ECE311 Electronic Circuit Homework 7

Due: **July 15th**

Note:

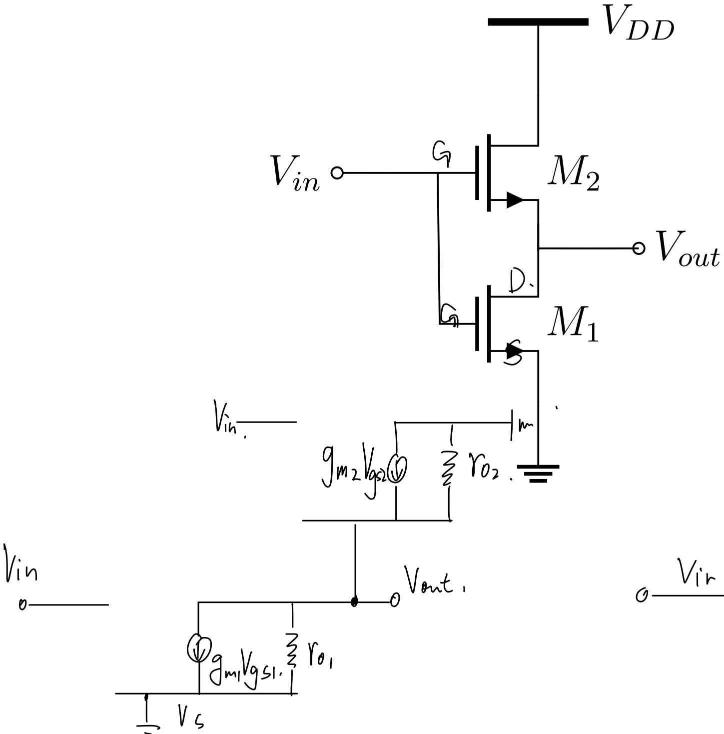
1) *Please use A4 size paper or page.*
2) *Please clearly state out your final result for each question.*

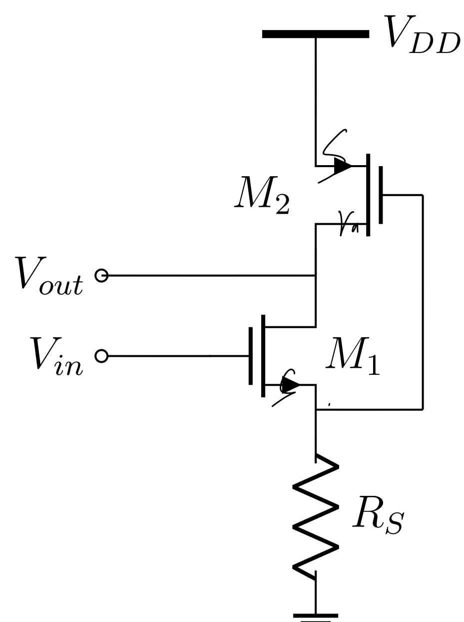
In the vibrant city of Innovatech, there was a skilled engineer named Sam who worked for a cutting-edge tech company called ElectroWorks.

At work, Sam was tasked with developing a new high-performance amplifier. Sam's journey began with the common source amplifier. which is known for its high gain, seeming like a great starting point. Sam set up the circuit, measuring its performance. The results were impressive, and the amplified signal was clear and strong. Sam felt a surge of excitement, seeing the theoretical knowledge come to life.

Question 1. Common Source

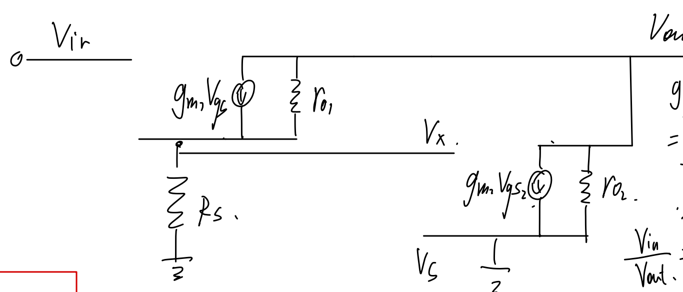
Assume that all MOSFETs are in saturation, calculate the small-signal voltage gain of each circuit. ($\lambda \neq 0, \gamma = 0$)





$$\begin{cases} g_{m1}(V_{in}-V_x)+\frac{V_{out}-V_x}{r_{o1}} = \frac{V_x}{R_S} \\ \frac{V_x}{R_S} + g_{m2}V_x + \frac{V_{out}}{r_{o2}} = 0 \end{cases}$$

$$\Rightarrow V_x = \frac{-1}{r_{o2}} \cdot \frac{1}{g_{m2}+\frac{1}{R_S}} V_{out}$$



$$g_{m1}V_{in} = \left(\frac{1}{R_S}+\frac{1}{r_{o1}}+g_{m1}\right)V_x - \frac{1}{r_{o1}}V_{out}$$

$$= \left[-\left(\frac{1}{R_S}+\frac{1}{r_{o1}}+g_{m1}\right)\frac{1}{r_{o2}(g_{m2}+\frac{1}{R_S})} - \frac{1}{r_{o1}}\right]V_{out}$$

$$\frac{V_{in}}{V_{out}} = \frac{\frac{-(\frac{1}{R_S}+\frac{1}{r_{o1}}+g_{m1})}{r_{o2}(g_{m2}+\frac{1}{R_S})} - \frac{1}{r_{o1}}}{g_{m1}}$$

$$= -\frac{r_{o1}+R_S+g_{m1}r_{o1}R_S+r_{o2}g_{m2}R_S+r_{o1}}{r_{o1}r_{o2}g_{m1}(g_{m2}R_S+1)}$$

$$g_{m1}V_{in} + \frac{V_{out}-V_{in}}{r_{o1}} = g_{m2}(V_{in}-V_{out}) + \frac{-V_{out}}{r_{o2}}$$

$$\boxed{A_V = \frac{V_{out}}{V_{in}} = \frac{(-g_{m1}+g_{m2})r_{o1}r_{o2}}{g_{m2}r_{o1}r_{o2}+r_{o1}+r_{o2}}}$$

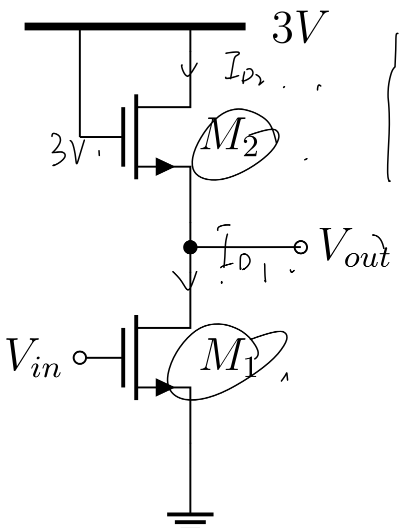
$$\boxed{A = \frac{V_{out}}{V_{in}} = -\frac{r_{o1} r_{o2} g_{m1}(g_{m2}R_s + 1)}{r_{o1} + R_s + g_{m1} r_{o1} R_s + r_{o2} g_{m2} R_s + r_{o2}}}$$

To have a better understanding of the quantified properties of the common source circuit, he built a simple model and decided to do some precise calculations with it. After careful considerations, he found out that his circuits perfectly satisfies the task.

Question 2. Common Source with Diode-connected Load (Lunatic)

Consider the following circuit with $(W/L)_1 = 50/0.5$ and $(W/L)_2 = 10/0.5$. Assume that $\lambda = \gamma = 0$. Then 3 V for V_{DD}, 0.7 V for V_{TH1}

1) At what input voltage is M_1 at the edge of the triode region? What is the small-signal gain under this condition?
2) What input voltage drives M_1 into the triode region by 50mV ? What is the small-signal gain under this condition?

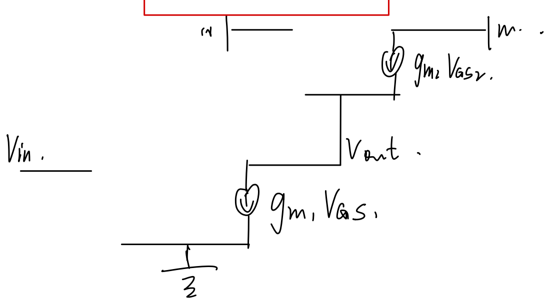


1).

$V_{GS} - V_T = V_{DS}$.

$$\begin{cases} V_{in} = 0.7V + V_{out}, \\ 100(V_{out})^2 = 20(3 - V_{out} - 0.7)^2. \end{cases}$$

$$\boxed{\begin{aligned} V_{out} &= 0.71\ V. \\ V_{in} &= 1.41 V. \end{aligned}}$$



$g_{m1}V_{in} = g_{m2}(\ -V_{out})$.

$\sqrt{2\mu_n C_{ox}(\frac{W}{L})_1 I_{D1}}\ V_{in} = \sqrt{2\mu_n C_{ox}(\frac{W}{L})_2 I_{D2}}\ (-V_{out})$.

$$\boxed{A = \frac{V_{out}}{V_{in}} = -\sqrt{\frac{100}{20}} = -2.23.}$$

(2).

$$\begin{cases} V_{GS1} - V_T - 0.05 = V_{DS1}. \\ V_{in} = V_{out} + 0.75, \\ 100(V_{in} - 0.7 - \frac{1}{2}V_{out})V_{out} = \frac{1}{2}\times 20(2.3 - V_{out})^2. \end{cases} \Rightarrow \boxed{\begin{cases} V_{out} = 0.68 \\ V_{in} = 1.43V. \end{cases}}$$

$10(V_{in} - 0.7 - 0.5V_{out}) = \frac{(2.3 - V_{out})^2}{V_{out}}$.

$V_{in} = \frac{(2.3 - V_{out})^2}{10 V_{out}} + 0.7 + 0.5V_{out}$,

$V_{out}^2 - 10V_{in}V_{out} - 4.6V_{out} + 7V_{out} + 5V_{out}^2 + 4.6^2 = 0.$

$6V_{out}^2 + 2.4V_{out} - 10V_{in}V_{out} + 2.3^2 = 0$.

$6\cdot 2V_{out}\cdot\frac{\partial V_{out}}{\partial V_{in}} + 2.4\frac{\partial V_{out}}{\partial V_{in}} - 10V_{out} - 10V_{in}\frac{\partial V_{out}}{\partial V_{in}} = 0$.

$\frac{\partial V_{out}}{\partial V_{in}}(12V_{out} + 2.4 - 10V_{in}) = 10V_{out}$.

$\frac{\partial V_{out}}{\partial V_{in}} = \frac{10V_{out}}{12V_{out} + 2.4 - 10V_{in}}$

$\boxed{= -1.8.}$

However, the project requirements demanded more than just high gain. The amplifier needed to handle high frequencies efficiently, which led Sam to explore the common gate amplifier, which has unique properties of low input impedance and good high-frequency response. Sam spent late nights in the lab, tweaking and testing, until they achieved a configuration that performed well at the required frequencies.

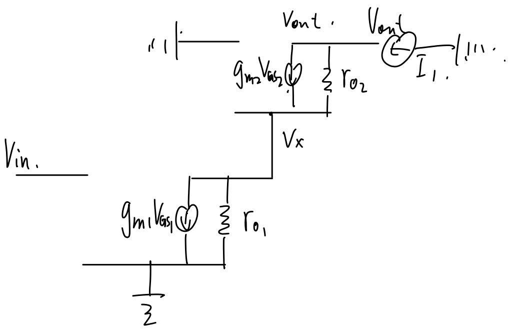
Question 3. Common Gate Common Source (Medium)

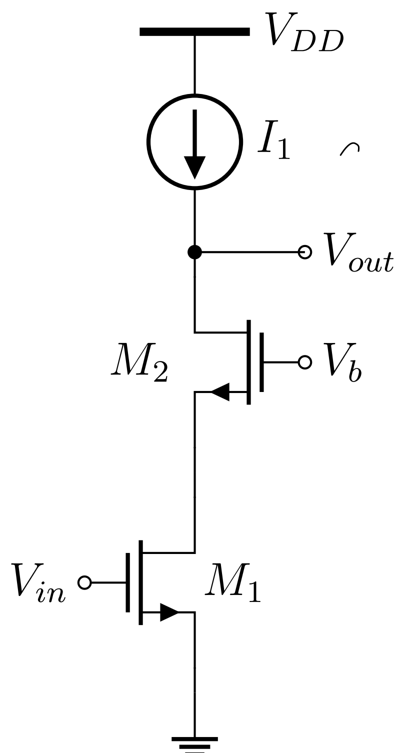
Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

Parameter for NMOS: $V_{THN} = 0.7V$, $K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V$, $K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

All the size of transistor is $W = 20\mu m, L = 1\mu m$





when $I_1 = 0.01$ mA.

$r_{o1} = r_{o2} = \frac{1}{\lambda I_1} = 2.5\times10^6\Omega$.

$g_{m1} = g_{m2} = \sqrt{2k_n'\frac{W}{L}I_D} = 2.1\times10^{-4}$

$\boxed{R_{out} = r_{o1}+r_{o2}+g_{m2}r_{o1}r_{o2} = 1.32\times10^9\Omega. \quad A_v = -g_{m1}g_{m2}r_{o1}r_{o2} - g_{m1}r_{o1} = -2.76\times10^5}$

when $I_1 = 0.1mA$.

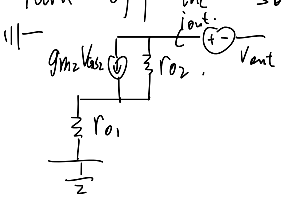
$r_{o1} = r_{o2} = 2.5\times10^5\Omega$.

$g_{m1} = g_{m2} = 6.6\times10^{-4}$

$\boxed{R_{out} = 4.2\times10^7\Omega \quad A_v = -2.77\times10^4}$

$g_{m1} = g_{m2} = \sqrt{2K_n'\frac{W}{L}I_D}$. $r_{o2} = r_{o1} = \frac{1}{\lambda I_d}$.

Turn off the source.



$$i_{out} = g_{m2}(-i_{out}r_{o1}) + \frac{V_{out} - i_{out}r_{o1}}{r_{o2}}$$

$$i_{out}\left(1+g_{m2}r_{o1}+\frac{r_{o1}}{r_{o2}}\right) = \frac{V_{out}}{r_{o2}}$$

$$R_{out} = \frac{V_{out}}{i_{out}} = r_{o1} + r_{o2} + g_{m2}r_{o1}r_{o2}.$$

$$\begin{cases} \frac{V_{out}-V_x}{r_{o2}} - g_{m2}V_x = I_1 \\ g_{m1}V_{in} + \frac{V_x}{r_{o1}} = I_1 \end{cases}$$

$$\Rightarrow V_x = \left(\frac{V_{out}}{r_{o2}} - I_1\right)\cdot\frac{1}{g_{m2}+\frac{1}{r_{o2}}}$$

$$g_{m1}V_{in} + \left(\frac{V_{out}}{r_{o2}} - I_1\right)\cdot\frac{1}{r_{o1}(g_{m2}+\frac{1}{r_{o2}})} = I_1$$

$$\frac{V_{out}}{r_{o2}} - I_1 = I_1\cdot r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right) - g_{m1}g_{m2}r_{o1}V_{in} - 3g_{m1}\frac{r_{o1}}{r_{o2}}V_{in}$$

$$V_{out} = I_1 r_{o1} + I_1 r_{o1}r_{o2}g_{m2} - g_{m1}g_{m2}r_{o1}r_{o2}V_{in} - g_{m1}r_{o1}V_{in}$$

$$A_v = \frac{dV_{out}}{dV_{in}} = -g_{m1}g_{m2}r_{o1}r_{o2} - g_{m1}r_{o1}$$

The real test came when Sam decided to combine the strengths of both configurations into a cascode amplifier. This was a complex task that required precise tuning and a deep understanding of both common source and common gate amplifiers. Sam dove into research papers, consulted with colleagues, and ran numerous simulations. Each iteration of the design brought them closer to the desired outcome. The lab became a second home, with Sam often working through weekends to perfect the design.

Question 4. Common Gate Common Source (Medium)

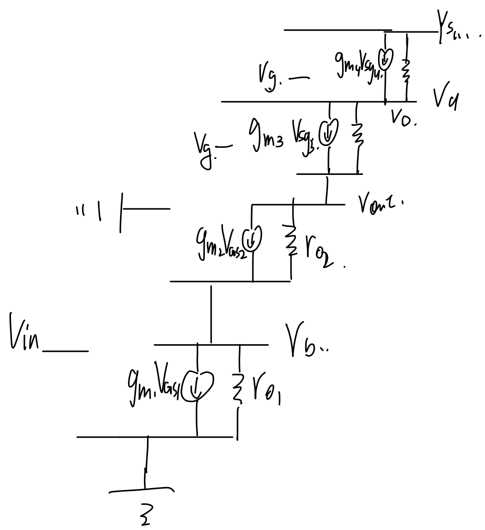
Find the intrinsic gain A_v and output impedance R_{out} for the amplifier when $I_1 = 0.01$ and $0.1mA$ respectively. (Neglect body effect)

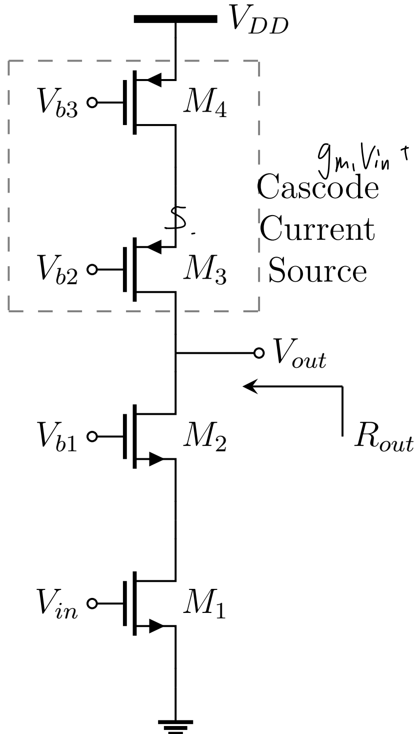
Parameter for NMOS: $V_{THN} = 0.7V,\ K_n = 110\mu A/V^2, \lambda = 0.04V^{-1}$

Parameter for PMOS: $V_{THP} = -0.7V,\ K_p = 50\mu A/V^2, \lambda = 0.05V^{-1}$

All the size of transistor is $W = 20\mu m, L = 1\mu m$

$$R_{out} = \frac{V_{out}}{i_{out}} = \frac{1}{\frac{1}{r_{o1}+r_{o2}+g_{m2}r_{o1}r_{o2}} + \frac{1}{r_{o3}+r_{o4}+g_{m3}r_{o3}r_{o4}}}$$





$$g_{m1}V_{in} + \frac{V_b}{r_{o1}} = \frac{V_{out}-V_b}{r_{o2}} - g_{m2}V_b = g_{m3}V_d + \frac{V_d - V_{out}}{r_{o3}} = -\frac{V_d}{r_{o4}}$$

when $I = 0.01mA$.

$r_{o1} = r_{o2} = 2.5\times10^6\ \Omega$.

$r_{o3} = r_{o4} = 2\times10^6\ \Omega$.

$g_{m3} = g_{m4} = 1.4\times10^{-4}$.

$g_{m1} = g_{m2} = 2.1\times10^{-4}$.

$\Rightarrow$ $\boxed{R_{out} = 4.0\times10^8\ \Omega. \quad A_v = -8.3\times10^4}$

when $I = 0.1mA$.

$r_{o1} = r_{o2} = 2.5\times10^5\ \Omega$.

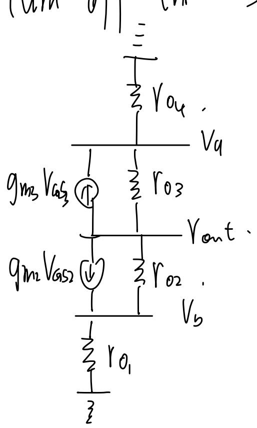
$r_{o3} = r_{o4} = 2\times10^5\ \Omega$.

$g_{m3} = g_{m4} = 4.5\times10^{-4}$.

$g_{m1} = g_{m2} = 6.6\times10^{-4}$

$\Rightarrow$ $\boxed{R_{out} = 1.27\times10^7\ \Omega. \quad A_v = -8.4\times10^3}$

Turn off the source,



$$i_{out} = g_{m3}(V_{out}-V_a) + \frac{V_{out}-V_a}{r_{o3}} + g_{m2}(V_{out}-V_b) + \frac{V_{out}-V_b}{r_{o2}}.$$

$$\begin{cases} g_{m2}(-V_b) + \frac{V_{out}-V_b}{r_{o2}} = \frac{V_b}{r_{o1}} \\ g_{m3}(-V_a) + \frac{V_{out}-V_a}{r_{o3}} = \frac{V_a}{r_{o4}} \end{cases}$$

$$\Rightarrow V_b\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right) = \left(\frac{1}{r_{o2}}\right)V_{out}.$$

$$V_b = \frac{\frac{1}{r_{o2}}}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}} V_{out} = \frac{r_{o1}}{r_{o1}+r_{o2}+g_{m2}r_{o1}r_{o2}} V_{out}.$$

$$V_a = \frac{r_{o4}}{r_{o3}+r_{o4}+g_{m3}r_{o3}r_{o4}} V_{out}.$$

$$i_{out} = \frac{V_b}{r_{o1}} + \frac{V_a}{r_{o4}} = V_{out}\left(\frac{+1}{r_{o1}+r_{o2}+g_{m2}r_{o1}r_{o2}} + \frac{+1}{r_{o3}+r_{o4}+g_{m3}r_{o3}r_{o4}}\right).$$