ECE3110J Electronic Circuit HW9
Due: **Aug 3rd 11:59 p.m.**

Note

1. Please use A4 size paper or page.
2. Please clearly state your final result for each question.
3. This is your last assignment:)
   Congratulations on surviving this course.

Introduction

As we all know, ECE2160J is the prerequisite of ECE3110J. While taking ECE3110J, Watt Nobody was also the Teaching Assistant of ECE2160J, where the differential op-amp is used to build the PD controller as well as the filter.

After taking ECE3110J toward the end of the semester, he finally realized that what he had learn throughout the semester was the physical foundation of analog signal processing. Thus, he wanted to build a real differential amplifier so that he could replace the op-amp used in ECE2160J with his own creation. Thus, he bought equipment and tried to build a differential amplifier from scratch.

Problem 1

Unfortunately, the noise was quite large. After changing all the wires he could change, Watt Nobody found that the voltage generator he bought can only work in an output load of $2R$ instead of high-z. Though the differential mode gain was not effected, he found the common mode gain was quite large. To compensate for the effect, thus reducing common mode gain, he connect a $2R$ resistor in the tail beside the ideal current source, as shown in Figure. 1.

Your Task:
Assume $\lambda = 0$ and $\gamma = 0$. Both M_1 and M_2 are in saturation. **M_1 and M_2 are identical with the same transconductance g_m.**

Hint: You **can** apply a half circuit here.

a) Write an expression for the low-frequency small-signal differential gain.
b) Write an expression for the low-frequency small-signal common-mode gain

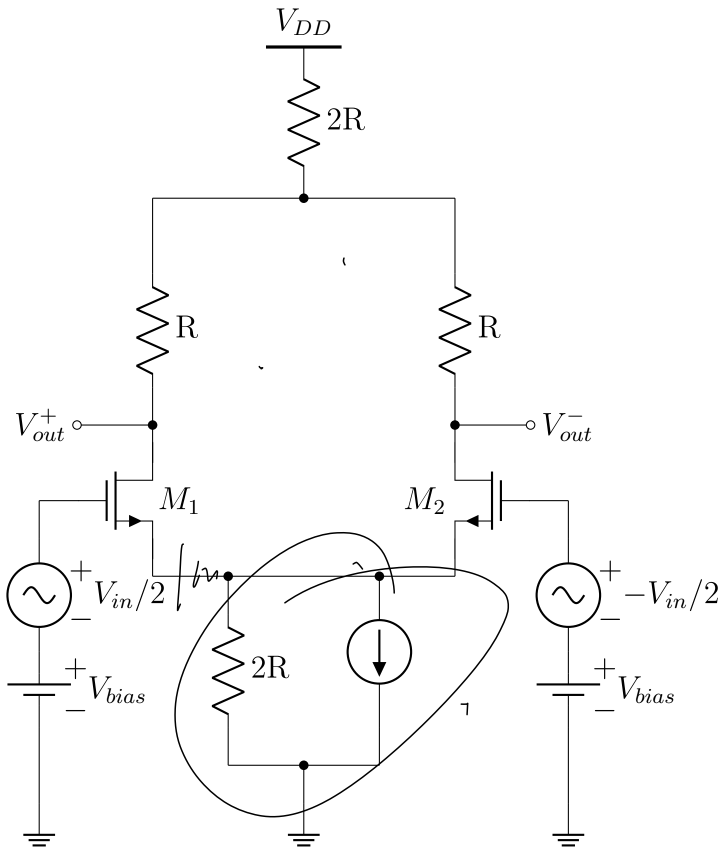

Figure 1: Differential Amplifier for Problem 1

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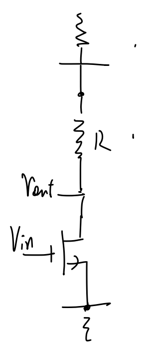
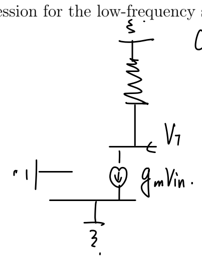
a)



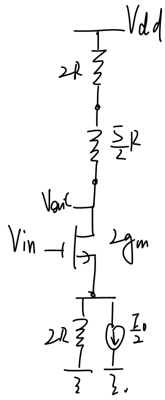


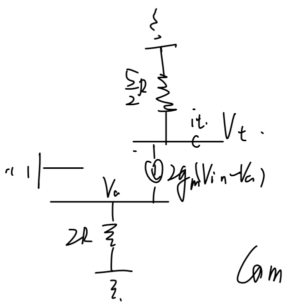
(a) $G_m = \frac{i_{out}}{V_{in}} = \frac{-g_mV_{in}}{V_{in}} = -g_m$.

$R_{out} = R$.

$A_v = G_mR_{out} = -g_mR$.

(b).





$i_{out} = -2g_m(V_{in}-V_a)$

$\frac{V_a}{2R} = 2g_m(V_{in}-V_a)$

$G_m = \frac{i_{out}}{V_{in}} = -\frac{V_a}{2R}\cdot\frac{1}{V_{in}} = -\frac{1}{2R}\frac{2g_m}{\frac{1}{2R}+2g_m} = \frac{-2g_m}{1+4g_mR}$

R_{out}: $V_a = 0$.

$R_{out} = \frac{5}{2}R$,

$A_v = G_mR_{out} = \frac{-5g_mR}{1+4g_mR}$.

Problem 2

That summer in Shanghai was extremely hot.

This week, Watt Nobody was doing Lab5 of ECE3110J. After doing the lab, he followed the lab to replaced the ideal current source with a rheostat and a negative bias.

When Nobody is testing his differential amplifier in and out of his room with the air conditioner, he found that the output fluctuates. Thus, he decided to use a current mirror to copy the reference current to provide a current that is insensitive to the temperature, as shown in Figure. 2.

Your task:

Here M_3 and M_5 double cascode the current source M_2. Transistors M_1-M_5 are identical, having width W and length L. Both M_6 and M_7 also have width W. This problem concerns the lengths of M_6 and M_7. Assume M4 is in saturation. Ignore the body effect. What are the minimum lengths for transistors M6 and M_7 so that the cascode transistors have the appropriate bias voltages?

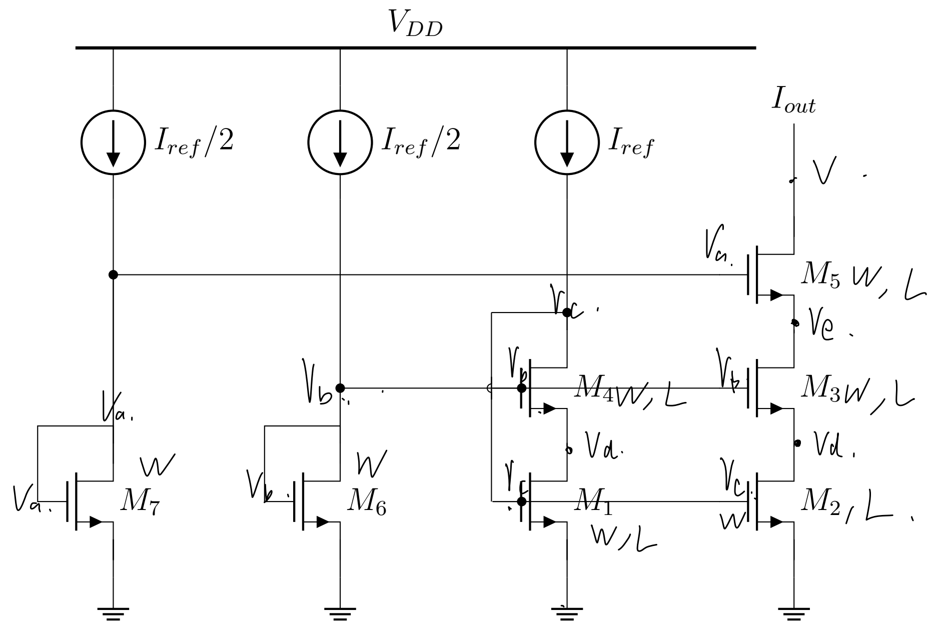


Figure 2: Current Mirror for Problem 2

$$\begin{cases} 0 \le V_b - V_T < V_e \\ 0 \le V_b - V_T < V_c \\ 0 \le V_c - V_T < V_d \\ V_a - V_e = V_b - V_d = V_c \end{cases}$$

$V_c - V_T = V_d$

$V_b - V_c + V_T = V_c$

$V_b = V_c + V_d$

$2V_c = V_b + V_T$

when

V_d reaches max

$V_b = V_c + V_T$

$V_a = V_c + V_e$

$V_c + V_b - V_T$

$V_c + 2V_c - V_T$

$$\frac{1}{L_7}(V_a - V_T)^2 = \frac{1}{L_6}(V_b - V_T)^2 = \frac{1}{2L}(V_c - V_T)^2$$

$$L_6 = 2L \frac{(V_b - V_T)^2}{(V_c - V_T)^2} = 2L\frac{(2V_c - 2V_T)^2}{(V_c - V_T)^2} = 8L$$

$$L_7 = 2L\frac{(V_a - V_T)^2}{(V_c - V_T)^2} = 2L\frac{(3V_c - 2V_T - V_T)^2}{(V_c - V_T)^2} = 18L$$

Problem 3

After using the current mirror to provide a constant current, the differential amplifier built by Nobody was so good that it could be considered an ideal op-amp. Then, he proceeded with the circuits that utilized the op-amp.

However, as the TA of ECE2160J, Nobody needed to change the input frequency of the signal from low to high so as to obtain a Power Density Spectrum. When the output is of high frequency, he found that the signal is attenuated more than expected.

Your Task:

In the circuit of Figure. 3, the voltage amplifier is ideal (i.e., it has an infinite input resistance and a zero output resistance).

(a) Use the Miller approach to find an expression for the input capacitance C_{in} in terms of A and C.

(b) Use the expression forC_{in} to obtain the transfer function $V_o(s)/V_{sig}(s)$.

(c)If $R_{sig} = 1\ k\Omega$, and the gain $V_o(s)/V_{sig}(s)$ is to have a dc value of 40 dB and a 3-dB frequency of 100 kHz, find the values required for A and C.

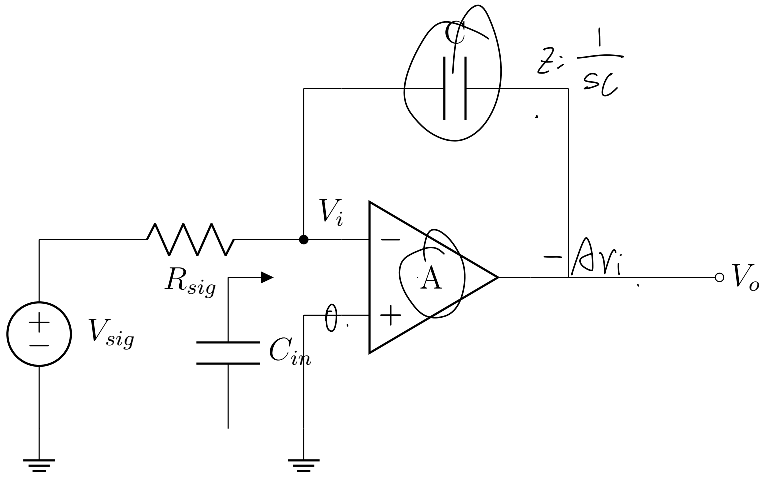
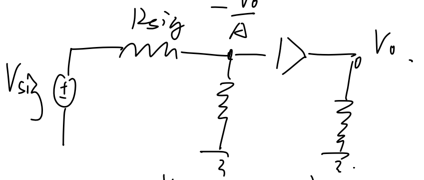
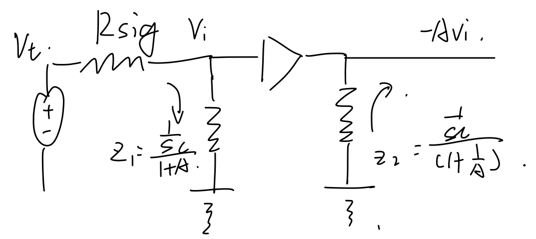


Figure 3: Ideal Op-amp Frequency Response for Problem 3

(ii).



(a).



$$\frac{V_{sig} + \frac{V_o}{A}}{R_{sig}} = \frac{-\frac{V_o}{A}}{\frac{1}{SC(1+A)}}$$

$$\frac{V_{sig}}{R_{sig}} = -\frac{V_o}{A} SC(1+A) - \frac{V_o}{A \cdot R_{sig}}$$

$$\frac{V_o}{V_{sig}} = \frac{1}{R_{sig}} \frac{1}{-\frac{1}{A}[SC(1+A)+\frac{1}{R_{sig}}]} = \frac{-A}{R_{sig}SC(1+A)+1}$$

$R_{sig} S (\frac{1}{A}+1)$

$$Z_{in} = \frac{1}{SC(1+A)}$$

$$C_{in} = C(1+A).$$

(iii)

DC:

$A_v = -A.$

$20 \lg A = 40$

$A = 100.$

$S = j10^5$

$$A_v = \frac{-100}{j\,101 \times 10^3 \times 10^5 \cdot C + 1}$$

$$|A_v| = \frac{100}{\sqrt{1^2 + (1.01 \times 10^{10} C)^2}}.$$

Problem 4

In ECE2160J, Nobody was asked to help the students build an RF tuner and Mixer for understanding the modulation and demodulation of signals. While the amplifier he built before was composed of MOSFET, he was told that BJT is more applicable in the high-frequency domain. Thus, he use a BJT for building a single stage amplifier and examines the frequency response.

Your Task:

In Figure. 4, assume that the amplifier is operating in FAR. Ignore all capacitance except that shown. Ignore r_o.

(a) Draw the small-signal model of the given circuit.

(b) Write an expression for the low-frequency small-signal gain, v_o/v_i.

(c) Next, write an expression for the frequency-dependent small-signal gain v_o/v_i. What are the pole/zero frequencies? (Consider only the capacitors shown in the figure. Use the open-circuit short circuit time constant method.

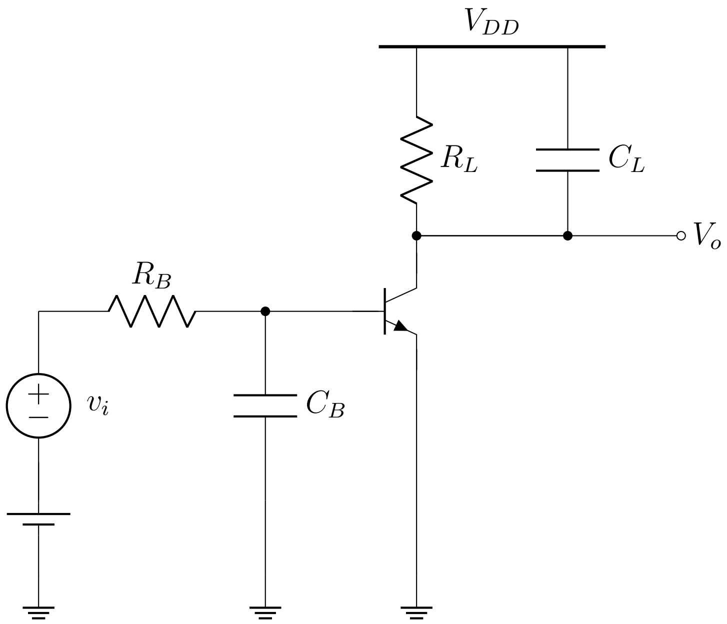
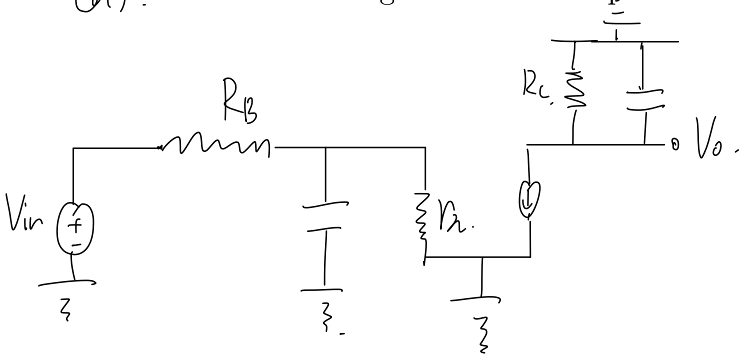


Figure 4: BJT Amplifier Frequency Analysis for Problem 4

(a).



(b).