ECE3110J Electronic Circuit Homework 1

Due: **Tuesday May 27th 11:59 a.m.**

Note.

1) Please use A4 size paper or page.
2) Please clearly state your final result for each question.
3) For questions asking for *plot*, you can either sketch or use computer software (Matlab, Python). But make sure to mark all the important values on the graph.

As a fan of photography, you decided to make your own camera based on knowledge you learned form VE311. Firstly, you have to obtain the desired voltage. Having built a circuit, please simplify it to verify the voltage between terminals.

Question 1. Thevenin Circuit

Obtain the Thevenin equivalent circuit at terminals a-b for the circuit shown in Fig. 1.

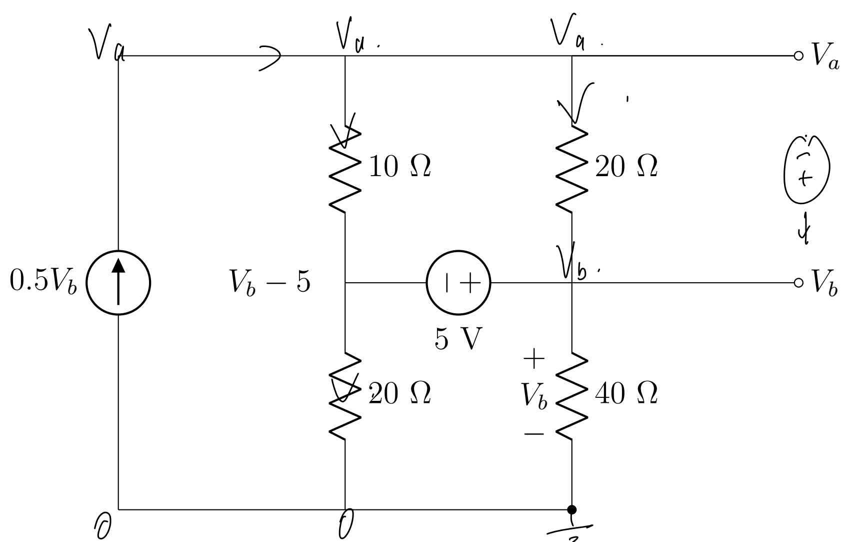


FIGURE 1. Thevenin equivalent circuit

$$\begin{cases} 0.5V_b = \dfrac{V_a - V_b + 5}{10} + \dfrac{V_a - V_b}{20} \\ \dfrac{V_a - V_b + 5}{10} + \dfrac{V_a - V_b}{20} = \dfrac{V_b - 5}{20} + \dfrac{V_b}{40} \end{cases}$$

$10V_b = 2V_a - 2V_b + 10 + V_a - V_b.$

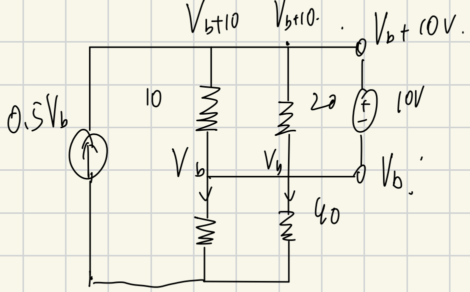
$$\begin{cases} 3V_a - 13V_b = -10. \\ 6V_a - 9V_b = -30. \end{cases}$$

$4V_a - 4V_b + 20 + 2V_a - 2V_b = 2V_b - 10 + V_b.$

$6V_a - 9V_b = -30$

$$\begin{cases} V_a = -5.88\text{ V}. \\ V_b = -0.59\text{ V}. \end{cases}$$

$$\boxed{V_{Th} = V_b - V_a = 5.29\text{ V}.}$$

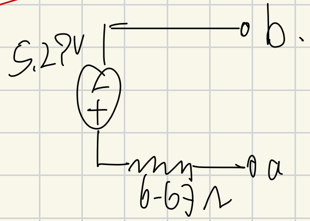


$$\begin{cases} i = \frac{10}{10} + \frac{10}{20} - 0.5V_b \\ 0.5V_b = \frac{V_b}{20} + \frac{V_b}{40} \end{cases}$$

$i = 1.5\,A$

$$\boxed{R_{th} = \frac{10}{1.5} = 6.67\,\Omega}$$

The equivalent circuit



After that, the image sensor drew your attention. To make it more sensitive to light, you looked into its structure to fully understand the imaging principle.

Question 2. CMOS Image Sensors (CIS)

Consider the structure shown below, where the green light is only incident on the gate area, and the rest of the area is shielded. Assume that the quantum efficiency is 30 %, which means only 30 out of 100 incident photons excite an electron-hole pair (An imaging-specific process nowadays typically has a quantum efficiency of 75%). The wavelength of green light $\lambda = 550$ nm. The size of the active area (Yellow area) $W = L = 10\ \mu m$. You looked into the textbook and found $h = 6.626 \times 10^{-34}$ J·s.

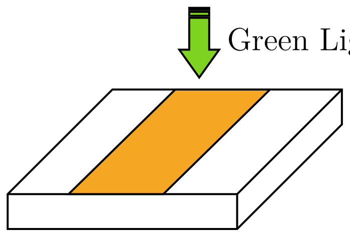


FIGURE 2. A sensor to capture green light

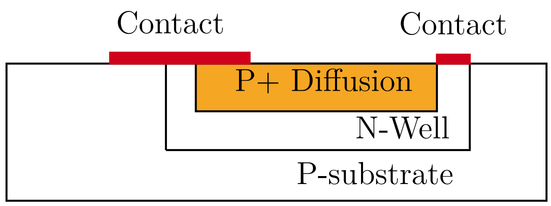


FIGURE 3. Side-view of an image sensor

(a) The CMOS camera is a PN junction. To understand how it works, you decide to revisit device physics. Consider a **p-n diode** with an abrupt junction. Assume that the doping densities on the p and n side are uniform and they are $N_A = 10^{19}/cm^3$ and $N_D = 10^{21}/cm^3$ respectively.
Calculate the majority and minority carrier density on each side at thermal equilibrium. Assume that the intrinsic carrier concentration (free electron and hole carrier concentration of pure silicon) $n_i = 1.5 \times 10^{10}/cm^3$ at room temperature of 25 $°C$.

(b) For a CMOS camera to operate properly, you will have to set the bias condition of the photodiode **which is a p-n diode**.
Calculate the built-in potential φ_0 when $V_d = 0V$. Based on φ_0, calculate the resulting potential barrier φ_+ when forward biased with $V_a = 0.5V$, and the potential barrier φ_- when reverse biased with $V_a = -1.7V$. (Hint: You can use $\varphi_0 = \frac{kT}{q}\ln(\frac{n_{n0}p_{p0}}{n_i^2})$)

(a) P: Holes: $N_A = 10^{19}/cm^3$.

Free Electrons: $\frac{n_i^2}{N_A} = 22.5 /cm^3$.

N: Holes $\frac{n_i^2}{N_D} = 0.225 /cm^3$.

Free Electrons $N_D = 10^{21}/cm^3$.

(b) $\varphi_0 = \frac{kT}{q} \ln\left(\frac{n_{no} p_{po}}{n_i^2}\right)$

$= 1.17 eV$.

when $V_d = 0V$, $\varphi = \varphi_0 = 1.17 eV$.

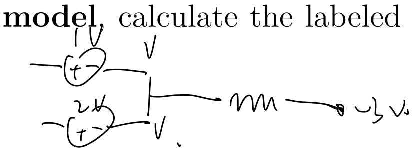
$V_d = 0.5V$, $\varphi = 0.67 eV$.

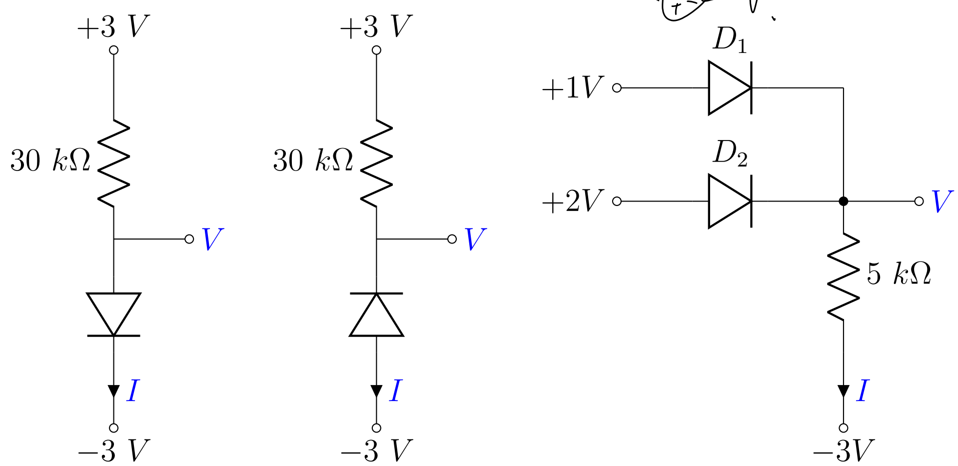
$V_d = -1.7V$, $\varphi = 2.87 eV$.

To connect different subsystems in your camera, you designed some diode circuits and analyzed them to see if they meet your requirements. To reduce computation, you tried to assume **ideal diodes** first.

"Error is gonna be within 5% and be ignored." you said to yourself.

Question 3. Diode Circuit Exercise 1
For each circuit in Figure. 4, assuming **ideal diode model**, calculate the labeled voltages and currents (V and I).





(A) (B) (C)

FIGURE 4. Ideal Diode Circuit DC Analysis

(A) $I = \frac{6V}{30k\Omega} = 2\times10^{-4}A$

$V = -3V$.

(b) $I = 0A$.

$V = 3V$.

(c) $V = 2V$

$I = \frac{6V}{30k\Omega} = 1\times10^{-4}A$.

Question 4. Diode Circuit Exercise 2
In each of the **ideal-diode** circuits shown in Figure. 5, v_I is a 1 kHz, 1 V peak sine wave with 0 initial phase. Sketch the output waveform v_O as a function of time, showing the important values, such as the peak voltage values (positive and negative).

$f = \frac{2\pi}{\omega} = 10^{-3}$.

$\omega =$

$V_I = \sin(2\times10^{3}\pi t)$.

1V.

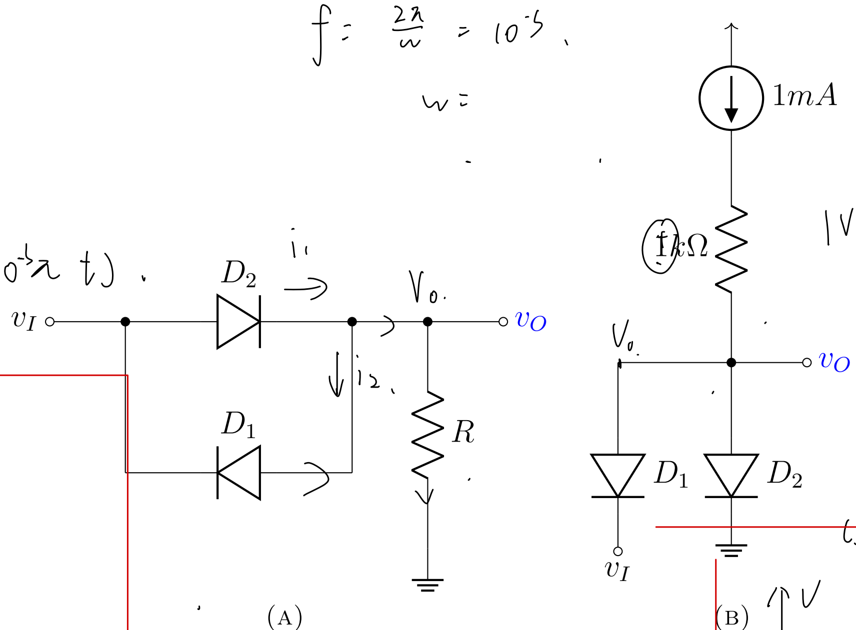
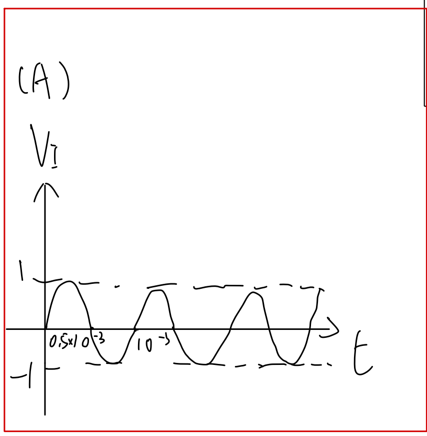
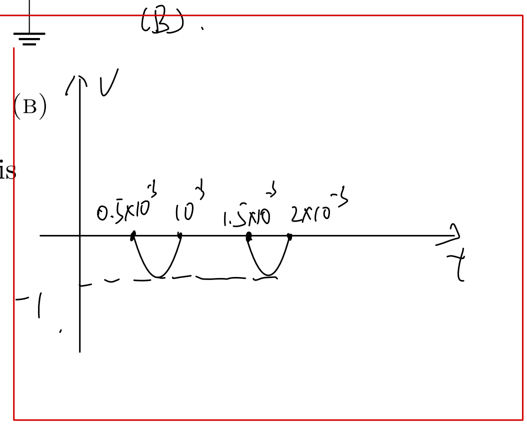


FIGURE 5. Ideal Diode Circuit AC Analysis





$V_0 = V_I = \sin(2\times10^{3}\pi t)$.

$T = \frac{2\pi}{\omega} = \frac{\pi}{\cos}$

Unfortunately, however, unexpected errors showed up. You re-analyzed some crucial parts of your circuits using the constant voltage drop model to see if any modifications are needed.

Question 5. Diode Circuit Exercise 3
In the circuit shown in Figure. 6, assume the **constant voltage drop model** for the diodes and assume the turn-on voltage is 0.7V. Calculate the values for current I_{R_2} and I_{D_2}.
(Hint: you can assume D_1 and D_2 is on and calculate V_0)

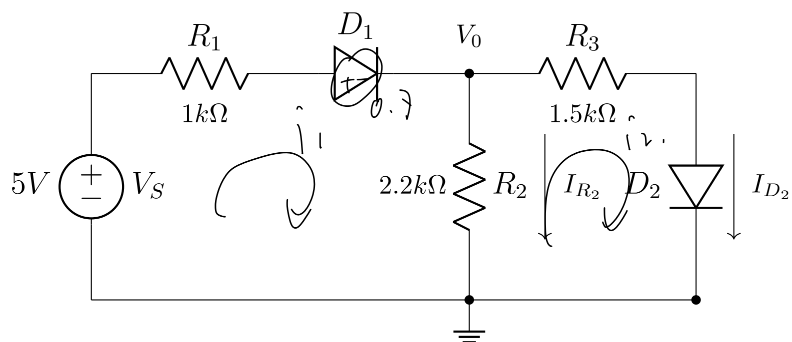


FIGURE 6. CVD Diode Circuit Analysis

$$\begin{cases} -5 + 10^3 \cdot i_1 + 0.7 + 2.2\times10^3(i_1 - i_2) = 0. \\ (i_2 - i_1)\cdot 2.2\times10^3 + 1.5\times10^3 i_2 + 0.7 = 0. \end{cases}$$

$$3.2\times10^3 i_1 - 2.2\times10^3 i_2 = 4.3.$$
$$-2.2\times10^3 i_1 + 3.7\times10^3 i_2 = -0.7.$$

$$\begin{cases} i_1 = 2.05\times10^{-3}\ A \\ i_2 = 1.03\times10^{-3}\ A. \end{cases}$$

The assumption is right.

$$\boxed{\begin{aligned} i_{R_2} &= i_1 - i_2 = 1.02\times10^{-3} A \\ i_{D_2} &= i_2 = 1.03\times10^{-3} A. \end{aligned}}$$

Question 6. Diode Circuit Exercise 4

Plot the input/output characteristics of the circuit shown in Figure. 7 using the **CVD model**. Label critical points and slopes clearly.

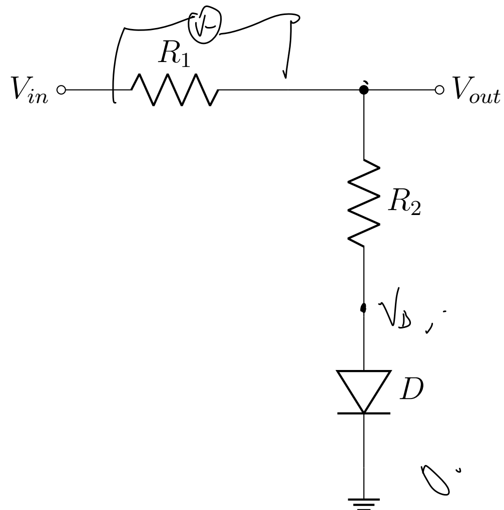
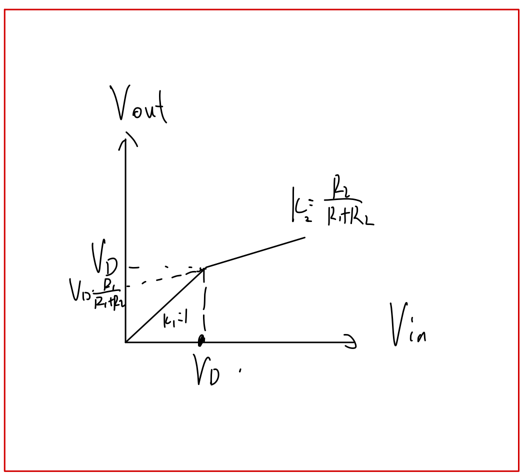


FIGURE 7. CVD Diode Circuit Characteristic Plotting



$$V_{out} = V_{in} - \frac{V_{in} - V_D}{R_1 + R_2} \cdot R_1$$

$$= V_{in} \cdot \frac{R_2}{R_1+R_2} + V_D \frac{R_1}{R_1+R_2}$$

Charging your camera requires DC input, while from the socket you can only get $220VAC$. Thus, you apply some rectifiers to transform the AC signal V_s into your desired DC signal.

Question 7. Half-Wave Rectifier
You designed a half-wave rectifier circuit, such as the one in Figure. 8, which can convert a sinusoidal voltage input, $V_s = 5\sin(2\pi \cdot 100 \cdot t)$, to an almost constant voltage output.

(a) Assuming $V_{on} = 1.0$ V and R = 100Ω, calculate C which makes the ripple voltage V_r is smaller than 0.1 V. Estimate $V_{dc}, I_{dc}, \theta_c, \Delta T, I_{peak}, I_{surge}$ and PIV of the designed half-wave rectifier.

(b) Plot V_s **and** V_o versus time on the sample graph.

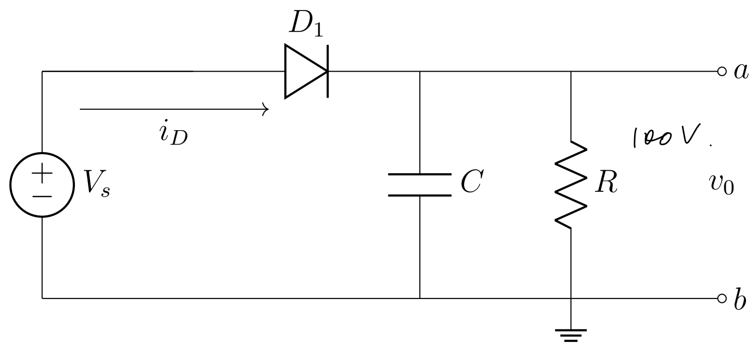


FIGURE 8. Half-Wave Rectifier

(a)

$V_{DC} = V_s - V_{on} = 5V - 1V = 4V$.

$I_{dc} = \frac{V_{dc}}{R} = \frac{4V}{100\Omega} = 0.04A$

$V_r = V_{DC} \cdot \left(\frac{T}{RC}\right) < 0.1$

$C_{min} = 10 \cdot 4 \cdot \frac{0.01}{100} = 4\times10^{-3} F$.

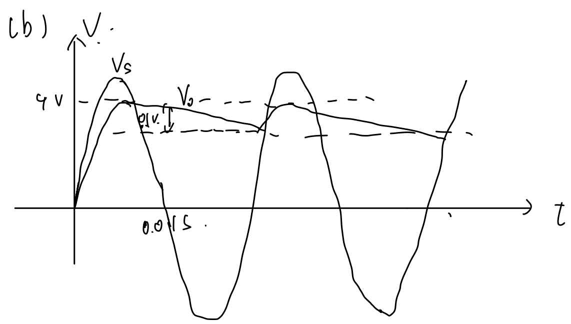
$\theta_c = \sqrt{\frac{2V_r}{V_s}} = \sqrt{\frac{0.2}{5}} = 0.2$.

$\Delta T = \frac{\theta_c}{\omega} = \frac{0.2}{200\pi} = 3.2\times10^{-4} s$.

$I_{peak} = \frac{2 I_{dc} T}{\Delta T} = \frac{0.08 \times 0.01}{3.2\times10^{-4}} = 2.5A$.

$I_{surge} = \omega C V_s = 200\pi \times 4\times10^{-3} \times 5 \approx 12.6A$.

$PIV = 2V_s - V_{on} = 2\times5 - 1 = 9V$.



PIV of the half-wave rectifier is quite high, which may potentially damage your camera. Thus, you applied the knowledge on the full wave rectifier you learned in ECE3110J and designed a more advanced circuit.

Question 8. Full-Wave Rectifier

You designed a full-wave bridge rectifier circuit, such as the one shown in Figure. 9, which can convert a sinusoidal voltage input, $V_s = 5\sin(2\pi \cdot 100 \cdot t)$, to an almost constant voltage output.

(a) Assuming $V_{on} = 1.0$ V and $R = 100\Omega$, calculate C which makes the ripple voltage V_r smaller than 0.1 V. Estimate $V_{dc}, I_{dc}, \theta_c, \Delta T, I_{peak}, I_{surge}$ and PIV of the designed full-wave bridge rectifier.

(b) Plot V_s **and** V_{out} versus time on the sample graph.

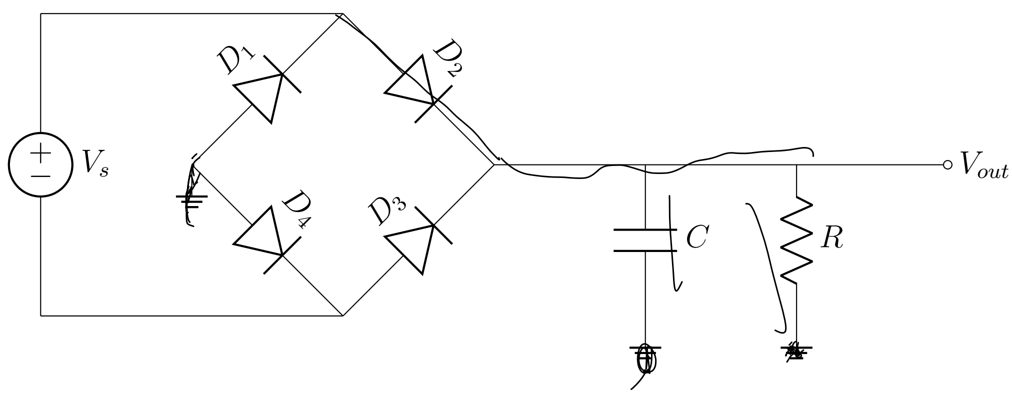


FIGURE 9. Full-Wave Rectifier

(a) .

$V_{DC} = V_s - V_{on} = 5V - 2V = 3V$.

$I_{DC} = \frac{V_{dc}}{R} = \frac{3}{100} = 0.03A$.

$V_r = V_{DC} \cdot \frac{T}{2RC} < 0.1$

$C_{min} = \frac{10 \cdot 3 \cdot 0.01}{2 \times 100} = 1.5 \times 10^{-3} F$.

$\theta_c = \sqrt{\frac{2V_r}{V_s}} = 0.2$.

$\Delta T = 3.2 \times 10^{-4} S$.

$I_{peak} = \frac{I_{dc} T}{\Delta T} = \frac{0.03 \times 0.01}{3.2 \times 10^{-4}} = 0.9375A$.

$I_{surge} = \omega C V_s = 200\pi \times 1.5 \times 10^{-3} \times 5 = 4.71A$.

$PIV = 2V_s - V_{on} = 8V$.

(b)