ECE3110J Final RC Part I

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key points for the exam:
I look at the exam problems one by one and tell you what you need to know in case you just started studying for ECE3110J. **Study before it is too late!**

1. MOSFET DC calculation
2. MOSFET region of operation
3. Small signal gain differential amp
4. PMOS DC calculation
5. Zero-frequency response
6. Finite frequency response
7. Transfer function and dominant pole
8. T model/ pi model
9. Small signal model and its calculation
10. gmRout method
11. DC and AC response
12. Virtual ground
13. CMRR
14. BJT current equation
15. BJT small signal

From Prof. Xuyang Lu

BJT

Current–Voltage Characteristics

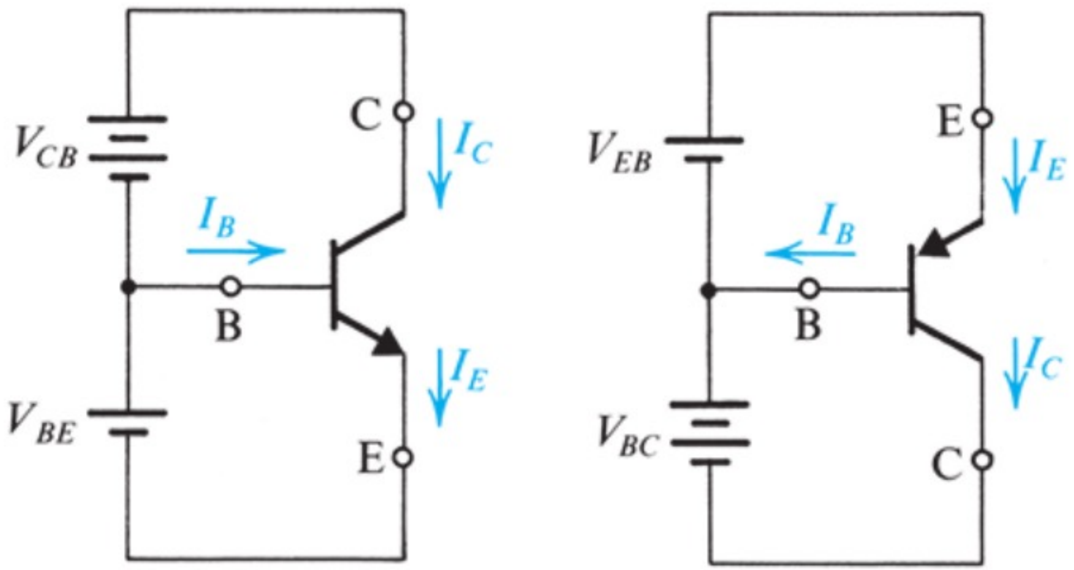


Figure: npn and pnp BJT

- $I_C = I_S e^{\frac{V_{BE}}{V_T}}$
- $I_B = \frac{I_C}{\beta}$, $i_E = I_B + I_C$
- β is called the common-emitter current gain.
- I_S is called the saturation current and is transistor parameter

02 BJT

Large Signal Voltage Relation

Table 6.2 Summary of the BJT Current–Voltage Relationships in the Active Mode

$$i_C = I_S e^{v_{BE}/V_T}$$

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T}$$

$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$$

Note: For the *pnp* transistor, replace v_{BE} with v_{EB}.

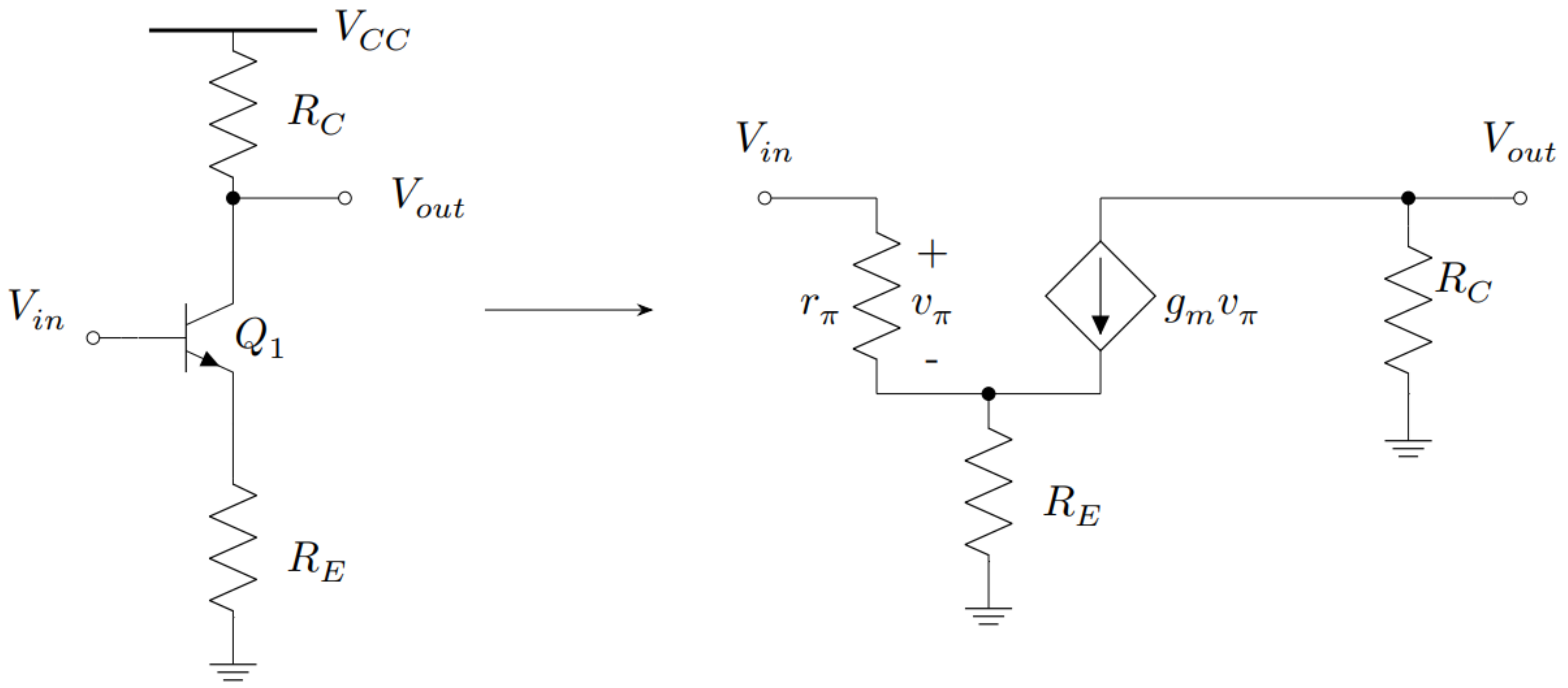
$$i_C = \alpha i_E \qquad i_B = (1-\alpha) i_E = \frac{i_E}{\beta + 1}$$

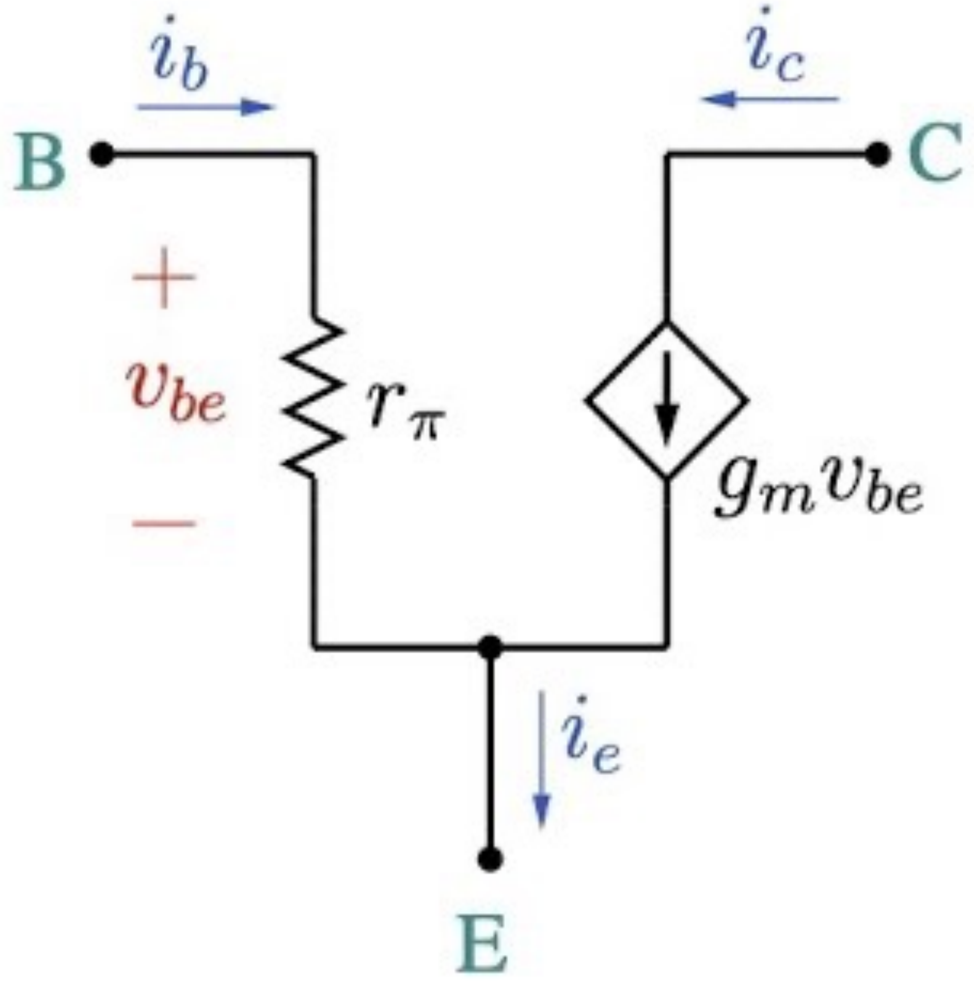
$$i_C = \beta i_B \qquad i_E = (\beta + 1) i_B$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta + 1}$$

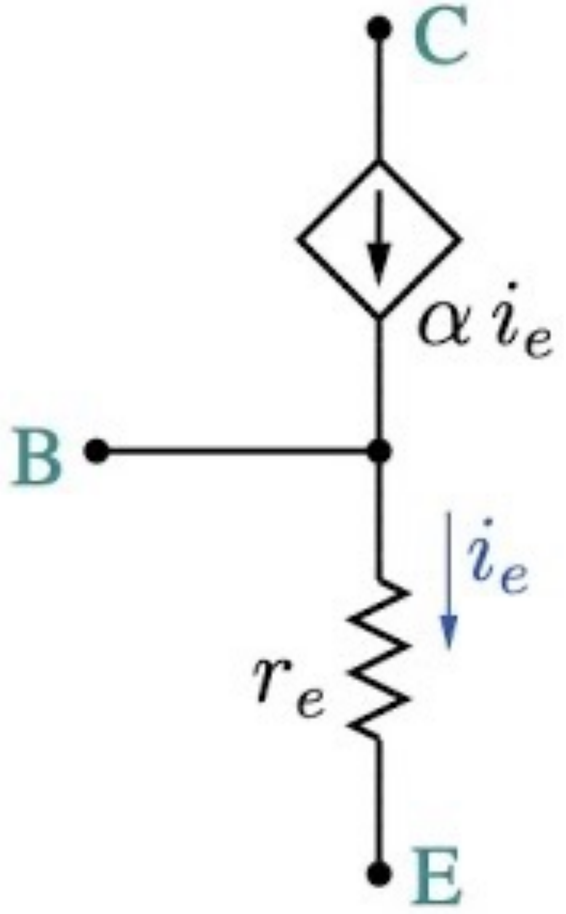
$$V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV} \quad \text{at room temperature}$$

CE Stage With Emitter Degeneration (Without Early Effect)





Hybrid-π model



T model

From https://www.youtube.com/watch?v=1qhUTvoBmiQ&ab_channel=LearnWithAmitBansal

Small Signal Model

Model Parameters in Terms of DC Bias Currents

$$g_m = \frac{I_C}{V_T} \qquad r_e = \frac{V_T}{I_E} = \alpha\frac{V_T}{I_C} \qquad r_\pi = \frac{V_T}{I_B} = \beta\frac{V_T}{I_C} \qquad r_o = \frac{|V_A|}{I_C}$$

In Terms of g_m

$$r_e = \frac{\alpha}{g_m} \qquad r_\pi = \frac{\beta}{g_m}$$

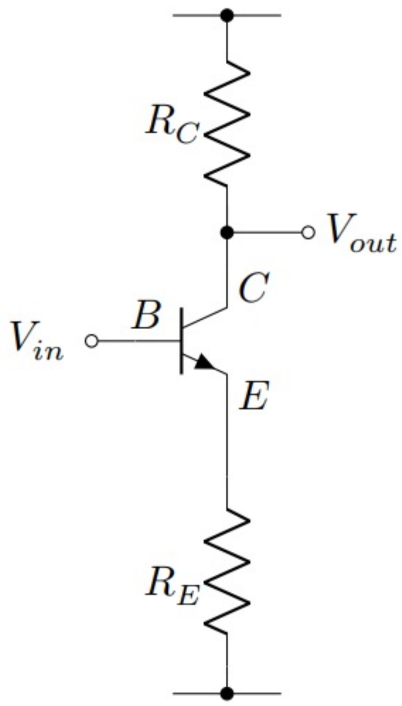
In Terms of r_e

$$g_m = \frac{\alpha}{r_e} \qquad r_\pi = (\beta+1)r_e \qquad g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

Relationships between α and β

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1} \qquad \beta+1 = \frac{1}{1-\alpha}$$

Summary

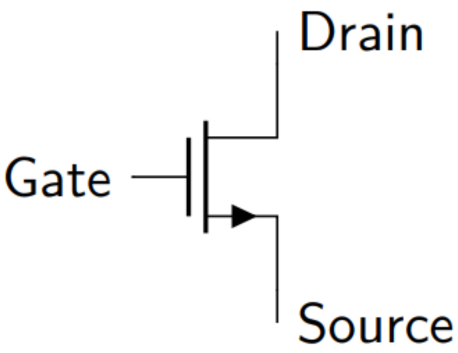


	Definition	Expression: With emitter degeneration	Expression: Without emitter degeneration; i.e., $R_E = 0$
Current gain	A_i	β	β
Voltage gain	A_v	$-\frac{\beta R_C}{r_\pi+(\beta+1)R_E}$	$-g_m R_C$
Input impedance	r_{in}	$r_\pi + (\beta+1)R_E$	r_π
Output impedance	r_{out}	R_C	R_C

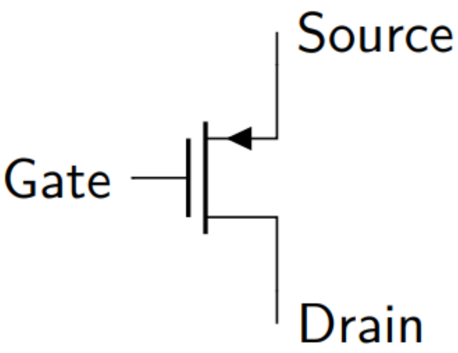
MOSFET

MOSFET Basics

NMOS



PMOS



I-V Characteristics (NMOS)

When $V_{DS} < V_{GS} - V_{TH}$, the NMOS is in triode region.

$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \tag{1}$$

When $V_{DS} \geq V_{GS} - V_{TH}$, the NMOS is in saturation region.

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2 \tag{2}$$

I-V Characteristics (PMOS)

When $V_{SD} < V_{SG} - |V_{TH}|$, the PMOS is in triode region.

$$I_D = \mu_p C_{ox} \frac{W}{L_{eff}} \left[\left(V_{SG} - |V_{TH}| \right) V_{SD} - \frac{1}{2} V_{SD}^2 \right] \tag{3}$$

When $V_{SD} \geq V_{SG} - |V_{TH}|$, the NMOS is in saturation region.

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L'} \left(V_{SG} - |V_{TH}| \right)^2 \tag{4}$$

Second Effects

Channel-Length Modulation

$$L' = L_{\text{eff}} - \Delta L \tag{5}$$

$$\begin{aligned} I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{\text{eff}}}\left(V_{GS} - V_{TH}\right)^2\left(1 + \frac{\Delta L}{L_{\text{eff}}}\right) \\ &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{\text{eff}}}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right) \end{aligned} \tag{6}$$

Body Effect

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}\right) \tag{7}$$

$$g_{mb} = g_m \cdot \eta, \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} \tag{8}$$

Transconductance

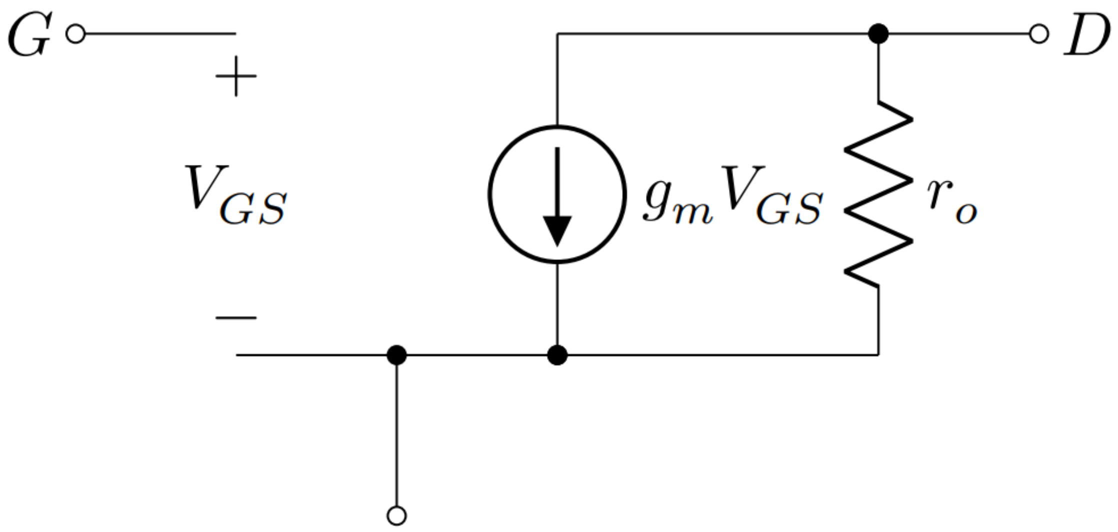
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox} \frac{W}{L'} I_D} = \frac{2I_D}{V_{GS} - V_{TH}} \tag{9}$$

With channel-length modulation:

$$\begin{aligned} g_m &= \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) (1 + \lambda V_{DS}) \\ &= \sqrt{2\mu_n C_{ox} (W/L) I_D (1 + \lambda V_{DS})} \end{aligned} \tag{10}$$

Small-Signal Model for NMOS

$\lambda \neq 0$, $\gamma = 0$

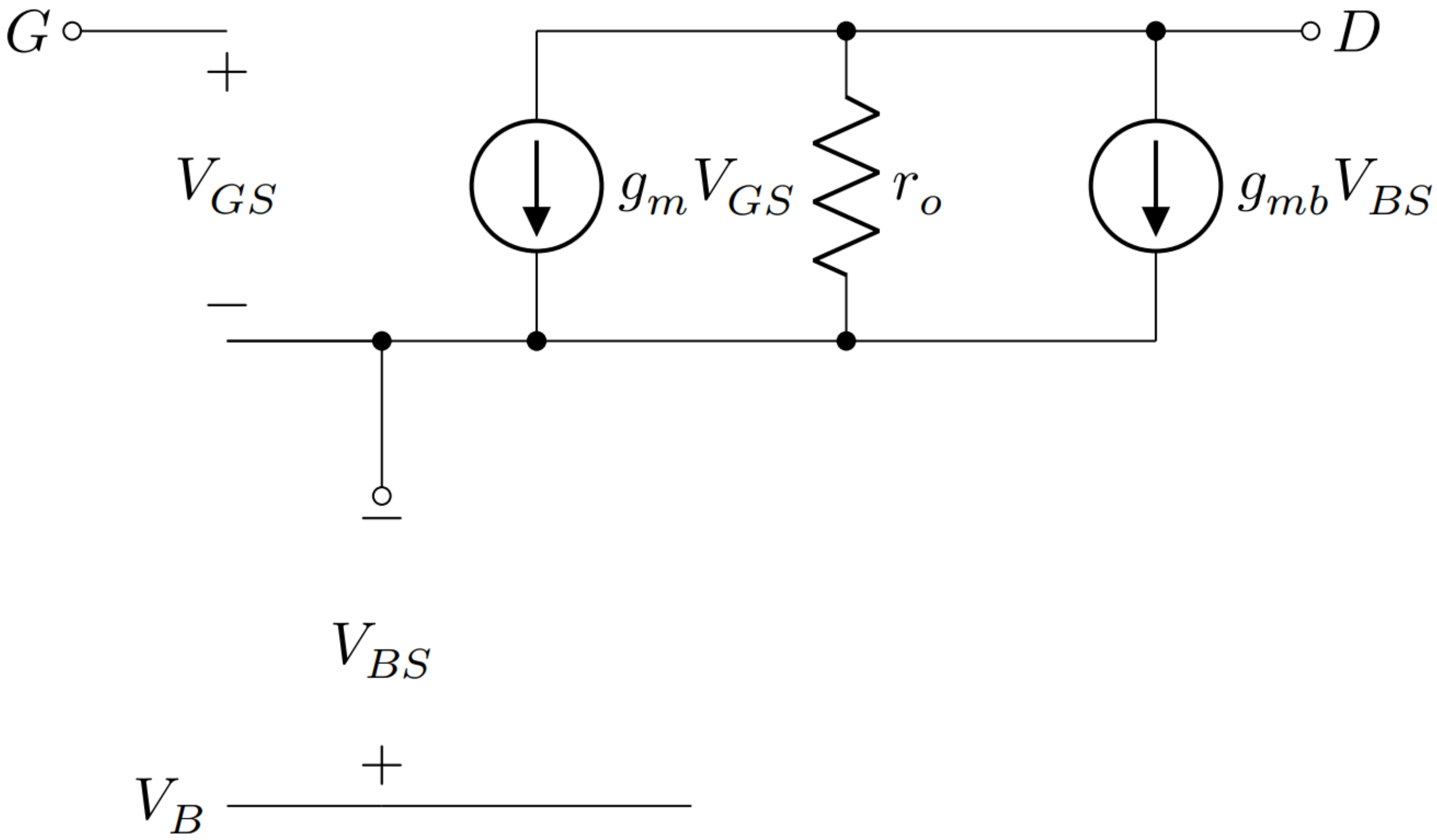


Where,

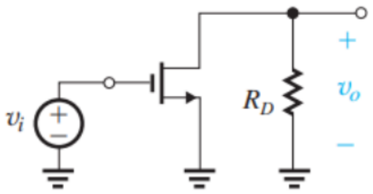
$$r_o = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 \cdot \lambda} \quad (11)$$

Small-Signal Model for NMOS

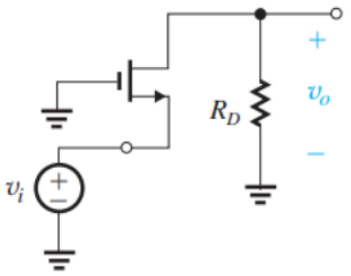
$\lambda \neq 0,\ \gamma \neq 0$



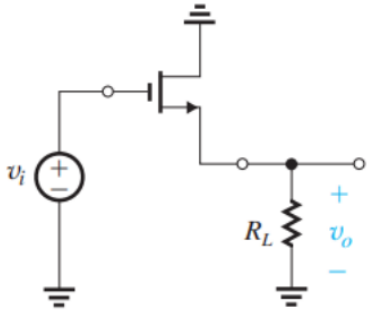
The basic configurations of transistor amplifiers



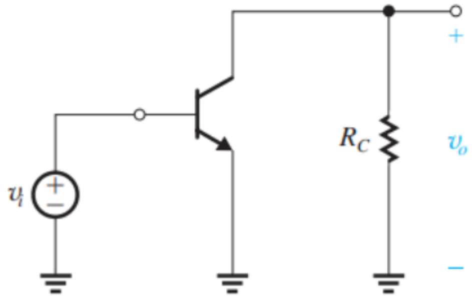
(a) Common Source (CS)



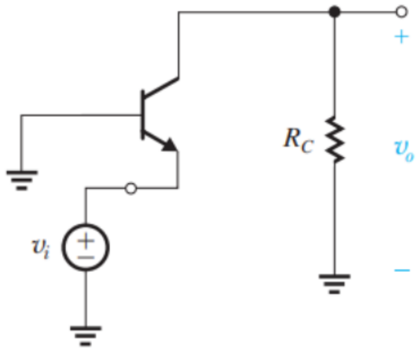
(b) Common Gate (CG)



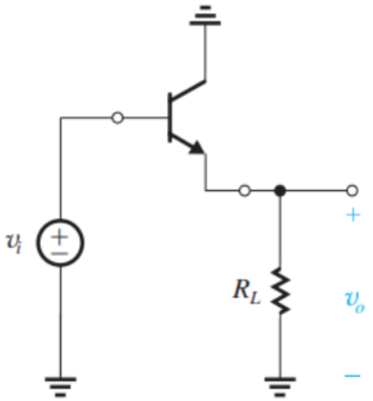
(c) Common Drain (CD)
or Source Follower



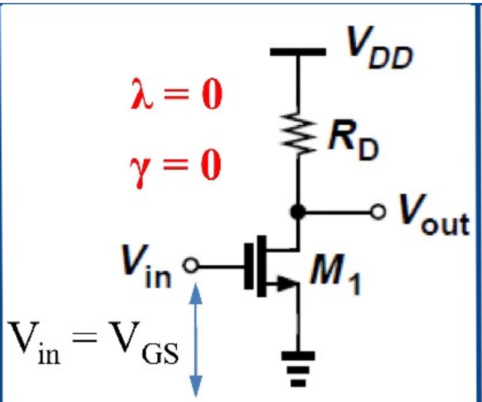
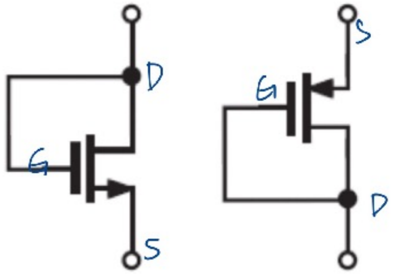
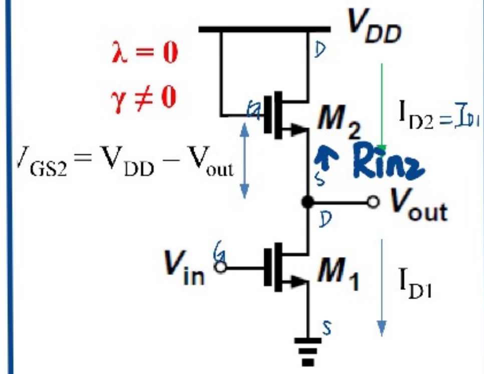
(d) Common-Emitter (CE)

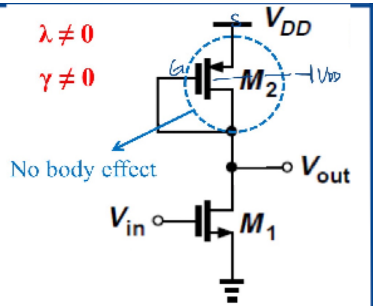
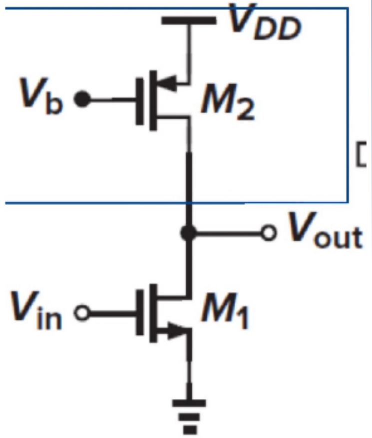
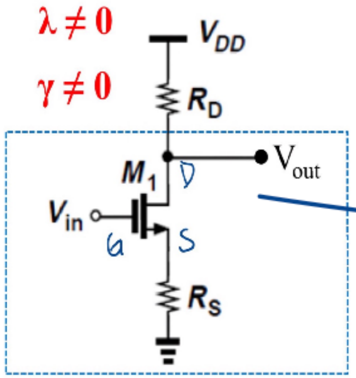


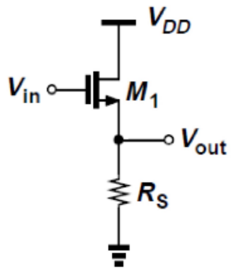
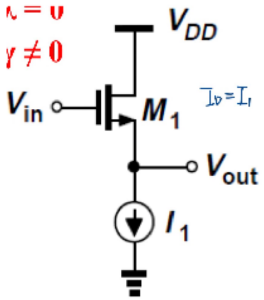
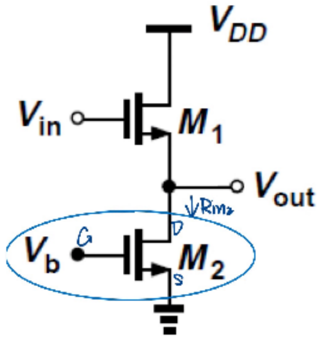
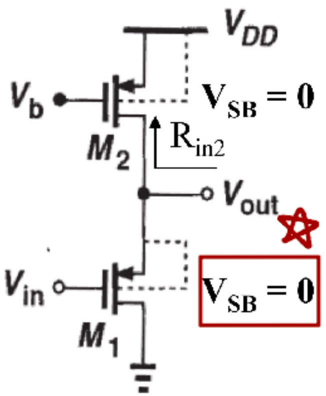
(e) Common-Base (CB)



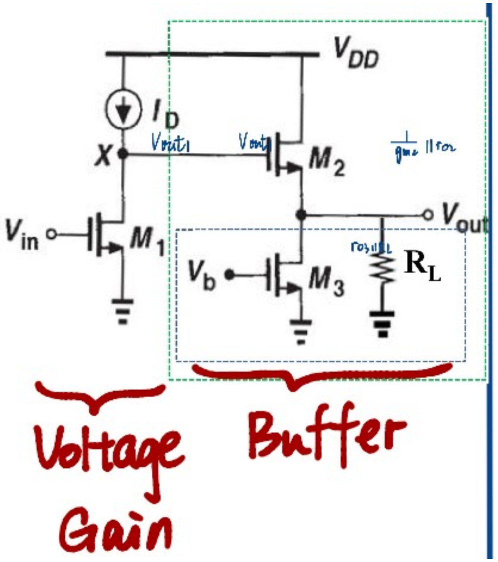
(f) Common-Collector (CC)

<table>
<tr>
<td>CS stage with resistive load</td>
<td>
</td>
<td>$\lambda = 0$. $\gamma = 0$
M_1: sat. $A_v = -g_m R_D$</td>
<td>$\lambda \neq 0$. $\gamma \neq 0$
M_1: sat. $A_v = -g_m(R_D \| r_o)$</td>
</tr>
<tr>
<td>Diode-connected load</td>
<td>
</td>
<td>$\lambda \neq 0$. $\gamma = 0$
D: $R = 1/g_m \| r_o$</td>
<td>$\lambda \neq 0$. $\gamma \neq 0$
S: $R = \frac{1}{g_m + g_{mb}} \| r_o$</td>
</tr>
<tr>
<td>NMOS</td>
<td>
</td>
<td>$\lambda = 0$. $\gamma \neq 0$
M_1: sat.
$A_v = -\frac{g_{m1}}{g_{m2} + g_{mb2}}$
$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1+\eta}$
$R_{in2} = \frac{1}{g_{m2} + g_{mb2}}$</td>
<td>$\lambda \neq 0$. $\gamma \neq 0$
$R_{in2} = \frac{1}{g_{m2}} \| \frac{1}{g_{mb2}} \| r_{o2}$
$A_v = -g_{m1}\left(\frac{1}{g_{m2}} \| \frac{1}{g_{mb2}} \| r_{o2} \| r_{o1}\right)$
$= -g_{m1}(r_{o1} \| R_{in2})$
$G_m = g_{m1}$</td>
</tr>
</table>

PMOS			$\lambda \neq 0.\ \gamma \neq 0$ $A_V = -g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1}\right)$ $\approx -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$ $G_m = g_{m1}$
Current-source		☆ $V_{out,max}$ $= V_{DD} - (V_{SG2} - \lvert V_{TH2} \rvert)$ $V_{out,min}$ $= V_{GS1} - V_{TH1}$	$\lambda \neq 0.\ \gamma \neq 0$ $A_V = -g_{m1}(r_{o1} \parallel r_{o2})$
source degeneration			$\lambda \neq 0.\ \gamma \neq 0$ $G_m = \frac{g_{m1} r_{o1}}{R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S}$ $R_{out} = [r_{o1} + R_S + (g_{m1} + g_{mb1}) r_{o1} R_S] \parallel R_D \approx R_D$ $A_V = \frac{-g_{m1} r_{o1} R_D}{[R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S] + R_D}$ $\approx -\frac{R_D}{R_S}$

Source Follower		$\lambda=0,\ \gamma\neq0$ M_1: sat. $A_v=\frac{g_m R_s}{1+(g_m+g_{mb})R_s}$ $\approx\frac{1}{1+\eta}$	$\lambda\neq0,\ \gamma\neq0$ M_1: sat. $G_m=-g_m$ $R_{out}=R_s \parallel \left(\frac{1}{g_m+g_{mb}}\right) \parallel r_o$ $A_v=\frac{g_m r_o R_s}{r_o+R_s+(g_m+g_{mb})r_o R_s}\approx\frac{1}{1+\eta}$
Current source		$\lambda=0\quad\gamma\neq0$ $A_v=\frac{g_m}{g_m+g_{mb}}=\frac{1}{1+\eta}$ $G_m=-g_m$ $R_{out}=\frac{1}{g_m+g_{mb}}$	$\lambda\neq0,\ \gamma\neq0$ $R_{out}=\frac{1}{g_m+g_{mb}} \parallel r_o$
NMOS			$\lambda\neq0,\ \gamma\neq0$ $A_v=\frac{g_{m1}}{g_{m1}+g_{mb1}+1/r_{o1}+1/r_{o2}}$ $G_m=-g_{m1}$ $R_{out}=r_{o1}\parallel r_{o2}\parallel\left(\frac{1}{g_{m1}+g_{mb1}}\right)$
PMOS			$\lambda\neq0,\ \gamma\neq0$ $A_v=\frac{g_{m1}}{g_{m1}+1/r_{o1}+1/r_{o2}}$ $G_m=-g_{m1}$ $R_{out}=r_{o1}\parallel r_{o2}\parallel\frac{1}{g_{m1}}$

CS + Source Follower



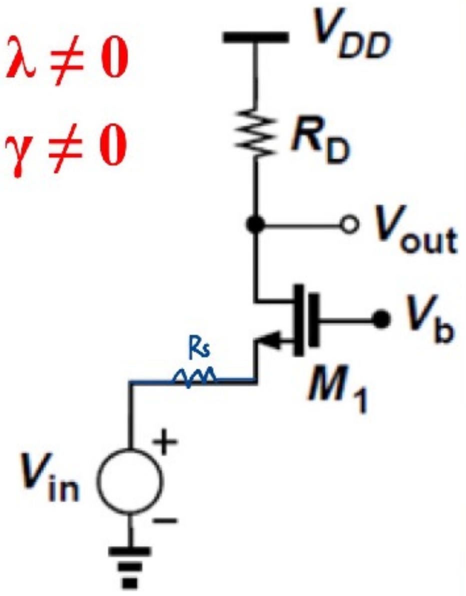
$\lambda \neq 0. \quad \gamma \neq 0$

$$A_v = \frac{V_{out}}{V_{out1}} \cdot \frac{V_{out1}}{V_{in}}$$

$$= -g_{m1} r_{o1} g_{m2} \left(r_{o2} \,\|\, \frac{1}{g_{m2}+g_{mb2}} \,\|\, r_{o3} \,\|\, R_L \right)$$

CG Stage

Rin↓ Av → Rout↑



$\lambda \neq 0$. $\gamma \neq 0$

$A_v = G_m R_{out}$

$$A_v = \frac{(g_m+g_{mb})r_oR_D + R_D}{r_o + R_s + R_D + (g_m+g_{mb})r_oR_s}$$

If $R_s = 0$. $r_o = \infty$. $A_v \approx R_D g_m(1+\eta)$

$$G_m = \frac{(g_m+g_{mb})r_o + 1}{r_o + R_s + (g_m+g_{mb})r_oR_s}$$

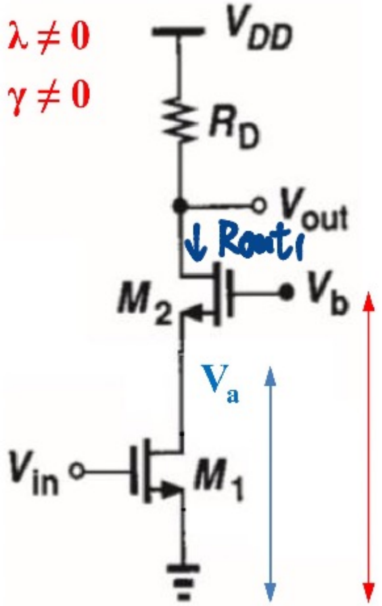
$$R_{out} = R_D \parallel [r_o + R_s + (g_m+g_{mb})r_oR_s]$$

$$R_{in} = \frac{R_D + r_o + R_s + R_sr_o(g_m+g_{mb})}{1+(g_m+g_{mb})r_o}$$

If $R_s = 0$. $R_{in} = \dfrac{R_D + r_o}{1+(g_m+g_{mb})r_o}$

Cascode:

CS stage + CG stage



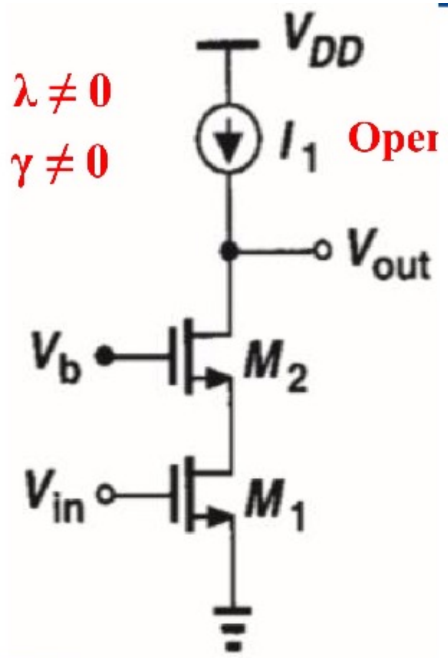
$\lambda \neq 0$. $\gamma \neq 0$

$$R_{out} = R_{out1} \parallel R_D$$
$$= [r_{o2} + r_{o1} + (g_{m2}+g_{mb})r_{o1}r_{o2}] \parallel R_D$$
$$\approx (g_{m2}+g_{mb})r_{o1}r_{o2} \parallel R_D$$

$$G_m = \frac{g_{m1}r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}})} \approx g_{m1}$$

$A_v = G_m R_{out}$ (R_D gnd x)

CS+CG with ideal current source load



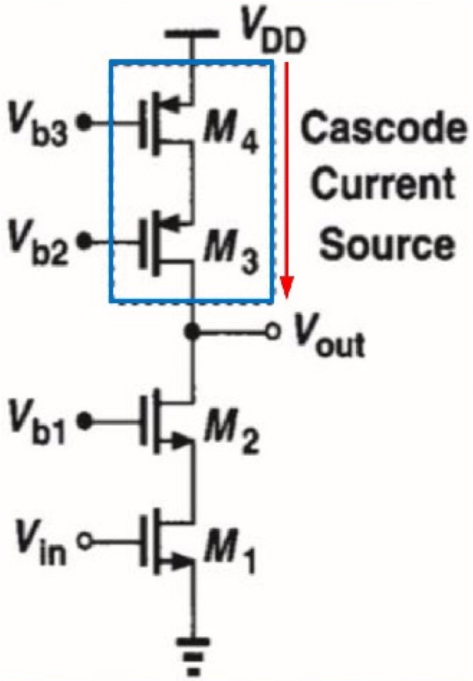
$\lambda \neq 0,\ \gamma \neq 0$

$$G_m = \frac{g_{m1} r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \approx g_{m1}$$

$$R_{out} = (g_{m2}+g_{mb}) r_{o2} r_{o1} \quad (R_D \to \infty)$$

$$A_v = G_m R_{out}$$

CS+CG with Cascode Current Source load



$\lambda \neq 0,\ \gamma \neq 0$

$$R_{out} = [r_{o1}+r_{o2}+(g_{m2}+g_{mb2}) r_{o2} r_{o1}] \parallel [r_{o3}+r_{o4}+(g_{m3}+g_{mb3}) r_{o3} r_{o4}]$$

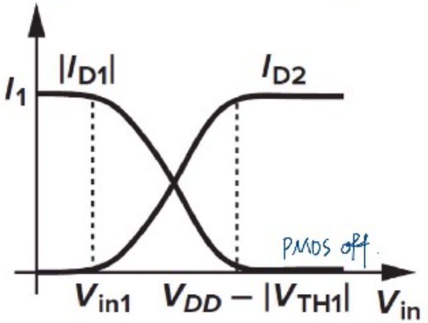
$$G_m = \frac{g_{m1} r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)}$$

(Vdd → gnd ⇒ won't affect Gm)

$$A_v = G_m R_{out}$$

Folded Cascode

$|I_{D1}| + I_{D2} = I_1$ constant

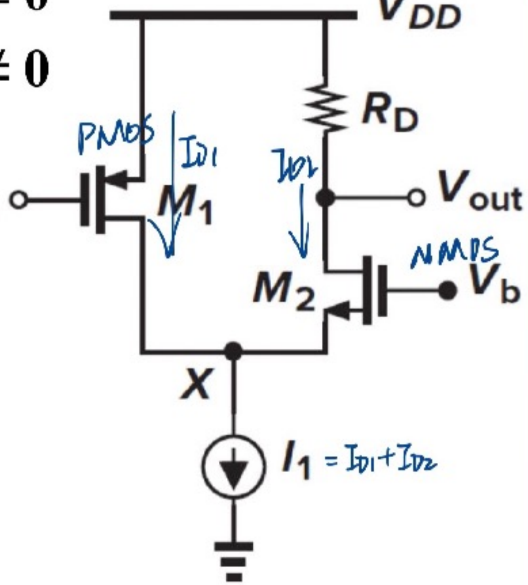


As V_{in} decreases I_{D1} goes into the saturation

$$V_{in1} = V_{DD} - \sqrt{\frac{2I_1}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} - |V_{TH1}|$$

$\lambda \neq 0$

$\gamma \neq 0$



$\lambda \neq 0.\ \gamma \neq 0$

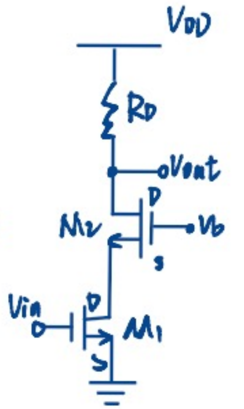
$$G_m = \frac{g_{m1} r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}}\right)} \cong g_{m1}$$

$$R_{out} = \left[r_{o2} + r_{o1} + (g_{m2} + g_{mb}) r_{o2} r_{o1}\right] \parallel R_D$$

$$\approx (g_{m2} + g_{mb}) r_{o2} r_{o1} \parallel R_D$$

$$A_v = G_m R_{out}$$

=



Differential Amp

CMRR of an amplifier [edit]

Ideally, a differential amplifier takes the voltages, V_+ and V_- on its two inputs and produces an output voltage $V_\mathrm{o} = A_\mathrm{d}(V_+ - V_-)$, where A_d is the differential gain. However, the output of a real differential amplifier is better described as :

$$V_\mathrm{o} = A_\mathrm{d}(V_+ - V_-) + \tfrac{1}{2}A_\mathrm{cm}(V_+ + V_-)$$

where A_cm is the "common-mode gain", which is typically much smaller than the differential gain.

The CMRR is defined as the ratio of the powers of the differential gain over the common-mode gain, measured in positive decibels (thus using the 20 log rule):

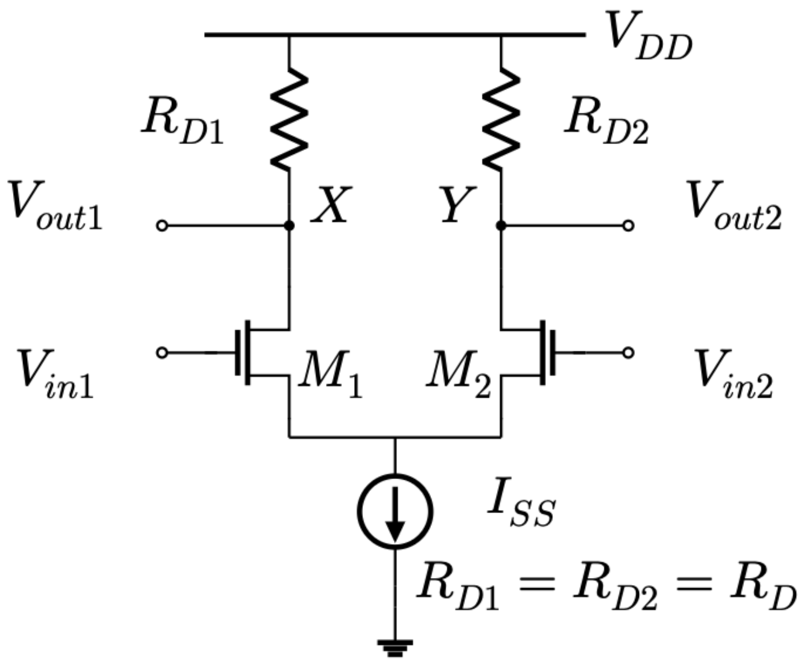
$$\mathrm{CMRR} = \left(\frac{A_\mathrm{d}}{|A_\mathrm{cm}|}\right) = 10\log_{10}\left(\frac{A_\mathrm{d}}{A_\mathrm{cm}}\right)^2 \mathrm{dB} = 20\log_{10}\left(\frac{A_\mathrm{d}}{|A_\mathrm{cm}|}\right)\mathrm{dB}$$

As differential gain should exceed common-mode gain, this will be a positive number, and the higher the better.

The CMRR is a very important specification, as it indicates how much of the unwanted common-mode signal will appear in the output, typically a measurement of some quantity. The value of the CMRR often depends on signal frequency, and must be specified as a function thereof.

From *Wikipedia*

Differential Pair with Resistor Load



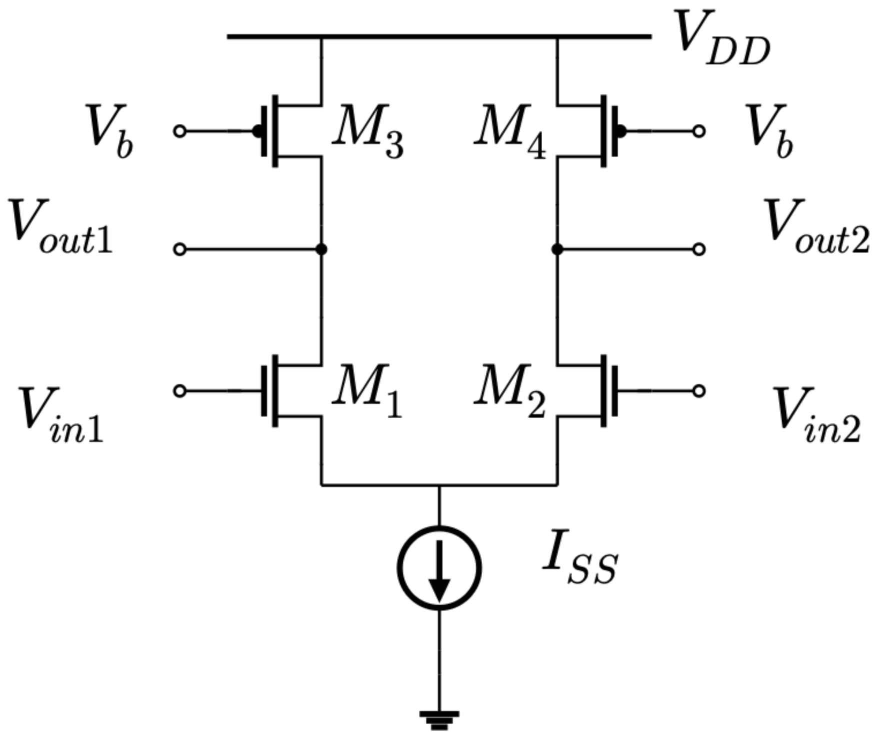
$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o) \qquad (1)$$

If the circuit is fully symmetric,

$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0 \qquad (2)$$

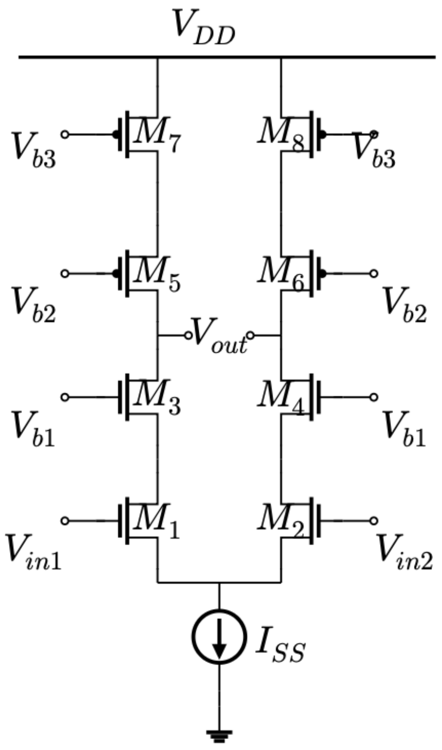
$$CMRR = \left| \frac{A_{DM}}{A_{CM}} \right| = \infty \qquad (3)$$

Differential Pair with MOS Loads



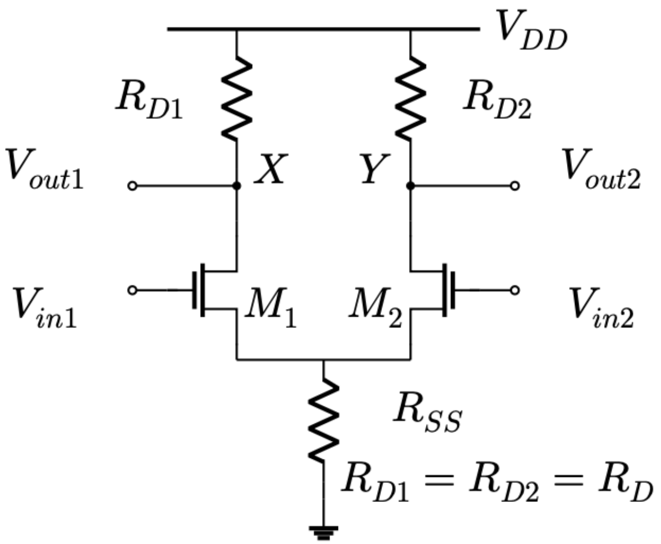
$$A_{DM} = g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \tag{4}$$

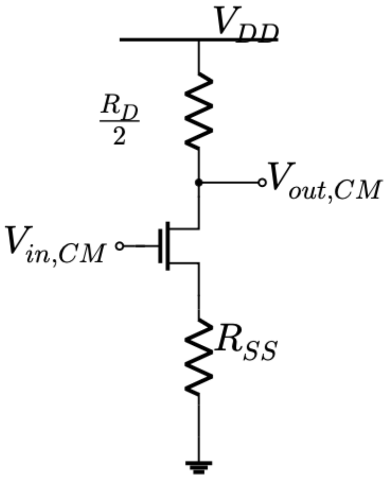
Differential Pair with Cascode Loads



$$
\begin{aligned}
A_{DM} &= G_m R_{out} \\
&\approx g_{m1,2}[(g_{m3,4}+g_{mb3,4})r_{o3,4}r_{o1,2} \parallel \\
&(g_{m5,6}+g_{mb5,6})r_{o5,6}r_{o7,8}]
\end{aligned} \tag{5}
$$

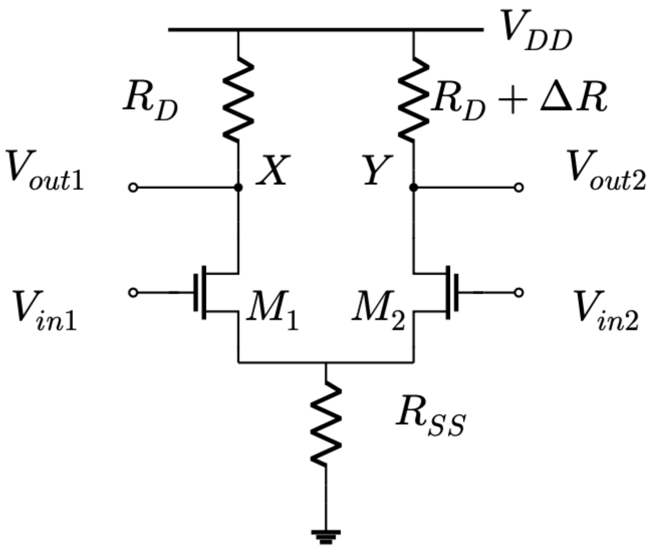
Common Mode Response

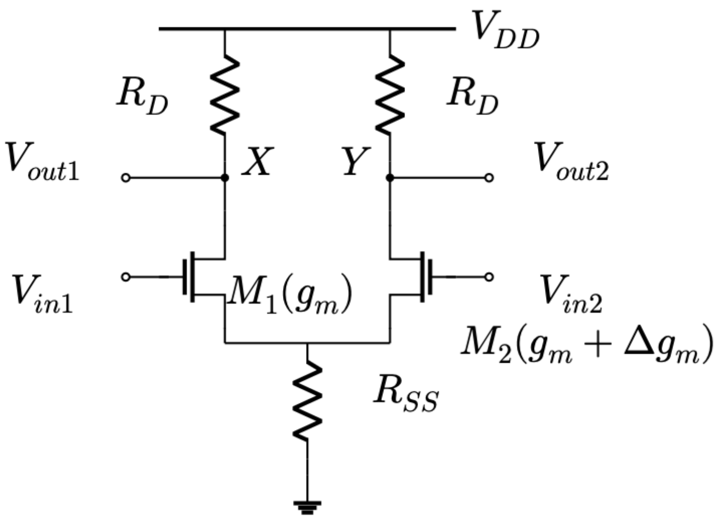




$$CMRR = \frac{|A_{vd}|}{|A_{vc}|} = 1+2g_m R_{SS} \tag{6}$$

Mismatch

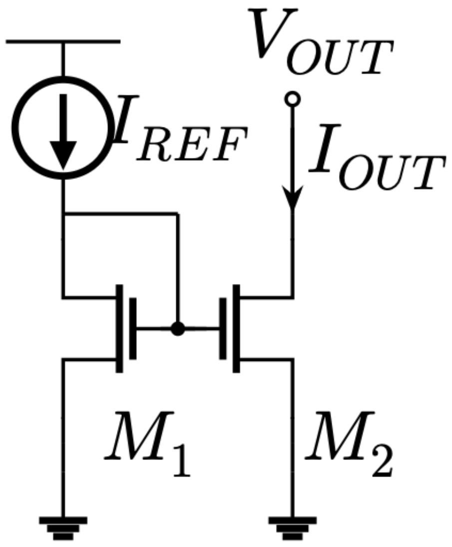




$$A_{cm-dm} = -\left(\frac{g_m \Delta R + \Delta g_m R}{2 g_m R_{SS} + 1}\right) \tag{7}$$

Current Mirror

Current Mirror

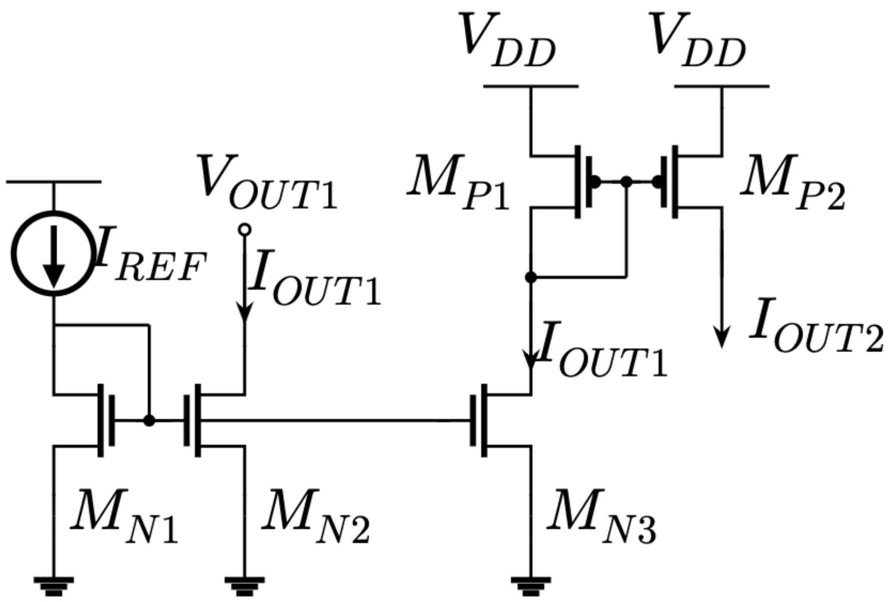


$$I_{REF} = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{GS} - V_{TH1})^2 \quad (8)$$

$$I_{OUT} = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_2(V_{GS} - V_{TH2})^2 \quad (9)$$

$$I_{OUT} = \frac{(\frac{W}{L})_2}{(\frac{W}{L})_1} I_{REF} \quad (10)$$

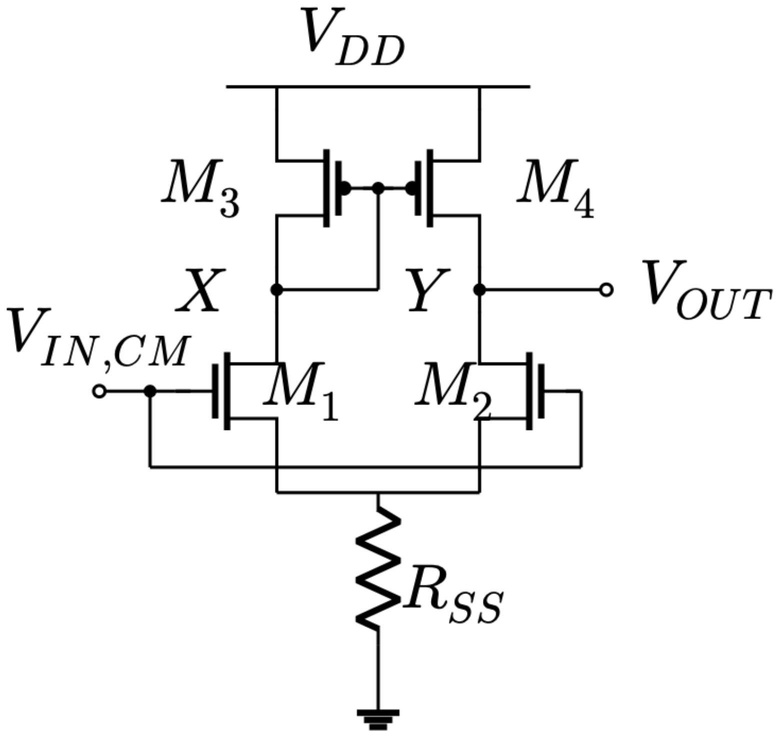
Current Mirror

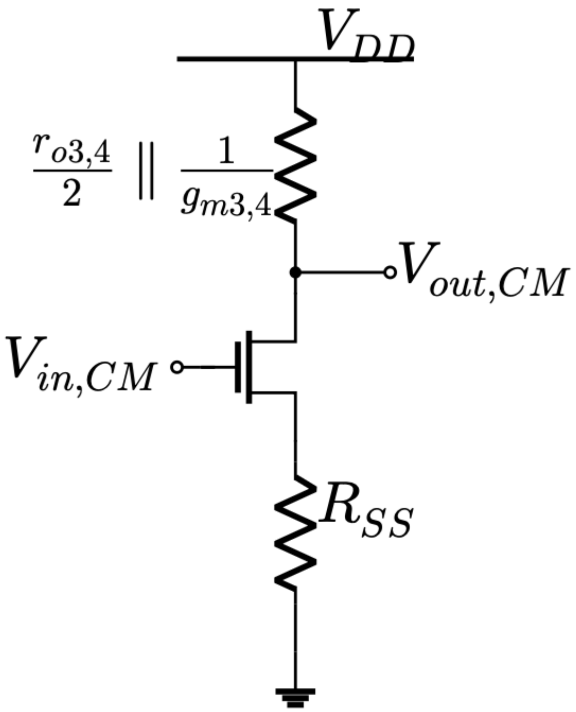


$$I_{OUT1} = \frac{(\frac{W}{L})_{N2}}{(\frac{W}{L})_{N1}} I_{REF} \tag{11}$$

$$I_{OUT2} = \frac{(\frac{W}{L})_{N3}}{(\frac{W}{L})_{N1}} \frac{(\frac{W}{L})_{P2}}{(\frac{W}{L})_{P1}} I_{REF} \tag{12}$$

Differential Pair with Current Mirror Load





Differential Pair with Current Mirror Load

$$A_{vd} = \frac{v_{out}}{v_d} = g_m(r_{o2} \parallel r_{o4}) \tag{13}$$

$$\begin{aligned} A_{vc} &\approx -\frac{\frac{r_{o3,4}}{2} \parallel \frac{1}{2g_{m3,4}}}{\frac{1}{2g_{m1,2}} + R_{SS}} \\ &\approx -\frac{1}{1 + 2g_{m1,2}R_{SS}} \frac{g_{m1,2}}{g_{m3,4}} \end{aligned} \tag{14}$$

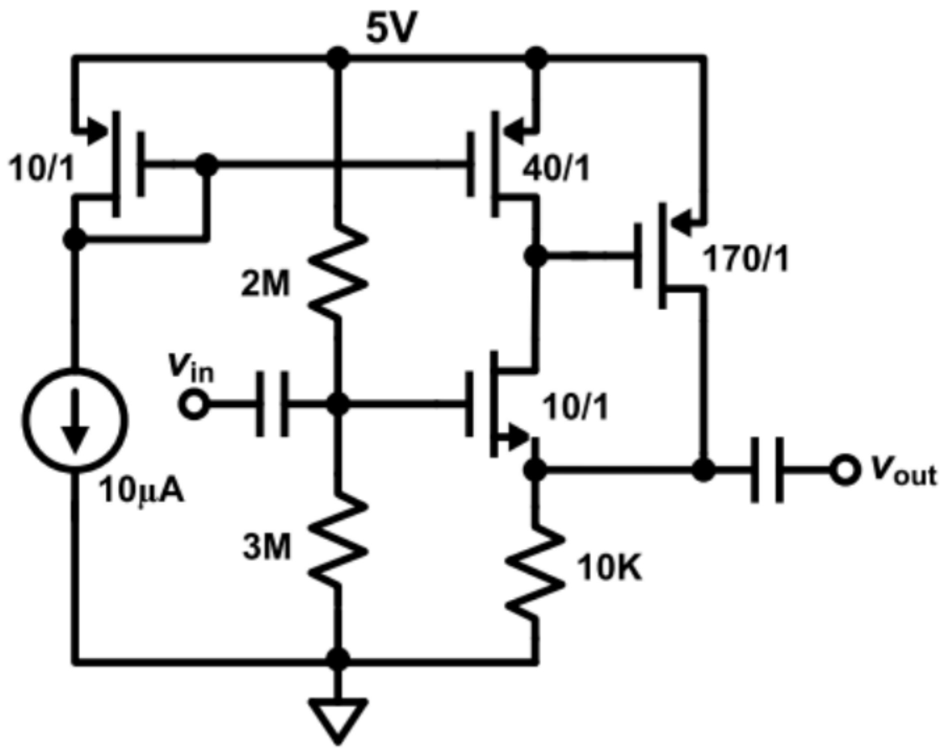
$$CMRR = \frac{| A_{vd} |}{| A_{vc} |} = (1 + 2g_{m1,2}R_{SS})g_{m3,4}(r_{o2} \parallel r_{o4}) \tag{15}$$

Exercises

Exercise (Hard)

(a) Calculate the DC operation voltage of all devices in the circuit. Ignore channel length modulation for this question only.

(b) Derive the small signal gain and input and output resistance.



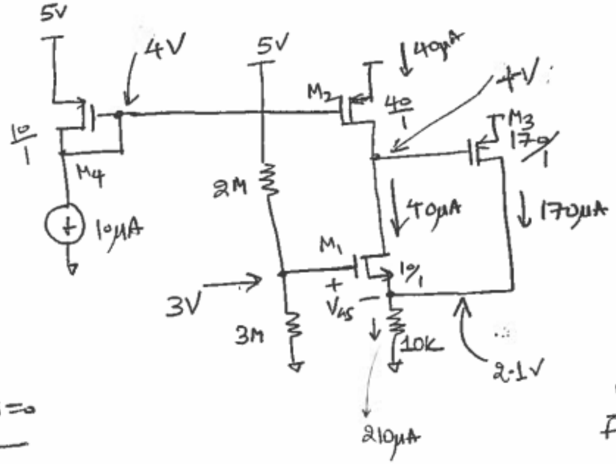
$$V_{THN} = 0.7V \; KP_n = 2 * 10^{-4}$$
$$V_{THP} = 0.8V \; KP_p = 5 * 10^{-5}$$

Problem 3

$KP_n = 250\frac{\mu A}{V^2}$, $V_{THN} = 0.7$

$KP_p = 50\frac{\mu A}{V^2}$, $V_{THP} = 0.8$

(a) DC picture:



Here $\lambda = 0$

M_1: for $I_D = 40\mu A$, $\left(\frac{W}{L}\right) = 10$

$\Rightarrow V_{GS_1} = \sqrt{\frac{2 I_D}{\beta}} + V_{THN}$

$= 0.9V$

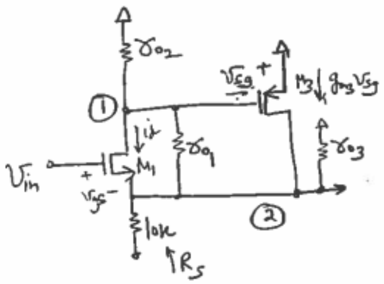
$\Rightarrow V_{GS} = 3 - 2.1V = \boxed{0.9V}$

$V_{DS} = 4.0V - 2.1V$

$\Rightarrow \boxed{V_{DS} = 1.9V}$

Feedback stabilizes gate of M_3 at 4V such that it sinks 170μA to set 40μA in M_1

(b)



$$\frac{v_1}{r_{o2}} + \frac{(v_1 - v_2)}{r_{o1}} + g_{m1}(v_{in} - v_2) = 0$$

$$\Rightarrow \frac{v_1}{r_{o1}\|r_{o2}} - v_2\left(g_{m1} + \frac{1}{r_{o1}}\right) = -g_{m1}v_{in} \longrightarrow (1)$$

$$\frac{v_2}{r_{o3}} + \frac{v_2}{R_s} + \frac{(v_2 - v_1)}{r_{o1}} - g_{m3}\underbrace{v_{sg}}_{-v_1} - g_{m1}(v_{in} - v_2) = 0$$

$$\Rightarrow \frac{v_2}{r_{o3}\|R_s\|r_{o1}} - g_{m1}v_2 + \left(g_{m3} - \frac{1}{r_{o1}}\right)v_1 = g_{m1}v_{in} \longrightarrow (2)$$

$$v_1 = v_2\left(g_{m1} + \frac{1}{r_{o1}}\right)r_{o1}\|r_{o2} - g_{m1}r_{o1}\|r_{o2} \cdot v_{in}$$

plug into (2)

$$v_2\left[\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2}\right] + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left[v_2 \times \left(g_{m1} + \frac{1}{r_{o1}}\right)r_{o1}\|r_{o2} - g_{m1}r_{o1}\|r_{o2}v_{in}\right] = g_{m1}v_{in}$$

$$\Rightarrow v_2\left[\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2} + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left(g_{m1} + \frac{1}{r_{o1}}\right)r_{o1}\|r_{o2}\right] = \left[g_{m1} + g_{m1}r_{o1}\|r_{o2}\left(g_{m3} - \frac{1}{r_{o1}}\right)\right]v_{in}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{v_2}{v_{in}} = \frac{g_{m1}\left[1 + \left(g_{m3} - \frac{1}{r_{o1}}\right)r_{o1}\|r_{o2}\right]}{\left(\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2}\right) + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left(g_{m1} + \frac{1}{r_{o1}}\right)r_{o1}\|r_{o2}} \longrightarrow (3)$$

Calculations:

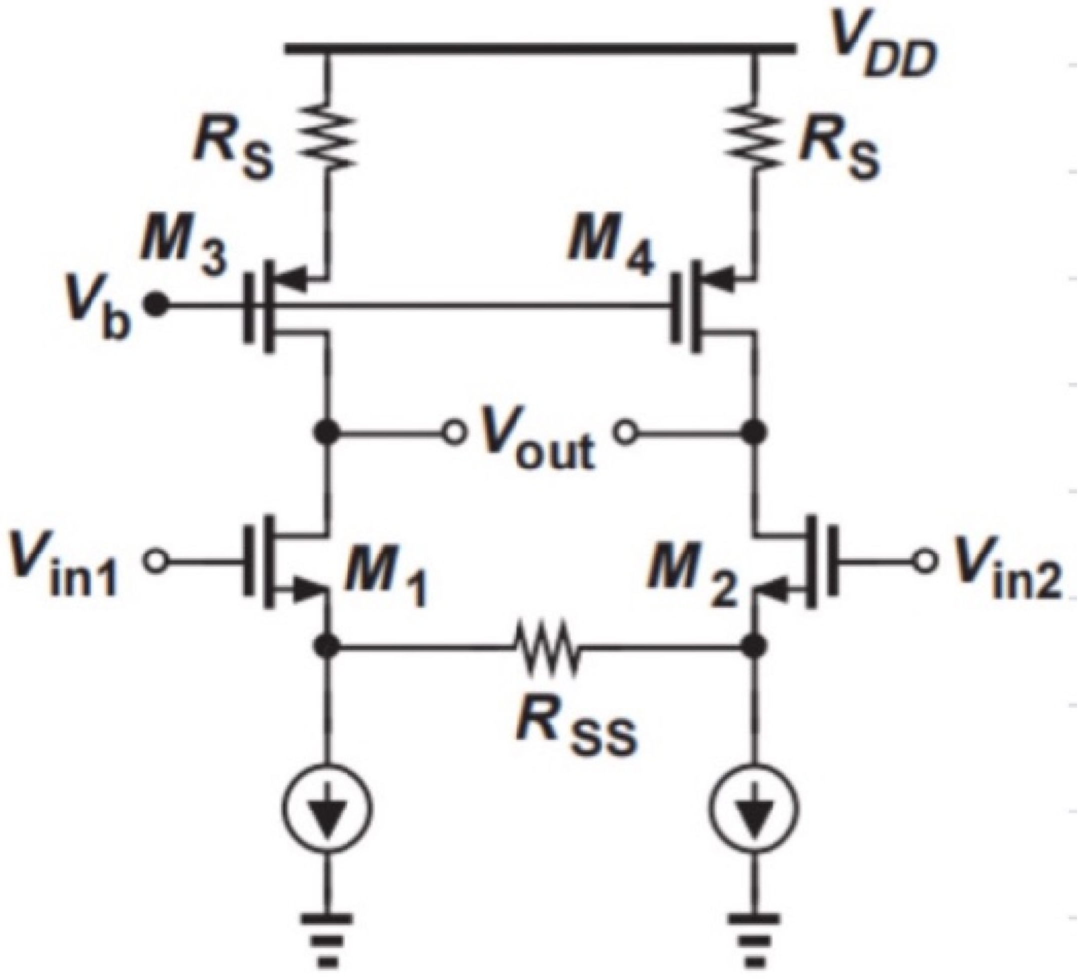
M_1: $g_{m1} = 0.4\frac{mA}{V} \Rightarrow r_{o1} = 250k\Omega$

M_2: $g_{m} = 0.4\frac{mA}{V} \Rightarrow r_{o2} = 250k\Omega$

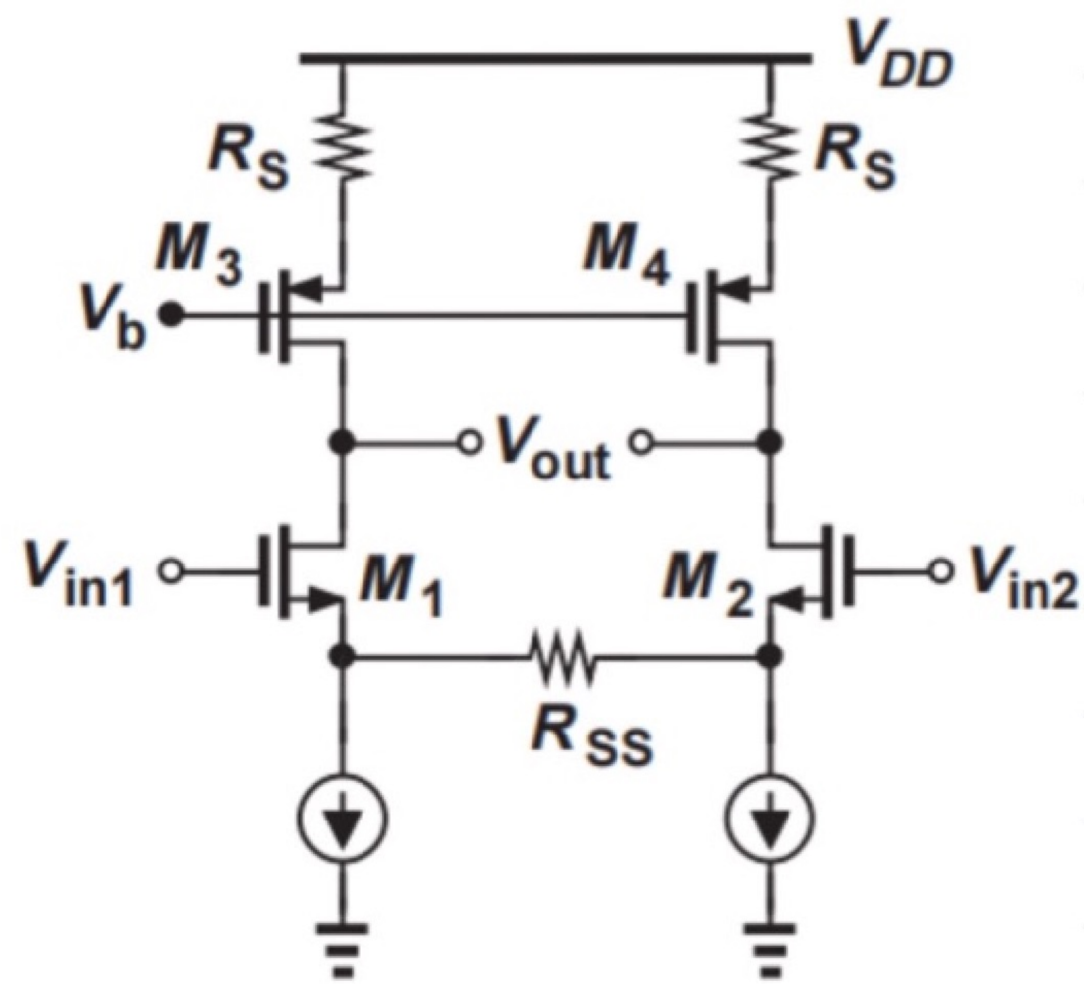
M_3: $g_{m3} = 1.7\frac{mA}{V} \Rightarrow r_{o3} = 58.82k\Omega$

plug these values into (3)

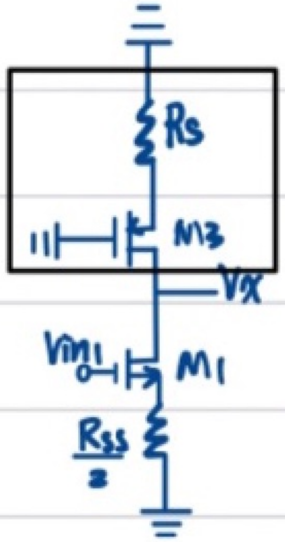
[40pts] Calculate the differential voltage gain of the circuits shown below. Assume perfect symmetry and $\lambda > 0$. You may need to compute the gain as $A_v = -G_m R_{out}$. Hint: use half circuit and R_{SS} can be divided into $\frac{R_{SS}}{2}$ in your half circuit.



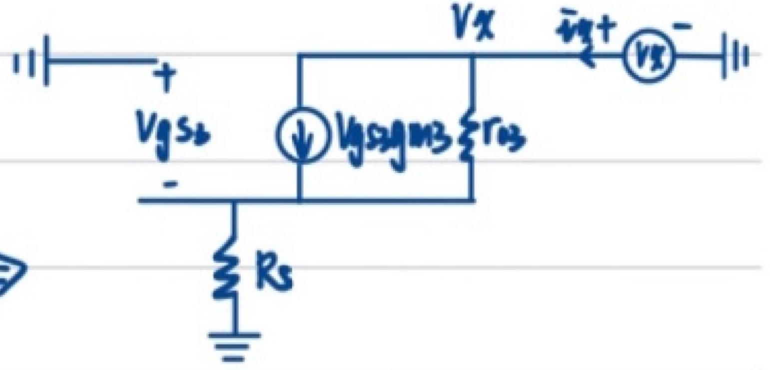
[40pts] Calculate the differential voltage gain of the circuits shown below. Assume perfect symmetry and $\lambda > 0$. You may need to compute the gain as $A_v = -G_m R_{out}$. Hint: use half circuit and R_{SS} can be divided into $\frac{R_{SS}}{2}$ in your half circuit.



Perfect symmetry: Half circuit.

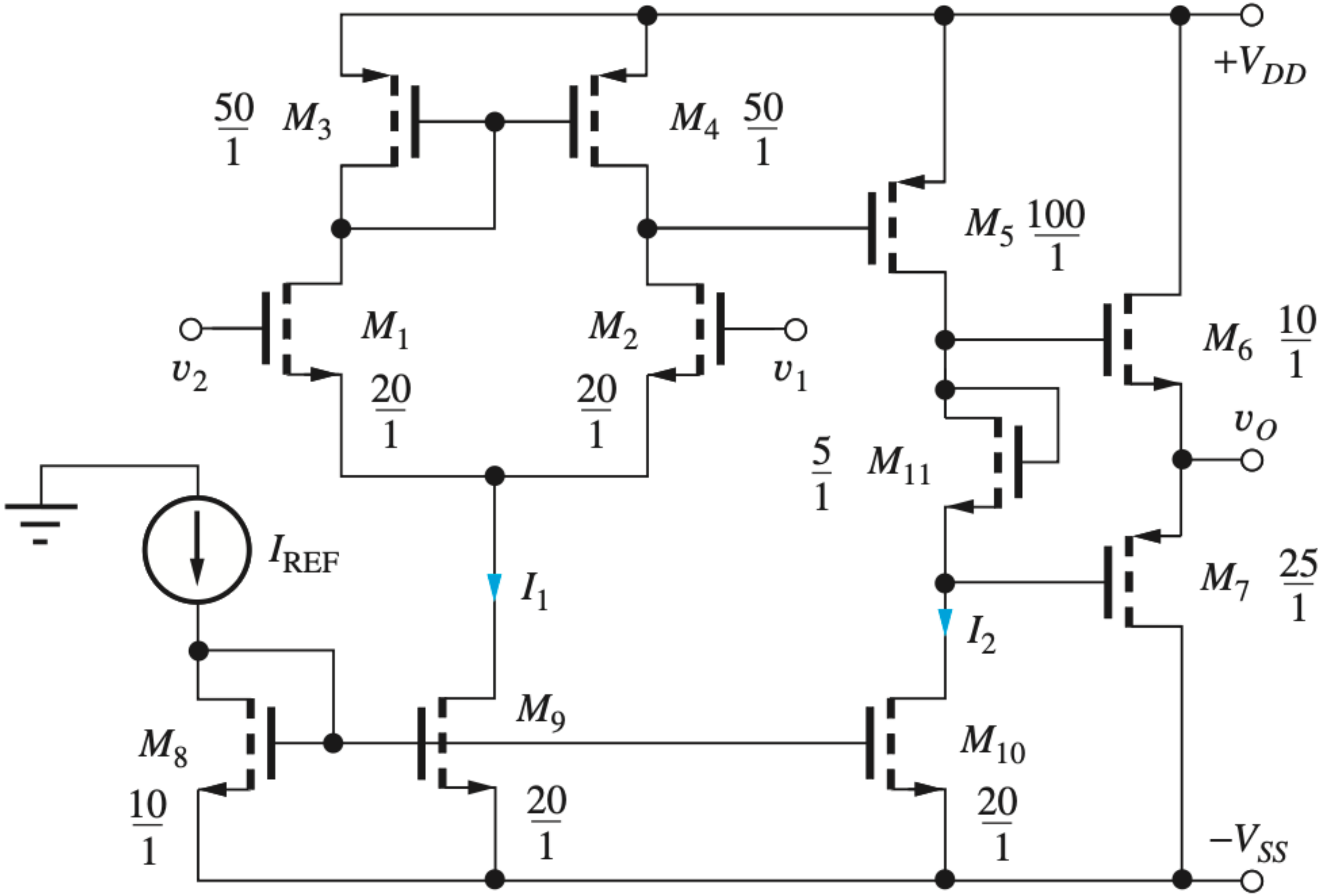


$\Rightarrow$



$$\begin{cases} -V_{gs3} = R_S i_X \\ i_X = V_{gs3} g_{m3} + \frac{V_X + V_{gs3}}{r_{o3}} \end{cases}$$

$$\Rightarrow R_{eq} = r_{o3} + R_S(g_{m3} r_{o3} + 1)$$



Find the voltage gain, input resistance, and output resistance of the amplifier in Fig. 16.42 if $K_n' = 25\ \mu A/V^2$, $K_p' = 10\ \mu A/V^2$, $V_{TN} = 0.75$ V, $V_{TP} = -0.75$ V, $\lambda = 0.0125\ V^{-1}$, $V_{DD} = V_{SS} = 5$ V, and $I_{REF} = 100\ \mu A$.

Analysis: The gain can be estimated using Eq. (16.94).

$$A_{dm} \cong \frac{\mu_{f2}\mu_{f5}}{4} = \frac{1}{4}\left(\frac{1}{\lambda_2}\sqrt{\frac{2K_{n2}}{I_{D2}}}\right)\left(\frac{1}{\lambda_5}\sqrt{\frac{2K_{p5}}{I_{D5}}}\right)$$

For the amplifier in Fig. 16.42,

$$I_{D2} = \frac{I_1}{2} = \frac{2I_{\text{REF}}}{2} = 100\ \mu\text{A} \qquad I_{D5} = I_2 = 2I_{\text{REF}} = 200\ \mu\text{A}$$

$$K_{n2} = 20K'_n = 500\frac{\mu\text{A}}{\text{V}^2} \qquad K_{p5} = 100K'_p = 1000\frac{\mu\text{A}}{\text{V}^2}$$

and

$$A_{dm} \cong \frac{\mu_{f2}\mu_{f5}}{4} = \frac{1}{4}\left(\frac{1}{0.0125}\right)^2 \text{V}^2 \sqrt{\frac{2\left(500\frac{\mu\text{A}}{\text{V}^2}\right)}{100\ \mu\text{A}}}\sqrt{\frac{2\left(1000\frac{\mu\text{A}}{\text{V}^2}\right)}{200\ \mu\text{A}}} = 16{,}000$$

The input resistance is twice the input resistance of M_1, which is infinite: $R_{id} = \infty$. The output resistance is determined by the parallel combination of the output resistances of M_6 and M_7, which act as two source followers operating in parallel:

$$R_{\text{out}} = \frac{1}{g_{m6}} \left\| \frac{1}{g_{m7}} \right. = \frac{1}{\sqrt{2K_{n6}I_{D6}}} \left\| \frac{1}{\sqrt{2K_{p7}I_{D7}}} \right.$$

To evaluate this expression, the current in the output stage must be found. The gate-source voltage of M_{11} is

$$V_{GS11} = V_{TN11} + \sqrt{\frac{2I_{D11}}{K_{n11}}} = 0.75\ \text{V} + \sqrt{\frac{2(200\ \mu\text{A})}{125\left(\frac{\mu\text{A}}{\text{V}^2}\right)}} = 2.54\ \text{V}$$

In this design, $V_{TP} = -V_{TN}$ and the W/L ratios of M_6 and M_7 have been chosen so that $K_{p7} = K_{n6}$. Because I_{D6} must equal I_{D7}, $V_{GS6} = V_{SG7}$. Thus, both V_{GS6} and V_{SG7} are equal to one-half V_{GS11}, and

$$I_{D7} = I_{D6} = \frac{250}{2}\frac{\mu\text{A}}{\text{V}^2}(1.27\ \text{V} - 0.75\ \text{V})^2 = 33.7\ \mu\text{A}$$

The transconductances of M_6 and M_7 are also equal,

$$g_{m7} = g_{m6} = \sqrt{2\left(2.50 \times 10^{-4}\frac{\mu\text{A}}{\text{V}^2}\right)(33.7 \times 10^{-6}\ \mu\text{A})} = 1.30 \times 10^{-4}\ \text{S}$$

and the output resistance at the Q-point is $R_{\text{out}} = 3.85\ \text{k}\Omega$.

Tips

- Remember to bring CTPP (one-page handwritten).
- Pay attention to the concepts (go back to slides).
- Check the key points list one by one carefully.
- Try your best to simplify the circuits (e.g. using Thevenin or drawing equivalent circuits) and write down all steps.
- Just avoid leaving blank on the answer sheet.

Good Luck!

References

- Prof. Xuyang Lu, Lecture Slides, 2025
- Yi Zhao, RC Slides, 2025
- Yifei Xu, RC Slides, 2025
- Jiale Chu, RC Slides, 2024
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- Tianrui Dang, RC Slides, 2024