UM-SJTU JOINT INSTITUTE

ECE3110J Mid RC

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Notations

Type	Letter Case	Subscript Case	Example Voltage	Example Current
DC	Uppercase	Uppercase	V_C, V_{BE}	I_C, I_{BE}
AC	Lowercase	Lowercase	v_c, v_{be}	i_c, i_{be}
DC+AC	Lowercase	Uppercase	v_C, v_{BE}	i_C, i_{BE}
AC RMS/Peak	Uppercase	Lowercase/ Uppercase	V_{rms}, V_p	I_{rms}, I_p

$$v_{OUT} = V_{OUT} + v_{out}$$

Decibels(dB) Representation of Gains

An amplifier receives a signal from a source and delivers it to a load. Gains are dimensionless, and are usually expressed in terms of decibels (dB):

$$\text{Voltage Gain} = A_v = \frac{v_o}{v_i} \tag{61}$$

$$\text{Current Gain} = A_i = \frac{i_o}{i_i} \tag{62}$$

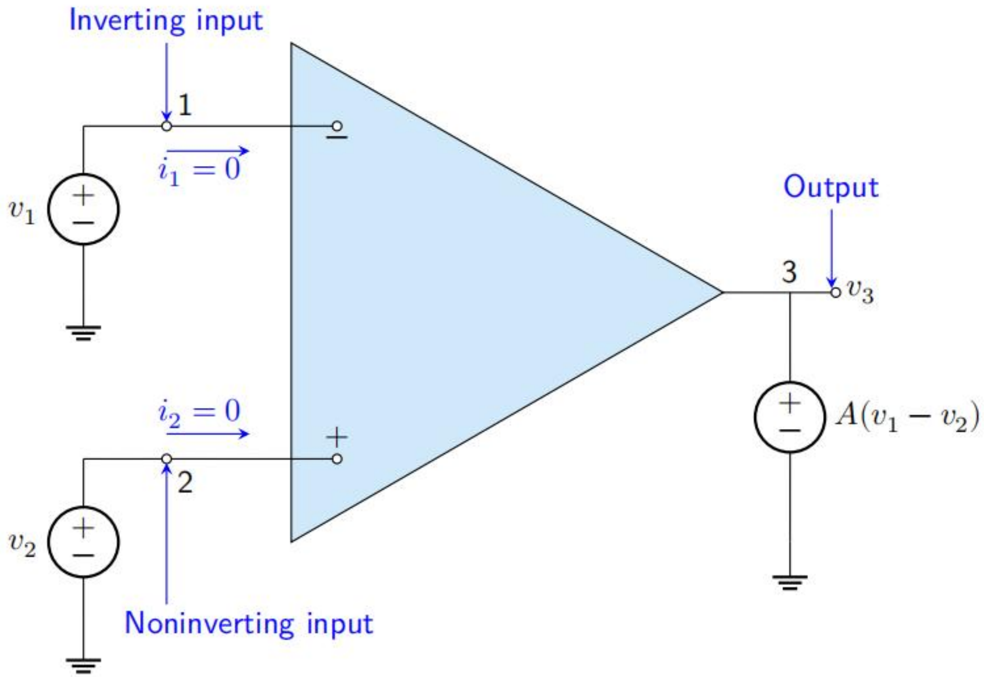
$$\text{Power Gain} = A_p = \frac{v_o i_o}{v_i i_i} = A_v A_i \tag{63}$$

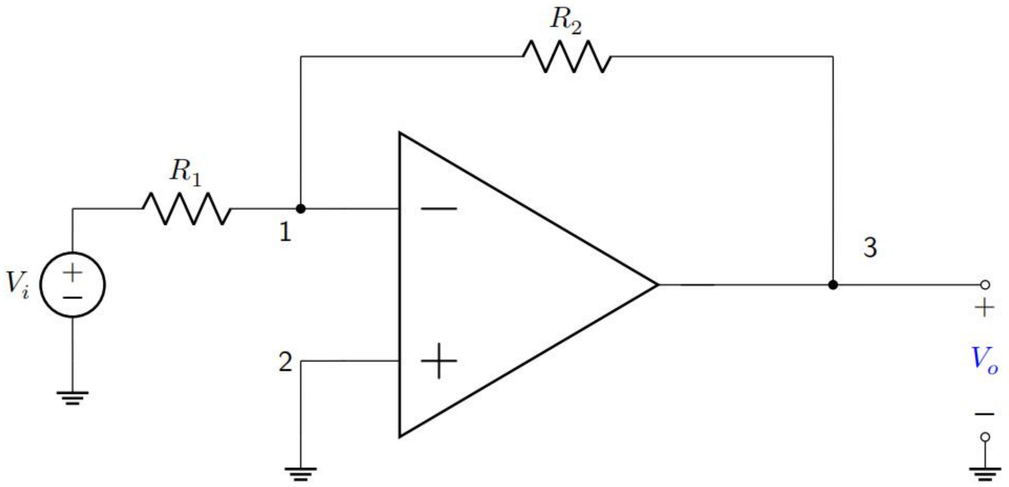
$$\text{Voltage Gain} = 20 \log A_v \tag{64}$$

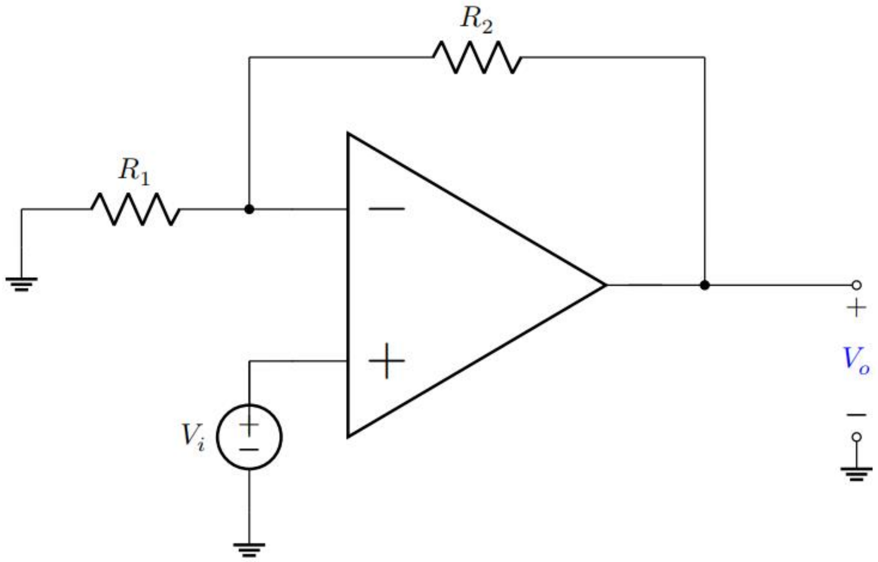
$$\text{Current Gain} = 20 \log A_i \tag{65}$$

$$\text{Power Gain} = 10 \log A_p \tag{66}$$

Op-amps







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Ideal Op-amps

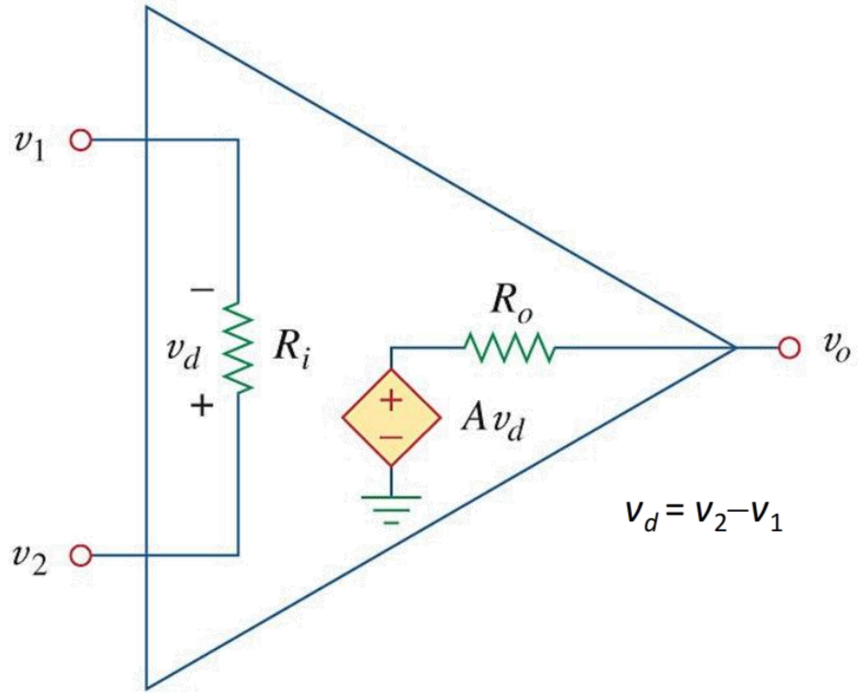
1. Infinite open-loop gain, $A \to \infty$

2. Infinite input resistance, $R_i \to \infty$

3. Zero onput resistance, $R_0 \to \infty$

4. $v_1 = v_2$

5. Current into both input terminals are zero.



Exercises(hw2, Q5)

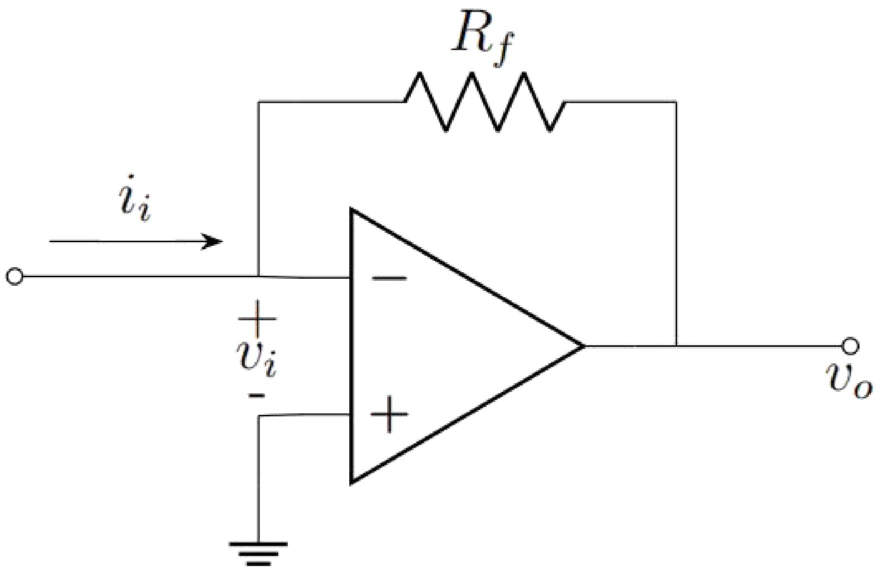
The circuit provides a current-voltage converter to convert input current i_i to output voltage v_o. Derive expressions for the transresistance $R_m = v_o/i_i$ and the input resistance $R_i = v_i/i_i$ for the following cases:

(a) The Op Amp is ideal.

(b) The Op Amp is non-ideal with a finite open-loop gain A.

(c) Op Amp is non-ideal with finite open-loop gain A, output impedance R_{out}, input impedance R_{in}, assuming the amplifier is a voltage amplifier.

(*Hint: draw equivalent circuit first.*)



Exercises(hw2, Q5)

(c) $V_2 = -A V_i$

$$i_2 = \frac{(A+1)V_i}{R_f + R_{out}}$$

$$i_1 = \frac{V_i}{R_{in}}$$

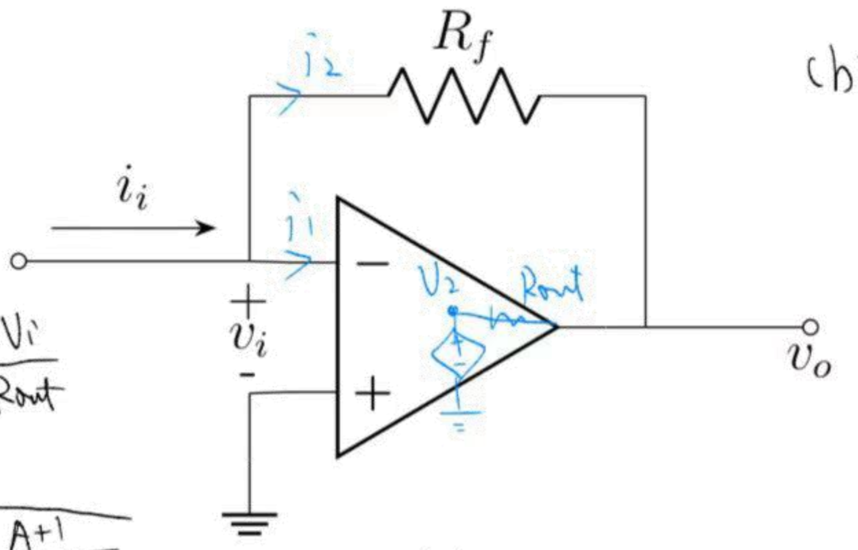
$$i_i = i_1 + i_2 = \frac{V_i}{R_{in}} + \frac{(A+1)V_i}{R_f + R_{out}}$$

$$R_i = \frac{V_i}{i_i} = \frac{1}{\frac{1}{R_{in}} + \frac{A+1}{R_f + R_{out}}}$$

$$= \frac{R_{in}(R_f + R_{out})}{R_f + R_{out} + (A+1)R_{in}}$$

$$V_o = V_i - i_2 R_f = V_i - \frac{(A+1)V_i}{R_f + R_{out}} R_f = V_i \frac{R_{out} - AR_f}{R_f + R_{out}}$$

$$R_m = \frac{V_o}{i_i} = \frac{V_o}{V_i} R_i = \frac{R_{in}(R_{out} - AR_f)}{R_f + R_{out} + (A+1)R_{in}}$$



(A)

(b) $R_{in} \to \infty$, $R_{out} \to 0$

$$R_i = \frac{R_f + R_{out}}{\frac{1}{R_{in}}(R_f + R_{out}) + A + 1} = \frac{R_f}{A+1}$$

$$R_m = \frac{R_{out} - AR_f}{\frac{1}{R_{in}}(R_f + R_{out}) + A + 1} = -\frac{AR_f}{A+1}$$

(a) $A \to \infty$

$$R_i = 0$$

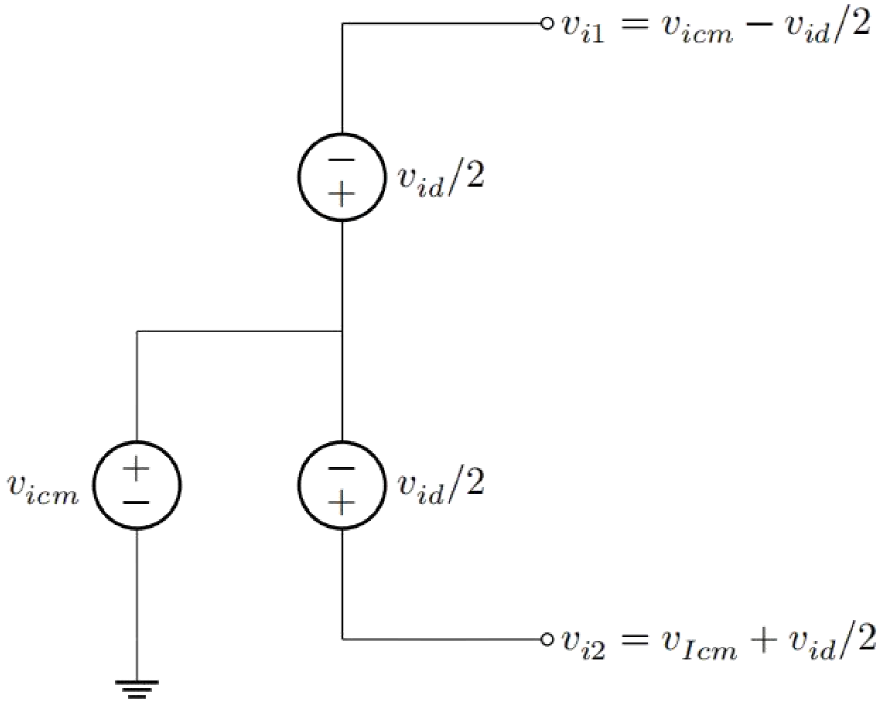
$$R_m = -\frac{R_f}{1 + \frac{1}{A}} \to -R_f$$

CMRR

$$V_{out} = A_d V_{id} + A_{cm} V_{icm}$$

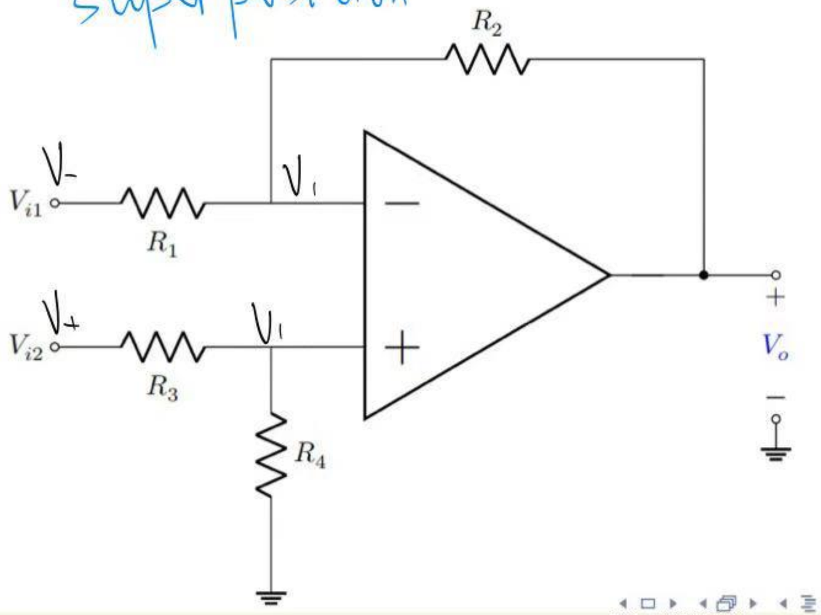
$$CMRR = 20\ log \frac{|A_d|}{|A_{cm}|}$$

Ideally, $CMRR \rightarrow \infty$



Example: Difference Amplifier

superposition



1° $V_+ = -V_-$

$$V_1 = \frac{V_+}{R_3+R_4}\cdot R_4$$

$$\frac{V_1 - V_o}{R_2} = \frac{0 - V_1}{R_1}$$

$$\frac{V_o}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1$$

$$= \frac{R_1+R_2}{R_1 R_2}\cdot\frac{R_4}{R_3+R_4} V_+$$

V_{o1}

$$V_o = \frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} V_+$$

V_{o2}

$$\frac{0 - V_o}{R_2} = \frac{V_- - 0}{R_1}, \quad V_o = -\frac{R_2}{R_1} V_-, \quad V_{out} = V_{o1} + V_{o2}, \quad V_{in} = V_+ - V_-$$

$$A_d = \frac{\frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} + \frac{R_2}{R_1}}{2} = \frac{R_4(R_1+R_2) + R_2(R_3+R_4)}{2R_1(R_3+R_4)}$$

2. $V_+ = V_- = V_{cm}$, $V_{in} = V_{cm}$

$$A_{cm} = \frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} - \frac{R_2}{R_1} = \frac{R_1 R_4 - R_2 R_3}{R_1(R_3+R_4)}$$

Input Resistance:

$$R_{in} = R_1 + R_3$$

Exercises(Sample Exam)

Blue Tiger Inc. tries to design a circuit to extract the desire signal out of an interference. two signals are getting measured,

$$v_1 = 3\sin(\omega t) + 5t \quad (1)$$

$$v_2 = -3\sin(\omega t) - 3t \quad (2)$$

As the Chief technology officer (CTO) of Blue Tiger Inc., you are working on a method to eliminate the kt term (the interference that increases with time) to isolate the $\sin(\omega t)$ component. The following circuit in Fig. 1 is designed for this purpose.

1. (Easy) Assume that the op-amp is ideal, determine the ratio of R_1 and R_2, find the transfer function between V_{out}, V_1 and V_2 of this circuit, pick $R_3 = R_1$.
2. (Hard, easy if you are lucky) You only have one type of resistor, namely a $1k\Omega$ one. you are going to use **Four** of them, with in series or in parallel operations, to create R_2. Find a topology to make it happen.
3. (Medium) Knowing the gain of the op-amp is finite ($A = 100$), without worrying about CMRR yet, what would be the differential impedance looking into the input.
4. (Easy) The CMRR of the op-amp is 10 dB, and $A_D = 100$, write the final expression V_{out}.

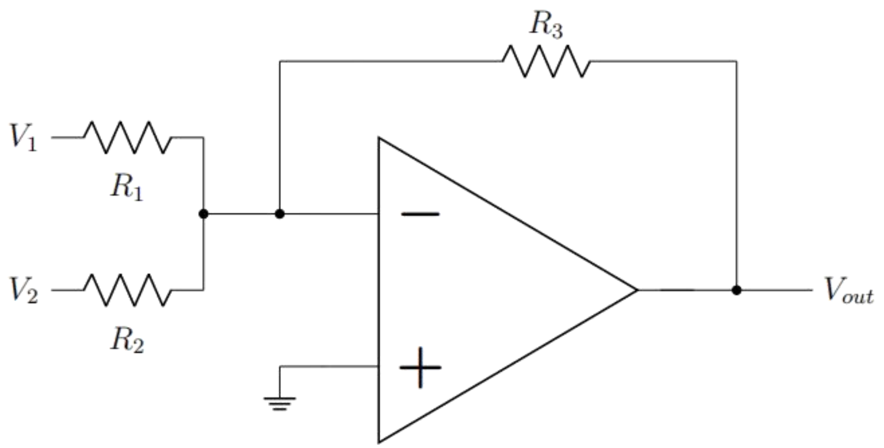


Figure 1: An op-amp interference suppressor.

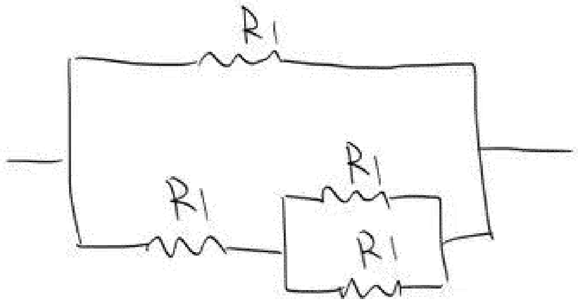
Exercises(Sample Exam)

1. $\frac{V_{out}}{R_3} = \frac{0 - V_1}{R_1} + \frac{0 - V_2}{R_2}$, $R_3 = R_1$, $\frac{R_1}{R_2} = \frac{5}{3}$, $R_2 = \frac{3}{5} R_1$

$\frac{V_{out}}{R_1} = -\frac{V_1}{R_1} - \frac{5}{3}\frac{V_2}{R_1}$

$V_{out} = -(V_1 + \frac{5}{3} V_2)$

2.



$R_2 = R_1 // (\frac{3}{2} R_1) = \frac{3}{5} R_1$

3. $R_{in} = R_1 + R_2$

4. $CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 10$

$\log \left| \frac{A_d}{A_{cm}} \right| = \frac{1}{2}$, $\frac{A_d}{A_{cm}} = \sqrt{10}$, $A_{cm} = 10\sqrt{10}$

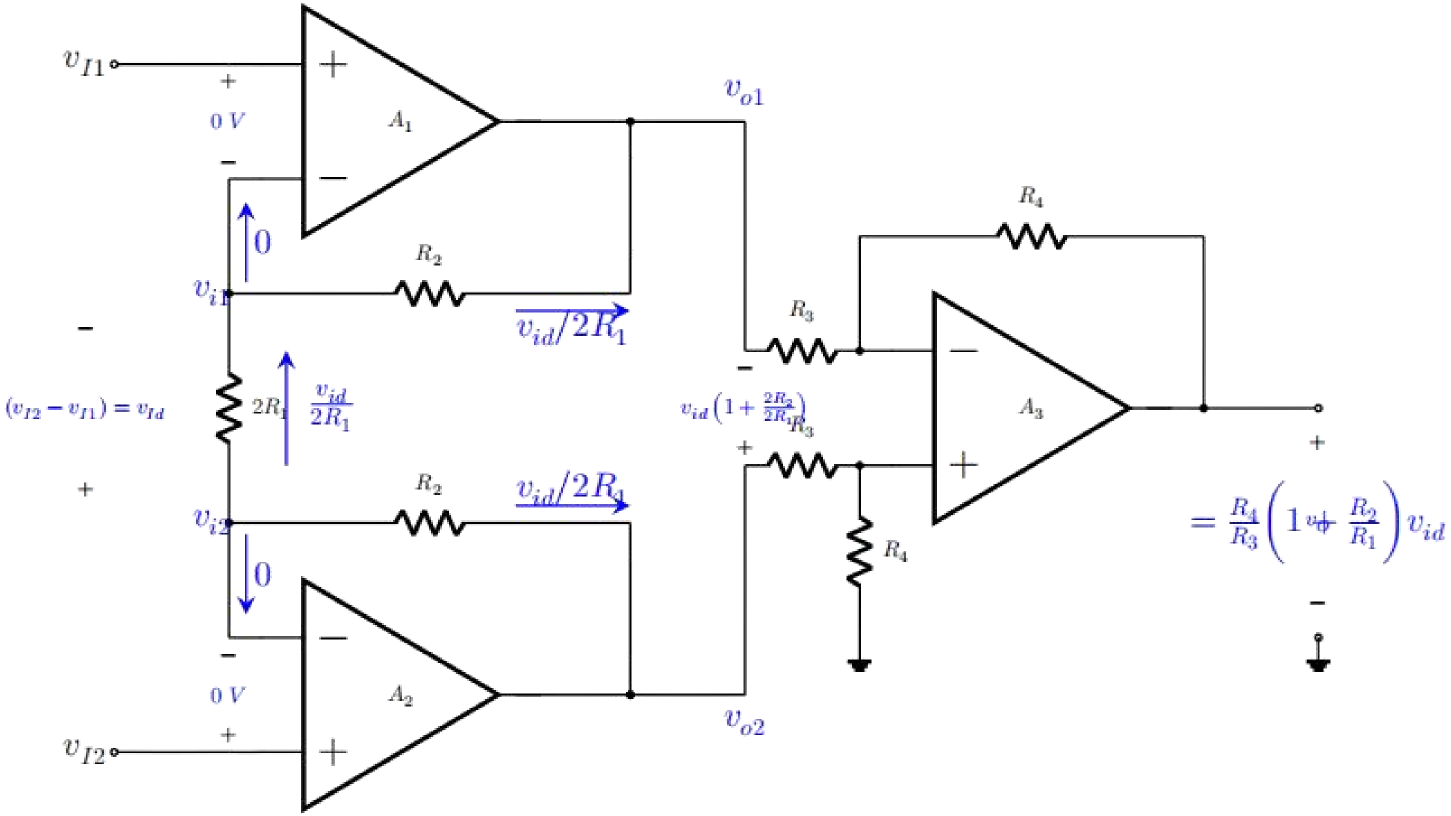
$V_{cm} = \frac{1}{2}(V_1 + V_2) = t$, $V_d = V_1 - V_2 = 6 \sin(wt) + 8t$

$V_{out} = A_{cm} V_{cm} + A_d V_d$

$= 10\sqrt{10}\, t + 100 (6 \sin wt + 8t)$

$= 600 \sin wt + (800 + 10\sqrt{10})\, t$

Instrumental Amplifier

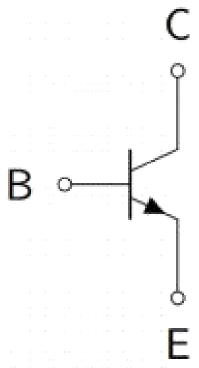


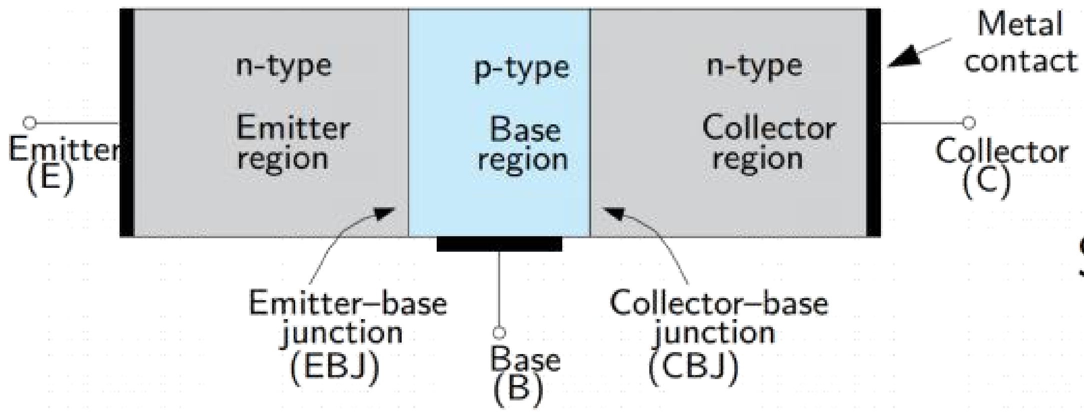
$$A_d = \frac{V_o}{V_{id}} \tag{13}$$

$$= \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) \tag{14}$$

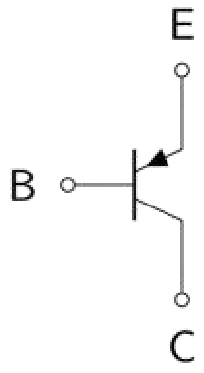
- Common mode first stage gain = 1

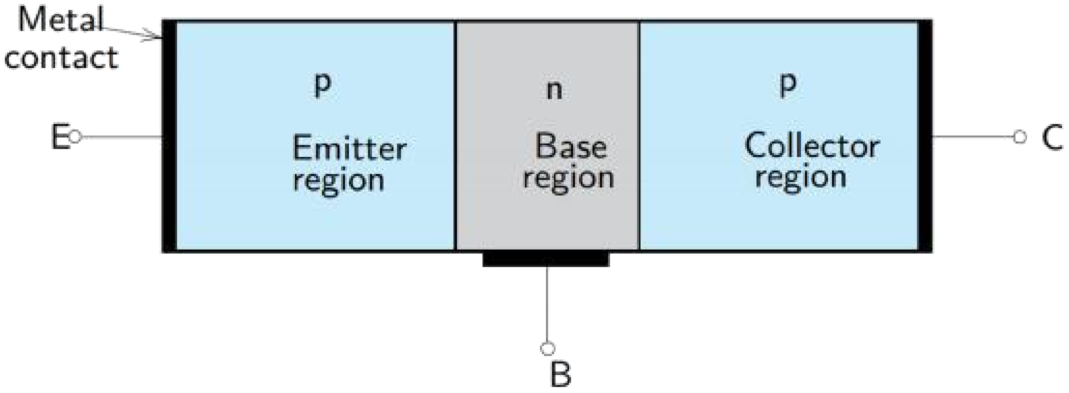
BJT Basics



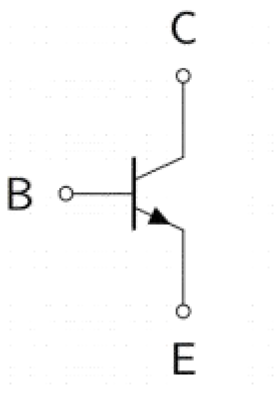


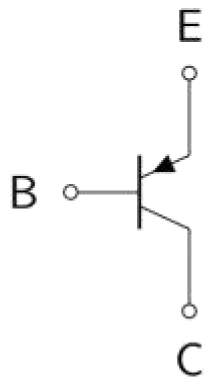
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

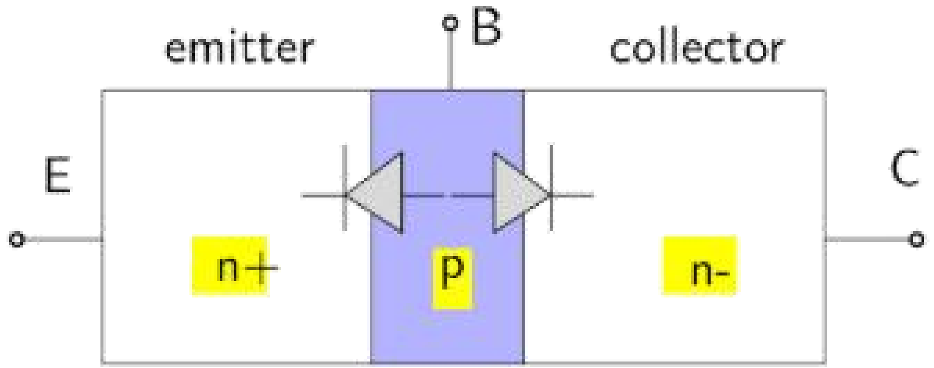


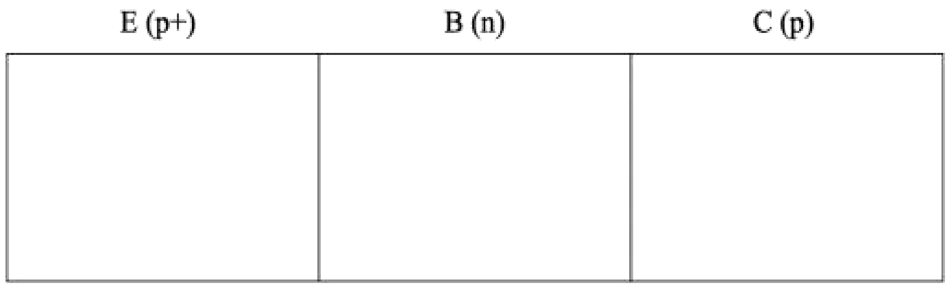


BJT Basics

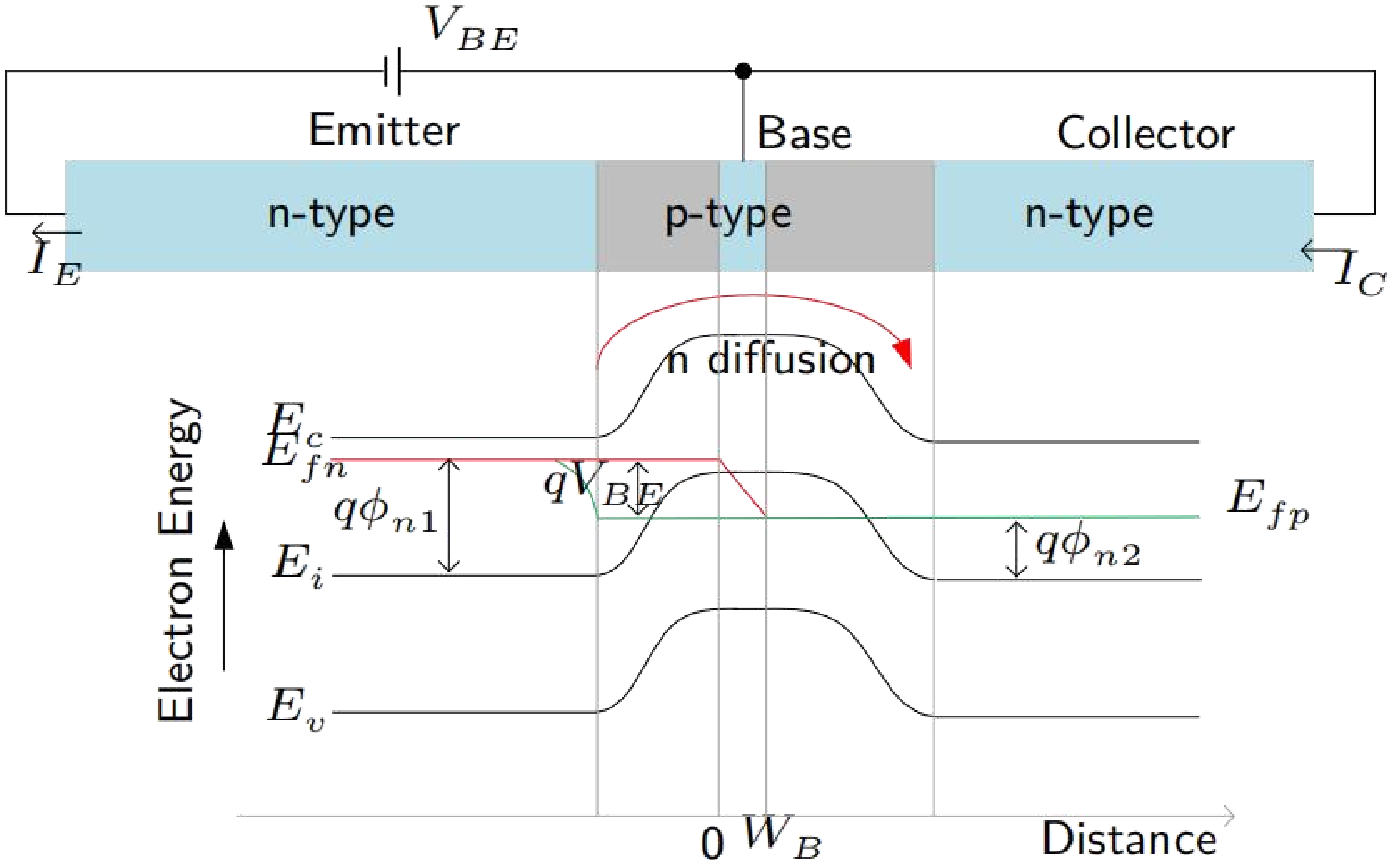




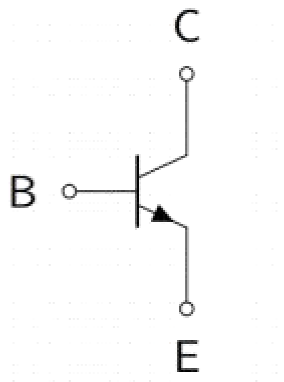




BJT Basics



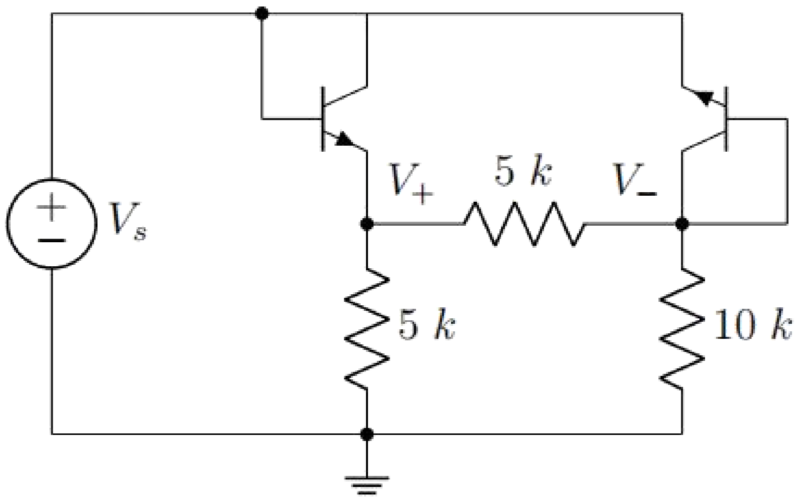
V_{BE}
Emitter
Base
Collector
n-type
p-type
n-type
I_E
I_C
n diffusion
E_c
E_{fn}
qV_{BE}
$q\phi_{n1}$
E_i
E_{fp}
$q\phi_{n2}$
E_v
Electron Energy
0
W_B
Distance



C
B
E

Exercises(Sample Exam)

One of Blue Tiger Inc.'s products transmits a specific type of waveform for pacemakers. You have discovered a clever method to generate this waveform using a sinusoidal source. The circuit is shown in Fig. 2.



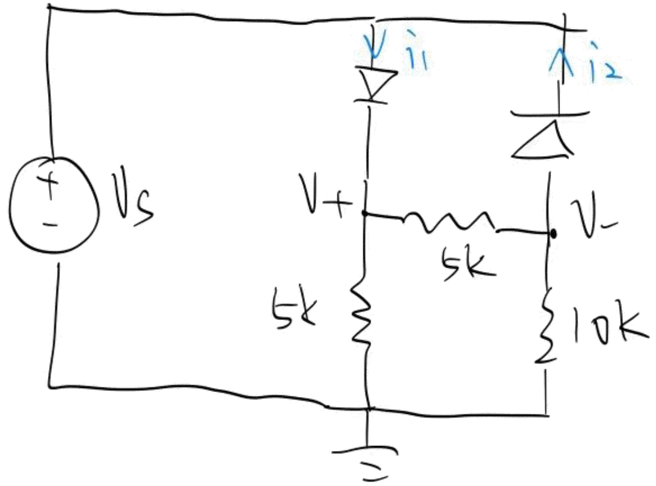
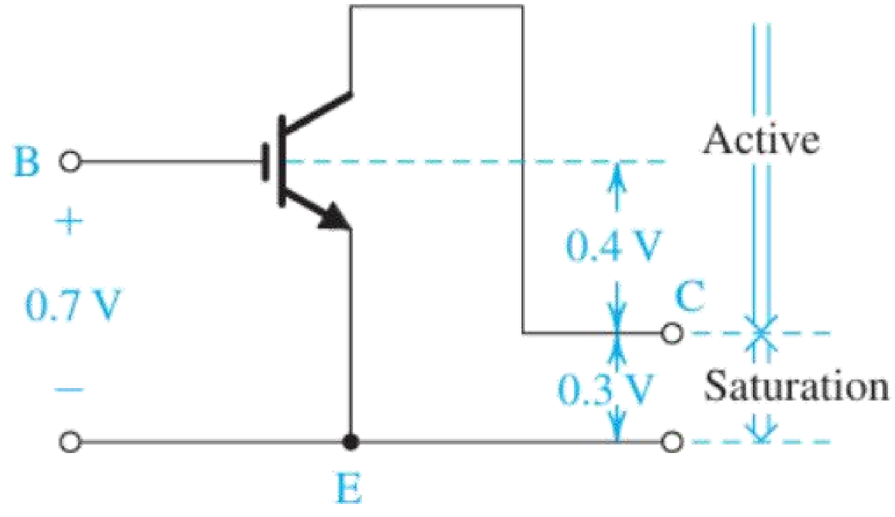


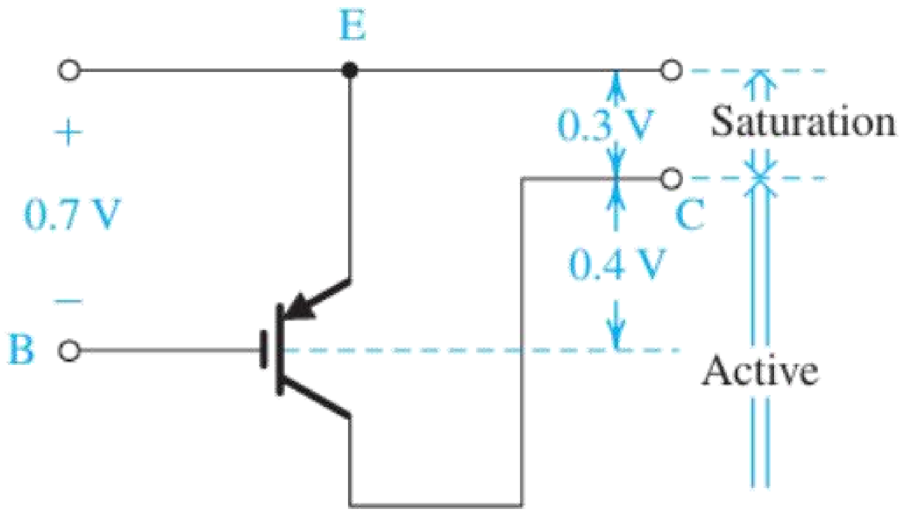
Figure 2: A pacemaker waveform generator.

1. (Easy) the company does not have diode in stock so that npn BJTs are used instead. Re-draw the circuit and identify the diodes (and their directions).

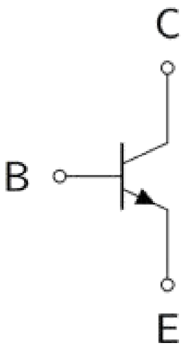
BJT Basics

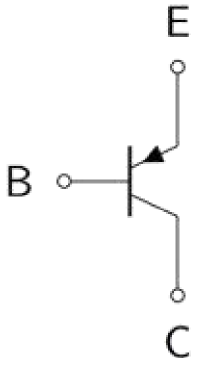


(a) *npn*



(b) *pnp*





Current in FAR(must know)

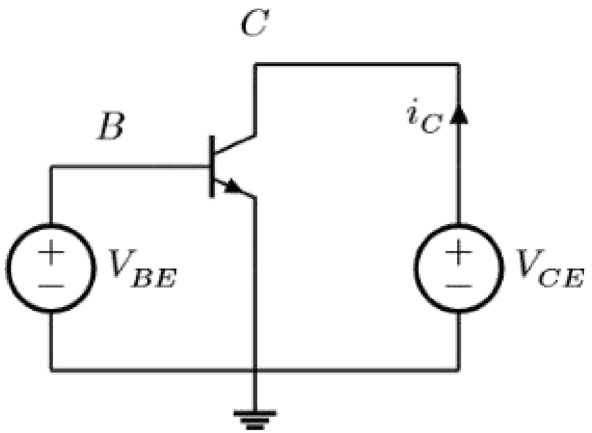
$$i_C = I_S\left(exp\frac{v_{BE}}{V_T} - 1\right) \approx I_S\, exp\frac{v_{BE}}{V_T}$$

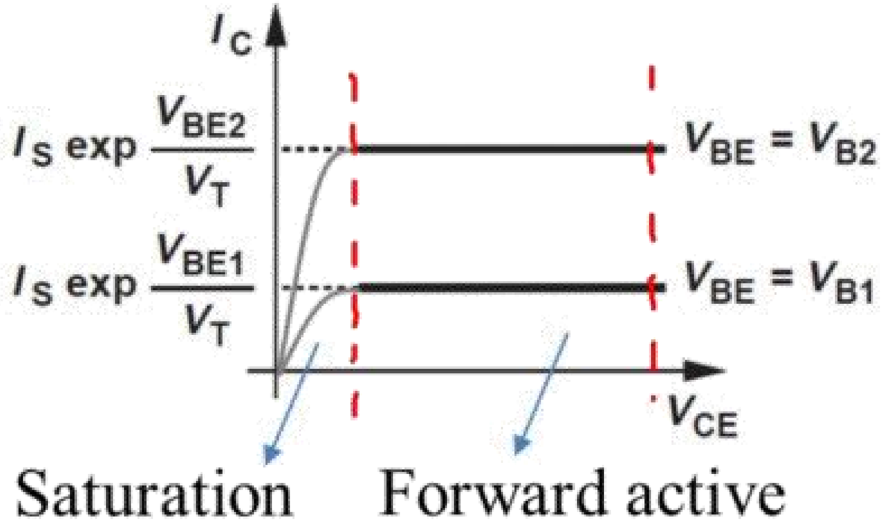
$$i_C = \beta i_B \qquad\qquad V_T \approx 25\, mV, 300K$$

$$i_C = \alpha i_E$$

$$i_E = i_C + i_B$$

$$\beta = \frac{\alpha}{1-\alpha}, \alpha = \frac{\beta}{1+\beta}$$



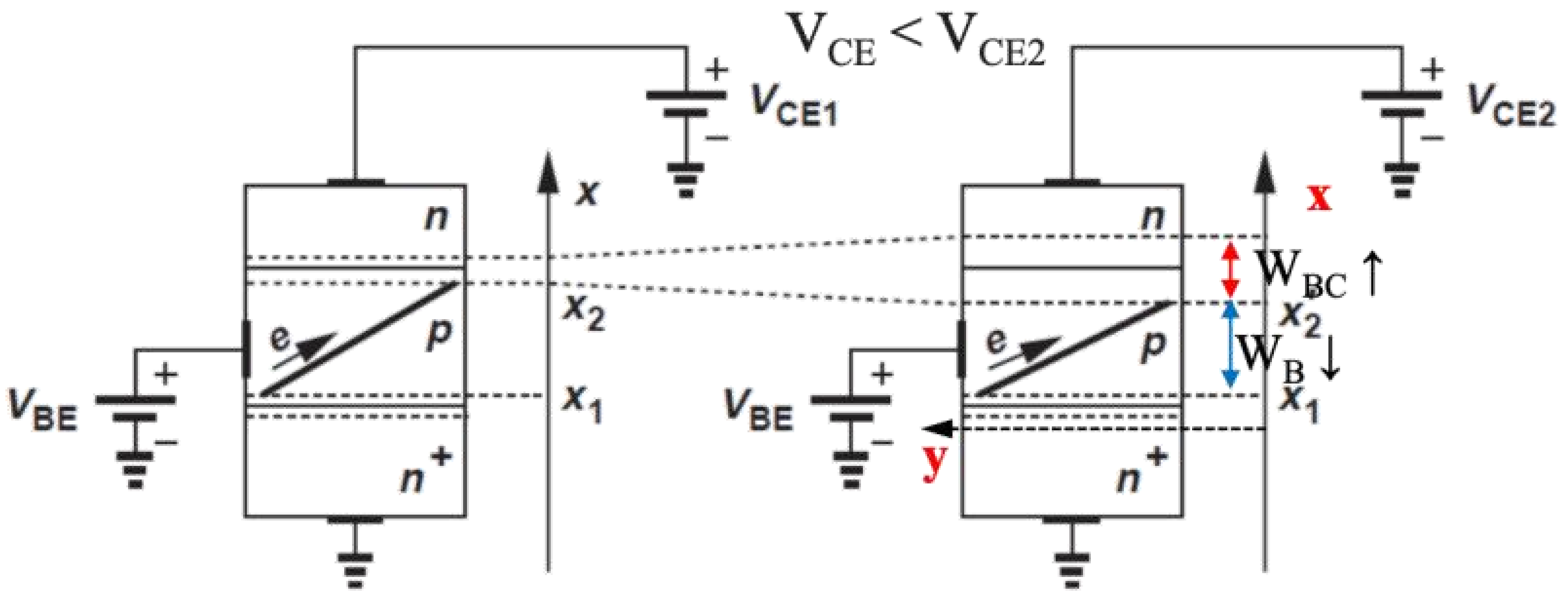


Works only in Forward Active Region!!!

Though we assume FAR mostly.

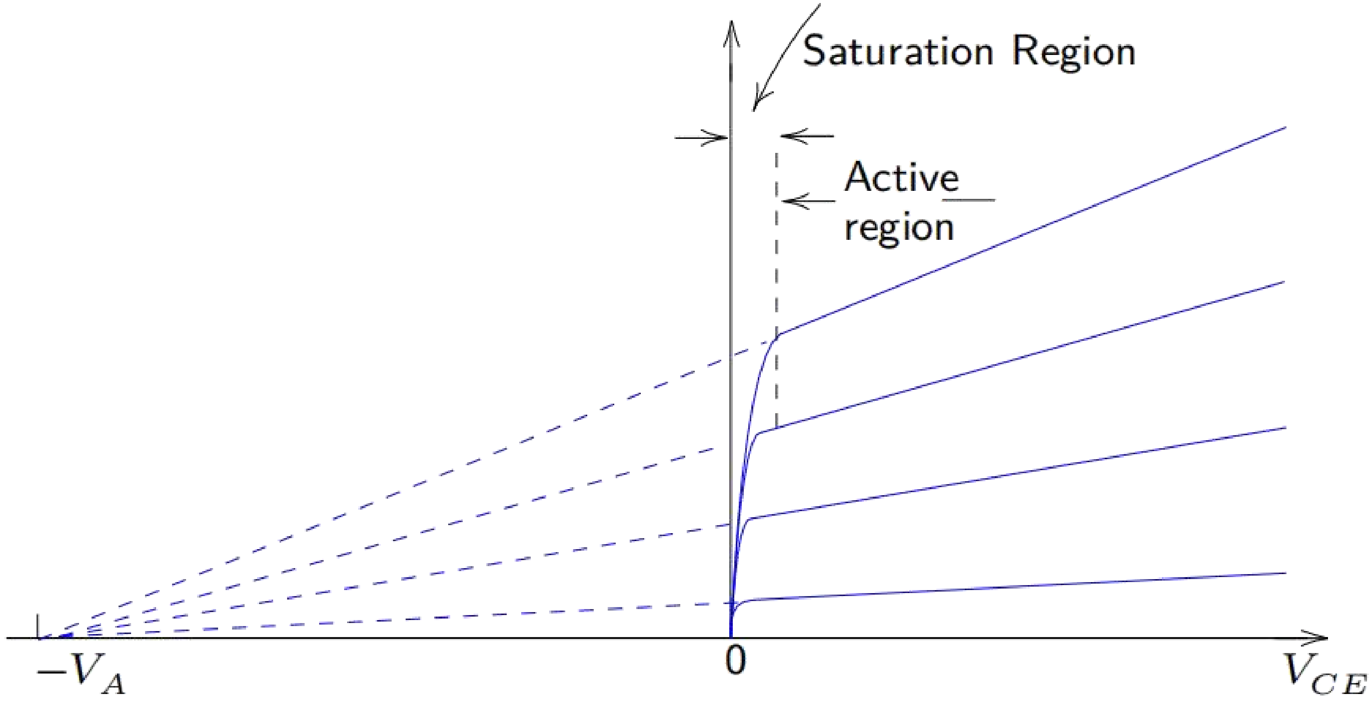
If not, you can assume FAR condition first, and then verify it, when solving unknown questions.

Early Effect



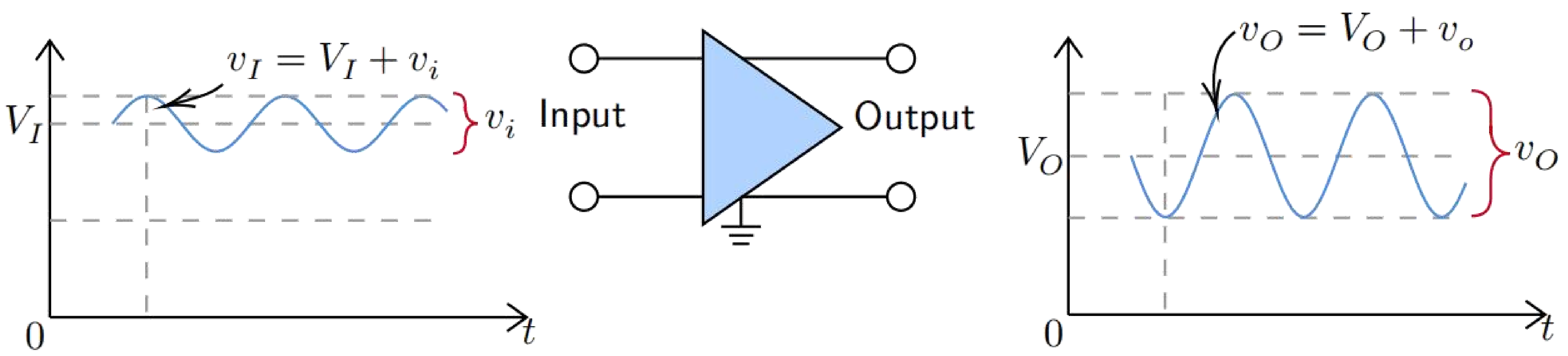
$$V_{CE2} > V_{CE1} \rightarrow W_{BC} \text{ increases}, W_B \text{ decreases} \rightarrow I_C \text{ increases}$$

Early Effect(must know)



$$I_C = I_S \, exp \frac{V_{BE}}{V_T}\left(1 + \frac{V_{CE}}{V_A}\right), V_A \text{ is early voltage.}$$

Why small signal?



- Amplifiers refer to circuits that amplify AC (time varying) signals; there is generally no interest in the DC component of a signal.

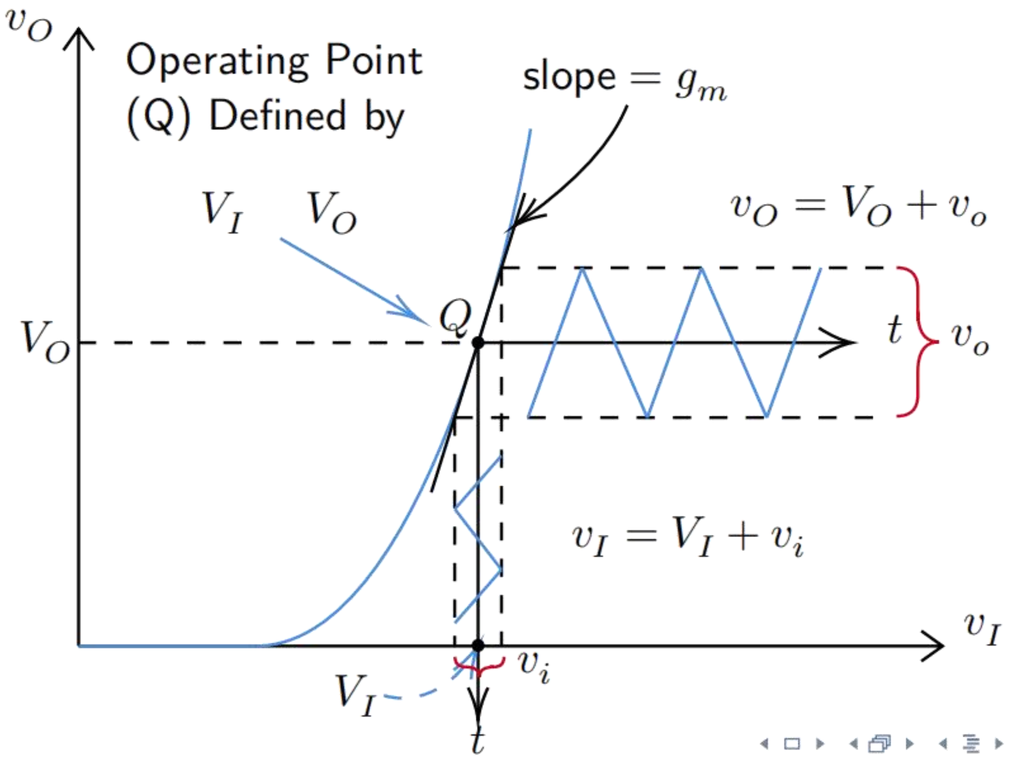
Why small signal?

Small signal model is nothing but an

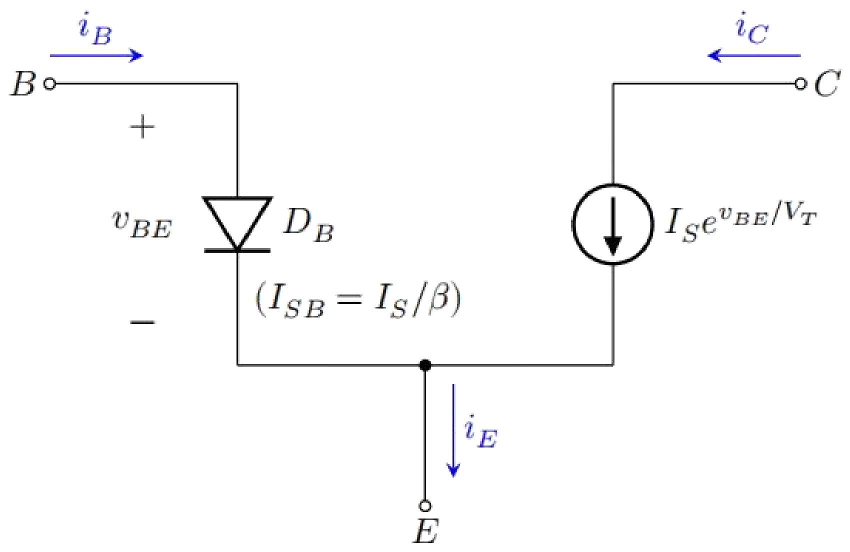
approximation model! It assumes and builds

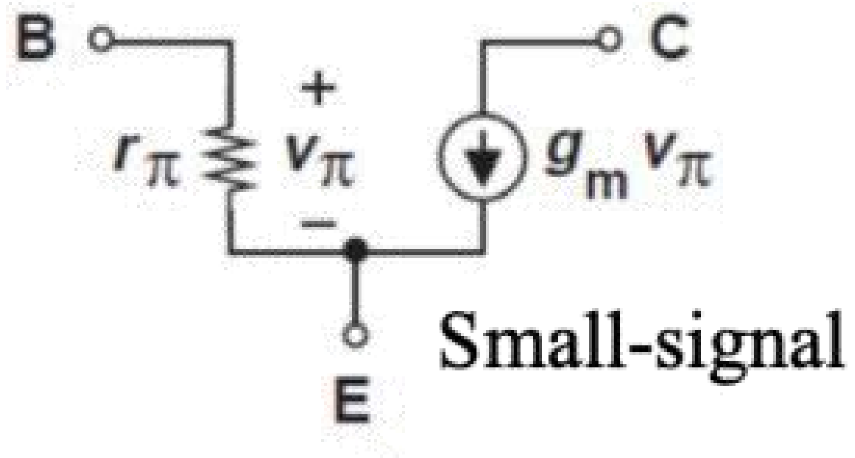
linear behavior on Quiescent Point(Q Point).

Preconditions on assumption......



Large Signal & Small Signal(must know)

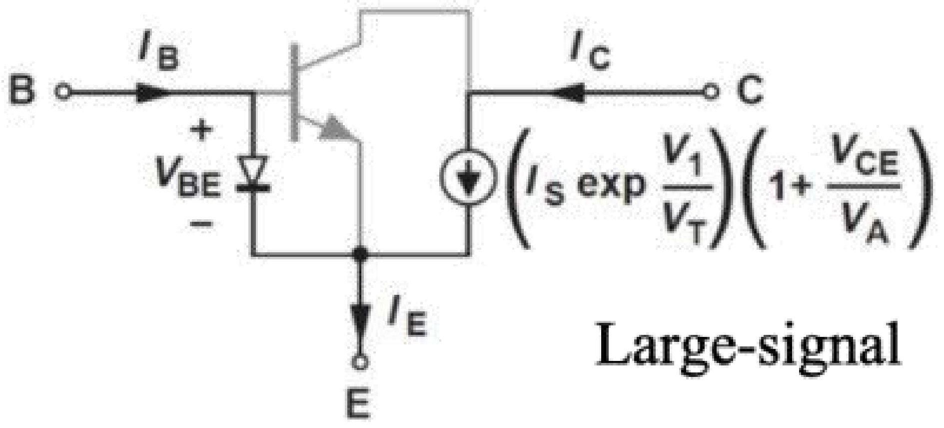


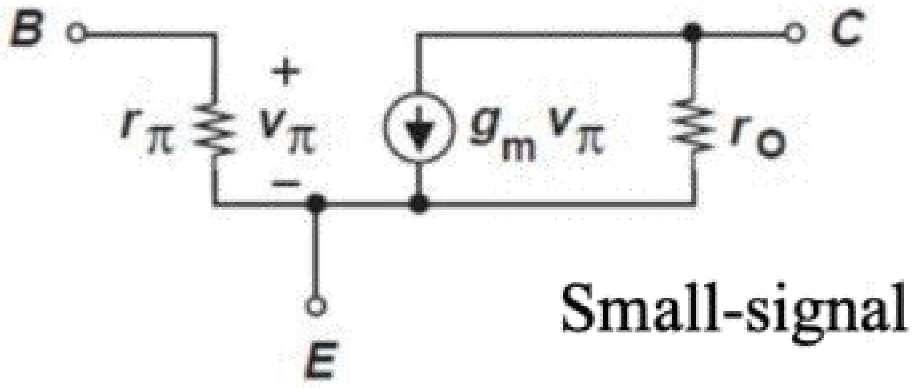


$$g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T} = \frac{I_C}{kT/q}$$

$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{\beta}{g_m}$$

Large Signal & Small Signal(with Early Effect)





$$g_m = \frac{dI_C}{dV_{BE}} = \frac{I_C}{V_T} = \frac{I_C}{kT/q}$$

$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{\beta}{g_m}$$

$$r_o = \frac{dV_{CE}}{dI_C} = \frac{V_A}{I_C}$$

How to draw small signal circuit(must know)

1. DC analysis, to calculate Operating Point(Q Point)

2. Apply BJT small signal models, we prefer Hybird- π model.

3. Consider independent DC Voltage Source as (AC) grounded, treat independent DC Current Source as open circuit.

4. Capacitors are open in DC, short in AC, if not specified. (Because we manually choose large capacitors)

Voltage Gain(A_v) Calculation(must know)

1. Draw small signal circuit.

2. Derive the relationship between v_{out} and v_{in}.

3. $A_v = \frac{v_{out}}{v_{in}}$.

Input Impedance(R_{in}) Calculation(must know)

1. Draw small signal circuit.

2. Connect a test volatge v_x on port with v_{in}. Do nothing on v_{out} port.

3. Calculate i_x, $R_{in} = \frac{v_x}{i_x}$.

Output Impedance(R_{out}) Calculation(must know)

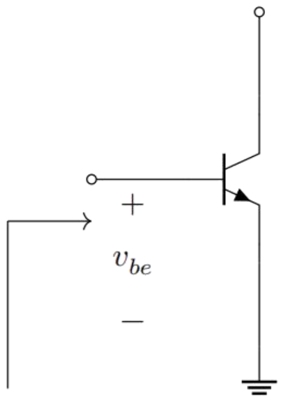
1. Draw small signal circuit.

2. Connect a test volatge v_x on port with v_{out}. Short v_{in} port to ground.

3. Calculate i_x, $R_{out} = \frac{v_x}{i_x}$.

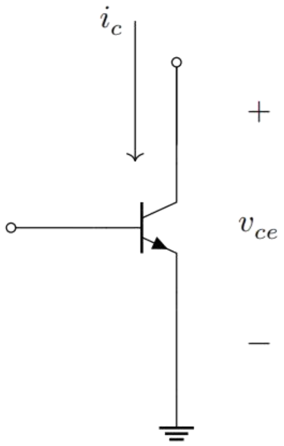
Impedance Models

Impedance Looking into Base



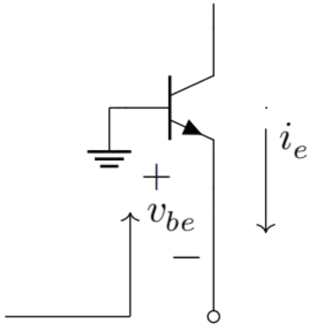
r_π

Impedance Looking into Collector



r_0

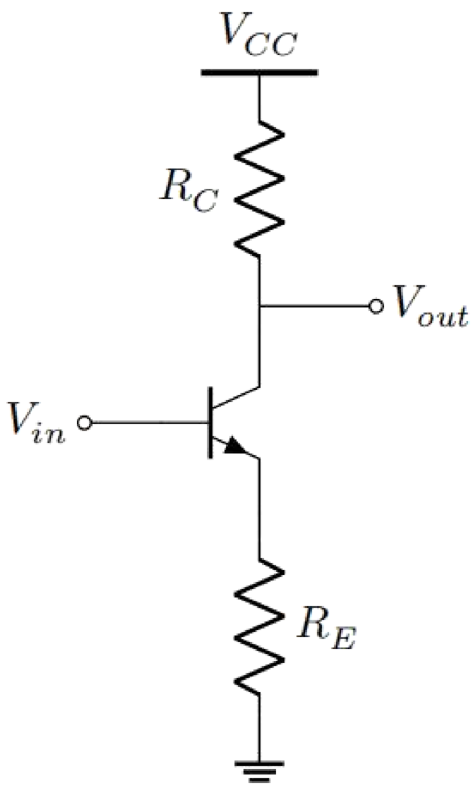
Impedance Looking into Emitter



$$\frac{1}{1/r_\pi + g_m}$$

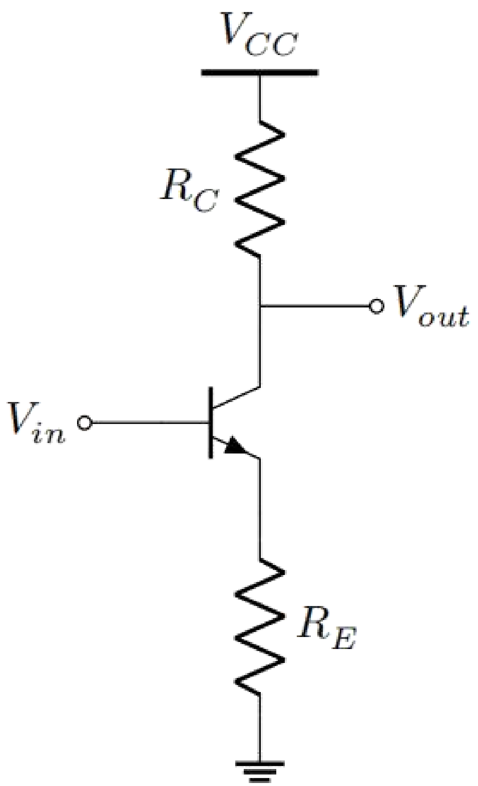
$$r_e = \frac{r_\pi}{(\beta + 1)}$$

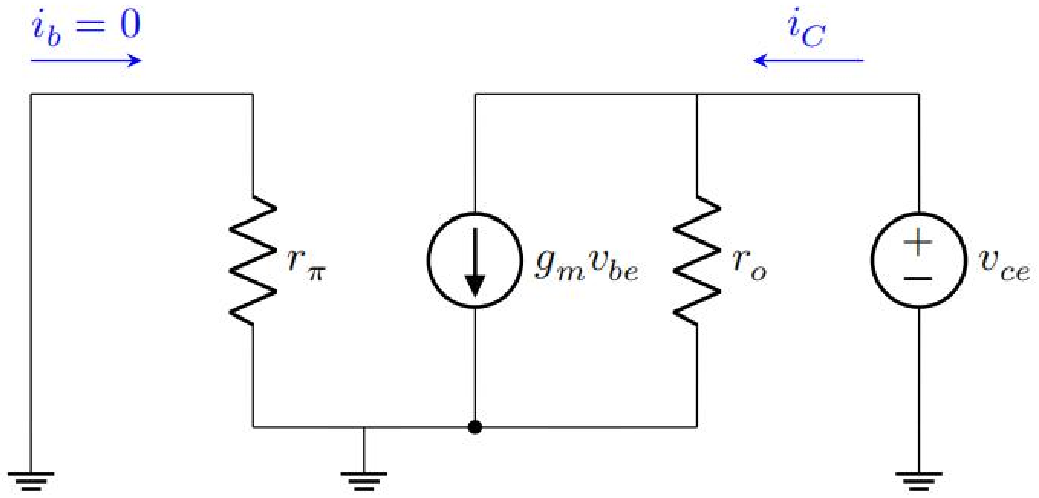
Common-Emitter Amplifier(must know)



	Definition	Expression	
		With emitter degeneration	Without emitter degeneration
Current gain	$A_i \triangleq \frac{i_{\text{out}}}{i_{\text{in}}}$	β	β
Voltage gain	$A_v \triangleq \frac{v_{\text{out}}}{v_{\text{in}}}$	$-\frac{\beta R_C}{r_\pi+(\beta+1)R_E}$	$-g_m R_C$
Input impedance	$r_{\text{in}} \triangleq \frac{v_{\text{in}}}{i_{\text{in}}}$	$r_\pi + (\beta+1)R_E$	r_π
Output impedance	$r_{\text{out}} \triangleq \frac{v_{\text{out}}}{i_{\text{out}}}$	R_C	R_C

Common-Emitter Amplifier(must know)



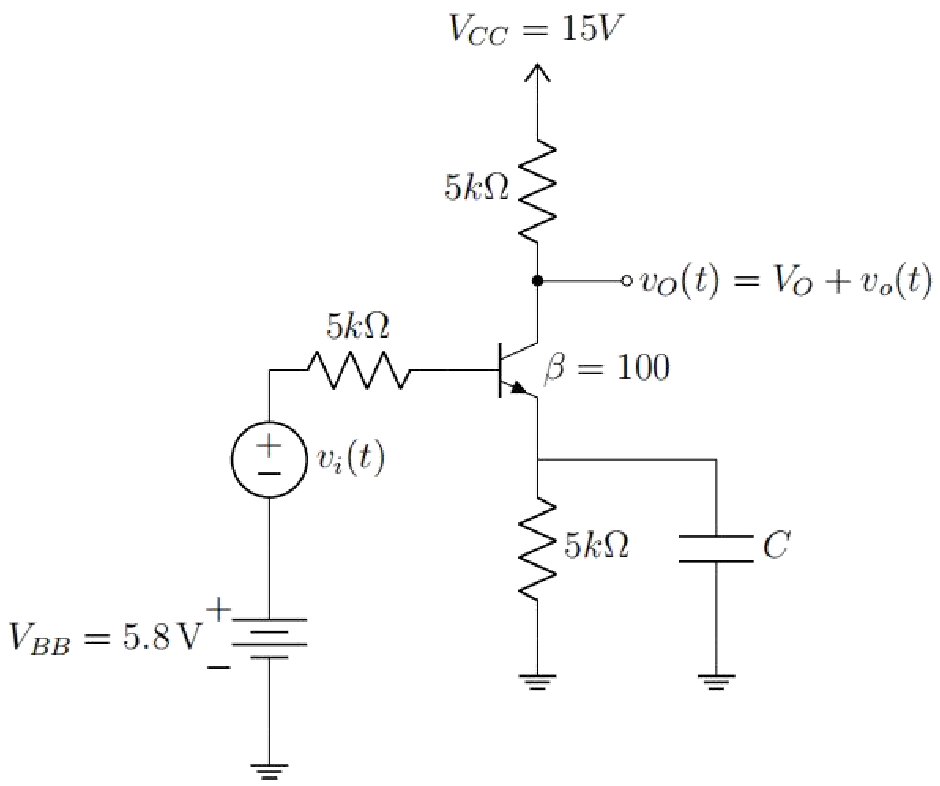


$$A_v = \frac{v_{out}}{v_{be}} = -g_m \left(R_C \parallel r_o\right)$$

$$= -g_m R_C \quad (\text{since } r_o = \infty)$$

Exercises

Calculate A_v, given $V_{BE} = 0.7V, V_T = 26mA$.(hw4)



Exercises

For problem (b),

$$I_B = \frac{5.8 - V_B}{5K}, I_E = (\beta + 1) I_B$$

$$V_E = V_B - 0.7, I_E = \frac{V_E}{R} = \frac{V_B - 0.7}{5K} = \frac{101 \times (5.8 - V_B)}{5K}$$

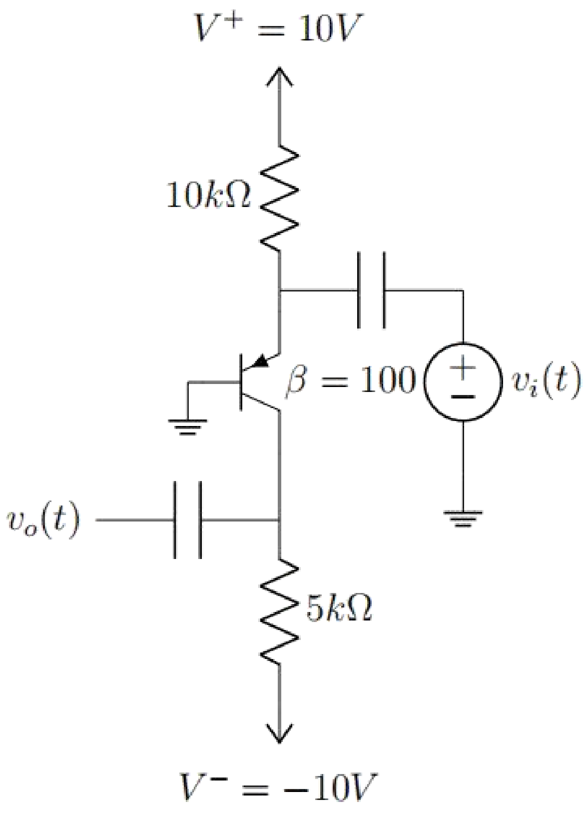
$$\Rightarrow V_B = 5.75V, V_E = 5.05V, I_B = 10\mu A, I_C = 1mA, V_c = 15 - I_C \cdot 5k = 10V$$

$$g_m = \frac{I_C}{KT/q} = \frac{1 \times 10^{-3}}{0.026} = 0.038\Omega^{-1}, V_\pi = \frac{\beta}{g_m} = 2.6k\Omega$$

$$V_\pi = \frac{r_\pi}{5k + r_\pi} V_i = \frac{13}{38} V_i, V_0 = -g_m V_x \cdot 5k = -65 V_i, A_v = \frac{V_0}{V_i} = -65$$

Exercises

Calculate A_v, given $V_{BE} = 0.7V$, $V_T = 25mA$.(hw4)



Exercises

For problem (c), the dc operating point can be determined as follows:

$$I_E = \frac{+10 - V_E}{R_E} \simeq \frac{+10 - 0.7}{10} = 0.93\text{mA}$$

Assuming $\beta = 100$, then $\alpha = 0.99$, and

$$I_c = 0.99 I_E = 0.92\text{mA}$$

$$V_c = -10 + I_c R_c$$

$$= -10 + 0.92 \times 5 = -5.4\text{V}$$

Thus the transistor is in the active mode.

We now determine the small-signal parameters as follows:

$$g_m = \frac{I_C}{V_T} = \frac{0.92}{0.025} = 36.8\text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{0.025}{0.92} = 27.2\Omega$$

$$\beta = 100 \quad \alpha = 0.99$$

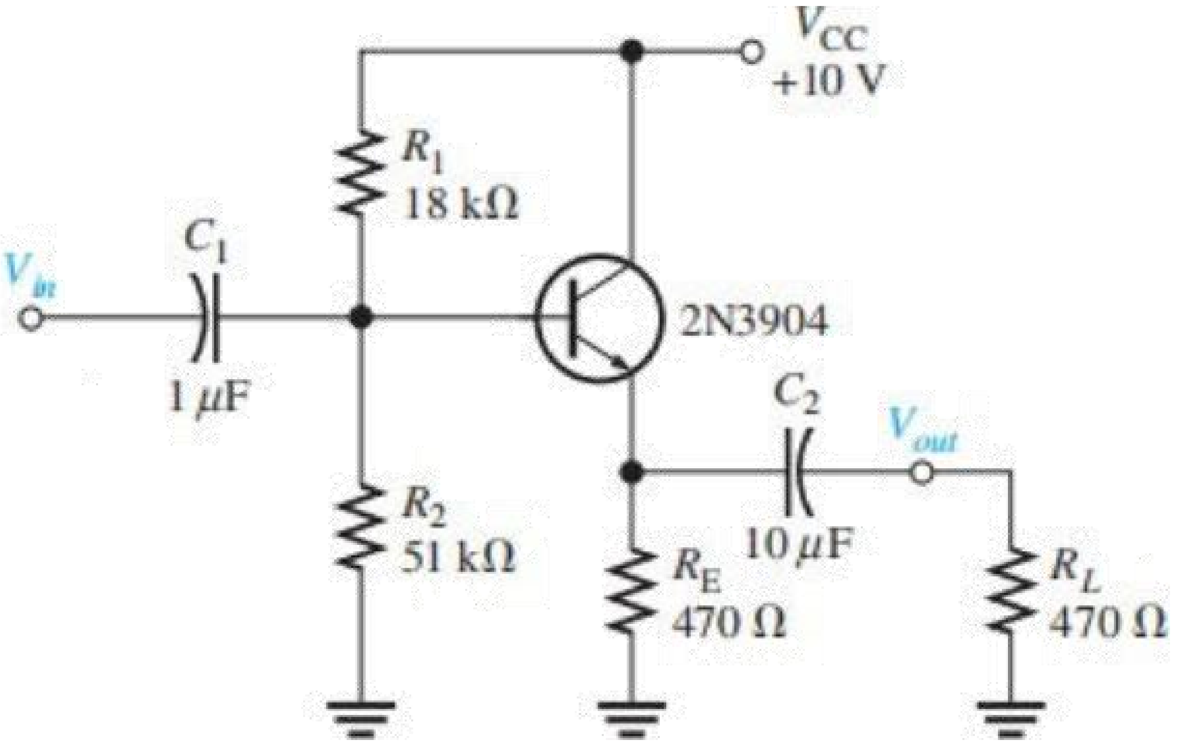
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{36.8} = 2.72\text{k}\Omega$$

To prepare the circuit for small-signal analysis, we replace the dc sources with short circuits.

$$A_v = \frac{v_o}{v_i} = \frac{\alpha R_C}{r_e} = \frac{0.99 \times 5}{0.0272} = 182$$

Exercises

[15pts] Determine the total input resistance R_{in} and voltage gain A_v of the emitter-follower in the circuit below. Assume $\beta = 175$ and that the capacitive reactance for all capacitors is negligible at the frequency of operation. Hint: $V_{BE} = 0.7$ V for the BJT in forward active region.



Exercises

1° DC. $V_C = 10$

$$\frac{10 - V_B}{18k} - \frac{V_B}{5.1k} = \frac{1}{\beta+1}\left(\frac{V_E}{470}\right)$$

$$V_B = V_E + 0.7$$

$$\Rightarrow V_E = 5.76V$$

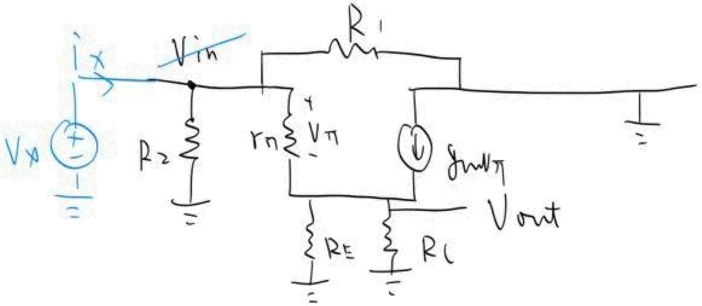
$$V_B = 6.46V$$

$$I_E = 12mA$$

$$I_B = 70\mu A$$

$$I_C = 12mA$$

2° small signal:



$$V_{out} - 0 = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E // R_L$$

$$V_{in} - V_\pi = V_{out}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{r_\pi} + g_m\right) R_E // R_L}{1 + \left(\frac{1}{r_\pi} + g_m\right) R_E // R_L} = \frac{\left(\frac{1}{\beta} + 1\right) g_m \cdot R_E // R_L}{1 + \sim}$$

$$= \frac{\left(\frac{176}{175}\right) \cdot \frac{12m}{0.026} \cdot 235}{1 + \sim}$$

$$= 0.991$$

4

Input Impedance R_{in}:

$$V_x - V_\pi = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E // R_L$$

$$i_x = \frac{V_x}{R_1 // R_2} + \frac{V_\pi}{r_\pi}$$

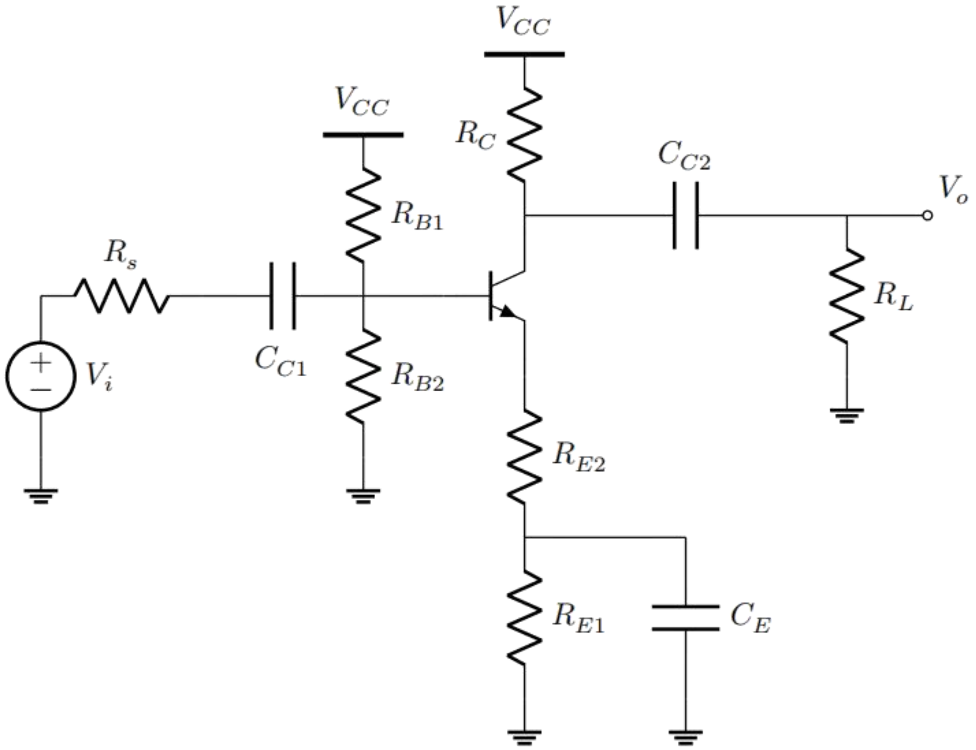
$$R_{in} = \frac{V_x}{i_x} = \frac{1 + \left(\frac{1}{\beta} + 1\right) g_m R_E // R_L}{\frac{1}{\beta} g_m + \frac{1}{R_1 // R_2}\left[1 + \sim\right]}$$

$$= 10088\ \Omega$$

$$= 10.1\ k\Omega$$

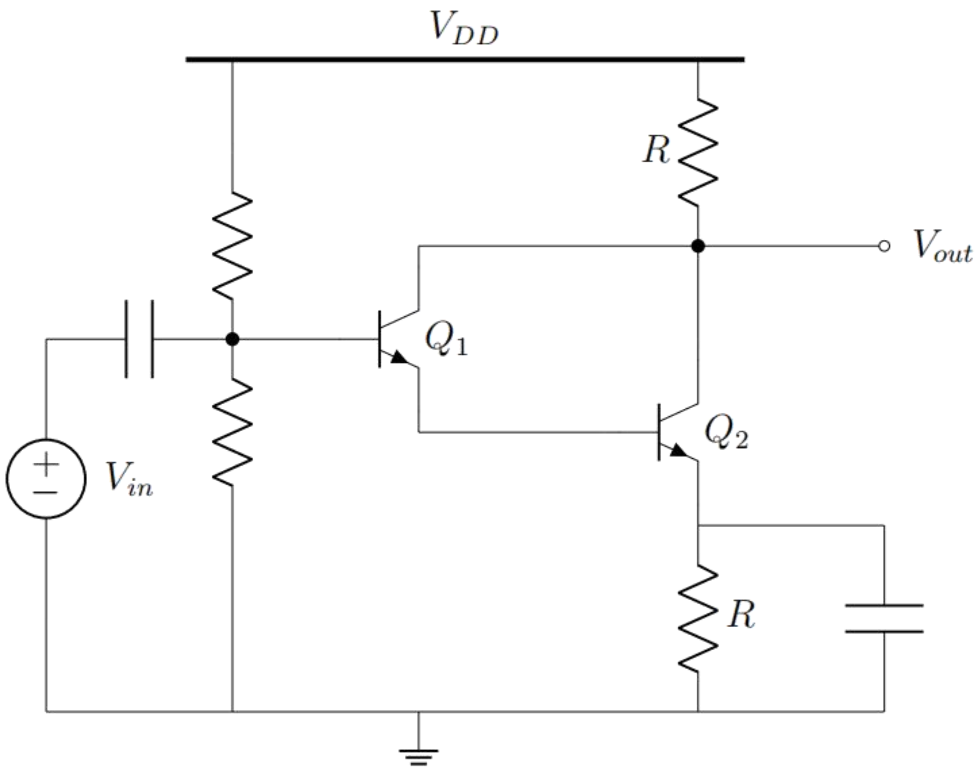
Exercises

Calculate A_v and R_{in}.($\beta = 100, assume\ always\ FAR$)



Exercises

(It is official name is the darlington pair) as follows. Assume the $\beta \gg 1$ for both BJTs, and we ignore early effect for simplicity. Assume both transistors are in Forward Active Region. Resistors used are large enough ($\gg 1\ K\Omega$), two marked resistors (R) have the same value.



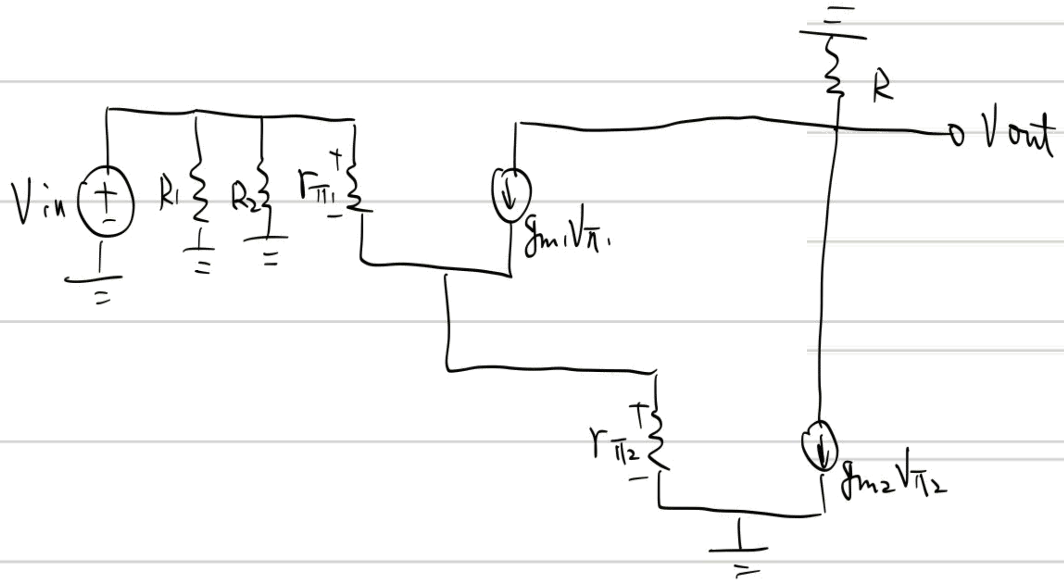
1. what is the relationship between g_{m1} and g_{m2}?
2. what is the relationship between $r_{\pi 1}$ and $r_{\pi 2}$?
3. Draw the small signal model for a high-frequency AC signal, assuming that the caps can be ignored.
4. Write the KCL for nodes (1) Emitter of Q_1. (2) Collector of Q_1.
5. (Hard) Write the expressions for the small signal gain. Assume $g_m r_{\pi 2} \gg 1$. How is the gain compared with a common emitter amplifier?

Exercises

1. $\dfrac{g_{m1}}{g_{m2}} = \dfrac{I_{C1}}{I_{C2}} = \dfrac{I_{C1}}{\frac{\beta+1}{\beta}(\beta+1)I_{C1}} = \dfrac{\beta}{(\beta+1)^2}$

2. $r_\pi = \dfrac{\beta}{g_m}$, $\dfrac{r_{\pi1}}{r_{\pi2}} = \dfrac{g_{m2}}{g_{m1}} = \dfrac{(\beta+1)^2}{\beta}$

3.



$$\begin{cases} g_{m1}V_{\pi1} + g_{m2}V_{\pi2} = \dfrac{0-V_{out}}{R} \\ \dfrac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \dfrac{V_{\pi2}}{r_{\pi2}} \\ V_{in} = V_{\pi1} + V_{\pi2} \end{cases} \Rightarrow$$

$$V_{\pi2} = \left(g_{m1} + \frac{1}{r_{\pi1}}\right) r_{\pi2} V_{\pi1}$$

$$V_{out} = -g_{m1}R\,V_{\pi1} + g_{m2}R\left(g_{m1} + \frac{1}{r_{\pi1}}\right) r_{\pi2} V_{\pi1}$$

$$V_{in} = V_{\pi1} + \left(g_{m1} + \frac{1}{r_{\pi1}}\right) r_{\pi2} V_{\pi1}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-g_{m1}R + g_{m2}\left(g_{m1} + \frac{g_{m1}}{\beta}\right)\cdot\frac{\beta}{g_{m2}}R}{1 + \frac{\beta+1}{\beta}g_{m1}\cdot\frac{\beta}{g_{m2}}}$$

$$= \frac{-g_{m1}R + (\beta+1)g_{m1}R}{1 + (\beta+1)\frac{g_{m1}}{g_{m2}}}$$

$$= \frac{-g_{m1}R + (\beta+1)g_{m1}R}{1 + \frac{\beta}{\beta+1}}$$

$$= g_{m1}R\,\frac{\beta(\beta+1)}{2\beta+1}$$

In the End...

1. I took VE311 in fall, so honestly I have no idea how difficult could it be in summer.

2. Review all the homeworks, and understand them. Contact TA if having problems.

3. Never overlook **CONCEPTS!!!** Like fundamentals of diodes, Op-amps, BJT... Try to read slides and learn device physics as much as possible.

4. Trust in your thoughts, even if you consider them unbelievable, since some questions simply pretend to be difficult...

Reference

1. VE311 Slides SU25, Xuyang Lu.
2. VE215 Slides FA23, Sungliang Chen.
3. VE311 RC FA24, Xuanmin Wang.
4. VE311 RC SU24, Tianrui Dang.

GOOD LUCK！

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