ECE311 FinalRC

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2025.8.1

Before we start…

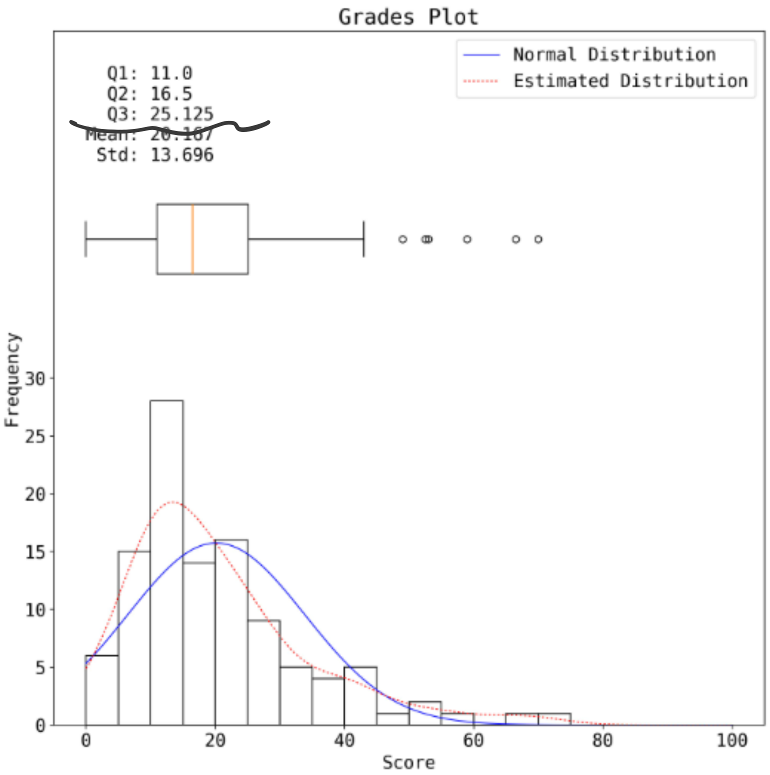
Reminders

1. Difficult exam!

2. Must go over all the questions!

3. Don’t stick on one question!

4. More about understanding rather than calculating!

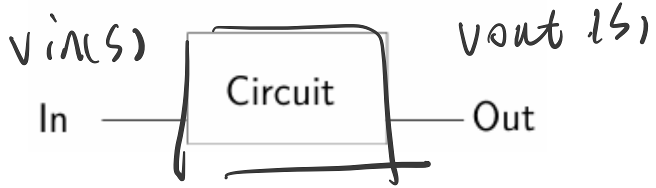


Contents

Frequency Domain

Definition Review

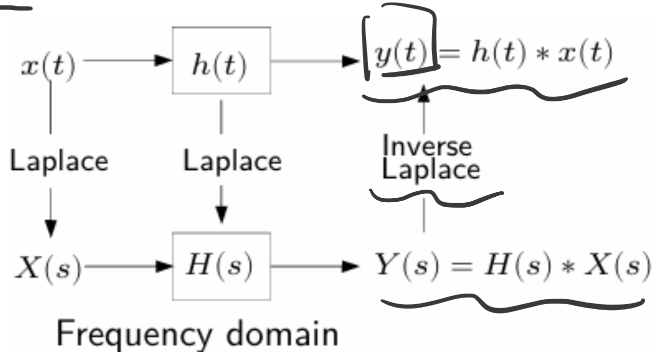
1. Transfer Function H(s)



$H(s) = \frac{V_{out}}{V_{in}}$

2. Use S-domain to Get Time Response

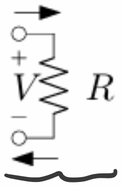
3. Partial Fraction Expansion



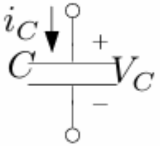
Frequency Domain

Definition Review

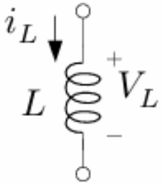
Capacitors and Inductors



R



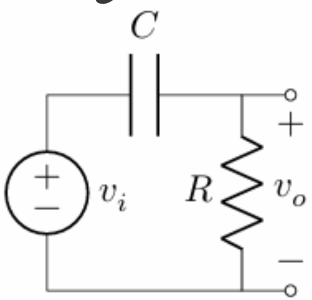
$\frac{1}{sC}$



$s \cdot L$

$\frac{1}{sC}$

vin



vout

$$vout = \frac{vin \cdot R}{\frac{1}{sC} + R}$$

Pole and Zero

$$|H(s_z)| = 0$$

$s \rightarrow H(s) = 0$

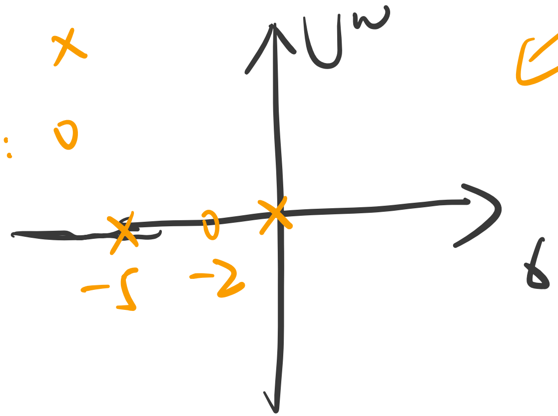
$$|H(s_p)| = \infty$$

$s \rightarrow H(s) = \infty$

pole : x

zero : o

G(s)



$$G_{(s)} = \frac{s+2}{s^2+5s}$$

$$G(s) = \frac{s+2}{s\cdot(s+5)}$$

pole : 0 and -5

zero : -2

Frequency Domain

Get Diff. Eq from H(s)

$$H(s) = 3\frac{(s+4)}{s^2+2s+5} \Rightarrow \frac{Y(s)}{X(s)}$$

$$s^n X(s) \xrightarrow{\text{Time domain}} \frac{d^n x(t)}{dx^n}$$

$$(s^2+2s+5)\cdot Y(s) = 3(s+4)\cdot X(s)$$

$$\frac{d^2y(t)}{dt^2} + 2\cdot\frac{dy(t)}{dt} + y(t) = 3\frac{dx(t)}{dt} + 12x(t)$$

Frequency Domain

Magnitude and Phase

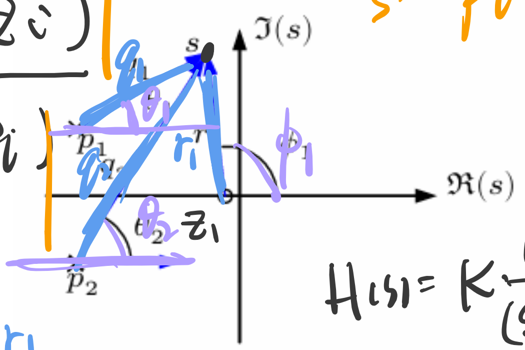
$$|H(s)| = K\frac{\prod_{i=1}^{m} |(s - z_i)|}{\prod_{i=1}^{n} |(s - p_i)|}$$

$$H(s) = K \cdot \frac{\prod_{i=1}^{m} |(s - z_i)|}{\prod_{i=1}^{n} |(s - p_i)|}$$

$s - z_i$

$s - p_i$

$$\angle H(s) = \sum_{i=1}^{m} \angle(s - z_i) - \sum_{i=1}^{n} \angle(s - p_i)$$



$$H(s) = K \cdot \frac{(s - z_1)}{(s - p_1)\cdot(s - p_2)}$$

$$|H(s)| = K\frac{r_1 \dots r_m}{q_1 \dots q_n}$$

$$|H(s)| = K \cdot \frac{r_1}{q_1 \cdot q_2}$$

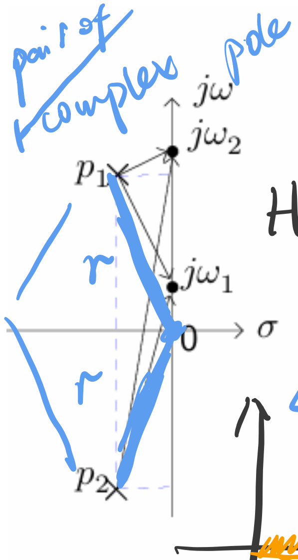
$P_1, P_2 \dots$

$$\angle H(s) = (\phi_1 + \dots + \phi_m) - (\theta_1 + \dots + \theta_n)$$

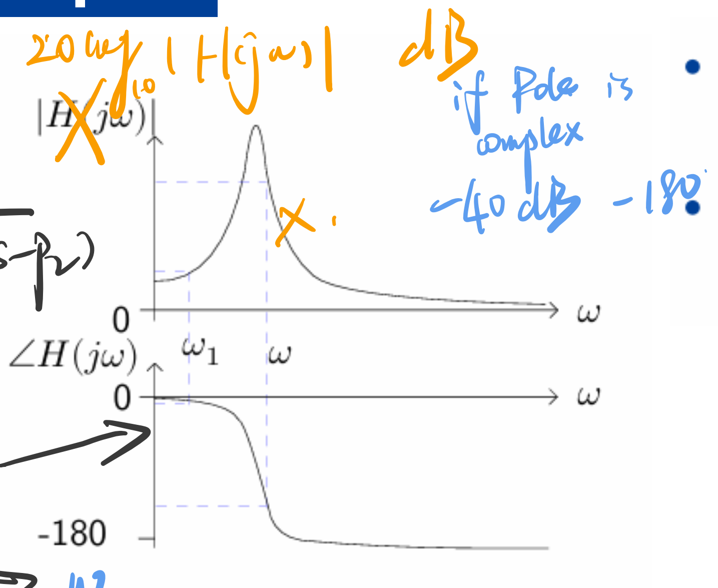
$$\angle H(s) = \phi_1 - \theta_1 - \theta_2$$

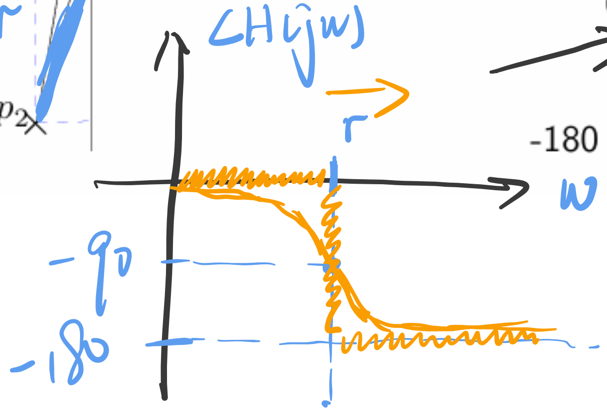
Bode Plot

Construction Example



$$H(s) = \frac{K}{(s-p_1)(s-p_2)}$$





Pole (left plane)

- For lefthand plane
  - Slope changes by −20 dB/decade
  - Phase decreases by 90°
- Zero
  - Slope changes by 20 dB/decade
  - Phase increase by 90°

right plane

slope 20 dB

increase 90°

slope −20 dB

decrease 90°

Construction Example

$$H(j\omega) = \frac{1}{j\omega + 1}$$

$$H(s) = s$$

$$H(s) = \frac{1}{s} \text{ and } s = j\omega$$

$$H(s) = \frac{s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^4}\right)}$$

$H(s) = \frac{1}{s+1}$

$= \boxed{A_0 = 1} \angle H(s)$

s for zero for pole in left plane.

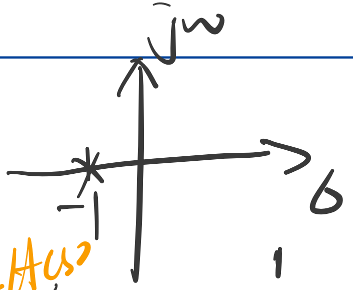
$\boxed{A_0 = 1} \rightarrow 0$

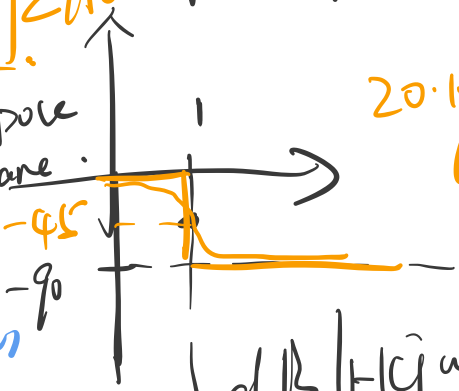
$H(s) = \boxed{A_0} \frac{(1-\boxed{\tau_1 s})\cdots(1-\tau_n s)}{(1-\boxed{\tau_{n+1} s})\cdots(1-\tau_{n+m} s)}$ zeros / poles

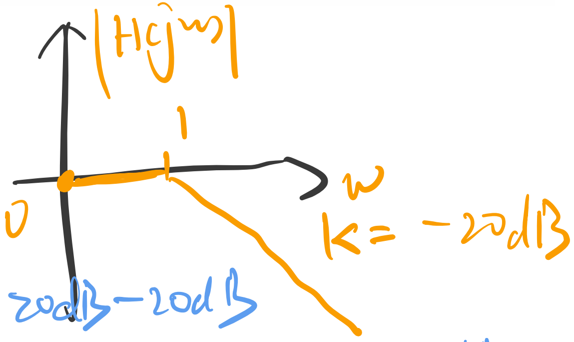
$|A_0|$

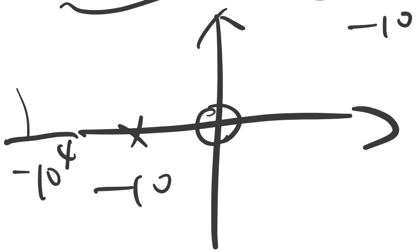
- For lefthand plane
  - Slope changes by –20 dB/decade
  - Phase decreases by 90°
- Zero
  - Slope changes by 20 dB/decade
  - Phase increase by 90°

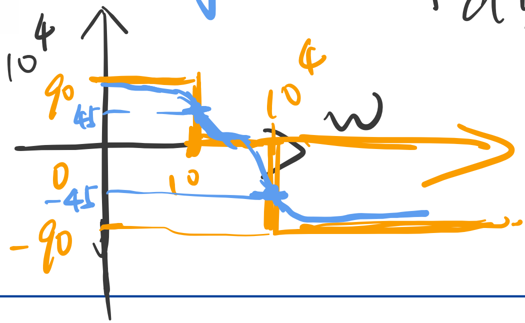
$20 \cdot \log_{10} 1 = 0$

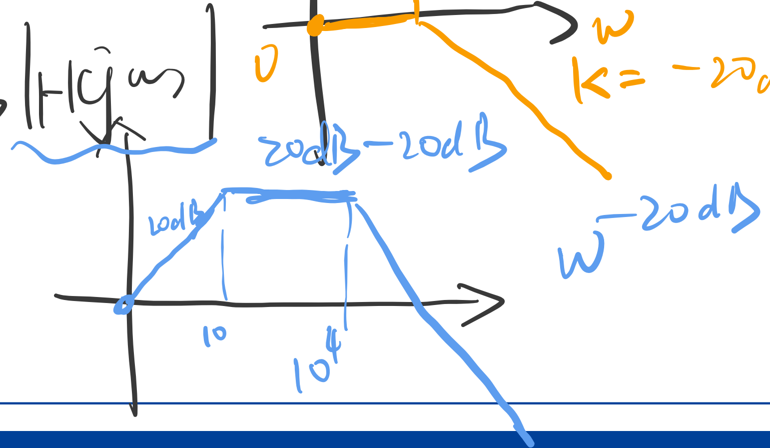






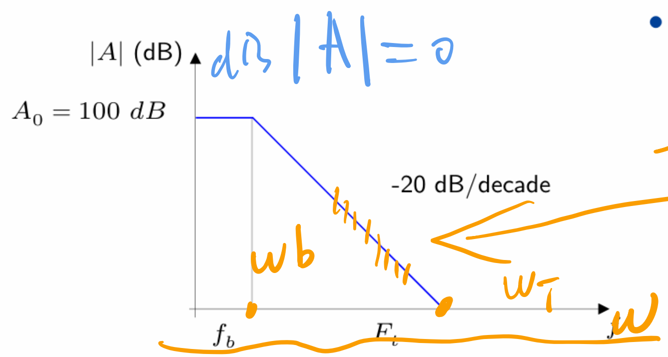






Bode Plot

Bandwidth and Gain-Bandwidth Product



- $f(t)$ (ω_t) is ω where $\boxed{|A(j\omega)| = 1}$

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b} = 1 \tag{24}$$

$$1 \approx \frac{A_0}{\omega_T/\omega_b} \Rightarrow \omega_T \approx A_0\omega_b \tag{25}$$

$$\boxed{\omega_T \approx A_0\omega_b} \tag{26}$$

- For lefthand plane
  - Slope changes by –20 dB/decade
  - Phase decreases by 90°
- Zero
  - Slope changes by 20 dB/decade
  - Phase increase by 90°

$$|A(j\omega)| = \frac{|A_0|}{\sqrt{2}}$$

$$\left|\frac{A_0}{1+\frac{s}{w_b}}\right|^2 = \left|\frac{A_0}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$\rightarrow \frac{1}{\left(1+\frac{js}{w_b}\right)\left(1-\frac{js}{w_b}\right)} = \frac{1}{1+\frac{s^2}{w_b^2}} = \frac{1}{2}$$

Recommendations for GBW calculation:
Draw the magnitude plot in dB directly to solve for bandwidth!
Or derive the equation and set it to 1
Both are OK!
Typically only one solution is needed to be considered!

$$\frac{s^2}{w_b^2} = 1 \rightarrow \boxed{s = w_b}$$

Bandwidth and Gain-Bandwidth Product

$$V_{out}(\omega) = A(\omega)V_{Id} = A(\omega)\left(V_{in} - V_n\right)$$

$$V_{out} = \frac{A_0}{1+\frac{s}{\omega_b}}\left(V_{in} - \beta V_{out}\right)$$

$$= A(\omega)\left(V_{in} - \beta V_{out}\right)$$

$$\left(\frac{A_0\beta}{1+\frac{s}{\omega_b}} + 1\right) V_{out} = \frac{A_0}{1+\frac{s}{\omega_b}} V_{in}$$

V_{out}:

$$= \frac{A_0}{1 + j\omega/\omega_b}\left(V_{in} - \beta V_{out}\right)$$

1. pole-zero plot
2. mag Bode plot
3. phase Bode plot

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\boxed{A_0}}{1+\frac{s}{\omega_b}} \cdot \frac{1}{\frac{A_0\beta}{1+\frac{s}{\omega_b}} + 1}$$

4. bandwidth
5. GBP

A_0'

First Order Systems

Definition

Contains 1 pole and at $\boxed{\text{most 1 zero}}$ 0 1

$$H(S) = \frac{a_0 + a_1 S}{1 + bS}$$

$$\tau = RC_1 \qquad \tau = \frac{L_1}{R_0}$$

$$\boxed{H(s) = \frac{H^0 + H^1 \tau s}{1 + \tau s}}$$

independent source.

To find the time constant, remove the cap/ind nulling all the sources, find the resistance. R τ

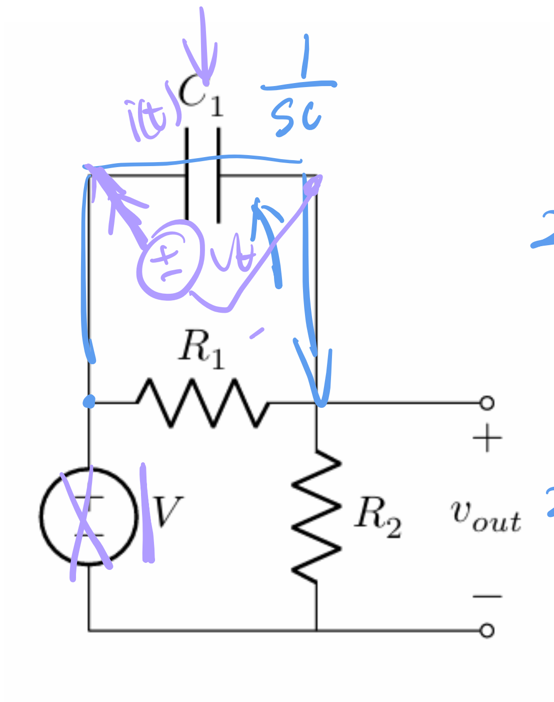
cap → open circuit.

To find transfer constant H^0, it is just the low frequency gain. ind → short circuit.

To find the transfer constant H^1, we look into high frequency response, so the cap shall be shorted. For inductor it is the opposite.

cap → short ind → open.

Example



$$H(s) = \frac{H^0 + H^1 \tau s}{1 + \tau s}$$

216 / 215 :

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{R_2}{\left(\frac{1}{sC} \| R_1\right) + R_2}$$

$$=$$

311 :

$$H^0 = \frac{v_{out}}{v_{in}} = \frac{R_2}{R_2 + R_1}$$

$$H^1 = \frac{v_{out}}{v_{in}} = 1$$

$$\tau = R \cdot C \qquad R = R_1 \| R_2$$

$$H(s) = \frac{\frac{R_2}{R_1 + R_2} + (R_1 \| R_2) C s}{1 + (R_1 \| R_2) C s}$$

First Order Systems

Example

H_0: $V_\pi = \dfrac{v_{in}\cdot r_\pi}{r_\pi + R_1}$

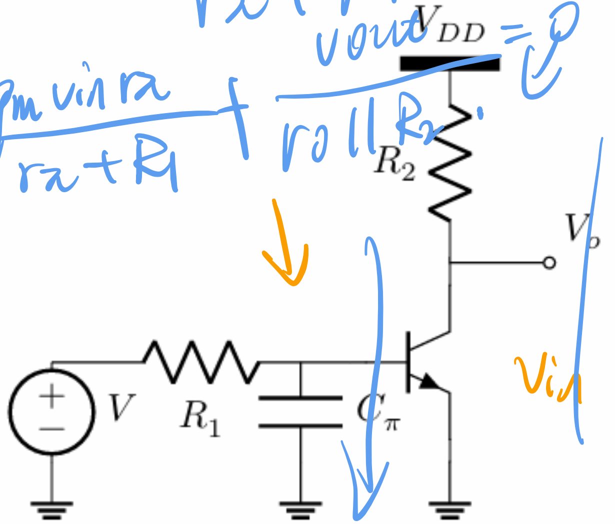
$\dfrac{g_m v_{in} r_\pi}{r_\pi + R_1} \cdot r_o \| R_2$

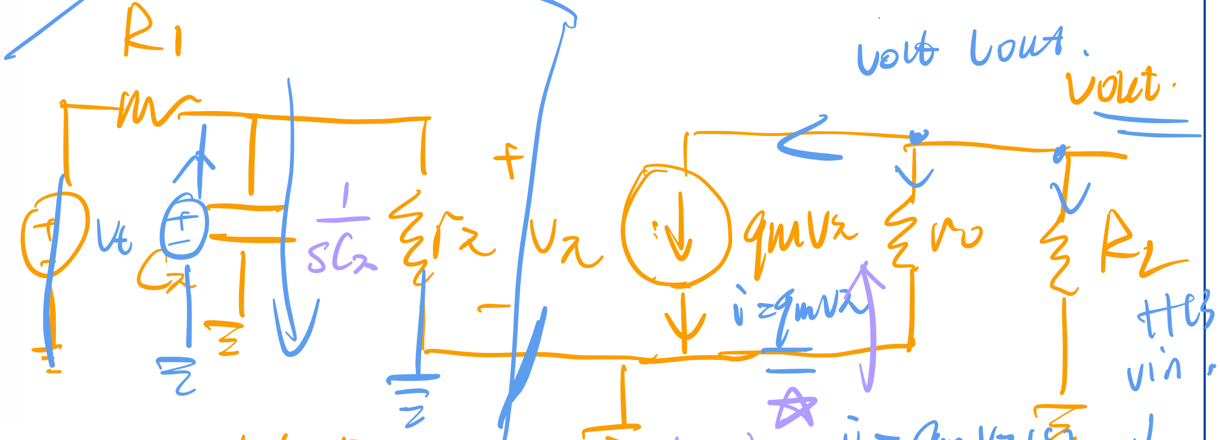
$v_{out} = 0$

$H_0 \cdot$

$$H(s) = \frac{H^0 + H^1 \tau s}{1 + \tau s}$$

Frequency





$H_0 =$

$H_1 =$ $v_\pi = 0$ $\quad v_{out} = 0$

$H_1 = 0$

$\tau = R \cdot C_\pi \quad R = R_1 \| r_\pi$

$v_\pi = \dfrac{v_{in}\left(r_\pi \| \frac{1}{sC_\pi}\right)}{R_1 + \left(r_\pi \| \frac{1}{sC_\pi}\right)}$

$i = g_m v_\pi(s)$

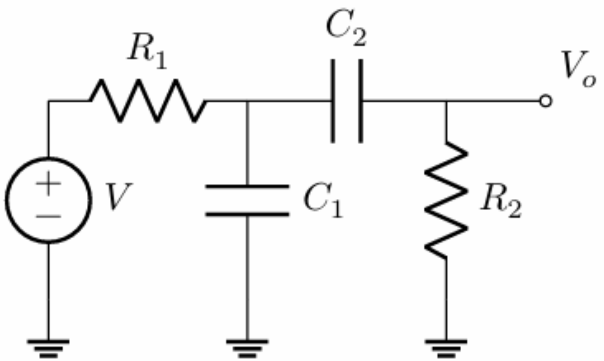
$g_m v_\pi = \dfrac{v_{out}}{r_o \| R_2}$

$\dfrac{H(s)}{v_{in}}$

Nth Order Systems

Example

$$H(s) = \frac{a_0 + a_1 S + a_2 S^2 + \ldots}{1 + b_1 S + b_2 S^2 + \ldots}$$



Too hard for me to explain it explicitly…(For myself I can not understand what prof is talking about…)

Too many parameters

Suggest ECE216 methods for analyzing and work out the parameter conversely

Tips: the voltage and current relationship in frequency domain still holds!

Bode Plot

Bandwidth Estimation

$$H(s) \approx \frac{a_0}{1 + b_1 s + b_2 s^2 + \cancel{\cdots} + b_n s^n}$$

$$H(s) \approx \frac{a_0}{1 + b_1 s}$$

$$\omega_h \approx \frac{1}{b_1} = \frac{1}{\cancel{\sum}_{i=1}^{N} \tau_i^0}$$

Parasitic Capacitance

Definition

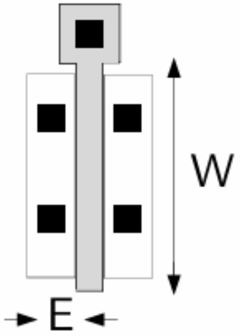
Triode

$$C_{GS} = W_{ov} + 1/2\,(WLC_{ox}) \quad (1)$$

$$C_{GD} = WC_{ov} + 1/2\,(WLC_{ox}) \quad (2)$$

$$C_{SB} = WEC_j + 2(W+E)C_{jsw} \quad (3)$$

$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \quad (4)$$



Saturation

$$C_{GS} = WC_{ov} + 2/3\,(WLC_{ox}) \quad (5)$$

$$C_{GD} = WC_{ov} \quad (6)$$

$$C_{SB} = WC_j + 2(W+E)C_{jsw} \quad (7)$$

$$C_{DB} = WEC_j + 2(W+E)C_{jsw} \quad (8)$$

Parasitic Capacitance

Parameter

$\varepsilon_{ox} = 3.9\,\varepsilon_0 \quad \varepsilon_0 = 8.85\times10^{-12}$ F/m

NMOS Model

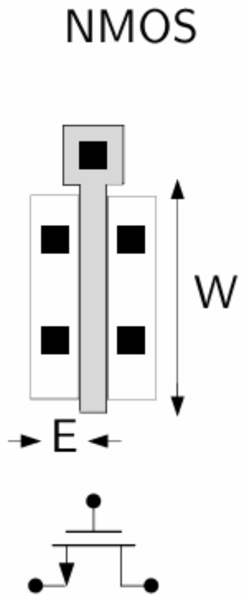
LEVEL $= 1$	VTO $=0.7$	GAMMA $= 0.45$	PHI $= 0.9$
NSUB $= 9e+14$	LD $= 0.08e-6$	UO $= 350$	LAMBDA $= 0.1$
TOX $= 9e-9$	PB $= 0.9$	CJ $= 0.56e-3$	CJSW $= 0.35e-11$
MJ $= 0.45$	MJSW $= 0.2$	CGDO $= 0.4e-9$	JS $= 1.0e-8$

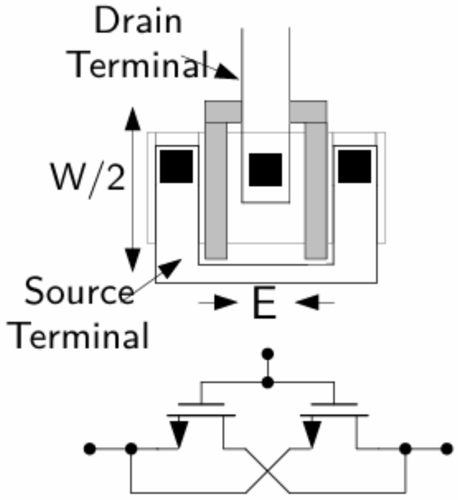
PMOS Model

LEVEL $= 1$	VTO $= -0.8$	GAMMA $= 0.4$	PHI $= 0.8$
NSUB $= 5e+14$	LD $= 0.09e-6$	UO $= 100$	LAMBDA $= 0.2$
TOX $= 9e-9$	PB $= 0.9$	CJ $= 0.94e-3$	CJSW $= 0.32e-11$
MJ $= 0.5$	MJSW $= 0.3$	CGDO $= 0.3e-9$	JS $= 0.5e-8$

Parasitic Capacitance

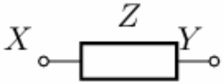
Example

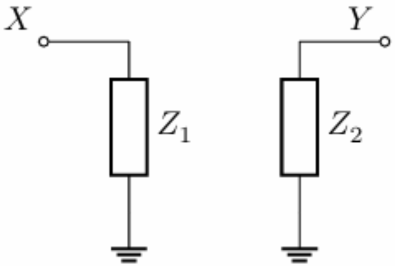




Miller Effect

Definition





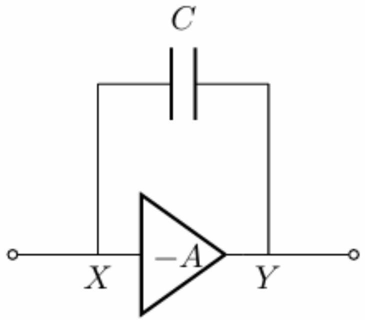
$$Z_1 = \frac{Z}{1 - A_v} \tag{31}$$

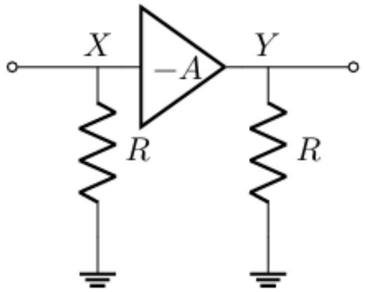
$$Z_2 = \frac{Z}{1 - \frac{1}{A_v}}$$

Note: There must be another way between X and Y

Miller Effect

Example





$$Z_1 = \frac{\frac{1}{SC_F}}{1+A} = \frac{1}{S(1+A)C_F}$$

$$Z_2 = \frac{\frac{1}{SC_F}}{1+\frac{1}{A}} = \frac{1}{S\left(1+\frac{1}{A}\right)C_F}$$

Miller Effect

Example

Output impedance

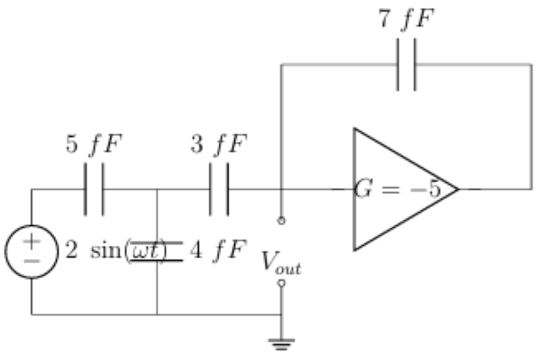


Figure 2: Miller

Thanks！