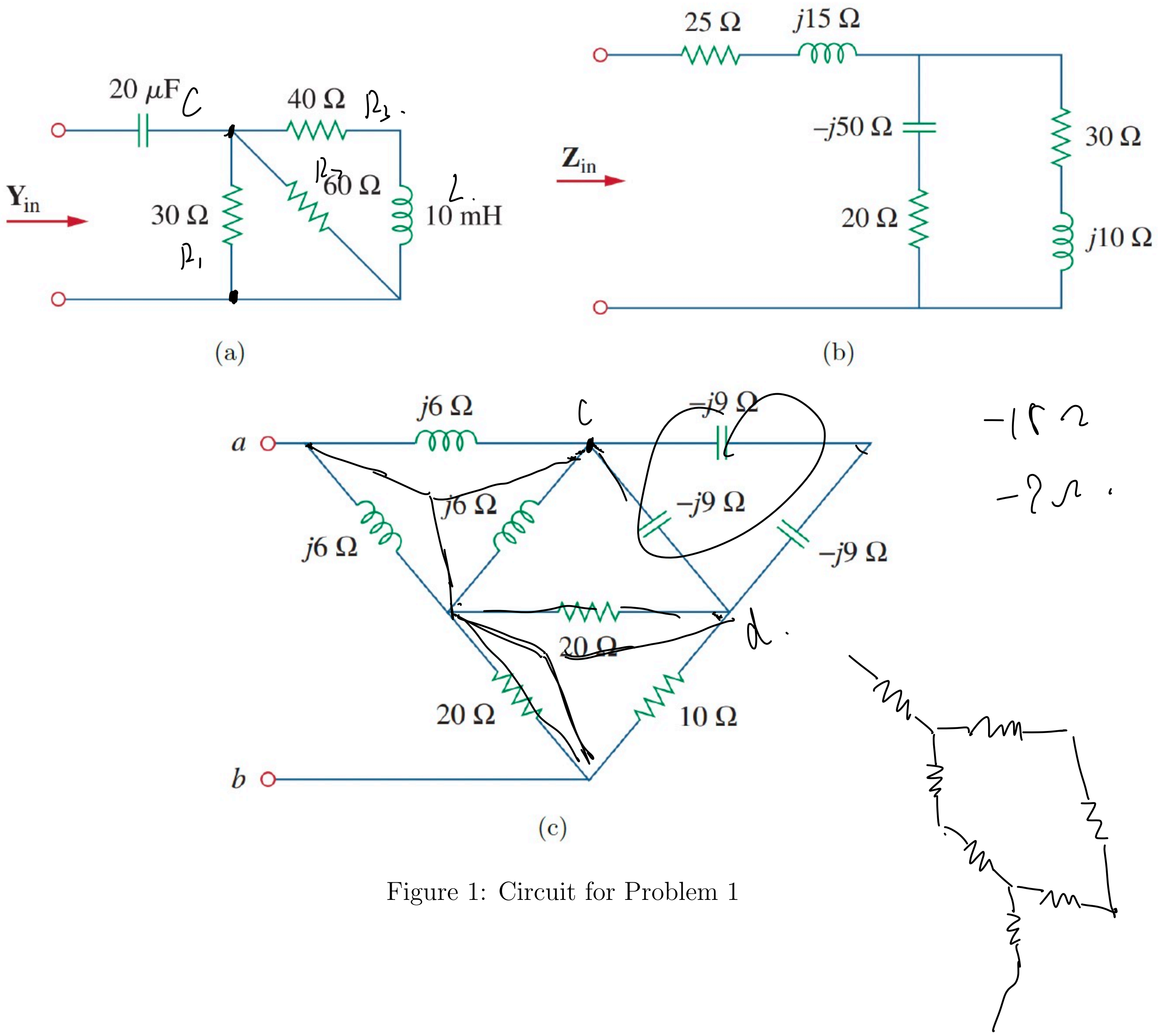


Figure 1: Circuit for Problem 1

(a)

$$C = \frac{1}{20\times10^{-6}\times10^{2}\,j} = -500j\ \Omega.$$

$$L = 10\times10^{2}\times10^{-3}\,j = 1j\ \Omega.$$

$$Z = -500j + \frac{1}{\frac{1}{30}+\frac{1}{60}+\frac{1}{40+j}} = 13.3-499.8j\ \Omega.$$

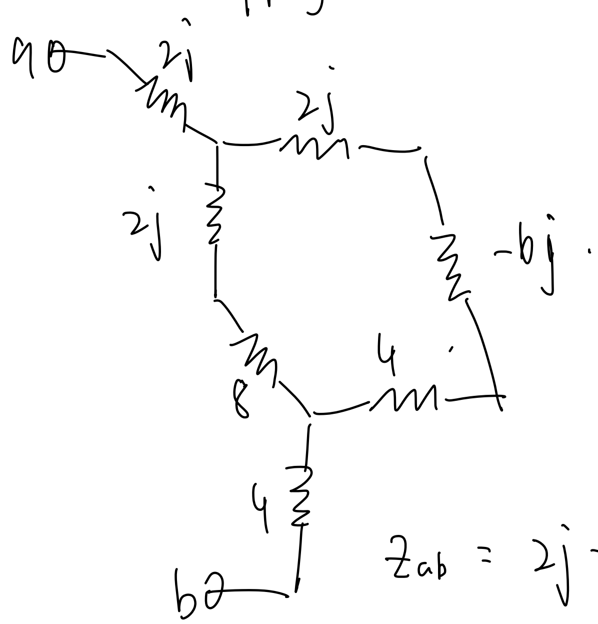
$$\boxed{Y = \frac{1}{Z} = 5.3\times10^{-5} + 2.0\times10^{-3}\,j\ \Omega.}$$

(b)

$$Z = 25+15j + \frac{1}{\frac{1}{20-50j}+\frac{1}{20+10j}}$$

$$\boxed{= 51.1+9.9j\ \Omega.}$$

(c) Apply Y-Δ, we can get the equivalent circuit.



$$Z_{ab} = 2j + \frac{1}{\frac{1}{8+2j}+\frac{1}{4-4j}} + 4 = \boxed{(7.57+0.6j)\,\Omega.}$$

Problem 2

(a) (30%) Please find the Thevenin equivalent circuits between terminal a and b under $\omega = 2000\,\text{rad/s}$ and $\omega = 4000\,\text{rad/s}$.

- **Hint:** Consider whether to turn on or turn off the voltage source and current source based on the frequency you choose.

(b) (10%) Please draw two phasor diagrams of the two Thevenin equivalent impedances under the two frequencies ($\omega = 2000\,\text{rad/s}$ and $\omega = 4000\,\text{rad/s}$).

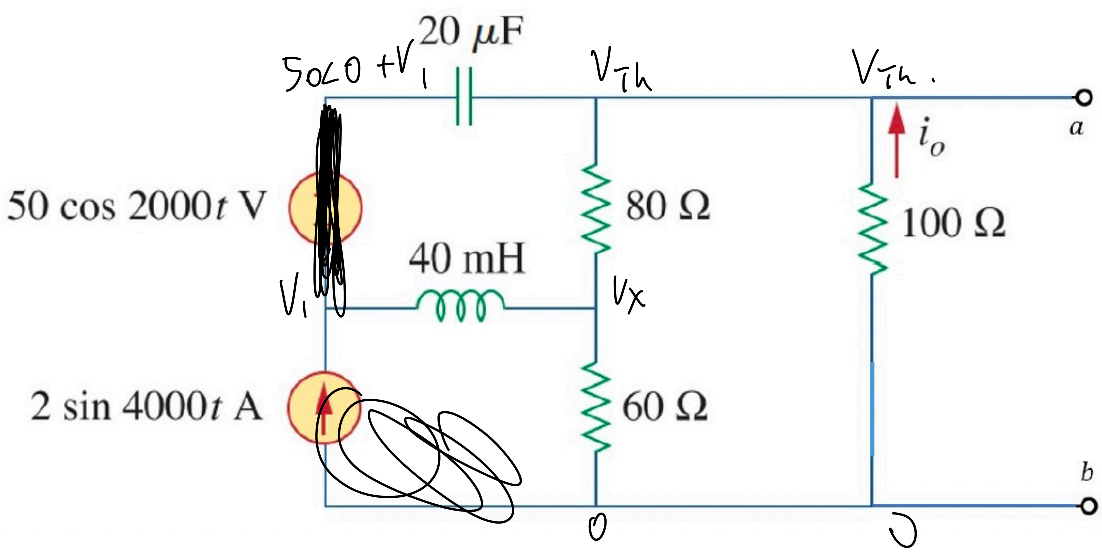
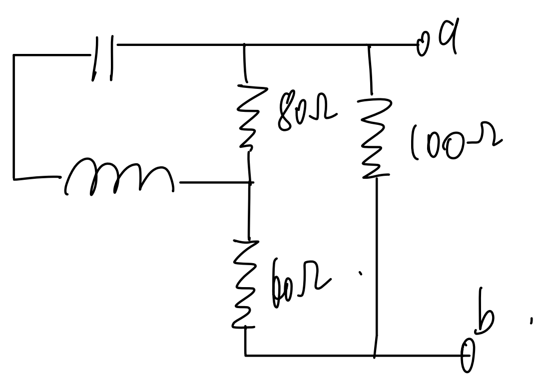


Figure 2: Circuit for Problem 2

Z_{Th}:



When $w = 2000$,

$$C = \frac{1}{20\times10^{-6}\times2\times10^{3}\,j} = -25j.$$

$$L = 40\times10^{-3}\times2\times10^{3}\,j = 80j.$$

$$Z_1 = \frac{1}{\frac{1}{80} + \frac{1}{80j-25j}} = 25.67 + 37.35j.$$

$$\boxed{Z_{Th} = \frac{1}{\frac{1}{100} + \frac{1}{25.67+60+37.35j}} = 48.24 + 10.4j\ \Omega}$$

$$\begin{cases} \dfrac{V_x - V_1}{80j} + \dfrac{V_x}{60} + \dfrac{V_x - V_{Th}}{80} = 0. \\[2ex] \dfrac{V_{Th} - 50\angle 0 - V_1}{-25j} + \dfrac{V_{Th} - V_x}{80} + \dfrac{V_{Th}}{100} = 0. \\[2ex] \dfrac{V_1 - V_y}{80j} + \dfrac{V_1 + 50\angle 0 - V_{Th}}{-25j} = 0 \end{cases}$$

$$V_{Th} = 21.75 \angle(-45.9^{\circ})$$

$$\boxed{V_{Th} = 21.75\cos(2000t - 0.8)}.$$

when $\omega = 4000$.

$C = -12.5j$. $L = 160j$.

$$Z_1' = \frac{1}{\frac{1}{80} + \frac{1}{160j - 12.5j}} = 61.8 + 33.5j .$$

$$\boxed{Z_{Th} = \frac{1}{\frac{1}{100} + \frac{1}{61.8 + 33.5j + 60}} = 55.9 + 6.65j .}$$

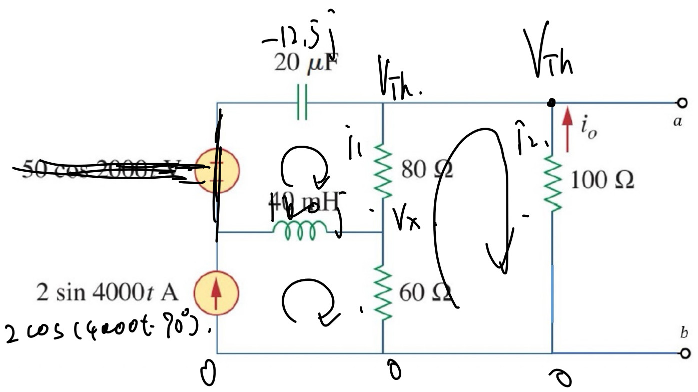


Figure 2: Circuit for Problem 2

$$\begin{cases} i_1 \cdot (-12.5j) + (i_1 - i_2) \cdot 80 + (i_1 - 2\angle -90^\circ) \cdot 160j = 0 . \\ 80(i_2 - i_1) + 100 i_2 + (i_2 - 2\angle -90^\circ) \cdot 60 = 0 . \end{cases}$$

$$(80 + 147.5j)\, i_1 - 80 i_2 = 320$$

$$-80\, i_1 + 240 i_2 = -120j ;$$

$$i_1 = 0.45 - 2j .$$

$$i_2 = 0.15 - 1.17j$$

$$= 1.17 \angle -82.6^\circ$$

$$\boxed{V_{Th} = 117 \cos(4000t - 1.44) .}$$

Problem 3

Given the circuit below, calculate the equivalent impedance in each of the blocks. Also find I_o and the overall complex power supplied.

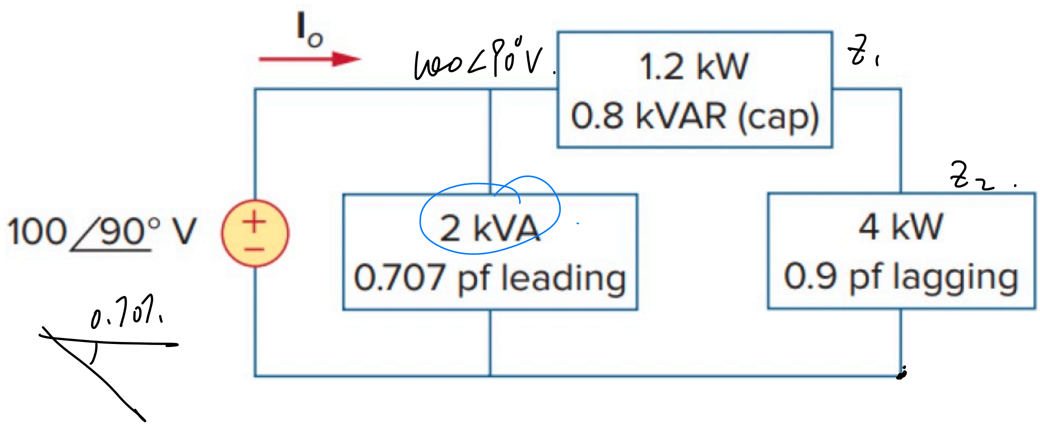
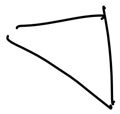


Figure 3: Circuit for Problem 3

Block 1.



$V_{rms} \cdot I_{rms}$

$P = 1.414\,k$ W.

$Q = -1.414k$ VAR.

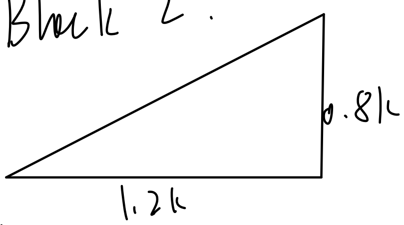
$P = Q.\quad \theta = 45°.$

$\tilde{V}_{rms} = 100 \cdot \angle 90°$

$\tilde{I}_{rms}^{*} = \frac{S}{\tilde{V}_{rms}} = \frac{2k\ \angle -45°}{100 \angle 90°} = 20 \cdot \angle -135°$ A.

$$\boxed{Z = \frac{\tilde{V}_{rms}}{\tilde{I}_{rms}} = \frac{100 \angle 90°}{20 \angle 135°} = 5 \angle -45°\ \Omega}.$$

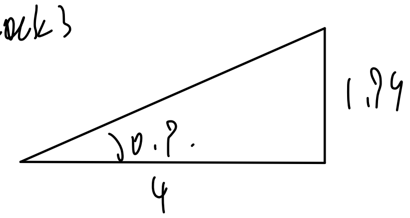
Block 2.



$I_{rms}^2 \cdot R_2 = 1.2k,$

$I_{rms}^2 \cdot X_2 = j0.8k.$

Block 3



$I_{rms}^2 \cdot R_3 = 4k.$

$I_{rms}^2 \cdot X_3 = j1.94k$

$S = 5.2k + j2.74k \quad = 5.9k \angle 27.8°.$

$\tilde{I}_{rms}^{*} = \frac{S}{\tilde{V}_{rms}} = \frac{5.9 \times 10^3 \cdot \angle 27.8}{100 \cdot \angle 90°} = 59 \angle -62.2°.$

$\tilde{I}_{rms} = 59 \angle 62.2°.$

$Z_{rms}^2 = 3481$.

$R_2 = 0.34\,\Omega$ $X_2 = j\,0.23\,\Omega$.

$\boxed{Z_2 = 0.41\angle 34.0^\circ\,\Omega}$.

$R_3 = 1.15\,\Omega$ $X_3 = j\,0.56\,\Omega$.

$\boxed{Z_3 = 1.28\angle 25.9^\circ}$.

$\boxed{I_0 = \tilde{I}_{1rms} + \tilde{I}_{2rms} = 67.7\angle 78.6^\circ\ A}$.

$\boxed{S = 6.614 + j1.33\ (kVA)}$.