# How are Pauli parameters inferred from syndromes?

A Pauli qubit channel is defined by 4 probabilities: .

1. – probability of bit flip in the computational basis (|0> ↔|1>)
2. – probability of simultaneous bit and phase flip in the computational basis
3. – probability of phase flip in the computational basis
4. – probability of state staying the same

Sum of these probabilities equal to one.

Pauli qubit channels act as binary symmetric channels (BSC) when inputs are the eigenstate of the Pauli operator.

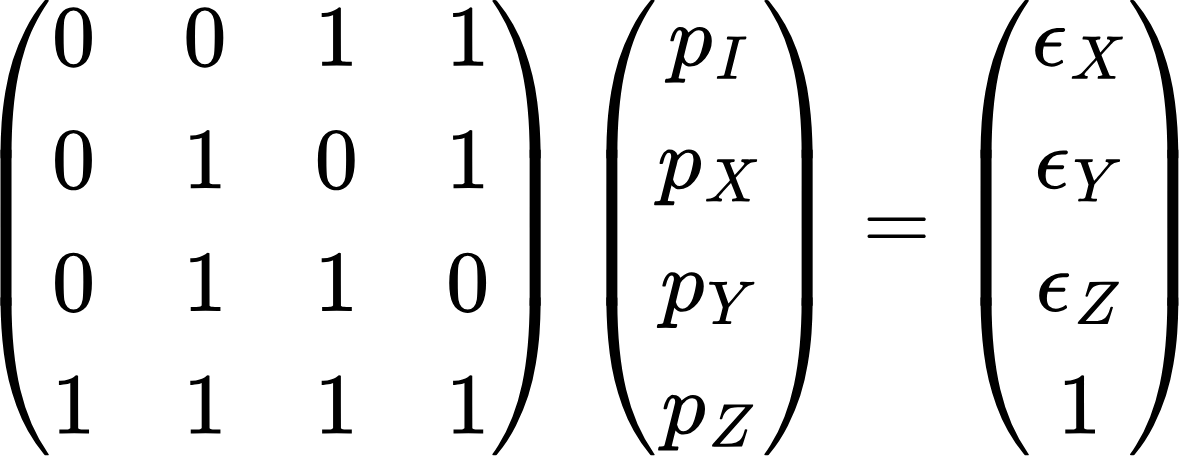
In the X-basis:

Here – error rate of the BSCX channel. Error rates for and can be described in a similar way.

## Entanglement free parameter estimation

In Entanglement free parameter estimation (EFPE) the pre-agreed message is being transferred, so the error rates can be measured directly.

The relation between these error rates and Pauli parameters can be written as a matrix equation:



This equation can be solved for , by multiplying the inverse matrix to the error rates vector. So each Pauli probability is just a linear combination of the observed error rates.

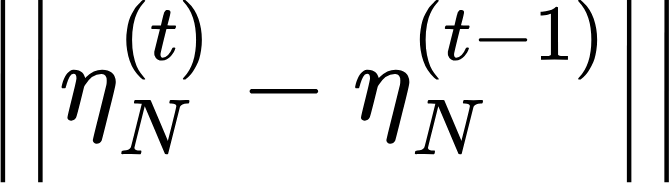
## Simultaneous communication and parameter estimation

SCAPE achieves Pauli parameters estimation during the communication. In SCAPE, the receiver decodes the k-repetition code, reencode the decision, and counts the amount of mismatched bits. Over the many samples the value of error rates can be estimated. Then the same matrix equation is used to obtain the estimated values of Pauli parameters.

# What performance metric is optimized?

The main metric used to compare the performance of the SCAPE protocol with EFPE protocol is diamond norm distance between true and estimated Pauli channels.

For time varying Pauli channels the choice of N (number of samples) is not trivial, since if N is too big the underlying distribution might change significantly. The choice of the optimal value of N is performed by minimizing the “average inter-estimate distance”, which is the mean distance between consecutive channel estimates.



Where *η* is the underlying quantum channel.

A high value of that value can be caused by N being too large so that underlying channel has changed significantly or from N being too small introducing significant statistical error. So minimization of that value determines the optimal value of N.