



THE UNIVERSITY OF QUEENSLAND
A U S T R A L I A

**Trajectory Optimisation of a Partially-Reusable
Rocket-Scramjet-Rocket Small Satellite Launch System**

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B.Eng. (Mechanical and Aerospace) (Hons. I) & B.Sc. (Physics) UQ

A thesis submitted for the degree of Doctor of Philosophy at
The University of Queensland in 2018

School of Mechanical Engineering

Abstract

Declaration by author

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Conference papers

S.O. Forbes-Spyratos, M.P. Kearney, M.K. Smart, and I.H. Jahn. “Trajectory design of a rocket-scramjet-rocket multi-stage launch system”. In: *21st AIAA International Space Planes and Hypersonics Technologies Conference, Hypersonics 2017*. Xiamen, China, 2017. ISBN: 9781624104633

Publications included in this thesis

This thesis comprises partly of publications, as allowed by University of Queensland Policy PPL 4.60.07. The papers that have been included have all been published in peer reviewed journals at the time of submission.

[make this table more compact](#)

Sholto O. Forbes-Spyratos, Michael P. Kearney, Michael K. Smart, and Ingo H. Jahn. “Trajectory Design of a Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: *Journal of Spacecraft and Rockets - Under Consideration* TBD (2018)

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Michael K. Smart	Analysis of results (%)
	Wrote and edited paper (%)

Contributions by Others to the Thesis

Except for the contributions by others to publications above, there are no contributions by others to this thesis.

Statement of Parts of the Thesis Submitted to Qualify for the Award of Another Degree

None.

Acknowledgements

Keywords

keyword, keyword, keyword, keyword, keyword

Australian and New Zealand Standard Research Classification (ANZSRC)

ANZSRC code: 090107 Hypersonic Propulsion and Hypersonic Aerodynamics, 20%

ANZSRC code: 090106 Flight Dynamics, 20%

ANZSRC code: 090108, Satellite, Space Vehicle and Missile Design and Testing, 20%

ANZSRC code: 090104 Aircraft Performance and Flight Control Systems , 20%

ANZSRC code: 010303 Optimisation , 20%

Fields of Research (FoR) Classification

FoR code: 0901, Aerospace Engineering, 100%

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CHAPTER 1

INTRODUCTION

Airbreathing engines are an ideal candidate for producing the next generation of space access systems. Airbreathing engines produce higher specific impulse than rockets, and require far less propellant to be carried on-board a launch vehicle. The mass savings gained by using airbreathing engines allow launch vehicles to be designed to be reusable, capable of manoeuvring aerodynamically and of landing similarly to a conventional aircraft. A three stage design utilising a scramjet second stage with rocket powered first and third stages is being developed by The University of Queensland. In previous studies it has been assumed that the optimal trajectory for a scramjet powered vehicle is at its maximum dynamic pressure and all other trajectory stages have conformed to this assumption. This study will aim to produce an optimal trajectory plan which may be applied to any rocket-airbreathing-rocket system for delivering small satellites to Earth orbit. The SPARTAN vehicle being developed by the university of Queensland will be used as a model for simulations. **focus on my achievements, rather than justifying small satellite launchers**

The SPARTAN is being developed to take advantage of a rapidly increasing market for small satellite launches. Many small satellites are currently launched with multiple small satellites piggy-backing onto the launch of larger satellites, comanifesting on a single launch to share costs. This has the result of smaller payloads being subject to the schedule of the other parties involved, and being delivered into an orbit determined by the requirements of the main payload. This dependence on a predetermined mission plan often has detrimental effects on the small satellite mission. Small satellites are often used in missions which require specific orbital and scheduling needs. Satellites to be used as part of constellations are especially sensitive to the orbit in which they are placed. Small satellites may be able to change orbits slightly, however larger changes mean additional fuel mass and potentially additional system mass if a larger drive is necessary. The production of a cheap and flexible small satellite delivery system would enable more small satellites to be delivered into customised orbits within relatively short time periods. **rephrase, comment on limited fuel availability**



Figure 1.1

Add image of the final trajectory

1.1 Research aims

The overall aim of this thesis is to apply state of the art numerical optimisation techniques to the trajectory of a rocket-scramjet-rocket small satellite launch system. The purpose of this optimised trajectory is to maximise the payload-to-orbit capabilities of the launch system, thereby also maximising the cost efficiency of the system.

add more, flexibility, turn around time, operability

This will be achieved through the following objectives: IJ - not objectives, redo

1. *Third stage redesign.*

To ensure that the simulated system is representative of an up-to-date, relevant design, the third stage of the rocket-scramjet-rocket launch system is to be redesigned to use the Kestrel pressure-fed engine. This redesign requires the third stage shape and internal layout to be changed considerably, as well as the second stage internal layout to be reconfigured.

2. *First stage design.*

The first stage has not yet been studied previously, and as such a representative design of a first stage rocket is created.

3. *Launch system simulation.*

A thorough numerical simulation of the launch system is created, which fits into the format required by the optimisation algorithm.

4. *Optimisation integration*

1.2 Thesis outline

Edit this as required. This thesis is organised in eight chapters which are outlined below. The appendices included at the end of this document contain further technical information and supplementary experimental details. **If you want to add colours to the text for some reason (supervisor reviewing etc.), use the `textcolor` command. Check the code here to see how.**

Chapter 2 -

This chapter presents a review of literature related to the different aspects of this thesis.

Chapter 3 -

This chapter provides details of the

Conclusions

The body of this thesis concludes by summarising the most significant findings from

CHAPTER 2

LITERATURE REVIEW

KEEP THIS SECTION AS BROAD AS POSSIBLE

This chapter examines the relevant literature associated with the different aspects of the work conducted as part of this thesis.

Explain order of literature and what was examined.

2.1 Scramjets

make this section more general , move discussion of launchers later

explain mechanisms

add research exploring min/max speeds - expected thrust / Isp

point of section - understanding expected performance of scramjets

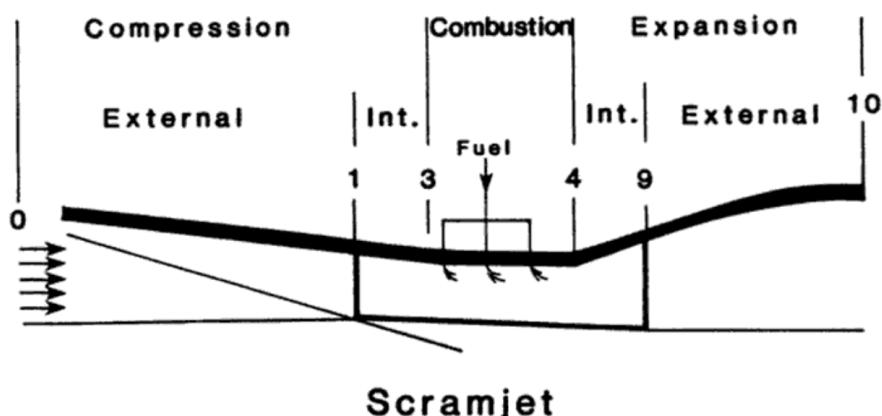


Figure 2.1: scramjet-CITE hypersonic airbreathing propulsion

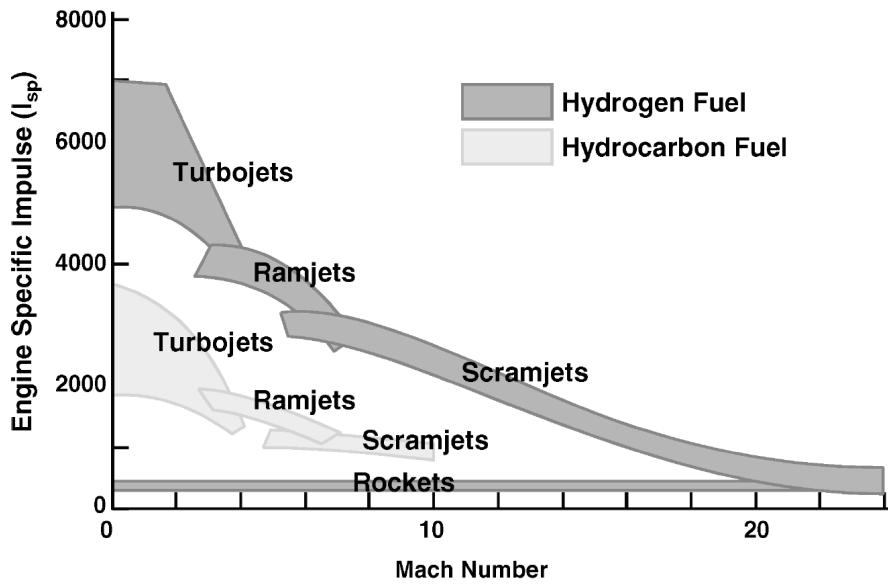


Figure 2.2: Characteristic performance for airbreathing and rocket engines with Mach number[Fry2004].

A Scramjet, or supersonic combustion ramjet, is an airbreathing engine design which combusts air at supersonic speeds and is capable of high Mach number operation. Scramjets compress air without moving parts, using geometry changes within the engine [Curran2001a], as well as on the forebody of the vehicle to create inlet shocks which provide the compression required for combustion[Smart2012]. This is similar in operation to a ramjet engine, however a scramjet does not generate a normal shock, allowing supersonic air to enter the combustor. Maintaining supersonic speeds throughout the engine allows scramjets to operate efficiently at Mach numbers of 5 and greater, with a large operable range. Scramjets were proposed in the 1940's [11] and found to be capable of positive net thrust in 1993 [43] but have yet to be developed to a level which would allow for commercial application.

Scramjets offer much higher specific impulse than rockets over their operating range [6] [10].

2.2 Reusable Satellite Launch Systems

Launch system technologies have progressed rapidly over the last 60 years. From the early vehicles based on intercontinental ballistic missile technology such as the Thor based launch systems, capable of launching 40-400kg to LEO in the 1960s, to the more modern Atlas V based systems of the 2000s capable of launching 9750-18500kg to LEO <http://www.aerospace.org/crosslinkmag/spring-2010/launch-vehicles-then-and-now-50-years-of-evolution/>. The materials, propulsion technology, aerodynamics and guidance algorithms have all improved significantly, enabling rockets to become more efficient, cheaper to produce, and more reliable. As the demand for satellite launches grows, and the cost of development of launchers becomes cheaper, the potential for profiting from space launches

increases. This has driven a large portion of the space flight industry to move towards privatisation, with a heavy focus on reusable technology.

Reusing launch vehicles allows the cost-over-time of the reused components to be reduced drastically, which subsequently allows the cost of individual launches to be reduced. Reduced costs minimises the barrier of entry for space launches, improving the diversity of space-based enterprises. Reusing launch system components also allows faster turnaround times for launches, as refurbishment of stages is much faster than manufacturing stages from scratch. Reduced turnaround times are key for improving mission scheduling, allowing satellites to be launched sooner, on a more flexible time frame.

Reusable launch technology is the focus of many of the largest private launch companies. The SpaceX Falcon 9 and Falcon Heavy have been demonstrated on multiple occasions, landing booster stages successfully, and re-flying reused boosters multiple times. In the near future the Blue Origin New Glenn is planned, with potentially the Airbus Adeline to follow (to be used on the Ariane 6).

For a launch vehicle to be reusable, it must necessarily have the ability to come back to Earth safely, without damage to major system components. This return flight requires the addition of system components which allow the reusable stage to fly to a specified landing point. Control surfaces, structural components, additional fuel, and in the case of the Adeline, additional engines, must be incorporated within a reusable launch vehicle design. The additional weight that these components contribute further increases the fuel and structural mass necessary to initially accelerate the reusable stage. The impact of reusability on the mass and cost of the vehicle is minimised when the stage is larger, and when the velocity at the initiation of the return trajectory is decreased. Because of this mass increase on any stage which is to be designed to be reusable, most current reusable launch vehicle designs include only reusable first stages, with later stages being expendable.

The structural mass fraction of a rocket stage increases as the gross mass decreases. This means that the additional mass necessary for reusability will have a larger effect on a small launch system. Due to this, many of the current developments in small satellite launch systems are focussed on expendable rockets.

2.2.1 Small Satellite Launchers

What point do I want to make in this section?-tie in to other launchers and reusability-point out gaps in technology ie reusable small satellite launchers

Too commercial - reframe in a more scientific way ie advancing human knowledge. cover alternate approaches.-i think focus on reusability-compare capability to similarly sized launchers

The vast improvements in computational technologies in recent years have allowed satellites to decrease in size and cost to a large degree. These factors have lowered the barrier of entry into small satellite manufacturing significantly, driving a surge in the demand for small satellite launches. Many

CHAPTER 2. LITERATURE REVIEW

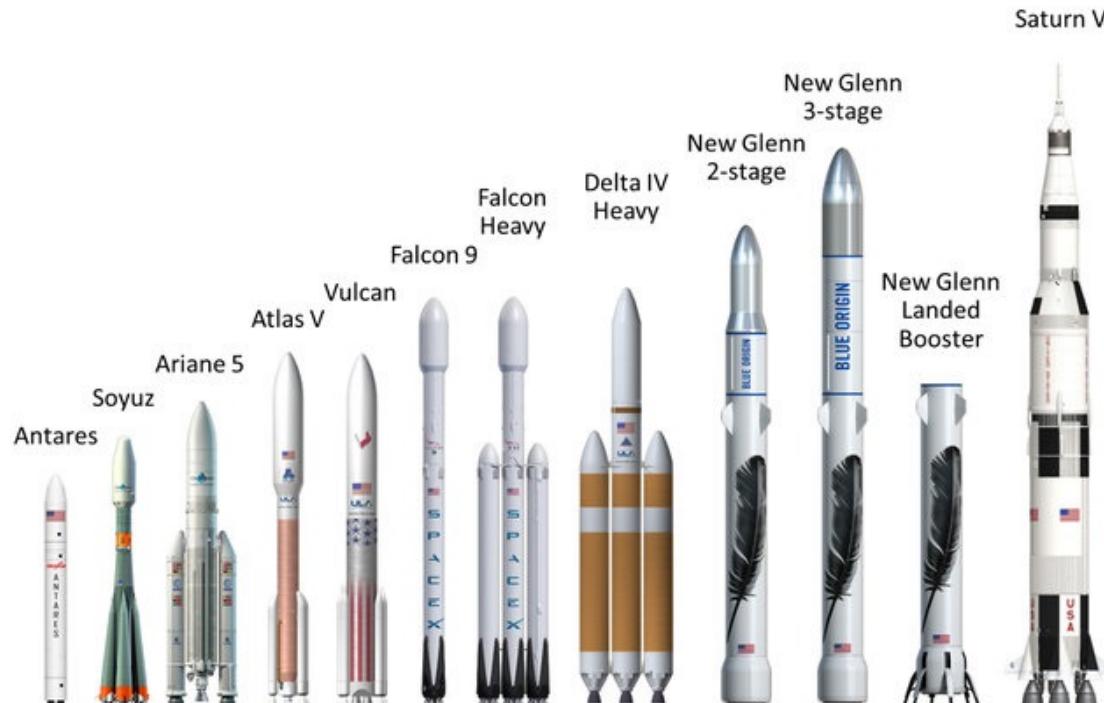


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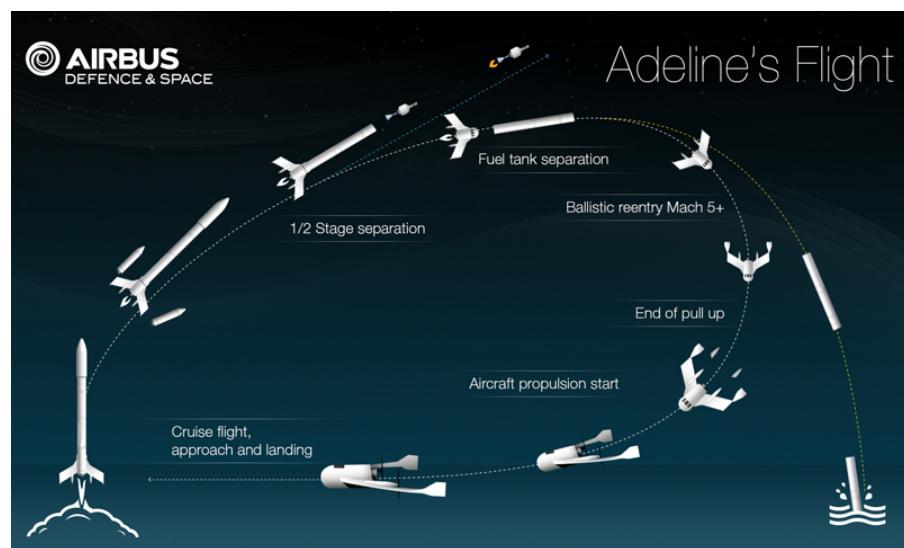


Figure 2.4

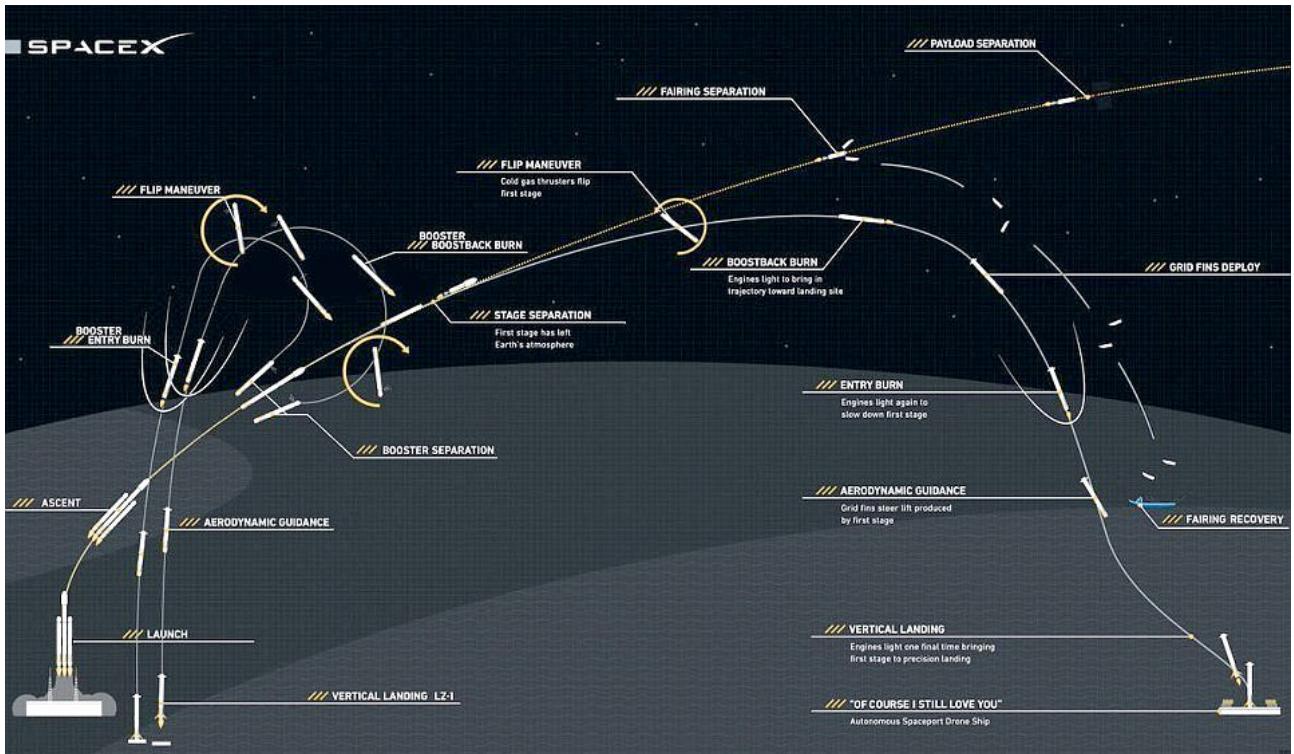


Figure 2.5: <http://www.dailymail.co.uk/sciencetech/article-5361789/Ride-Elon-Musks-Starman-travels-space.html>

private and public companies are currently developing small satellite launchers to tap into this new market. These organisations are currently developing launchers to offer bespoke launches with low costs and relatively rapid turn around times. A summary of the small satellite launchers currently in active development is shown in Table ?? . Many of these launchers are projected to be available within the next few years, and will offer cost-per-kg comparable to piggybacking on larger launches.

The majority of the small satellite launchers in development are non-reusable. Non-reusable small satellite launch systems aim to reduce costs by creating a launch system which is as cost-efficient as possible to produce and launch. This generally entails making use of conventional, well-tested designs, combined with state of the art manufacturing techniques, such as 3-D printing. This method allows for rapid development, although it has an intrinsic cost limit due to the requirement of manufacturing a new launch system for each launch. Reusable small satellite launchers have far higher initial development costs, but also have the potential for large advantages in the long term. Reusable small satellite launchers have the potential for lower cost-per-launch than expendable systems, with increased launch flexibility. However, for small satellite systems, novel approaches are required for reusability. It is notable that the launchers with re-use capabilities are entirely within the early stages of development, with no currently projected completion dates.

Much of the market niche fulfilled by each launch system is determined by the payload to orbit capabilities of the vehicle. Although the launchers shown are all classed as 'small satellite launchers',

CHAPTER 2. LITERATURE REVIEW

organisations seeking a launch service will choose a launcher which is designed to launch payloads of comparable size to their desired payload, for maximum cost efficiency. There is a large variation in the payload-to-orbit capabilities of the small satellite launchers in development, ranging between 50kg to 500kg. The rocket-scramjet-rocket system studied in this thesis will target approximately 150kg of payload to orbit, most similar to the Vector-H[71], Haas2CA[3], Bloostar[74] and Electron[51].

The majority of the small satellite launch systems have projected launch costs between \$20000-\$40000USD/kg. The outlier is the Haas2CA[3], developed by ARCA, which has the ambitious target of \$10000USD/kg. The Haas2CA is a single stage to orbit launcher, powered by a hydrogen peroxide/kerosene aerospike engine.

Launcher	Company	Country	Payload Capacity	Cost/Kg (USD)	Availability	Stages & Propulsion	Reusability
Electron [51]	RocketLab	NZ/USA	150Kg to SSO	\$32,600	Available	Rocket-rocket	No
Lynx [1]	XCOR	USA	-	-	-	rocket-rocket	First stage
LauncherOne [72]	Virgin Orbit	UK	300kg to SSO	\$33,000	-	Aircraft-rocket-rocket	Aircraft
Bloostar [74]	Zero2Infinity	Spain	100kg to SSO	\$40,000	-	Balloon-rocket-rocket-rocket	No
XS-1 [42]	Boeing	USA	-	-	-	-	First Stage
Eris [21]	Gilmour Space Technologies	Aus/SG	380kg to LEO	\$23,000-38,000	Q4 2020	-	No
Black Arrow 2 [25]	Horizon	UK	350kg to SSO	-	2019	Rocket-rocket	No
Haas 2CA [3]	ARCA	USA	100Kg to LEO	\$10,000	2018	Rocket	No
Intrepid-1 [52]	Rocket Crafters	USA	376kg to SSO	\$23,936	Q1 2019	Rocket-rocket	No
KZ-1A [31]	CASIC	China	250kg to SSO	-	-	Rocket-rocket	No
500R [41]	Orbital Access	UK	500kg to SSO	-	-	Aircraft-Rocket	Fully Reusable
Vector-H [71]	Vector Space Systems	USA	160kg to LEO	\$21,875	2018	rocket-rocket-(third rocket optional)	No
SMILE [32]	NLR	EU	50kg	<\$50,000	-	-	-

2.2.2 Airbreathing Access to Space Systems

-need to generalise this section and add more

The possibility of using airbreathing stages in access to space systems has been studied in some detail. These systems have been investigated in various forms including; single stage[45, 73, 70], dual stage [68][37] and tri stage [48] designs. The most prominent of the airbreathing access to space systems currently in development is the SKYLON Spaceplane being developed by Reaction Engines Limited [69]. The SKYLON is a spaceplane which utilises SABRE, a combined cycle airbreathing rocket propulsion system potentially reusable for 200 flights, and is being developed to deliver payloads on the order of 2800kg to heliosynchronous orbit [24].

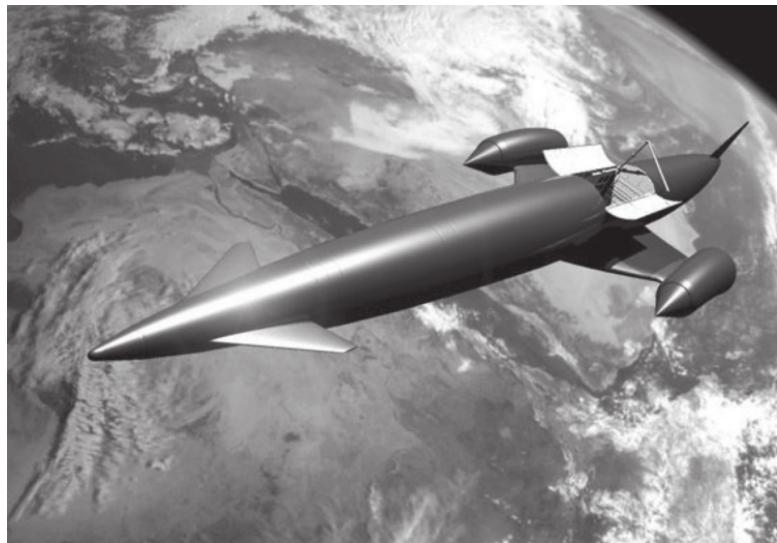


Figure 2.6: The SKYLON spaceplane [70].

A single stage design has the advantage of being fully contained within one vehicle, which is convenient for reusability and return trajectories however it has been suggested by Smart & Tetlow [59] that these designs suffer from severe limitations as they must contain multiple engines which add mass at later stages of the trajectory and decrease the efficiency of the vehicle. Smart & Tetlow suggest that multistage systems offer significant improvements in payload mass fractions, and have the advantage of using airbreathing stages only within their operable range. Dual stage designs have been investigated in some detail using the 'spaceplane' concept by Mehta & Bowles [37] using life cycle cost analysis in order to take flexibility and reusability into account. Mehta & Bowles conclude that a two stage design is the optimal configuration for reusable hypersonic space access systems, however this study is only based on comparison with single stage to orbit systems, and it is more useful to consider their conclusions as an endorsement of multi stage airbreathing designs in general. They find that multi stage vehicles have higher potential for payload than single stage to orbit (SSTO) systems and have less propellant requirements, partly due to a greater atmospheric cruise capability.

make the point that all of the proposed launchers are large, but that scramjets are easily scalable and efficient at small scales, connect with the next section

2.2.3 Small Airbreathing Launchers

Figure 2.7 shows the operating corridor for scramjet engines, indicating the point at which transition to rocket stage would occur, the lower dynamic pressure limit on engine operation and the upper dynamic pressure limit on the aircraft structure.

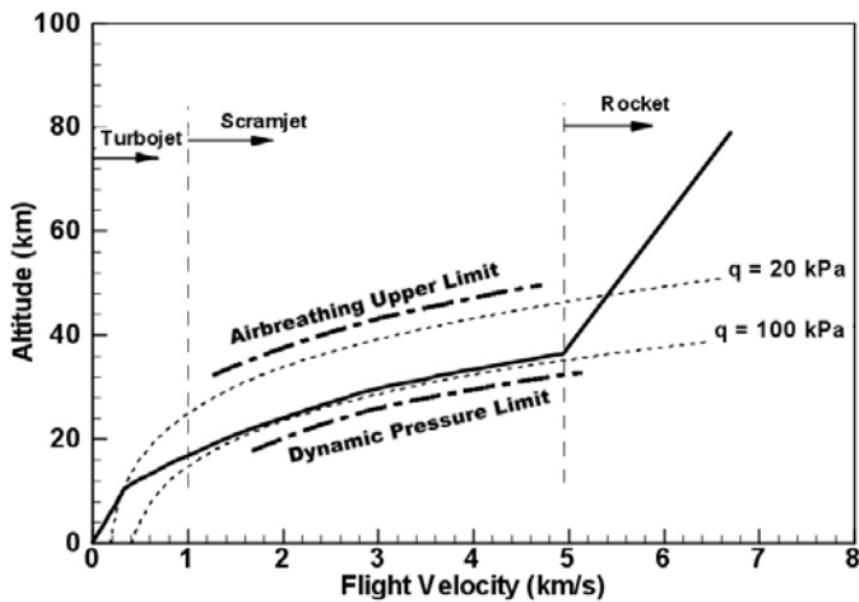


Figure 2.7: The airbreathing vehicle flight corridor [57].

Smart & Tetlow [59] have found that the fuel savings achieved by using a scramjet engine within these limits, coupled with small necessary payload mass, may enable the development of a partially or fully reusable space access system utilising a scramjet powered stage in the near future. Simulations carried out for three stage systems utilising scramjet and rocket engines for small payload delivery show favourable payload mass fractions with a reusable scramjet stage [59]. Scramjet powered vehicles may also offer the ability to put small payloads into orbit with greatly increased flexibility and launch window when compared to similarly sized rocket systems. This has been assessed in a study by Flaherty [18] comparing the United States Air Force's Reusable Military Launch System all-rocket launch vehicle RMLS102 against the Alliant Techsystems rocket/scramjet launch system ATK-RBCC. These vehicles are similarly sized and comparisons were made for payloads launched to rendezvous with satellites in randomly generated orbits. These vehicles were compared using the range of orbital trajectories that each vehicle was able to rendezvous with within one day, determined by launch vehicle range. The rocket/scramjet ATK-RBCC was found to have a large advantage in trajectory flexibility over the rocket only vehicle, in a large part due to the scramjets ability to fly

fuel efficiently over long distances. **put in some quantitative data abot this study** This means in general that a partially scramjet powered accelerator is able to fulfil the specific delivery needs of small payloads over a wider range of orbits within smaller time periods when compared to a fully rocket powered accelerator. This can be advantageous for time critical and orbit dependant payloads which have specific mission requirements to be met.

2.3 The SPARTAN

reorder until after trajectory of 1,2,3 stage vehicles

The three stage, partially reusable, access to space system under development at The University of Queensland utilises the SPARTAN[26] scramjet powered vehicle as the reusable second stage, shown in Figure 2.8. This system is considered in this study as a representative model for three stage, air-breathing access to space system designs. This launch system is designed for small payload deliveries to orbit and will in the future utilise a fly-back rocket booster to accelerate the SPARTAN stage to minimum Mach number required for stable burn, at which point separation occurs and the second stage uses a scramjet engine to accelerate to between approximately Mach 5-9. The first and second stages are to be reusable, the first stage via conversion into a propeller powered drone, and the second stage through either a glide or extra scramjet powered flight to a suitable landing site. **However, the first stage booster considered in this study is a simple disposable design. The ascent profile is of interest to this study, however development of the fly-back trajectory profile requires detailed first stage vehicle design which is outside of the scope of this study. DONT MIX LIT REVIEW AND STUDY**

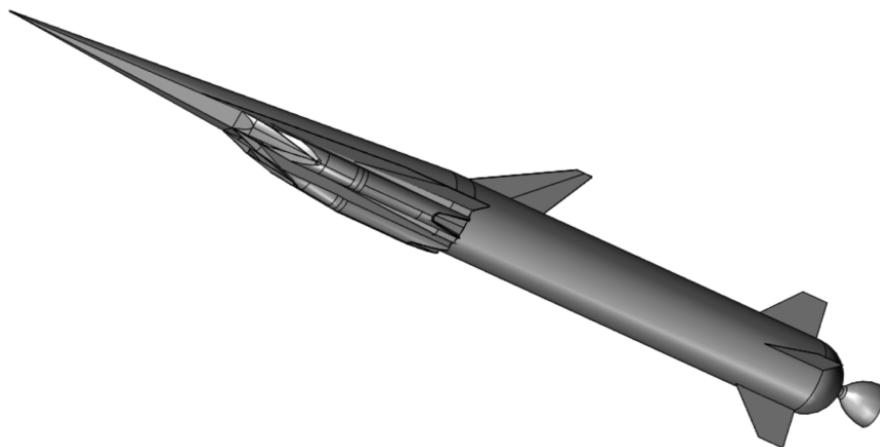


Figure 2.8: An early design of the socket-scramjet-rocket launch system incorporating the SPARTAN [26].

The third stage will be a disposable rocket stage, which will then deliver the payload to orbit, exiting the atmosphere and performing a Hohmann transfer.

This system is in the preliminary design stage, with estimates of payload mass fraction indicating that 1.26% PMF is possible when delivering a 257.4kg payload to low earth orbit [47]. This compares well with existing space systems of similar size, with the advantage of being designed for reusability. The second stage vehicle has been designed to a detailed level, and optimised for payload delivery to heliosynchronous orbit.

The SPARTAN has been studied in some detail, simulated over a constant dynamic pressure trajectory with variation in the aerodynamic surface properties of the vehicle.

2.3.1 Scramjet Engine Model

Much more info, how is off design performance calculated, what model is used.

To deliver a payload to orbit, the SPARTAN uses four Rectangular-to-Elliptical Shape Transition (REST) scramjet engines, with inlets configured to allow installation on a conical forebody. This study uses engines configured to fly between Mach 5 and 10, this type of engine is known as a C-RESTM10 engine[48]. The REST model has been studied experimentally for flight at off design conditions by Smart & Ruf [58]. The REST engine has been used as it has been proven to operate successfully at off-design conditions, an extremely important property for this study as the scramjet stage experiences a range of Mach number flight conditions, and significant variation in dynamic pressure.

A propulsion database of the C-RESTM10 has been provided for this study by Prof. Michael Smart.

2.4 First Stage Rocket Booster

review engine options and modelling/scaling techniques

The SPARTAN scramjet vehicle must be accelerated to its minimum operating speed by a first stage rocket booster. This booster must be capable of efficient operation from launch at sea level until staging. It is envisioned that the SPARTAN system will eventually use a reusable flyback booster[48]. However the flyback of the first stage is outside the scope of this study. Instead, a model of an existing first stage booster will be used, for which the Falcon-1e was chosen.

The SpaceX Falcon-1e launch system was a proposed small satellite launch system which would have entered use after 2010[61], but was discontinued and superseded by the Falcon-9 project. The Falcon-1e was designed to launch payloads between 600-900kg to low Earth orbit, with flexible orbit altitude and inclination. The Falcon-1e was to be the next generation of the Falcon-1[61], a launch tested, modern launch system.

The Falcon-1e was chosen for its appropriate size and the proven flight effectiveness of the Falcon-1. The first stage of the Falcon-1e is powered by a Merlin 1C LOX/Kerosene turbopump engine. The

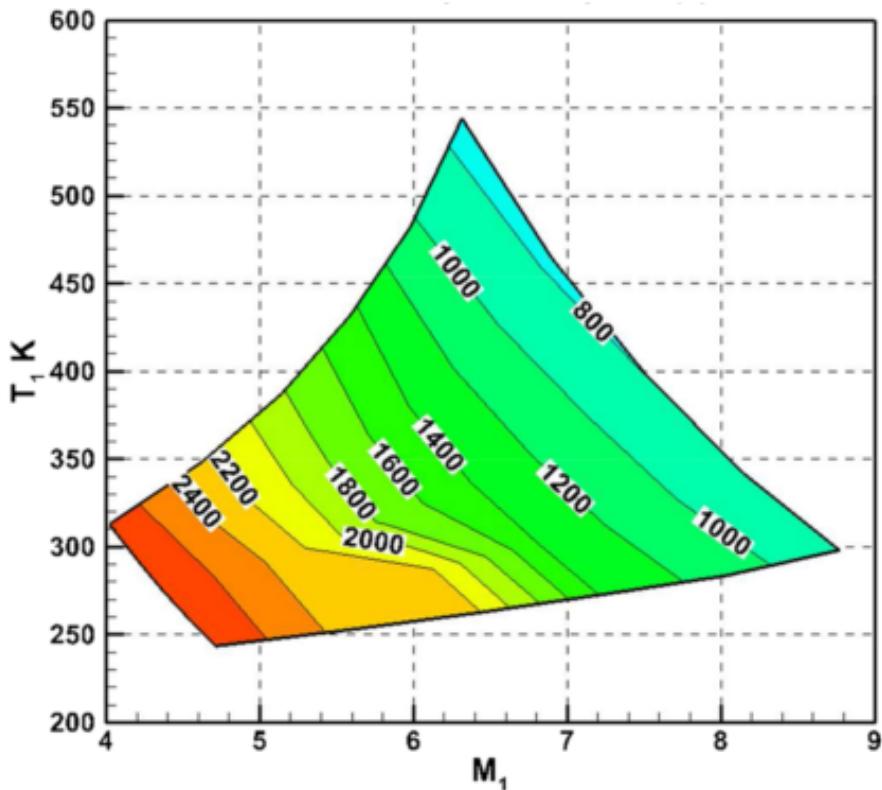


Figure 2.9: The C-RESTM10 propulsion database, specific impulse.

first stage rocket is designed to be a Falcon-1e first stage scaled lengthwise to 50% of its original length. REMOVE THIS - do not describe design in lit review

2.5 Exoatmospheric Rocket Engines

expand this to include control of third stage, identify losses/penalties

After scramjet stage burnout, the upper stage rocket must exit the atmosphere, and accelerate the payload to orbital velocity. This requires a rocket engine with sufficient thrust to accelerate the third stage rocket out of the atmosphere, and a diameter small enough to allow the rocket to fit within the fuselage of the SPARTAN. Table 2.1 shows a comparison study of upper stage rocket engines.

Engine	Fuel Supply	Fuel	Thrust	Isp	Mass	Diameter	Length	Thrust Vector Capability
Aestus	Pressure-fed	MMH/NTO	27.5kN	320s	110kg	1.27m	2.2m	4° & 4° by mechanical adjustment
OMS	Pressure-fed	MMH/NTO	26.7kN	316s	118kg	1.168m	1.956m	8°
Aestus II	Pump-fed	MMH/NTO	46kN	337.5s	148		2.2m	6°
RS-72	Pump-fed	MMH/NTO	55.4kN	338s	154kg		2.286	6°
ATE	Pump-fed	MMH/NTO	20kN	345s	57.9kg	0.38m	1.4m	15°
AJ10-118K	Pressure-fed	A-50/NTO	43.3kN	320.5s	124.5kg	1.53m	2.7m	Fixed
Kestrel	Pressure-fed	LOX/Kerosene	30.7kN	317	52kg	1.1m	1.9m	Yes, Unknown limits
R1-10-3A								

Table 2.1: Comparison of upper stage rocket engines.

2.6 Partially-Airbreathing Launch Vehicle Ascent Trajectories

discuss trim - investigated by Dawid

Current simulations of the SPARTAN vehicle have been carried out with the assumption of a 50kpa dynamic pressure trajectory, a likely design point of the vehicle and scramjet engine. Constant dynamic pressure trajectories have been used for airbreathing vehicle simulation due to the trade-off between structural loading and engine performance for hypersonic vehicles [40]. As dynamic pressure increases so does the structural loading on the vehicle, however scramjet thrust is directly reliant on dynamic pressure ie. an increase in dynamic pressure directly means more air into the engine inlet. A constant dynamic pressure is viewed as being an acceptable compromise between these two factors. This form of trajectory has so far been simulated using proportional integral derivative (PID) feedback control, directly investigated since 1998 by Olds & Budianto [40]. This is a simple and effective form of control for systems being simulated over a constant dynamic pressure path [46]. This form of control utilises minimises an error function as the vehicle moves along its trajectory. This error function causes the controls of the vehicle to be modified by a feedback term which is a function of the error including proportional, integral and derivative terms. The result is a trajectory which is suitably close to the objective design point, with minimal overshoot and steady state error which can cause oscillations around the specified design value. Figure 2.10 shows an example of a constant gain PID controller as applied to the SPARTAN vehicle being simulated to a design point of 50kPa dynamic pressure. Oscillations can be observed around the design point due to overshoot and steady state error, factors which are prevalent in PID control systems.

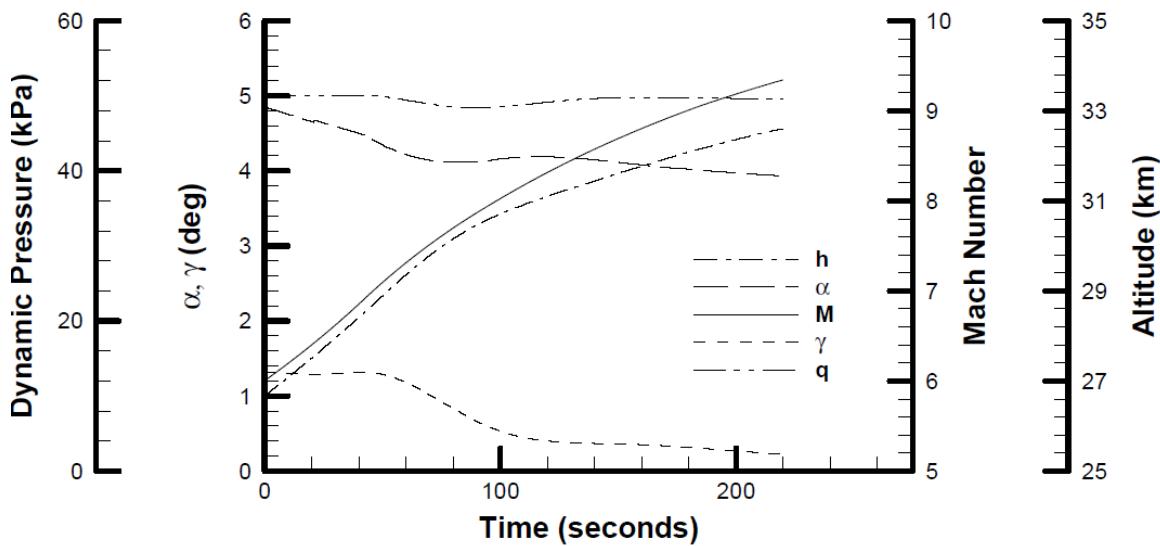


Figure 2.10: An example of a constant gain PID controller, controlling the SPARTAN trajectory around a 50kPa dynamic pressure design point [46].

It is possible that a constant dynamic pressure trajectory may not be the most appropriate trajectory

for the second stage vehicle, as there is a variety of aerodynamic factors that must be considered in designing optimal payload to orbit flight. It may be optimal for the vehicle to fly at less than maximum dynamic pressure at point throughout the trajectory, most likely at the first to second and second to third stage separation points. For a constant dynamic pressure trajectory the second-third stage transition will occur at a very low trajectory angle. This is suboptimal for the third stage rocket, which needs to perform a large pull up manoeuvre at high dynamic pressure. Launching this system at higher angle or altitude may improve third stage fuel usage greatly, at a small cost to the second stage fuel, allowing a greater payload to be delivered to orbit.

2.6.1 Single-Stage Vehicles

Optimal trajectories have previously been developed for launch systems integrating airbreathing and rocket propulsion within single-stage-to-orbit (SSTO) vehicles [45, 34, 67]. These optimal trajectory studies found unanimously that a pull-up manoeuvre before the end of the airbreathing engine cut-off was the optimal flight path for the SSTO airbreathing-rocket vehicles being investigated. A pull-up was found to be optimal for vehicles where the rocket engines are not ignited until circularization altitude [45, 34] as well as vehicles where the rocket engine is ignited immediately after airbreathing engine cut-off [67]. For SSTO vehicles a pull-up manoeuvre is a simple trade-off between the altitude at airbreathing engine cut-off and the velocity achievable at cut-off. Due to the entire vehicle being lifted into orbit, this becomes a relatively simple problem of engine efficiency. The airbreathing engine is used for its high efficiency, until the dynamic pressure drops below the operable limit of the airbreathing engine, or until the thrust provided by the airbreathing engine is significantly counteracted by the effects of drag and gravity.

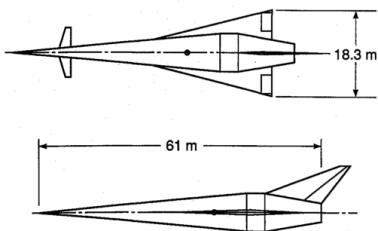


Figure 2.11

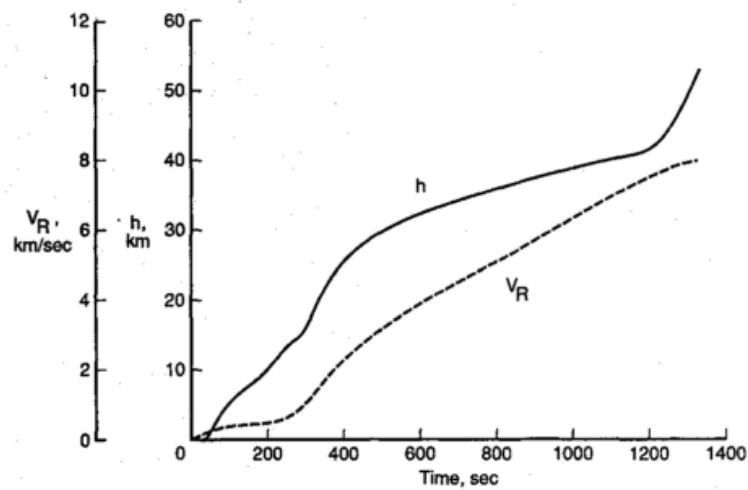


Figure 2.12: Powell

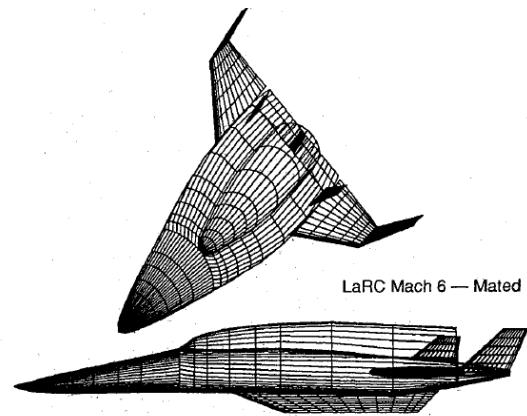


Figure 2.13

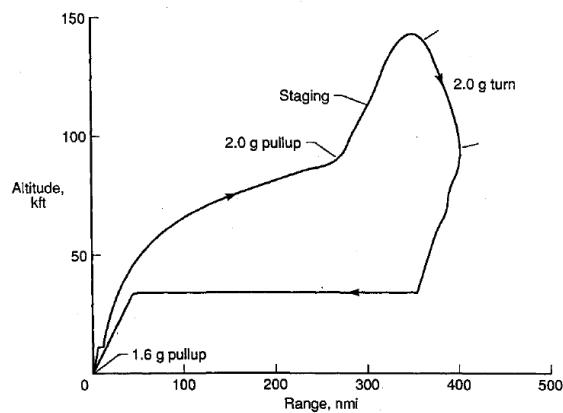


Figure 2.14: Wilhite

2.6.2 Multi-Stage Vehicles

For a multi-stage to orbit vehicle, calculating the optimal trajectory for maximum payload flight is significantly more difficult. A multi-stage vehicle has one or more stage transition points, where the vehicle separates a component which is discarded or reused later, and does not continue to orbit. At a stage transition point there is a large change in the mass and aerodynamics of the launch system. This change in flight dynamics makes finding the optimal stage transition point more complicated. To find the optimal separation point there is a trade-off between: I. The high efficiency of the scramjet engines, II. The thrust produced by the scramjet engines, III. The potential thrust of the rocket engines, IV. The energy necessary to increase the altitude of the scramjet stage, V. The aerodynamic efficiency when performing the required direction change. All of these factors must be considered in order to generate an optimal trajectory.

There has been a number of studies which have identified a pull-up manoeuvre as being advantageous for a multi-stage system [68, 73, 37]. However, in these studies a pull-up manoeuvre has been specified in order to decrease the dynamic pressure of the vehicle at airbreathing-rocket stage separation.

In the studies by Tsuchiya et al.[68] and Wilhite et al.[73], decreased dynamic pressure is necessary for the successful operation of the orbital rocket stages, of the systems under investigation. In these studies the airbreathing stages pull-up to the maximum allowable dynamic pressure for the rocket-powered orbital stages. When the orbital stages are able to operate, stage separation occurs. These pull-up manoeuvres demonstrate the advantages of a pull-up for the operation of the orbital stages, allowing the aerodynamic and thermal loading on the vehicle to be reduced. However these pull-up manoeuvres are not performed as part of optimal trajectories, instead they are designed to ensure that the performance constraints of the systems are met.

Mehta & Bowles [37] prescribe a 2g pull-up at flight conditions of Mach 10, 95000 ft for an airbreathing stage in order to "lower dynamic pressures and to achieve the optimal launching flight

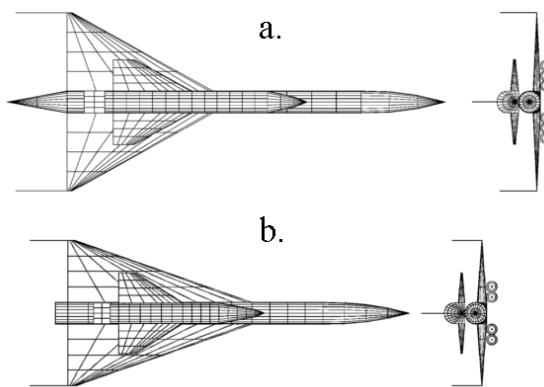


Figure 2.15: a) Airbreathing b) Airbreath- Rocket

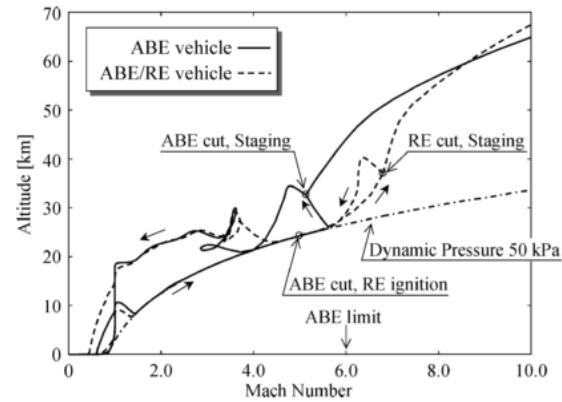


Figure 2.16: The trajectory of the launch system developed by Tsuchiya and Mori [68]



Figure 2.17

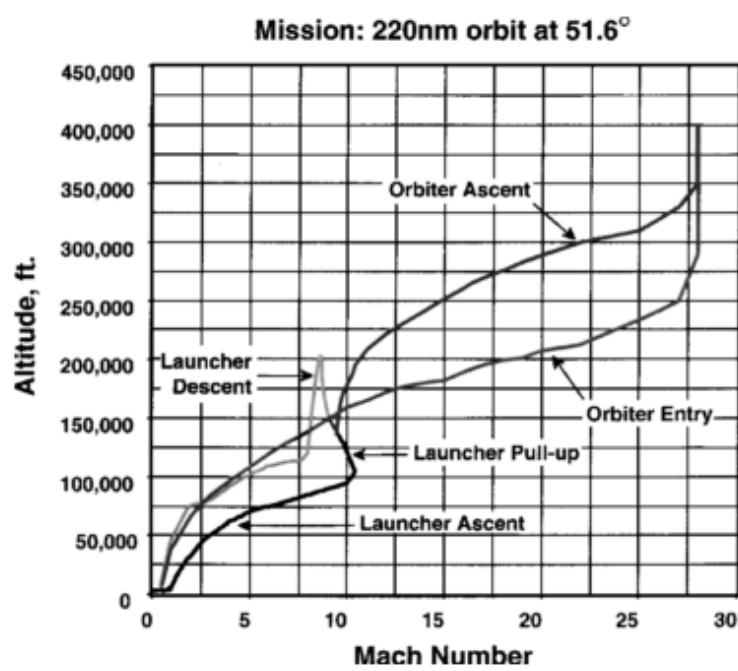


Figure 2.18: The trajectory of the launch system developed by Mehta and Bowles [37].

path angle for the orbiter vehicle”. This indicates that a pull-up manoeuvre before airbreathing-rocket transition is considered the optimal trajectory, however this study does not optimise the shape or magnitude of the pull-up manoeuvre, only considering the increased performance of the rocket vehicle.

2.7 Hypersonic Vehicle Flyback Trajectories

Past studies of the SPARTAN vehicle have assumed that a fly-back to launch site is possible after third stage separation[48]. However, this fly-back has not yet been simulated. A simulation of the fly-back of the SPARTAN is necessary in order to assess the viability of the SPARTAN concept. The fly-back of the SPARTAN is a crucial component to the viability of the rocket-scramjet-rocket launch system. The SPARTAN must return to the initial launch site to enable refurbishment to begin immediately, and so that there is no significant transportation of the SPARTAN required between launches. Due to the second-third stage staging velocity, it is complicated for the SPARTAN to cover the necessary fly-back distance. To maximise fuel efficiency, it is desirable for the SPARTAN to perform a minimum-fuel fly-back to the initial launch site, with the best-case scenario being for the fly-back to use no fuel at all.

There are three main methods that have been studied for potential hypersonic vehicle return; glide-back, cruise-back and boost-back. Glide-back involves the hypersonic vehicle returning to base and landing entirely using its aerodynamics. This requires sufficient lift to sustain the hypersonic vehicle over the entire return range, as well as the controllability to land the hypersonic vehicle in level flight. For a hypersonic trajectory a fully glide-back return flight is most likely unobtainable. This is due to the large downrange distance flown, and the large initial velocity at the beginning of the fly-back trajectory, when the vehicle is oriented away from the landing site. Multiple studies have investigated the maximum staging velocity allowable for the glide-back flight of a booster. In these studies, the maximum separation velocity for glide-back to be feasible has been found to be between Mach 3-4 at 30km-120km downrange distance, with higher initial velocities or longer downrange distances requiring fly-back under power[23, 65].

Cruise-back involves the inclusion of subsonic engines, which are used to power the fly-back of the hypersonic vehicle until landing similar to a conventional aircraft. These engines may be included solely for the fly-back[23], or used in the acceleration phase for low velocity acceleration[37, 65, 73]. The addition of subsonic engines powering a constant velocity cruise-back phase allows the accelerator to return to base with a similar trajectory to that of traditional aircraft, allowing the velocity and altitude of the accelerator to be precisely controlled. However, the addition of subsonic engines necessary for cruise-back increases the mass of the vehicle significantly, leading to decreased mass efficiency and increased design complexity[23].

A preferable mode of powered fly-back is to use the existing hypersonic airbreathing engines dur-

ing the return trajectory in a boost-back trajectory. Using the existing airbreathing engines allows for range to be added to the return trajectory, without the inclusion of additional engines. The hypersonic airbreathing engines can be operated at appropriate times during the fly-back, when they will be most impactful on the return trajectory range. However, the hypersonic airbreathing engines may only be used within their operating region, and vary in thrust and efficiency dependent on flight conditions. Hypersonic airbreathing engines have maximum efficiency at low Mach numbers[48], with the thrust produced depending on the dynamic pressure and inlet conditions, which vary with the trajectory path and angle of attack of the vehicle. This added complexity requires the use of trajectory optimisation methods to find the most efficient flight path for return to the launch site, and to ensure that the return of the vehicle under its own power is viable.

2.7.1 Examples of Optimised Fly-Back Trajectories

Tetlow et al. [65] compare powered cruise-back to glide-back flight for the return of the first stage booster of a rocket powered two-stage launch vehicle. The powered flyback vehicle uses airbreathing engines to cruise back to the launch site at relatively low speeds. The powered flyback and the glide-back cases are analysed separately to determine the difference in the optimal staging velocities. Figure 2 shows the optimised deceleration phase for the cruise-back vehicle, terminating at cruise-back conditions. This trajectory shows three distinct skips, with the minimum trajectory angle progressively decreasing between each. Tetlow et al. conclude that the 'skips' are due to the rate of density increase being greater than the deceleration rate 'causing increased lift and levelling out of the altitude profile'[65]. This trajectory was optimised for minimum cruise-back fuel mass, indicating that the 'skips' are the optimal trajectory shape to maximise range in high initial velocity fly-back. The powered cruise-back vehicle is able to stage at 3000m/s due to the ability to cover a large distance during cruise-back. The glide-back vehicle was optimised to find the maximum possible staging velocity at which a turn and glide-back is still possible. This was found to be at 1200m/. Figure 3 shows the corresponding optimal deceleration trajectory, terminating at sustainable glide conditions. This trajectory again shows multiple skips. The low allowable separation velocity of 1200m/s for the glide-back booster potentially suggests that the SPARTAN will require powered flyback. Tetlow et al. assume an admittedly optimistic L/D of 7 at Mach 0.9, and maximum angle of attack of 40 deg. However, the mass of the vehicle in this study is up to 1182 tons. A lower L/D may be acceptable for a lighter vehicle.

A maximum staging velocity at which glide-back is possible is also investigated by Hellman [23]. Hellman uses a suboptimal, scheduled trajectory simulation to investigate the flyback capabilities of a reusable rocket booster. Hellman finds that Mach 3 is the upper staging limit for a glide-back booster. This agrees with Tetlow et al. that high staging velocities are not possible for glide-back boosters, suggesting that the SPARTAN will require powered return flight.

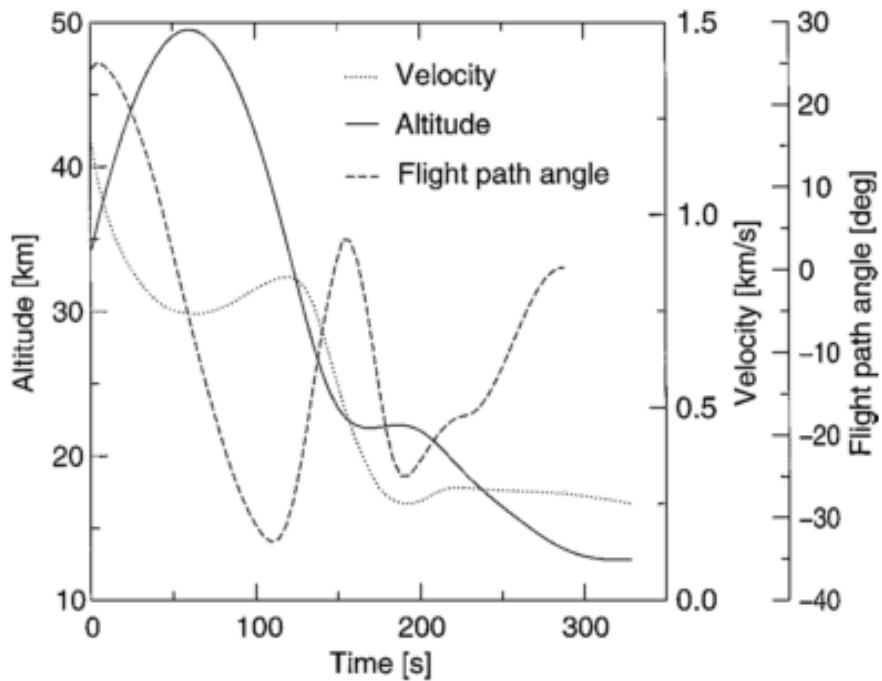


Figure 2.19: The flight path of a glide-back booster, developed by Tetlow et al. CITEXX.

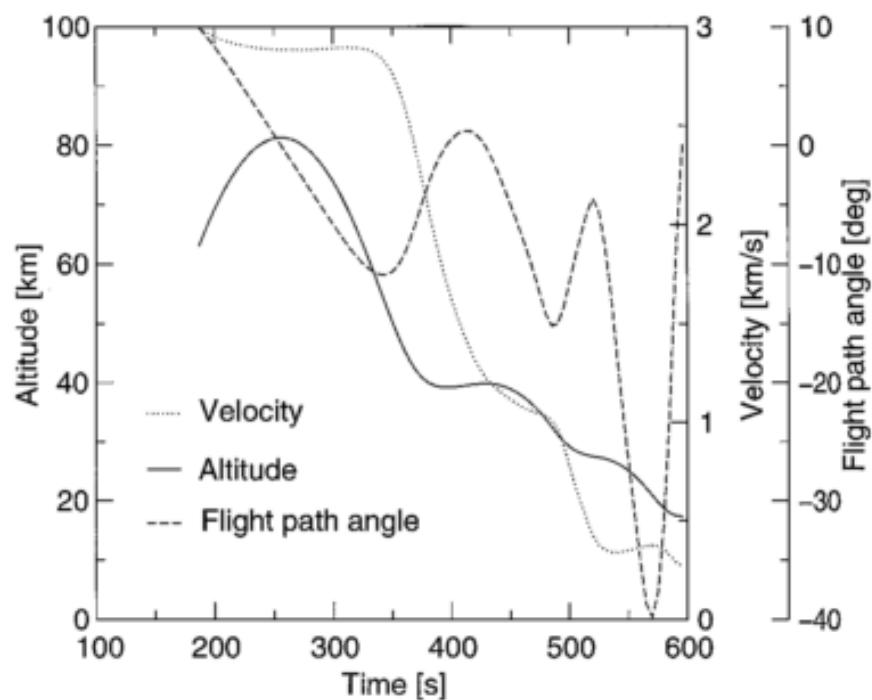


Figure 2.20: The flight path of a powered fly-back booster, developed by Tetlow et al. CITEXX.

The possibility of an airbreathing vehicle reigniting high speed airbreathing engines for short periods has been investigated by Tsuchiya and Mori [68]. Tsuchiya and Mori investigate two conceptual launch vehicles; a vehicle powered solely by airbreathing propulsion returning after separation of an orbital stage at Mach 5.1, and an airbreathing/rocket vehicle returning after a separation at Mach 6.8[68]. For the purposes of fly-back these vehicles are very similar, both using the high speed airbreathing engines during return flight. The optimal launch and return trajectories for these vehicles are shown in Figure 2.16. The separation for the airbreathing vehicle occurs at Mach 5.1, and the separation for the airbreathing/rocket vehicle at Mach 6.8. Both optimised return trajectories follow a relatively similar path. A long skip is performed, with both vehicles hitting the dynamic pressure limit. Both vehicles ignite the airbreathing engines at around Mach 3.5 for several tens of seconds to extend the range of the fly-back manoeuvres. After this, the vehicles descend and land at the launch site. The velocity at the landing point is not constrained, causing the airbreathing vehicle to land at slightly under Mach 1, and the airbreathing-rocket vehicle to land at approximately Mach 0.5. The latter is comparable with the landing velocity of the space shuttle [53]. These boosters fly to a downrange distance of 600-625km from the launch site, and separate from the orbital accelerator at a dynamic pressure of 15kPa[68]. At the start of their respective return trajectories, both boosters turn with a bank angle of 130-145°. Both the fully-airbreathing and partially-airbreathing vehicles ignite their airbreathing engines for 'several tens of seconds' at approximately Mach 3.5, in order to extend the flight range of the vehicles and return to the initial launch site[68]. Less than 5% of the vehicles initial propellant was required to return the vehicles to the initial launch sites[68].

These optimised return trajectories show three distinct sections; a turn manoeuvre, a skipping phase, and a high L/D descent. The skipping phase is similar to the boost-skip trajectory of the space shuttle CITEXX, which uses repeated entries into the atmosphere, and skips out, to extend the range of an atmospheric entry. A skipping trajectory has been shown to be range optimal for hypersonic vehicles able to skip out of the atmosphere [39], as well as vehicles flying entirely within the atmosphere[39, 12]. A skipping trajectory has also been shown to be optimal for an air-breathing hypersonic vehicle thrusting throughout the trajectory[9]. The range optimal operation of the air breathing engine is shown to be repeated ignitions at the trough of each skip. The scramjets are ignited as the vehicle climbs after the through, as the Mach number decreases to the minimum operable conditions of the scramjet engines. Minimising the Mach number during operation in this way maximises the efficiency of the scramjet engines.

When compared to the vehicles investigated by Tsuchiya and Mori[68], the SPARTAN separates from its third stage rocket at a considerably higher Mach number of Mach 9.1, as well as at a considerably higher dynamic pressure of 33.9kPa. The SPARTAN must also cover a longer fly-back range of 878km. In addition, the C-REST scramjet engines are limited to operating at hypersonic speeds of Mach 5.1 or higher. These design differences create substantially more challenging conditions than those studied by Tsuchiya and Mori. Consequently, it is necessary to investigate the ability of the

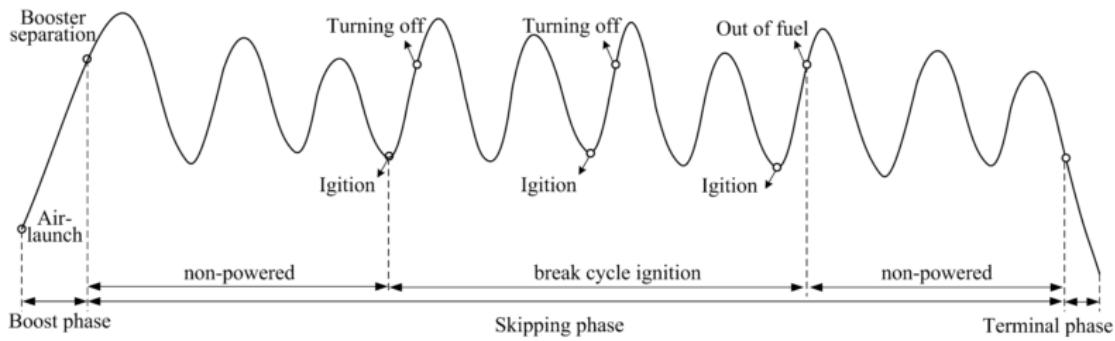


Figure 2.21

SPARTAN to return to the launch site following separation. From comparisons of the studies within this section it is evident that an optimisation of the return flight of the SPARTAN is required, and that a simply defined trajectory is not appropriate. TALKIGN ABOUT MY WORK

2.8 Optimal Control

KEEP THIS SECTION AN BROAD AS POSSIBLE - describe all alternatives

be more generic here To this point analysis of the SPARTAN trajectory has been performed using PID feedback control, assuming a constant dynamic pressure trajectory [46]. PID feedback control simulates the vehicle flight and manoeuvres the vehicle so that the trajectory conforms to a defined design point. Currently the trajectory of the SPARTAN has been simulated to align as closely as possible with 50kPa flight, a problem for which PID control is well suited.

more concise This study aims to find the optimal trajectory path for the SPARTAN that will produce maximum payload to orbit capability for the three stage system. Feedback control is infeasible for finding an optimal trajectory shape as there is no single design point at each stage of the simulated trajectory to calibrate towards. A control method is required that can take into account all aerodynamic factors at every timestep in order to find an optimal trajectory. Defining the trajectory simulation as an optimisation problem allows the trajectory to be solved at every point simultaneously, producing an optimal trajectory over the entire simulation space. Optimal control theory is widely used in situations where an optimal trajectory path must be found, and has been used widely in aerospace applications. However a trajectory optimisation has not been attempted on a three stage airbreathing system, and is usually reserved for simpler systems as there can be issues with convergence and long computation times [13]. This does not rule out using an optimisation method on a complex aerodynamic system however it does reinforce that choosing the correct optimisation procedure and defining the problem in the correct way is integral to developing an accurate solution method. This study aims to find a method of optimisation that is applicable to a complex aerodynamic trajectory and to apply this to the SPARTAN vehicle to produce an optimal trajectory shape for

maximum payload delivery to heliocentric orbit.

For an optimisation of a complex trajectory there are a variety of optimal control methods that are useful for specific problem types. These are separated into two categories: direct and indirect solution methods. Indirect methods are based on the calculus of variations or minimum principle model, and generally result in high accuracy solutions to optimisation problems [8]. However indirect models suffer from the drawbacks of small radii of convergence and the fact that the equations to be solved often exhibit strong nonlinearity and discontinuities. This means that indirect methods will not be solvable unless the problem is very well defined with a minimum of nonlinearity, making indirect methods unsuitable for many complex optimisation problems, such as aerospace vehicle simulations which can exhibit strong nonlinear behaviour and have a wide solution space.

Direct methods transform an optimisation problem into a nonlinear programming (NLP) problem which can be solved computationally [62]. NLP solvers solve the optimisation problem defined as [4]:

$$\text{Minimise} \quad f(x) \quad (2.1)$$

$$\text{Subject to} \quad g_i(x) \leq 0 \quad \text{for } i = 1, \dots, m \quad (2.2)$$

$$\text{and} \quad h_j(x) = 0 \quad \text{for } j = 1, \dots, n \quad (2.3)$$

An optimisation problem that has been discretised in this form can thus be solved using any of a variety of NLP solvers. One of the most effective methods of solving twice differentiable NLP problems is sequential quadratic programming (SQP) [7] for which there is a variety of commercial solvers available such as NPSOL, SNOPT and packages within MATLAB.

more concise In order for these packages to be able to solve an optimisation problem it must be presented in discretised form, and as such must be transformed using approximation techniques. The task of approximating a continuous optimisation problem in discrete NLP solvable form is not simple. SQP solvers can very easily run into convergence issues when provided with an optimisation problem which has not been well defined over a logical solution space. Also, any approximation must be carried out with care that the accuracy of the solution is not compromised. There are multiple ways to approximate a continuous optimisation problem directly as an NLP problem, the most common of which are shooting and collocation methods. The differences in the behaviour of each method are related to the interaction between the SQP solver and the discretisation method by which the problem is defined, and can affect the stability and accuracy of the solution as well as the solution time of the problem. For this study of in atmosphere trajectory optimisation with complex atmospheric and vehicle properties it was desired that a method be found with maximum stability and accuracy for a relatively large solution space, while solution time is a secondary priority.

2.8.1 The Single Shooting Method

The oldest and simplest method of approximating continuous optimisation problems as NLP problems is the direct single shooting method. Direct single shooting discretises the control function over the solution space, and solves this directly as an NLP by integrating the vehicle dynamics, or state variables, along the trajectory at each trajectory guess. Single shooting is simple to apply and has been used since the 1970s for rocket trajectory optimisation [27]. Single shooting methods suffer from nonlinearity problems, ie. an optimisation problem solved using the single shooting method will potentially struggle to solve if the problem exhibits even small nonlinearities, due to being unable to converge to an optimal solution. This makes the single shooting method unsuitable for complex problems such as a scramjet model, as there are many nonlinear factors inherent in atmosphere and airbreathing engine modelling.

2.8.2 The Multiple Shooting Method

Direct multiple shooting is a popular solution to trajectory optimisation problems. This method solves some of the instabilities of the single shooting method by splitting the trajectory into multiple shooting arcs, and collocating these at specific time points. This creates a system of discontinuities, illustrated in Figure 2.22, which are gradually removed by the solver algorithm until the trajectory is continuous. These discontinuities allow greater flexibility for the solver than is afforded by the single shooting method. The multiple shooting problem is solved as an NLP through discretisation of the state and control variables at each time node and integration for the state variables x over each shooting arc t_k [63]:

$$\dot{x} = v = f[x(t), u(t)] \quad (2.4)$$

With the state variables subject to the boundary conditions:

$$r[x(t_0), x(t_f)] = 0 \quad (2.5)$$

And solving for the unknown values s_i :

$$x(t_i) = s_i, \quad i = 1, 2, \dots, N \quad (2.6)$$

The control variables at the nodes are guessed and the state variables are integrated along the trajectory, with each node segment being considered separately. A matching condition is introduced that must be met between each segment; ie. the trajectory must be continuous. [38]

$$\mathbf{X}(\mathbf{s}) = \begin{Bmatrix} x(t_1; s_0, v_0) - s_1 \\ x(t_2; s_1, v_1) - s_2 \\ \vdots \\ x(t_N; s_{N-1}, v_{N-1}) - s_N \\ r[x(s_0), x(s_N)] \end{Bmatrix} = \mathbf{0} \quad (2.7)$$

This is now in the form of an NLP problem which may be solved in a standard NLP solver. ie. minimise:

$$\min J(s, v) = \sum_{i=0}^{N-1} J_i(s_i, v_i) \quad (2.8)$$

subject to:

$$x[t_{i+1}, s_i, v_i] - s_{i+1} = 0 \quad (2.9)$$

$$r[x(s_0), x(s_N)] = 0 \quad (2.10)$$

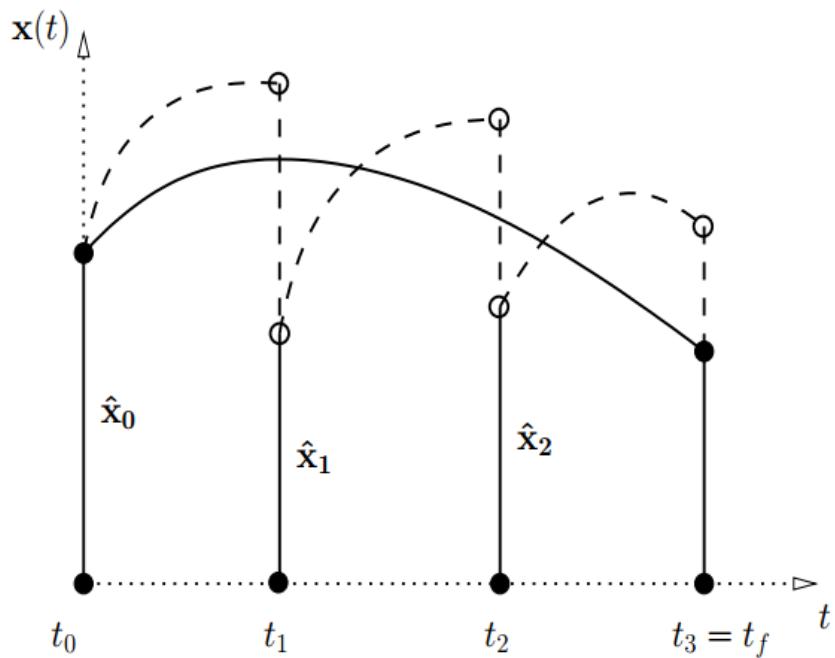


Figure 2.22: An illustration of the subdivision associated with multiple shooting [38]. Dashed lines illustrate the initial trajectory guess and the solid line indicates the final trajectory once the joining conditions are met.

The multiple shooting method has greatly improved convergence compared to the single shooting method, removing much of the susceptibility to instabilities resulting from nonlinear effects. How-

ever, the multiple shooting approach still suffers from a relatively small radius of convergence and slow computation times. Radius of convergence is extremely important to this study as the optimal solution cannot be approximated to a great degree of accuracy, and as such multiple shooting was deemed inappropriate for this study. It was desired to find a method with a global radius of convergence to apply to the optimisation problem being considered.

2.8.3 The Pseudospectral Method

check some of my claims about the pseudospectral method and cite GPOPS papers

GPOPS USES HP-ADAPTIVE GAUSS, WHICH HAS BEEN SHOWN TO improve(?) GAUSS
I should show the hp-adaptive gaussian method

Direct Trajectory Optimization and Costate Estimation via an Orthogonal Collocation Method - benson is quite a good paper (the first?) explaining gaussian ps method

also look at gpops 1 manual

The most promising method found to address the issue of radius of convergence is the direct collocation approach, which provides global radius of convergence with the additional advantage of smaller computational time [17]. Direct collocation methods discretise the control and state equations of the optimal control problem, and use these as constraints in an NLP problem similarly to the multiple shooting approach. However the state functions in collocation methods are approximated by polynomial functions over the solution space, inherently being continuous at each collocation node rather than the state functions being integrated over each timestep. The derivative of the state functions then become a constraint within the NLP, being equated to the polynomial approximation functions by the solver algorithm. The method used to discretise the derivative functions is very important to the accuracy of the collocation method, with trapezoidal, Hermite-Simpson and pseudospectral methods being popular choices.

The most accurate form of collocation method is the pseudospectral, or orthogonal collocation method[14], which uses some elements of spectral method approximation to accurately approximate derivative terms. The pseudospectral method was first introduced in 1972 by Kreiss & Oliger [30] as an efficient way to compute meteorology and oceanography problems. The pseudospectral method is now garnering a large amount of attention for its ability to rapidly and accurately solve a wide variety of optimal control problems. The pseudospectral method employs the use of orthogonal polynomials such as Legendre or Chebychev polynomials to approximate the state and control functions at a specific set of collocation points[Huntington2007, 14]. The pseudospectral method has the property that when evaluated at the specific collocation points, the functions approximating approximate state and control are exactly equal to the continuous function being approximated at that point.

A large usability advantage of the pseudospectral method is the ability to generate Hamiltonian and costate values easily[22, 15]. The Hamiltonian and costate values allow a solution to easily and

quickly be checked to determine if some of the necessary conditions for optimality are being met. This is useful to determine initially if the optimal solution calculated by the pseudospectral solver is valid.

The pseudospectral method discretises the optimisation problem for application of an NLP solver. The initial form of the optimisation problem is that of a generic Bolza optimisation problem, described by a continuous Bolza cost function:

$$J(\mathbf{u}, \mathbf{x}, \tau_f) = M[\mathbf{x}(\tau_f), \tau_f] + \int_{\tau_0}^{\tau_f} L[\mathbf{x}(\tau), \mathbf{u}(\tau)] d\tau \quad (2.11)$$

Subject to a set of state dynamics, which describe the behaviour of the system over the solution space:

$$\dot{\mathbf{x}}(\tau) = f[\mathbf{x}(\tau), \mathbf{u}(\tau)] \quad (2.12)$$

These are constrained by boundary conditions of the system at the initial and final time points:

$$\psi_0[\mathbf{x}(\tau_0), \tau_0] = \mathbf{0} \quad (2.13)$$

$$\psi_f[\mathbf{x}(\tau_f), \tau_f] = \mathbf{0} \quad (2.14)$$

Three primary pseudospectral methods are described in this chapter. All of these methods make use of Lagrange interpolating polynomials. These methods use different collocation points.

The Legendre Pseudospectral Method

The Legendre polynomial method[16] uses Legendre-Gauss-Lobatto points for collocation. These create efficient and simple relationships for the evaluation of time-continuous optimisation problems [14]. The Legendre pseudospectral method uses the Legendre polynomial, defined as L_N :

$$L_N = \frac{1}{2^N N!} \frac{d^N}{dt^N} (t^2 - 1)^N \quad (2.15)$$

The Legendre-Gauss-Lobatto points, $t_l, l = 0, \dots, N$, are given by:

$$t_0 = 0$$

$$t_N = 1$$

and $t_l, l = 1 \leq l \leq N - 1$ are the zeros of \dot{L}_N .

The Legendre pseudospectral method transforms the problem to lie on the interval $t \in [-1, 1]$, so that the cost function (Equation 2.11) is discretised as follows:

$$J[\mathbf{x}(t_k), \mathbf{u}(t_k)] = \frac{\tau_f - \tau_0}{2} \sum_{k=0}^N L[\mathbf{x}(t_k), \mathbf{u}(t_k)] w_k + M(\mathbf{x}(t_N), \tau_f) \quad (2.16)$$

where w_k is a weighting function defined as:

$$w_k = \frac{2}{N(N+1)} \frac{1}{[L_N(t_k)]^2} \quad (2.17)$$

The state equations (Equation 2.12, in this case the system dynamics) are described at each LGL point by:

$$\frac{(\tau_f - \tau_0)}{2} \mathbf{f}[\mathbf{x}(t_k), \mathbf{u}(t_k)] - \sum_{l=0}^N D_{kl} \mathbf{x}(t_l) = \mathbf{0} \quad (2.18)$$

Where D is the differentiation matrix, defined at each entry k, l by:

reduce the spacing between equations...

$$k \neq l \quad D_{kl} = \frac{L_N(t_k)}{L_N(t_l)} \frac{1}{t_k - t_l} \quad (2.19)$$

$$k = l = 0 \quad D_{kl} = -\frac{N(N+1)}{4} \quad (2.20)$$

$$k = l = N \quad D_{kl} = \frac{N(N+1)}{4} \quad (2.21)$$

$$otherwise \quad D_{kl} = 0 \quad (2.22)$$

The boundary conditions are expressed analogously to the continuous problem:

$$\psi_0[\mathbf{x}(t_0), \tau_0] = \mathbf{0} \quad (2.23)$$

$$\psi_f[\mathbf{x}(t_N), \tau_f] = \mathbf{0} \quad (2.24)$$

The Chebychev Pseudospectral Method

CGL collocation points [22] $t_k = \cos(\pi(N-k)/N), k = 0, \dots, N$

The Gauss Pseudospectral Method

A large advantage of the pseudospectral method is the ability to generate Hamiltonian and costate values easily, detailed by Gong et al. [22] and Fahro & Ross [15]. The Hamiltonian and costate values allow a solution to easily and quickly be checked for accuracy. The Hamiltonian equalling zero is a necessary (but not sufficient) condition for optimality. In order to make a preliminary test for an optimal solution, the Hamiltonian must be observed to be close to zero along the entire trajectory.

expand on this, and optimality conditions

2.8.4 Pseudospectral Examples

this section needs to be linked with the hypersonic vehicle trajectories that use the pseudospectral method

need a six dof example, and a multiple state example

In order to assess the applicability of the pseudospectral method to an in atmosphere trajectory optimisation problem, the range of existing solutions utilising the pseudospectral method has been investigated. The pseudospectral method has been proven to be extremely effective for simulations in aerospace applications and has been proven in flight applications such as the zero propellant manoeuvre of the International Space Station in 2007, where the ISS was rotated 180 degrees without any propellant used following a pseudospectral method solution [5]. The pseudospectral method has been used successfully in a multitude of studies for the trajectory optimisation of hypersonic vehicles[33, 28, 75, 66, 12, 9, 50, 39].

The pseudospectral method has been used for the guidance of re-entry vehicles including in-atmosphere dynamics, to keep a vehicle on a desired three degree of freedom path in real-time in a study by Tian & Zong [66]. The pseudospectral method was found to generate an accurate trajectory around the desired reference trajectory, satisfying all necessary constraints in real-time for six state variables with minimal error. Nonlinear effects are an intrinsic part of simulating a complex aerodynamic system, and this study indicates that the pseudospectral method will be able to simulate the SPARTAN vehicle and is appropriate for use in this study.

Chai et al.[9] investigate the range optimal trajectory of a scramjet-powered hypersonic missile. Chai et. al successfully compute an optimal trajectory for an airbreathing hypersonic vehicle in three degrees of freedom[9]. The hp-adaptive pseudospectral method is compared to the Gauss pseudospectral method and the direct shooting method. Both pseudospectral methods show greater precision and robustness than the direct shooting method, and are reported to be much more computationally efficient.

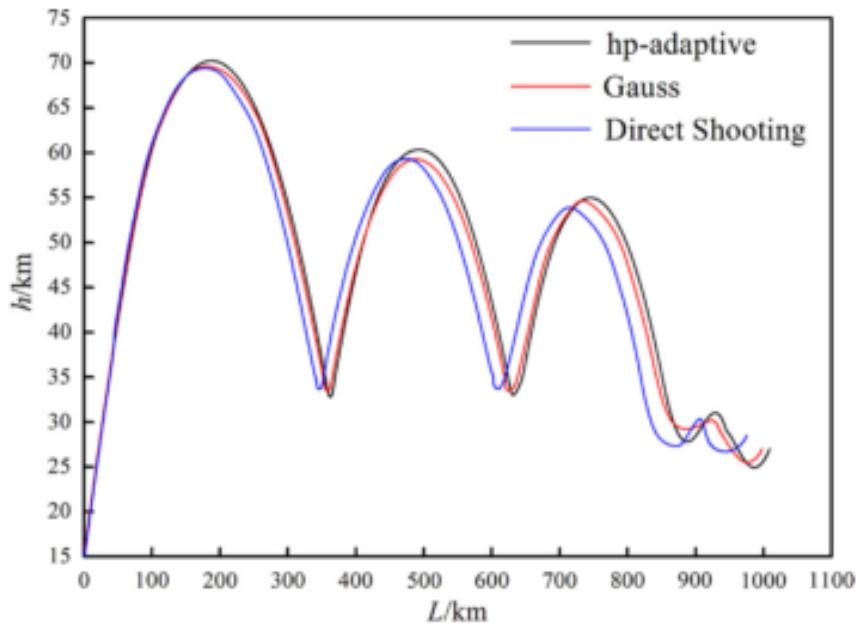


Figure 2.23

ally efficient. The direct shooting method takes 268.2s to complete, whereas the Gauss pseudospectral method completes optimisation in 12.7s, and the hp-adaptive pseudospectral method takes only 5.3s[9]. The pseudospectral methods produce similar results. The hp-adaptive pseudospectral method shows a slight range increase over the Gauss pseudospectral method due to its ability to redistribute the collocation points adaptively, which improves robustness.

These results indicate that the pseudospectral method does not exhibit instabilities with nonlinear effects such as atmospheric density varying the dynamics of the system being optimised, and that the pseudospectral method can be used for systems with at least up to six state variables.

Based on the proven track record of the pseudospectral method in simulation situations, it was decided that using the pseudospectral method for the optimisation of the rocket-scramjet-rocket system was a sound and feasible decision. It has been necessary to investigate the range of solvers available that will utilise a pseudospectral method to solve an optimisation problem with the desired capabilities and usability.

2.8.5 Available Solvers

change this section to reflect more current perspectives. expand on what each program actually does.
Add programs like POST/POST2 (direct shooting?), OTIS, TRANSWHORP, ASTOS

The pseudospectral method has a number of solvers available commercially, the foremost of which are DIDO, produced by Elissar Global [54], GPOPS II [49] and PROPT, a module integrated with TOMLAB [56]. These solvers utilise nonlinear programming techniques to solve optimal control

problems in discretised form and are all similar in their operation. These have been investigated for a variety of considerations with details available in Section ??.

There are a number of existing programs that can complete optimisation problems using the pseudospectral method and as such creating an optimiser from scratch was considered to be overly time consuming and not a useful contribution. A number of programs were found, which are detailed in Table 2.2. An effort was made to find an open source program that could complete the desired op-

Software	Platform	Cost	Notes
DIDO	MATLAB	\$50 (Student)	Simple and automated with free trial available. Documentation readily available. Can perform required problem. Provides costates and Hamiltonian values.
GPOPS II	MATLAB	\$100	Free trial available, but was unable to obtain. Some documentation available. Can perform required problem.
ROC-HJ Solver	C++	Free	A basic solver for some specific optimal control problems. Does not perform collocation.
PROPT (IPOPT)	MATLAB	\$500	No free trial. Documentation available. Can perform required problem.
Imperial College OCS	MATLAB	Free	Documentation and download unavailable
PSOPT	C++	Free	Open source and can perform required problem. Not updated, many modules out of date.

Table 2.2: Summary of programs capable of pseudospectral optimisation.

timisation however it was found that all open source programs studied were out of date or not well supported. Out of the premium packages available GPOPS II and DIDO were considered the best options, both being proven in aerospace applications, with available free trials and affordable price points. Free trials were requested from both, however the free trial for GPOPS II was not received while the free trial for DIDO was received with good customer service provided. On reception of the DIDO trial it was found that DIDO provides the required optimiser package in an easy to use form, with sufficient documentation and examples provided to streamline the simulation process. DIDO also provides costate and Hamiltonian values, used for verifying that the optimised solution is accurate and physically sound, as well as the lowest price point and benefit of being proven in both real space applications and in-atmosphere simulations. After weighing these considerations DIDO was selected as the best program to use for the pseudospectral optimisation and simulation.

2.9 Aerodynamic Analysis

- preliminary design requires fast and easy aerodynamics

Simulating the trajectory of access to space vehicles requires the aerodynamics of the launch vehicle to be characterised accurately. This entails the creation of large aerodynamic coefficient databases, which cover the operable region of the vehicle, and include the effect of control surface deflections. The liability of the vehicle design to change during the preliminary design phases renders highly accurate CFD or experimental studies expensive and inefficient CITATION. Instead, solutions are found which are able to approximate the aerodynamic characteristics to an appropriate accuracy, while providing fast set-up speeds as well as computation times.

2.9.1 HYPAERO

need to expand to be a general review of solvers available (unbiased). include brief discussion on turbulence. discuss viscous effects

Hypaero is an aerodynamic calculation tool developed for the preliminary design of hypersonic vehicles. HYPAERO uses strip theory, dividing the surface of the vehicle into panels to calculate aerodynamic properties along the aircraft as illustrated in Figure 2.24. In strip theory two dimensional sections are created running the length of the vehicle, defining streamlines along the vehicle faces. Pressure, Mach number and skin friction are calculated along these streamlines to compute the aerodynamic coefficients of the vehicle. The Aerodynamic analysis of the SPARTAN has been performed using HYPAERO by Jazra et al. [26]. The aerodynamic coefficients of the SPARTAN vehicle have been provided for this study by Dawid Preller.

NEED TO CHANGE THIS to clarify where I'm using this

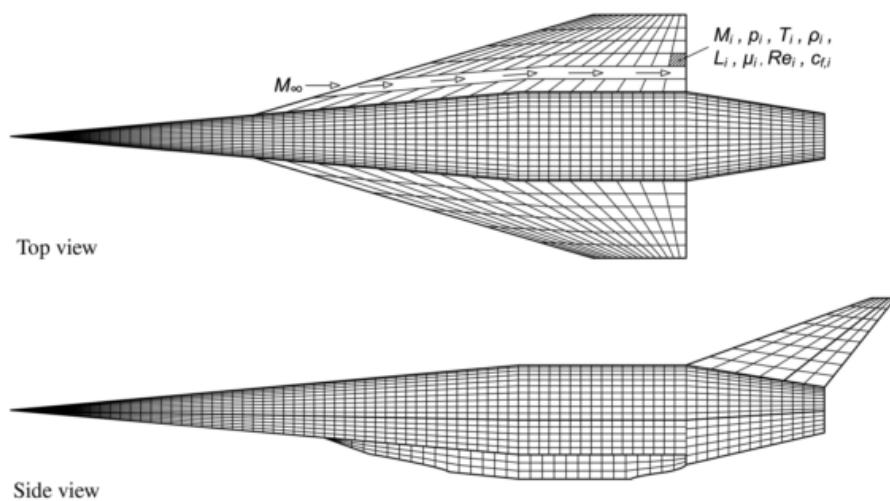


Figure 2.24: HYPAERO analysis surface grid [26].

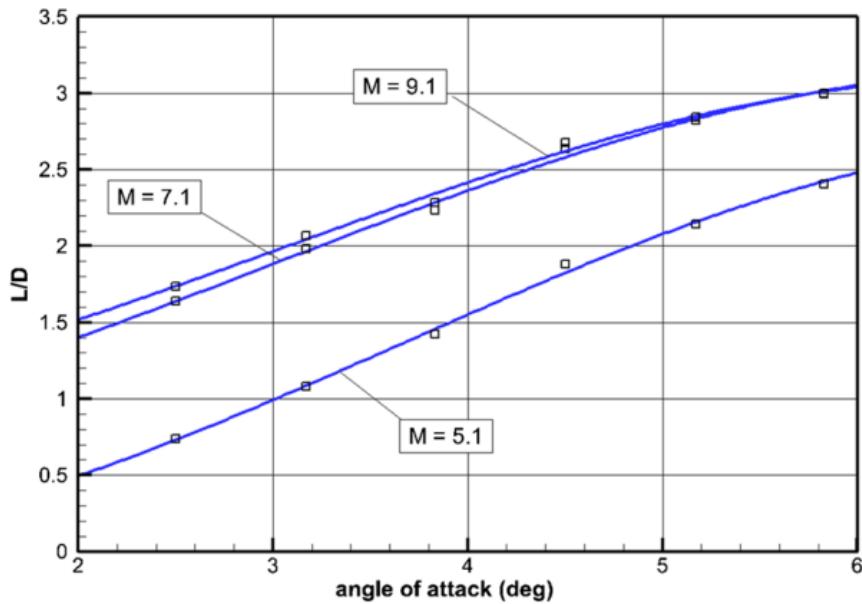


Figure 2.25: SPARTAN aerodynamics, developed using HYPAERO [48].

2.9.2 CART3D

Insert an example of a validation of CART3D

CART3D is an inviscid CFD package, designed for use during preliminary vehicle design and analysis CITATION. CART3D requires only a surface triangulation of the vehicle being analysed. CART3D features adjoint mesh adaption, and uses cartesian 'cut-cells' which intersect the surface, allowing complex geometries to be analysed. The mesh automatically refines as the simulation progresses, reducing error. The absence of a requirement for a user generated mesh allows CART3D to be easily applied to complex launch vehicle designs, as well as allowing for simple modification of control surface deflections and flight conditions. CART3D has been used extensively for aerodynamic simulations in preliminary design, including analysis of the Skylon spaceplane[36], HIFiRE-5[29], and in low sonic boom shape optimisations[2]. Mehta et al. used CART3D to analyse the Skylon spaceplane, in a study which investigated the aerodynamics of the vehicle as well as the plumes produced by the Reaction Engines LTD SABRE engines[36]. The Skylon was analysed at a range of mach numbers from 2.625 to 16.969, at altitudes from 15.326km to 75.771km. These results indicate the applicability of CART3D over a wide range of Mach numbers, which is of particular use to this study. During a study by Aftosmis & Nemec where CART3D was applied to low sonic boom shape optimisation, CART3D was shown to have good agreement with experimental results in tests at Mach 1.6.

The wide Mach number range of CART3D, along with its ease of use and demonstrated accuracy, makes CART3D a useful tool for this study.

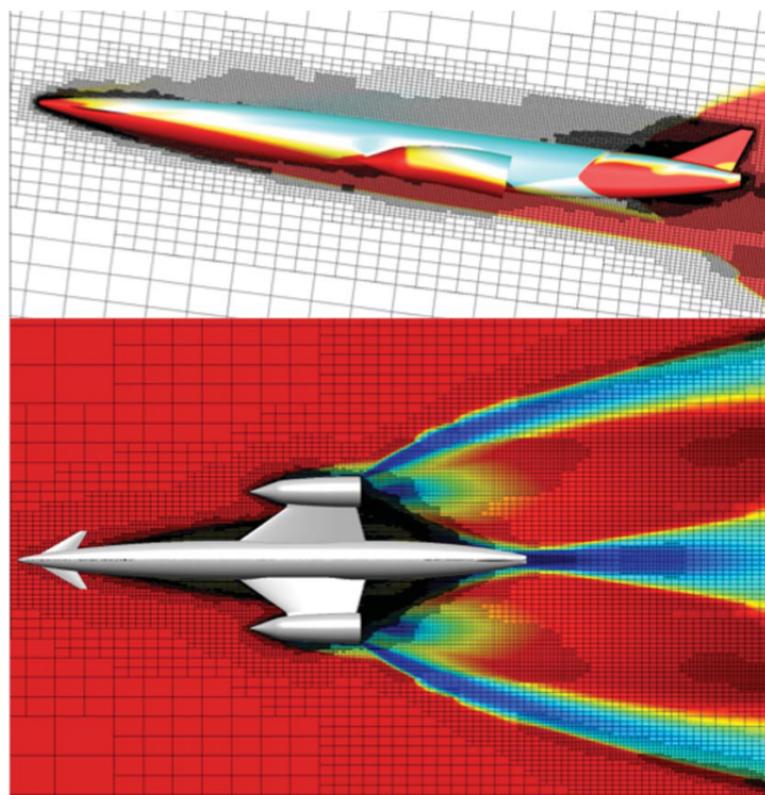


Figure 2.26: The Skylon spaceplane, simulated using CART3D at Mach 12.189, $\alpha = 7.512^\circ$ [36]. Cell distribution produced by mesh adaption is shown.

2.9.3 Missile DATCOM

Missile DATCOM is a widely used, semi-empirical, aerodynamic prediction tool for missile configurations. Missile DATCOM is capable of calculating the aerodynamic forces, stability derivatives and moments over a range of angle of attack and Mach number values, allowing an aerodynamic database to be generated simply and rapidly. Missile DATCOM has been shown to produce close agreement with experimental wind tunnel data for normal force and pitching moment coefficients, and reasonable agreement for axial force coefficients [60].

2.10 Summary

This chapter has reviewed literature relevant to this thesis. Add more details.

CHAPTER 3

LAUNCH VEHICLE BASELINE DESIGN

move tables to appendix, think of either moving performance maps to appendix, or including smaller images

This chapter presents the baseline three stage small satellite launch system utilised in this thesis. The launch system is launched vertically under rocket power, from a traditional small rocket launch facility. The SPARTAN vehicle is mounted to the front of the first stage rocket. This configuration allows the SPARTAN to take the brunt of the aerodynamic forces and heating, as well as allowing the use of the control surfaces of the SPARTAN. The SPARTAN is accelerated by the first stage rocket, to its minimum operating velocity, at which point separation occurs. The SPARTAN's four scramjet engines are ignited, and SPARTAN accelerated through the atmosphere, reaching approximately Mach 9. At this point, the specific impulse of the scramjet engines, and thus the efficiency of the SPARTAN, have decreased, and the third stage rocket is separated. The third stage rocket accelerates for a time, before cutting its engine and coasting out of the atmosphere. Once the rocket is exoatmospheric, the engine is reignited multiple times, performing first a circularisation burn, and then a Hohmann transfer to the intended orbit. Meanwhile, the SPARTAN banks and executes a fly-back manoeuvre to return to its initial launch site. The SPARTAN extends landing gear, and lands on a traditional runway in the style of a conventional aircraft. The SPARTAN is able to be rapidly refurbished and remounted for further launches.

The design for the first stage rocket-powered vehicle, second stage scramjet-powered vehicle (the SPARTAN), and third stage rocket-powered vehicle are presented. This baseline design is used for the initial trajectory analysis and optimisation in this study. The launch system has been designed based on the SPARTAN vehicle developed by Preller & Smart. The size and external design of the SPARTAN are used exactly as defined by Preller & Smart, and are not modified in any way. The internal component masses of the SPARTAN are either identical to those defined by Preller & Smart, or are scaled if the internal component is modified. The first stage rocket, third stage rocket, and

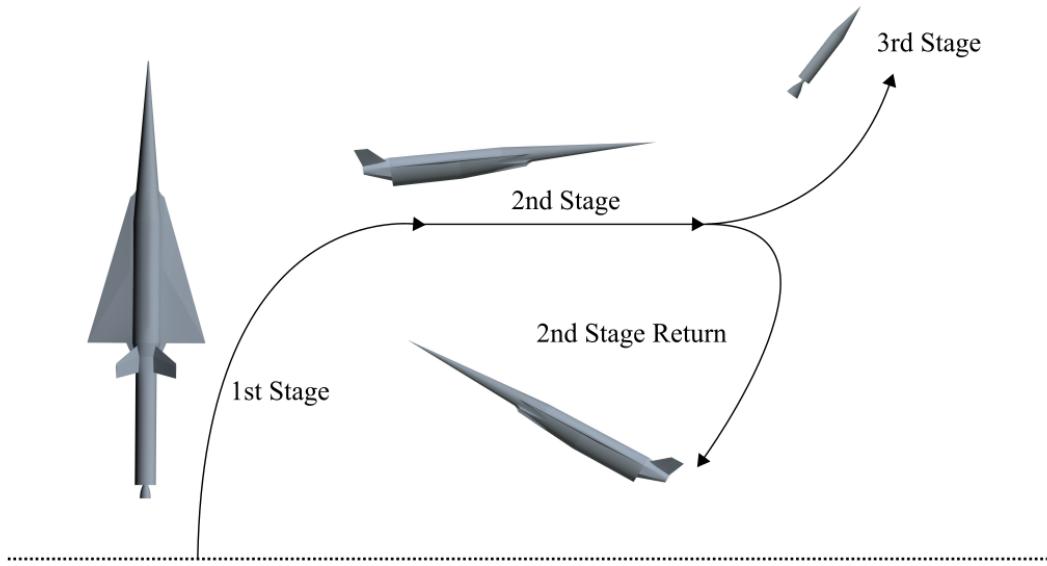


Figure 3.1: The launch process of the rocket-scramjet-rocket launch system, presented in simplified form.

internal components have been designed to conform to the size and shape of the baseline SPARTAN vehicle as defined by Preller & Smart.

This chapter also describes the aerodynamic models of the vehicles used in this study, including the engine models.

3.1 Second Stage Scramjet

3.1.1 Baseline Vehicle

The SPARTAN vehicle in this study has been designed based on the work by Preller & Smart CITATION.

The baseline SPARTAN has been designed to hold a 9 m long third stage. Two cylindrical tanks underneath the third stage and a conical tank situated in the nose have a total volume of 22.0m³ and hold a total of 1562kg of LH₂ fuel. This assumes an LH₂ density of 71kg/m³, slightly denser than LH₂ at phase transition point at 1 atm.

The fuel tanks are sized to fit around the kestrel-powered third stage, described in Section XX. The fuel tanks have a total tank volume of 22.0m³, containing a fuel mass of 1562kg. The mass of the fuel tanks is scaled from Dawid Prellers model of the SPARTAN, giving a total fuel tank mass of 179.41kg.

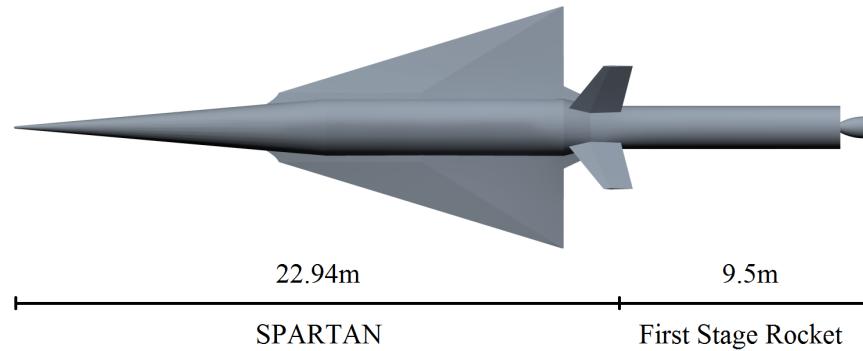


Figure 3.2: The rocket-scramjet-rocket launch system, top view, showing the SPARTAN and first stage.



Figure 3.3: The rocket-scramjet-rocket launch system, side view, showing the SPARTAN and fuel tanks, along with the third and first stages.

-detail individual fuel tank masses (and how they were designed)

3.1.2 Aerodynamics

The aerodynamics of the SPARTAN have been calculated using CART3D, an inviscid CFD package used in the preliminary design of aerospace vehicles. Cart3D utilises adjoint mesh adaption with a Cartesian cut-cells approach to produce an iteratively refined mesh to fit a flow solution. CART3D has been used to generate the aerodynamic database of the SPARTAN vehicle due to its applicability in both the subsonic and supersonic regimes, and its robustness across multiple flow solutions [16]. CART3D has previously been used to analyse hypersonic vehicles, and has shown good agreement with experimental data across multiple studies [24, 25]. The SPARTAN has been simulated in CART3D across Mach numbers from 0.2 to 10, and for angle of attack values from 0° to 10° . Example cases are shown in Figure 3.5, 3.6 and 3.7. Figures XXX and XXX show the lift and drag coefficients of the SPARTAN vehicle over the range of Mach numbers and angle of attack values analysed.

method of calculating aerodynamics for trim at required lift

-detail hypaero vs CART3D?

conical shock calculation

An initial centre of gravity of 14.0m was calculated using CREO, using the mass fractions calculated by Preller [CITATION]. This centre of gravity calculation assumes that the structural mass

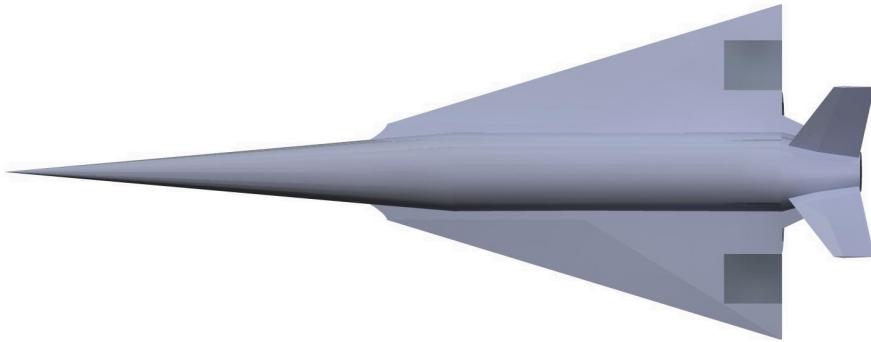


Figure 3.4

of the SPARTAN is evenly distributed throughout the body volume. As the SPARTAN is still in the preliminary design stages, it has been assumed that the centre of gravity is slightly variable. This assumption allows the centre of gravity to be varied slightly to improve the stability of the SPARTAN, as well as positioning the moment centre to minimise necessary flap deflection over the trajectory of the SPARTAN, improving the aerodynamic characteristics of the SPARTAN.

The SPARTAN is trimmed using control surfaces on the wings, shown in figure 3.4.

The centre of gravity of the SPARTAN is varied as fuel is depleted throughout the acceleration phase, as well as when the third stage rocket is released. A point mass model is used in conjunction with the aerodynamic database, and atmospheric properties obtained from the U.S Standard Atmosphere 1976[26]. The SPARTAN is assumed to be trimmed at all conditions during flight.

Aerodynamics of SPARTAN Alone

NOTE calculated SPARTAN aero using missile datcom (something close) and lift is very similar to CART3D, with drag being in between/a little lower than CART. Seems to support CART.

This section details the aerodynamics of the SPARTAN, calculated using CART3D. A SPARTAN model was provided for this study by Mr. Joseph Chai. A surface grid was created on this model using Pointwise[44]. This model includes engine flowpaths.

Images of pointwise mesh

CART3D lift/drag

Images of CART3D results for a range of Mach numbers

details of CART3D setup (no adapt levels / radius etc)

CART3D validation

14.5m set as CG at end of burn, after third stage release. This is observed to produce a relatively stable trimmed acceleration and return.

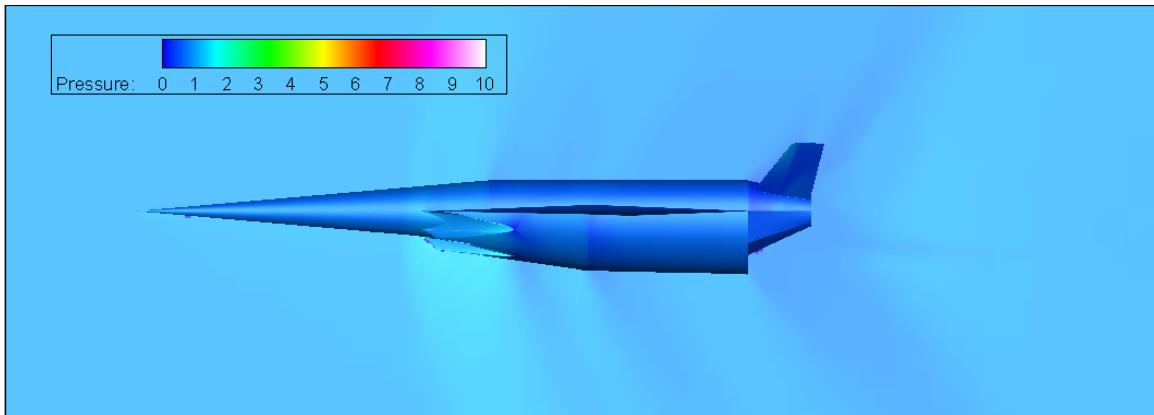


Figure 3.5: CART3D flow result for the SPARTAN, at Mach 1.1, 6° angle of attack.

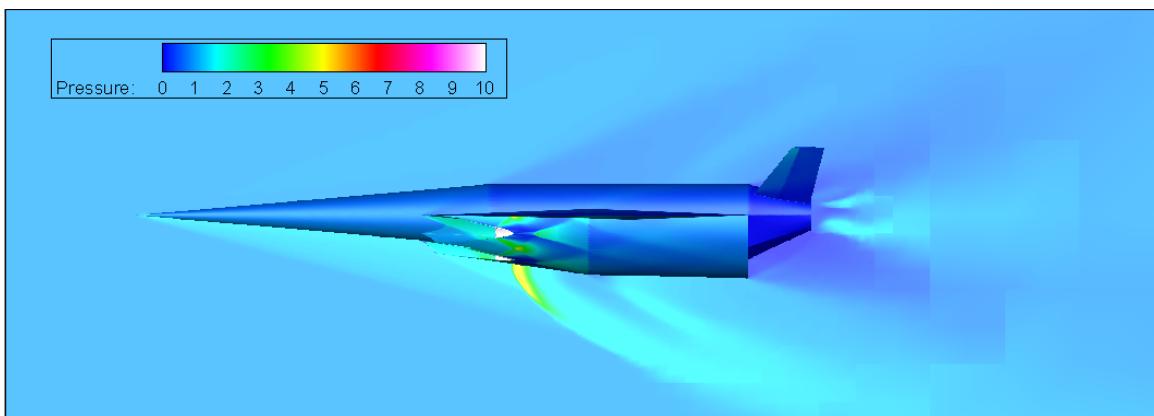


Figure 3.6: CART3D flow result for the SPARTAN, at Mach 3, 6° angle of attack.

A total of 9 aerodynamic matrices are created -CG no third stage, no fuel (assumed fly-back fuel does not change CG) -CG third stage, no fuel -CG third stage, full fuel

for each of these

-engine off coeffs -engine on coeffs -Flap Deflection coeffs

need to include cliccalc and CART3D into my process diagrams?

-moment of inertia

-show side and bottom results for M=0.5,1.1,5,9 (appendix)

need engine off, engine on and flap deflection comparisons CHANGE THESE GRAPHICS

These results show a distinct maximum region in the L/D of the SPARTAN at high Mach numbers, within the hypersonic regime. Below Mach 5, the L/D of the SPARTAN decreases sharply.

For the engine-on case, the engines and boat tail are removed from the aerodynamics of the vehicle. In the engine-on case the drag is decreased for all conditions. The lift is increased at low angle of attack, and increased at high angle of attack.

-put in plots for moments, think about best way to present trim?

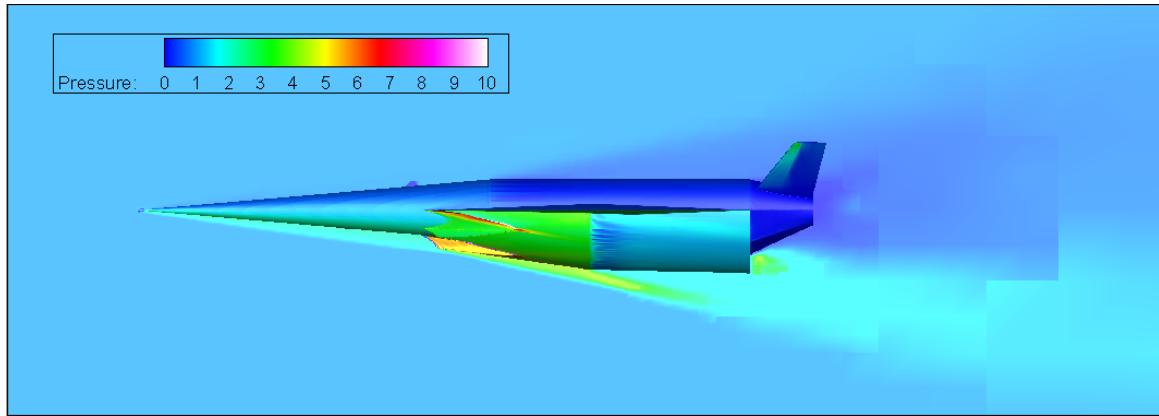
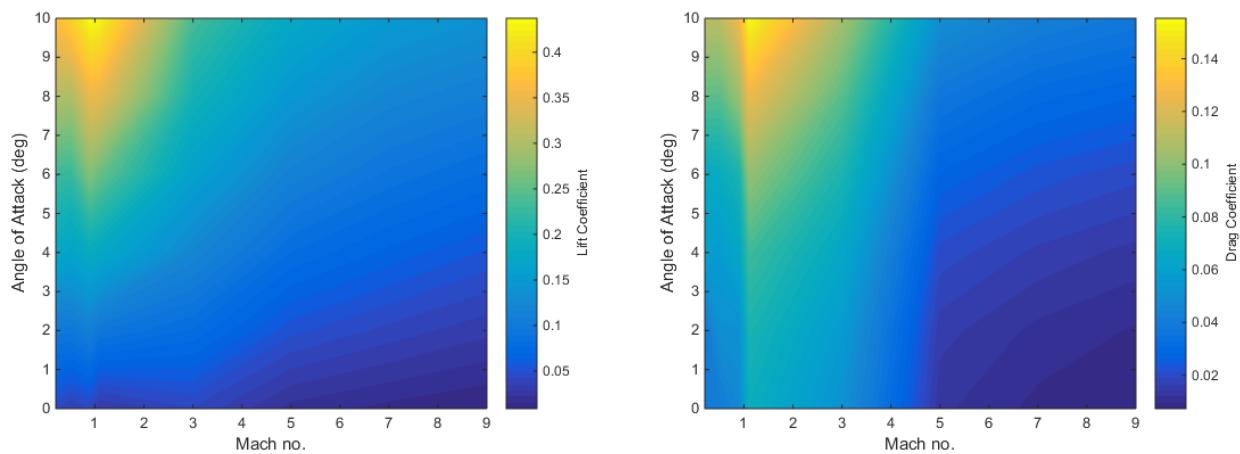
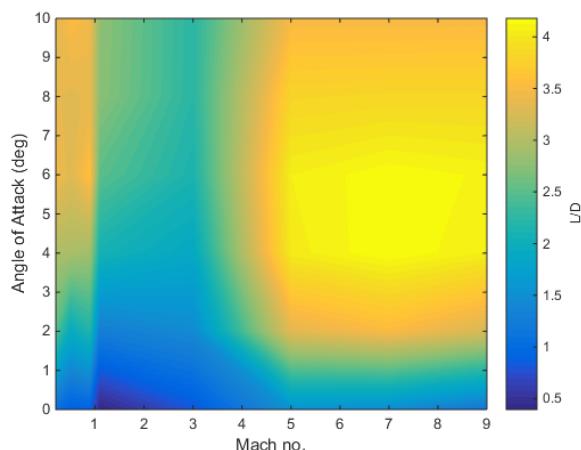


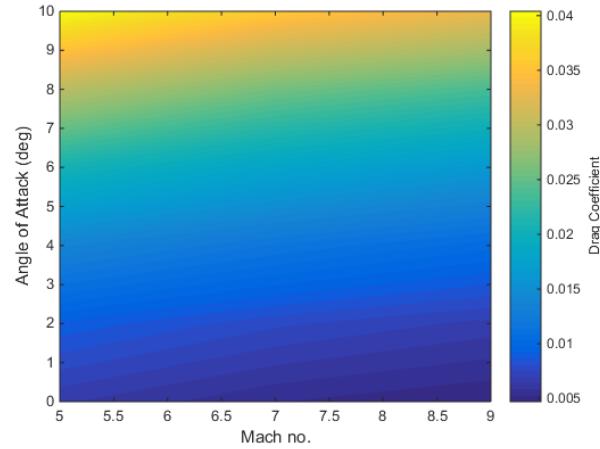
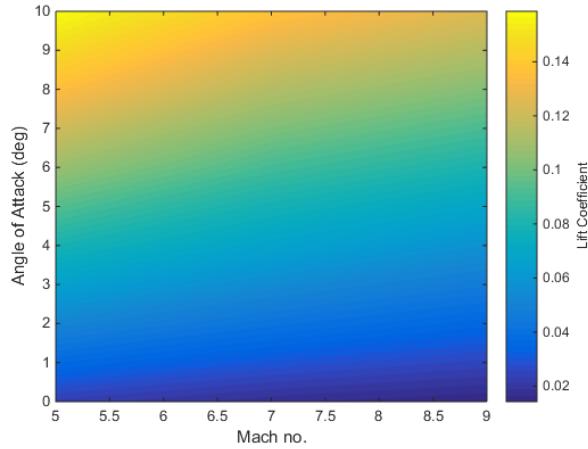
Figure 3.7: CART3D flow result for the SPARTAN, at Mach 7, 6° angle of attack.



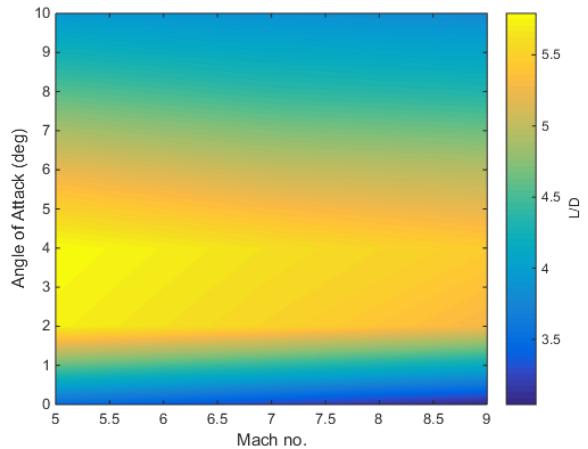
(a) Coefficients of lift of the SPARTAN, calculated us- (b) Coefficients of drag of the SPARTAN, calculated us-
ing CART3D. using CART3D.



(c) L/D of the SPARTAN.



(a) Coefficient of lift of the SPARTAN with the C-REST engines powered-on. This is obtained by removing the REST engines powered-on. This is obtained by removing engines and boat-tail from the CART3D simulation results.



(c) L/D of the SPARTAN with the C-REST engines powered on.

3.1.3 Propulsion Modelling

conical shock calc

C-REST thrust calculator method?

be more quantitative, include control law for mass flow rate (eq=1?) The SPARTAN is powered by four underslung scramjet engines. These engines are Rectangular To Elliptical Shape Transition (REST) engines, configured to allow for a conical forebody (C-REST). The engine model used is a CRESTM10 database[48], analysed using quasi-1D simulation. This database provides data points of engine performance over inlet conditions within the operational range, at 50kPa dynamic pressure equivalent conditions. The specific impulse and equivalence ratio data sets are shown in Figure XXX. This data is interpolated for the given inlet conditions to calculate the exit conditions and the specific impulse produced by the engine. The specific impulse is obtained by spline interpolation. The thrust, T , is then obtained by inclusion of the mass flow rate (\dot{m}) obtained via the inlet conditions, ie. $T = g_0 \dot{m} I_{sp}$. The C-REST engine is a fixed geometry engine, designed for operability at high Mach numbers[48]. At lower Mach numbers, the addition of excessive fuel may cause the engine to choke and unstart, resulting in total loss of thrust[48]. To avoid unstart, an equivalence ratio (ϕ) of less than 1 is set at low Mach numbers. As the equivalence ratio is equal to 1 in the majority of the Mach number and temperature regime, only the regions in which it is less than 1 are used for the interpolation. The equivalence ratio interpolation is linear, as the number of data points available for interpolation is low.

During flight the C-REST inlet conditions will stay within the region bounded by the available data. However, for the purposes of the pseudospectral method solver, it is necessary for the vehicle model to be able to extrapolate for ISP and equivalence ratio data.

Put Plot of eq Here too

3.1.4 Engine-On Plume Model

The operation of the scramjet engines changes the aerodynamic characteristics of the SPARTAN significantly. The plumes of the scramjet engines exit the nozzle of the SPARTAN, and are further expanded onto the boat tail on the rear of the SPARTAN fuselage. This expansion causes significant force on the boat tail of the SPARTAN, generating additional lift, thrust and moment forces. The plumes of the SPARTAN have been simulated using CART3D, using surface boundary conditions which produce inflow and outflow conditions at the inlet and exit of the scramjet engines. The CRESTM10 database has been used to calculate the engine conditions at the exit of the scramjet engines, and this has been set as the outflow condition. These conditions are normalised for CART3D as follows:

$$P_e^* = P_e / (\gamma_e P_0) \quad (3.1)$$

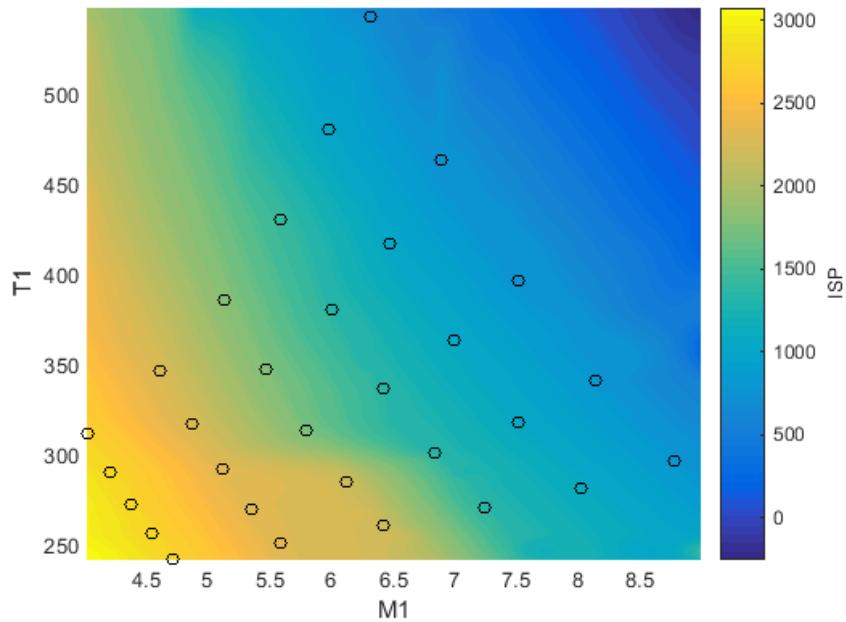


Figure 3.10

$$\rho_e^* = \rho_e / \rho_0 \quad (3.2)$$

$$M_e^* = \sqrt{\gamma_e / \gamma_0} (M_e \sqrt{P_e^* / \rho_e^*})^2 \quad (3.3)$$

This includes a correction on the Mach number to account for γ_e variation, which is not possible to include directly in CART3D.

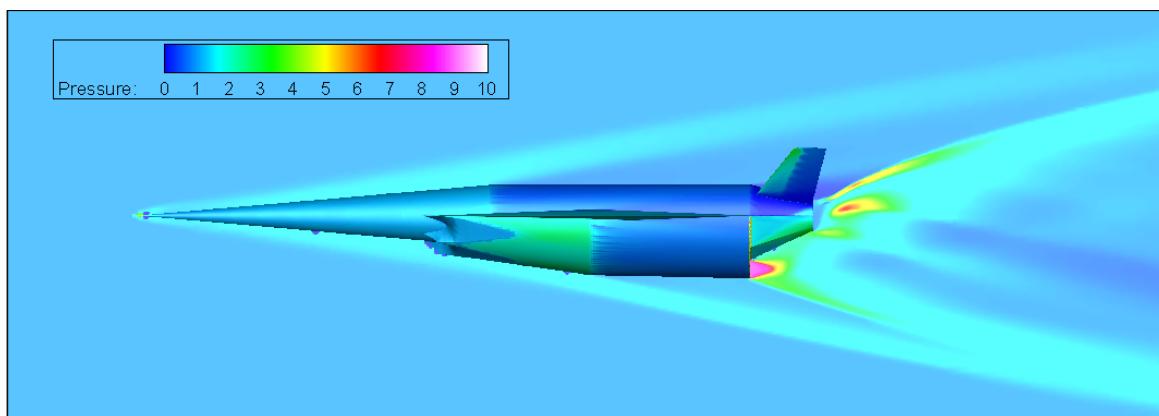


Figure 3.11

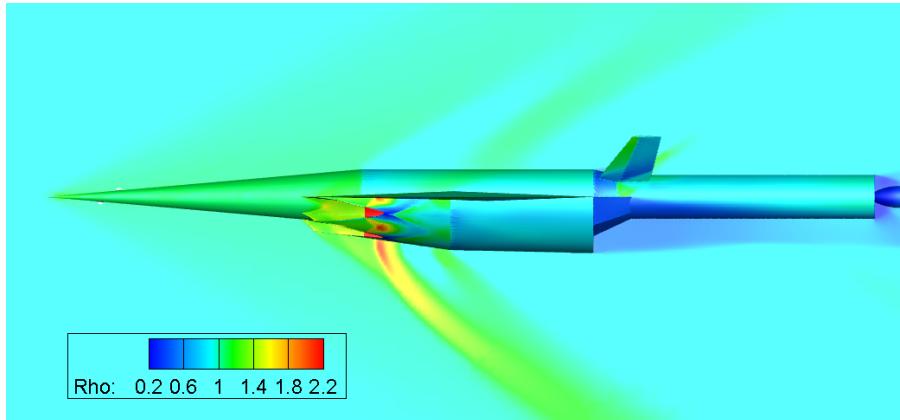


Figure 3.12: CART3D result for the SPARTAN and first stage vehicles at Mach 2, -1° angle of attack.

3.2 First Stage Rocket

The first stage rocket-powered vehicle is based on the first stage of the SpaceX Falcon-1e. The Falcon-1e has been chosen due to its appropriate scale, and the proven flightworthiness of the Falcon-1.

- detailed design
- First Stage aerodynamic coefficients calculated in CART3D
- details of CAD, pointwise mesh, and CART3D gridding settings

Aerodynamics Including First Stage

The aerodynamics of the SPARTAN and first stage rocket have been calculated using CART3D. For the purposes of these simulations, a connecting cowl has been modelled between the first stage rocket and the SPARTAN, to improve aerodynamics. The first stage aerodynamics have been modelled between angles of attack of 0° to -5°, as the first stage will be flying at negative angle of attack to induce faster pitch-over. Mach numbers from 0.2 to 5.1 (second stage separation velocity) have been simulated. Figure 3.12 shows an example CART3D simulation case, at Mach 2, -1° angle of attack. Figure 3.13 shows the lift and drag coefficients of the first stage, as well as the lift-over-drag, across the simulated Mach Numbers and angles of attack.

3.3 Third Stage Rocket - Baseline

In this study the third stage rocket has been designed to accommodate a SpaceX Kestrel engine. In previous studies, the third stage was designed to be powered by a Pratt & Whitney RL-10-3A pump-fed engine, weighing 52kg. The Kestrel has been used over the RL-10-3A for its cost effectiveness. As a pressure-fed engine, the Kestrel trades off specific impulse for weight and cost savings when

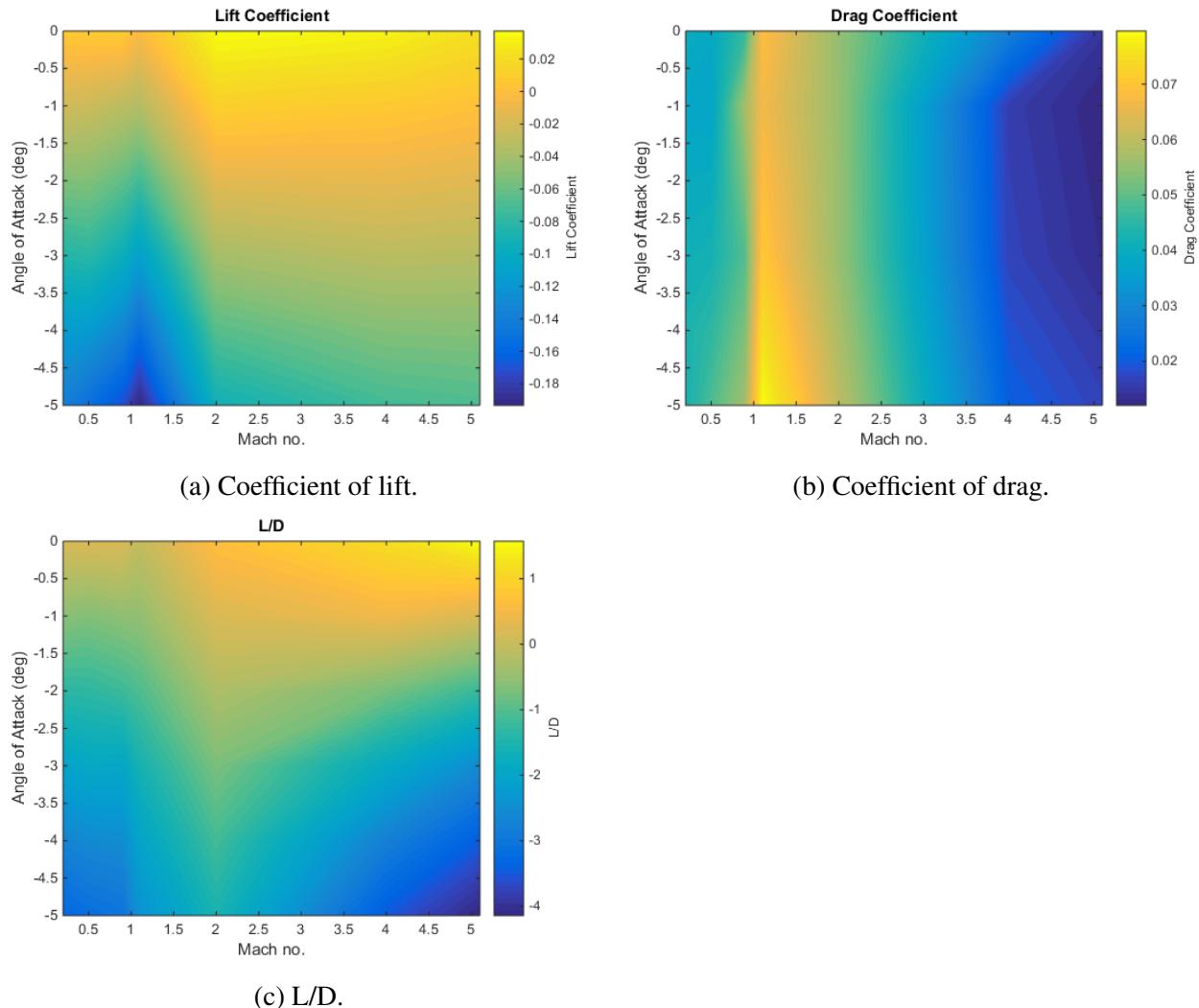


Figure 3.13: Aerodynamic characteristics of the SPARTAN including the first stage rocket.



Figure 3.14

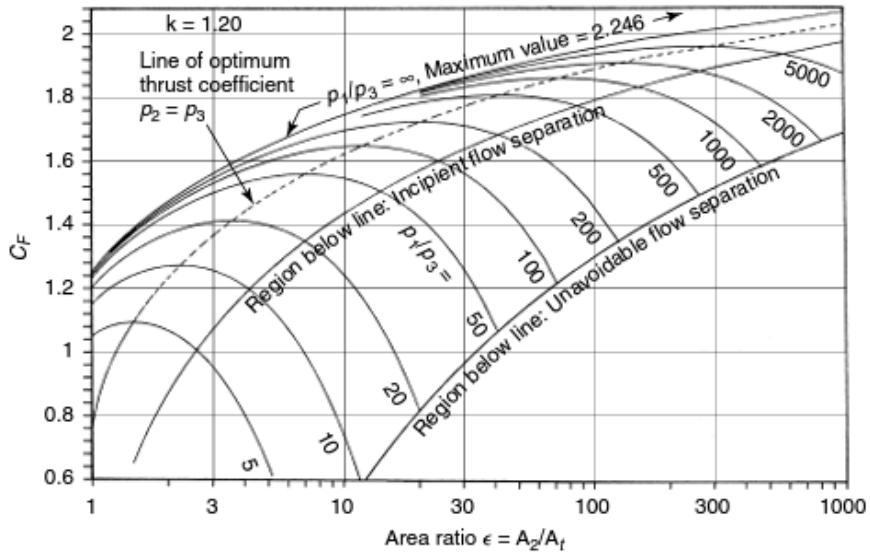


Figure 3.15: [64]

compared to the RL-10-3A. As the only expendable portion of the system; the cost of the third stage is one of the main drivers of overall system cost. Reducing the cost of the third stage allows the cost of launch to be directly reduced.

The third stage rocket is released at the end of the scramjet accelerator burn, and lifts the payload out of the atmosphere and into the desired orbit. The third stage weighs a total of 3300kg. This was chosen as a design weight, to fit the fuel necessary to achieve orbit with an acceptable payload while also allowing for ample payload volume. The third stage has a structural mass fraction of 0.09, similar to the Falcon 1 second stage [61]. This gives a total structural mass of 285.7kg.

The kestrel engine has been modified to have 50% increased propellant mass flow rate, giving a mass flow rate of 14.8kg/s. The nozzle of the Kestrel engine has been kept at 1.1m diameter. This increase in mass flow results in a 2% loss of efficiency from the nozzle[64], due to the thrust coefficient decreasing as shown in Figure 3.15. The modified specific impulse of the engine is 310.7s.

Arat initial is 60 (reference, measured I think), after thrust increase Arat is 40

The third stage has a total length of 9m, with a 3m long nose, 4.5m long centrebody and 1.5m long engine.

The centre of gravity is determined using CREO, and is at XXm from the nose. It is assumed that the mass of the structure of the rocket (excluding fuel tanks, heat shielding, engine and payload) is distributed homogeneously, for simplicity.

3.3.1 Heat Shield Sizing

present more of the logic which led to this heat shield design

The third stage rocket is separated from the SPARTAN at a high dynamic pressure, after which

it spends a considerable amount of time accelerating in-atmosphere before reaching exoatmospheric conditions. This release into a high dynamic pressure environment creates a large amount of heating, which must be mitigated by heat shielding. The third stage is protected while in-atmosphere by a heat shield, weighing 130.9kg. This heat shield is constructed from a phenolic cork cylinder, a reinforced carbon-carbon nose cone, and a tungsten nose tip. The tungsten nose is 50mm diameter, at the end of a 50mm cylinder. The density of tungsten is $\rho_{Tungsten} = 19.25 \text{ g/cm}^3$, giving a total mass for the nose of $m = 12.6\text{kg}$. The carbon-carbon shell has a density of $\rho_{CC} = 1800 \text{ kg/m}^3$. The carbon-carbon shell has a thickness of 10mm. The mass of the carbon-carbon is 89.3kg. The cork cylinder has a density of $\rho_{Cork} = 320 \text{ kg/m}^3$, a thickness of 5mm and a total mass of 23.7kg. The total mass of the heat shield is 125.6kg.

put info in table

3.3.2 Fuel Tank Sizing

The fuel tanks have been sized assuming 100kg of payload-to-orbit. Note that the method of calculating final payload-to-orbit relies on using left over 'fuel mass' as effective payload mass. Realistically this would cause the fuel tanks to be resized slightly. For the purposes of this study the fuel tanks have been assumed to be of constant size for simplicity. Currently this is a reasonable assumption as the internals of the rocket are very simplified. The structural mass is held constant at 9%. The third stage carries a total propellant mass of 2736.7kg. Table XX breaks shows the component break-down of the LOX oxidiser and RP1 fuel.

	LOX	RP1
Ratio	2.56	1
Density	1141kg/m ³	813kg/m ³ [35]
Volume	1.7248m ³	0.9455m ³
Mass	1968.0 kg	768.7 kg

it needs to be clearer here that the fuel tanks have been approximately sized, and that some fuel / payload is interchangeable

3.3.3 Aerodynamics

The third stage aerodynamics have been calculated using Missile DATCOM [REFXX], a preliminary design tool for estimating the aerodynamic characteristics of missile and rocket vehicles. Missile DATCOM utilises empirical methods, along with various estimation techniques, to compute the aerodynamics of missile-like vehicles across the subsonic, supersonic and hypersonic regimes. The code used to compute the aerodynamics of the third stage rocket is detailed in Appendix XX.

Put plots of aerodynamics here

3.3.4 Thrust Vectoring

The third stage rocket is trimmed during the in-atmosphere portion of its ascent trajectory via thrust vectoring. The centre of pressure is calculated using missile DATCOM. The thrust vector is set so that the moment generated by the engine matches the lift force acting at the centre of pressure. The maximum thrust vector limit has been set to 8° . As no data on the maximum thrust vectoring capabilities of the kestrel engine was able to be found, this was set to the maximum gimbal range of the Aestus pressure-fed engine and OMS, similar pressure fed engines.

[add a diagram](#)

3.4 Third Stage Sizing

-diameter and length of centrebody changed independently

-length varied

-changed mass of cork cylinder -changed aerodynamics -changed total mass

-diameter varied -changed mass of cork cylinder and C-C cone -changed aerodynamics -changed total mass -changed engine efficiency from nozzle size

CHAPTER 4

LODESTAR

This chapter covers the optimal control program LODESTAR, which has been used to simulate the optimal trajectories of the rocket-scramjet-rocket system. The basics of optimal control and the optimal control methods used are presented, as well as the specific problem formulation used by LODESTAR.

The program LODESTAR (Launch Optimisation and Data Evaluation for Scramjet Trajectory Analysis Research) has been developed to aid with the simulation and trajectory optimisation of space launch systems. LODESTAR is a MATLAB based trajectory optimiser which utilises GPOPS-2, a proprietary pseudospectral method optimisation package as well as MATLAB's inbuilt SQP solver. LODESTAR optimises a trajectory towards a user-defined objective function, such as constant dynamic pressure or maximum payload-to-orbit. LODESTAR accurately models both rocket-powered and scramjet-powered vehicles in 5 degrees of freedom. LODESTAR contains multiple modules configured for the SPARTAN launch system, which are able to optimise trajectories for;

1. The ascent of the first stage rocket.
2. The ascent of the second stage scramjet-powered accelerator.
3. The flyback of the second stage scramjet-powered accelerator.
4. The ascent of the third stage rocket.

LODESTAR performs optimisation using four classes of variables, primals, controls, constraints and costs, which define the optimisation problem being solved. Primals are the variables which define the physical trajectory and are dependent on other primals and the control variable. The control variables are independent in the solution space and can be modified freely by the solver algorithm within the prescribed limits. Constraints bound the primal and control variables to a well defined solution space and events confine the primal and control values to a specified value at a specific point

in time. The cost variables define the target of the optimisation problem. A solution is found when the cost is minimised to a suitable accuracy.

GPOPS-II and DIDO require these variables to be precisely described. LODESTAR contains multiple modules that calculate the vehicle aerodynamic properties and run external simulations which then feed data back to the pseudospectral solver for evaluation. Modules calculate the vehicle aerodynamic and engine output at each point along the trajectory. The vehicle model module takes this data and calculates the mass of the vehicle as well as the 6 degree of freedom motion derivatives of the vehicle along the trajectory. As this is an iterative process, the calculated motion derivatives do not match the motion approximation exactly until convergence is achieved (This is a fundamental property of the pseudospectral method, see Section 2.8.3).

-XX define the iterative process here, include evaluation of the optimisation variables, and sequential quadratic programming

The cost is calculated using the primal variables. The trajectory is evaluated to produce the final cost of the trajectory. The final step of each iteration is updating the primal derivatives, which is performed by the dynamics file. The primal derivatives and cost are evaluated by DIDO to determine if the trajectory has produced a suitable result.

4.1 Optimal Control

The pseudospectral method and direct single shooting techniques used by LODESTAR are described in detail in sections XX. Practically, the implementation of these techniques involves the specification of the set of constraints and objectives which govern the optimisation problem. These constraints inform the optimiser of the bounds of the optimisation, and perform the functions of limiting the search space to the physically possible (eg. constraining altitude to be greater than ground level) as well as constraining the vehicle within its performance limits (eg. limiting the angle of attack). These constraints also come in the form of initial or terminal constraints, which define the initial conditions of the trajectory as well as any conditions which the trajectory must meet at termination.

The pseudospectral method requires the specification of a set of 'primal variables'. These primal variables describe the physical dynamics of the system. In the pseudospectral method, the dynamics of the system are used as constraints on the optimal control problem;

$$\dot{\mathbf{x}}(\tau) = f[\mathbf{x}(\tau), \mathbf{u}(\tau)]. \quad (4.1)$$

Implementing the dynamics as constraints allows the optimiser to explore each primal variable independently, greatly increasing the robustness of the optimal control problem. However, the constraints may be violated by the optimiser in the process of searching for an optimal solution. A violation of the physical dynamics constraints means that the dynamics of the system may not hold throughout the

Primal Variables	x Position y Position Velocity
Control Variables	Angle of Descent
Initial Constraints	Velocity x Position y Position
Terminal Constraints	x Position y Position
Path Constraints	None
Target Cost	Minimum Time

solution process, causing potential complications for the computational model of the vehicle. Much of the design of the vehicle simulation in this study is driven by the need for smooth, continuous interpolation schemes, and viable extrapolation regions ie. even if the solution is well within the range of all input data sets, the solver must be able to explore all regions within the set bounds.

4.1.1 GPOPS-2 Example - Brachistochrone Problem

This section describes a short example optimal control problem solved in GPOPS-II. The purpose of this example is to demonstrate the effectiveness of the pseudospectral method and GPOPS-II, and to provide a simple example case to establish the terminology of an optimal control problem.

The brachistochrone (from the Greek for 'shortest time') problem is a simple optimal control problem, which describes a ball rolling in two dimensions under gravity. The objective is to find the curve of descent which will minimise the time from point *a*, where the ball is at rest, to point *b*. It is assumed that gravity is constant and that there is no forces other than gravity acting on the ball.

The analytical solution of this problem can be computed using the Euler-Lagrange equation as the equations describing a cycloid:

$$x = A(\theta - \sin \theta),$$

$$y = A(1 - \cos \theta)$$

This problem has been solved using GPOPS-2. Table XX describes the set-up of the optimal control problem in GPOPS-2. The dynamic equations for the Brachistochrone problem are:

$$\dot{x} = v * \cos(u),$$

$$\dot{y} = v * \sin(u),$$

$$\dot{v} = -g * \sin(u).$$

The GPOPS-2 solution to the Brachistochrone problem is shown in Figure XX. This is overlaid with a plot of the optimal solution.

- xx put optimal solution in here **compare to analytical solution (dont include, just have plot)**

The dynamics of the basic Brachistochrone problem are very simple. As the dynamic become

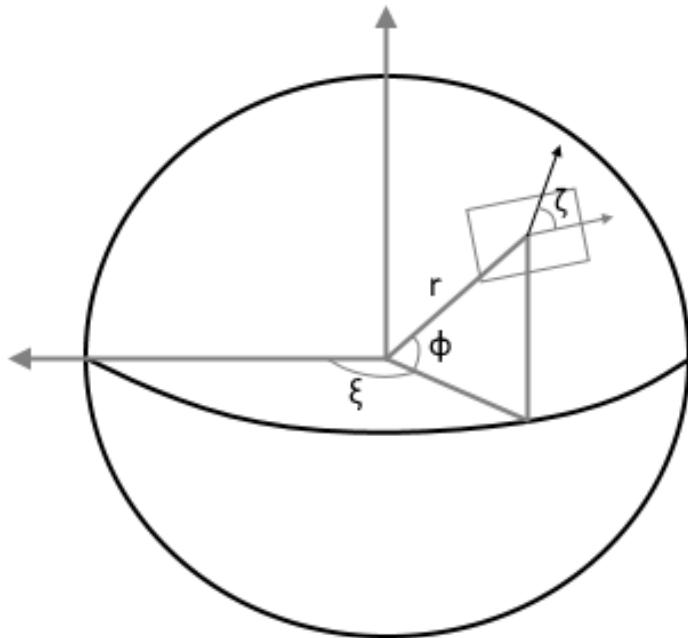


Figure 4.1

more complex, it is no longer possible to obtain an analytical solution.

4.2 The Trajectory Optimal Control Problems

4.2.1 Dynamic Model

The drag and lift produced by each stage of the vehicle are calculated using the standard definition of the aerodynamic coefficients:

$$F_d = \frac{1}{2} \rho c_d v^2 A, \quad (4.2)$$

$$F_L = \frac{1}{2} \rho c_L v^2 A. \quad (4.3)$$

The dynamics of the vehicle are calculated in six degrees of freedom, with pitch constrained to zero. The dynamics of all stages are calculated using an geodetic rotational reference frame, written in terms of the angle of attack α , bank angle η , radius from centre of Earth r , longitude ξ , latitude ϕ , flight path angle γ , velocity v and heading angle ζ . The equations of motion are [28]:

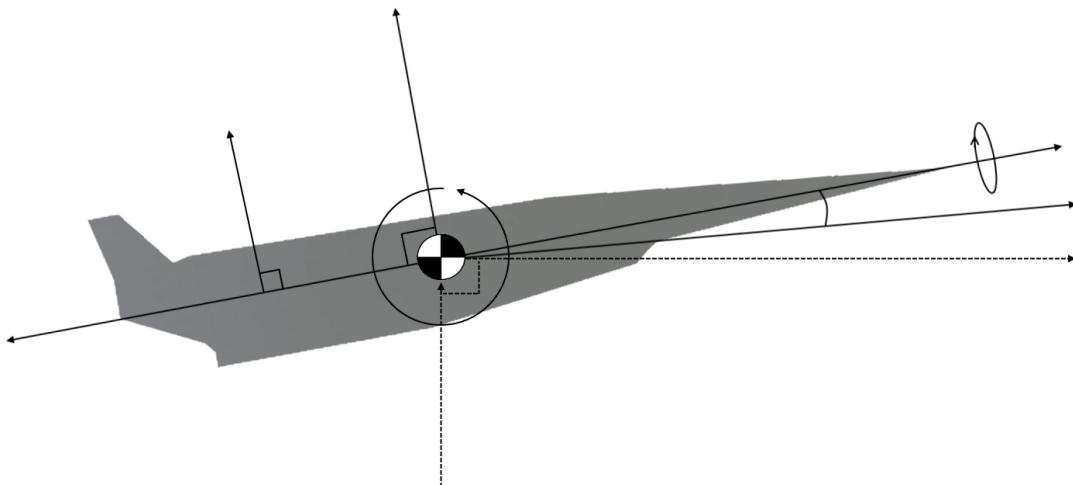


Figure 4.2

$$\dot{r} = v \sin \gamma \quad (4.4)$$

$$\dot{\xi} = \frac{v \cos \gamma \cos \zeta}{r \cos \phi} \quad (4.5)$$

$$\dot{\phi} = \frac{v \cos \gamma \sin \zeta}{r} \quad (4.6)$$

$$\dot{\gamma} = \frac{T \sin \alpha \cos \eta}{mv} + \left(\frac{v}{r} - \frac{\mu_E}{r^2 v} \right) \cos \gamma + \frac{L}{mv} + \cos \phi [2\omega_E \cos \zeta + \frac{\omega_E^2 r}{v} (\cos \phi \cos \gamma + \sin \phi \sin \gamma \sin \zeta)] \quad (4.7)$$

$$\dot{v} = \frac{T \cos \alpha}{m} - \frac{\mu_E}{r^2} \sin \gamma - \frac{D}{m} + \omega_E^2 r \cos \phi (\cos \phi \sin \gamma - \sin \phi \cos \gamma \sin \zeta) \quad (4.8)$$

$$\dot{\zeta} = \frac{T \sin \alpha \sin \eta}{mv} - \frac{v}{r} \tan \phi \cos \gamma \cos \zeta + 2\omega_E \cos \phi \tan \gamma \sin \zeta - \frac{\omega_E^2 r}{v \cos \gamma} \sin \phi \cos \phi \cos \zeta - 2\omega_E \sin \phi \quad (4.9)$$

Flow chart of modules details of simulation (5DOF geodetic rotational) details of limits

verification methods -hamiltonian/costates -complementary conditions -forward sim (for sanity checking, will need to detail deficiencies in this) -forward integration -logic check (ie solver is still a heuristic process, run multiple times with varying guess. Is solution logical?)

- outline the pseudospectral method, as implemented in GPOPS? ie with stage definitions

4.2.2 Trajectory Connection Points

The optimisation of a large, nonlinear system is a complex and time-consuming task. The optimisation of the trajectory of the rocket-scramjet-rocket launch system considered in this study is an extremely

Section	Initial Constraint	End Constraint
1 st Stage Vertical Ascent (I-II)	Constrained to start at a velocity of 0m/s.	Constrained to fly to 90m altitude and 30m/s velocity.
1 st Stage Pitching Ascent (II-III)	Constrained to start at 90m altitude and 30m/s velocity	Constrained entirely to 2 nd stage initial conditions
2 nd Stage Ascent (III-IV)	Constrained to 1520m/s velocity.	Constrained to 102.0° heading angle, the approximate angle which allows the third stage to reach sun synchronous orbit.
2 nd Stage Return (IV-VI)	Constrained entirely to 2 nd stage ascent end conditions.	Constrained to conditions approaching landing at the initial launch site.
3 rd Stage Ascent (IV-V)	Constrained entirely to 2 nd stage ascent end conditions.	Flight parallel with Earth's surface.
3 rd Stage Hohmann Transfer (V)	Constrained entirely to 3 rd stage ascent end conditions.	Constrained to sun synchronoud orbit.

Table 4.1

complex problem, if simulated in its entirety. It is a potentially unmanageable task to produce an optimised trajectory by considering the entire launch trajectory as a single nonlinear programming problem. To mitigate the complexity of the simulation, the trajectory of the rocket-scramjet-rocket launch system has been broken down into subsections, shown in Figure 4.3. These contiguous subsections are then able to be analysed and optimised independently. The subsections of the trajectory are connected through the use of initial and end constraints on the independent optimisation problems. These constraints are described in Table ??.

4.2.3 First Stage Trajectory

LODESTAR is able to optimise the first stage of a launch vehicle, for an angle of attack controlled trajectory, from launch to a pre-defined end point. LODESTAR is able to optimise for either a maximum velocity or minimum mass optimisation objective.

A maximum velocity case is desired when a specific first stage vehicle design is being investigated.

A minimum mass objective is applicable when the first stage trajectory has a pre-defined end goal. This is the case with the SPARTAN vehicle where the SPARTAN scramjet accelerator is to be released at its minimum operating conditions at close to horizontal flight. A variable mass for the first stage launch vehicle is desired as the mass has large effects on the dynamics of th vehicle, effecting the trajectory angle change rate, as well as the acceleration and time of flight of the vehicle. It is useful in the preliminary design stages to be able to optimise the mass of the first stage vehicle, allowing a less trial-and-error approach. In the minimum mass case, the launch altitude is slightly variable, as LODESTAR starts optimisation from a set pitchover altitude and velocity, and the pre-pitchover

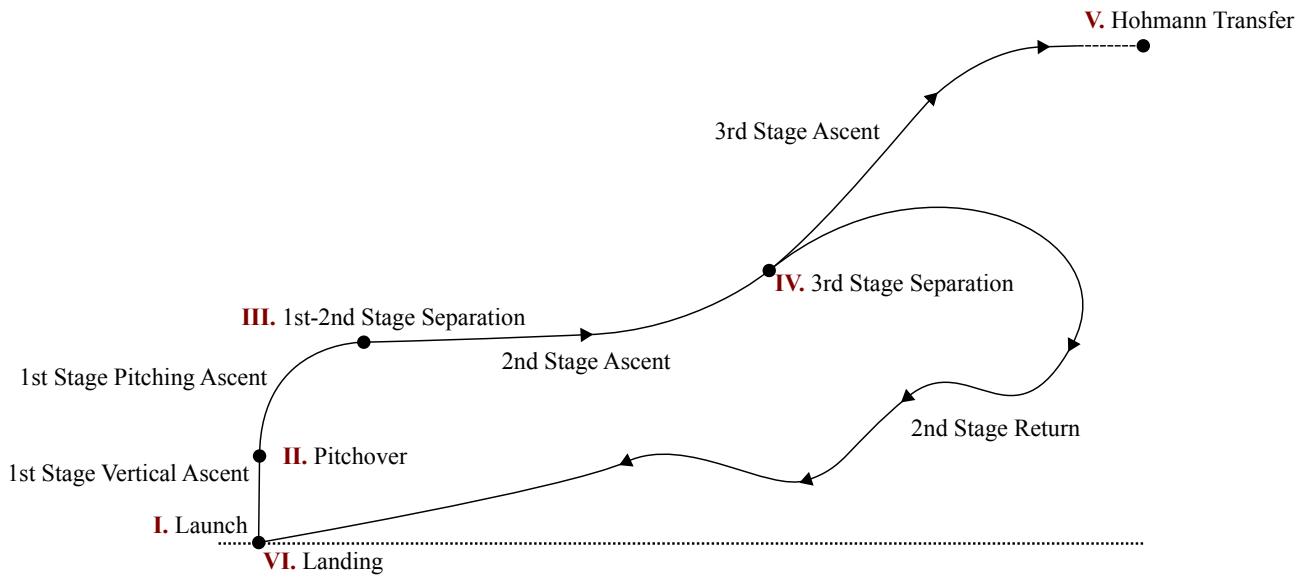


Figure 4.3

trajectory is calculated to match the pitchover mass.

Input	Contains
Aerodynamic Database	

The first stage is launched from an area in northern Queensland.

Initial Constraints	
Terminal Constraints	
Path Constraints	
Target Cost	

Control Variables

Primal Variables

4.2.4 Second Stage Trajectory

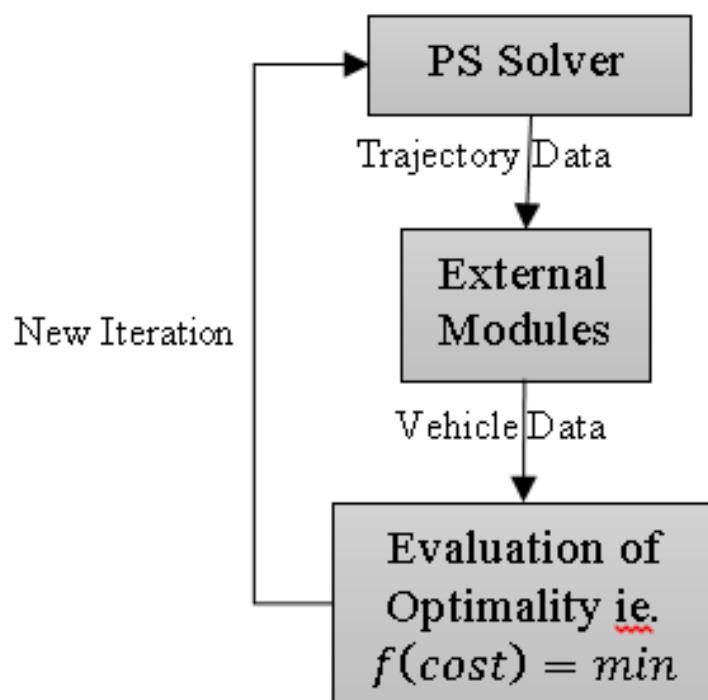


Figure 4.4

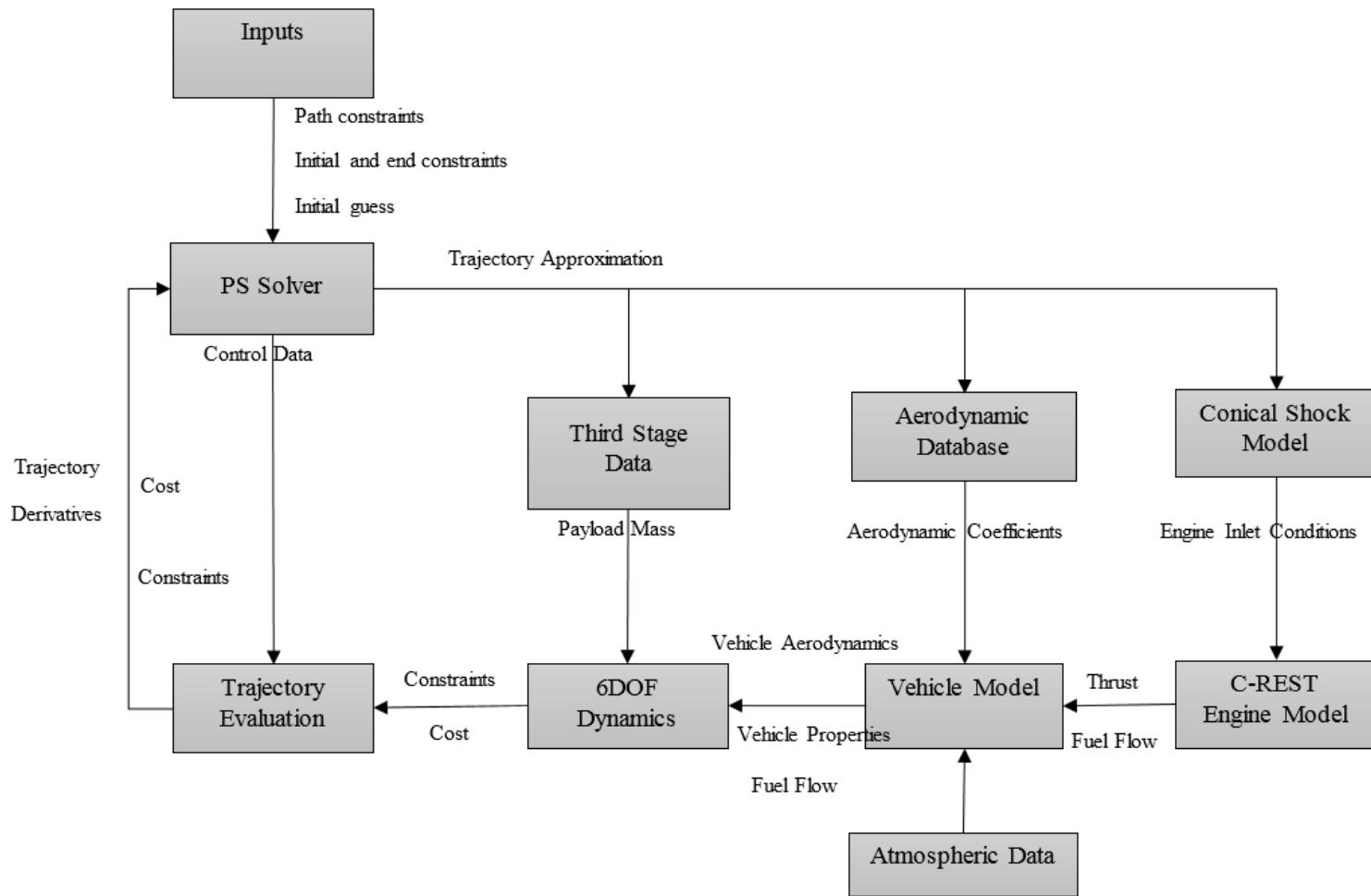


Figure 4.5

LODESTAR is able to optimise the trajectory of airbreathing accelerators in the supersonic or hypersonic regime. LODESTAR is able to optimise for a range of optimisation metrics, including a maximum payload-to-orbit and constant dynamic pressure. The trajectory of the SPARTAN scramjet-powered accelerator has been simulated in LODESTAR.

Initial Constraints	Velocity Fuel Mass Latitude Longitude
Terminal Constraints	Fuel mass Heading Angle
Path Constraints	Dynamic Pressure
Target Cost	Maximum Payload-to-Orbit

Control Variables

Primal Variables

primal variable limits:

4.2.5 Second Stage Return Trajectory

After releasing the third stage rocket, the scramjet-powered second stage must return back to an area close to the initial launch site. During the flyback, the SPARTAN cannot exceed its dynamic pressure limit of 50kPa. The SPARTAN must land on the ground with minimum velocity, within close proximity to the launch area. It is assumed that a landing strip is available at the spot where the SPARTAN lands. The SPARTAN is required to land within a set radius of the launch site. This constraint is;

$$(\phi_{end} - \phi_{launch})^2 + (\xi_{end} - \xi_{launch})^2 - r^2 \leq 0 \quad (4.10)$$

Summary Table

Initial Constraints	Altitude Velocity Flight Path Angle Heading Angle Latitude Longitude
Terminal Constraints	Distance From Launch Site
Path Constraints	Dynamic Pressure
Target Cost	Minimum End Velocity

Control Variables

4.2.6 Third Stage Trajectory

-detail the vehicle dynamics with a free body diagram

Input	Contains
Aerodynamic Database	Mach Number, AoA, CA, CN, CD, CL, cP
Output	Contains

The third stage is required to deliver the payload into heliosynchronous orbit. The heliosynchronous orbit chosen is 566.89km.

Initial Constraints	
Terminal Constraints	
Path Constraints	Angle of Attack
Target Cost	

The third stage trajectory angle is initiated at a heading angle of 102.0° . This is chosen as the heading angle which allows the third stage to closely match the required orbital inclination angle for sun synchronous orbit at 567km, of 97.63° .

-Detail the hohmann transfer

Control Variables

4.2.7 Combined Second Stage Ascent & Return Trajectory

4.2.8 Validation

Optimality Conditions

LODESTAR provides the capacity to partially validate the optimal solution provided by the pseudospectral method solver. This partial validation is used to determine whether the pseudospectral method solver has converged close to an optimal solution of the nonlinear programming problem. It is particularly useful to validate that the optimality and constraint tolerances which have been chosen are sufficiently small, or to check whether the pseudospectral method solver has approached an optimal solution in the case that the defined tolerances are not able to be reached. This partial validation is achieved through the examination of some of the necessary conditions for optimality. The necessary conditions

The control Hamiltonian is investigated to verify that the first order necessary conditions hold. Due to the unconstrained end time of the trajectory problems, $H \equiv 0$ [Pucci2007]. The Hamiltonian is defined as: $H(x(t), u(t), \lambda(t), t) = \lambda^T(t)f(x(t), u(t)) + L(x(t), u(t))$. Make sure this is consistent with format in lit review, link This is calculated using LODESTAR and the Hamiltonian condition

is able to be verified. The Hamiltonian will likely not be exactly equal to zero along the trajectory. This is due to the heuristic nature of the solver, which will approach close to an optimal solution, but never reach it exactly. A sufficiently small Hamiltonian indicates that the end solution approaches an optimal solution, and may be a candidate as an optimised trajectory case.

Costates Complementary conditions

To assess the quality of the optimisation problem solved using the pseudospectral method,

The primal feasibility of the solution is checked through a comparison of the state derivatives, $\dot{x} = f(x, u)$. \dot{x} is first determined through numerical differentiation of the primal variables over the solution time. Then $f(x, u)$ is determined using the dynamics of the system and vehicle model, in the same way that $f(x, u)$ is input to the pseudospectral solver. Examination of the error between the 'expected' state derivatives, and the numerical approximation of the derivatives, $\dot{x} - f(x, u)$, allows the accuracy of the system dynamics to be verified.

GPOPS - output.result.solution.phase.pathmultipliers I believe these should be 0, as they are the mu associated with path constraints (and the function C;0, meaning mu=0)

output.result.solution.phase.costate gives costates, not sure how to use these

include IPOPT infeasibilities infpr and infdu (need to mention GPOPS-2 is based on IPOPT)

Forward Simulations

-both the control check and derivative check

The pseudospectral method considers the dynamics of the system as constraints on the optimal control problem, and solves across the entire trajectory simultaneously. This causes the physical system dynamics to have an associated margin of error, ie. $\dot{x} = f(x)$ will only hold to a certain degree of accuracy. For a well converged solution, this margin of error will be negligibly small, and the dynamics of the system will be consistent with realistic Newtonian dynamics. However, when the problem is not well converged, the dynamics of the system may have a large error. It is possible to make a preliminary check of the system dynamics using the XX (complementary conditions?). However, to be certain that the system is behaving as it should be, a full forward simulation is necessary. This forward simulation starts at the initial conditions prescribed by the pseudospectral method solver, and propagates the dynamics of the system forward in time using numerical approximation. The forward simulation uses only inputs of the control sequence, as solved for by the pseudospectral method.

CHAPTER 5

TRAJECTORIES INVOLVING FLYBACK

5.1 Flyback trajectories

- Details of bounds & guesses, with reasoning.
- Fuel optimised fly-back of the SPARTAN.

5.2 Sensitivity Analysis

- Details of bounds & guesses, with reasoning.
- Variation of L/D & ISP.
- Heat flux analysis

CHAPTER 6

FULL TRAJECTORY OPTIMISATION

add third stage, matrix and trajectory

- Details of bounds & guesses, with reasoning.
- Optimisation of the combined ascent and fly-back trajectories of the SPARTAN.
- Potential Abort Analysis
 - sonic boom estimation?
 - alternate trajectory recommendation?

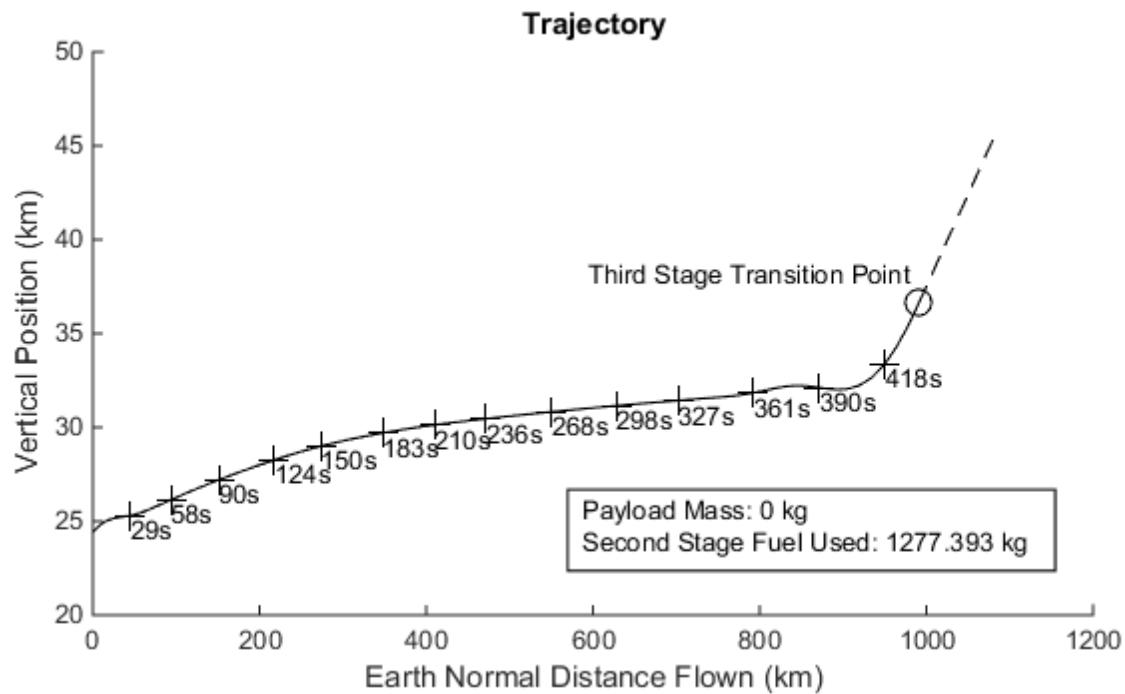


Figure 6.1

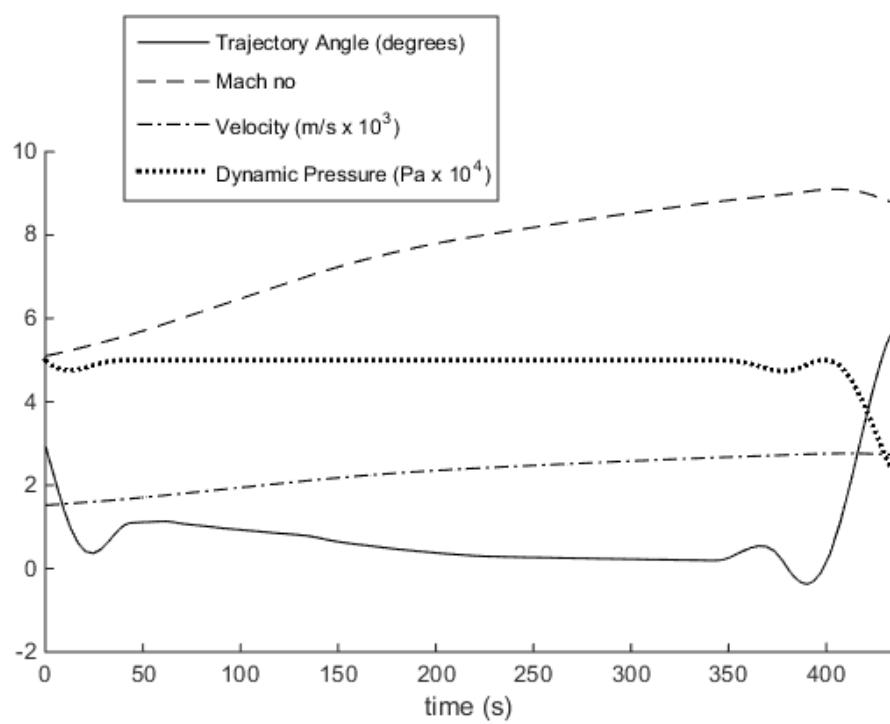


Figure 6.2

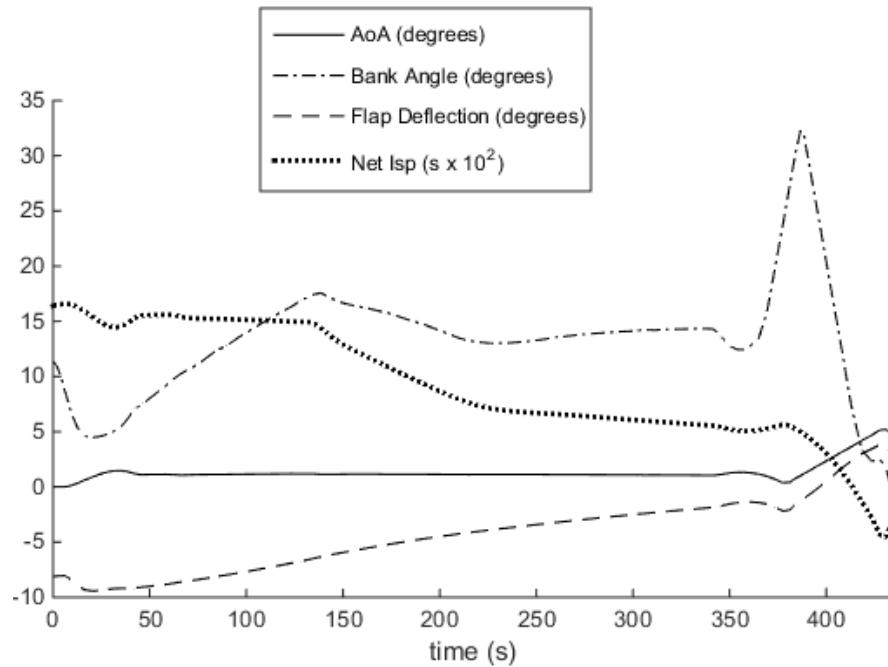


Figure 6.3

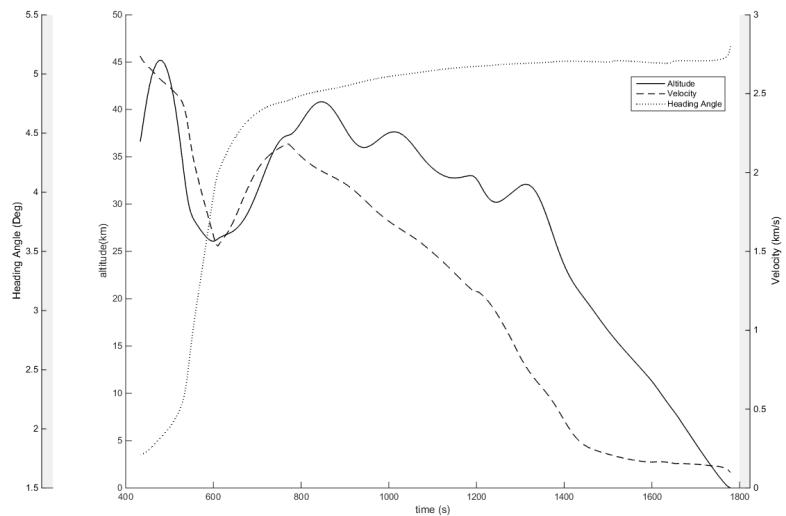


Figure 6.4

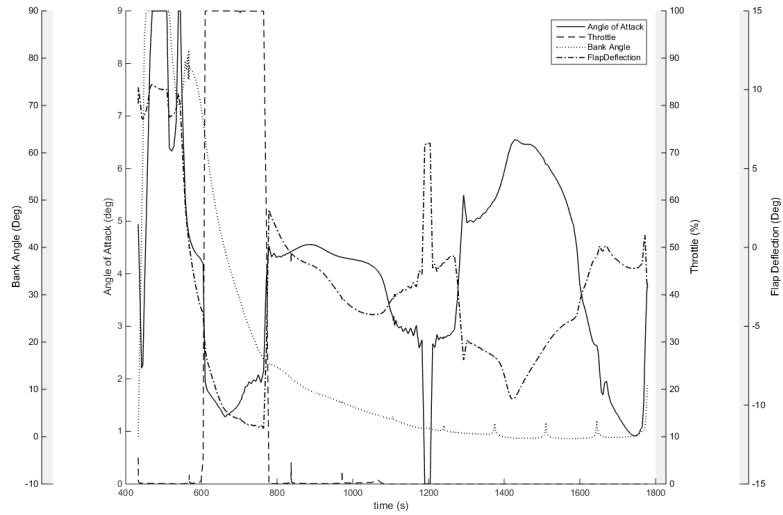


Figure 6.5

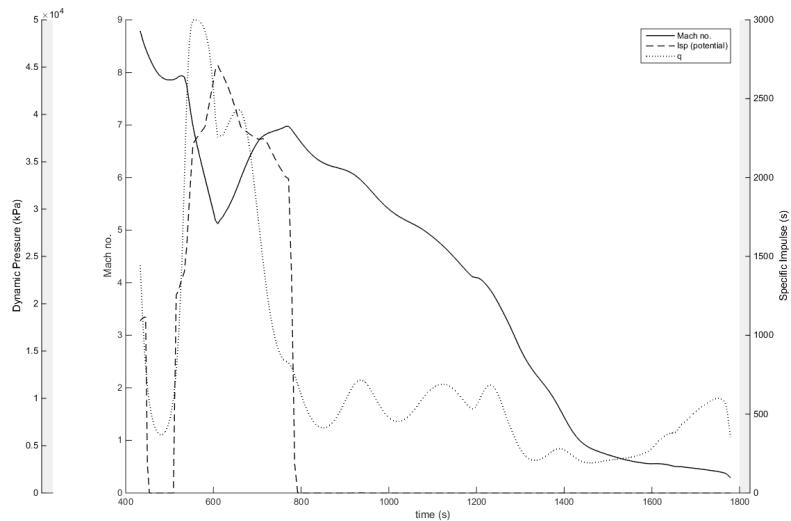


Figure 6.6

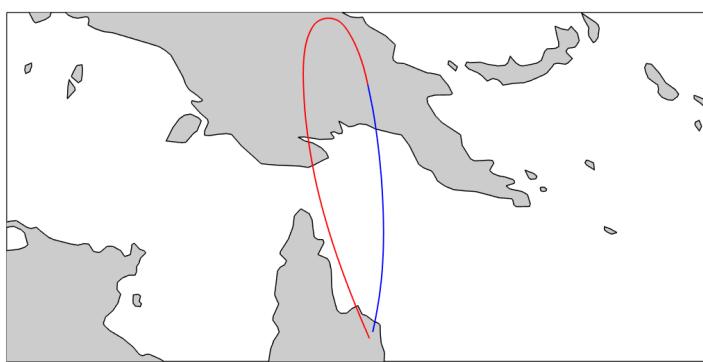


Figure 6.7

CHAPTER 7

THIRD STAGE SIZING STUDY

- I need to frame this well, so that it does not seem to invalidate my previous work
 - Variation of third stage design, changing width, length, thrust, CL and CD of the third stage, and recalculating optimised trajectories.
 - Determine the optimal sizing of the third stage
 - Analyse the variation in the optimal trajectory with variation in third stage design
 - could do a small cost analysis of the third stage using transcost, compare to payload-to-orbit

CHAPTER 8

CONCLUSIONS

- The optimal ascent trajectory for the rocket-scramjet-rocket three stage launch system involves a pull-up before third stage release.
- The optimal fly-back of the SPARTAN involves several distinct stages, including a boost phase, skip phase and glide phase.
- Description of conclusions from the combined ascent/fly-back trajectory. The combined ascent and fly-back trajectory involves the SPARTAN banking during ascent, to mitigate the distance flown during the ascent trajectory.
- Summary of conclusions from third stage sizing analysis.

8.1 Recommendations for future work

- Design optimisation incorporating trajectory optimisation.
- Sizing of entire system for optimal cost efficiency.

LIBRARY

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APPENDIX A

CART3D RESULTS

A.1 Engine-On Plume Check

-simulate engine-on conditions to check that the plumes do not adversely affect the tail of the vehicle
(justify that I can just remove engines/boattail)

-Mach 5,7,9 at 50kPa

A.2 CART3D Results

-include mesh here

APPENDIX A. CART3D RESULTS

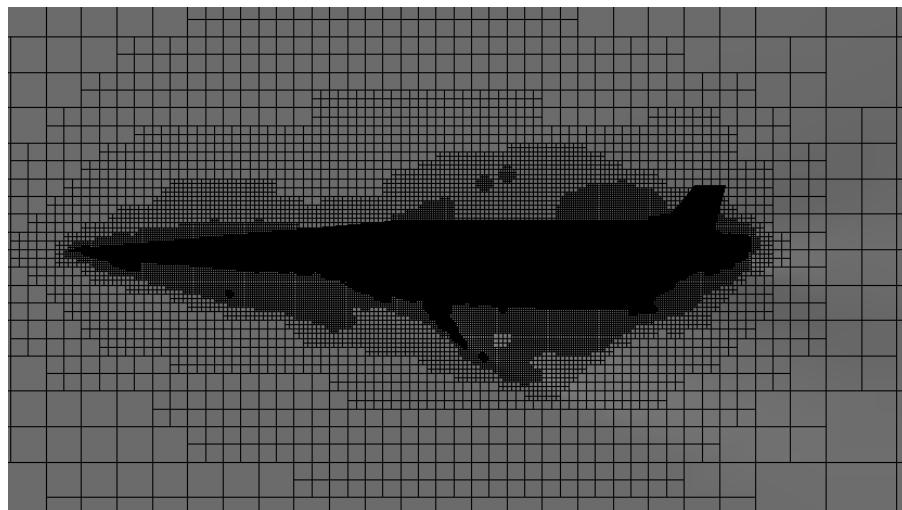


Figure A.1

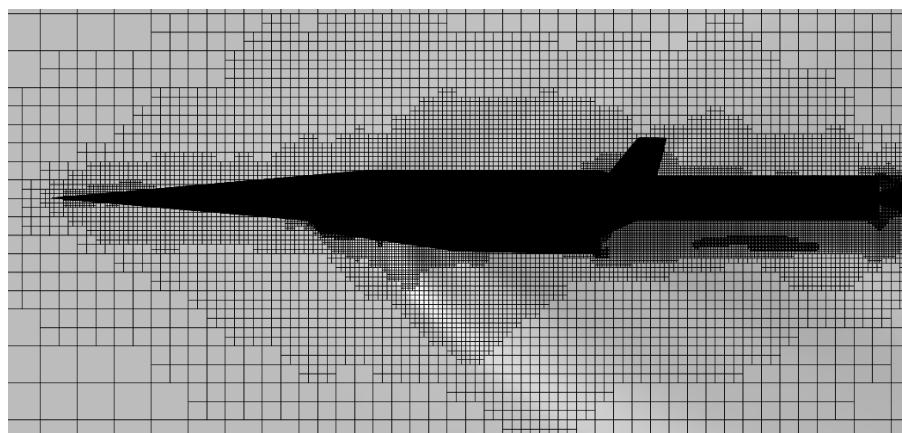


Figure A.2: Mesh generated by CART3D around the SPARTAN and first stage vehicles.