VE311 Electronic Circuit Homework 8

Due: **Jul 22nd 11:59a.m.**

Note:

1) Please use A4 size paper or page.

2) Please clearly state your final result for each question.

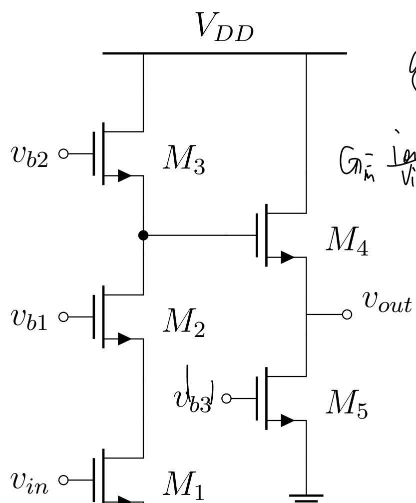
Many years ago, there was a student named Watt Nobody, who was enrolled in ECE3110J course. The course was very interesting, but hard to understand. Mr. Nobody found himself hard to grasp the cutting-edge directions.

HW8 would be due at noon the next day, yet he still had no clue where to begin. Suddenly, Watt Nobody's Apple Pencil broke and could no longer write. With less than a day left before the deadline, there was no time to buy a new one. So, Watt decided to seek help from Professor Xuyang Lu.

After inspection, Xuyang told him that the issue was likely a malfunction in the amplifier module of the Apple Pencil, which prevented the output signal generated by the tip's pressure sensor from being properly amplified. As a result, Watt decided to redesign the amplifier module using MOSFETs.

Question 1. Cascode Amplifier

He tried to cascade the cascode amplifier with a source follower first. Please help him derive the expression of voltage gain A_v and output impedance R_{out} of the cascaded circuit. Assume all MOSFETs are in saturation. ($\lambda \neq 0, \gamma = 0$)

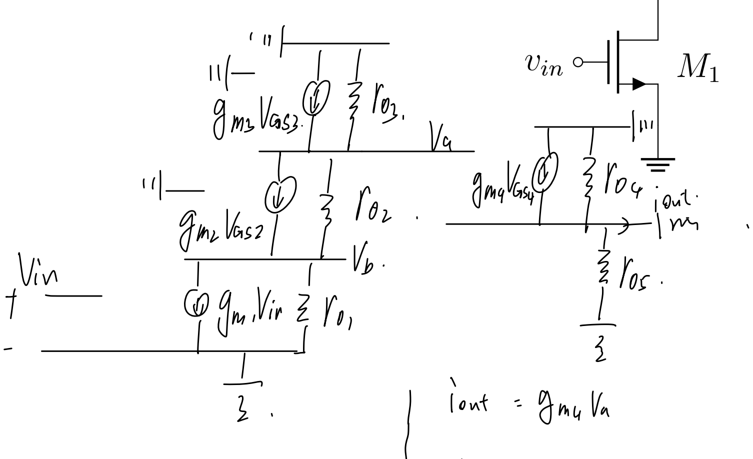


$$g_{m1} V_{in} = \left(-\frac{1}{r_{o3}} - g_{m3} - \frac{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}+g_{m3}}{r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)}\right) V_a.$$

$$G_m = \frac{i_{out}}{V_{in}} = \frac{g_{m4} V_a}{V_{in}} = \frac{g_{m4}}{\frac{1}{g_{m1}}\left(-\frac{1}{r_{o3}} - g_{m3} - \frac{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}+g_{m3}}{r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)}\right)}.$$

$$A_v = G_m R_{out}$$

$$= -\frac{g_{m1} g_{m4} r_{o4} r_{o5}}{\left(\frac{1}{r_{o3}} + g_{m3} + \frac{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}+g_{m3}}{r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)}\right)(r_{o4}+r_{o5})}$$



$$i_t = \frac{V_t}{r_{o5}} + \frac{V_t}{r_{o4}}$$

$$R_{out} = \frac{V_t}{i_t} = \frac{1}{\frac{1}{r_{o5}}+\frac{1}{r_{o4}}} = \frac{r_{o4} r_{o5}}{r_{o4}+r_{o5}}.$$

$$\begin{cases} i_{out} = g_{m4} V_a \\ -\frac{V_a}{r_{o3}} - g_{m3} V_a = -g_{m2} V_b + \frac{V_a - V_b}{r_{o2}} = g_{m1} V_{in} + \frac{V_b}{r_{o1}} \end{cases}$$

$$V_a\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}} + g_{m3}\right) = \left(g_{m2} + \frac{1}{r_{o2}}\right) V_b.$$

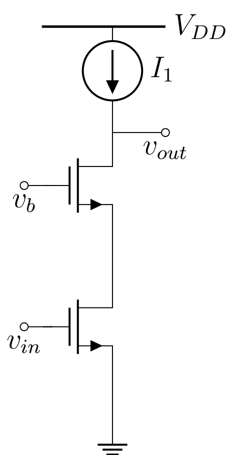
$$V_b = \frac{g_{m2} + \frac{1}{r_{o2}}}{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}+g_{m3}} V_a.$$

After reviewing the first design proposal, Xuyang pointed out that while it could provide high gain and low output impedance, it might not be suitable for mobile devices due to limited space, high power consumption, and insufficient output swing.

As a result, Watt Nobody decided to modify the design by folding the cascode structure to optimize performance within the constraints. Since Watt wasn't very familiar with this approach, he first designed a simpler two-MOSFET circuit to test the concept.

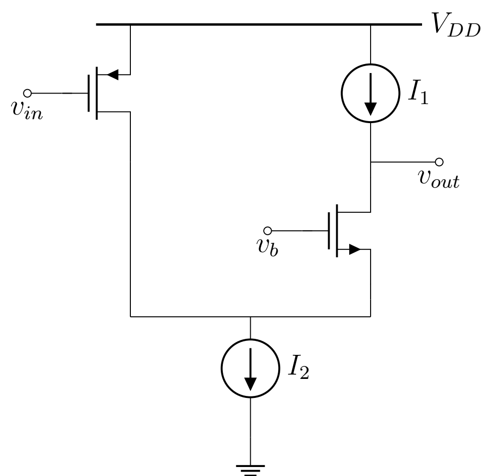
Question 2. Folded Cascode

A cascode MOSFET structure can be converted to its equivalent folded cascode topology. The PMOS and current source I_2 replace an NMOS in the simple cascode structure.



(A)

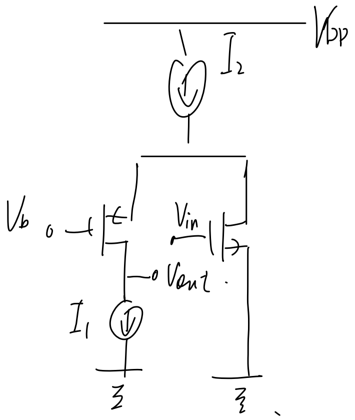
FIGURE 1. Cascode Structure

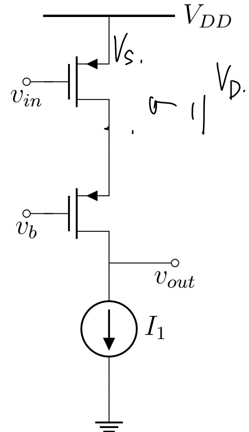


(A)

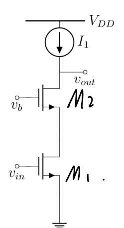
FIGURE 2. folded Cascode Structure

(a) Draw the equivalent folded cascode structure for the PMOS cascode structure.



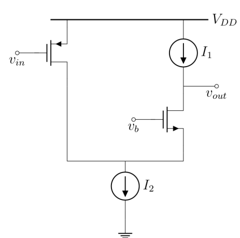


(b) Find the gain for small-signal and R_{out} of the cascode Figure.1 and folded-cascode Figure.2. Assume that the current sources are ideal. Assume you already know g_m and r_o for each MOSFET, you don't need to consider the body effect.



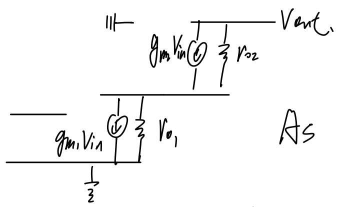
(A)

FIGURE 1. Cascode Structure



(A)

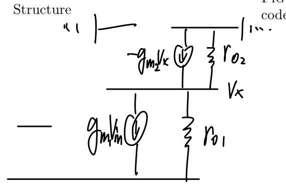
FIGURE 2. folded Cascode Structure



As the small signal model is equivalent in Figure 2, it's the same as that in figure 1.

$R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$.

$A_v = -(g_{m1} g_{m2} r_{o1} r_{o2} + g_{m1} r_{o1})$.



Turn off Vin.

$i_t = -g_{m2}V_x + \frac{V_t - V_x}{r_{o2}} = \frac{V_x}{r_{o1}}$.

$\Rightarrow \quad \frac{V_t}{r_{o2}} = \left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right) V_x$.

$V_t = r_{o2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right) V_x$.

$R_{out} = \frac{V_t}{i_t} = \frac{r_{o2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right)}{\frac{1}{r_{o1}}} = r_{o1} r_{o2}\left(\frac{r_{o1}+r_{o2}}{r_{o1} r_{o2}} + g_{m2}\right) = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$.

Vout connect to ground.

$-g_{m2}V_x - \frac{V_x}{r_{o2}} = g_{m1}V_{in} + \frac{V_x}{r_{o1}} = -i_{out}$.

$g_{m1}V_{in} = \left(-g_{m2} - \frac{1}{r_{o1}} - \frac{1}{r_{o2}}\right) V_x$

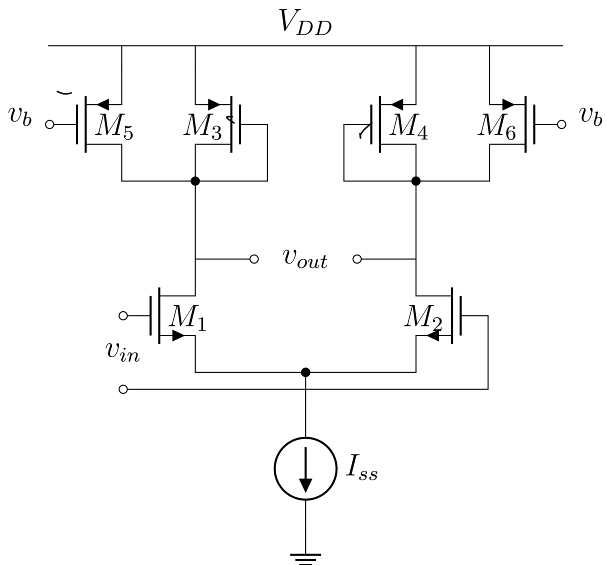
$G_m = \frac{i_{out}}{V_{in}} = \frac{(+g_{m2} + \frac{1}{r_{o2}})}{\frac{1}{g_{m1}}\left(-g_{m2} - \frac{1}{r_{o1}} - \frac{1}{r_{o2}}\right)} = -\frac{g_{m1}\left(g_{m2} + \frac{1}{r_{o2}}\right)}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$

$A_v = G_m R_{out} = -g_{m1}\left(g_{m2} + \frac{1}{r_{o2}}\right) r_{o1} r_{o2} = -(g_{m1} g_{m2} r_{o1} r_{o2} + g_{m1} r_{o1})$.

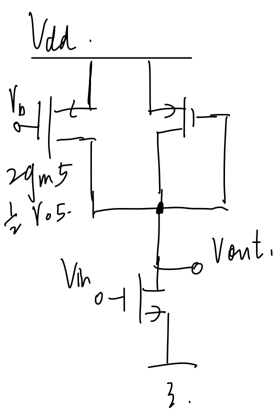
Xuyang was pleased with the design but pointed out that, as a single-ended circuit, it might suffer from noise susceptibility. To improve noise immunity, he suggested Watt consider switching to a differential amplifier configuration.

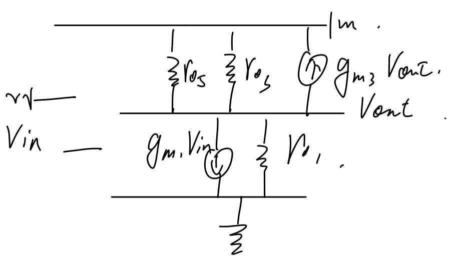
Question 3. Differential Circuit

So Watt Nobody raised out the new design. Consider the following differential circuit. M_1 is same as M_2, M_3 is same as M_4, M_5 is same as M_6. Calculate differential gain of the circuit using variables including g_m and r_o. Consider the channle length modulation for all MOSFETs and ignore body effect. Half-circuit Method is recommended.



Common Mode.





$$\frac{V_{out}}{r_{o5}} + \frac{V_{out}}{r_{o3}} + g_{m3} V_{out} = -\left(g_{m1} V_{in} + \frac{V_{out}}{r_{o1}}\right)$$

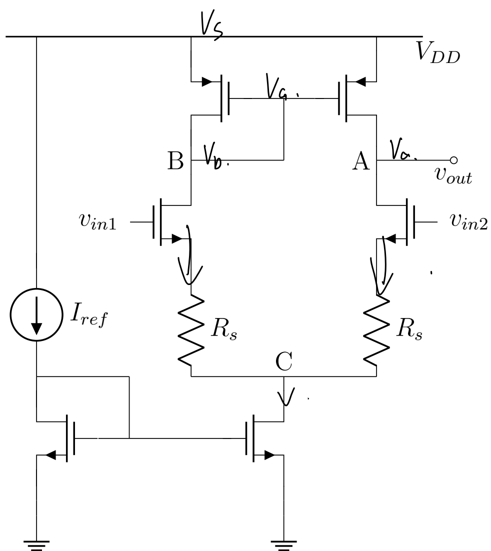
$$V_{out}\left(\frac{1}{r_{o5}} + \frac{1}{r_{o3}} + g_{m3} + \frac{1}{r_{o1}}\right) = -g_{m1} V_{in}$$

$$A_v = \frac{-g_{m1}}{\frac{1}{r_{o5}} + \frac{1}{r_{o3}} + g_{m3} + \frac{1}{r_{o1}}}$$

Xuyang praised Watt Nobody's progress, saying, "This is already excellent, but there's room for optimization. In practice, we don't always need to pursue the highest gain, while sacrificing some gain in exchange for better linearity, stability, and improved CMRR can be a worthwhile trade-off."

Watt Nobody had a lightbulb moment and proceeded to enhance the differential input pair by adding Source Degeneration R_S.

Question 4. Differential Pairs with Source Degeneration



Assume All MOSFET have identical $\frac{W}{L}$ and $\frac{1}{2}\mu_n C_{ox}\frac{W}{L} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L} = 10mA \cdot V^2$, $\lambda = 0.1V^{-1}$, $V_{TH} = 1V$, $R_s = 10k\Omega$, $I_{ref} = 10\mu A$, $V_{DD} = 5V$

$10^{-2}A$

(a). Determine the DC currents and voltages at A and B. Ignore channel length modulation for this question only.

(b). Determine the differential gain for the small signal by using the half circuit. C can be regarded as the ground for AC small signals.

(c). Determine the common mode gain for a common mode small signal input and derive the value of CMRR.

(a)

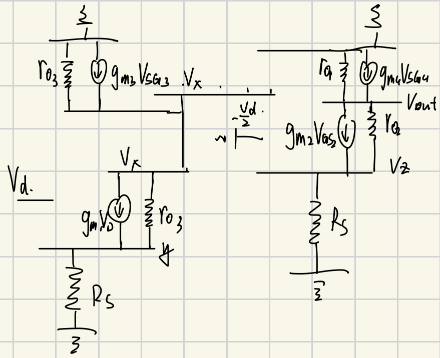
$I_c = I_{ref} = 10^{-5}A$

$I_A = I_B = \frac{1}{2} I_{ont} = 0.5 \times 10^{-5} A$

$I_A = I_B = \frac{1}{2} \mu_n C_{on} \frac{W}{L} (4-V)^2 = 0.5 \times 10^{-5} A$

$V_A = V_B = V = 3.98V$

(b)



M_1:

$$r_o = \frac{1}{I_D\lambda} = 2\times 10^6\ \Omega.$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left((5-V_G)-1\right)^2\left(1+\lambda(5-V_G)\right).$$

$$\Rightarrow V_G = 3.98V. \quad V_{SD} = 1.02V.$$

for M_3, M_4, $V_S = 1.08V$.

M_1, M_2, $g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}\,(1+\lambda V_{DS}) = 4.68\times 10^{-4}$.

M_3, M_4: $g_{m3} = g_{m4} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}\,(1+\lambda V_{DS}) = 5.08\times 10^{-4}$

$$\begin{cases} -\frac{V_x}{r_{o3}} - g_{m3}V_x = \frac{V_y}{R_S}. \\ g_{m1}\frac{V_D}{2} + \frac{V_x - V_y}{r_{o3}} = \frac{V_y}{R_S}. \\ g_{m2}(-\frac{V_D}{2} - V_z) + \frac{V_{out} - V_z}{r_{o2}} = \frac{V_z}{R_S}. \\ -\frac{V_{out}}{r_{o4}} - g_{m4}V_x = \frac{V_z}{R_S}. \end{cases} \Rightarrow A_D = \frac{V_{out}}{V_{in}} = 143.3.$$

(1)
$$\begin{cases} -\frac{V_x}{r_{o3}} - g_{m3}V_x = \frac{V_y}{R_S} \\ g_m V_c - \frac{V_x - V_y}{r_{o3}} = \frac{V_y}{R_S} \\ g_m(V_c - V_z) + \frac{V_{out} - V_z}{r_{o2}} = \frac{V_z}{R_S} \\ -\frac{V_{out}}{r_{o4}} - g_{m4}V_x = \frac{V_z}{R_S} \end{cases} \Rightarrow A_c = -0.13.$$

$$CMRR = 20\lg\left|\frac{A_d}{A_c}\right| = 58.12\ dB.$$

In the end, Watt Nobody successfully repaired his Apple Pencil and finished HW8 on time. Thanks to this experience, Nobody no longer found 311 as suffering and ultimately achieved an excellent grade in the course.

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